Lagrangian Approach to Modeling the Biodynamics of the Upper Extremity: Applications to Collegiate Baseball Pitching

Matthew J. Solomito
University of Connecticut - Storrs, msolomito@ccmckids.org

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Biomechanical modeling can provide information regarding movement patterns and allow for an improved understanding of injury mechanisms. Biomechanical models typically rely on Newtonian mechanics leading to computationally intensive models; however, an often overlooked means of solving these same modeling needs is to apply the Lagrangian approach. This approach allows for a more direct means of solving the equations of motion; leading to more accurate models of human motion. The purpose of this project was to create a Lagrangian based model of the upper extremity to describe joint kinematics and kinetics as well as muscle kinetics during complex motion tasks.

The model developed in this work was a four segment, 18 degree of freedom model capable of providing joint moments as well as muscle force estimates. The model was then applied to data collected from 33 collegiate level baseball pitchers. The results of this project indicated that the model was capable of accurately predicting the motion profiles of the upper extremity joints. The results also indicated that the model was able to accurately estimate muscle forces leading to a greater understanding of pitching related injuries. The adaptability of the model offers the ability to create patient specific treatment plans or modify a pitcher’s mechanics to decreased joint moments. Finally, the results of this work illustrate the utility of using Lagrangian based biomechanics models to create accurate and adaptable models to describe complex human movement.
Lagrangian Approach to Modeling the Biodynamics of the Upper Extremity: Application to Collegiate Baseball Pitching

Matthew John Solomito

B.S.B.E, Western New England College, 2007

M.S., University of Connecticut, 2011

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

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2015
Doctor of Philosophy Dissertation

Lagrangian Approach to Modeling the Biodynamics of the Upper Extremity: Application to Collegiate Baseball Pitching

Presented by
Matthew John Solomito, B.S.B.E, M.S.

Major Advisor ________________________________________________________________
Donald R. Peterson

Associate Advisor _____________________________________________________________
Carl W. Nissen

Associate Advisor _____________________________________________________________
Kevin S. Brown

Associate Advisor _____________________________________________________________
David M. Pierce

Associate Advisor _____________________________________________________________
Pouran D. Faghri

University of Connecticut
2015
Dedication

To my parents for everything you have done. I love you.

and

To Tina, my wife, love you.
Acknowledgments

Throughout the process of completing this dissertation I have been lucky enough to have a considerable amount of help, guidance, and encouragement. While there have been many and I thank you all I want to take some time to appropriately thank those of you who have done so much.

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Executive Summary

Biomechanical modeling can provide an enormous amount of information regarding the movement patterns of a person and allow for the understanding of injury mechanisms. It can also be a means of improving the efficiency of tasks. Currently, biomechanical modeling for the human body relies almost entirely on Newtonian mechanics resulting in extremely complex models that require a large amount of computational time to solve. This has led to simplified models that either ignore or make many assumptions regarding physical properties in an attempt to reduce the models complexity. An often overlooked means of solving these same modeling needs is to use the Lagrangian approach in which motion is modeled based on the transfer of kinetic and potential energy. Lagrangian based models allow for a more direct solution to the modeling of complex motion as the mathematics is limited to scalar rather than vector quantities, and constraint forces do not need to be solved for, as generalized coordinates allow for these constraints to fall out of the equations. Once the Lagrangian has been developed it is a relatively straightforward process to solve for the equations of motion, and solving these equations requires far less computational power and computing time. Furthermore, this more direct approach allows for the inclusion of additional model elements to better reflect the movement of the human body; creating more accurate and informative models compared to those developed using the Newtonian approach.

The purpose of this work was to create a Lagrangian based model of the upper extremity that could describe upper extremity kinematics and kinetics during complex motion tasks. The model developed was also designed to provide information about the underlying soft tissue structures that are typically excluded from motion based models. This model was also designed to be used with previously collected motion capture data so that researchers could use it without the need to collect additional motion capture data. The model was then applied in a sports biomechanics situation to provide information on describing the kinematics and kinetics of motions associated with baseball pitching.

The model developed in this work included a total of four segments (i.e. the thorax, upper arm, lower arm, and hand) and a total of 13 degrees of freedom (i.e. three degrees of freedom at the wrist and
forearm, one degree of freedom for the elbow joint, three degrees of freedom of the glenohumeral joint, and six degrees of freedom for the thorax, three rotational and three linear). Also included in this model were five muscle elements modeled using a three element muscle model to represent the Biceps, Triceps, Deltoid, Pectorialis Major, and Infraspinatus. Each of these muscle elements added one degree of freedom to the model. The model was designed to provide joint moment estimations for the glenohumeral joint and the elbow joint as well as to estimate the contribution of the muscle elements to these joint moments. A Lagrangian approach was used to create the equations of motion for the model described in this work and the muscle elements were added using a generalized force approach.

The model was validated, using a motion capture system along with surface EMG data, to illustrate that it produced results that were both accurate and physiologically consistent. Motion capture data was then used as the input to the model to analyze the pitching mechanics of 33 collegiate aged pitchers and to understand how changes in pitching mechanics affected the joint kinetics of these pitchers. Motion capture data was collected using a Vicon Motion capture system using a previously validated model and methodology. The equations of motion were solved used using custom Matlab programs.

The results of this work indicated that the model was capable of accurately predicting the motion profiles of simple motions such as elbow flexion and extension with a 0.97 correlation to actual motion data collection. The model was also capable of predicting the motion of glenohumeral vertical abduction and glenohumeral adduction. The results of the validation testing also showed that model was able to accurately estimate the muscle forces required to perform various tasks both in simplistic motions such as elbow flexion and extension, as well as more complex upper extremity motions that were associated with baseball pitching. Finally, the validation results indicated that the model was capable of accurately assessing joint kinetics at the elbow and glenohumeral joints as compared to currently used and validated Newtonian based models.

The results of analyses on the 33 collegiate baseball pitchers indicated that the model showed similar results to those previously determined using the available Newtonian based models. The model was also able to provide additional information that is not currently available using Newtonian based
models most notably how the inclusion of soft tissue elements can provide a deeper understanding of the potential causes of pitching injury. The results of this model also provided a greater understanding of the effects of pitching mechanics on the soft tissue structures included in this model and how these soft tissue structures can be affected by changes in pitching mechanics. Finally this paper provides examples as to how this model could be extended to produce patient specific modeling to determine how modifications to a pitcher’s mechanics, injury, or surgical intervention could affect joint and muscle moments.

This work provides the frame work for future modeling, to create more accurate detailed models that can be used to better describe human movement. The outcomes of this project indicate that a Lagrangian based approach is capable of producing a valid model of the upper extremity that can be used both in a research setting as well as a clinical setting. More importantly the results of this work also illustrate the utility of applying a Lagrangian based approach to create accurate and adaptable models describing human complex human movement that are currently overlooked in favor of the more computationally intensive Newtonian approach.
1. Introduction

The study of human biodynamics, how the body moves to accomplish any number of tasks, has been a major focus of researchers for decades. The understanding of how a person moves in various tasks can help to define common patterns of motion, which can then be studied to determine which motions lead to increased risk of injury. Essentially, biodynamic studies provide quantifiable data concerning joint ranges of motion and joint velocities, as well as providing data about muscles and soft tissue forces that help to drive these motions. This quantifiable data can then be used primarily for two distinct purposes, medical science and sports performance. In medical science, it can be used to describe how pathology effects human movement as well as helping to make recommendations for surgical intervention. In sports biomechanics, the data can be used to increase performance, understand injury mechanisms, and reduce the risk of injury.

The study of human movement in not new and has been studied for over a century; there has always been a directed focus to understand human gait [1]. Over the past three decades, improvements in computer software and computational algorithms have led to the advent of three-dimensional computerized motion analysis [2]. These improvements have greatly advanced the utility and availability of human gait analysis. The advances in technology and understanding of human movement has also led to a growing use of motion analysis techniques in other areas of human movement, specifically, upper extremity motion and sport’s mechanics analysis.

The investigation of upper extremity motion especially in overhead sports such as baseball pitching, has become a major focus in research partly due to the fact that 5.7 million children and adolescents, high school aged and younger are involved in baseball, where a large majority of these children are baseball pitchers [3]. Furthermore the NCAA estimates that 3.3% of these 5.7 million children will be involved in collegiate level baseball and a larger percentage will join intramural or under 30 recreational leagues. Studies have shown that upwards of 50% of this population involved in pitching will complain of shoulder or elbow pain at least once during the season [4]. What is of greater concern is that 5 to 15% of those that are involved in regular pitching will sustain serious injury of their dominate
arm, which often times will require surgical intervention, the cessation of pitching for a season or in some cases a lifetime and, in some cases, reduced upper extremity mobility in daily life [5]. These numbers have continued to increase and, at Connecticut Children’s Medical Center in Hartford, CT, between 1994 and 2012, there was a tenfold increase in the number of ulnar collateral ligament reconstructions in adolescent aged children. There have also been similar trends in other institutions throughout the United States.

Many researchers have postulated that this rise in injury rate can be attributed to overuse, poor mechanics, and pitching prior to having the required skeletal maturity for certain pitch types (i.e. fastball, curveball, slider, and change-up) [6-9]. The idea of overuse comes from the fact that, with the opportunity for year round play, adolescents and young adults are choosing to specialize in a single sport and do not allow their arm to rest during the off season. Poor pitching mechanics can place increased stresses on the soft tissue around the shoulder and elbow and place these pitchers at a greater risk of injury. Pitchers with poor muscular conditioning do not have the appropriate soft tissue foundation to sufficiently absorb the stresses generated during the pitch, which leads to a greater risk of injury. Studies have suggested that over half of these injuries could be prevented by either having pitchers pitch less often or use better mechanics [10]. Telling an adolescent or collegiate level pitcher to pitch less often is often difficult and this advice is often ignored. By gaining a better understanding of pitching mechanics it is possible to find motion patterns that can be associated with a greater risk of serious injury. These results can then be used to better teach these young athletes in hopes of reducing the risk of serious injury in this population.

Over the past decade, a large quantity of research has been conducted using motion analysis techniques in the hopes of elucidating the causes of pitching injury. Although the advances in computing power have advanced the use of motion analysis many of the skeletal models that have been developed are lacking. All of the current models used in upper extremity sport analysis are Newtonian based vector models, which are able to adequately define the pitching motion and provide joint moments; however, these models, due to their vector based math, tend to be extremely cumbersome and make it difficult to
provide data about soft tissue structures or allow exploration of the biological systems response to changes in the input parameters. Therefore, the purpose of this work is to develop a Lagrangian based dynamic model of the upper extremity to provide a more direct description of the motion of the upper extremity and to demonstrate how this modeling can be paired with soft tissue models to provide a more complete description of the forces and moments acting on the upper extremity joints. The results of these models can then be analyzed to help determine possible injury mechanisms and attempt to reduce the risk of injury to this population.

1.1 Purpose

The purpose of this project is to develop a Lagrangian based model of the upper extremity that can be used to describe the motion of the human upper extremity. The model will incorporate both joint kinematics and joint kinetics, as well as muscle elements, to provide a more complete model of the upper extremity than is currently available. This model will also be able to be used with previously collected motion capture data of the upper extremity so that investigators would not have to collect large quantities of new data for this model to be applied. This model will then be applied to describing the upper extremity motion of baseball pitchers to determine if a Lagrangian based model of this nature is capable of providing additional clinically relevant data beyond what is currently presented in literature. The broader range goal is that this model can then be used to provide additional insight into the pitching motion to help identify possible injury mechanisms.

1.2 Specific Aims

Primary Aim: To develop a Lagrangian based upper extremity model that will produce similar results as current Newtonian based models.

Currently there are only a few Lagrangian based models for the upper extremity and even fewer that provide a multi-plane description of the kinematics and kinetics of the upper extremity. Therefore this work aims to develop a Lagrangian based model that can accurately represent both simple and complex motions of the human upper extremity. The results of this model will be compared to currently
available Newtonian models in an effort to validate the new Lagrangian based model as well as to illustrate differences between the two approaches.

**Secondary Aim:** Assess muscle force contributions using the Lagrangian based model.

Most biomechanical models calculate joint level kinetics only, which describe the moments at the joint; however, these models do not provide an understanding of how the underlying soft tissues contribute to these joint moments. Therefore a secondary aim of this work is to show how a Lagrangian based model can be extended to allow for a deeper understanding of sub-joint level moments, specifically the muscle force contributions driving the movement of the upper extremity.

2. Background

2.1 Anatomy

2.1.1 Joint Anatomy

The human upper extremity, when excluding the hand, initially appears to be a relatively simple construct of three joints (i.e. the shoulder, elbow and wrist) and three bones (i.e. the humerus, ulna, and radius); however, with closer examination the human upper extremity is surprisingly complex, and is comprised of numerous joints and a total of five bones (i.e. the humerus, ulna, radius, clavicle and scapula) [11] as shown in Figure 1. The complex architecture of the joints and bones of the upper extremity allows the upper extremity to move through an extremely large range of motion and perform a myriad of complex tasks [12].
Figure 1: Illustration of the bony anatomy of the upper extremity

2.1.1.1 Shoulder Joint

The shoulder joint is often considered a ball and socket type joint. It is one of the most complex joints in the human body and can produce the greatest arc of motion of any joint [13]. The shoulder joint can more accurately be described as three separate joints (i.e. the glenohumeral joint, scapulothoracic joint, and the acromioclavicular joint) and is constructed of three bones (i.e. the clavicle, the scapula, and the humerus) [11, 12] as seen in Figure 2.
The acromioclavicular joint, or AC joint, is the upper joint of the shoulder complex and is found at the junction of the acromion process of the scapula and the distal end of the clavicle [11]. The AC joint aids in the sliding movement of scapula allowing for additional extension and rotation of the arm. The glenohumeral joint, which is the primary joint of the shoulder complex, is a ball and socket joint that is constructed by the humeral head resting within the glenoid cavity of the scapula [11]. Interestingly, the glenoid cavity of the scapula is relatively shallow and does not completely encase the humeral head which allows for the large rotational range of motion seen at the shoulder joint [11]. Given the shallowness of the glenoid cavity, the glenohumeral joint would be unstable if not for a series of tendons and ligaments, the labrum, and negative capsular pressure that all serve to restrict the humeral head from excessive slipping [11] as shown in Figure 3.
The ligaments of the shoulder include the capsular ligament, coracohumeral ligament, transverse ligament, and the glenoid ligament. The capsular ligament circles the entire joint, and although it restricts excessive slipping of the joint, it is loosely attached to the glenoid cavity and humeral neck, which allows for some translational movement of the shoulder [11]. The remaining three ligaments function to restrict slipping and limit excessive motions of the joint.

The shoulder joint has a total of six degrees of freedom with substantial rotational range of motion but limited translational motion. The primary motions of the joint are vertical abduction/adduction, which is a coronal plane rotation of the glenohumeral joint, horizontal abduction/adduction, which is transverse plane rotation of the glenohumeral joint, and finally internal and external rotation, which is a sagittal plane rotation of the glenohumeral joint. Each of these motions are driven by the contraction of muscles attached to the humerus, scapula, and clavicle and are discussed in detail in a later section of this work.

2.1.1.2 Elbow Joint

The elbow, in comparison to the shoulder, is a much simpler joint. The elbow functions as a simple hinge joint and is constructed with three bones (i.e. the humerus, ulna, and radius) as depicted in
Figure 4. The joint is primarily formed by the ulnohumeral joint but is also formed by the connection of the trochlea of the humerus to the greater sigmoid cavity of the ulna, and the radial head of the humerus sits in a cup shaped depression in the radius[11].

Figure 4: Illustration of the bony anatomy of the elbow

Similar to the shoulder, the elbow joint is connected by four major capsular ligaments, (i.e. the anterior ligament, posterior ligament, internal ligament, and the external ligament) as shown in Figure 5. The purpose of these four ligaments are to connect the three bones constructing the elbow joint, and provide stability to the joint [11].
Although the elbow is primarily a single degree of freedom hinge joint, with most of the motion occurring in the sagittal plane, the joint is capable of allowing some rotational motion due to the sliding of the ulna and radius against the distal end of the humerus therefore providing an additional degree of freedom [11]. To best describe the motion of the elbow, it is best to consider the elbow joint as two separate joints, the connection between the ulna and humerus and the connection between the radius and humerus [11]. The ulnar-humerus joint is the largest of the two joints and is a simple hinge joint allowing the joint to flex and extend [11]. The radiohumeral joint is much smaller than the ulnar-humeral joint and forms a ball and socket joint [11]. Although a ball and socket joint typically has six degrees of freedom, the restrictions created by the ligaments limits the motion of this joint to axial rotation only [11]. Therefore, the radialhumeral joint allows the forearm to rotate causing some of the supination and pronation that the forearm is capable of performing. Again, the motions of these two joints are driven by the contraction of the attached muscles and will be discussed in a later section.

2.1.1.3 Wrist Joint

The architecture of the wrist joint is very similar to that of the elbow joint, and is the last major joint of the human upper extremity. The wrist joint is considered a condyloid articulation that is
comprised of five bones, the distal ends of the radius and ulna, as well as the scaphoid, semilunar, and carpal bones [11] as depicted in Figure 6.

![Figure 6: Illustration of the bony anatomy of the wrist](image)

The joint is formed by the ulna and radius entering into the elliptical depression created by the scaphoid, semilunar, and cuneiform bones. This construction allows the joint to have two degrees of freedom. The wrist joint motion is restricted by a total of four ligaments (i.e. the external radio-carpal ligament, internal ulnocarpal ligament, the posterior ligament, and anterior ligament) as seen in Figure 7 [11]. Similar to the previous two joints discussed, the ligaments serve to restrict motion and provide stability to the joint.

![Figure 7: Depiction of the ligaments of the wrist](image)
The wrist joint can again be best described by separating the joint into two distinct joints the radio-ulnar joint and the radio-carpal joint [11]. The radio-ulnar joint is proximal to the radio-carpal joint and is constructed by the connection of the distal end of the radius and distal end of the ulna. The radio-ulnar joint acts as a simple pivot joint, allowing the distal end of the ulna to rotate against the radius [11]. The motion of this pivot joint aids the radiohumeral joint in forearm supination and pronation. The radio-carpal joint is a two degree of freedom joint that allows coronal and sagittal plane motion but completely restricts any rotational motion. The elliptical shape of the receiving portion of the joint allows for flexion and extension of the wrist, with a slightly greater amount of flexion permitted, and allows for coronal plane motion (i.e. ulnar and radial deviation), with a greater amount of ulnar deviation permitted [11]. It is important to note that the radiocarpal joint does not allow any rotation; therefore, the supination and pronation of the hand is caused by the motion of the radio-humerus and radio-ulnar joints.

2.1.2 Muscle Anatomy

There are three types of muscle in the human body, smooth, cardiac, and skeletal and this work will be limited to a brief discussion of skeletal muscle. Skeletal muscle makes up 40% of the human body and is responsible for providing both stabilizing forces to the underlying skeleton and driving forces to allow for joint motion [13]. Muscles are arranged in a hierarchical structure in which the muscle body is the largest muscle unit as depicted in Figure 8. The muscle body can then be further divided into individual muscle fibers, which are made up of myofibrils that are made up of actin and myosin [13].
Each myofibril is made up of 1500 myosin filaments and 3000 actin filaments and each of the filaments are arranged in a specific geometry as seen in Figure 9 which is termed a sarcomere [13]. The sarcomere is arranged in an alternating pattern of actin and myosin, where the actin is termed the light band, or I band, and the myosin is termed the dark band, or A band, this structure gives the muscle it’s striated appearance [13]. Each actin filament is connected to the adjacent sarcomere via the z-line.

The sarcomere is important to muscle physiology because the muscle contractions, which drive the movements of the joints, occur at the level of the actin and myosin filaments. A muscle contraction begins with an action potential from a motor nerve which causes an influx of acetylcholine in the muscle tissue, this then causes a rapid diffusion of sodium ions in the muscle which causes the muscle action potential to begin[13]. The action potential depolarizes the muscle membranes which releases calcium ions into the muscle tissue creating an attractive force between the myosin and actin filaments. In the presence of the calcium ions, the actin and myosin filaments create cross bridges contracting the
sarcomeres and in turn contracting the muscle body [13]. As the calcium ions dissipate the contraction ends and the sarcomeres relax to their original resting lengths [13].

In this work the muscles of the human upper extremity are of interest and for completeness the major muscles that help to drive the joints of the upper extremity are briefly described. An understanding of the function of the upper extremity muscles as well as their origins and insertions is important for the modeling described later in this work.

2.1.2.1 Muscles of the Shoulder

There are 10 major muscles that help to both stabilize and drive the shoulder joint; however, this work will only focus on those that are primarily responsible for the movement of the humerus in relation to the thorax (i.e. the Pectoralis Major, the Deltoid, the Subscapularis, the Infraspinatus, and Teres Major) as depicted in Figures 10 and 11.

![Figure 10: Illustration of the anterior view of the shoulder and chest](image)
The Pectoralis Major is a broad triangular shaped muscle that originates at the anterior surface of the sternal half of the clavicle and extends from the upper portion of the sternum to the 6th or 7th rib [11]. The fibers then travel across the upper anterior portion of the chest to the insertion on the bicipital ridge of the humerus [11]. The Pectoralis Major functions primarily to aid in the horizontal adduction of the humerus, it also functions as a weak internal rotator for the humerus [14]. The Deltoid is also a triangular shaped muscle, which gives the edge of the shoulder its rounded appearance. The Deltoid originates from the upper edge of the distal third of the clavicle, the acromium process, and the lower tip of the scapular spine [11]. The muscle fibers travel around the shoulder joint and converge to a single insertion point located at a prominence on middle third of the humeral shaft [11]. The deltoid has two primary functions, the first is to abduct the arm to a 90° angle with the thorax. The second function of the deltoid is to assist the pectoralis major and tres major in flexion and extension of the humerus [14]. The Deltoid can also aid the suprasinatus with abduction. The subscapularisis also a large triangular muscle that originates along the medial and lower two-thirds of the subscapular fossa of the scapula and inserts into the lesser tuberosity of the humerus [11]. The subscapularis allows the internal rotation at the shoulder and also prevents dislocation of the humeral head from the glenoid [14]. The infraspinatus is a thick muscle that
has its origin at the infraspinatus fossa of the scapula it separates the Teres Major and Teres Minor and passes across the scapular where it inserts into the middle facet of the greater tuberosity of the humerus [11]. The Infraspinatus is the major external rotator of the shoulder and also assists with extension of the humerus [14]. Finally, the last major driving muscle for the shoulder joint is the Teres Major. The Teres Major is a thick flat muscle that originates from the dorsal inferior angle of the scapula and inserts into the bicipital ridge of the humerus [11]. The Teres Major does not act as a primary driver for the shoulder joint but rather assists the other muscles in internal rotation, adduction, and also serves to aid in stabilizing the joint [14].

2.1.2.2 Muscles of the Elbow Joint

Since the elbow joint has fewer degrees of freedom than the shoulder joint, the elbow requires fewer major muscles to drive the joint’s motion. There are four major muscles that aid in joint flexion and extension and assist in joint stabilization as seen in Figures 10 and 11; however, two of the muscles, the biceps and triceps, can be considered the primary drivers for elbow flexion and extension.

The Biceps is a long muscle that covers the anterior portion of the upper arm and is divided into two major portions, or heads [14]. The shorter head originates from the apex of the coracoid process and the long head originates from the upper margin of the glenoid cavity, the muscle fibers travel along the humerus and insert in the tuberosity of the radius [11]. The Biceps is the main flexor of the elbow and assist in the supination of the forearm [14]. The Triceps is the other major muscle body of the upper arm and covers the posterior portion of the upper arm. The Triceps is divided into three heads each with their own origin, the middle head of the triceps originates on the scapula below the glenoid, the origin of the middle head is found on the shaft of the humerus near the insertion of the Teres Minor, and the internal head of the Triceps also originates from the shaft of the humerus [11]. The three heads of the Triceps have a common insertion in the upper back portion of the olecranon process [11]. The Triceps is the main extensor for the elbow and when the arm is fully extended the triceps can aid in adduction of the arm [14].
The Biceps and the Triceps aid in the flexion and extension of the elbow. The supination and pronation of the forearm are driven primarily by two other muscles the pronator radii teres and the supinator longus as shown in Figure 12.

The Pronator Radii Teres has two heads the first originates from the medial epicondyle of the humerus and the other head originates from the coronoid process of the ulna and both heads share a common insertion on the lateral border of the radius [11]. The Pronator Radii Teres helps to pronate the forearm by rotating the radius on the ulna [14]. The Supinator is the most superficial muscle along the radial border of the forearm and originates at the lateral supracondylar ridge of the humerus and runs along the forearm where it inserts into the radial styloid [11]. The primary function of the supinator longus is to aid in both supination of the forearm and flexion of the elbow [14].
2.1.2.3 Muscles of the Wrist

There are four major muscles of the wrist (i.e. the flexor carpi radialis, flexor carpi ulnaris, extensor carpi radialis, and extensor carpi ulnaris) as shown in Figures 13a and 13b, that are responsible for providing the driving forces to move the joint in both the coronal and sagittal planes.

![Figure 13a and 13b: Illustration of the forearm and wrist musculature](image)

The flexor carpi radialis, originates from the medial epicondyle of the humerus and runs the length of the forearm and inserts into the base of the second and third metacarpal bones [11]. The flexor carpi radialis serves two functions, the first is as one of the two flexors of the wrist, and the second function is to aid in radial deviation of the wrist [14]. The flexor carpi ulnaris has two heads and runs along the ulnar side of the forearm [11]. Both of the flexor carpi ulnaris’ heads arise from the same origin at the medial epicondyle of the humerus and terminate at the insertion point on the base of the 5th metacarpal [11].

Similarly to the flexor carpi radials, the ulnaris serves two functions; to flex the wrist and assist in ulnar deviation [14]. The wrist also has two primary extensor muscles. The extensor carpi radialis originates from the lateral supracondylar ridge of the humerus and runs along the radial side of the forearm and then inserts into the second metatarsal [11]. As the name of this muscle would suggest, the extensor carpi
radials functions as one of the primary extensors of the wrist and also aids the flexor carpi radialis in radial deviation of the wrist [14]. Finally, the last major driving muscle of the wrist is the extensor carpi ulnaris. The extensor carpi ulnaris has its origin at the lateral epicondyle of the humerus and then travels down the ulnar side of the forearm where it inserts into the fifth metacarpal [11]. The extensor carpi ulnaris along with the extensor carpi radialis functions to extend the wrist, as well as aiding the flexor carpi ulnaris to cause ulnar deviation at the wrist [14].

2.2 Modeling Muscle Forces

Describing the activation and force generation of individual skeletal muscles is an ever expanding area of research, as the results of these studies can be used to better quantify muscle coordination, improve neuro-prosthetic designs, and define muscle fiber contributions to drive human movement [15-17]. The identification of muscle fiber contributions is extremely important as it can help to identify potential injury mechanisms and future orthopedic issues [15]. Muscle activation patterns can be easily obtained through the use of either surface electromyography (EMG), in which an electrode is placed on the skin directly over the muscle belly of superficial muscles, or with fine wire electromyography, in which a wire electrode is inserted directly into the muscle. The electrodes measure the electrical impulses created by the muscle action potentials that drive the muscle contraction [18]; however, EMG signals cannot directly provide a means of determining the force generation of the muscle being measured. Although some researchers have developed models that can be used to estimate muscle forces and moments, these models are only limited to thoracic models [19], while methods that compare EMG signals collected during isolated maximal voluntary contractions to signals obtained during activity only provide a percent of total muscle activation [20]. Furthermore, EMG signals can also be negatively influenced by cross talk between two different muscles, skin motion artifact, and electrode placement.

There are a few methods available that allow for the direct measurement of muscle forces in-vivo; however, these methods are very evasive and require the surgical implantation of strain gauges [16]. In an attempt to non-invasively estimate muscle forces, many researchers have begun to develop
computational models to describe the viscoelastic properties of muscles in conjunction with EMG recordings.

The current models used to describe muscle force generation are based on the macroscopic descriptions of muscle mechanics defined by A.V. and viscoelastic models structured by Maxwell and Voigt [21]. Interestingly, the models created by Maxwell and Voigt were never originally intended to describe muscle mechanics, instead Maxwell’s model was developed to describe the elastic nature of various materials and fluids, and Voigt’s model was developed to better describe the viscosity of crystalline substances [22]. Throughout the literature, there are some inconsistencies in the terminology associated with the various muscle models, such as the Maxwell model, the three-element Hill model, or the Kelvin-Voigt model and, in this work, the various models and their names are based on the descriptions published by Bradey et al. [15] and Fung et al. [22] and are included below for completeness.

The original Hill model, shown in Figure 14, was described as a single elastic element, or spring, connected in series with a contractile element, a dashpot, to describe both the passive force generation and contractile or active force generation.

![Figure 14: Mechanical analogue of the Hill muscle model [15]](image)

This model was deemed to be too simplistic to accurately describe the passive force generation of the muscle while also being able to adequately describe the force-velocity relationship seen in skeletal muscle [15]. Therefore, the more common macroscopic models that are used to estimate muscle forces are the three-element Voigt model shown in Figure 15 and the three-element Maxwell model shown in Figure 16.
Both of these models incorporate an additional elastic element to the other elements in the Hill model. It has been shown that a three-element model is the minimum number of elements necessary to accurately describe muscle force production [23]. In both models, the contractile element (i.e., dashpot) provides a description of the velocity dependence of force generation and the series elastic element (i.e., spring connected in series with dashpot) provides the resistance of muscle to velocity and aids the description of the force length relationship of skeletal muscle. The final element of both models is the parallel elastic element (i.e., spring placed in parallel with two other elements), which provides the passive force generation for the models [15, 23]. The major difference between these two models is the positioning of the series elastic element, as shown in the figures above. This difference translates into the ability of the model to support both compressive and extensive forces [15]. In both models, the contractile element is assumed to be highly damped and, therefore, a change in length cannot occur instantly; therefore, changes in length are initially dependent on the elastic structures [15]. In the Voigt model, the two elastic elements provide the resting tension as well as describing the length of the muscle. In this configuration
with the assumption that all activation forces are dependent on the contractile element, compressive forces are not adequately accounted for [15]. The Maxwell model is setup in such a way that all three elements are capable of supporting both extensive and compressive length changes, especially the series elastic element, allowing the model to better handle compressive loads [15]. It is for this reason that many researchers suggest using the Maxwell model when describing the force generation of skeletal muscle [15, 23].

A number of studies have been conducted to test the utility of the Maxwell muscle models. Thelen et al. conducted a study to determine if adjustments in the parameters governing the elements of the Maxwell model could successfully describe the effects of aging on force production in skeletal muscle [24]. They modeled four of the major muscles involved in that plantar flexion and dorsiflexion of the ankle joint. Using this model, the researchers adjusted the constants associated with the contractile element to mimic the reduction in contractile strength of aging muscles as well as to reduce the force-velocity relationship and they also adjusted the spring constants associated with the elastic elements in an effort to mimic the increased stiffness associated with muscles in an elderly population [24]. The results of adjusted models were compared to isokinetic dynamometer measures as measured from 24 young adults and 24 older adults. Since the isokinetic testing provided a total joint torque only, the results of four individual muscles were summed and compared to the measured total joint torque [24]. The results of the work indicated that the models were within one standard deviation of the experimentally collected data leading to the conclusion that constant parameters can be changed to effectively mimic the muscle force generation in aging muscle. Thelen et al. caution that there is a great deal of uncertainty in the model and that the results should be compared to a known joint torque in order to make physiologically appropriate changes to the model parameters [24].

A similar study was conducted by Hof et al., where the Maxwell model was used to describe the individual muscle force contributions required to plantarflex and dorsiflex the ankle. Unlike Thelen et al., Hof et al. used surface EMG signals normalized to maximum voluntary contractions to validate his model. In their study, subjects were moved through a specified range of motion and EMG signals were
recorded using surface electrodes and joint angles were computed using electrogoniometry [20]. The results of the work showed a root mean square error ranging from 2.8 to 6.7 between the calculated force and the estimated force as measured from the EMG signals [20]. It was noted that the error increased when the velocity of the ankle movement increased or decreased from the mean velocity. They concluded that the calculated muscle force contributions, determined using the Maxwell model, were comparable to those forces measured experimentally [20]. However, Hof et al. caution that a careful description of the model assumptions and the inclusion of appropriate anatomical properties of the muscles being modeled are required to ensure the validity and proper use of the developed model [20].

More recently, a study was conducted by Holzbaur et al. to validate a complex upper extremity muscle model. This model uses motion data as an input to drive the muscle force production. Their model incorporated a total of 50 muscles to drive the motion of the shoulder, elbow, wrist, thumb, and index finger [17]. The model is based on the Maxwell model, as well as the A.V. Hill description of muscle mechanics, and includes anatomically appropriate muscle lengths, pinnation angles, tendon slack length, and peak forces typically seen in these 50 muscles. The stiffness for the lower arm muscles, those driving the forearm, wrist, and fingers, was set at 45 Ncm$^2$ and, for those muscles involved in moving the shoulder and elbow joints, the stiffness was set to 140 Ncm$^2$ [17]. To validate the model, Holzbaur et al. used a similar technique to that of Thelen et al., where the individual muscle forces were summed for each joint and then compared to an experimentally determined joint torque. In Holzbaur et al.’s work, the joint torques were provided from motion analysis data and their results showed that the calculated muscle forces were able to accurately estimate the true muscle force contributions with an $r$ value of 0.81 for the elbow and 0.62 for the shoulder joint musculature [17]. They concluded that this model does appropriately estimate muscle force contributions to joint torques and is capable of accounting for changes in muscle length and angle from the movement of surrounding joints; however, the model is validated for adult males moving through a simple motion only and, therefore, is not reflective of adolescent or female populations [17].
It has been shown that a three-element muscle model can provide a reasonable estimation of the individual muscle forces required to produce a joint torque to move a specified joint. In this work, the three-element muscle model described by Peterson and Adrezin was used [25], which utilizes two spring elements and a dashpot that are connected in parallel. Peterson and Adrezin suggest that, by incorporating a similar model into a Lagrangian-based model of the upper extremity, it may be possible to more completely describe the motion of the arm and understand the effects of individual muscle contributions on the overall joint torque. Data such as this can then be used to help assess injury mechanisms and eventually be used to help develop training programs to reduce the injury potential in a specified patient or athletic population.

2.3 Lagrange Mechanics

Determining the equations of motion is paramount to understanding the dynamic behavior of a biomechanical system. Understanding the movement of a biomechanical system can lead to improvements that can increase efficiency and reduce injury to the biological structures. The equations of motion can be determined using any of four methods (i.e., Newton-Euler, Lagrange’s Equation, D’Alembert’s method of virtual work, and Principle of virtual power using Kane’s equation), with the most common being the Newtonian approach and the application of Lagrange’s Equation [25-27].

2.3.1 Differences between Newtonian and Lagrangian Approaches

The Newtonian method is by far the most common approach of the four methods and greatly relies on Newton’s Second Law of Motion, which states that force is equal to the mass multiplied by the acceleration of a system as shown in Equation 1,

\[ F = ma, \]  

Eq.1

where \( F \) is the resultant force, \( m \) is the particles mass, and \( a \) is the acceleration of the particle. The Newtonian method requires the determination of all applied forces, constraint forces, and moments on each of the bodies that comprise the system, as well as the vector-based accelerations. Once the forces,
momen
ts, and accelerations are determined, it is possible to establish the equations of motion for the
system [26].

The Lagrange method is not as commonly employed as the Newtonian method and utilizes
Lagrange’s Equation, which makes use of scalar quantities of velocity. Once the velocities for each of the
bodies of the system are determined, it is possible to find the kinetic and potential energy of the system
and, through the application of Lagrange’s Equation, the equations of motion can then be derived [25].

The primary difference between these two methods is that the Newtonian method requires all of
the forces and moments for all the bodies making up the system to be balanced with the kinematic
description of the system and then, using the systems constraints, it is possible to reduce the number
equations defining the motion of the system [26]. The Lagrange method requires that the systems
constraints and kinematic expressions are found prior to balancing the force applied to the system [26]
and allows for a more direct method of determining the equations of motion for the dynamic system.
Another difference between the two methods is that the Newtonian method relies on vector-based
quantities and accelerations, while the Lagrange method relies on scalar quantities and velocities [25, 26].
Since the Lagrange equations rely on scalar quantities, they require less vector algebra, which is
convenient when there are a number of bodies comprising the system or the geometry of the system is
complex. The last major difference between the two methods is that constraint forces are eliminated in
the Lagrange method. If constraint forces are of interest, then the forces must be solved for and added
back into the Lagrange-based equations [25-27].

2.3.2 Lagrange’s Equation

Prior to delving into the derivation of Lagrange’s equation, there are two concepts that must be
understood in order to accurately describe the equation. These concepts are generalized coordinates and
generalized forces.
2.3.3 Generalized Coordinates

In every mechanical system, there are a minimum number of variables called degrees of freedom that are needed to describe the movement of the system. For every rigid body, there are a total of six degrees of freedom: three spatial variables and three orientation variables. There are instances in which less than six variables are required to completely describe a system of rigid bodies; however, this is dependent upon the number of constraints for the system and the application of a specific coordinate system, such as a spherical coordinate system instead of the Cartesian coordinate system [27, 28]. A set of generalized coordinates are the independent variables required to fully describe the motion of a system and are often considered the degrees of freedom for the system [27, 28]. It is important to stress that these variables must be independent, in order to be considered as a generalized coordinate. For instance, the position of a point, A, in spherical coordinates can be defined by the angles, \( \theta \) and \( \phi \), along with a distance parameter, \( r \), as shown in Equation 2,

\[
A = A(\theta, \phi, r, t) = A(q_1, q_2, q_3, t),
\]

Eq. 2

where \( A \) is the particle, \( \theta \) and \( \phi \) are the angular positions of point A, \( r \) is the distance coordinate for the position of point A, and \( t \) is the temporal parameter. The variables \( q_1, q_2, \) and \( q_3 \) are the generalized coordinates corresponding to \( \theta, \phi, \) and \( r \), respectively. If for some reason the two angular parameters were related so that as one changed the other also changed then the system’s generalized coordinates would reduce to \( q_1 \) and \( q_2 \), where \( q_1 \) would be an independent variable describing the movement of both \( \theta \) and \( \phi \).

Lagrange’s equation makes use of independent generalized coordinates, in order to derive the equations of motion of a dynamic system.

2.3.4 Generalized forces

The concept of generalized forces requires a brief description of the principle of virtual work, which states that in any system of N particles the system is said to be in a state of static equilibrium if the
work performed over an infinitesimally small displacement of the system is equal to zero [28]. Virtual work can be calculated using Equation 3,

\[ \delta W = \sum_{i=1}^{N} F_i \cdot \delta r_i, \]

where \( \delta W \) is the virtual work, \( F \) is the applied force, \( \delta r \) is the virtual displacement of position vector \( r \), and \( i \) is the \( i^{th} \) particle or body within the system. Note that if the virtual work of the system is not equal to zero, then the system is not considered to be in a state of static equilibrium and can therefore undergo a positional change caused by an applied force [27].

Generalized forces make use of the principle of virtual work by assuming that the system is not in static equilibrium and undergoes a known virtual displacement; therefore, the force applied to the system can be solved for using the generalized coordinates[28]. To solve for the generalized forces on a dynamic system, a single generalized coordinate position is changed by \( \delta q \) while all other generalized coordinates are held fixed. In using Equation 3, it is possible to determine the virtual work performed on the system by the driving forces in the direction of the general coordinate [26, 28]. This method can then be repeated for all the remaining generalized coordinates, in order to balance the equations of motion for the system with the applied driving forces. It is also important to note that although the term “generalized forces” is used, generalized forces also include applied moments.

### 2.3.5 Derivation of Lagrange’s Equation

The derivation of Lagrange’s Equation can be performed a number of ways. The most rigorous derivation starts with a single particle and the application of Newton’s Second Law of Motion [29]. Another common method of deriving Lagrange’s Equation is to start with the summation of kinetic energy of a system of particles as presented by Greenwood [27]. In this work the more rigorous derivation is presented in an effort to present a clear understanding of the physical meaning of the equation.

To begin the derivation, assume a single spherical particle can roll across a plane without slipping and without friction as seen in Figure 17.
From Newton’s Second Law, $F = ma$, the force and movement of the particle can be written using Equations 4, 5, and 6,

$$F_x = \ddot{x}m,$$  \hspace{1cm} \text{Eq. 4}
$$F_y = \ddot{y}m,$$  \hspace{1cm} \text{Eq. 5}
$$F_z = \ddot{z}m,$$  \hspace{1cm} \text{Eq. 6}

where $F_x$, $F_y$, and $F_z$ are the force applied to the particle in the x, y and z directions, respectively, $m$ is the total mass of the particle, and $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ are the second derivatives with respect to time of the particle’s position, or the particle’s acceleration, in each of the respective directions. For simplicity, only the x direction will be used in the remaining derivation.

The next step requires that the virtual work done by the applied force on the particle be calculated using Newton’s laws and the concept of virtual work in Equation 7,

$$\delta W = F_x \delta x,$$  \hspace{1cm} \text{Eq. 7}

where $\delta W$ is the virtual work performed on the particle, $F_x$ is the externally applied force to the particle, and $\delta x$ is the virtual displacement of the particle in the x direction caused by the applied force. Using this application of the principle of virtual work, it is now possible to rewrite Equation 4 as shown in Equation 8,

$$m\ddot{x}\delta x = F_x \delta x,$$  \hspace{1cm} \text{Eq. 8}
where \( m \) is the mass of the particle, \( \delta x \) is virtual displacement of the particle in the \( x \) direction caused by the applied force, \( F_x \), and \( \ddot{x} \) is the acceleration of the particle in the \( x \) direction. This equation shows that the work done by the applied force \((F_x \delta x)\) is equal to a small change in the kinetic energy of the particle \((m \ddot{x} \delta x)\) [29]. This is the basis for Lagrange’s equation and the remainder of the derivation transforms Equation 8 into the standard form of Lagrange’s Equation.

The first step in converting Equation 8 into the standard form of Lagrange’s equation is to introduce the concept of generalized coordinates so that the position variables are replaced by the generalized coordinates. Through the introduction of the generalized coordinates the virtual displacement can be written as shown in Equation 9,

\[
\delta x = \frac{\partial x}{\partial q_1} \delta q_1 + \frac{\partial x}{\partial q_2} \delta q_2, \quad \text{Eq. 9}
\]

where \( \delta x \) is the virtual displacement of the particle, \( \frac{\partial x}{\partial q_1} \) and \( \frac{\partial x}{\partial q_2} \) are the partial derivatives of \( x \) with respect to the generalized coordinate \( q_1 \) and \( q_2 \), respectively and \( \delta q_1 \) and \( \delta q_2 \) are the virtual displacement of the particle using the generalized coordinates. Using Equation 9 and substituting it into Equation 8, it is possible to define the virtual work performed on the particle using only the independent generalized coordinates as seen in Equation 10,

\[
F_x \frac{\partial x}{\partial q_1} \delta q_1 + F_x \frac{\partial x}{\partial q_2} \delta q_2 = m \ddot{x} \frac{\partial x}{\partial q_1} \delta q_1 + m \ddot{x} \frac{\partial x}{\partial q_2} \delta q_2, \quad \text{Eq. 10}
\]

where \( F_x \) is the externally applied force on the particle, \( m \) is the mass of the particle, \( \ddot{x} \) is the acceleration of the particle in the \( x \) direction, and the summation of \( F_x \frac{\partial x}{\partial q_1} \delta q_1 \) and \( F_x \frac{\partial x}{\partial q_2} \delta q_2 \) is equal to the total work done on the particles using generalized coordinates. Equation 10 can be further simplified through the use of a few mathematical identities shown in equations 11, 12, 13, and 14.

\[
\dddot{x} \frac{\partial x}{\partial q_i} = \frac{d}{dt} \left( \frac{\partial x}{\partial q_i} \right) \frac{dx}{dt} \frac{d}{dx} \left( \frac{\partial x}{\partial q_i} \right), \quad \text{Eq. 11}
\]

\[
\dot{x} = \frac{d}{dx} \left( \frac{1}{2} \dot{x}^2 \right), \quad \text{Eq. 12}
\]
\[
\frac{\partial x}{\partial q_i} = \frac{\partial \dot{x}}{\partial q_i}, \quad \text{Eq. 13}
\]
\[
\frac{d}{dt} \left( \frac{\partial x}{\partial q_i} \right) = \frac{\partial \dot{x}}{\partial q_i}, \quad \text{Eq. 14}
\]
Substituting equations 11, 12, 13, and 14 into right side of Equation 10 allows Equation 10 to be written to show the kinetic energy more clearly as seen in Equation 15,
\[
F_x \frac{\partial x}{\partial q_1} \delta q_1 = \left[ \frac{d}{dt} \left( \frac{\partial}{\partial q_1} \left( \frac{m \dot{x}^2}{2} \right) \right) - \frac{\partial}{\partial q_1} \left( \frac{m \dot{x}^2}{2} \right) \right] \delta q_1. \quad \text{Eq. 15}
\]
The left side of the equation, \( F_x \frac{\partial x}{\partial q_1} \delta q_1 \), is the work performed on the particle by the applied force \( F \), as a function of the generalized coordinates. Whereas, the right side of Equation 14 shows that the work performed is caused by a difference in kinetic energy, \( \frac{m \dot{x}^2}{2} \), as a function of the virtual displacement of the generalized coordinates. Finally, Equation 15 can be simplified by replacing the kinetic energy term, with a single variable, \( T \), as shown in Equation 16,
\[
F_x \frac{\partial x}{\partial q_1} = \frac{d}{dt} \left( \frac{\partial T}{\partial q_1} \right) - \frac{\partial T}{\partial q_1}. \quad \text{Eq. 16}
\]
The final step of achieving the general form of Lagrange’s equation is to introduce a generalized force to replace the virtual work term on the left side of Equation 16, which yields Equation 17,
\[
Q_i = \frac{d}{dt} \left( \frac{\partial T}{\partial q_i} \right) - \frac{\partial T}{\partial q_i}, \quad \text{Eq. 17}
\]
where \( Q \) is the generalized force acting on the particle and \( i \) is the \( i \)th generalized coordinate. Equation 17 is the considered the general form of Lagrange’s equation. The equation describes the motion of the particle by showing that the difference between the generalized momentum on the particle, \( \frac{d}{dt} \left( \frac{\partial T}{\partial q_i} \right) \), and the inertial forces on the particle due to the motion of the particle in generalized coordinates, \( \frac{\partial T}{\partial q_i} \), is equal to the applied force on the particle [27].

The final steps to deriving the standard form of Lagrange’s equation requires that the general form of the equation be extended to a system of particles or rigid bodies, in which the potential functions,
such as gravitational forces and spring forces, are taken into account[26-28]. The potential function can be written using general coordinates as seen in Equation 18,

\[ F_i = -\frac{\partial V}{\partial q_i}, \]  

Eq. 18

where \( F_i \) is the potential force, and \( \frac{\partial V}{\partial q_i} \) is the partial derivative of the potential energy of the system with respect to the generalized coordinates. The potential function can be added to the general form of Lagrange’s Equation to yield Equation 19,

\[ -\frac{\partial V}{\partial q_i} + Q_i = \frac{d}{dt} \left( \frac{\partial T}{\partial q_i} \right) - \frac{\partial T}{\partial q_i}, \]  

Eq. 19

The final step in deriving the standard form of Lagrange’s equation is to introduce the Lagrangian, which is defined as the difference in kinetic and potential energy as seen in Equation 20,

\[ L = T - V, \]  

Eq. 20

where \( T \) is the kinetic energy acting on the system, \( V \) is the potential energy acting on the system, and \( L \) is the Lagrangian. For conservative systems, the Lagrangian illustrates that energy is not dissipated but rather is transferred between potential and kinetic energy. The standard form of Lagrange’s equation can finally be written after rearranging the terms in Equation 19 and substituting in the Lagrangian as shown in Equation 21,

\[ Q_i = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i}. \]  

Eq. 21

2.4 Uses of Lagrangian Mechanics

As mentioned previously, Lagrangian mechanics provides a more direct method of determining the governing equations for a dynamic system and allows for a more complete theoretical understanding of the system through the ease at which a single parameter can be varied and the output analyzed. Although Lagrange’s Equation provides a powerful means to describe and analyze a system, there are only a handful of researchers that have used Lagrange’s equation to describe the mechanics of human
motion. Almost all of the researchers that relied on Lagrangian mechanics conducted their research in the late 1960’s and throughout the 1970’s, when the computational power of computers was limited and researchers required a more direct means of developing and analyzing the equations of motion governing the system. As computational power increased throughout the 1980’s and 1990’s, almost no biomechanical papers were published in which the Lagrangian approach was utilized. Instead, many authors used the Newton-Euler method that could provide similar results with less upfront work required to set up the Lagrangian. A small handful of researchers have more recently turned back to using Lagrange mechanics in combination with improved computational power to provide highly detailed biomechanical models to gain insight into motor control and disease states. This section will detail the applications of Lagrange’s Equation in biomechanics.

In 1968, Beckett et al. realized that the human body attempts to use the least possible energy when performing mechanical work and therefore believed that a model of human motion could best be approached by applying Lagrange’s equation [30]. Beckett et al. created a simple sagittal plane model of the leg in which the hip, knee, and ankle were all assumed to be single degree of freedom joints. The driving force for the model was assumed to be the hip moment. Once the model was completed, they applied Lagrange mechanics to the model to create a system of equations that described the motion of the leg [30]. These equations were then applied to the describe the swing phase of gait, at the point at which the foot is not in contact with the ground, in order to understand how perturbations to the system (i.e., changes in cadence and body weight) could affect the joint angles of the leg [30]. Specifically, Beckett et al. wanted to find the optimum walking speed that allowed for smooth joint motion and minimization of the total energy required to move the leg. They assumed that the joint angles followed a sinusoidal pattern and provided initial conditions for each joint and the hip moment was derived from EMG recordings [30]. Using this model, Beckett et al. was able to show that the optimum walking speed was 73 steps per minute and that, above 84 steps per minute, the joint moment at the knee, which he equated to the energy expenditure, increased dramatically [30].
A few years later in 1973, Chao et al. applied optimization principles to a Lagrange based model of the lower body to assess how joint kinematics affected joint moments. Chao proposed that by using optimization principles along with a model of the lower body, it would be possible to learn more about the joint moments of the hip, knee, and ankle by systematically varying the input joint angles and velocities [31]. Similar to Beckett et al.’s work, Chao et al. chose to use a simple sagittal plane only model of the lower extremity and again assumed that each joint was only a single degree of freedom and relied on Lagrange’s equation to develop the governing equations of motion. Unlike Beckett et al., they decided to input actual joint angles and velocities into the equations from previously collected gait data, which would provide a more accurate description of the joint moments in comparison to a sinusoidal wave input [31]. Once the model was validated by comparing the theoretical joint moments with the experimentally determined joint moments, they began to vary the input joint angles and velocities until they were able to minimize the energy expenditure of the system. Chao et al. concluded that, although the methods presented in his work provided insight into finding an optimal solution of joint angles and velocities, without a known physiological optimum solution the model presented in his work would only have theoretical applications [31].

In 1976, there was a shift away from looking solely at human gait and a turn towards describing motion of the human body in general. Huston et al. developed a complex generalized model that was far more advanced than the models proposed by Beckett et al. and Chao et al. Huston et al. realized that many of the models lacked the ability to be fully generalizable and that, with advances in computing power, it was possible to create advanced models to provide a better understanding of human motion [32]. Huston et al. proposed a 34 degree of freedom model in which joint angles could be input to calculate joint moments or the model could be adapted to allow forward dynamic solutions, where joint moments could be input to solve for joint angles. Huston et al. based their model on a 15-segment model previously described by Hanavan et al., in which joints were defined either as hinge joints (i.e., knee, ankle, and elbow) or ball and socket joints (i.e., shoulder and hip) [32]. Although they state that the model was more restrictive than human motion, the equations required to describe the motion were
“unwieldy” when using the Newton-Euler method. Since constraint forces were of little or no consequences, Huston et al. used Lagrange methods to provide a more direct solution [32]. The final model they presented was generalizable for a number of different applications; however, the weakness of the model was in the fact that the coefficients describing the anatomical parameters must be assumed if not previously measured limiting the utility of the model [32]. They validated model by using experimentally collected data from vehicular crashes and was able to describe the body’s response to impulsive forces [32].

A year later, in 1977, Hatze et al. further expanded upon Huston et al.’s work, where they realized that Huston et al.’s model provided a general model of human motion but did not allow for the predication or input of individual muscle torques. Therefore, Hatze et al. proposed a model that could more fully describe the workings of the human musculoskeletal system [33]. Similar to Huston et al., they started with the model described by Hanavan et al. and used Lagrange’s Equation because it was a “more sophisticated method” of developing the equations of motion without the need to solve for constraint forces [33]. Once the skeletal model was developed, they went on to include muscle forces by including them as generalized forces helping to drive the system. They state that the inclusion of muscles into the model must take into account the origin and insertion points, the line of action for the muscle fibers, and include both the active contractile and passive elastic components of the muscle [33]. Hatze et al. used a muscle model in which the active components were modeled by a dashpot and spring arranged in series and a second spring connected in parallel to simulate the passive portion of the muscle as seen in Figure 17 [33].
The muscle model developed was then tested using known joint angles and moments and then solving for the resultant muscle forces. The theoretical output provided by the model was then compared to the actual experimentally collected EMG recordings. The results showed that the model provided a good estimate of the actual muscle force and activation pattern collected using the EMG [33]. This was one of the first models to attempt to assess muscle force generation using Lagrange methods. Using this model Hatze et al. stated that it would be possible to gain a better understanding of how muscle force production can affect joint kinematics and kinetics [33].

Jackson et al., in 1978, turned his attention to describing the motion of the upper extremities during gait. They realized that there were a number of studies describing the movements of the lower extremities but there were very few papers that looked into how the motion of the upper extremity contributed to human gait [34]. Jackson et al. proposed a model to describe the swing of the arm while walking using only joint moments and shoulder and elbow joint angles and velocities. Similar to the early lower extremity models, they chose to create a sagittal plane model since the primary motion of the arm when walking occurs in the sagittal plane [34]. Jackson et al. assumed that the glenohumeral joint
and elbow joint could be modeled as a pivot joint and, by applying Lagrange’s Equation, the governing equations of the model could be developed following Equations 22 and 23,

\[
\ddot{\theta} = f_1(\theta, \dot{\theta}, \phi, \dot{\phi}, D_s, V_s, E_a, H, t), \quad \text{Eq. 22}
\]

\[
\ddot{\phi} = f_2(\theta, \dot{\theta}, \phi, \dot{\phi}, D_e, V_e, E_a, J, t), \quad \text{Eq. 23}
\]

where \( \theta \) and \( \phi \) are joint angles for the shoulder and elbow, respectively, \( \dot{\theta} \) and \( \dot{\phi} \) are the angular velocities of the shoulder and elbow respectively, \( \ddot{\theta} \) and \( \ddot{\phi} \) are the angular acceleration of the shoulder and elbow, respectively, \( D_s \) and \( D_e \) are the elastic components of the shoulder and elbow, \( T \) is the resistive torque of the elbow, \( V_s \) and \( V_e \) and \( H \) are the vertical linear accelerations of the shoulder and elbow and the horizontal linear accelerations of the shoulder, respectively, \( H \) and \( J \) are the joint moments for the shoulder and elbow, respectively, and \( t \) is time [34]. \( D_s \) and \( D_e \) can be further defined as seen in Equation 24. Only the equation for \( D_s \) is provided, since it is nearly identical to \( D_e \) with the only difference being the related joint angle,

\[
D_s = \beta(-0.9\theta - 0.1\dot{\theta}), \quad \text{Eq. 24}
\]

where \( \beta \) is the joint stiffness and \( \theta \) and \( \dot{\theta} \) are the joint angle and angular velocity, respectively. Using this model Jackson et al. used initial values collected from gait data to determine how joint angles varied with changes in elbow position and joint stiffness [34]. The results of Jackson et al.’s work indicated that, if the elbow remained in a locked position, then the glenohumeral moment increased and that, if joint stiffness increased, then the angular displacement of the joint decreased while the joint moment increased [34]. This model is important to illustrate how additional information, joint stiffness beyond joint kinematics and kinetics, can be easily integrated into a Lagrange based model.

In 1979, Winters et al. realized that there were few researchers that were modeling human movement using direct dynamics without an optimization scheme, which simplified the system too much and could not accurately assess the higher harmonics of a moving system (when only inputting sinusoidal waveforms) [35]. Winters et al. attempted to create a model in which the motion of the human lower
extremity during gait could be predicted by knowing only the joint moments. This would allow clinicians to alter joint moments to help predict surgical outcomes [35]. They initially created a seven-segment model with a total of six joints (i.e. two hips, two knees, and two ankles) in which the sagittal plane motion only was described using coordinate transforms and Newtonian mechanics. This created numerous equations and multiple constraint equations and led Winters et al. to use a Lagrangian approach [35]. The Lagrange method allowed for a more direct approach at solving for the dynamic equations to govern the proposed model. Once the equations of motion were solved, Winters et al. used data gathered from a motion analysis laboratory to provide the initial conditions to solve for the resulting joint kinematics [35]. The results of the model showed that, given the initial kinematic position of the joints along with the time history of the joint moments, it was possible to predict the joint angles. The predicted joint angles provided by the model were found to be very close to the joint kinematics as captured using motion analysis [35]. Once the model was validated, the research group modified the input moments to gain a better understanding of the resulting joint angles. Winter et al. showed that, when the ankle moment was increased by 20%, the left heel strike occurred earlier and propelled the body faster whereas a decrease in ankle moment required shortening the step lengths [35]. Their paper illustrates the utility of Lagrange’s equation to develop a forward dynamic model.

Throughout the 1980’s and 1990’s, Lagrange mechanics was not as readily used as they were throughout the 1970’s. Instead, many of the papers in this period relied on computational power of computers and Newton-Euler methods to describe the dynamics of biological systems. Other papers focused on optimization methods and used Lagrangian multipliers to find various solutions to muscle force based problems. In the early 2000’s and more recently, a few researchers have combined Lagrange methods with computational power available to create models that are extremely complex.

In 2013, Hong et al. applied Lagrange methods to investigate the effects of stiffening in joints and tissues as a person ages and how this affects the gait of this population [36]. The model helped describe how diseased states could progress with increasing joint stiffness. Hong et al. modeled the sagittal plane motion of the lower extremities; however, the major difference in this model is that the legs were modeled
as a spring dashpot system shown in Figure 18 to mimic the effects of tissues and the elastic elements of the joints [36].

Hong et al. created a system of equations describing the dynamics of the lower extremities using Lagrange’s equation. The Lagrange method was used since it is capable of providing a theoretical model that could be more rapidly manipulated to understand the dynamics of the system [36]. Once the model was created, Hong et al. used results from gait analysis and walking velocities to predict ground reaction forces during gait. The results of the validation testing showed that the r squared value was 0.77 when comparing the experimental data and the theoretical data [36]. Hong et al. then used simulated data to change various parameters to better describe how increases in age, walking velocities, and muscle activation affected joint stiffness [36].

Although Hong et al.’s model illustrates how Lagrange’s equations can be applied to show how disease states can change over time, the most complete human motion modeling using a Lagrangian approach is described by Ivancevic et al. [37, 38]. Ivancevic set out to create a model of “realistic humanoid movement” that not only included a description of human motion but also included a description of the effects of muscle forces and neurological control [38]. The model presented in the series by Ivancevic started in 2006 and is an extremely complex model. It combines aspects of Newtonian, Hamiltonian, and Lagrangian mechanics, as well as Lie-group algebroids to simulate real human biodynamics for both illustrative and predictive analysis [37, 38]. The model consist of 150 rigid
bodies, and 270 degrees of freedom (135 rotational degrees of freedom and 135 translation degrees of freedom), 26 flexible bodies, and a total of 270 non-linear actuators to simulate the driving forces of muscles [38]. The early versions of this model were developed using Hamiltonian mechanics as well as Newton-Euler equations; however, this was abandoned since the Lagrangian approach could be more easily applied and reduce computational times needed to solve the resultant equations [37]. The model can be broken down into three primary components: the skeletal, the muscle, and the neurological. The primary component, the skeletal model, consists mainly of the 135 rotational degrees of freedom with the 135 translational degrees of freedom limited to minor motion of the joints, so that they are free to translate and provide a more accurate representation of human movement [38]. Here, Ivancevic uses the Lagrangian approach to develop the equations of motion to describe the joint movements. To further provide a more realistic description of human motion, Ivancevic includes a dissipative force for all synovial joints to mimic the “stabilizing effect” [37]. This dissipative force is incorporated in the model using Rayleigh’s dissipation function. The second major component of the model is highly integrated with the first component and describes the models driving forces using 270 muscles as well as external forces and moments. The muscle modeling is based on the classic Hill muscle model and also includes dissipative forces using Rayleigh’s dissipation function [37]. Again, the muscle modeling is accomplished using a Lagrangian approach. The final component of the model takes into account the neurological control over the entire system and, though outside the scope of this work, it will be discussed for completeness. Ivancevic turns to Lie-algebroids and Hamiltonian mechanics to model cerebellum control and reflex control respectively [37]. Modeling the neurological control patterns provides a more realistic description of human motion, which allows an increased ability to describe and predict musculoskeletal injury [38]. Ivancevic’s model was validated using gait analysis and EMG data and comparing the experimentally collected data to the model output. Although Ivancevic created the most complete full body model to describe human motion, almost all of the applications of this model so far have been to describe spine motion. Ivancevic does believe that future applications do not need to be
limited solely to one joint or segment but could be used to describe complex whole body human biodynamics [37].

2.5 Motion Analysis

As shown in the previous section when Lagrange’s equation is applied to human movement, investigators often use motion analysis techniques to obtain joint angles, velocities, accelerations or joint moments. This motion data can then be used to either provide a basis for solving the equations of motion or help validate the derived equations. Typically, there are two groups of people interested in human motion; athletes and coaches who seek to optimize performance and avoid injury, and physicians and therapists who seek to identify abnormalities in motion and determine corrective actions [2]. Motion analysis techniques to describe human motion have been around for centuries with the first reported study of human movement by Borelli in 1680 [39]. There are a number of motion analysis techniques that have been developed to measure the joint motion of the human body since Borelli’s work. Modern motion analysis techniques are based on the works of Muybridge in 1887 who published a photo series of walking to provide an observational description of human gait [39]. These techniques include the use of accelerometers, film digitization, and optoelectronic motion analysis. In 1973, the foundations for computerized optoelectric motion analysis were laid and provided the most quantifiable description of human motion [39]. The scope of this work has been limited to optoelectronic motion analysis.

Optoelectronic motion capture systems (OMC) track the three dimensional movements of a person in space using markers affixed to various bony landmarks [40]. These markers can be either passive, retro-reflective markers, plastic spheres typically covered with a tape containing glass prisms, that reflect infrared light back to motion cameras to track the movement of various body segments, or active markers. Active markers are LED diodes affixed to the subject that emit light and this light is tracked by the motion cameras [40]. The motion camera systems are capable of tracking the positions of these markers through a calibrated area. Computer programs can then use the positional data of the markers to derive the kinematic description of the motion occurring within the motion capture volume.
The calculations that are commonly used in optoelectronic motion analysis were published by Kadaba et al. [42]. In their work, they described, in detail, the methods of determining segment rotations based on a specific marker configuration. The marker system described by Kadaba et al. is one of the most commonly employed marker sets used in clinical gait analysis. Motion analysis models assume that each body segment is a rigid body whose relative rotations occur around a single point at the center of the joint [42]. Using a global coordinate system based on the configuration of a motion laboratory and local coordinate system defined by a given marker configuration, it is possible to described the motion of each joint using Euler rotation sequences [42]. Customarily the global coordinate system is defined as the x-axis along the long axis of a motion laboratory, the z-axis pointing in the vertical direction and the y-axis being perpendicular to the other two axes. The primary axis for the local coordinate systems is commonly defined as the axis in which the initial motion would rotate about [42]. To accurately describe the relative rotations of the body segments and joints, it is essential that the coordinate systems be accurately defined.

While motion analysis can provide a great deal of information about the motion of the human body, there are a number of drawbacks such as skin motion artifact, caused by the movement of the skin and underlying musculature that can introduce noise into the data [41]. Another drawback is the location of the markers themselves. Markers must be placed on specific bony landmarks, and deviations in these positions can create inaccurate data [41].

Motion analysis systems are used in a variety of disciplines that attempt to describe human motion including medical science and sports biomechanics [41]. In medical science, motion analysis is commonly used for both clinical research and clinical decision making. In both cases, markers are affixed to patients and the motion of a patient is then studied to determine the outcomes of surgical interventions or the progression of a disease state [2]. In sports biomechanics, the same systems are used with more complex marker sets and are used to track the motion of various sports activities, such as baseball pitching, jumping tasks, and spinning sports including figure skating and gymnastics, in an effort to improve performance and reduce injury [41].
A number of researchers have begun to use motion analysis techniques to study the motion of the upper extremity, which this is especially true in sports biomechanics. A major area of research in sport biomechanics, and an area in which this current work will be applied, is attempting to understand the motion of the upper extremity during baseball pitching. Over the past decade there has been an alarming increase in the number of pitching related injuries that require surgical intervention[4, 5]. Many researchers have turned to motion analysis techniques to attempt to better understand this motion in the hopes of reducing the risk of injury to this population.

The earliest papers that used optoelectronic motion capture methods focused on describing the kinematics of baseball pitching. Escamilla et al. studied the differences in joint kinematics of 26 collegiate level pitchers among various pitching types (i.e. fastball, curveball, slider, and change-up) [43]. They used computerized motion analysis to track the motion of the 26 pitchers while pitching within their lab space. Once the motion was tracked, they calculated the joint angles and joint angular velocities for the entire body for each pitcher with particular attention was placed on the shoulder and elbow [43]. Using this data, Escamilla et al. was able to compare the kinematics of each pitch type to determine if there were unique joint angles that could be used to describe the motion of each pitch type. The results of his work showed that there are minimal differences between kinematics between pitch types [43]. They concluded that this was an obvious finding since pitchers strive to minimize the visual differences in pitching mechanics so as not to indicate the type of pitch being thrown to the batters. Nissen et al. performed a similar study to that of Escamilla et al. in which they aimed to describe the pitching motion of adolescent pitchers [44]. In this study, Nissen et al. focused on describing the upper extremity joint kinematics of adolescent pitchers as well as describing the elbow and shoulder joint kinetics. This study allowed for future research to build upon these findings to better describe differences in joint kinetics as well as describing differences in joint kinematics based on different modeling techniques [44].

Following the early descriptive papers, many researchers began to focus on determining how various pitch types could increase the risk of injury to this population. More researchers wanted to study
the effects of breaking pitches (i.e. curveballs and sliders) on joint kinetics, since coaches believed that throwing these pitches placed pitchers at an increased risk of injury. Fleisig et al. compared the shoulder and elbow kinetics among the fastball, curveball, slider, and change-up in collegiate level pitchers [45]. They used motion analysis techniques to study the collegiate pitchers and found that it was the fastball and slider and not the curveball that created the greatest joint stresses [45]. Fleisig et al. concluded that, based on these results, the curveball, when thrown properly, did not increase the risk of injury to collegiate level pitchers. Solomito et al. repeated Fleisig et al.’s study using 36 collegiate level pitchers to determine the differences in shoulder and elbow kinetics among the fastball, curveball, slider, and change-up [9]. Similar to Fleisig et al.’s results, Solomito et al. found that the fastball produced the greatest shoulder and elbow moments. Unlike Fleisig et al., they showed that the slider produced moments similar to that of the curveball and not the fastball as originally stated [9]. The author concluded that, once again, the curveball, when thrown properly, was not as dangerous as originally believed and also stated that the differences in results for the slider could be attributed to sample size difference, where Fleisig et al. had only six pitchers throw a slider while Solomito et al. had 20 [9]. A similar study was performed by Dun et al. in which he used motion analysis techniques to determine the joint kinetics of the fastball and curveball in 24 youth baseball pitchers [46]. The results of Dun et al. study were very similar to that of Fleisig et al. showing that once again the fastball produced the greatest stresses on the shoulder and elbow compared to the curveball. Dun et al. concluded that, if properly thrown, then it was safe for youth pitchers to throw a curveball [46]. Nissen et al. also looked at differences in shoulder and elbow moments when pitchers threw a fastball and curveball [7]. In their study, 32 high school level pitchers were asked to pitch both a curveball and a fastball. The results of Nissen et al.’s study showed that it was the fastball and not the curveball that created the greatest shoulder and elbow moments, which are consistent with the previously published findings [7].

Multiple studies have proven that it was not the type of pitch thrown that increased the risk of injury to the pitcher; therefore, many research teams began looking at the motion analysis data for more
causal relationships with the intentions of being able to suggest alternative pitching mechanics to help reduce injury. Fleisig et al. began using motion analysis techniques to describe variability in pitching mechanics between various skill levels of pitching [47]. In this study, Fleisig et al. studied repeated fastball pitches thrown by youth, adolescent, collegiate, minor league, and major league pitchers to try and understand if variations lead to increased injury risk. Fleisig et al.’s results focused on elbow flexion, shoulder maximum external rotation, and trunk rotation and he found that, as a pitchers skill level increased, the variability between pitches decreased [47]. Fleisig et al. concluded that, although within pitcher variation decreased, there were no substantial changes in joint kinetics, leading to their theory that much of the pitching injuries were caused by overuse and not attributable to a single mechanical flaw [47].

Escamilla et al. attempted to look at how fatigue could increase the risk of injury to a baseball pitcher [48]. In this study, they had 10 college level pitchers pitch a simulated game (i.e., 105 to 135 pitches) in his motion analysis laboratory and collected joint kinematics and kinetics of pitches early in the collection period, toward the middle of the collection, and at the completion of the simulated game. Escamilla et al.’s results showed that, as fatigue set in, the ball velocity decreased and there was less trunk tilt; however, there were no other statistically significant differences found [48]. This led Escamilla to conclude that fatigue may not contribute to injury in college aged pitchers.

Aguinaldo et al. used previously collected motion analysis data to determine if certain pitching motions were associated with increased elbow moments [49]. In this work, they showed that some variables, such as early trunk rotation, increased the elbow joint moment and, therefore, was a potential factor that could increase injury risk to pitchers [49]. They theorized that, by training pitchers to remain closed to home plate longer and begin their trunk rotation after initial foot contact, it would be possible to reduce the risk of injury to some pitchers.
Matsuo et al. attempted to use motion analysis data collected from pitchers, along with optimization methods, to determine the best position for a pitcher’s trunk and shoulder when pitching to reduce injury to the elbow [50]. In this study, they looked specifically at motion analysis data collected from college level pitchers and varied the vertical abduction of the shoulder as well as lateral trunk lean of the pitcher to determine the effects of these two variables on the elbow varus moment. Matsuo et al. found that, as trunk lean away from the pitching arm increased, so did the elbow varus moment [50]. Using his optimization protocol, Matsuo et al. found that, with 10° of trunk lean away from the pitching arm along with 100° of vertical adduction of the shoulder, a pitcher could maintain an age appropriate ball velocity and minimize their elbow joint moment [50]. Matsuo et al. is the only author to have attempted any type of optimization or simulation of pitching mechanics to try and reduce the risk of injury to this population.

Using motion analysis techniques along with Lagrangian mechanics will provide an improved method of understanding how certain pitching mechanics could potentially lead to injury. The equations developed using Lagrange equations can be used to easily analyze the system by varying specific parameters to test the outcomes of the dynamic system. In this way, it is possible to determine how specific differences in pitching mechanics can lead to injury and, more importantly, by creating pitcher specific models, it may be possible to develop individualized coaching plans specific to the findings of a motion based assessment to improve performance while reducing the risk of injury.
3. Pilot Work

Pilot work on this topic included the derivation and exploration of the equations of motion for a simple spring pendulum. The spring pendulum was chosen for a number of reasons. First, the dynamics of the system are well known and using the spring pendulum as a means of introducing the Lagrangian approach was an appropriate first step. Second, the spring pendulum can also serve as an analogue for a passive muscle model in which only a singular resistive element is modeled. This passive model can then easily be expanded to an active muscle model similar to the one described later in this work. The pilot work consisted of deriving the equations of motion using the Lagrangian approach and then using these equations to predict the angular and radial position of the pendulum given specified initial conditions. These equations were validated through the collection of actual data using a motion capture system to record the motion of a spring pendulum, which was compared to the theoretical model. Finally, the initial conditions were varied to determine how the changes would influence the output of the system and to show the utility of Lagrangian-based equations.

The spring pendulum model illustrated in Figure 19 was used in the pilot work and it was assumed that the system was constrained to move within the $r$-$\theta$ plane only, where the motion of the mass is limited to the radial direction, $r$, or the change of length of the pendulum, and the transverse direction, $\theta$, or the angular position of the pendulum.
The coordinate system for the spring pendulum, shown in Figure 19, was placed at the fixed end of the pendulum as denoted by point A. The primary axis for the system, $A_1$, was defined along the axis of the spring and always points through the center of gravity of the system, CG. The second axis, $A_2$, was defined as a vector orthogonal to axis $A_1$. The spring has a natural length of $L$ and an external gravitational force, $g$, was included that acted in the negative vertical direction. In this model, the mass of the spring was neglected.

The potential energy of the system was solved for both the spring and the gravitational field as given by Equations 24 and 25, respectively.

\[
V_s = \frac{1}{2} k(r - L)^2, \quad \text{Eq. 24}
\]

\[
V_g = mgr \cos \theta, \quad \text{Eq. 25}
\]

where, $V_r$ and $V_s$ are the resultant potential energy for the spring and gravitational field, respectively, $m$ is the mass of the pendulum, $r$ is the total length of the pendulum, $\theta$ is the angular position of the pendulum, $g$ is the gravitational force constant (9.81 m/s$^2$), $L$ is the natural length of the spring, and $k$ is the spring constant. Combining Equations 24 and 25 provides the total potential energy of the system as shown in equation 26.
Next, the kinetic energy of the system was determined as shown in Equation 27,

\[ T = \frac{1}{2} m v^2, \]  

Eq. 27

where \( T \) is the resultant kinetic energy, \( m \) is the mass of the pendulum, and \( v \) is the velocity of the system given by Equation 28.

\[ v = \dot{r}_i + r \dot{\theta}_j, \]  

Eq. 28

where \( v \) is the velocity of the system, \( \dot{r}_i \) is the radial velocity of the system along the vector \( A_1 \) and is caused by the motion of the spring, and \( r \dot{\theta}_j \) is the velocity along the \( A_2 \) vector and describes the angular velocity of the system. By substituting Equation 28 into Equation 27, the kinetic energy for the system is provided as is shown in Equation 29,

\[ T = \frac{1}{2} m [\dot{r}^2 + (r \dot{\theta})^2]. \]  

Eq. 29

The Lagrangian for the system was then created by combining Equations 26 and 29 as shown in Equation 30,

\[ L = \frac{1}{2} m [\dot{r}^2 + (r \dot{\theta})^2] + m g r \cos \theta - \frac{1}{2} k (r - L)^2. \]  

Eq. 30

Applying Lagrange’s equation for the two degrees of freedom of the system yielded the equations of motion for the radial and angular components as seen in Equations 31 and 32, respectively,

\[ m \ddot{r} - m r \ddot{\theta} - m g \cos \theta + k (r - l) = 0, \]  

Eq. 31

\[ \ddot{\theta} m r^2 + 2 m r \dot{\theta} + m g r \sin \theta = 0. \]  

Eq. 32

These equations were then simplified to yield the system of equations describing the motion of the system as shown in Equations 33 and 34,

\[ \ddot{r} - r \dot{\theta}^2 = g \cos \theta - \frac{k}{m} (r - L), \]  

Eq. 33

\[ \ddot{\theta} r + 2 \dot{\theta} \dot{r} = g \sin \theta. \]  

Eq. 34
where $\ddot{r}$ is the radial acceleration of the system in the r direction, $r \dot{\theta}^2$ is the tangential acceleration of the pendulum, and $\ddot{\theta}$ is the angular acceleration. The resultant equations provided by Equations 33 and 34 describe the ideal conditions for the spring pendulum. They neglect the presence of friction in the spring, air resistance, and general dissipation of motion. Inclusion of these dissipative terms allows for a more realistic description of the physical behavior of the system. To account for these dissipative forces, Rayleigh’s dissipation function was used and is given by Equation 35,

$$\mathcal{F} = \frac{1}{2} c \dot{q}_i^2,$$

where $c$ is a dampening coefficient and $\dot{q}_i$ is the velocity of the generalized coordinate that is being dampered. In the case of the spring pendulum described above, the major source of dissipation is caused by the frictional forces within the spring. Therefore, the Rayleigh dissipation function was applied to only the radial direction, $r$, and is given by equation 36,

$$\mathcal{F} = \frac{1}{2} c \dot{r}^2,$$

where $c$ is the dampening coefficient and $\dot{r}$ is the radial velocity to the system. Rayleigh’s dissipation function can be combined and included into Lagrange’s Equation by combining Equations 17 and 35 as shown in equation 37,

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \frac{\partial \mathcal{F}}{\partial \dot{q}_i} = Q_i.$$

Applying this form of Lagrange’s equation to Equation 30 and Equation 37 yielded the final equations describing the motion of the system shown in Equations 38 and 39. Since the dissipation function was only applied in the radial direction, there was no change in the equation governing the angular direction, therefore, Equations 34 and 39 are identical:

$$\ddot{r} - r \ddot{\theta}^2 = g \cos \theta - \frac{k}{m} (r - L) + \frac{c}{m} \dot{r},$$

Eq. 38

$$\dot{\theta} r + 2 \dot{\theta} \dot{r} = g \sin \theta.$$

Eq. 39
The resulting equations account for a more realistic description of the motion of the pendulum.

To test the output of the theoretical spring pendulum model detailed above, an actual spring pendulum was constructed as shown in Figure 20. The pendulum was constructed using a weighted bracket, 25 cm of string (allowing a total length of 10 cm for movement), a series of brass plumb bobs ranging in weight between 0.25 kg and 1.7 kg, and a 15 cm spring scale.

![Figure 20: Picture of the completed spring pendulum](image)

Motion data was collected for the spring pendulum described above using a 12-camera Vicon MX Motion capture system (Vicon Motion Systems, Los Angeles, CA) sampling at 120 Hz. A total of four retro-reflective markers were placed on the pendulum in specific locations to describe both its radial and angular motion as seen in Figure 21.
The results of the comparison between the theoretical model and the actual data collected indicated good agreement between the models, with a correlation coefficient of 0.96 for the angular position and 0.85 for the radial position, as seen in Figure 22.
The largest differences noted were that the model slightly over estimated the upswing of the pendulum (i.e., negative angle) and under estimated the backswing (i.e., positive angle). Also of note is that the theoretical model showed a consistent reduction of the angle from the initial start point to the final angle measure. The actual behavior of the pendulum, however, has a slightly inconsistent dampening effect in which a later maximum angle is slightly greater than the preceding one. This behavior could be attributed to the spring scale used to construct the pendulum. The initial instability of the system noted in the first two seconds of the trial for the radial position was somewhat exaggerated in the theoretical data. Also of note was the presence of a slight catch in the waveform of the actual data when the spring reaches its maximum length that is not present in the Lagrangian based model. This can be attributed to a limitation in the model in that the actual behavior of the spring scale was not fully described by the model. The spring scale has a physical block that will not allow the spring to extend beyond the radial force associated with 2.0 kg. Given that the mass of this trial was near the maximum scale weight, it was possible that the radial force when the pendulum swings to its most extended position during the upswing exceeds this limit and therefore causes an abrupt catch in the radial position of the pendulum. Since the
Lagrangian model did not include a scenario in which a force threshold could be exceeded, it produced a smooth motion for the radial position of the pendulum. It was this difference that is the primary cause of the low agreement.

The exploration of the model provided results that were expected in that increased mass created increased radial motion and a reduction in angular motion, where an increase in the spring constant produced the opposite effect. While the information here was not new or unexpected, it does prove that the Lagrangian method produced equations that could accurately represent the behavior of the system.

The pilot work described here provided a foundation for deriving the equations of motion for a system and how those equations could then be manipulated to better understand the behavior of the system. While the system described here is simpler than what will be discussed throughout the remainder of this work, it is important to note that the methods discussed here can easily be extended to the system required to describe upper extremity motion. Furthermore, the equations describing the movement of the upper extremity that are developed in this work can also be used to address how changes in movement could cause reductions in joint moments. The equations can also address the moments caused by the muscles that could then be used to reduce injury risk. In the same way an individual could take the equations governing the spring pendulum and derive specified motion profiles by modifying different parameters of the system.
4. Methods

4.1 Lagrange Modeling and Derivations

4.1.1 Lagrange Model

The model developed for this work, shown in Figure 23, was a four-segment model describing the upper extremity and trunk. The first segment describes the trunk, while the remaining three segments describe the upper extremity including the upper arm, lower arm, and hand. Each segment was assumed to be a rigid body with fixed heights and radii that can be entered by the user into a custom-written MATLAB (MathWorks, Natick, MA) program. This allowed for the model to be appropriately scaled for the individuals being modeled. The four rigid bodies were connected using three joints to represent the shoulder, elbow, and wrist. The resulting model has 13 degrees of freedom, where a total of six degrees of freedom describe the movement of the trunk segment (i.e., three rotational degrees of freedom and three linear degrees of freedom), the glenohumeral joint was defined as a ball and socket joint (i.e., three rotational degrees of freedom), the elbow joint was defined as a hinge joint (i.e., a singular rotational degree of freedom), and the wrist joint was also modeled as a ball and socket joint (i.e., three degrees of rotational freedom). Five muscle elements were also included in this model and these elements were used to estimate muscle forces from specified kinematic patterns, where, in future work, they could be used to drive the kinematic patterns of the model using forward dynamic techniques. The muscle elements were modeled using a three-element model to better represent the actions of skeletal muscle as previously described by Peterson and Adrezin and illustrated in Figure 24 [25]. The inclusion of the muscles into the model increases the degrees of freedom for the entire system to 18 as each muscle element adds an additional degree of freedom.
The trunk segment was described as a cylinder of height, $L_1$, and radius, $L_2$. The coordinate system for the trunk segment was established at point A, which is located in the center of the base of the
cylinder, and was defined in such a way that vector $A_1$ points out of the page, vector $A_3$ was defined so
the vector always passes through the center of gravity of the trunk segment, $G_T$, and vector $A_2$ was
perpendicular to both vectors $A_1$ and $A_3$. The upper arm segment was also assumed to be a cylinder of
length, $L4$, and radius, $L3$. The coordinate system for the upper arm segment was located at point $C$ at the
center of the proximal portion of the upper arm cylinder and was defined in such a way that vector $C_1$
points out of the page, vector $C_2$ was defined in such a way as to always pass through the center of gravity
of the upper arm segment, $G_S$, and vector $C_3$ was perpendicular to both vector $C_1$ and $C_2$. The lower arm
segment was also modeled as a cylinder of length, $L5$, and radius, $L6$. The coordinate system for the
lower arm segment was located at point $D$ at the center of the proximal portion of the lower arm cylinder.
The coordinate system was defined so that vector $D_2$ points out of the page, vector $D_3$ always passes
through the center of gravity, $G_L$, for the lower arm segment, and vector $D_1$ was perpendicular to both
vector $D_2$ and $D_3$. Finally, the hand segment was similarly defined using a cylinder of length, $L7$, and
radius, $L8$. The coordinate system for the hand was located at point $E$ in the center of the proximal end of
the hand segment. Similar to that of the lower arm, vector $E_2$ points out of the page, $E_3$ was defined so
that it always passes through the center of gravity, $G_W$, and vector $E_1$ was defined as being perpendicular
to both $E_2$ and $E_3$ vectors.

The five muscle elements included in this work were meant to serve as analogues to three of the
muscles activating the glenohumeral joint (i.e, Pectorialis Major, Deltoid, and Infraspinatus). The
remaining two muscles were included to mimic the Triceps and Biceps acting on the elbow joint. The
five muscle elements each contain three elements connected in parallel with one another. Two of the
elements were spring elements that were meant to mimic the passive resistance and elasticity of the
muscle and the third element was a dashpot that was meant to mimic the active resistance encountered
during a muscle contraction. The locations of each of the five muscles were meant to mimic the
anatomical origin and insertion points for each of the muscles, which were previously described in the
background section of this work. The muscle lengths were calculated based on the joint angle, the length
of the segment, and adjacent segment, and the estimated insertion point of the muscle in relationship to
the rotation center of the joint, as shown in Figure 25. Using the law of cosines, as described by Equation 40, it is possible to calculate the muscle lengths as a function of the joint angle,

\[
L = \sqrt{A^2 + B^2 - AB \cos(\theta)},
\]

where \(L\) is the length of the muscle, \(A\) is the length of the adjacent segment, \(B\) is the distance between the muscle insertion and the rotation center of the joint, and \(\theta\) is the angle of the joint.

![Figure 25: Geometric system describing muscle attachments (Black lines are the segments and red and blue lines represent muscles)](image)

The muscle lengths were then compared to the resting length of the muscle as described in the literature [51] and Table 1 refers to the resting muscle lengths and muscle cross sectional areas for the muscles of interest in this dissertation. The spring constants and dampening coefficients were determined through experimental iteration during the validation testing described later in this section.

<table>
<thead>
<tr>
<th>Muscle Characteristics</th>
<th>Cross Sectional Area (cm(^2))</th>
<th>Resting Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pectoralis Major</td>
<td>15.9 ± 8.3</td>
<td>20.2 ± 2.2</td>
</tr>
<tr>
<td>Deltoid</td>
<td>25.0 ± 8.7</td>
<td>18.1 ± 1.8</td>
</tr>
<tr>
<td>Infraspinatus</td>
<td>11.9 ± 4.2</td>
<td>14.0 ± 1.0</td>
</tr>
<tr>
<td>Biceps</td>
<td>8.2 ± 3.4</td>
<td>27.0 ± 2.6</td>
</tr>
<tr>
<td>Triceps</td>
<td>40.0 ± 15.4</td>
<td>27.0 ± 3.2</td>
</tr>
</tbody>
</table>

The trunk coordinate system was allowed to freely rotate within space and Euler angles were used to describe the orientation of the trunk segment in relation to the fixed global coordinate system. In this
case, an YXZ Euler rotation sequence, as shown in Equation 41, was chosen to describe the motion of the trunk segment and to mimic the rotation sequence used in the motion capture based pitching model previously described.

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3
\end{bmatrix} = \begin{bmatrix}
C\theta C\psi + S\theta S\phi S\psi & C\phi S\psi & -S\theta C\phi + C\theta S\phi S\psi \\
-C\theta S\psi + S\theta S\phi C\psi & C\phi C\psi & S\theta S\psi + C\theta S\phi C\psi \\
S\theta C\phi & -S\phi & C\theta C\phi
\end{bmatrix}
\begin{bmatrix}
R_x \\
R_y \\
R_z
\end{bmatrix}, \text{ Eq. 41}
\]

where \(R_x, R_y,\) and \(R_z\) are the \(X, Y,\) and \(Z\) coordinates of the reference coordinate system, respectively, \(C\) is cosine, \(S\) is sine, \(\theta\) is a rotation about the \(x\) axis, \(\phi\) is a rotation about the \(y\)-axis, \(\psi\) is a rotation about the \(z\) axis, and \(T_1, T_2,\) and \(T_3\) are the coordinates of the transformed coordinate system corresponding to the \(x, y\) and \(z\) axes, respectively. For the trunk, a rotation about the \(A_2\) axis corresponds to sagittal plane motion of the trunk (i.e., anterior and posterior tilt). A rotation about the \(A_1\) axis corresponds to coronal plane motion of the trunk, which describes the lateral lean of the trunk segment while a rotation about the \(A_3\) axis corresponds to axial rotation of the trunk.

The upper arm segment coordinate system uses the trunk as its reference and the orientation of the segment is also described using Euler angles. The upper arm segment utilizes a XYZ Euler rotation sequence as shown in Equation 42, which was chosen to most closely mimic the rotation sequences used in the motion capture pitching model.

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3
\end{bmatrix} = \begin{bmatrix}
C\psi C\phi & S\theta S\phi C\psi - C\theta S\psi & C\theta S\phi C\psi + S\theta S\psi \\
C\phi S\psi & S\theta S\phi S\psi + C\theta C\psi & C\theta S\phi S\psi - S\theta C\psi \\
-S\phi & C\phi S\theta & C\phi C\theta
\end{bmatrix}
\begin{bmatrix}
R_x \\
R_y \\
R_z
\end{bmatrix}, \text{ Eq. 42}
\]

where \(R_x, R_y,\) and \(R_z\) are the \(X, Y,\) and \(Z\) coordinates of the reference coordinate system, respectively, \(C\) is cosine, \(S\) is sine, \(\theta\) is a rotation about the \(x\) axis, \(\phi\) is a rotation about the \(y\)-axis, \(\psi\) is a rotation about the \(z\) axis, and \(T_1, T_2,\) and \(T_3\) are the coordinate of the transformed coordinate system corresponding to the \(x, y\) and \(z\) axes, respectively. For the upper arm, a rotation about the \(C_1\) axis corresponds to coronal plane motion of the glenohumeral joint, which describes vertical abduction and adduction of the upper arm segment. A rotation about the \(C_2\) axis corresponds to sagittal plane motion of the glenohumeral joint.
which describes the internal and external rotation of the upper arm segment, while a rotation about the $C_3$ axis corresponds to transverse plane motion of the glenohumeral joint, which corresponds to horizontal abduction, and adduction of the upper arm segment.

The coordinate system of the lower arm segment uses the upper arm segment as its reference and the orientation of the lower arm segment is defined using an YXZ rotation sequence identical to that described for the trunk segment. Although the lower arm segment uses the YXZ rotation sequence, the X and Z axes are fixed as the joint only contributes sagittal plane motion. In future versions of this model, this joint could easily be extended to allow limited motion in the other two planes. Based on these definitions, a rotation about the $D_2$ axis describes the flexion and extension of the elbow joint.

Finally, the hand segment coordinate system uses the same YXZ rotation sequence of the lower arm segment and trunk segments with the major difference being the lower arm segment as the reference for the hand segment. For the hand segment, a rotation about the $E_1$ axis corresponds to coronal plane motion, which describes wrist ulnar and radial deviation. A rotation about the $E_2$ axis corresponds to sagittal plane motion describing the flexion and extension of the wrist joint and, finally, a rotation about the $E_3$ axis corresponds to transverse plane motion describing forearm supination and pronation. The wrist joint was chosen to describe the forearm rotation as a greater amount of forearm motion comes from the motion of the wrist than it does with the rotation of the humeral head against the ulna and radius.

Prior to developing the Lagrangian, the angular velocities for each segment were determined to facilitate the subsequent calculations. Since Euler angles were used to describe the motion of each segment, the angular velocities had to be determined by taking into account the angular displacement of the other axes [26-28]. Therefore the angular velocity associated with an YXZ rotation is provided by Equations 43a, 43b, and 43c, and the angular velocities associated with a XYZ rotation sequence is provided by Equations 44a, 44b, and 44c.

$$\omega_1 = \dot{\phi}C\theta + \dot{\psi}S\theta C\phi,$$  \hspace{1cm} Eq. 43a

$$\omega_2 = \dot{\theta} - \dot{\psi}S\phi,$$  \hspace{1cm} Eq. 43b
\[ \omega_3 = -\dot{\phi}C\theta + \dot{\psi}C\theta C\phi, \quad \text{Eq. 43c} \]
\[ \omega_1 = \dot{\theta} + \dot{\psi}S\phi, \quad \text{Eq. 44a} \]
\[ \omega_2 = \dot{\phi}C\theta - \dot{\psi}S\theta C\phi, \quad \text{Eq. 44b} \]
\[ \omega_3 = \dot{\phi}S\theta + \dot{\psi}C\theta C\phi, \quad \text{Eq. 44c} \]

where \( \omega_1, \omega_2, \) and \( \omega_3 \) correspond to the resulting total angular velocities about the X, Y, and Z axes, respectively, \( C \) is cosine, \( S \) is sine, \( \theta \) is the angular displacement about the X axis, \( \phi \) is the angular displacement about the Y axis, \( \dot{\theta} \) is the time rate of change of the angular displacement about the X axis, \( \dot{\phi} \) is the time rate of change of the angular displacement about the Y axis, and \( \dot{\psi} \) is the time rate of change of the angular displacement about the Z-axis. Following the calculations of the angular velocities, they were squared, which, again, was done to facilitate the subsequent calculations of kinetic energy for each of the segments. The squared angular velocity for the YXZ rotation are shown in Equations 45a, 45b and 45c, and those for the XYZ rotation are shown in Equations 46a, 46b, 46c,

\[ \omega_1^2 = \dot{\phi}^2 C^2 \theta + 2\dot{\phi}\ddot{\phi}C\theta C\phi + \dot{\psi}^2 S^2 \theta C^2 \phi, \quad \text{Eq. 45a} \]
\[ \omega_2^2 = \dot{\theta}^2 - 2\dot{\theta}\ddot{\theta}S\phi + \dot{\psi}^2 S^2 \phi, \quad \text{Eq. 45b} \]
\[ \omega_3^2 = \dot{\phi}^2 S^2 \theta - 2\dot{\phi}\ddot{\phi}C\theta S\theta + \dot{\psi}^2 C^2 \theta C^2 \phi, \quad \text{Eq. 45c} \]
\[ \omega_1^2 = \dot{\theta}^2 + 2\dot{\theta}\ddot{\theta}S\phi + \dot{\psi}^2 S^2 \phi, \quad \text{Eq. 46a} \]
\[ \omega_2^2 = \dot{\phi}^2 C^2 \theta - 2\dot{\phi}\ddot{\phi}C\theta C\phi S\theta + \dot{\psi}^2 S^2 \theta C^2 \phi, \quad \text{Eq. 46b} \]
\[ \omega_3^2 = \dot{\phi}^2 S^2 \theta + 2\dot{\phi}\ddot{\phi}S\theta C\theta C\phi + \dot{\psi}^2 C^2 \theta C^2 \phi. \quad \text{Eq. 46c} \]

4.1.2 Derivation of the Equations of Motion

It is important to note, prior to the derivation of the model that the primary assumption for the model is that it is holonomic in nature and, therefore, there is one generalized coordinate to describe each degree of freedom. To derive the equations of motion, the Lagrangian must be constructed. The first step in constructing the Lagrangian was to determine the potential energy for the system, which required that the potential energy for each segment be defined first. The potential energy of the trunk segment, \( V_T \), was
taken at the center of gravity of the trunk segment. It was assumed that the height of the trunk segment’s center of gravity was only influenced by the lateral shifting and the anterior/posterior movement of the trunk and was not influenced by the axial rotation of the trunk. Therefore, the potential energy for the trunk segment is given by Equation 47,

\[ V_T = -\frac{1}{2} m_T g L_1 C \theta_T C \phi_T, \]  \hspace{1cm} \text{Eq. 47}

where \( V_T \) is the potential energy of the trunk, \( L_1 \) is the height of the trunk segment in meters, \( m_T \) is the mass of the trunk segment, \( g \) is the acceleration due to gravity, \( C \) is cosine, and \( \theta_T \) and \( \phi_T \) are the angular displacements of the trunk about the A1 and A2 axes, respectively. The potential energy of the upper arm was assumed to be influenced by the position of the trunk segment as well as by the vertical abduction and adduction of the glenohumeral joint. Therefore, the potential energy of the upper arm segment is given by Equation 48,

\[ V_S = -m_T g L_1 C \theta_T C \phi_T - \frac{1}{2} m_S g L_4 S \theta_S, \]  \hspace{1cm} \text{Eq. 48}

where \( V_S \) is the potential energy of the upper arm segment, \( L_1 \) is the length of the trunk segment, \( L_4 \) is the length of the upper arm segment, \( \theta_T \) and \( \phi_T \) are the angular displacements of the trunk in the coronal and sagittal planes, respectively, \( \theta_S \) is the angular displacement of the upper arm segment in the coronal plane, \( m_T \) and \( m_S \) are the masses of the trunk segment and upper arm segment, respectively, and \( g \) is the acceleration due to gravity. The potential energy of the lower arm segment was assumed to be influenced by the position of the two preceding segments as well as by the sagittal plane motion of the lower arm segment. The potential energy of the lower arm segment is given by Equation 49,

\[ V_L = -m_T g L_1 C \theta_T C \phi_T - m_S g L_4 S \theta_S - \frac{1}{2} m_L g L_5 S \phi_L, \]  \hspace{1cm} \text{Eq. 49}

where \( V_L \) is the potential energy of the lower arm segment, \( L_5 \) was the length of the lower arm segment, \( \phi_L \) is the angular displacement of the lower arm segment in the sagittal plane, \( m_L \) is the mass of the lower arm segment, and the other variables are as previously described. Finally, the potential energy of the
hand segment was calculated. Only the sagittal plane motion of the wrist and the previous three segments influenced the potential energy for this segment. Therefore, the potential energy for the hand segment is given by Equation 50,

\[ V_w = -m_T g L_1 C \theta_T C \phi_T - m_5 g L_4 S \theta_5 - m_6 g L_5 S \phi_L - \frac{1}{2} m_w g L_7 S \phi_w , \]  

Eq.50

where \( V_w \) is the potential energy of the hand segment, \( L_7 \) is the length of the hand segment, \( \phi_w \) is the angular displacement of the hand segment in the sagittal plane, \( m_w \) is the mass of the hand segment and the remaining variables are as previously defined. Finally, the summation of the Equations 47, 48, 49, and 50 provided the total potential energy for the system without the muscle elements. The total completed potential energy is given by Equation 51,

\[ V = \frac{1}{2} g (-7m_T g L_1 C \theta_T C \phi_T - 5m_5 g L_4 S \theta_5 - 3m_6 g L_5 S \phi_L - m_w g L_7 S \phi_w ) . \]  

Eq.51

The inclusion of the muscle elements provides additional terms to the potential energy equation as the resistive elements and the springs also provide an additional potential term due to their ability to store energy. Therefore, each of the potential terms for the muscle elements follow a similar equation to that shown in Equation 52,

\[ V = \frac{K_1 + K_2}{2} (E \theta - L) , \]  

Eq. 52

where \( K1 \) and \( K2 \) are the spring constants for the two spring elements, \( L \) is the resting length for the muscle element, \( E \) is the length of the muscle at any given time, and \( \theta \) is the joint angle that is associated with the muscle element. Including the potential terms for each of the five muscle elements into Equation 51 provides the total potential energy for the system as given by Equation 53,

\[ V = \frac{1}{2} g (-7m_T g L_1 C \theta_T C \phi_T - 5m_5 g L_4 S \theta_5 - 3m_6 g L_5 S \phi_L - m_w g L_7 S \phi_w ) + \frac{K_{T1} + K_{T2}}{2} (E_T C \phi_L - L_T) + \frac{K_{P1} + K_{P2}}{2} (E_P C \phi_S C \psi_S - L_P) + \frac{K_{D1} + K_{D2}}{2} (E_D C \psi_S C \theta_5 - L_D) + \frac{K_{B1} + K_{B2}}{2} (E_B C \phi_L - L_B) . \]  

Eq. 53

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Following the determination of the potential energy for the system the kinetic energy for each of the four segments was solved for using Equation 54,

$$\ T = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2, $$  \hspace{1cm} \text{Eq. 54}

where $T$ is kinetic energy, $\omega$ is the angular velocity of the segment, $I$ is the mass moment of inertia of the segment, $v$ is the velocity of the segment, and $m$ is the mass of the segment. Given that each of the segments are represented by solid cylinders, the mass moment for each segment are very similar and can be found in Table 2.

<table>
<thead>
<tr>
<th>Segment</th>
<th>$I_x$</th>
<th>$I_y$</th>
<th>$I_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trunk</td>
<td>$I_{T1} = \frac{1}{12} m_T (3L2^2 + L1^2)$</td>
<td>$I_{T1} = \frac{1}{12} m_T (3L2^2 + L1^2)$</td>
<td>$I_{T3} = \frac{1}{2} m_T L2^2$</td>
</tr>
<tr>
<td>Upper Arm</td>
<td>$I_{U1} = \frac{1}{12} m_S (3L3^2 + L4^2)$</td>
<td>$I_{U2} = \frac{1}{2} m_S L3^2$</td>
<td>$I_{U3} = \frac{1}{12} m_S (3L3^2 + L4^2)$</td>
</tr>
<tr>
<td>Lower Arm</td>
<td>$I_{L1} = \frac{1}{12} m_L (3L6^2 + L5^2)$</td>
<td>$I_{L1} = \frac{1}{12} m_L (3L6^2 + L5^2)$</td>
<td>$I_{L3} = \frac{1}{2} m_L L6^2$</td>
</tr>
<tr>
<td>Hand</td>
<td>$I_{w1} = \frac{1}{12} m_w (3L8^2 + L7^2)$</td>
<td>$I_{w1} = \frac{1}{12} m_w (3L8^2 + L7^2)$</td>
<td>$I_{w3} = \frac{1}{2} m_w L8^2$</td>
</tr>
</tbody>
</table>

In Table 2, $I_{T1}$, $I_{T2}$, and $I_{T3}$ are the mass moments of inertia about the $A_1$, $A_2$, and $A_3$ axes of the trunk, respectively, $I_{U1}$, $I_{U2}$, and $I_{U3}$ are the mass moments of inertia about the $C_1$, $C_2$, and $C_3$ axes of the upper arm, respectively, $I_{L1}$, $I_{L2}$, and $I_{L3}$ are the mass moments of inertia about the $D_1$, $D_2$, and $D_3$ axes of the lower arm, respectively, and $I_{w1}$, $I_{w2}$, and $I_{w3}$ are the mass moments of inertia about the $E_1$, $E_2$, and $E_3$ axes of the hand segment, respectively. $m_T$, $m_S$, $m_L$, and $m_w$ are the masses of the trunk segment, upper arm segment, lower arm segment, and hand segment, and the $L$ terms representing the heights and radii of each of the cylinders have all been previously defined. The velocity of each segment can be calculated using Equation 55,

$$\ V_{G_1} = \omega_{Se} \times R_{G_1fp} + V_p, $$  \hspace{1cm} \text{Eq.55}

Where $V_{G_1}$ is the velocity of a specified segment taken at the center of gravity of that segment, $\omega_{Se}$ is the angular velocity of the specified segment, $V_p$ is the reference point’s velocity given by Equation 56, and
$R_{G_jP}$ is the vector between the reference point and the center of mass for the segment of interest, which are given in Table 3.

$$V_p = \omega_{\text{Ref}} \times R_{\Omega/\alpha} + V_\alpha,$$  
Eq.56

where $\omega_{\text{Ref}}$ is the angular velocity of the specified segment vector crossed with $R_{\Omega/\alpha}$, which is between the reference point $\alpha$ and final point $\Omega$ for the segment of interest and given in Table 4, and $V_\alpha$ is the velocity at the reference point. The reference point for the trunk segment is point A, for the upper arm segment, point C, the lower arm segment, point D, and the hand segment, point E. All length terms, L, are as previously described.

Table 3: Vectors between reference point and center of mass for each segment

<table>
<thead>
<tr>
<th>Segment</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trunk</td>
<td>$\left( \frac{1}{2}L1 \right)_{A3}$</td>
</tr>
<tr>
<td>Upper Arm</td>
<td>$\left( \frac{1}{2}L4 \right)_{c2}$</td>
</tr>
<tr>
<td>Lower Arm</td>
<td>$\left( \frac{1}{2}L5 \right)_{d3}$</td>
</tr>
<tr>
<td>Hand</td>
<td>$\left( \frac{1}{2}L7 \right)_{e3}$</td>
</tr>
</tbody>
</table>

Table 4: Reference vectors between points of interest

<table>
<thead>
<tr>
<th>Reference</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{C/A}$</td>
<td>$L1_{A3} + L2_{A2}$</td>
</tr>
<tr>
<td>$R_{C/D}$</td>
<td>$L4_{c2}$</td>
</tr>
<tr>
<td>$R_{D/E}$</td>
<td>$L5_{d3}$</td>
</tr>
</tbody>
</table>

$V_p$ is the velocity taken at the reference point for each segment and is calculated using a similar equation to that shown in Equation 56. The only exception to this is that the velocity of point A is taken in reference to the global coordinate system and, therefore, is given by Equation 57. The velocities for each center of mass are given in Table 5.

$$V_A = \dot{X}_{A1} + \dot{Y}_{A2} + \dot{Z}_{A3},$$  
Eq. 57

where $\dot{X}, \dot{Y}, \dot{Z}$ were the linear velocities of point A in the $A_1, A_2,$ and $A_3$ directions, respectively.
Table 5: Velocities taken at the center of mass for each segment

<table>
<thead>
<tr>
<th>Center of Mass</th>
<th>Velocity at the Center of Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trunk</td>
<td>$V_{GT} = \frac{1}{2}L1\dot{\phi}_t - \frac{1}{2}L1\dot{\theta}_T + \dot{X}(C\theta_T C\psi_T + S\theta_T S\phi_T S\psi_T) + \dot{Y}C\phi_T C\psi_T + \dot{Z}C\theta_T C\phi_T$</td>
</tr>
<tr>
<td>Upper Arm</td>
<td>$V_{GU} = -\frac{1}{2}L4\dot{\psi}_s + \frac{1}{2}L4\dot{\phi}_s + (L1\dot{\phi}_T - L2\dot{\psi}_T)C\phi_s C\psi_s - L1\dot{\theta}_T(C\theta_s C\psi_s + S\theta_s S\phi_s S\psi_s) + L2\dot{\theta}_T C\theta_s C\phi_s + X\dot{C} \phi_s C\psi_s (C\theta_T C\psi_T + S\theta_T S\phi_T S\psi_T) + \dot{Y}C\phi_T C\psi_T (C\theta_s C\psi_s + S\theta_s S\phi_s S\psi_s) + \dot{Z}C\theta_T C\phi_T C\theta_s C\phi_s$</td>
</tr>
<tr>
<td>Lower Arm</td>
<td>$V_{GL} = \frac{1}{2}L5\dot{\phi}_L - \frac{1}{2}L5\dot{\theta}_L - L4\dot{\psi}_s (C\theta_L C\psi_L + S\theta_L S\phi_L S\psi_L) + L4\dot{\theta}_s C\theta_L C\phi_L + (L1\dot{\phi}_T - L2\dot{\psi}_T) C\phi_s C\psi_s (C\theta_L C\psi_L + S\theta_L S\phi_L S\psi_L) - L1\dot{\theta}_T (C\theta_s C\psi_s + S\theta_s S\phi_s S\psi_s) C\phi_L C\psi_L + L2\dot{\theta}_T C\theta_s C\phi_s C\theta_L C\phi_L + \dot{X}C\phi_s C\psi_s (C\theta_T C\psi_T + S\theta_T S\phi_T S\psi_T) (C\theta_L C\psi_L + S\theta_L S\phi_L S\psi_L) + \dot{Y}C\phi_T C\psi_T (C\theta_s C\psi_s + S\theta_s S\phi_s S\psi_s) C\phi_L C\psi_L + \dot{Z}C\theta_T C\phi_T C\theta_s C\phi_s C\theta_L C\phi_L$</td>
</tr>
<tr>
<td>Hand</td>
<td>$V_{GW} = \frac{1}{2}L7\dot{\psi}_w - \frac{1}{2}L7\dot{\theta}_w + L5\dot{\phi}_t (C\theta_w C\psi_w + S\theta_w S\phi_w S\psi_w) - \frac{1}{2}L5\dot{\phi}_L C\phi_w C\psi_w - L4\dot{\psi}_s (C\theta_L C\psi_L + S\theta_L S\phi_L S\psi_L) (C\theta_w C\psi_w + S\theta_w S\phi_w S\psi_w) + L4\dot{\theta}_s C\theta_L C\phi_w C\psi_w + (L1\dot{\phi}_T - L2\dot{\psi}_T) C\phi_s C\psi_s (C\theta_L C\psi_L + S\theta_L S\phi_L S\psi_L) (C\theta_w C\psi_w + S\theta_w S\phi_w S\psi_w) - L1\dot{\theta}_T (C\theta_s C\psi_s + S\theta_s S\phi_s S\psi_s) C\phi_L C\psi_L C\phi_w C\psi_w + L2\dot{\theta}_T C\theta_s C\phi_s C\theta_L C\phi_w C\psi_w + \dot{X}C\phi_s C\psi_s (C\theta_T C\psi_T + S\theta_T S\phi_T S\psi_T) (C\theta_L C\psi_L + S\theta_L S\phi_L S\psi_L) (C\theta_w C\psi_w + S\theta_w S\phi_w S\psi_w) + \dot{Y}C\phi_T C\psi_T (C\theta_s C\psi_s + S\theta_s S\phi_s S\psi_s) C\phi_L C\psi_L C\phi_w C\psi_w + \dot{Z}C\theta_T C\phi_T C\theta_s C\phi_s C\theta_L C\phi_w C\phi_L C\theta_w C\phi_w$</td>
</tr>
</tbody>
</table>

By substituting the Equations from Tables 2, 3 and 5, as well as Equation 55 into Equation 54, it is possible to determine the kinetic energy for each of the segments as listed in Table 6.
Table 6: Kinetic energy equations for each segment

<table>
<thead>
<tr>
<th>Segment</th>
<th>Kinetic Energy Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trunk</td>
<td>$T_T = \frac{1}{2} m_T A (\dot{\theta}_T^2 + \dot{\phi}_T^2) + \frac{1}{4} m_T L_2^2 \dot{\psi}_T^2 + \frac{1}{4} m_T (L_1 (\dot{\phi}_T - \dot{\theta}_T) + 2x(C\theta_T C\psi_T + S\theta_T S\phi_T S\psi_T) + 2\dot{y} C\phi_T C\psi_T + 2z C\theta_T C\phi_T)^2$</td>
</tr>
<tr>
<td>Upper Arm</td>
<td>$T_s = \frac{1}{2} m_s B (\dot{\theta}_s^2 + \dot{\phi}_s^2) + \frac{1}{4} m_s L_3^2 \dot{\psi}_s^2 + \frac{1}{4} m_s (L_4 (\dot{\theta}_s - \dot{\psi}_s) + 2L_1 (\dot{\phi}_T C\phi_s C\psi_s - \dot{\theta}_T C\theta_s C\psi_s - \dot{\phi}_T S\theta_s S\phi_s S\psi_s) + 2L_2 (-\psi_T C\phi_s C\psi_s + \dot{\theta}_T C\theta_s C\phi_s) + 2x(C\phi_s C\psi_s C\theta_T C\psi_T + C\phi_s C\psi_s S\theta_T S\phi_T S\psi_T) + 2\dot{y} C\psi_s C\phi_T C\psi_T + S\theta_s S\phi_s S\psi_s C\theta_T C\psi_T) + 2z C\theta_s C\phi_s C\theta_T C\phi_T)^2$</td>
</tr>
<tr>
<td>Lower Arm</td>
<td>$T_L = \frac{1}{2} m_L D (\dot{\phi}_L^2 + \dot{\psi}_L^2) + \frac{1}{4} m_L (L_5 \dot{\phi}_L + 2L_4 (\dot{\theta}_s C\phi_L - \dot{\psi}_s) + 2L_1 (-\dot{\theta}_T C\phi_L C\theta_s C\psi_s + \dot{\phi}_L C\theta_s C\phi_s C\psi_s - \dot{\phi}_T C\phi_s S\theta_s S\phi_s S\psi_s) + 2L_2 (-\psi_T C\phi_s C\psi_s + \dot{\theta}_T C\phi_L C\phi_s C\psi_s) + 2x(C\phi_s C\psi_s C\theta_T C\psi_T + C\phi_s C\psi_s S\theta_T S\phi_T S\psi_T) + 2\dot{y} (C\phi_s C\theta_s C\psi_T C\psi_T + S\theta_s S\phi_s S\psi_s C\theta_T C\psi_T) + 2\dot{z} C\phi_s C\psi_s C\phi_T C\psi_T) + 2z C\phi_L C\theta_s C\phi_s C\theta_T C\phi_T)^2$</td>
</tr>
<tr>
<td>Hand</td>
<td>$T_w = \frac{1}{24} m_w E (\dot{\phi}_w^2 + \dot{\psi}_w^2) + \frac{1}{4} m_w L_7^2 \dot{\psi}_w^2 + \frac{1}{4} m_w (L_7 (\dot{\phi}_w - \dot{\psi}_w) + 2L_5 (\dot{\phi}_L S\theta_s S\phi_w S\psi_w) + 2L_4 (\dot{\theta}_s C\phi_L C\theta_w C\psi_w - \dot{\psi}_s C\theta_w C\phi_w + S\theta_w S\phi_w S\psi_w) + 2L_1 (-\dot{\theta}_T C\phi_L C\theta_s C\psi_w + \dot{\phi}_L C\theta_s C\phi_s C\phi_w C\psi_w + \dot{\phi}_T (C\theta_s C\psi_s C\theta_w C\psi_w + C\phi_s C\psi_s S\theta_w S\phi_w S\psi_w) - \dot{\theta}_T C\phi_L S\theta_s S\phi_s S\psi_s C\phi_w C\psi_w) + 2L_2 (-\psi_T C\phi_s C\psi_s C\theta_T C\psi_T - \dot{\psi}_T C\phi_s C\theta_s S\phi_w S\psi_w + \dot{\theta}_T C\phi_L C\phi_s C\psi_s C\theta_w C\phi_w) + 2x(C\theta_w C\psi_w C\phi_s C\psi_s C\theta_T C\psi_T + C\phi_s C\psi_s S\theta_T S\phi_T S\psi_T C\theta_w C\phi_w + C\phi_s C\psi_s S\theta_s C\phi_T C\psi_T C\theta_T C\phi_w C\psi_w) + 2\dot{y} (C\phi_L C\phi_s C\psi_T C\psi_T C\theta_T C\phi_w C\psi_w + C\phi_L S\theta_T S\phi_s S\psi_s C\theta_T C\phi_w C\psi_w) + 2\dot{z} C\phi_L C\theta_s C\phi_s C\theta_T C\phi_T C\phi_w C\phi_T)^2$</td>
</tr>
</tbody>
</table>

Finally, the Lagrangian was created by summing each of the kinetic energy equations and then subtracting the sum of the potential energy equations as shown in Equation 58,

$$L = \Sigma T - \Sigma V,$$  Eq.58

where $T$ is the kinetic energy of each segment, $V$ is the potential energy for each segment, and $L$ is the Lagrangian. Therefore, the final Lagrangian is solved to become Equation 59,
\[ \mathcal{L} = \frac{1}{24} m_T A (\dot{\theta}_T^2 + \phi_T^2) + \frac{1}{24} m_s B (\dot{\theta}_s^2 + \dot{\psi}_s^2) + \frac{1}{24} m_l D (\dot{\phi}_l^2) + \frac{1}{24} m_w E (\dot{\theta}_w^2 + \dot{\phi}_w^2) + \frac{1}{4} (m_T L^2 \dot{\psi}_T^2 + m_s L^3 \dot{\phi}_s^2 + m_w L^3 \dot{\psi}_w^2 + \frac{1}{4} m_s (L_4 (\dot{\theta}_s - \dot{\psi}_s)) + 2L1 (\dot{\phi}_T C \phi_s C \psi_s - \dot{\theta}_T C \theta_s C \psi_s - \dot{\theta}_T \theta_s S \phi_s \psi_s) + 2L2 (-\dot{\psi}_T C \phi_s C \psi_s + \dot{\theta}_T \theta_s S \phi_s \psi_s) + 2\dot{x} (C \phi_s C \psi_s C \theta_T C \psi_T + C \phi_s \psi_s S \theta_T S \phi_T S \psi_T) + 2\dot{y} (C \phi_s C \psi_s C \theta_T C \psi_T + S \theta_s S \phi_s S \psi_s C \theta_T C \psi_T) + 2\dot{z} (C \phi_s C \psi_s C \theta_T C \psi_T + C \phi_s \psi_s S \theta_T S \phi_T S \psi_T) + 2\dot{z} (C \phi_s C \psi_s C \theta_s S \phi_s S \psi_s S \phi_T C \psi_T) + 2\dot{z} C \phi_s C \theta_s C \phi_T C \theta_T C \psi_T + 2\dot{z} C \phi_s \psi_s S \theta_T S \phi_T S \psi_T)^2 + \frac{1}{4} m_w (L_5 (\dot{\phi}_w - \dot{\psi}_w)) + 2L5 (\dot{\phi}_L S \theta_w S \phi_w S \psi_w) + 2L4 (\dot{\theta}_s C \phi_l C \theta_w \phi_w - \dot{\psi}_s (C \theta_w C \psi_w + S \theta_w S \phi_w S \psi_w)) + 2L1 (-\dot{\theta}_T C \phi_l C \theta_s C \phi_s C \psi_w + \dot{\phi}_T (C \theta_s C \psi_s C \theta_T C \psi_T + C \phi_s C \theta_s S \psi_s S \phi_s F \psi_w + \dot{\theta}_T \theta_s S \phi_s S \psi_s S \theta_T S \phi_T S \psi_T) + 2\dot{x} (C \theta_w C \psi_w C \phi_s C \psi_s C \theta_T C \psi_T - C \phi_s C \psi_s S \theta_T S \phi_T S \psi_T C \theta_w C \psi_w + C \phi_s C \psi_s C \phi_T C \theta_T C \psi_T S \theta_T S \phi_T S \psi_T S \phi_w S \psi_w) + 2\dot{y} (C \phi_l C \theta_s C \phi_s C \psi_s C \theta_T C \psi_T C \phi_w C \psi_w + C \phi_l S \theta_s S \phi_s S \psi_s C \phi_T C \theta_T C \psi_T C \phi_w C \psi_w) + 2\dot{z} C \phi_l C \theta_s C \phi_s C \theta_T C \phi_T C \theta_T C \psi_T C \phi_w C \psi_w) \right] - \frac{1}{2} \alpha (-7 m_T g L_1 C \theta_T C \phi_T - 5 m_s g L_4 S \theta_T - 3 m_w g L_5 S \phi_T - \frac{K_{p1} + K_{p2}}{2} (E_I C \psi_s - L_I) + \frac{K_{p1} + K_{p2}}{2} (E_P C \phi_s C \psi_s - L_P) + \frac{K_{D1} + K_{D2}}{2} (E_D C \psi_s C \theta_s - L_D) + \frac{K_{B1} + K_{B2}}{2} (E_B C \phi_L - L_B) + \frac{K_{B1} + K_{B2}}{2} (E_T C \phi_L - L_T) \right].}
In the previous section, the equations of motion were derived using the Lagrangian approach; however, those equations did not include the driving forces nor did they include the non-conservative forces associated with the muscles of the upper extremity. This section details the derivation of the muscles force equations using generalized forces along with the Lagrangian approach. Each of the muscle forces can easily be included into Equations 60 through 63 by substituting the newly derived muscle force equations for the Q terms in each equation. Prior to deriving the muscle force equations, it was necessary to first determine the equations governing the muscle length as a function of the joint angle. To calculate the average muscle lengths, the length of the associated segment, the muscle action, and the muscle attachments were taken into account. For example, the associated segment for the biceps and triceps muscles were the upper arm, with the action was through the elbow joint and the attachments on the forearm. A geometric system, as illustrated in Figure 25, was created in order to depict the Biceps and Triceps. The Law of Cosines was then applied to determine the muscle lengths as shown in Equation 40 and the muscle length equations are included in Table 7.

### Table 7: Equations governing muscle lengths

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Length Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biceps</td>
<td>$LB = \sqrt{LA^2 + 0.02^2 - 0.04LA \cos(3.14 - \phi_L)}$</td>
</tr>
<tr>
<td>Triceps</td>
<td>$LT = \sqrt{LA^2 + 0.02^2 - 0.04LA \cos(6.28 - \phi_L)}$</td>
</tr>
<tr>
<td>Deltoid</td>
<td>$LD = \sqrt{LC^2 + 0.04^2 - 0.08LC \cos(3.14 - \theta_s)}$</td>
</tr>
<tr>
<td>Pec Major</td>
<td>$LP = \sqrt{LS^2 + 0.06^2 - 0.12LS \cos(6.28 - \theta_s)}$</td>
</tr>
<tr>
<td>Infraspinatus</td>
<td>$LI = \sqrt{LS3^2 + 0.03^2 - 0.06LS3 \cos\psi_s}$</td>
</tr>
</tbody>
</table>

In Table 7, $LB$ is the length of the Biceps, $LT$ is the length of the Triceps, $LD$ is the length of the Deltoid, $LP$ is the length of the Pec Major, $LI$ is the length of the Infraspinatus, $LC$ is the length of the clavicle, $LS$ is the length between the base of the scapula and the shoulder (representing a muscle whose insertion and origin are from the base of the scapula to the shoulder), $LS3$ is $3/4$ of the length of $LS$, and all other variables are as previously defined. The subtraction of the joint angles was performed to ensure that the
muscles were shortening and lengthening when appropriate. Once the equations for the muscle lengths were completed, the derivation of the muscle forces could begin.

Each muscle element in the model described in this work consists of two spring elements and a singular dashpot element all connected in parallel as illustrated in Figure 24, where the spring elements can each be modeled using Equations 64 and 65,

\[ V_1 = \frac{1}{2}k_1(x_1 - L_1)^2, \quad \text{Eq. 64} \]
\[ V_2 = \frac{1}{2}k_2(x_2 - L_2)^2, \quad \text{Eq. 65} \]

where \( V_1 \) and \( V_2 \) are the potential energy associated with the spring elements, \( k_1 \) and \( k_2 \) are the spring constants, \( x_1 \) and \( x_2 \) are the length of the spring element at any given time, and \( L_1 \) and \( L_2 \) are the resting lengths of the springs. Since both spring elements are considered potential functions, these values are included in the potential function of the equations of motion as previously described. The dashpot was modeled using Equation 66.

\[ F = \frac{1}{2} \mu \dot{x}, \quad \text{Eq. 66} \]

where \( F \) is the force associated with the dashpot, \( \mu \) is the dampening coefficient, and \( \dot{x} \) is the time rate of change of the system’s length. Since the dashpot is considered a dissipative force, it must be included in the equations of motion as a generalized force. The simplest means of doing this was to apply the concept of virtual work to derive the equations governing the behavior of the dashpot within the muscle elements, as shown in Equation 67,

\[ \delta W = M = -N + \mu(\dot{x}\delta x + \dot{\xi}\delta \zeta), \quad \text{Eq. 67} \]

where \( \delta W \) is the virtual work of the summation of muscle elements for a single joint, \( M \) is the total joint moment, \( N \) is the moment that the muscle contributes to the total joint moment, \( \mu \) is the dampening coefficient, \( \dot{x} \) is the time rate of change of the muscle length, \( \dot{\xi} \) is the angular velocity of the joint, \( \delta \xi \) and \( \delta x \) are the virtual displacements associated with the angular motion of the joint being acted on and the
length of the muscle, respectively. Equation 67 can then be expanded by replacing the virtual
displacement terms with the actual displacements associated with each generalized coordinate resulting in
Equation 68,

$$N = \mu \left( x \frac{L_s}{L_s-i} C\xi + \dot{\xi} \frac{L_s^2}{L_s^2-i^2} C^2\xi \right),$$  \hspace{1cm} \text{Eq. 68}

where $x$ is the length of the muscle, $L_s$ is the length of the segment associated with the muscle, $i$ is the
distance between the muscle rotation center and the insertion point of the muscle, $\xi$ is the angular position
of the joint acted on by the specified muscle, $\dot{\xi}$ is the angular velocity of the joint acted on by the
specified muscle, and $C$ represents the cosine. This method was repeated for each of the five muscle
elements included in the model described in this work and the final equations with their associated
generalized coordinates are included in Table 8.

<table>
<thead>
<tr>
<th>Generalize Coordinate</th>
<th>Generalized Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_s$</td>
<td>$Q\theta_s = M_D - \alpha_{D1}\mu_D \left( L_D \frac{L_C}{L_C-0.04} C\theta_s + \dot{\theta}_s \frac{L_C^2}{L_C^2-0.04^2} C^2\theta_s \right)$</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>$Q\phi_s = M_p + M_I - \alpha_{I1}\mu_I \left( L_I \frac{L_{S3}}{L_{S3}-0.03} C\phi_s + \phi_s \frac{L_{S3}^2}{L_{S3}^2-0.03^2} C^2\phi_s \right) - \alpha_{p1}\mu_p \left( L_P \frac{L_S}{L_S-0.06} C\phi_s + \phi_s \frac{L_S^2}{L_S^2-0.06^2} C^2\phi_s \right)$</td>
</tr>
<tr>
<td>$\psi_s$</td>
<td>$Q\psi_s = M_p + M_D - \alpha_{D2}\mu_D \left( L_D \frac{L_C}{L_C-0.04} C\psi_s + \psi_s \frac{L_C^2}{L_C^2-0.04^2} C^2\psi_s \right) - \alpha_{p2}\mu_p \left( L_P \frac{L_S}{L_S-0.06} C\psi_s + \psi_s \frac{L_S^2}{L_S^2-0.06^2} C^2\psi_s \right)$</td>
</tr>
<tr>
<td>$\phi_L$</td>
<td>$Q\phi_L = M_B + M_T - \alpha_B\mu_B \left( LB \frac{L_S}{L_S-0.02} C\phi_L + \phi_L \frac{L_S^2}{L_S^2-0.02^2} C^2\phi_L \right) - \alpha_T\mu_T \left( LT \frac{L_S}{L_S-0.02} C\phi_L + \phi_L \frac{L_S^2}{L_S^2-0.02^2} C^2\phi_L \right)$</td>
</tr>
</tbody>
</table>

The subscripts of $B$, $T$, $I$, $P$, and $D$ correspond to the Triceps, Biceps, Infraspinatus, Pectorialis Major, and
the Deltoid, respectively. The dampening coefficients, $\mu$, for each of the specified muscles described in
this dissertation were experimentally determined that was described in the validation portion of this work.
The $\alpha$ terms were constants added to the equations as a means of adjusting for the force velocity
relationship of the muscle and were determined using the actual EMG signal reduction relationship to the
angular velocity of the associated joint. Each of the $\alpha$ terms are included in Table 9.
### Table 9: Definitions for the Alpha constants

<table>
<thead>
<tr>
<th>α-constant</th>
<th>Definition</th>
</tr>
</thead>
</table>
| α_B        | \[
|           | \begin{cases} 0.6\dot{\phi}_L & \text{when } \ddot{L}B \leq 0 \\ 0.24\dot{\phi}_L & \text{when } \ddot{L}B > 0 \end{cases} \] |
| α_T        | \[
|           | \begin{cases} 0.1\dot{\phi}_L & \text{when } \ddot{L}T \leq 0 \\ 0.06\dot{\phi}_L & \text{when } \ddot{L}T > 0 \end{cases} \] |
| α_D1       | \[
|           | \begin{cases} 0.2\dot{\theta}_s & \text{when } \ddot{L}D \leq 0 \\ 0.02\dot{\theta}_s & \text{when } \ddot{L}D > 0 \end{cases} \] |
| α_D2       | \[
|           | \begin{cases} 0.1\dot{\psi}_s & \text{when } \ddot{L}D \leq 0 \\ 0.01\dot{\psi}_s & \text{when } \ddot{L}D > 0 \end{cases} \] |
| α_P1       | \[
|           | \begin{cases} 0.1\dot{\phi}_s & \text{when } \ddot{L}P \leq 0 \\ 0.02\dot{\phi}_s & \text{when } \ddot{L}P > 0 \end{cases} \] |
| α_P2       | \[
|           | \begin{cases} 0.4\dot{\theta}_s & \text{when } \ddot{L}P \leq 0 \\ 0.08\dot{\theta}_s & \text{when } \ddot{L}P > 0 \end{cases} \] |
| α_I        | \[
|           | \begin{cases} 0.1\dot{\phi}_s & \text{when } \ddot{L}I \leq 0 \\ 0.11\dot{\phi}_s & \text{when } \ddot{L}I > 0 \end{cases} \] |

The equations described in this section fully describe the muscle forces associated with the glenohumeral and elbow joints of the model detailed in this work. Using the methods described in this section, it is possible to expand this work to include additional muscle elements and describe the motion of the wrist or forearm should those be of interest in later works.
4.3 Protocol

4.3.1 Subjects

This study was conducted with the approval of the Institutional Review Board at Connecticut Children’s Medical Center (IRB 01-095 and 13-095). A total of 181 baseball pitchers were recruited over the course of seven years at Connecticut Children’s Medical Center. The pitchers were between the ages of 8 and 24, and all pitchers had at least one year of experience pitching for an organized baseball team. At the time of the test, none of the pitchers indicated that they had any shoulder or elbow pain. Subjects were excluded if they had a history of surgery to their pitching arm and if they had any injury or pain in the pitching arm that caused them to miss at least one game or practice in the six months preceding the pitching analysis. All subjects signed consent prior to the pitching analysis and, for those subjects that were under the age of 18, parental permission and assent from the subject was obtained. For this study, the subjects were limited to the collegiate aged population only resulting in a possible 99 subjects. To be considered for this work, the subjects had to have been capable of pitching both a fastball and a curveball. Subjects also had to have three trials for each tested condition (i.e. fastball and curveball) in which all motion data was available. This resulted in a pool of 47 subjects from which 33 subjects were randomly selected using a random number generator in Excel.

A 29 year-old male subject was used for the initial validation testing. The final validations used an adult male pitcher, who was collected under the Connecticut Children’s Medical Center’s IRB number 13-095, and had consented to additional data collection using surface EMG.

4.3.2 Calibration

The Vicon Motion Capture System (Vicon Motion Systems, Los Angeles, CA) was calibrated once per day prior to data collection. Additional calibrations were collected if it was believed the cameras may have been moved between subjects. The system calibration was divided into two portions, a dynamic calibration and a static calibration. The dynamic calibration was performed first. A calibration wand, shown in Figure 27, was waved through the data collection area (5 meters in length, 3.5 meters wide, and
2.5 meters in height) within the Center for Motion Analysis, at the Connecticut Children’s Medical Center. The calibration was performed at 120Hz and required each of the 12 cameras to collect 2000 frames of data in which the calibration wand was fully visible. Once all data was captured, the Vicon calibration pipeline was run. The calibration pipeline synchronized the position of the markers on the wand based on their coordinates from each camera and optimized their position by reducing the residuals for the best fit measure. To be considered a successful calibration, the residuals for each camera had to be below 0.2. If the residuals were higher than 0.2, then the system was recalibrated.

![Calibration wand used for all data collections](image)

Following the completion of the dynamic calibration, the static calibration was completed. The static calibration was performed to ensure the origin of the system was correctly positioned within the data collection space. The origin for all data collection was located on the left upper corner of the second force plate as shown in Figure 28. To perform the data collection, four 1” retro-reflective markers were placed in the screw wells of the large force plates as shown in Figure 29. Using the position of these markers, it was possible to establish the correct position and orientation of the origin. To ensure a proper calibration was achieved, an additional 1” marker was placed at a known location within the data collection space. The Vicon system was then used to locate the marker and measure the distance from the origin. The additional marker must be within 1mm of the known position to be considered an acceptable calibration. If the marker was not within 1mm of the known position, then the static calibration was
performed a second time. If the marker position was still incorrect, then both the dynamic and static calibrations were repeated.

![Figure 28: Location of laboratory origin](image)

4.3.3 Validation Marker Placement and Modeling

The Lagrangian based model previously derived was an untested model; therefore, ensuring that the model outputs were correct was extremely important. A simplistic upper extremity marker configuration was developed to evaluate various aspects of the model prior to using the full pitching model. The initial validation testing evaluated elbow flexion and only required the use of three markers where the first marker, M1, was placed over the humeral head, the second marker, M2, was placed over

![Figure 29: Static calibration marker locations](image)
the lateral epicondyle of the elbow, and the final marker, M3, was placed over the radial styloid of the wrist, as depicted in Figure 30.

![Figure 30: Marker positions for initial validation testing](image)

The second validation test was designed to evaluate a single plane of motion at the GH joint, specifically vertical abduction/adduction, as well as elbow flexion and extension. An additional marker, designated as M4, was added to the previous three markers and was placed on the 6th rib of the right side of the subject.

Using the three dimensional coordinates of each of the markers, it was possible to calculate the joint angles. The elbow joint angle was constructed using the M1, M2, and M3 markers, which were used to create two vectors. The first vector, the reference vector, was constructed using the M1 and M2 markers using Equation 69, and the second vector was constructed between the M2 and M3 markers, using Equation 70,

\[
V_1 = (M1_x - M2_x)_i + (M1_y - M2_y)_j + (M1_z - M2_z)_k, \quad \text{Eq. 69}
\]

\[
V_2 = (M2_x - M3_x)_i + (M2_y - M3_y)_j + (M2_z - M3_z)_k, \quad \text{Eq. 70}
\]

where \(V_1\) is the reference vector, \(V_2\) is the second vector, \(M1\) and \(M2\) are the marker designations, and the \(x, y, \) and \(z\) subscripts correspond to the \(x, y, \) and \(z\) coordinates of the markers, respectively. Once the two
Vectors were created the magnitude of the vectors were determined using the general equation given by Equation 71,

$$
||V_m|| = \sqrt{(V_x)^2 + (V_y)^2 + (V_z)^2}, \quad \text{Eq. 71}
$$

where $||V_m||$ is the magnitude of the specified vector, $V_x$, $V_y$, and $V_z$, are the x, y, and z coordinates of the specified vector. The dot product of the two vectors was then determined using equation 72,

$$
D_e = (V_{1,x} \cdot V_{2,x}) + (V_{1,y} \cdot V_{2,y}) + (V_{1,z} \cdot V_{2,z}), \quad \text{Eq. 72}
$$

where $D_e$ is the dot product for the elbow vectors, and all other variables have been previously defined.

Finally, the joint angle, in radians, was determined using Equation 73,

$$
\theta_e = \cos^{-1}\left(\frac{D_e}{||V_1|| ||V_2||}\right), \quad \text{Eq. 73}
$$

where $\theta_e$ is the joint angle and each of the other variables has been previously defined. To calculate the glenohumeral joint vertical abduction angle the markers designated as M1, M2, and M4 were used. A vector was created between the M4 and M1 marker which served as the reference vector and the second vector created between the M1 and M2 makers were previously defined in the description for the elbow joint angle. The vector between the M4 and M1 markers was created using Equation 74,

$$
V_3 = (M_{1,x} - M_{4,x})_i + (M_{1,y} - M_{4,y})_j + (M_{1,z} - M_{4,z})_k, \quad \text{Eq. 74}
$$

where $V_3$ is the reference vector, M1 and M4 are the marker designations, and the x, y, and z subscripts correspond to the x, y, and z coordinates of the markers, respectively. The joint angle was then calculated using a similar process as described for the elbow joint angle with the noted exception that the $V_3$ was used instead of the $V_1$ vector.
4.3.4 Pitching Marker Placement and Modeling

The pitching model used to collect data for this work has been previously described by Nissen et al. [44]. A complete description of the pertinent portions of this model are included in this work for completeness. The pitching model utilizes a total of 38 retro-reflective markers that, as shown in Figure 31, were placed on specific bony landmarks to create a 16 segment biomechanical model that was capable of describing the motion of the head, trunk, upper extremities, pelvis, and lower extremities.

![Figure 31: Marker placement for the complete pitching model used at Connecticut Children’s Medical Center](image)

An additional two markers were placed on the ball which allowed for inverse dynamic calculations and to calculate ball velocity. Appendix B contains a complete description of the marker names and alignment goals. The following description of the model is a detailed description that is pertinent to the portions of the model used in this work, specifically the trunk and pitching arm segments.

A total of four markers were used to describe the motion of the trunk within the laboratory space. One marker was placed over the spinous process of the C7 vertebra, a second marker was placed over the spinous process of the T10 vertebra, the third marker was placed at the clavicular notch, and the last of the four markers was placed at the xyphoid process. See Figure 32 for placement.
The trunk coordinate system was created using these four markers. The origin of this coordinate system was centered at the intersection of the vector between the T10 marker and clavicle marker and the vector between the C7 marker and the xyphoid marker. The primary axis, the Z-axis, was parallel to the vector between the C7 and T10 markers, and was centered at the origin. The Y-axis, was formed by the cross product of the Z-axis and vector connecting the C7 and clavicle markers. Finally, the X-axis was created from the cross product of the Y-axis and the Z-axis. Figure 33 depicts the position and orientation of the trunk coordinate system.

The thoracic angles were defined using an YXZ Euler rotation sequence, and the lab coordinate system was considered the reference coordinate system. The coronal plane angles described thoracic lean, where the angle had a positive value when the right side of the shoulder girdle was above the left.
The sagittal plane motion described the thoracic tilt, where an anterior position was considered as a positive value, and the transverse plane motion described the thoracic rotation of the trunk, where internal rotation (i.e. the right side is in front of the left) was considered a positive angle, as depicted in Figure 34.

![Trunk Angle Definitions](image)

Figure 34: Thoracic angle definitions: A: lean, B: anterior/posterior tilt, C: internal/external rotation

The shoulder joint in this model describes the motion of the glenohumeral joint (GH) only. This was done to simplify the shoulder model and based on the assumption that the majority of the shoulder motion during pitching comes from the rotation of the GH joint. A total of three markers were used to define the coordinate system of the GH joint. The first marker was placed over the most lateral aspect of the scapular spine, the second marker was placed over the acromioclavicular joint, and the last of the three markers was placed over the coracoid process, as shown in Figure 35.

![Marker Placement](image)

Figure 35: Marker placement and designations for the three shoulder markers
The initial origin of the coordinate system was centered at the marker placed over the acromioclavicular joint. The primary axis, the Z-axis, was defined as the vector parallel to the vector created between the center point of the vector between the scapular spine marker and the coracoid marker, and the acromioclavicular joint, and perpendicular to the vector, between the scapular spine marker and the coracoid marker. The Y-axis was formed by the cross product of the Z-axis and the vector between the scapular spine marker and the coracoid marker. The X-axis was then formed by the cross product of the previously defined Z and Y axes, as shown in Figure 36. Once the coordinate system had been constructed, the origin for the glenohumeral joint was shifted from the acromioclavicular joint marker to the center of rotation for the glenohumeral joint. To shift the center, a dynamic centering technique was used to determine the rotation center of the glenohumeral joint.

![Coordinate System Illustration](image)

Figure 36: Illustration of the coordinate system for the GH joint

The dynamic centering technique required each subject to fully flex and extend their shoulder in the sagittal plane, and then fully abduct and adduct their shoulder in the coronal plane, where the motion “drew a sphere” around their shoulder with the glenohumeral joint as point of rotation. A vector was then defined between the acromioclavicular joint marker and the elbow joint center, which was the center point of the vector between the two elbow markers. The length of this newly formed vector was assumed to be the length of the humerus and, therefore, should have a minimal length change over the dynamic motion. A Gaussian optimization was then performed to minimize the change in humeral length, while remaining within the confines of the sphere of motion created by the subject’s shoulder range of motion. This
spherical centering technique has been shown to reduce the humeral length change over the entire pitch to 7% compared to 21% and 16% by the Golem and Mesker shoulder joint centers respectively[52].

To define the angles of the glenohumeral joint a XYZ Euler sequence was used, especially since it produced the most clinically appropriate angle measures. The reference coordinate system for the glenohumeral coordinate system was the trunk coordinate system. The coronal plane motion described the vertical abduction and adduction of the glenohumeral joint and abduction was considered to be a positive value. The sagittal plane motion of the glenohumeral joint described the internal and external rotation of the glenohumeral joint, where internal rotation was considered as a positive value. The transverse plane of motion described the horizontal abduction and adduction of the glenohumeral joint, where abduction was considered a positive value. Figure 37 contains the joint angle definitions for the glenohumeral joint.

Figure 37: GH joint angle definitions: A: Vertical ab/adduction, B: internal/external rotation, C: Horizontal ab/adduction

The motion of the elbow joint and forearm were combined in this model allowing for a description of forearm rotation and elbow joint flexion and extension only. This model assumes that there was minimal coronal plane motion in the elbow joint. A total of four markers were used to construct the elbow joint and forearm in this model, where the first two markers were placed over the medial and lateral condyles of the elbow and the other pair of markers were placed over the medial and lateral styloids of the wrist, as shown in Figure 38.
The elbow joint center was the midpoint of the vector between the medial and lateral condylar markers, and the wrist joint center was defined as the midpoint of the vector between the medial and lateral styloid markers. The elbow angle was calculated as the cross product of the vector between the shoulder joint center and elbow joint center, and the vector between the elbow joint center and the wrist joint center as shown in Figure 39.

The elbow angle was only described in the sagittal plane and flexion was considered to be positive, as seen in Figure 40.
The forearm angle was more complicated than the elbow angle and required the creation of two coordinate systems, the proximal radial coordinate system and the distal radial coordinate system. The proximal radial coordinate system has its origin at the elbow joint center. The primary axis, the Y-axis, was parallel to the Y-axis of the GH coordinate system, with its origin at the elbow joint center, the X-axis, was the cross product of the Y-axis and the vector between the elbow joint center and the wrist joint center. The X-axis was the cross product of the Y and Z axes as shown in Figure 41.

Figure 40: Angle definitions for elbow flexion and extension

Figure 41: Illustration of the distal radial coordinate system (left) and proximal radial coordinate system (right) for the forearm segment
The distal radial coordinate system had its origin at the wrist joint center. The primary axis, the Z-axis, was parallel to the vector between the elbow joint center and the wrist joint center with its origin as the wrist joint center. The Y-axis was the cross product of the Z-axis and the vector between the medial and lateral styloid markers and the X-axis was the cross product of the Y and Z axes, as depicted in Figure 41 above.

The forearm angle was calculated as the angle between the X-axis of the distal radial coordinate system and the X-axis of the proximal coordinate system. The forearm angle measured the supination and pronation of the forearm, where pronation was considered a positive value, as shown in Figure 42.

![Figure 42: Angle definition for forearm rotation](image)

The last part of the model discussed in this work was a description of the wrist angles. A total of three markers were required to define the coordinate systems necessary to create the wrist angles. The first two markers were those markers placed on the medial and lateral styloids of the wrist with the third marker placed proximal to the knuckle of the 3rd digit, as shown in Figure 43.
The origin of the coordinate system was centered at the midpoint of the vector connecting the 3rd digit marker and the wrist joint center. The primary axis, the Z-axis, was defined as the vector parallel to the vector between the origin and the wrist joint center, the X-axis was formed by the cross product of the Z-axis and the Y-axis of the distal radius coordinate system, and the Y-axis was formed by the cross product of the previously defined Z and X axes, as shown in Figure 44.

The wrist angles were described using an YXZ Euler rotation sequence with the reference coordinate system being the distal radius coordinate system. The wrist angles were only described in the coronal and sagittal planes. The coronal plane angles described the ulnar and radial deviation of the wrist, where an ulnar deviation was considered to be positive. The sagittal plane angles described the flexion and extension of the wrist, where flexion was considered to be positive, as shown in Figure 45.
4.3.5 EMG Placements and Signal Quality Assurance

The surface EMG protocol for skin preparation and quality assurance testing was the same for both the validation testing and pitching protocols. Prior to the application of the surface electrodes, the skin was cleaned using an alcohol wipe and then abraded using a skin prep pad soaked in alcohol. This was performed to ensure the best possible contact between the electrodes and the skin. The alcohol was allowed to dry and the electrodes were then affixed to the subject using tape and secured using a rubber strap, as per the standard operating procedures in place at the Center for Motion Analysis at Connecticut Children’s Medical Center. In some instances, with very rapid motions such as those encountered during pitching, additional fixation methods beyond the tape and rubber straps were required. In these instances, the additional fixation was achieved by wrapping the strap and electrode in two to three layers of coband. All electrodes were placed over the muscle bellies of each of the muscles of interest and the exact placements are described later in this section. Following the placement of all of the electrodes, a quality assurance protocol was followed to ensure that the electrodes were appropriately placed and provided a good signal. The signal was assessed visually using an electronic strip chart recorder that showed the EMG signal in real time. The subjects were then instructed to sit comfortably, without moving, to assess...
that the baseline signal was quiet. If there was an excessive amount of noise on the EMG channel, then the electrode was taken off and then skin was cleaned again. Once the baseline signals were deemed appropriate, the subject was instructed to perform certain tasks that would isolate each muscle group of interest. As the subject performed the task, the tester added resistance in which the subject had to push against, similar to performing a maximum voluntary contraction but for a shorter duration. During each of the tasks, the electronic strip chart was assessed to ensure that the appropriate signal was appearing on the correct channel and with the correct timing. If the channel was incorrect then the EMG electrode was moved to the correct channel. If there was cross talk between two muscle groups the EMG electrodes were removed and then replaced and checked again to ensure that they signals were reading appropriately.

The electrode placement and fixation for the validation testing that recorded surface EMG signals from the Biceps and Triceps are depicted in Figure 45. The Biceps EMG was placed over the muscle belly of the long head of the Biceps and the Triceps EMG was placed over the muscle belly of the long head of the Triceps. The Deltoid EMG, not shown in Figure 46, was positioned over the muscle belly of the Deltoid over the anterior/lateral portion of the Deltoid.

![Figure 46: EMG placement for initial validation testing](image)

The electrode placement and fixation for the pitching testing recorded EMG signals from the Biceps, Triceps, Deltoid, and Pectorialis Major, as shown in Figures 47 and 48. The surface EMGs were
again placed over the muscle bellies of the long heads of the Biceps and Triceps, as well as the anterior/lateral portion of the Deltoid, as described in the validation placement portion of the protocol. An additional surface EMG was also placed over the muscle belly of the Pectoralis Major with a more lateral bias so as not to interfere with the pitching motion. Due to the depth of the Infraspinatus a surface EMG could not accurately collect the signal and, since the IRB does not cover the use of fine-wire EMGs, the Infraspinatus was not collected.

Figure 47: EMG placement for Biceps and Triceps, for the pitching validation test

Figure 48: EMG Placement for Pectoralis Major and Deltoid for the pitching validation test
4.3.6 Data Collection for Validation Testing

Once the markers and EMGs were attached, the validation testing began. The initial validation test was created to serve a dual purpose to determine the immeasurable values of describing muscle resistance and muscle dampening and to ensure that the model was capable of accurately predicting joint motion when provided with a specified initial condition. For the initial validation, the elbow joint’s motion was chosen as the majority of the motion from this joint is in a singular plane. In an effort to limit cross talk within the model and ensure that only sagittal plane motion at the elbow was produced, a series of splints were used to restrict motion at both the shoulder and wrist. A splint was placed on the wrist restricting motion in both the sagittal and coronal planes as seen in Figure 49. As an application of a splint at the shoulder was nearly impossible, a series of ace bandages were wrapped around the subject’s chest and the middle of the upper arm as shown in Figure 50. This did not completely restrict shoulder motion and the subject was also instructed to keep his shoulder still during the collection of data.

Figure 49: Depiction of the wrist splint used in the validation testing
Following the application of the splints, maximum voluntary contractions for the biceps and triceps were collected. The subject was then instructed to perform a series of elbow flexion and extension tasks at three speeds (i.e. slow, normal, and rapid) with a variety of weights that were tested in random order. The initial test was unweighted, followed by a 10 lb weight and then 15 lb, 20 lb, and 5lb weights, respectively. Motion data were collected during each trial using a Vicon MX motion capture system (Vicon Motion Systems, Los Angeles, CA). Motion data was collected at 120 Hz and Vicon Nexus was used to identify the marker trajectories and save the data to a C3D file. The EMG data were collected using an integrated Motion Analysis Lab EMG system (Motion Lab Systems, Baton Rouge, LA) that allowed the EMG data to be collected simultaneously with the motion data using the Vicon system. All EMG data were collected at 1080Hz. Simultaneous EMG data were collected at 1800 Hz using the Vicon Nexus interface. The gain settings for each of the electrodes attached to the Biceps and Triceps were set to 4000 to ensure that the signal was not oversaturated, which would cause the maximum amplitudes to be lost as they would be above the voltage rails. The maximum voltage scale for the EMG data collection was 10v (-5 V to 5 V).

The second validation was designed to mimic the first two validation tests; however, in this test, instead of restricting motion at the shoulder, the subject was allowed to move their shoulder in the coronal
plane. This validation tested the ability of the model to predict the motion at two different joints, the glenohumeral and the elbow joint, as well as give an estimation of the muscle moments. The gain setting for the Deltoid EMG was set to 6000 as the signal was of much lower amplitude than the Biceps and Triceps signal. A wrist splint was again used to restrict wrist motion. Once all markers were placed and the splints were applied MVCs were collected for the three muscles of interest and then motion data collection began. This validation test required the subject to flex his elbow while simultaneously vertically abducting their glenohumeral joint and then returning to their initial starting position. This was repeated four times: unweighted slow movement, unweighted rapid movement, weighted slow movement, and weighted rapid movement. For the weighted trials, the subject was provided a 5lb weight. Motion data and EMG data were collected as described in the other validation testing.

The final validation was designed to fully test all aspects of the model. A single adult pitcher was asked to pitch a fastball toward a designated target set 60’ 6'” away. The subject had the 38 markers required for the pitching model attached, as previously described, as well as four surface electrodes placed over the muscle bellies of the long head of the Triceps, the long head of the Biceps, the Pectorialis Major, and the Deltoid. (The gain setting for the Pectorialis Major was set to 6000.) Motion data were collected using a Vicon MX motion capture system at 250 Hz and EMG data were collected at 1800 Hz as described previously. The purpose of the final validation testing was to determine if the muscle moments for each of the four muscles tested could be estimated, as well as the total joint moments given the motion capture data.

4.3.7 Data collection for Pitching Motion

Prior to data collection all subjects were given a brief medical and pitching history. Anthropometric measures (i.e. subject height, weight, wrist diameter, elbow diameter, pelvic width, and pelvic depth) were obtained from each subject to aid in the calculation of the inertial properties for the pitching model. The 38 retro-reflective markers were then attached as described in the modeling section
of this work. Prior to the analysis, all subjects were given as much time as they required to warm up and feel comfortable pitching within the laboratory environment.

All subjects pitched from a 10” regulation mound toward a designated target with a strike zone 60’ 6” away as per baseball regulations as shown in Figure 51.

![Figure 51: Example of pitching data collection](image)

All subjects pitched the pitch types (i.e. fastball, curveball, slider, cutter, and change-up) that they felt comfortable pitching in a game setting. After deciding on the pitch types that they felt comfortable pitching, each subject pitched seven of each pitch type in randomly assigned order to simulate a game like setting. Following the analysis, all of the markers were removed from the subject and the subject was allowed to leave.

All data was collected using a Vicon 512, 12 camera motion capture system. The 12 cameras were positioned circumferentially around the lab and each one was placed along 30° radials around the origin of the laboratory. All data were collected at 250Hz using Vicon Workstation software (Vicon Motion Systems, Los Angeles, CA).
4.4 Data Analysis

4.4.1 Validation Testing

Following the data collection for each of the validation tests, the initial data processing was performed using Vicon Nexus that allowed for the identification of the marker trajectories. The identified marker data were then saved to a C3D file and the three dimensional coordinates for each of the markers were exported to Excel. The only exception was the final test in which the pitching data was collected as this was processed using the same techniques required for the pitching data collection. The joint angles were calculated as described earlier and the resulting joint angles for both the elbow joint and glenohumeral joints were considered to be the gold standard to compare the model results against.

The EMG data analysis was initially processed using MATLAB, where a custom program was developed to read the EMG data from the C3D file for further processing. The raw data was converted into a voltage scale using Equation 75,

\[ EMG_v = (EMG - offset) \cdot scale, \]

where \( EMG_v \) is the voltage based EMG signal, \( EMG \) is the raw signal, \( offset \) is the DC offset value contained in the C3D file for each individual analogue channel, and \( scale \) is the analogue scale contained in the C3D file for each channel. The EMG signal was then rectified by taking the absolute value of the voltage based EMG signal. The rectified signal was then filtered by using an RMS filter with a window of 30 (0.27 s). Finally, the signal was down sampled to 120Hz to match the motion data collection rate. The EMG signals were then exported into Excel where the muscle moment was estimated using the equation described by Marras et al. and provided in Equation 76 [19],

\[ MM = \left( \frac{EMG_s}{EMG_m} \right) \cdot M_{force} \cdot CA \cdot GL \cdot fV, \]

where \( MM \) is the estimated muscle moment for a specified muscle, \( EMG_s \) is the EMG amplitude from the EMG signal for a specified muscle, \( EMG_m \) is the maximum amplitude of the signal for a specified model collected during an MVC, \( M_{force} \) is the maximum force capacity for a specified muscle and can be...
calculated using Equation 77, \( CA \) is the cross sectional area of the muscle, \( GL \) is the coefficient for length modulation which is given by Equation 78, and \( fV \) is the coefficient for velocity modulation as given by Equation 79,

\[
M_{force} = 30 \cdot CA ,
\]

\[
GL = -3.2 + 10.2 \cdot \frac{L}{LR} - \left( 10.4 \cdot \frac{L}{LR} \right)^2 + \left( 4.6 \cdot \frac{L}{LR} \right)^3 ,
\]

\[
fV = 1.2 - 0.99V + 0.72V^2 ,
\]

where \( M_{force} \) is the maximum force capacity for a specified muscle, and \( CA \) is the cross sectional area for that muscle. \( GL \) is the coefficient for length modulation, \( L \) is the length of the specified muscle, and \( LR \) is the resting length of that muscle. \( fV \) is the coefficient for the velocity modulation and \( V \) is the velocity of the contraction as determined as the time rate of change of the change of length of the muscle. This estimated muscle moment was then considered to be the gold standard to compare the current model to as this method is based on collected data as well as published methodology.

For the initial validation, the equations of motion governing the system developed using the Lagrangian approach were used, specifically the equation describing the motion of the elbow joint. As the motion at the wrist and glenohumeral joint were restricted, it was assumed that these angles in the equation were zero and, therefore, the equation was modified as shown in Equation 80,

\[
\ddot{\phi}_L = \begin{bmatrix}
\text{moment} + \\
3gL5 \cos \phi + \frac{2KB}{mt} (LB - 0.29) \sin \phi - \dot{\phi}DB \left( \frac{L^5}{L5^2 - 5^2} \right) \cos \phi^2 - LBDB \left( \frac{L^5}{L5 - 5} \right) \cos \phi + \frac{2KT}{mt} (LT - 0.26) \sin \phi - \dot{\phi}DT \left( \frac{L^5}{L5^2 - 5^2} \right) \cos \phi^2 - LTDT \left( \frac{L^5}{L5 - 5} \right) \cos \phi \end{bmatrix} \cdot [0.5(L_6^2) + 0.1667(L_5^2) + L5]^{-1} ,
\]

where all variables have been previously defined. This was then programmed into Matlab as an initial value problem and the elbow joint angle was solved for using the ODE23 solver in Matlab. The initial conditions including initial angle, initial angular velocity, initial muscle lengths and velocities were entered into the program from the results calculated from the actual motion data. The constants
associated with the spring and dashpot elements were experimentally determined as previously described. The equation was first solved with all of the constants set to zero to ensure that the model was producing the correct values (i.e., the maximum and minimum joint angles). Then each of the constants, one at a time, were set to 50 and the model was re-run to determine the effect that each of the constants had on the model output. Finally, the constants were modified from 0 to 30 in increments of five and then further refined to bring the results into better agreement with the actual data. The final modification to the model was to change the constants after the physiological maximum angle was reached. The reason this was included in the model was that, as the subject lowered his arm, additional resistance was required to produce a controlled descent; therefore, the model was adjusted to reflect this physiological property. The results were then compared against the actual data collected during the motion capture trials.

The next step of the analysis was to focus on the generalized forces describing the actions of the Biceps and Triceps. Using the actual motion data collected and the constants determined during the initial validation, the estimated muscle moments for the Biceps and Triceps were calculated using the equations described in Table 7. The constants (i.e., the spring constants and dampening coefficients) were determined by choosing a value and then visually comparing the model output to the reference waveform. The initial value was then varied until the models came into good visual agreement, and the final value had to be within an acceptable physiological range as described in literature. The results were then compared against the estimated muscle moments from the Lagrange based modeling described earlier.

The next portion of the validation analysis was carried out exactly as described in the initial validation analysis. The only difference was the addition of the coronal plane glenohumeral angle. The other glenohumeral angles (i.e., the sagittal and rotational angles) were still considered to be zero along with the wrist angles. These assumptions again required modification to the original equations of motion developed using the Lagrangian approach, which resulted in the following two equations given by Equation 81 and 82,

Eq. 81
\[ \ddot{\theta}_s = \left[ M_D - \alpha_D \mu_D \left( L_D \frac{L C}{L C - 0.05} C \theta_s + \dot{\theta}_s \frac{L C^2}{L C^2 - 0.082} C^2 \theta_s \right) + m_w \left( 2l_1l_4 \phi_L S \phi_L - 2\dot{\theta}_s (C^2 \phi_L - S \phi_L C \phi_L) \right) - \frac{5}{2} m_s gl_4 C \theta_s - \frac{2}{m_s} lps \theta_s - \frac{2}{m_s} lds \theta_s \right] \cdot \left[ m_s \left( \frac{1}{8} L^2 + \frac{1}{24} L^2 + \frac{1}{2} L \dot{A} \right) + 2m_l L^2 C^2 \phi_L + m_w L^2 C^2 \phi_L \right]^{-1}, \]

Eq. 82

\[ \ddot{\phi}_L = \left[ -M_B - M_T - \alpha_B \mu_B \left( L_B \frac{L^4}{L^4 - 0.02} C \phi_L + \phi_L \frac{L^4}{L^4 - 0.02} C^2 \phi_L \right) - \alpha_T \mu_T \left( L_T \frac{L^4}{L^4 - 0.02} C \phi_L + \phi_L \frac{L^4}{L^4 - 0.02} C^2 \phi_L \right) \right] + m_l \left( l_4 \theta_s C \phi_L - l_4 \theta_s^2 C \phi_L S \phi_L \right) + m_w L^2 \ddot{\theta}_s \phi_L C \phi_L - \frac{3}{2} m_l g L 5 C \phi_L - \frac{2}{m_l} L B S \phi_L - \frac{2}{m_l} K T L T S \phi_L \right] \cdot \left[ \frac{1}{4} L^2 + \frac{1}{12} L^2 + \frac{1}{2} L \right]^{-1}. \]

Each of the variables given in Equations 81 and 82 have been previously defined. These equations were then programmed into Matlab as initial value problems and both the coronal plane glenohumeral angle, when applicable, and elbow angles were calculated. A similar process was conducted to determine the muscle constants for the deltoid as its contribution affected the motion of the glenohumeral joint. The muscle moment created by the Deltoid was also estimated in a similar fashion to the procedure outlined in the initial validation test. Again the results from the model output were compared to the actual motion and EMG data collected.

The final validation test required the use of a previously created Matlab program to determine the joint angles and glenohumeral and elbow joint moments of the subject as he pitched. The EMG data collected during this validation trial was also processed as described in the previous analyses. A Matlab program was created containing the equations developed using the Lagrangian approach for the model described in this work. The joint angles, angular velocities, and accelerations, as well as the estimated muscle lengths and anthropometric measurements, were all entered into this Matlab program and the joint moments for the elbow and glenohumeral joints were calculated. These moments were then compared to the joint moments calculated using the validated pitching model. The muscle moments for each of the four muscles were also estimated using the procedures outlined in the second validation test described above and compared to the EMG data collected from the subject.
4.4.2 Data Analysis for Pitching Data

The initial data processing, including tracking and labeling the marker trajectories, was performed using Workstation software. Vicon Bodybuilder was then used to calculate the joint angles using the Euler rotation sequences described above in the description of the pitching model. Following the calculation of the joint angles the C3D file was then read into Matlab, where a custom program was used to filter the data using a fourth order zero-lag Butterworth filter with a cutoff frequency of 15 Hz [44]. The Matlab program was also used to perform all of the calculations for the joint angular velocities and accelerations as well as to calculate the joint moments at the wrist, elbow, and glenohumeral joints. All joint moments calculated in the Matlab program for both the Lagrangian and Newtonian models are internal moments. Finally, the Matlab program normalizes the data to the pitch cycle.

The pitch cycle discussed in this work was originally described by Fleisig et al. [6]. The pitching cycle begins at the point of lead foot contact with the mound (FC) and ends with the beginning of the follow through at the maximum internal rotation of the glenohumeral joint (MIR). The pitching cycle is further broken up using two intermediate time points, where the first of which is the point of maximum external rotation of the glenohumeral joint (MER) and the second is the point of ball release (BR). The pitching cycle is normalized to these time points, as shown in Figure 52, where foot contact is 0% and maximum internal rotation is 100%.
The results from the Matlab program were then entered into the Matlab program created for the final validation testing where the total joint moments, and estimated muscle moments, were calculated for each of the 33 subjects. This data (i.e., the muscle moments for the Pectoralis Major, Deltoid, Infraspinatus, Biceps, and Triceps and gleno-humeral and elbow joint moments) were then exported to an excel file and combined with the data exported from the pitching data collection which included the following variables of interest: ball velocity, trunk angular position, glenohumeral angular position and angular velocity, elbow angular position and angular velocity, and elbow and gleno-humeral joint moments.

### 4.4.3 Statistical Analysis

To compare the model output to the physical data created using previously validated methods for all of the validation tests a Pearson correlation was used. A correlation coefficient of 0.80 or greater was considered as good agreement between the two models.

As an additional comparison of the models, the Lagrangian based model was used to repeat the findings in select previously published papers to determine if the new model produced similar findings to the currently validated model. To determine if the findings of the pitching analysis provided similar messages between the two models, a mixed model random effects regression model was used to determine the relationship between the variables of interest. This model was chosen for a number of
reasons. First, the model has been extensively used in previous pitching analyses [7, 9, 53], and secondly, the model is able to take into account all available trials for each pitcher and not just an averaged trial allowing the model to be more accurate. Furthermore, this model can be extended when needed to control for confounding variables. When the p-value for the regression model was 0.05 or less, the association between the two tested variables was considered to be statistically significant. If an association was found to be statistically significant, the beta value was used to determine the relationship between the two variables.

The same mixed model (i.e., random effects regression model) was also used to determine how the relationship between specified variables and the muscle moments were associated. Again a p-value of 0.05 or less was considered to be a statistically significant finding. All statistical analyses for this work were conducted using SAS 9.3 (SAS, Cary, NC).

4.5 Model Exploration

The final analysis provided in this work is an application of how this model can be used in a clinical setting. The current models available for analyzing pitching data can only provide information based on the motion data collected. To provide feedback to a pitcher on how to modify their pitching mechanics to reduce joint moments is difficult. The Lagrangian based model developed in this work can allow the user to modify input parameters and determine how these modifications can affect the model output, mainly the joint and muscle moments. Therefore, the final analysis randomly chose a single subject and the input variables were modified to understand how the pitcher could modify their pitching mechanics to reduce this joint moment. The model inputs were varied in sequential order starting at the trunk and progressing to the glenohumeral joint. Specifically, the trunk coronal plane lean was modified so as to understand what would happen as the subject increase their lean toward the pitching arm or away from their pitching arm, as well as increasing or decreasing their vertical abduction of the glenohumeral joint, and increasing their glenohumeral internal rotation velocity. Additional anthropometric variables were also varied to show what would happen if the subject gained or lost weight or had surgery or injury.
to the infraspinatus which would change the muscles properties. The model outputs were then analyzed to understand the affect the changes had on the model output and what recommendations could then be made to these subjects.
5.0 Results

The results of this work are presented in five major sections for ease of understanding. These sections include examples of how the equations were used to obtain the results, the results of the validation work, the results of the comparison between the Lagrange model and the validated pitching model, the results of the muscle force analysis, and finally a clinical example showing the utility of the model.

5.1 Examples

5.1.1 Validations

To illustrate how the validation results were obtained the elbow flexion and extension task with a 10lb weight with slow extension was used. The goal was to determine how to calculate the elbow position at any given time during the flexion and extension task. It is important to note that it was assumed that there was no movement at the subject’s trunk, glenohumeral joint, and wrist joint, and that these joints were held in a neutral position.

The first step in solving this problem was to draw the diagram of the system, as shown in Figure 53.
In this figure, mL is the mass of the forearm and hand segments, mA is the additional mass of the 10lb weight, L5 is the length of the forearm, L4 is the length of the upper arm, LB is the length of the Biceps, LT is the length of the Triceps, point A is the glenohumeral joint representing the motion capture marker at the shoulder, point B is the elbow joint representing the motion capture marker at the elbow, point C is the wrist joint representing the motion capture marker at the wrist, and ϕ is the elbow joint angle.

The necessary anthropometric measures for this example were, the subject’s weight (95kg), the subject’s upper arm length as measured from the humeral head to the lateral elbow condyle (0.28m), and the subject’s forearm length as measured from the lateral elbow condyle to the ulnar styloid (0.30m). Each of these measures were taken prior to the start of the data collection. To calculate the segment masses, the table provided by Clauser et al. was used as it provided the estimated weight of each body segment as
a percentage of total body weight [54]. Using Equation 83, it was possible to determine that the mass of the forearm and hand were 1.62kg and, with the additional 10lb weight, the total mass was 6.04kg.

\[ \text{Mass} = (0.017 \times \text{Total Body Weight}) \quad \text{Eq. 83} \]

The next step was to calculate the initial angle of the elbow joint, where the vector between point A and point B, as well as the vector between point B and point C, were required. The coordinates obtained from the C3D file and the initial positions for each marker were: point A (707.6, 59.2, 1414.3) mm, point B (677.0, 62.5, 1117.2) mm, and point C (771.4, -23.7, 863.8) mm. Using these points and Equations 69, 70, and 71, it was possible to determine that the vector from A to B was (30.6, -3.4, 297.1) mm and had a magnitude of 298.6 mm, and that the vector from B to C was (-94.4, 86.2, 253.5) mm and had a magnitude of 283.87. The dot product between the two vectors was calculated to be 72109.9 mm and, when multiplying the magnitudes of both vectors together, it was calculated as 84778.3 mm. Applying Equation 73, it was determined that the initial elbow angle was 0.55rad. Following the calculation of the initial joint angle, the initial joint velocity was calculated. In this case, as the system began at rest, the initial joint velocity was assumed to be zero.

Once the initial elbow angle and joint velocity were determined, the muscle lengths were calculated using the equations for the Biceps and Triceps listed in Table 7. As all variables in this equation have been previously calculated or measured, as described above, the values were substituted and the equations were solved. For the Biceps, the initial length was noted to be 0.297 m and the initial length of the Triceps was 0.263 m. Again, the initial velocity for both the Biceps and Triceps were assumed to be zero.

The constants associated with muscle stiffness, KB and KT, as well as the dampening coefficients, Db and Dt, were experimentally determined through using the process outlined in the previous section in which an initial value was chosen and varied until the model was in good agreement visually with the reference waveform. As previously described the constants were found to be 0.68, 0.14,
0.9, and 1.5, respectively. The resting length of the Biceps was assumed to be 0.29 m and the Triceps was assumed to be 0.26 m based on the measures provided by Holzbaur et al. [51].

The last step prior to solving the differential equation was to calculate the estimated joint moment occurring at the elbow joint as shown in Equation 84,

\[
\text{Moment} = \text{trimoment} + \text{bimoment} + \text{armmoment},
\]

Eq. 84

where \text{Moment} is the estimated joint moment, \text{trimoment} is the estimated triceps moment given by Equation 85, \text{bimoment} is the estimated biceps moment given by Equation 86, and \text{armmoment} is the estimated moment of the arm as given by Equation 87,

\[
\text{trimoment} = mt \cdot (Ll + iT) \cdot \sin(6.28 - \phi),
\]

Eq. 85

\[
\text{bimoment} = mt \cdot (Ll + iB) \cdot \sin(3.14 - \phi),
\]

\[
\text{armmoment} = mt \cdot Ll \cdot \sin(\phi),
\]

Eq. 86

Eq. 87

where \(mt\) is the total mass of the lower arm, hand, and weight, \(Ll\) is the length of the forearm and hand segment as measured from the lateral condyle of the elbow to the knuckle of the third digit (0.38 m), \(iB\) and \(iT\) are the distance of the insertion points from the center of the rotation of the elbow (\(iB=0.02\) and \(iT=-0.02\)), and \(\phi\) is the elbow joint angle. The initial joint moment was calculated to be 1.06 Nm.

Once all these values were determined, the second order differential equation was solved to determine the joint angle. This was done using MATLAB’s ODE23 solver (Mathworks, Natick, MA). The initial conditions determined above and the system of equations in Equations 88a and 88b were solved. A system of equations was necessary as Matlab is unable to directly solve a second order differential equation, and as such, must solve a system of two first order equations. Also of note, a loop is required to produce smooth controlled motion to move the elbow back to its initial position. This loop indicates that, once the elbow angle reaches a value of 2.3 rad, the constants change as there is increased resistance and dampening by the muscles for a controlled decent. The new constants were 4, 6, 15, and 15 for Db, Dt, KB, and KT, respectively.
\[ x(1) = \dot{\phi} \quad \text{Eq. 88a} \]

\[ x(2) = \left[ \text{moment} + 3gl5 \cos \phi + \frac{2KB}{mt} (LB - 0.29) \sin \phi - \dot{\phi}Db \left( \frac{L5}{L5 - iB^2} \right) \cos \phi^2 - LBDb \left( \frac{L5}{L5 - iB} \right) \cos \phi + \frac{2KT}{mt} (LT - 0.26) \sin \phi - \dot{\phi}Dt \left( \frac{L5}{L5 - iT^2} \right) \cos \phi^2 - LTTd \left( \frac{L5}{L5 - iT} \right) \cos \phi \right] \cdot [0.5(L6^2) + 0.1667(L5^2) + L5]^{-1} \quad \text{Eq. 88b} \]

5.1.2 Pitching Model

To demonstrate how the joint moments were calculated for the upper extremity model developed in this work, the calculation for the glenohumeral internal rotation joint moment calculation was used. This example steps through the necessary calculations for the anthropometric measures, joint angle and angular velocities, and the necessary muscle components, where the joint moment for the glenohumeral internal rotation moment was calculated.

The first step to solving this problem was to create a diagram of the geometry of the system. Unlike the validation, which was for understanding the free body system, this diagram was used to better understand the geometry of the system in order to calculate the anthropometric measures required to solve the overall problem. This diagram is seen in Figure 54.
In Figure 54, where C7, T10, STRN, and CLAV are the four markers placed on the subjects thorax, SPC and SAA are the markers placed on the subject’s shoulder, ELB and EMP are the markers placed on the subject’s elbow, WRA, WRB, and FIN are the markers placed on the subject’s hand, as previously described. LS is the length from the sternum to the shoulder. The remainder of the lengths are as previously defined in the methods section of this work. Each of these lengths could be measured directly from the subject but for completeness, each of the measures were calculated using only maker data. The calculations for each of the lengths are listed in Table 10.

Table 10: Calculations for segment lengths

<table>
<thead>
<tr>
<th>Segment</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>$abs(C7_x - T10_x)$</td>
</tr>
<tr>
<td>L2</td>
<td>$abs(\overline{C7}_x - \overline{LSH}_y)$</td>
</tr>
<tr>
<td>L3</td>
<td>$abs(SAA_y - SPC_y)$</td>
</tr>
<tr>
<td>L4</td>
<td>$abs(SJ\overline{C}_x - E\overline{C}_x)$</td>
</tr>
<tr>
<td>L5</td>
<td>$abs(\overline{ELB}_z - WRA_x)$</td>
</tr>
<tr>
<td>L6</td>
<td>$abs(\overline{ELB}_x - EMP_x)$</td>
</tr>
<tr>
<td>L7</td>
<td>$abs(FIN_x - WRA_y)$</td>
</tr>
<tr>
<td>L8</td>
<td>$abs(WRB_y - WRA_y)$</td>
</tr>
<tr>
<td>LS</td>
<td>$abs(T10_x - \overline{LSH}_y)$</td>
</tr>
</tbody>
</table>
In Table 10, the bar over the marker name indicates the mean position over the pitch cycle, the subscript \( x, y, \) and \( z \) indicate the component of the marker coordinate, and \( \text{abs} \) is the absolute value. Following the length calculations, each of the segment masses were calculated in similar manner to that described in the first validation example. The subject’s body weight for this example was 86 kg. The segment masses were calculated using the equations contained in Table 11, where \( ms, ml, \) and \( mw \) are the masses of the upper arm, lower arm, and hand segments, and \( BW \) is the total bodyweight of the subject.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Arm</td>
<td>( ms = 0.029BW )</td>
</tr>
<tr>
<td>Lower Arm</td>
<td>( ml = 0.017BW )</td>
</tr>
<tr>
<td>Hand</td>
<td>( mw = 0.007BW )</td>
</tr>
</tbody>
</table>

The next step in solving the equation for the joint moments was to calculate the joint angles, which were constructed using Euler rotation sequences based on the coordinate systems for each segment and rotation matrices were solved to determine the joint angles based on the most appropriate joint rotation. The joint angle equations used for this example are listed in Table 12, where \( P \) is the proximal coordinate system, \( D \) is the distal coordinate system, and the subscripts \( x, y, \) and \( z \) represent the joint axis of rotation for each coordinate system.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Euler Equations</th>
</tr>
</thead>
</table>
| \( \theta \) | \( \sin^{-1} P_x \cdot D_z \) | \( \sin^{-1} \frac{P_x \cdot D_z}{\cos \phi} \) \\
| \( \phi \) | \( \sin^{-1} \frac{P_y \cdot D_z}{\cos \theta} \) | \( \sin^{-1} P_y \cdot D_z \) \\
| \( \psi \) | \( \sin^{-1} \frac{P_x \cdot D_y}{\cos \theta} \) | \( \sin^{-1} \frac{P_y \cdot D_x}{\cos \phi} \) |

Using these equations, it was possible to calculate the joint angles of the thorax, glenohumeral joint, elbow joint, and wrist. Following the calculation of the joint angles, the angular velocities were then calculated using a simple differentiation as described in Equation 89.
\[ \omega = \frac{y(i+1) - y(i)}{\Delta t}, \]

Eq. 89

where \( \omega \) represents the joint angular velocity, \( y \) is the joint angle at a given instant, \( i \) is the count variable, and \( \Delta t \) is the change in time. In this example, the time step used was 0.004. Using this same method, the angular accelerations were also solved. The next step involved the calculation of the linear velocity of the trunk reference system. This was done by taking the positional data from the motion capture system for the center point between the STRN marker and the C7 marker. The center point was calculated using Equation 90,

\[ \{CP\} = \frac{C7_x + STRN_x}{2} + \frac{C7_y + STRN_y}{2} + \frac{C7_z + STRN_z}{2}, \]

Eq. 90

where \( C7 \) and \( STRN \) are the markers of interest, \( x, y, \) and \( z \) are the \( x, y, \) and \( z \) coordinates of each marker respectively, and the \( \{CP\} \) vector contains the coordinates for the center point between the two markers. Once the center point was established, the linear velocity and accelerations were calculated in a similar manner as described previously using Equation 89.

The last step prior to solving the equation governing the glenohumeral internal rotation moment was to calculate the length and angular velocity for the infraspinatus, which was calculated using the equations previously described in Table 7 of the methods section,

\[ LI = \sqrt{LS3^2 + 0.03^2 - 0.06LS3 \cos \psi_s}. \]

The velocity of the muscle length change was also calculated using a method similar to that described earlier in Equation 89. Finally, the constants for the muscle were obtained. As described earlier, these constants were determined during validation work and it was assumed that the spring constant for the infraspinatus was 11 and the coefficient of dampening was 0.12. Once all of the variables described above in this example were calculated, it was a simple matter of substituting in the appropriate values into equation A3 from the previous section and Appendix A. As in the first example, the most efficient means of solving this problem was through the use of MATLAB.
5.1.3 Muscle Forces

To demonstrate how the muscle force calculations were obtained, the calculation for the Biceps forces collected during the same validation trial, as described in the validation example, was used. In this example, the estimation of the moment created by the Bicep during the elbow flexion and extension task was calculated using the collected anthropometric measures, joint angles and velocities, and muscle lengths and velocity of shortening and elongation. The goal was to determine the Bicep force at a given point within the flexion and extension curve.

The first step in solving the given problem was to draw a diagram of the system, which was the same as the one used to determine the equation to calculate the Biceps moment,

\[
M_B = \text{abs} \left( \alpha_B LBS \phi - \alpha_B \mu_B \left( L_B \frac{L_5}{L_5 - 0.02 \phi L} \phi_L \frac{L_5^2}{L_5^2 - 0.02^2 \phi L} \right) \right)
\]

The next step, once the equation has been derived, is to calculate the necessary anthropometric measures to determine the variables required for calculating the subject’s forearm length (L5), upper arm length (L4 required for a later calculation), and Bicep velocity (LB dot). In this case, the subject’s measures were physically taken and the upper arm segment was found to be 0.28m and the subject’s forearm length was measured at 0.30m. If the subject’s measurements were not available, it was possible that these measurements can be calculated using the marker data. Using marker M1, positioned at point A in Figure 55, and M2, positioned at point B in Figure 55, it is possible to calculate the upper arm length by calculating the distance between the two markers in the Z direction. Using this method, the upper arm segment was found to be 0.275 m. A similar approach can be used for the length of the forearm as well. In this example, the 0.28 m was used.
Once the anthropometric variables have been calculated, the next step is to determine the joint angle and velocity of the change of length of the Bicep. The joint angles were calculated using the approach described in the validation example. Alternatively, if the joint angles were already available from a motion capture system, then those angles could be used directly without the need to calculate the angles. Once the joint angles were calculated, the length of the Bicep needed to be calculated using the equations shown in Table 7 in the methods section and reproduced here.

\[
LB = \sqrt{L_A^2 + 0.02^2 - 0.04LA \cos(3.14 - \phi_L)}
\]

In this example, the joint angle (\(\phi_L\)) used was taken from the 300th frame of data where the joint angle was 0.67 rad. Using 0.28 m for the length of the upper arm (L4), it was determined that the length of the Bicep was 0.26 m.

Following the calculation of the joint angle and the length of the Biceps for the 300th frame of data, the next step was to determine the angular velocity and rate of change of the Biceps length at the 300th frame. To accomplish this, a simple differentiation was used resulting in Equation 91,

\[
V = \frac{y(i+1)-y(i)}{\Delta t}, \quad \text{Eq. 91}
\]
where \( V \) represents the velocity (either angular for the joint or linear for the Bicep), \( y \) is the value of the either the joint angle or Bicep length, \( i \) is the count variable for this example \( i \) is 300, and \( \Delta t \) is the time change in time in this example the time step is 0.008. Using this formula and the previously calculated values, the angular velocity of the elbow joint was calculated as 0.20 rad per second and the velocity of the Bicep was -0.031 m/s, which indicated that the Bicep was shortening.

The last step in calculating the estimated Bicep moment was to determine the coefficients. Using the results from the validation portion of this work the spring stiffness was 0.68 and the dampening coefficient was 0.9, and the alpha coefficient was determined using the equations from Table 9 and reproduced here.

\[
\alpha_B = \begin{cases} 
0.6\phi_l & \text{when } LB \leq 0 \\
0.24\phi_l & \text{when } LB > 0 
\end{cases}
\]

Since \( LB \) is -0.031, \( \alpha_B \) was calculated to be 0.12. The final step was to substitute all of these calculated values into the equation for the Bicep moment and the estimated Bicep moment at the 300\(^{th} \) frame of data is 0.136Nm.

### 5.2 Validation Results

The validation results of this model were based on a single 29 year old male and a single 32 year old right handed male pitcher. The demographic information for both subjects is given in Table 13.

<table>
<thead>
<tr>
<th>Table 13: Subject Demographics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Validation Subject</strong></td>
</tr>
<tr>
<td>Age</td>
</tr>
<tr>
<td>Weight (kg)</td>
</tr>
<tr>
<td>Height (m)</td>
</tr>
</tbody>
</table>

Each subject repeated three trials and the results were averaged for all validation results. The initial validation results compared the models ability to predict simple elbow flexion and extension to a validated measurement obtained from the Vicon motion capture system (Vicon Motion Systems, Los Angeles, CA). This task was repeated for a number of trials with varying weights as previously described. The results of this initial validation testing are presented in Figures 56 through 63 and the
correlation coefficients between the model and optoelectronic motion capture (OMC) data are presented in Table 14. It is important to note that the x-axis on each figure is slightly different to better represent the speed of the motion while still being able to accurately display the characteristics of the waveform.

Figure 56: Comparison between OMC measured position (blue) and Model predicted position (red) for the slow velocity elbow flexion trial with 10 lbs

Figure 57: Comparison between OMC measured position (blue) and Model predicted position (red) for the normal velocity elbow flexion trial with 10 lbs
Figure 58: Comparison between OMC measured position (blue) and Model predicted position (red) for the rapid velocity elbow flexion trial with 10 lbs

Figure 59: Comparison between OMC measured position (blue) and Model predicted position (red) for the slow velocity elbow flexion trial with 15 lbs
Figure 60: Comparison between OMC measured position (blue) and Model predicted position (red) for the rapid velocity elbow flexion trial with 15 lbs

Figure 61: Comparison between OMC measured position (blue) and Model predicted position (red) for the slow velocity elbow flexion trial with 20 lbs
Figure 62: Comparison between OMC measured position (blue) and Model predicted position (red) for the normal velocity elbow flexion trial with 20 lbs

Figure 63: Comparison between OMC measured position (blue) and Model predicted position (red) for the rapid velocity elbow flexion trial with 20lbs
Table 14: Correlations between OMC measured motion and model predicted motion

<table>
<thead>
<tr>
<th>Trial</th>
<th>Correlation between Model and Vicon Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elbow Angle 10lbs- slow</td>
<td>0.96</td>
</tr>
<tr>
<td>Elbow Angle 10lbs</td>
<td>0.98</td>
</tr>
<tr>
<td>Elbow Angle 10lbs- fast</td>
<td>0.99</td>
</tr>
<tr>
<td>Elbow Angle 15lbs- slow</td>
<td>0.97</td>
</tr>
<tr>
<td>Elbow Angle 15lbs- fast</td>
<td>0.98</td>
</tr>
<tr>
<td>Elbow Angle 20lbs- slow</td>
<td>0.95</td>
</tr>
<tr>
<td>Elbow Angle 20lbs</td>
<td>0.96</td>
</tr>
<tr>
<td>Elbow Angle 20lbs- fast</td>
<td>0.98</td>
</tr>
</tbody>
</table>

During the initial validation trials, simultaneous EMG signals were collected, with an example of a raw EMG signal presented in Figure 64.

The EMG estimated muscle forces were compared to the muscle model forces for each of the above tasks and are presented in Figures 65 through 80, with the results of the correlations between the EMG estimated force and the muscle model force are presented in Table 15. It is important to note that the x-axis on each figure is slightly different to better represent the speed of the motion while still being able to accurately display the characteristics of the waveform.
Figure 65: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the slow velocity elbow flexion trial with 10 lbs.

Figure 66: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for the slow velocity elbow flexion trial with 10 lbs.
Figure 67: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the normal velocity elbow flexion trial with 10 lbs

Figure 68: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for the normal velocity elbow flexion trial with 10 lbs
Figure 69: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the rapid velocity elbow flexion trial with 10lbs

Figure 70: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for the rapid velocity elbow flexion trial with 10 lbs
Figure 71: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the slow velocity elbow flexion trial with 15 lbs

Figure 72: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for the slow velocity elbow flexion trial with 15 lbs
Figure 73: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the rapid velocity elbow flexion trial with 15 lbs.

Figure 74: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for the slow velocity elbow flexion trial with 15 lbs.
Figure 75: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the slow velocity elbow flexion trial with 20 lbs

Figure 76: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for the slow velocity elbow flexion trial with 20 lbs
Figure 77: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the normal velocity elbow flexion trial with 20 lbs

Figure 78: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for the normal velocity elbow flexion trial with 20 lbs
Figure 79: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for the rapid velocity elbow flexion trial with 20 lbs

Figure 80: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for rapid slow velocity elbow flexion trial with 20 lbs
Table 15: Correlations between EMG predicted muscle moment and model predicted muscle moments for the Biceps and Triceps

<table>
<thead>
<tr>
<th>Trial</th>
<th>Correlation Coefficients between EMG based muscle moment and Model based muscle moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Biceps</td>
</tr>
<tr>
<td>10lbs-slow</td>
<td>0.93</td>
</tr>
<tr>
<td>10lbs</td>
<td>0.93</td>
</tr>
<tr>
<td>10lbs- rapid</td>
<td>0.86</td>
</tr>
<tr>
<td>15lbs-slow</td>
<td>0.92</td>
</tr>
<tr>
<td>15lbs-rapid</td>
<td>0.86</td>
</tr>
<tr>
<td>20lbs-slow</td>
<td>0.94</td>
</tr>
<tr>
<td>20lbs</td>
<td>0.90</td>
</tr>
<tr>
<td>20lbs-rapid</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The results from the second portion of the validation, the trials in which both elbow flexion and extension occurred simultaneously with glenohumeral vertical abduction and adduction, were again based on the average of three repeated trials. The comparisons between the motion from the predicted model for both the elbow and glenohumeral joints and the OMC output can be found in Figures 81 through 86 and in Table 16. It is important to note that the x-axis on each figure is slightly different to better represent the speed of the motion while still being able to accurately display the characteristics of the waveform.

Figure 81: Comparison between OMC measured position (blue) and Model predicted position (red) for the rapid velocity elbow flexion trial with 5 lbs
Figure 82: Comparison between OMC measured position (blue) and Model predicted position (red) for the rapid velocity glenohumeral vertical abduction trial with 5 lbs

Figure 83: Comparison between OMC measured position (blue) and Model predicted position (red) for the slow velocity elbow flexion trial with 10 lbs
Figure 84: Comparison between OMC measured position (blue) and Model predicted position (red) for the slow velocity glenohumeral vertical abduction trial with 10 lbs.

Figure 85: Comparison between OMC measured position (blue) and Model predicted position (red) for the rapid velocity elbow flexion trial with 10 lbs.
Figure 86: Comparison between OMC measured position (blue) and Model predicted position (red) for the rapid velocity glenohumeral vertical abduction trial with 10 lbs.

Table 16: Correlation between OMC measured motion and model predicted motion for the elbow and glenohumeral joints

<table>
<thead>
<tr>
<th>Trial</th>
<th>Correlation between Model and OMC Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elbow</td>
</tr>
<tr>
<td>10 lbs- Fast</td>
<td>0.95</td>
</tr>
<tr>
<td>10 lbs- Slow</td>
<td>0.97</td>
</tr>
<tr>
<td>5 lbs- Fast</td>
<td>0.97</td>
</tr>
</tbody>
</table>

The EMG data that was collected during these trials for the Biceps, Triceps, and Deltoid were also compared to determine how similar the EMG estimated force was to the predicted force using the model. The results of the EMG analysis for this portion of the results can be found in Figures 87 through 95 and in Table 17. It is important to note that the x-axis on each figure is slightly different to better represent the speed of the motion while still being able to accurately display the characteristics of the waveform.
Figure 87: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for slow velocity dual motion trial with 5 lbs

Figure 88: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for slow velocity dual motion trial with 5 lbs
Figure 89: Comparison between EMG based Deltoid moment (blue) and Model predicted Deltoid moment (red) for slow velocity dual motion trial with 5 lbs.

Figure 90: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for slow velocity dual motion trial with 10 lbs.
Figure 91: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for slow velocity dual motion trial with 10 lbs.

Figure 92: Comparison between EMG based Deltoid moment (blue) and Model predicted Deltoid moment (red) for slow velocity dual motion trial with 10 lbs.
Figure 93: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for rapid velocity dual motion trial with 20 lbs.

Figure 94: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for rapid velocity dual motion trial with 20 lbs.
Table 17: Correlations between EMG estimated moments and model predicted muscle moments for the
dual motion validation trials

<table>
<thead>
<tr>
<th>Trial</th>
<th>Correlation Coefficients between EMG based muscle moment and Model based muscle moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Biceps</td>
</tr>
<tr>
<td>5lbs</td>
<td>0.91</td>
</tr>
<tr>
<td>10lbs-slow</td>
<td>0.85</td>
</tr>
<tr>
<td>20lbs-rapid</td>
<td>0.93</td>
</tr>
</tbody>
</table>

These initial validation tests were also used to calculate the spring and dashpot constants for the
muscle elements included in these models. Each constant was initially set to a value of 0.1 and the
motion of the joints were then plotted and assessed. The constants were then varied one at a time in
increments of 0.1 and the behavior of the model was assessed through visual comparison with the known
joint angle collected using the Vicon system. Once the gross adjustments were made, minor adjustments
were made to bring the model into closer agreement with the known joint angle. A sample of the constant
determination work is included in Figures 96 and 97.
The final validation was based on the result from a single male pitcher and were based on an average of three pitching trials. The results comparing the predicted moments to the moments calculated using the validated pitching model developed by Nissen et al.[44] are presented in Figures 98 through 101 and in Table 18.
Figure 98: Comparison of the glenohumeral coronal plane moment between the Newtonian based pitching model (red) and the Lagrangian based model (blue) (solid vertical line indicates ball release)

Figure 99: Comparison of the glenohumeral sagittal plane moment between the Newtonian based pitching model (red) and the Lagrangian based model (blue) (solid vertical line indicates ball release)
Figure 100: Comparison of the glenohumeral transverse plane moment between the Newtonian based pitching model (red) and the Lagrangian based model (blue) (solid vertical line indicates ball release)

Figure 101: Comparison of the elbow sagittal plane moment between the Newtonian based pitching model (red) and the Lagrangian based model (blue) (solid vertical line indicates ball release)
Table 18: Correlations between the Newtonian pitching model and Lagrangian upper extremity model

<table>
<thead>
<tr>
<th>Joint Moment</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenohumeral Vertical Ab/Adduction</td>
<td>0.51</td>
</tr>
<tr>
<td>Glenohumeral Horizontal Ab/Adduction</td>
<td>0.95</td>
</tr>
<tr>
<td>Glenohumeral Rotation</td>
<td>0.90</td>
</tr>
<tr>
<td>Elbow Flexion/Extension</td>
<td>0.63</td>
</tr>
</tbody>
</table>

The EMG muscle force estimations based on the surface EMG collected during the pitching trials were also compared to the predicted muscle forces using the Lagrangian approach, as shown in Figures 102 through 105 and in Table 19.

![Figure 102: Comparison between EMG based Deltoid moment (blue) and Model predicted Deltoid moment (red) for pitching validation trials](image)
Figure 103: Comparison between EMG based Pectoralis Major moment (blue) and Model predicted Pectoralis Major moment (red) for pitching validation trials

Figure 104: Comparison between EMG based Biceps moment (blue) and Model predicted Biceps moment (red) for pitching validation trials
5.3 Comparison of Pitching Model

As the results from the model developed in this work were somewhat different than the results from the currently used Newtonian based model, it was important to determine if using this model could produce similar results to those currently published in the literature. Since the majority of published works are descriptive in nature, the group data for all 33 pitchers for both the fastball and curveball were compared using the Lagrange based model and the currently used Newtonian based models. These results are included in Figures 106 through 113 and the correlations are presented in Table 20.

Table 19: Correlations between EMG based muscle moments and model predicted muscle moments

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biceps</td>
<td>0.73</td>
</tr>
<tr>
<td>Triceps</td>
<td>0.62</td>
</tr>
<tr>
<td>Deltoid</td>
<td>0.74</td>
</tr>
<tr>
<td>Pectorialis Major</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Figure 105: Comparison between EMG based Triceps moment (blue) and Model predicted Triceps moment (red) for pitching validation trials

Figure 106: Comparison of the fastball glenohumeral coronal plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue)
Figure 107: Comparison of the fastball glenohumeral sagittal plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue).

Figure 108: Comparison of the fastball glenohumeral transverse plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue).
Figure 109: Comparison of the fastball elbow sagittal plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue)

Figure 110: Comparison of the curveball glenohumeral coronal plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue)
Figure 111: Comparison of the curveball glenohumeral sagittal plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue)

Figure 112: Comparison of the curveball glenohumeral transverse plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue)
Figure 113: Comparison of the curveball elbow sagittal plane moment for the group data between the Newtonian based pitching model (red) and the Lagrangian based model (blue)

Table 20: Correlations between the Newtonian Pitching Model and Lagrange Upper Extremity Model using group data

<table>
<thead>
<tr>
<th>Joint Moment</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenohumeral Vertical Ab/Adduction</td>
<td>0.11 (0.88)</td>
</tr>
<tr>
<td>Glenohumeral Horizontal Ab/Adduction</td>
<td>0.65</td>
</tr>
<tr>
<td>Glenohumeral Rotation</td>
<td>0.97</td>
</tr>
<tr>
<td>Elbow Flexion/Extension</td>
<td>0.10</td>
</tr>
</tbody>
</table>

*Values in parentheses are correlations based on Foot contact to Ball release only*

It is important to note that the moments calculated using the Lagrangian approach include the muscle elements as well. When removing the muscle elements from the joint moments, especially at the elbow as both primary drivers (i.e., the Biceps and Triceps) were included, the results of the model significantly change as noted in Figure 114 and the improved correlation value of 0.54.
As many of the currently published works are descriptive observational studies, there were only a small handful of parameters that could be easily repeated and verified using this model, mainly the comparison between joint moments based on different pitch types and the effect of coronal plane trunk lean on the glenohumeral internal rotation moment. Therefore, the model results for the glenohumeral internal rotation moment for the fastball and curveball were compared to determine if there was a statistically significant difference between the two pitch types. The results showed that there was a statistically significant difference between the two pitch types, with the fastball producing a greater internal rotation moment than the curveball (p=0.002). This finding was then compared with currently published literature to ensure that the statistical findings were similar. The comparison between results from this model and the published works can be found in Table 21.

Table 21: Comparison among this work and currently published works comparing the fastball and curveball Glenohumeral internal rotation moment

<table>
<thead>
<tr>
<th></th>
<th>Fastball Moment Internal Rotation Moment (Nm)</th>
<th>Curveball Moment Internal Rotation Moment (Nm)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current work</td>
<td>83.5 ± 18.4</td>
<td>79.4 ± 17.6</td>
<td>0.002</td>
</tr>
<tr>
<td>Solomito et al. [9]</td>
<td>80.5 ± 15.5</td>
<td>76.3 ± 15.8</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Fleisig et al. [45]</td>
<td>84 ± 13</td>
<td>81 ± 14</td>
<td>&lt;0.01</td>
</tr>
</tbody>
</table>
A similar comparison was used to determine if trunk lean affected the joint moments. The results of the regression analysis showed that, for every 10° increase in lateral trunk lean away from the pitching arm, there was a 1.5Nm increase in the glenohumeral joint moment (p<0.001). This finding is also consistent with Solomito et al. whose results indicated that for every 10° increase in lateral trunk lean away from the pitching arm there was a 2.5Nm increase in the glenohumeral joint moment (p<0.001) [53].

5.4 Muscle Force Analysis Results

The muscle force descriptive results were calculated for the total group of pitchers for both the fastball and the curveball. These results are presented in Figures 115 through 124.

Figure 115: Biceps moment for the fastball pitches based on group data
Figure 116: Triceps moment for the fastball pitches based on group data

Figure 117: Deltoid moment for the fastball pitches based on group data
Figure 118: Pectoralis Major moment for the fastball pitches based on group data

Figure 119: Infraspinatus moment for the fastball pitches based on group data
Figure 120: Biceps moment for the curveball pitches based on group data

Figure 121: Triceps moment for the curveball pitches based on group data
Figure 122: Deltoid moment for the curveball pitches based on group data

Figure 123: Pectorialis Major moment for the curveball pitches based on group data
As there is currently no published data describing the contributions of the various muscles to the total joint moments, this section does not compare the results to other validated results; rather, this section follows a similar analysis pattern that is commonly used to analyze the joint moments in pitching analyses.

The first comparison was between the muscle moments between the fastball and the curveball. The results indicated that the fastball produces the greatest muscle moments for all muscles analyzed in this work with the noted exception of the infraspinatus, as presented in Table 22.

Table 22: Comparison between peak muscle moments of the fastball compared to the curveball

<table>
<thead>
<tr>
<th></th>
<th>Peak Fastball Moment (Nm)</th>
<th>Peak Curveball Moment (Nm)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biceps</td>
<td>30.4 ± 4.2</td>
<td>28.3 ± 4.1</td>
<td>0.008</td>
</tr>
<tr>
<td>Triceps</td>
<td>31.4 ± 4.4</td>
<td>28.6 ± 4.2</td>
<td>0.006</td>
</tr>
<tr>
<td>Deltoid</td>
<td>38.5 ± 4.9</td>
<td>36.4 ± 4.3</td>
<td>0.003</td>
</tr>
<tr>
<td>Pectoralis Major</td>
<td>36.9 ± 9.9</td>
<td>30.8 ± 9.0</td>
<td>0.030</td>
</tr>
<tr>
<td>Infraspinatus</td>
<td>42.7 ± 3.5</td>
<td>45.7 ± 3.2</td>
<td>0.031</td>
</tr>
</tbody>
</table>

As the fastball muscle moments were found to be greater than the curveball moments in most of the muscles groups and most current literature assumes that pitching the fastball presents a greater chance for the pitcher to be injured than when they are pitching the curveball, the remaining analyses in this
section were limited to analysis of the fastball data only. Regression analyses were performed to determine how ball velocity, segment angles (i.e., thorax angles), joint angles, joint angular velocities, and joint moments affected the five muscle elements included in the model. Ball velocity was found to be significantly associated with the muscle contributions to the joint moments for all of the muscle elements except for the Pectoralis Major and Infraspinatus, as shown in Table 23. Similarly the majority of the joint moments were found to be highly associated with the muscle contributions to those moments again with the noted exceptions of the Pectoralis Major and Triceps, as shown in Table 24.

<table>
<thead>
<tr>
<th></th>
<th>p-value</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biceps</td>
<td>0.019</td>
<td>0.14</td>
</tr>
<tr>
<td>Triceps</td>
<td>0.032</td>
<td>0.22</td>
</tr>
<tr>
<td>Deltoid</td>
<td>0.047</td>
<td>1.7</td>
</tr>
<tr>
<td>Pectorialis Major</td>
<td>0.098</td>
<td>--</td>
</tr>
<tr>
<td>Infraspinatus</td>
<td>0.078</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 24: Results of regression analysis between the peak muscle moments and joint moments.

<table>
<thead>
<tr>
<th></th>
<th>p-value</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biceps</td>
<td>0.011</td>
<td>0.19</td>
</tr>
<tr>
<td>Triceps</td>
<td>0.056</td>
<td>--</td>
</tr>
<tr>
<td>Deltoid</td>
<td>0.035</td>
<td>0.06</td>
</tr>
<tr>
<td>Pectorialis Major</td>
<td>0.783</td>
<td>--</td>
</tr>
<tr>
<td>Infraspinatus</td>
<td>0.006</td>
<td>0.24</td>
</tr>
</tbody>
</table>

When looking at the Biceps moment, it was found to be highly associated with elbow flexion at foot contact for every 10° increase in elbow flexion increased the Biceps moment by 1.9Nm (p=0.024). Similarly, increased elbow flexion at the instant of ball release was also found to have a significant association with the Biceps moment indicating a 1.4 Nm increase in moment for every 10° increase elbow flexion (p=0.011). It was also noted that those pitchers with greater peak elbow flexion velocities had increased Biceps moments with an increase of 2 Nm per 100%/s increase in peak flexion velocity (p=0.005). The Triceps moment had similar associations as described for the Biceps. Increased elbow flexion at foot contact was shown to increase the peak Triceps moment and, for every 10° of increased flexion, the Triceps moment increased by 2.2 Nm (p=0.022). At the instant of ball release, the Triceps showed an inverse relationship with elbow flexion and, as elbow flexion was increased at ball release, the
Triceps moment was decreased. For every 10° increase in elbow flexion the Triceps moment was reduced by 5 Nm (p=0.013). Finally, as elbow extension speed increased, so did the Triceps moment. The association indicated that, for every 100°/s increase in elbow extension velocity, the Triceps moment increased by 3 Nm.

The Deltoid moment showed no associations with the coronal position of the trunk at any time in the pitching cycle and was found to be associated with the vertical abduction of the glenohumeral joint, where for every 10° increase in glenohumeral vertical abduction, the Deltoid moment increased by 1.7 Nm (p=0.049). Notably, the Pectorialis Major was found to have no associations with ball velocity or joint moments, as described earlier, nor did it have any associations with coronal plane trunk position or glenohumeral horizontal abduction or adduction.

Finally, the Infraspinatus had no association with glenohumeral rotational position; however, it had a significant association with glenohumeral internal rotation velocity. The results of the regression analysis indicated that, for every 100°/s increase in glenohumeral internal rotation velocity over the mean, the Infraspinatus moment increased by 2 Nm (p<0.001).

5.5 Clinical Example

A subject was selected at random using a random number generator in Excel and the subject’s information is included in Table 25.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>1.74</td>
</tr>
<tr>
<td>Weight (kg)</td>
<td>81.8</td>
</tr>
<tr>
<td>Hand</td>
<td>Right</td>
</tr>
<tr>
<td>Age</td>
<td>19.5</td>
</tr>
<tr>
<td>Experience</td>
<td>Division I college (12 years pitching experience)</td>
</tr>
<tr>
<td>Ball Velocity (mph)</td>
<td>74</td>
</tr>
</tbody>
</table>

Various input parameters were then varied to determine the response of the model to the variations, specifically the effects of the variations on the glenohumeral rotation moment, elbow sagittal plane moment, Infraspinatus moment, and Triceps moment. The results indicated that variations in the
coronal plane trunk position influenced the joint moments but did not influence the contributions of the two muscles included in this analysis, as shown in Table 26.

Table 26: Results of modifications in coronal plane thoracic position

<table>
<thead>
<tr>
<th></th>
<th>Reference</th>
<th>Increased Trunk Lean</th>
<th>Decreased Trunk Lean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenohumeral Internal Rotation Moment</td>
<td>53.9</td>
<td>59.8</td>
<td>41.1</td>
</tr>
<tr>
<td>Elbow Moment</td>
<td>16.2</td>
<td>20.1</td>
<td>13.9</td>
</tr>
<tr>
<td>Triceps Moment</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Infraspinatus Moment</td>
<td>54.3</td>
<td>54.3</td>
<td>54.3</td>
</tr>
</tbody>
</table>

Variations in the glenohumeral joint angle and glenohumeral joint angular rotation velocity were also shown to influence the glenohumeral rotation moment and the contribution from the Infraspinatus but did not influence the elbow joint moments nor the contributions from the Triceps, as shown in Table 27.

Table 27: Results of modifications in glenohumeral parameters

<table>
<thead>
<tr>
<th></th>
<th>Reference</th>
<th>10° increase glenohumeral coronal plane angle</th>
<th>10° decreased glenohumeral coronal plane angle</th>
<th>500°/s increased glenohumeral rotation velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenohumeral Internal Rotation Moment</td>
<td>53.9</td>
<td>54.2</td>
<td>53.4</td>
<td>70.9</td>
</tr>
<tr>
<td>Elbow Moment</td>
<td>16.2</td>
<td>11.8</td>
<td>18.8</td>
<td>16.2</td>
</tr>
<tr>
<td>Triceps Moment</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Infraspinatus Moment</td>
<td>54.3</td>
<td>54.3</td>
<td>54.3</td>
<td>68.1</td>
</tr>
</tbody>
</table>

Variations in the subject’s anthropometric measures, such as changing the subject’s weight, had substantial changes in the joint moments but did not seem to directly influence the peak moments for either the infraspinatus nor the Triceps as shown in Table 28. Changes in the stiffness and dampening of the infraspinatus influenced the glenohumeral internal rotation moment as well as the Infraspinatus moment, as described in Table 29, but did not influence either the elbow moment or the Triceps moment. Finally, a change in length of the infraspinatus influenced the Infraspinatus’ moment but did not seem to influence any of the three other moments of interest, as presented in Table 30.
### Table 28: Results of modifications in modifying weight parameters

<table>
<thead>
<tr>
<th></th>
<th>Reference</th>
<th>10 kg increase in subject weight</th>
<th>10 kg decrease in subject weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenohumeral Internal Rotation Moment</td>
<td>53.9</td>
<td>60.5</td>
<td>47.3</td>
</tr>
<tr>
<td>Elbow Moment</td>
<td>16.2</td>
<td>18.2</td>
<td>14.2</td>
</tr>
<tr>
<td>Triceps Moment</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Infraspinatus Moment</td>
<td>54.3</td>
<td>54.3</td>
<td>54.3</td>
</tr>
</tbody>
</table>

### Table 29: Results of Modifications to muscle constants

<table>
<thead>
<tr>
<th></th>
<th>Reference</th>
<th>100% increase in Infraspinatus stiffness</th>
<th>50% decrease in Infraspinatus stiffness</th>
<th>100% increase in Infraspinatus dampening</th>
<th>50% decrease in Infraspinatus dampening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenohumeral Internal Rotation Moment</td>
<td>53.9</td>
<td>55</td>
<td>51</td>
<td>98</td>
<td>41</td>
</tr>
<tr>
<td>Elbow Moment</td>
<td>16.2</td>
<td>16.2</td>
<td>16.2</td>
<td>16.2</td>
<td>16.2</td>
</tr>
<tr>
<td>Triceps Moment</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Infraspinatus Moment</td>
<td>54.3</td>
<td>60.0</td>
<td>51.4</td>
<td>102.9</td>
<td>30.0</td>
</tr>
</tbody>
</table>

### Table 30: Results of modifications in Infraspinatus length

<table>
<thead>
<tr>
<th></th>
<th>Reference</th>
<th>2cm decrease in Infraspinatus length</th>
<th>2cm increase in Infraspinatus length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenohumeral Internal Rotation Moment</td>
<td>53.9</td>
<td>53.9</td>
<td>53.9</td>
</tr>
<tr>
<td>Elbow Moment</td>
<td>16.2</td>
<td>16.2</td>
<td>16.2</td>
</tr>
<tr>
<td>Triceps Moment</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Infraspinatus Moment</td>
<td>54.3</td>
<td>54.9</td>
<td>53.6</td>
</tr>
</tbody>
</table>
6. Discussion

The purpose of this work was to create and validate a Lagrangian based model to describe the motion of the human upper extremity, as well as to estimate the muscle moments driving these motions. The model created was designed to be a simple, accurate model that could be easily modified or extended for a number of different biomechanical applications. Furthermore, the model was designed to be used with previously collected motion capture data to allow for additional information to be gathered without the need to recollect data.

While there are a number of models described in current literature; almost none of these models provide the ability to estimate muscle forces or the contributions of muscles to the total joint moment. Those models that have been previously described in the literature are typically two-dimensional models, or are very limited models that have specific applications rather than providing an adaptable model. There is one model created by Ivancevic et al. that is very complex and capable of taking into account a number of different variables including muscles and cognitive control; however, their model has not provided examples for the model’s utility, beyond the use for describing spine biomechanics. Commercially available models are extremely complex and difficult to utilize without some familiarity with the program. Furthermore, these commercially available models are typically based on idealized models, and creating patient specific models can become cumbersome due to the number of parameters that need to be adjusted. Therefore the model developed in this work was designed to be simple enough to be easily modified for individuals, or the model can be based on an idealized data as well. This model can also be used to understand how variations in a pitcher’s mechanics can lead to a reduction in joint stresses and their risk of injury.

The results of this work indicated that the Lagrangian based upper extremity model was capable of producing the desired results. It accurately predicted simple upper extremity motions based on initial conditions, as well as accurately calculating joint moments and muscle moments in complex motions (i.e. baseball pitching). The application of this model to previously collected pitching based data indicated that it was possible to use the Lagrangian based model to provide additional information to better
understand how pitching mechanics could lead to injury, by gaining an understanding of the stresses placed on the soft tissue structures response to the motions of the upper arm required for baseball pitching. Finally, the results of this model also indicated that the applications of this model could be clinically utilized to reduce joint moments, and to understand how surgical intervention or injury to the muscles could affect the joint stresses and the muscle forces driving the pitching motion. It is also important to note that the programming of the Lagrangian model was simpler and more direct allowing for shorter run times than those associated with the Newtonian based pitching model. Overall there was a 37% decrease in the number of lines required to code the Lagrangian model.

6.1 Validation

The validation studies in this work were essential for two reasons. The first was to determine the constants required for the muscle stiffness and dampening, and the second purpose was to prove that the model was able to provide the expected results in a number of different scenarios. As such the validation work was designed to become progressively more taxing to the model.

Prior to discussing the results of the validation work, it is important to point out that the determination of the muscle coefficients was necessary to ensure that the validation results were properly calculated. The validation model for the elbow flexion extension task was originally modeled without any muscle attachments which resulted in a repeated undamped pattern similar to what one would see from a pendulum swinging without resistance. Once the muscle elements were added this oscillation was reduced and became more in line with what was expected. The initial coefficients for the muscle elements were set to previously published data and then varied slightly from there until the motion patterns came into alignment with the expected motion profile. This was accomplished through modification of an initial value and visual inspection, and as such required a number of attempts to fine tune the coefficients. Once the coefficients were set for the initial portion of the motion, the coefficients were then varied for the second portion of the motion. For example, the coefficients related to the Biceps were first solved for during the flexion portion of the task and then the coefficients for the extension
portion of the task were solved. The reason for this two-step approach was to attempt to mimic the change from an agonist to an antagonist within the movement. If the models had included a parameter for neuromuscular control this two-step process may not have been necessary; however, since control mechanisms were outside the scope of this work this method was deemed an appropriate analogue to the neuromuscular control mechanism. Without the change in coefficients the motion profiles dampened quickly and never returned to their resting states.

Following the calculations of these coefficients the parameters were checked by using these same coefficients to calculate the muscle moments during the same tasks. The EMG based muscle moment estimate was then compared to the calculated muscle moment. In every case, the coefficients did not need to be modified as the agreement between the EMG estimate and model estimate were satisfactory. These results allowed for the assumption that the muscle coefficients were appropriately determined for each muscle group.

The elbow flexion task was the simplest of the validation tests and was designed to reduce the model created in this work to a simple two dimensional model in which only one parameter, the sagittal plane elbow angle, was predicted for a number of different scenarios including different flexion velocities and various external weights. The elbow flexion results, as described earlier, showed excellent correlations between the predicted motion and that of the motion measured using the motion capture system. With each of the correlation coefficients being greater than or equal to 0.95, the model was capable of accurately determining the joint motion given a set of initial conditions. On closer examination of the plots used to compare the model output to that of the motion capture system there are a number of differences. Firstly, in each of the rapid trials with the exception of the rapid 10 lb elbow flexion task, the model under estimated the peak flexion angle indicating that the system may have been slightly over damped. The reason for the excellent agreement in the 10 lb elbow flexion task could be attributed to the fact that the muscle coefficients were calculated using the 10 lb flexion tasks; therefore, it stands to reason that these trials would have better agreement. This result also points to the fact that the
coefficients do not behave in a linear fashion as assumed in this model; this finding is further discussed in the limitations portion of this work.

It is also important to point out that in each of the elbow flexion tasks; the velocity of flexion predicted by the model was more rapid than the actual motion profile determined through the use of the motion capture system. This can most easily be visualized by looking at the slope of the slow elbow angles for both the 15 and 20 pound weight conditions. A similar finding can also be noted for the extension velocity for each trial in which the model again predicts a more rapid return to resting position than the actual motion. This finding could be attributed to the lack of neuromuscular control loops within the model. The model predicts the motion based on a set of initial conditions and with a linearized function governing the muscle element coefficients. What the model does not take into account are, the number of small neuromuscular control loops that occur in the human body to ensure a steady controlled motion that will not cause undo stresses on the soft tissue. Since this control mechanism was not included in the model, the predicted motion profiles are based solely on movement patterns.

The next validation testing was designed to further stress the model by including two moving joints, though these joints were still moving in only one plane of motion. Therefore, this test determined the ability of the model to predict the motion of both the elbow in the sagittal plane and the glenohumeral joint in the coronal plane. This validation testing also increased the number of muscle elements to three as there were two muscle elements helping guide the motion of the elbow as in the previous test and an additional element helping guide the motion of the glenohumeral joint. Again this validation testing was carried out for a number of different scenarios including different weights and speeds. The results indicated that once again the model’s ability to predict the motion of the elbow was excellent, with correlations of 0.95 and greater. However, the model’s ability to predict the motion of the glenohumeral joint was not as accurate as the elbow motion with correlations of 0.58 to 0.91. Interestingly, the model was able to accurately predict the abduction of the glenohumeral joint but it was not able to predict the movement back toward the resting position as accurately. It is important to note that during the abduction
phase of the motion the correlations were much higher and in line with those seen with the movement of the elbow with correlations ranging between 0.83 and 0.93.

Much of the inaccuracies with the prediction of the positioning of the glenohumeral joint can be attributed to the inclusion of the muscle elements at the glenohumeral joint. As previously described, the glenohumeral joint is an extremely complex joint that requires a number of soft tissue elements including tendons, ligaments, and muscles to provide the joint with stability. Since the model used in this specific validation test only tested coronal plane motion the primary muscle, the Deltoids, involved in coronal plane glenohumeral motion was included. Therefore a number of soft tissue structures that help provide stability and control the motion of this joint were excluded, and this is believed to produce the underdamped appearance occurring during the adduction phase of the motion. This theory is further supported by the fact that the Deltoid is the primary muscle for abducting the glenohumeral joint, and so it’s inclusion in the model produces a very accurate representation of the joints abduction as shown by the high correlation values. In contrast, the Deltoid does not aid in adduction of the glenohumeral joint nor is it the primary antagonist for this motion, and as such the ability to predict the adduction of the joint’s motion was reduced. The initial agreement within the glenohumeral adduction phase could be caused by the resistance to the motion provided by the Deltoid; however, at a certain point within the motion the resistance of the Deltoid was not enough to fully dampen the motion and adduct the humerus. This example of a limitation in the model is ideal for illustrating the need to understand how limitations in the model could affect both the accuracy of the model as well as how to interpret the results of the model.

In contrast to the glenohumeral joint, the elbow motion was accurately predicted by the model in both the flexion and extension phases of the motion. This can be attributed to the fact that the primary flexor of the elbow, the Biceps, and the primary extensor, the Triceps, were both included in the model and thus the motion profile is a more accurate representation of how the elbow was moving. This fact lends further support to the idea that much of the inaccuracies in the adduction phase of the glenohumeral joint can be attributed to the limited number of muscle elements used. Therefore, to improve the accuracy
of this model future work should be directed to including additional soft tissue elements at the shoulder joint complex to provide a more physiologically appropriate action.

It is also important to note that when reviewing the plots of the glenohumeral joint motion the model tends to overestimate the peak abduction angle which could again be attributed to the fact that the muscle elements included in this model do not provide enough resistance and dampening to the motion. Furthermore, as seen in the elbow flexion tasks previously discussed the velocity of the abduction and adductions motions were slightly greater than those noted in the actual motion of the joint as measured by the motion capture system. This can again be attributed to the lack of neuromuscular control elements within the model.

In both of these validation tests EMG data was also collected. This EMG data was then used to estimate the muscle moments driving the motion of the joints. The initial comparisons between the calculated muscle moments and the EMG estimated muscle moments were based on the first validation testing looking at the elbow flexion and extension task; therefore the initial tests looked at the Biceps and Triceps only. The muscle moments for Biceps show a similar waveform for each flexion and extension trial. There is an initial large moment during the early portion of the wave form that coincides with the rapid flexion of the elbow. As the elbow reaches its maximum flexion the moment drops to zero, this is expected since the line of action moves through the rotation center of the joint. Then as the elbow moves into the elbow extension phase there was a secondary moment peak that coincides with the resistance to the extension movement of the joint. This secondary peak was expected as the Biceps were providing passive resistance as the elbow was extended; therefore, this peak should be lower than the initial peak which was created by the active contraction of the Biceps. The muscle moment for the Triceps also show a consistent pattern from trial to trial, and were the inverse of the Biceps pattern with a larger secondary moment and lower initial moment. Again, this makes sense as the initial moment coincides with the passive resistance of the Triceps to the initial flexion of the elbow joint. The moment again goes to zero at the peak of the flexion curve, followed by the second larger moment that coincides with the extension of the arm as the Triceps are active.
The correlations between the Biceps EMG based moment and calculated Biceps moment were excellent with correlation coefficients ranging between 0.86 to 0.94, the only exception to this was the 20lb rapid flexion trial in which the correlation coefficient was only 0.72. Overall the model showed excellent agreement with the measured values and also maintained a similar waveform pattern. The model showed good agreement with the initial burst of activity from the Biceps, though the more rapid the motion the more the model tended to underestimate the peak force. Likewise the model tended to always underestimate the second burst of activity from the Biceps. Interestingly, the lowest correlation values were all found in the rapid flexion trials, and the highest correlations were noted in the slow flexion trials. This finding could be attributed to the idea that the muscle constants were not linear as assumed in this model and since the constants were initially established using the slow velocity trials it stands to reason that these would be most accurate. As the speed of the contraction for the Biceps increased, the model parameters did not accurately reflect the non-linear nature of the muscle tissue and therefore underestimated the secondary burst attributed with the antagonist action of the muscle. This underestimation of the secondary burst was the primary cause for the lower correlation values seen in the rapid flexion and extension trials. Another cause for the lower correlation values especially in the rapid trials could also be attributed to the presence of the increased signal noise from the EMG collected data which was harder to mimic in the model.

The agreement between the Triceps EMG based moment and calculated Triceps moment were well correlated but not as well correlated as the Biceps moments were, with correlation values ranging between 0.59 and 0.93. Similar to the Biceps’ results the moments for the Triceps maintained similar waveforms between the EMG based moments and the model calculated moments. Unlike the Biceps, the Triceps moments lowest correlations were with the slow trials, especially with the higher weights (i.e. 15 and 20lbs trials). One factor that could be attributed to this was that the slower motions required much greater control during both the flexion and extension phases of the trial. Therefore, the linear functions used to calculate the coefficients of the muscle overestimated the passive resistance producing an increased moment for a greater duration than was actually present.
The second set of validation tests, in which the model analyzed both the coronal plane motion of the glenohumeral joint as well as elbow flexion, included EMG collection for the Biceps, Triceps, and Deltoid of the subject. The results were very similar to the previous analysis looking solely at the Biceps and Triceps. In this analysis the Biceps and Triceps again had similar waveforms as described in the initial validation. The Deltoid moment waveform was consistent from trial to trial. The Deltoid also had a two burst pattern moment in which the first larger burst coincided with the abduction of the glenohumeral joint. As the glenohumeral joint neared its maximum abducted position the Deltoid moment dropped to nearly zero; again this makes intuitive sense as the line of force is moving closer to the rotation center of the joint and so the moment nears zero. As the glenohumeral joint moves into adduction and returns to its initial position, the Deltoid was no longer active but rather provided passive resistance to control the descent of the humerus. This adduction movement coincides with a much reduced second moment peak in comparison to the initial moment peak.

The Biceps, Triceps, and Deltoid calculated moments were all well correlated with the EMG estimated moments. The Biceps and Deltoids showed nearly equal agreement with correlations ranging between 0.85 to 0.93 and 0.86 to 0.92 respectively. The lowest correlations were associated with the Triceps in which the correlation values ranged from 0.75 to 0.88. Overall, the Deltoid waveforms from the EMG based estimate and the model estimate were nearly identical, the only difference was that the model estimate tended to under estimate the peak moments in both the initial and secondary bursts. Again, these under estimated peaks could be caused by the muscle constants as previously discussed in the preceding paragraphs. The other possible cause for the lower correlations in the Deltoid was the presence of the noise in the EMG signal. The patterns of the Biceps and Triceps were similar to those already discussed with the initial validation test.

The final validation testing in this work was, to use the model for its initial purpose of calculating joint moments as well as estimating the muscle moments during complex upper extremity motions such as those found when a person is pitching. The data collected during this validation testing included both motion capture data as well as surface EMG from four muscle groups, the Biceps, Triceps, Deltoid, and
the Pectoralis Major. The joint moments at the glenohumeral joint for each of the three planes of motion as well as the sagittal plane elbow joint moment were calculated using the currently validated and published model by Nissen et al. [44]. The EMG data was then processed in a similar manner as previously described using the equations by Marras et al. to calculate the EMG estimated muscle moments[19]. These data were then compared to the current model output. The model results showed good agreement with the validated model output when comparing the glenohumeral rotation moment and the glenohumeral horizontal abduction and adduction moments. However, the glenohumeral vertical abduction and adduction moment as well as the sagittal plane elbow moment were noted to have much lower correlations with the validated moment output.

The results of the validation testing for the glenohumeral vertical abduction and adduction moment had a correlation coefficient of only 0.51 when compared to the Newtonian based model. However, the initial portion of the pitching cycle from the instant of foot contact through the early acceleration phase showed a very similar pattern between both models. Similar patterns were also noted in the deceleration phase from around 80% of the pitching cycle through maximum internal rotation of the glenohumeral joint (MIR). During this phase the correlation improves to 0.91. The main difference between the two model outputs occurs from about 60 to 75% of the pitching cycle. Interestingly, this time period typically corresponds to the maximum external rotation of the glenohumeral joint (MER) through ball release. Ball release occurs at 68% of the pitching cycle for this subject and this corresponds with the greatest difference in the models. There are two potential causes for this firstly the Newtonian model was specifically designed for pitching and therefore takes into account the mass of the ball as well as removing the mass of the ball following ball release. As the current model developed in this work was initially intended to work for any upper extremity motions, the mass of the ball was neglected. When a pitcher reaches MER the ball is as far behind the rotation center of glenohumeral joint as possible, and at the instant of ball release the ball is as far forward of the rotation center of the glenohumeral joint that it would have the greatest impact on the calculated joint moment. Therefore, by neglecting the weight of the ball in the Lagrangian based model, the joint moment was representative of the stresses occurred from
the upper extremity only and not additional outside weights. The second potential cause is that the Newtonian based model’s glenohumeral joint calculation is slightly different than the one created in this work, as the glenohumeral moment in the Newtonian based model takes into account a thoracic-scapular component as well, whereas this model only takes into account the motion of the glenohumeral joint. Furthermore the Newtonian based model does not take into account any of the soft tissue elements at the shoulder, whereas the Lagrangian model includes three muscle structures, two of which act in the coronal plane.

The results of the validation testing for the glenohumeral horizontal abduction/adduction moment showed much better agreement between the two models with a correlation of 0.95. The model in this work tends to under estimate the minimum value of the moment that occurs around 40 to 50% of the pitching cycle. The improved correlation between the two models can be attributed to the fact that the range of motion that occurs in this plane is negligible in comparison to the other two planes of motion. Much of the motion occurs due to the glenohumeral joint and only some scapular motion, which is more consistent with how the Lagrangian based model developed in this work was created. The improvement in the model agreement can also be attributed to the fact that the muscle elements included in the current model do not factor into this motion as much as in the glenohumeral rotation and vertical abduction, and therefore the modeling here is much more similar to that in the Newtonian model.

The results of the validation testing for the glenohumeral rotation moments also showed excellent agreement between the Lagrange model and the Newtonian based model with a correlation of 0.90. Interestingly, there were two major deviations of note between the two models. The Lagrangian model indicated a more exaggerated moment during the first 30% of the pitching cycle, and the moment reached its peak amplitude more rapidly than in the Newtonian model. Again these differences could be attributable to the fact that the mass of the ball was neglected in the Lagrangian model. Furthermore, if this subject relies on scapular motion to assist with the rotation of his arm, specifically in the early portions of the pitching cycle, the Lagrangian based model does not include this scapular motion and as
such would produce a somewhat different motion; however, overall the two moment profiles were similar and suggest similar loading patterns.

The results of the elbow sagittal plane moment produced one of the lower correlations between the Newtonian model and the Lagrangian based model with a correlation of 0.63. The overall shapes of the two moment profiles were somewhat similar; however, there were differences noted in the early portion of the pitching cycle, from 0 to 30% of the pitching cycle as well as from 60 to 90% of the pitching cycle. While the modeling for the elbow was nearly identical between the two models, the Lagrangian based model does include both the Triceps and Biceps as additional elements which add both resistance and dampening to the motion and therefore could be the cause of some of the differences noted between the two models. This theory was tested using the group based data in which the muscle moments for the Biceps and Triceps were subtracted from the total sagittal plane elbow moment. The resulting moment when compared to the Newtonian based model showed a much higher correlation. This is discussed further in the following section.

The muscle moment estimates showed moderate to excellent agreement between the EMG based estimates and the calculated moments from the model. It is important to note that the same process that was described during the initial validation to determine the muscle coefficients was used again for each of the muscle elements within the pitching validation. The starting points for each of the coefficients were based on the values determined during the simple validation tasks. The coefficients were then varied slightly to bring the estimated muscle moments more in line with the EMG based estimates. These coefficients were then used for all of the subsequent muscle moment calculations. Overall the muscle moments had correlation coefficients between 0.61 and 0.89, with the lowest correlation found with the Triceps and the highest correlation with the Pectorialis Major.

The comparison between the Triceps EMG based moment and the model calculated moment showed the lowest correlation. The model results produced a small time shift in the data producing a delayed peak. The model results also had a delayed rise in the peak, the EMG estimated moment showed that the peak began around 58% of the pitching cycle and quickly rose to the peak value. The calculated
Triceps moment had a slow rise from 58% to 71% and then quickly rose to the peak amplitude. It is important to note that in both calculations the peaks were the same as was the rate that the moment decreases back to zero prior to having a small secondary rebound late in the pitch cycle. The differences in timing can be attributed to the fact that the EMG signal was collected at 1800Hz while the motion data, on which the calculated muscle moment was based on, was collected at 250Hz. During the EMG based calculations the signal was down sampled so as to be on the same time scale as the calculated moment. Therefore, this could have led to the time shift. The differences in the rate at which the two models reach the peak moment could be attributed to the fact that the muscle coefficients were assumed to be constant and linearly related to the angular velocity of the joint, where in reality this relationship is non-linear.

The comparison between the Biceps EMG based moment and the model calculated moment produced a higher correlation than found in the comparison between the Triceps moments with a correlation of 0.73. Again the behavior of the model showed nearly identical differences between the EMG based calculations and the model calculations produced muscle moments as was seen with the Triceps. The same time shift and slow rise to the peak were present again indicating that the time shift could be caused by the down sampling of the EMG signal and the difference in the rate at which the peak moment was reached. This could be caused by the physiological differences between actual muscle and how the muscle elements were modeled.

The glenohumeral muscles indicated slightly better agreement than was noted with the Biceps and Triceps. The Deltoid showed a correlation of 0.78 between the EMG based calculation and the model based calculation. Interestingly the time shift that was present in the muscle elements associated with the elbow was also present here; however the model peak occurred slightly earlier than the EMG estimated peak rather than later as with the Triceps and Biceps. This could be potentially caused by the conversion in timing between 250Hz motion data and EMG data collected at 1800Hz. It is also important to note that, the model calculation slightly overestimated the peak moment but the rates at which the peak was reached and returned back to zero were nearly identical between the EMG based calculation and the model based calculation. This could again be attributed to the fact that the muscle coefficients are based
on a linear relationship. However, given that the peak was overestimated, but the rate at which peak was reached was similar seems to suggest that it was the dampening coefficients and not the spring constants that would need to be improved on in subsequent attempts at modeling the functions of the Deltoids.

Finally, the Pectoralis Major indicated the best agreement between the EMG based moment calculation and the model based moment calculation, with a correlation of 0.89. The results showed that there was no time delay present. However, the rate at which the moment returned to zero was slightly slower than the EMG based estimation indicating that the spring constants included in the muscle model may be the main cause of the differences between the two models. It is also important to note that the early portion of the pitch cycle indicated a very low but sustained moment from 10 to 38% of the pitching cycle when looking at the EMG based estimation; however, it was not present in the model based calculation. This may be caused by the fact that Pectoralis Major provides stabilization at this point, and therefore has an active EMG signal. Since this period of time has limited motion occurring in which the Pectoralis Major is the primary driver, the calculated model may underestimate this phase of activation for the muscle.

Overall the Lagrangian based model showed moderate to good agreement with the estimation of muscle moments, good agreement with the calculation of joint moments, and excellent agreement with the prediction of joint angles. The validation work did point out a few weaknesses in the model mainly that the calculations for muscle moments could be improved in future work. Some of these improvements could be improvements in calculating the muscle coefficients to better mimic the non-linear behavior of the muscle tissue and muscle-tendon interaction. Another important point to note was that the EMG based estimations of the joint moments were calculated using an equation originally designed to estimate muscle moments and forces of thoracic muscles, and therefore may not be completely appropriate for some of the muscles modeled in this work. This could also explain why the Pectorialis Major showed the best agreement between the two muscle moment estimates and the Triceps and Biceps had the lowest agreement. With these limitations aside, the results showed that the Lagrange based model was valid and
capable of performing as expected in each of the tested conditions and therefore could be used to gain additional insights into complex motions of the upper extremity.

6.2 Comparison of Pitching Data

Following the validation work to ensure that the model was capable of performing as expected, all 33 pitchers were analyzed in an effort to develop group data profiles describing the joint moments calculated using the Lagrangian based model. Since the validation work showed that the moments were slightly different than the currently used validated model it was important to prove that the Lagrangian based model was capable of producing similar results to the currently validated models. These comparisons included comparing the group data for the joint moments based on the Lagrangian model to the validated Newtonian based pitching model, as well as repeating the statistical findings of a small number of previously published studies. Repeating these studies allowed for the model to be further validated as it proved that even with differences the Lagrangian based model was capable of showing the same statistical relationships that were previously established using the validated models.

When comparing the means of the group data the results were somewhat surprising because of how low the correlations were for the moments associated with the glenohumeral horizontal and vertical abduction/adduction moments and the elbow sagittal plane moment. The correlations for these values ranged from 0.10 through 0.65 when reviewing the data from both the fastball and the curveball. Interestingly the glenohumeral rotation moment had near perfect correlations between the two models with 0.98 and 0.97 for the fastball and curveball data respectively. While initially surprising, these results were not completely unexpected given the results from the validation work which showed differences consistent with the differences noted during the validation work, and the increased patient numbers compared to a singular patient amplified these differences.

The glenohumeral vertical abduction/adduction moment behaved in a very similar manner to what was previously discussed in the validation work of this section. The major differences between the Newtonian based pitching model and the current Lagrangian based model occurred around the instant of
ball release in which the Lagrangian model shows a rapid abduction moment while the Newtonian model shows a more gradual return toward an abduction moment. Although different, the Lagrangian model result was consistent with the fact that as this abduction moment was occurring the arm was moving into its peak adducted position, and therefore an abduction moment would be required to stabilize this motion. As stated previously the presence of the ball weight in the Newtonian model as well as a scapular segment and absence of muscle elements could all factor into these large differences between the two models. To further prove this theory, when looking only at the models between foot contact and ball release, prior to the large adduction motion and when the muscle elements, and ball weight would play the greatest role, the correlation improves considerably to 0.88 from the initial 0.11. When reviewing the data from the curveball dataset for this same moment the results were nearly identical to those noted in the fastball dataset. The only noted exception was the reduced peak moment; however, this finding is consistent with previously published papers indicating the moments at the glenohumeral joint and elbow joint are lower in the curveball than in the fastball [7, 9, 45].

The glenohumeral horizontal abduction/adduction moment also behaved in a very similar manner as previously described in the validation work of this section. The major difference between the two models occurred in the early portion of the pitching cycle prior to the time of ball release. The Lagrangian based model tends to overestimate the adduction moment in the first 50% of the pitching cycle, but then matches the moment well in the second half of the pitching cycle. The curveball group data also indicated a similar behavior in the model. The differences in the early portion of the pitch cycle were not completely unexpected as the early phase of the pitching cycle has the greatest range of motion variability in this plane of motion, as some pitchers tend to drag their elbow behind the plane of their thorax (increased glenohumeral horizontal abduction) while other pitchers lead with their elbow maintain the arm position in front of their thorax (increased glenohumeral horizontal adduction). Most pitchers tend to drag their elbows’ to some extent as their thorax rapidly internally rotates at a faster rate than they can horizontally adduct their arm. As a result, the pitchers’ arms are in a more abducted position leading
to an increased adduction moment to counteract this positioning. Therefore the model results were consistent with the motion of the pitcher and were physiologically reasonable.

Unlike the other two glenohumeral moments, the rotation moment of the glenohumeral joint correlated extremely well for both the fastball and curveball datasets with correlation values of 0.98 and 0.97. As this moment is typically the most reported moment in literature and has also been previously correlated with risk of injury, the finding that the Lagrangian model developed in this work was so comparable to the currently used Newtonian model also helps to lend credibility to the Lagrangian based model. The fact that these models were so comparable was not surprising given the physiological motions associated with this position. The rotational velocity of the glenohumeral joint during baseball pitching is one of the fastest motions occurring at any joint during any task with rotational velocities falling between 4000 and 6000°/s [55]. Therefore, with these velocities, the main contributor to the joint moment would be from the inertial properties included in the model, and so the differences caused by the inclusion of muscle elements or the exclusion of ball weight would have a negligible impact on the joint moment. It is important to note that there were slight differences in the early portion of the pitching cycle between foot contact and around 40% of the pitching cycle, in which the model results indicated a larger external rotation moment compared to the currently utilized pitching model. During this period of the pitching cycle, typically referred to as the arm cocking phase, the pitcher’s arm is externally rotating at a much slower rate than during the acceleration phase in which the arm is rapidly internally rotated as previously described. During this phase the inertial properties are not necessarily the primary driving force associated with the moment and the additional ball weight can somewhat offset the joint moment. As the pitcher moves from an initially internal position towards the external position the moment would start as an external moment to offset the internal movement, and then rapidly progress to an internal moment with the greater external position of the glenohumeral joint as the Lagrangian model predicts. Then as the glenohumeral joint rapidly internally rotates the moment returns toward a more external moment. Again this was in agreement with the results of the model. Interestingly the peak moment through maximum internal rotation of the glenohumeral joint indicated much better agreement between
the two models as this was the period in which the inertial properties were believed to play a major role in the joint moment. It is also important to note that during the deceleration portion of the pitch cycle, post ball release, the two models also showed excellent agreement, lending support to the belief that the additional mass from the ball affected the early portion of the joint moment.

The elbow sagittal plane moment was noted to have one of the lowest correlations between the two models in both the fastball dataset and the curveball data set. The Lagrangian based model indicates that the sagittal plane moment begins as an extension moment and moves into a flexion moment prior to returning to an extension moment prior to ball release. Following ball release the model indicates that there was a significant flexion moment followed by a return to an extension moment during later follow through. In contrast the Newtonian based pitching model indicates a slow gradual extension moment until just around ball release where there is a rapid movement toward a flexion moment followed by a return to an extension moment. Analysis of this motion associated with this moment during the pitching cycle reveals a pitcher’s arm begins flexed and moves into greater flexion which would cause a minor extension moment. Then as the pitch cycle continues the elbow slowly extends until around 60% of the pitching cycle when the elbow is rapidly extended as the ball is released. Throughout this motion there would be an increasing flexion moment. Finally, the after ball release pitchers typically begin to flex their elbows again as they follow through creating the final extension moment again. Based on these findings the Newtonian based model better describes the joint stress at the elbow in the sagittal plane, as the Lagrangian based model tends to over exaggerate the flexion portions of the moment. This could be caused by the inclusion of the muscle elements that help to drive this motion as the current model contains muscle elements for both the primary extensor and flexor. When the moments associated with these two muscle elements were subtracted from the total joint moment calculated by the Lagrangian based model, the resulting moment was in much greater agreement with the Newtonian based model with the correlation coefficient increasing from 0.11 to 0.54; therefore, indicating that the currently used Newtonian based pitching model is based completely on position and inertial properties rather than including soft tissue elements as the Lagrangian based model developed in this work portrays. Therefore,
the Lagrangian based model may produce a more accurate representation of the actual joint stresses experienced by the joints during the pitching motion.

Since a number of differences were noted between the resulting joint moments based on which model was used to calculate the moment’s, additional comparisons using group data were performed to determine if the model produced similar findings to the currently used and accepted pitching model. One of the most studied and agreed on findings in the literature was that the fastball and not the curveball, as suspected by many pitchers and pitching coaches produced the greatest joint moments. Therefore the data obtained using the Lagrangian based model was compared to determine which pitch type, either the fastball or curveball, produced the greatest moments and if these findings were in line with the previously published works. The results based on the Lagrangian model for the glenohumeral internal rotation moment indicated that there was a statistically significant difference between the fastball and curveball, with the fastball having a larger moment. The differences and standard deviation noted using the Lagrangian based model were also noted to be very similar to previously published works; thus lending credibility to the Lagrangian based model.

Interestingly when reviewing the differences between the elbow sagittal plane motion the data showed no statistically significant difference between the two pitch types with the fastball having a peak moment of 4.9 ± 2.7 Nm and the curveball moment around 5.2 ± 2.9 Nm (p=0.375), yet the curveball shows a slightly greater moment. While the Newtonian model also shows no statistically significant difference with a p-value of 0.123, the actual peak moments indicate that the fastball and not the curveball had a greater moment with 21.2 ± 4.3 Nm and 19.0 ± 5.1 Nm respectively. Although this finding is not statistically significant it does raise a number of questions. As has been previously shown the rotational moment seemed to be based more on inertial properties rather than the soft tissue elements as the elbow moment was noted. Therefore much of the pitching related literature indicates that the stresses on a pitcher’s joints during pitching are caused by the motion and speed of the joints which would logically produce the greatest moments during the fastball which produces the greatest speeds. Interestingly the moment most driven by the soft tissue structures indicates that the curveball was causing
a greater stress on the joint, and this result would be consistent with what pitchers typically note. Therefore it may be possible that the soft tissue structures are stressed to a greater extent pitching the curveball compared to that of that fastball which could potentially be a cause of pitching related injury. However, further extensive work is necessary to confirm this finding.

A second comparison was also conducted based on another well documented finding in the pitching mechanics literature. Many pitchers believe that leaning away from their pitching arm can increase their pitching velocity, and therefore a number of researchers looked into whether this commonly held belief was true. The results of the previous studies that looked into this finding indicated that ball velocity did increase; however, at the same time the glenohumeral internal rotation moment as well as the elbow varus moment increased to a greater extent. One study noted that while there was a 1.5% increase in ball velocity the peak glenohumeral internal rotation moment increased by 3.2% and therefore indicated that leaning away from the pitch produced a greater chance of a pitcher injuring themselves than increasing ball velocity [53]. Therefore, the studies looking into this finding were again repeated using the results from the Lagrangian based model. The results indicated that increased lateral trunk lean away from the pitching arm was associated with an increased glenohumeral internal rotation moment, and this finding showed nearly the same statistical significance. Interestingly the only difference between the findings was that the beta value was slightly smaller using the results from the Lagrangian based model compared to the Newtonian results. None the less the results between the two models were consistent and therefore indicated that the results of the Lagrangian model were valid.

6.3 Muscle Force Analysis

Currently there are no models available in the pitching literature to discuss the contributions of soft tissue to the total joint moment; nor are there any models that have discussed the moments placed on specific muscles due to a pitcher’s mechanics. The Lagrangian model developed in this work was able to estimate muscle moments for the Biceps, Triceps, Deltoid, Pectorialis Major, and the Infraspinatus. As this is the first time any of this type of data has been obtained the analysis methods for these data
followed the typical analysis pattern that has been used throughout the pitching literature. Initially the group data was reviewed and descriptively analyzed, then the muscle moments between the fastball and curveball were compared to determine if there were any statistically significant relationships, and then finally, regression based analyses were performed to determine if there were variables of interest that were highly associated with the peak muscle moments.

The results of the Biceps indicated that the majority of the moment occurred between 50 and 100% of the pitching cycle with the peak moment occurring around 78% of the pitching cycle which occurs around the same time as ball release. The initial 50% of the pitching cycle shows almost no Bicep moment which is not unexpected as pitchers typically start with their arm in the most flexed position at initial foot contact and there is typically very little elbow flexion activity until the pitcher begins to rapidly extend his arm around 50% of the pitch cycle. It is important to note that the generation of the Biceps moment begins around the same time that the pitchers begin to extend their arms. Although the Biceps muscle is primarily responsible for flexion, the initial portion of the moment is most likely caused by the fact that the Biceps is acting as an antagonist to the Triceps and helping to control the rapid extension of the arm. The peak moment occurs just after ball release. Again this makes intuitive sense as the pitcher is beginning to flex his pitching arm again during early follow-up to help himself get back into a fielding position. Therefore a large moment is required to slow the extension velocity and begin to flex the arm, which holds with the plot of the Biceps moment. Finally, there is a slow return back to a resting state at the point of MIR indicating that the pitchers leave their arm in a more neutral position. It is important to note that there was a small secondary peak that occurred at around 89% of the pitching cycle. The timing of this peak is consistent with the initial movement back into flexion of the pitching arm. The behavior of the Biceps during the curveball trials were nearly identical to that of the fastball trials; the only difference being a slightly lower moment throughout the pitching cycle in comparison to the fastball.

The results of the Triceps data was very similar to that of the Biceps which was not unexpected given the two muscles are antagonists. Similar to that of the Biceps, the Triceps data shows very little force generation during the early portion of the pitching cycle between foot contact and around 40% of
the pitching cycle. Again this low moment time period corresponds with the lack of elbow motion during the early portion of the pitching cycle. The Triceps moment generation begins slightly earlier than that of the Biceps muscle moment and this occurs peaks around 66% of the pitching cycle. The initial generation of the moment corresponds with the start of the extension motion of the joint and peaks around the same time that the peak extension velocity is reached. The Triceps moment then decreases following the peak extension moment as the Biceps begin to fire to slow the extension of the arm and begin to move back into flexion. Interestingly there was a secondary peak noted around the same time the Biceps reached the peak moment; potentially indicating that both muscles were recruited to reduce the rapid elbow extension and begin to move the arm back into flexion. Again the Triceps moment calculated based on the curveball data was similar in behavior but had a slightly lower moment throughout the pitching cycle in comparison to the fastball data.

The results from the analysis of the Deltoid again showed that the moment generated by the muscle occurred near the end of the pitching cycle. This finding was not unexpected as the range of motion for glenohumeral vertical abduction and adduction is around 10°, and much of the glenohumeral motion and velocity is transferred from the trunk and lower extremities when a pitcher is using a correct pitching technique. The Deltoid moment begins around 60% of the pitching cycle and reaches its peak moment late in the pitch cycle around 80 to 88% with an average of around 83%, which is after the timing of ball release. This peak corresponds with the vertical abduction motion occurring at the glenohumeral joint. Therefore it seems that during the pitching cycle the Deltoid produces the greatest moment in the deceleration phase in which it is eccentrically loading to slow and reverse the motion of the glenohumeral joint to help the pitcher obtain fielding position after the pitch. Again the moment behavior of the Deltoid for the curveball data showed similar findings with the only noted exception that the moment was lower throughout the entire pitching cycle for the curveball compared to that of the fastball.

The results obtained from the Pectorialis Major shows a different moment pattern that in the other three muscles previously discussed. This muscle shows to separate peaks one in the early portion of the pitching cycle and a second larger peak later in the pitching cycle. The first moment peak begins very
early in the pitching cycle at around 5% just after foot contact and peaks around 40% of the pitching cycle before tapering off around 67% of the pitching cycle which corresponds with the timing of maximum external rotation of the glenohumeral joint. This initial moment corresponds with the horizontal abduction of the glenohumeral joint. As stated earlier most pitchers allow their pitching arm to trail behind the plane of their thorax as the thorax is rotating more rapidly than they can horizontally adduct their arm. During this portion of the pitch cycle the Pectoralis Major’s moment corresponds with the pitcher attempting to horizontally adduct their arm in an effort to bring it into a neutral position prior to ball release so that they can accurately deliver their pitch. Another plausible explanation for this moment initial burst as that the rotation of the trunk can passively move the arm and the pectoralis major moment is occurring as a result of the muscles attempt to stabilize and control the forward motion of the arm. As this initial moment tapers around the timing of MER, the moment generated by the Pectoralis Major is consistent with the motion of most pitchers. The second larger moment begins around 70% of the pitching cycle and peaks around 80% of the pitching cycle before tapering off again at MIR. Similar to the other muscles previously discussed, this period corresponds with the deceleration phase of the pitching cycle in which the pitcher is slowing the forward pitching motion in an effort to achieve fielding position. During this phase the pitcher is adducting and internally rotating their arm as well, both motions are consistent with the an increased moment at the Pec as well as the fact that the Pec is acting again to slow the motion of the arm during the deceleration phase. Again, as seen with the previous three muscles, the Pectoralis Major has a nearly identical pattern when looking at the curveball data with the noted exception that the curveball moment is lower than that of the fastball data.

The final muscle modeled in this work was the Infraspinatus which is one of the muscles responsible for glenohumeral external rotation, and therefore would contribute to an internal rotation moment. It is important to note that this was the only muscle in this work that EMG data was not obtained from a pitcher; this was due to the fact that the Infraspinatus is a deep muscle and the only way to accurately measure the EMG activity of this muscle was through the use of fine wire EMG. Since the IRB approval for this study did not allow for any invasive collection of data, the data from the
Infraspinatus was estimated and assumed to be correct based on the amplitude of the moment as well as the behavior of the moment which was consistent with the other muscles. Similar to that of Biceps, Triceps, and Deltoid, the Infraspinatus did not have a large moment until late in the pitching cycle. Nearly the entire pitch cycle from foot contact until after ball release, the Infraspinatus had almost no generated moment. The moment begins around the timing of ball release and peaks around 85 to 90% of the pitching cycle. The fact that the Infraspinatus was quiet through the early portion of the pitch cycle was not unexpected, as the action of this muscle is to internally rotate the humerus, and during this early portion of the pitching cycle through MER, the majority of the motion is external rotation. However, once the maximum external rotation of the glenohumeral joint was reached and muscle moment begins to develop corresponding with the internal rotation of the joint. Again it seems that this muscle was most active during the deceleration portion of the pitching cycle. A secondary finding was that the peak of this moment coincides with the presence of a second lower glenohumeral internal rotation moment burst that is sometimes noted in pitchers. Although there has been limited work analyzing this secondary moment it has been theorized that this moment occurs in pitchers that are rapidly slowing their pitching motion to obtain fielding position more quickly than other pitchers. When analyzing the Infraspinatus data from the curveball the shape of the moment curves were again noted to be similar between the two pitch types; however, the curveball moment, not the moment associated with the fastball data was noted to be greater.

It is important to note that the data from each of the muscle moments estimated using the Lagrangian based model were very consistent from pitcher to pitcher as noted by the tight standard deviation bands around the mean curve, with the noted exceptions of the Pectorialis Major and the Deltoid. These two muscles had much larger standard deviation bands indicating that the moments generated by these muscles were more variable from pitcher to pitcher. Since the standard deviations of the motions associated with the actions of these muscles have much tighter standard deviation bands it stands to reason that the recruitment of these two muscles may indicate a difference in pitching mechanics that cannot easily be noted from motion data alone. This may give further insight into the potential cause of injury in pitchers. For instance, a pitcher with an increased Pectoralis Major in the early portion of the
pitching cycle may be attempting to force his arm into a more aligned position which could indicate that he is over rotating his trunk and therefore placing additional stresses on his joints.

Another important finding regarding the five muscles modeled in this work was that nearly all of the muscles were active during the deceleration phase of the pitching cycle, indicating the greatest stresses on these soft tissue structures were occurring during this time period as well. Interestingly almost every pitching based paper currently published focuses on the acceleration phase of the pitching cycle and the time prior to ball release, and yet currently there has been no singular variable that has been found to be associated with pitching related injury. Potentially with the data shown in this work, using a Lagrangian based approach, the focus should shift from the acceleration phase of the pitching cycle to the deceleration phase as the soft tissue structures, for at least these five muscle elements, experience the greatest moments following ball release.

When comparing the fastball muscle moments to the curveball muscle moments the results of this analysis showed that in all cases, except for the Infraspinatus, the peak moment was statistically significantly greater in the fastball than the curveball. These findings were consistent with previously established pitching literature and the results within this work indicating that the fastball joint moments were consistently greater in the fastball than in the curveball. This makes intuitive sense given that the muscle moment calculations were highly dependent on the angular velocity of the joints. Given that the joint angular velocities are greater when a pitcher pitches a fastball then it stands to reason that the muscle moments were greater for the fastball.

Interestingly the Infraspinatus showed a statistically significant finding showing that the moment was greater during the curveball than during the fastball. This finding was somewhat surprising as it did not agree with the other muscles nor did it agree with the joint moments. However as discussed previously the Infraspinatus acts as an internal rotator for the glenohumeral joint; therefore, the peak moment along with the timing of action for the Infraspinatus occurs during the deceleration phase which has not been studied to any extent previously. Furthermore this finding could indicate that while pitching the curveball, pitchers must internally rotate at the glenohumeral joint to a greater extent than they would
when pitching a fastball, potentially as a compensatory motion caused by the increased forearm supination. This finding also lends some credibility to the anecdotal evidence that pitching coaches and pitchers indicate that when throwing the curveball they feel greater strains on their joints than when throwing other pitches, which up until this work has never been shown biomechanically. Therefore additional work should be undertaken specifically looking at both the deceleration phase of the pitching cycle as well as the soft tissue moments to better understand injury mechanisms in baseball pitching, and to determine if the curveball may in fact place additional stress on upper extremity structures that were not previously studied.

A regression analysis was performed to determine how specific variables were associated with the five muscle elements. While there could be an unlimited number of variables used in these regression analyses, the variables of interest were limited to those variables typically studied in pitching literature, such as joint angles at specified times during the pitching cycle, joint moments, peak values, and ball velocity. The variables were further limited to those that were directly related to the muscle elements. For example, the Biceps and Triceps were analyzed along with the elbow joint variables but were not analyzed with the glenohumeral joints variables. Future work in this area could easily expand on the number of variables examined, or provide a very in depth analysis of a single muscle element and how the muscle moment may be associated with other pitching variables or how specific variables could affect the muscle moment.

The results of the regression analysis concerning the Biceps indicated that there were a number of statistically significant associations with ball velocity, elbow joint angle, elbow flexion velocity, and the elbow moment. The results indicated that the Biceps moment was only associated with the elbow joint angle at the instant of foot contact and ball release. The regression analysis indicated that the greater the amount of flexion at foot contact the greater the Biceps moment, and that for every 10° increase above the mean elbow flexion angle at foot contact resulted in a 1.9 Nm increase in the Biceps moment. Therefore, those pitchers that tended to maintain more elbow flexion, which could indicate they were trying to shield the ball from the batter for a longer time by keeping it behind their head, at foot contact placed a greater
stress on their Biceps than those that did not. A similar result was also noted at the instant of ball release where a 10° increase in elbow flexion at ball release increased the Biceps moment by 1.4 Nm. This makes intuitive sense since as the pitcher releases the ball they typically reach their maximum elbow extension. By maintaining a more flexed arm position at this point the Biceps would need to increase the force exerted on the arm to ensure that it does not over extend and maintain a flexed position. The peak elbow flexion velocity was also noted to be associated with the Biceps moment. The relationship indicated that a 100°/s increase in peak elbow flexion velocity increased the Biceps moment by 2.0 Nm. Typically the peak elbow flexion velocity occurs following ball release as the pitcher is attempting to slow his arm and return to a more neutral position; therefore, those pitchers that try to decelerate more rapidly are at a greater risk of injury. To a pitching coach or trainer these findings would provide potential means to reduce risk of injury to a pitcher or to help rehabilitate a pitcher that complains of Bicep tenderness during pitching. To reduce stresses on the Biceps, coaches and trainers should teach pitchers to pitch with as extended an arm as possible at ball release, and during follow through to use the entire follow-through to slow their pitching arm rather than abruptly stopping the arm. Both of these suggestions are easily coached and implemented for a pitcher.

The results of the Biceps analysis also indicated that the greater the elbow flexion moment the greater the Biceps moment, with a 10 Nm increase in the flexion moment resulting in a 1.9 Nm increase in the Biceps moment. This finding is intuitive as an increased flexion moment is a direct result of an increased Biceps moment as discussed previously. However, this finding though intuitive serves as check to the model to ensure that the model is working in a predictable and physically plausible way. Finally, the Biceps moment was also shown to be highly associated with ball velocity; those pitchers with increased ball velocity showed greater Biceps moments. For example, a 1m/s increase in ball velocity resulted in a 1.9Nm increase in the Biceps moment. Again this finding is intuitive as greater ball velocity requires a pitcher to more rapidly accelerate their arm and pitch with greater force leading to greater stresses on the Biceps. To maintain the stability of the arm during the extension phase of the pitching
cycle, and since the arm has reached a higher angular velocity, the Biceps is required to decelerate more rapidly as well.

The results for the Triceps moments indicated very similar results to those found with the Biceps and as expected indicated opposite relationships as those found for the Biceps. The Triceps moment was also found to be associated with the elbow joint angle only at foot contact and at ball release. The results indicated that those pitchers who had greater elbow extension, essentially a less flexed elbow, at foot contact had greater Triceps moments. Those pitchers that had 10° less of elbow flexion at foot contact under the mean value were noted to have a Triceps moment 2.2Nm greater than the mean Triceps moment. Those pitchers with greater elbow extension at ball release were noted to have increased Triceps moments with a 10° increase in elbow extension resulting in a 5Nm increase in the Triceps moment. Finally the peak elbow extension velocity was also noted to have a significant relationship with the Triceps moment; for every 100°/s increase in elbow extension velocity there was a 3Nm increase in Triceps moment. This finding regarding the elbow extension velocity is somewhat intuitive as those pitchers that more rapidly extend their arms, typically associated with pitchers that are arm throwers, rather than relying on the power developed by the lower extremity and trunk, would have greater stresses on their Triceps as they are muscling the ball towards the target.

Overall these findings were very similar when compared to those regarding the Biceps which was not surprising as the two muscles function as antagonists. The fact that the relationships shown in this analysis were opposite as those noted in the Biceps results, for instance increased flexion at lead foot contact increased the Biceps moment while decreased flexion results in an increased Triceps moment, indicates that there is a potential optimum solution for the pitchers elbow positioning at foot contact to produce the least amount of stress on both muscles. Given the fact that increased flexion results in a 1.4 Nm increase in the Biceps moment, and decreased flexion results in a 2.2 Nm increase in Triceps moment, the results indicate that pitchers should pitch with a more flexed arm position at foot contact. However, it is also important to take into account what the maximum moment that each muscle can sustain prior to failure as a 2 Nm increase in the Triceps may be less damaging than a 1 Nm increase in
the Biceps moment. Therefore future work could be directed toward finding the optimal positioning of the arm to reduce muscle strain. This knowledge would aid coaches in teaching pitchers the basic pitching mechanics to reduce the risk of injury later in their careers.

The results of the Triceps analysis also indicated that those pitchers with increased ball velocity had greater Triceps moments; for every 1 m/s increase in ball velocity the Triceps moment increased by 2.2 Nm. As discussed earlier with the Biceps results this finding is not surprising and helps to validate the model’s accuracy. As a further means of validating the model the Triceps moment was related to the elbow flexion moment. This was intentionally done as it was expected that a muscle that acts in extension should not be related to the flexion moment, and the results of the analysis supported this finding showing no statistically significant association between the flexion moment and the Triceps moment. This finding once again indicated that the model was working in a predictable manner.

The results of the regression analysis concerning the Deltoid showed a limited number of associations between the positioning of the glenohumeral joint and the Deltoid moment. These results indicate that increased vertical glenohumeral joint abduction at ball release was associated with an increased Deltoid moment. Those pitchers that pitched with 10° greater abduction at ball release had a 1.7Nm increase in their glenohumeral coronal plane moment. This makes intuitive sense as a pitcher’s arm would be in a higher vertical position at ball release, which would create a greater stress on the soft tissue structures surrounding the shoulder. For a pitching coach this finding would indicate that those pitchers pitching with increased vertical abduction of their arm would experience greater stresses on the Deltoid. Interestingly this was the only finding that was associated with an increased Deltoid moment when analyzing the glenohumeral coronal plane positioning and angular velocities. Given that there is limited coronal plane motion and the coronal plane motion is relatively uniform between pitchers, it stands to reason that unless there was a large excursion of motion there would be little associations found given the sample size in this work.

Additionally there were no significant associations found between the coronal plane positioning of the trunk and the Deltoid moment. Originally it was believed that increased trunk lean away from the
pitching arm would result in an increased Deltoid moment similar to that noted with the glenohumeral joint moment. The results of the regression analysis proved this hypothesis to be false, and given the muscle physiology, this finding does make sense. The insertion and origin of the muscle would not allow the muscle to elongate with additional trunk lean and therefore would not affect the muscle moment. Though initially surprising, this finding further proves the ability of the model to correctly discern physical relationships that are anatomically sound.

Similar to the findings of both the Biceps and Triceps, the Deltoid moment was found to be significantly associated with both the peak glenohumeral vertical abduction moment and ball velocity; where a 1 Nm increase in the glenohumeral vertical abduction moment resulted in a 0.06 Nm increase in the Deltoid moment, and a 1m/s increase in ball velocity would increase the Deltoid moment by 1.7Nm. As with the previous results these findings were not unexpected. An increased glenohumeral vertical abduction moment would increase the Deltoid moment. Similarly a pitcher with a greater velocity would need to recruit additional muscle fibers to pitch at a more rapid velocity, and therefore generate a greater Deltoid moment.

The results of the regression analysis concerning the Pectorialis Major were somewhat surprising. Unlike the previous three muscles discussed, the Pectorialis Major showed no associations with glenohumeral positioning at any point within the pitching cycle nor did the Pectorialis Major show any association with the coronal plane trunk position at any point within the pitch cycle. These findings were somewhat surprising given that the Pectorialis Major aids in the horizontal adduction of the humerus and therefore, it was assumed that there would be an association between the Pectorialis Major’s moment and the horizontal adduction angle of the glenohumeral joint. During the pitch, much of the horizontal adduction comes from passively allowing the arm to swing around as the pitcher’s trunk externally rotates towards home plate; therefore the Pectorialis Major does not contribute a large force to pull the arm into further adduction, rather the Pectorialis Major is aiding in stabilizing the joint movement throughout much of the pitching cycle. For this reason, it is possible that the Pectorialis Major is not associated with the joint angular positions and velocities. Pitchers that have poor rotation or arm throw may have
increased Pectoralis Major moments; however, given that the study group in this work were all actively pitching for Division I and Division III collegiate teams they could have better pitching mechanics than younger less experienced pitchers. Therefore, future work should also be focused on looking at younger age groups. As discussed earlier much of the activity of the Pectoralis Major occurs in two separate time periods. The first period occurs during the initial portion of the pitching cycle which as stated may coincide with the pitcher trying to control the forward motion of the pitching arm. The second period, which was noted to be the larger of the two moment peaks, occurs much later in the pitching cycle when the variables that were studied along with the Pectoralis Major may not be as prevalent. While these findings may not aid a coach in training pitchers, these results could be useful for researchers studying the biomechanics of baseball pitching in an effort to reduce pitching injury. Those pitchers that show a greater moment during the initial portion of the pitching cycle may be requiring the use of the Pectoralis Major due to a mechanical flaw somewhere else in their pitching mechanics, potentially in the transfer of motion from the lower extremity and trunk to the pitching arm.

Interestingly the regression analysis also indicated that there were no associations between ball velocity nor with the glenohumeral horizontal abduction and adduction moments. This finding helps to confirm the theories discussed previously; during the pitching cycle the Pectoralis Major acts more as a stabilizer than playing an active role in developing joint velocities to increase the speed of the pitch. Since the Pectoralis Major is assumed to perform a stabilizing role the glenohumeral horizontal adduction and abduction moment would not be as affected if the contractions of the muscle were not being used to generate force to drive the forward motion of the arm but rather to support it.

Finally the results of the regression analysis for the Infraspinatus were more consistent with those results noted for the Biceps, Triceps, and Deltoid than the results for the Pectoralis Major. The regression analysis showed that the Infraspinatus was not associated with the glenohumeral rotational position at any of the major time points within the pitching cycle. This was not an unanticipated finding. The majority of the glenohumeral rotation within the pitching cycle is dominated by marked external rotation of the joint and since the Infraspinatus is an internal rotator, the muscle would not be associated
with a majority of the motion of the glenohumeral joint. The regression analysis did show that there was a statistically significant positive association with the peak glenohumeral internal rotation velocity and the Infraspinatus’ moment. The results indicated that for every 100°/s increase over the mean in peak glenohumeral internal rotation velocity, the Infraspinatus moment increased by 2 Nm. This finding again was expected, as an increased internal rotation velocity would require greater force generation by the Infraspinatus.

The results of the regression analysis for the Infraspinatus also indicated a statistically significant association between the peak glenohumeral internal rotation moment and the moment of the Infraspinatus, where a 1 Nm increase in the peak glenohumeral internal rotation moment coincided with a 0.24 Nm increase in the Infraspinatus moment. Again, as stated previously, this finding is intuitive and does help to show the validity of the model as an increased internal rotation moment at the glenohumeral moment should be associated with an increased moment from a muscle responsible for external rotation, since the moment is presented as an internal moment. A somewhat interesting finding was that the Infraspinatus moment was not associated with ball velocity. However, further investigation indicated that there was no association between the peak glenohumeral internal rotation velocity and ball velocity (p=0.079), so it stands to reason that the muscle moment which was highly associated with the peak glenohumeral rotation velocity would show a similar relationship with ball velocity as the peak internal rotation velocity showed. Based on this information ball velocity does not increase with a more rapid internal rotation of arm, but the stress placed on the Infraspinatus does increase with a more rapid internal rotation of the arm. So coaches should be aware of this and coach pitchers to not expect an increase in ball velocity by more rapidly internally rotating their arms. Rather, it could place them at a higher risk of injury.

Interestingly the results of the muscle force analysis indicated that all of the muscles chosen in this work provided the largest moment contributions late in the pitching cycle during the deceleration phase of the pitch. If these findings hold true for other muscle groups as well the results of this work, they could indicate that the pitching related injuries may occur during the deceleration phase and not the acceleration phase as currently believed. Furthermore while most of the results discussed in this section
were not unexpected, there were a number of findings that challenge currently held findings. While this work is not suggesting that these findings are incorrect it does raise questions that should be explored further to better understand injury mechanisms in pitching, specifically concerning the stresses placed on soft tissue structures during the pitch.

6.4 Clinical Example

The final portion of this work concerned a clinical example, which served two purposes. The first reason a clinical example was chosen, using a singular randomly selected pitcher was to demonstrate the utility of using a model of this nature to explore how changing various parameters would affect the model’s output specifically joint and muscle moments. This has the benefit of not only aiding in the models validity but also illustrating how the model could be used to help solve clinically based research questions as to how modifying a pitcher’s mechanics would affect their risk of injury. The second reason this example was chosen was to highlight the functionality of this model in a clinical setting. To this point this work has discussed the strengths of the model both from a validity standpoint and from that of a research standpoint, but one of the greatest strengths of this model is its ability to be utilized clinically as well. The ability of the model to quickly adjust parameters allows for users to test outcomes using the model that cannot easily be tested in either a laboratory setting or in real life. For instance, understanding how muscle lengthening may affect a joint muscle or a pitchers mechanics could not be easily tested. Also coaches could determine if changing a pitcher’s mechanics could improve a pitcher's outcomes or risk of injury.

The results were based on a collegiate pitcher chosen randomly from the group of 33 pitchers analyzed in this work. The pitcher was from a Division I college and had 11 years of pitching experience. A series of parameters typically discussed by pitching coaches were selected to be varied to determine what the effect would be on the pitcher’s peak glenohumeral internal rotation moment, their peak elbow flexion moment, their Triceps moment, and their Infraspinatus moment. The changes in the peak moments were compared to the pitcher’s moments based on their typical pitching mechanics.
As discussed a number of times throughout this work lateral trunk lean is extremely important to a pitcher’s mechanics, and has been shown in group data to have a statistically significant association with increased moments especially at the glenohumeral rotation joint and the elbow varus moment. Therefore in this example a pitching coach may be interested in modifying their pitcher’s trunk position to improve the pitcher’s delivery. Using this model a coach could make modifications to the pitcher’s mechanics and determine if these changes would have a beneficial or deleterious effect. By increasing the pitcher’s trunk lean by 10° away from their pitching arm the pitcher’s moment was increased by 5.9 Nm and increased the pitcher’s elbow flexion moment by 1.1 Nm. On the other hand having the pitcher pitch more upright by 10° actually reduced the pitchers glenohumeral internal rotation joint by 4.1Nm and reduced the elbow flexion moment by 0.3Nm. Interestingly, both changes indicated that it had no effect on the muscle moments included in this clinical example. The results of this were not unexpected as increased lateral trunk lean is associated with an increased moment. However, the results were representative of this pitcher rather than assuming the increase was the same as found using the group based results. Similarly the results showed that the modifications in the pitcher’s trunk position did not affect the moments of the muscles as expected given the results discussed earlier. The results of this analysis were expected and further validated the model.

The most important result of this analysis was the one in which the pitcher’s mechanics were changed to determine how a lean towards the pitching arm might affect the moment. While it has been theorized that a pitcher pitching more upright would reduce the moments on the joints there have been no results to prove this, as most pitcher’s typically fall away from their pitching arm during the pitch. Therefore the Lagrangian based model is capable of allowing for a better understanding of pitching mechanics. For a coach or a trainer, this information would be invaluable as modifications could be made to the pitcher’s trunk position to reduce the stresses placed on the pitchers joints. Furthermore, using this information that indicates pitching in a more upright position coaches and trainers could focus on core strengthening exercises to reduce the lean away from a pitcher’s arm during the pitch.
Another commonly discussed area of pitching concerns the proper positioning of the arm in what is termed the power position. The power position is described by pitching coaches as having the arm in 90° of abduction so that it the humerus appears parallel to the ground when viewed from the frontal plane. Pitching coaches will often say that if the pitcher is not in the power position then he will have a reduced pitch velocity and place greater stresses on his arm; however there has been little scientific evidence to support or refute this claim. The results found in this work using this individual pitcher indicated that with a 15° increase in glenohumeral vertical abduction the pitcher’s glenohumeral internal rotation moment increased by 0.3Nm and the elbow flexion moment decreased by 0.1Nm. While a 15° decrease in glenohumeral vertical abduction decreased the pitcher’s glenohumeral internal rotation moment by 0.1Nm and increased the elbow flexion moment by 0.15Nm. Again the results showed no effect on the moments of the Infraspinatus or the Triceps which was consistent with the other findings in this work. The results of this analysis indicated that there may be some support for the beliefs held by pitching coaches that the power position does in fact influence the stresses on a pitcher’s joints, at least for this individual pitcher. Although the results do indicate that the changes in the total joint moment caused by a 15° change in horizontal glenohumeral abductions resulted in a relatively small change in the joint moments, and though the results support coaching theory it may not be as influential as many coaches believe.

It is important to note that the results of this scenario indicated that there was an inverse relationship for this pitcher in that a change in glenohumeral positioning increased the glenohumeral rotation moment while decreasing the elbow flexion moment, and vice versa. Therefore if these results were to hold consistent for all pitchers an equilibrium point would need to be found that would allow for an optimal power position for an individual pitcher that would minimize both joint moments. Additional work on this model could also expand the results of the model to estimate ball velocity and therefore this model could not only be used to reduce the risk of pitching injury but also help coaches determine methods for improving a pitchers performance as measured by ball velocity.
The final scenario explored in this work concerning data that may be helpful to a pitching coach was to determine what the effect would be on the joint and muscle moments if the pitcher was able to increase their glenohumeral internal rotation velocity. Adjusting pitcher’s mechanics, essentially the positioning of their joints, is relatively easy as a pitcher could be asked to pitch more upright, release earlier, or increase or decrease their stride; however exploring what would happen if the pitcher was able to more rapidly rotate their arm is far more difficult to assess. One of the reasons for this is that increasing rotational velocities requires a great deal of strengthening and conditioning, and therefore if a coach is thinking that by increasing a pitcher’s rotational velocity it may improve their pitching mechanics it would be beneficial to know if it would be successful prior to spending months training. Using the model developed in this work it would be possible to perform these types of explorations. The results of this analysis showed that a 500°/s increase in glenohumeral internal rotation velocity increased the glenohumeral internal rotation moment of this subject by 16.9 Nm and increased the moment at the Infraspinatus by 13.8 Nm. The results also indicated there were no changes in the elbow flexion moment or for the Triceps moment. These results were not unexpected as the more rapid the motion of the glenohumeral joint the greater the force at the joint which would eventually translate into an increased joint moment, and the results were also consistent with the other findings in this work suggesting an increase in glenohumeral joint moment was associated with an increase in the moment at the Infraspinatus. Similarly a change effecting only glenohumeral rotation should not affect elbow joint kinematics or kinetics. Given these results along with the knowledge that an increase in glenohumeral internal rotation velocity does not always translate into an increased ball velocity, a pitching coach would probably recommend against training specifically to increase the glenohumeral joint velocity given the substantial increases in the associated joint and muscle moments.

While the above discussed results were based on a singular pitcher and therefore these results can be extended only to this specific pitcher this gives an example of the utility of this model for both a coach and athletic trainer. Using this model coaches or trainers could evaluate ideas for a pitcher prior to implementing these changes to determine if the changes they are considering would place the pitcher at a
greater risk of injury. Conversely a pitcher returning from an injury could be studied at a motion analysis laboratory and then specific recommendations could then be made as to what could be done to reduce their joint loads and reduce their chances of future injury. These recommendations could then be made to a pitching coach or trainer so that they could come up with a pitcher specific training regiment to address the needs of a pitcher. Furthermore this model could also be used by other researchers as well as coaches to solve a series of optimization problems to find the ideal pitching mechanics for a pitcher. As the results showed many of the modifications to a pitcher’s mechanics has the potential to either reduce or increase joint moments, and more than likely effect the pitcher’s ball speed. These results are perfect to create a bounded optimization problem in which the minimum joint moment is solved for while attempting to maximize ball velocity within a confined physiological range of motion. Although this could be done using group data it would be far more accurate and effective to be solved for individual pitchers, since the range of motion of a pitcher’s joints as well as their flexibility would vary from pitcher to pitcher and would need to be taken into account while solving the optimization problem.

The three examples above were tailored specifically to modifying a pitchers mechanics; however, the model also has the ability to be useful for clinicians and surgeons to better understand how changes in the patient’s health, injury, or surgery may affect a pitcher’s joint stresses. To this end the following examples were created to show how this model could also be extended for clinical uses.

In the first scenario a pitcher comes to clinic with complaints of shoulder and elbow pain during pitching that has just started during the past year; however, over the past year the pitcher has also gained 10 kg. While it may be obvious that the additional weight can be attributed to increased joint moments, this model can be used to better understand to what extent the mass has affected the joint moments. The results showed if the selected pitcher gained 10 kg their glenohumeral internal rotation moment increased by 6.6Nm and their elbow flexion moment also increased by 2.0 Nm. The results also showed that the if the same pitcher were to lose 10 kg their glenohumeral internal rotation moment would decrease by 6.6Nm and their elbow flexion moment would decrease by 2.0 Nm. It is important to note how linear the effects of this result are showing that an increase or decrease of 1 kg results in a 0.66 Nm change at the
glenohumeral joint and a 0.2 Nm change at the elbow joint. Results of this nature could be used to help
patients understand how their weight could affect their risk of orthopedic injury. When counseling a
patient as to the benefits of weight loss, providing concrete evidence such as these presented here may be
able to help a patient better realize the implications of how weight may affect their day to day life.

A second scenario was performed to understand how an injury to the pitcher’s muscle that
affected either the muscle elasticity or viscous dampening of the Infraspinatus may affect the
glenohumeral joint moment and Infraspinatus moment. The results of this scenario showed that when the
subject’s Infraspinatus stiffness was doubled the glenohumeral internal rotation moment increased by 1.1
Nm and the Infraspinatus moment increased by 5.7 Nm. While a decrease in stiffness by 50% decreased
be glenohumeral internal rotation moment by 2.1 Nm and the moment at the Infraspinatus decreased by
2.9 Nm. These results indicate that an increase in the stiffness of the muscle causes a greater influence on
the resulting moments than a decrease in muscle stiffness. Also these results indicated that if this pitcher
had more muscle stiffness he may be at a greater risk of injury as his muscles and joints were noted to
have greater stresses. The results of this exploration illustrate how stretching and keeping the muscles
flexible can help to alleviate the potential for injury. This information could also be used to develop a
patient specific stretching protocol that could be implemented as either a home stretching program or with
a physical therapist, should a pitcher be noted to have tight muscles.

The results of modifying this pitcher’s viscous dampening showed extremely large effects to both
the glenohumeral joint moment and that of the Infraspinatus, where doubling the dampening coefficient
resulted in a 44.2 Nm increase in the glenohumeral internal rotation moment and a 46.6 Nm increase in
the Infraspinatus moment. Likewise a decrease by 50% of the dampening coefficient for the Infraspinatus
resulted in a decrease in the joint moment by 12.9 Nm and a 14.2 Nm reduction in the muscle moment.
The results of this scenario indicated that changes in the dampening coefficients affected the moments to
a much greater extent than changes in the spring constants. This result is not unexpected as an increase in
the dampening coefficient would represent an increased muscle resistance to a rapid stretch, which would
require a much greater force to overcome this resistance to perform the deserve motion. In this case, if
this pitcher presented at a clinic and it was noted they had such a severe response to a quick stretch, stretching programs as suggested for the pitcher who had stiff muscles, may not be adequate to address the patient’s problems and surgical intervention may be needed to address this problem.

The final example performed as part of the clinical examples and model exploration in this work was to determine what the effects of surgically lengthening or shortening the Infraspinatus by 2 cm would have on the joint moments and muscle moments. When the pitcher’s Infraspinatus was elongated by 2 cm the glenohumeral internal rotation moment showed an decrease of 0.01 Nm and was therefore determined to be insignificant; however the Infraspinatus moment decreased by 0.6 Nm. Similarly shortening the Infraspinatus by 2 cm did not affect the joint moment but resulted in a 0.9 Nm increase in the muscle moment. While the results of this analysis may not have a large impact on this pitcher’s joint and muscle moments, this example does illustrate additional uses for this model in a clinical setting in that surgeons could use this model to gain a better understanding of how surgical interventions to soft tissue structures could affect muscle force post-surgery. While this example only illustrated the effect of a change in the length of the resting length of the muscle, this model could easily be extended to provide surgeons with a better understanding of how lengthenings, and transfers may affect joint movements or muscles forces.

Although this model and the results of the clinical examples were confined to a single subject, based on a pitching motion, this model can be easily adapted to any upper extremity motion profile which greatly increases its utility in a clinical setting. Future work on this model could easily expand on the understanding of how soft tissue structures allow clinicians, therapists, trainers, and coaches to better address a patient or pitcher’s needs. Furthermore, the ability of this model to be scaled to a specific subject and therefore create individualized models provides an enormous strength over the currently available models which are based on idealized parameters.

6.5 Limitations

The majority of this work has expounded on the strengths of the Lagrangian, energy based, approach to modeling human biodynamics; however, there are a number of limitations using this method
as well as a number of limitations that must be acknowledged within this work. In this work, the model developed was slightly different than the original pitching model that was developed at Connecticut Children’s Medical Center, most notably the exclusion of the thoracic scapular joint, which can explain some of the differences between the two models as previously discussed.

The modeling used in this work for the calculation of the joint angles was based on Euler rotation sequences which have a number of inherent problems, this can be especially evident at the shoulder joint complex where certain rotation combinations could have the potential to create gimbal lock or a situation in which two different angle measures can solve a single equation. Therefore, additional checks were put into place within the Matlab coding to address these potential problems. In future work, the angles of the shoulder may be more accurately modeled using a combination of Euler rotations, projection angles, and quatrains to better represent the physical motion of the shoulder.

The model results in this work were also based on previous measurement and as such some of the measurements required had to be estimated using the previously collected measures. However, in a few cases the subject was available to be called back and the measurements required were more accurately measured. The results showed minimal differences between the estimated parameters and the actual measures, with the differences typically noted to be around 0.5cm. Therefore, while this was a limitation to the model the estimates were reflective of the actual measurements and could be considered as an adequate representation of the data.

Another important aspect of this model is the fact that the muscle parameters for each subject were based on subject measures that were essentially idealized as the actual muscle parameters were unable to be obtained for each subject. Some of these parameters include the muscle lengths and cross sectional areas. While the muscle lengths were estimated using each subject’s segment lengths, there could still be a number of differences in the geometry of the muscle which could have affected the muscle lengths. Furthermore, the cross sectional areas were unable to be assessed due to the absence of MRI data for each subject and therefore the cross sectional areas were based on measures from previously published
studies. It is important to point out that this model can include these input parameters should they be available; thereby increasing the accuracy of the model.

A major limitation regarding the muscle force analysis used in this work is that the muscle constants; the spring/elastic coefficients as well as the viscous dampening coefficients, were created from multiple trials from a single subject who had consented to the EMG portion of the study. Therefore, the constants may not be completely reflective of each individual subject. However, given that the study group is homogenous since it is made up of young adults of similar weight, height, and athletic build the muscle constants may be reflective of the population used in this work. Future work using different populations would require the modifications of these constants to be reflective of the subjects used.

Another limitation surrounding the muscle force analysis was that the validation of the muscle forces were based on comparisons to estimated muscle forces from EMG signals as described in the methods section of this work. However, the equation developed by Marras et al. was designed to estimate muscle forces from the musculature surrounding a subject’s core, specifically those muscles originating and inserting into the spine. Therefore, the application of this equation to the upper extremity may not be completely accurate as the muscle architecture of core musculature is somewhat different than that of the upper extremity. However, as the results of the muscle force analysis are consistent with the actual physical behavior of the upper extremity muscles the use of this equation is adequate. In future work a different reference equation may provide a slightly more accurate muscle force estimation.

Finally, the last major limitation for the muscle modeling again revolves around the constants. In this work the elastic elements’ constants were functions of the joint angular position and the viscous components coefficients were functions of angular velocities, as such a major assumption made in this work were that the functions reflected a linear relationship between stiffness and joint angles and dampening and joint angular velocities. However, actual muscle tissue does not follow a linear relationship but rather a non-linear relationship as evident by the stress-strain curves for soft tissues and the force-velocity curves describing the behavior of muscles. This work did include additional constants within the equations to help improve the accuracy of the models; however, they are not completely
representative of this non-linear relationship. As this work was a first attempt to model muscle forces and moments, this method proved to be adequate, but future work should be directed to improving the equations governing the muscle constants to reflect their non-linear behaviors of the muscle and differences in tissue properties between muscles, tendons, and ligaments.

The final specific limitation for this work is that the model described in this work details a limited number of muscles and does not completely take into account the joint frictional forces or viscous dampening occurring within the joint itself or as a result of those muscles that were not included in the muscle modeling. Therefore, when interpreting the results from this model it was important to realize that the model was only accurate for the elements represented and not fully reflective of the human upper extremity.

There are also a number of general limitations to this work that need to be addressed. With all human modeling, mathematically based models are reflective only of that which is included in the model and a number of assumptions need to be made to create a manageable model as described earlier in this section. Therefore, when comparing or interpreting the results of these mathematical models it is essential to understand the limitations of the model so that these limitations can be taken into account.

Another general limitation to this work was that the data was based on motion capture data collected in a laboratory environment and therefore the subjects may have modified some of their pitching mechanics due to the environment. To reduce the effect of this limitation all pitchers were provided as much time as they required to pitch and warm up until they felt comfortable in the laboratory space prior to data collection. Furthermore all pitchers, pitched from a 10” regulation indoor pitching mound toward a pitching target with a designated strike zone set 60’6” away to better simulate a typical pitching environment.

The final limitation that needs to be addressed is the Lagrangian approach itself. One of the largest limitations is in creating the Lagrangian. A great deal of thought must be taken at the outset of the model to understand the most efficient and complete way of setting up the generalized coordinates for the system. It is actually the construction of the Lagrangian that is the most difficult part of using this
method as the subsequent math is straightforward and relatively easy. While the mathematics are relatively straightforward, requiring only the use of partial derivatives, the math can be extremely tedious requiring a very careful accounting of each term as one progresses further through the model. It is therefore very easy to miss or repeat a term in an early step and then carry this mistake throughout the remainder of the solution. Therefore using a systematic approach as described in the discussion section of this work can help prevent these problems.

6.6 Future Work

The goal of this project was to provide evidence that the Lagrangian approach could be a useful tool in creating accurate biomechanical models to describe complex motions of the upper extremity. This project has provided this evidence but there are a number of improvements and modifications that could be expanded on to improve the utility of this approach for biodynamic modeling. The model is currently able to estimate joint moments as well as estimate the contributions of individual muscle groups to these moments. Through the use of the equations by Marras et al. it was possible to validate the muscle moments from EMG signals obtained from a subject. Using this technique with the current muscle model allowed for a 0.43 correlation between the estimated EMG signal and the actual EMG signal as shown in Figure 125.
Future work may be able to modify this equation to allow researchers to take the motion profiles and joint moments output from the model and reverse the equation by Marras et al. to estimate EMG signals. This method could allow researchers to obtain EMG signals from subjects without the need for collecting actual EMG data but basing the EMG signal from motion data alone. This would provide huge benefits in both cost and the time required to conduction testing as well as allowing for the collection of estimated EMG signals in instances where the environment may make it impossible to collect EMG data.

Additional topics for future work that could improve the accuracy of the muscle model would be to improve the muscle constants as discussed in the limitations section. Finally, the inclusion of additional muscle elements, dampening caused by the joint fluid, and the inclusion of translational motion at each of the joints would improve the accuracy of the model. Furthermore, the inclusion of these elements would allow for a far more detailed model that would provide greater insight into the working of the biomechanical system. It may be more efficient to focus on a single joint each time rather than the entire upper extremity system as the mathematics would quickly become extremely tedious. The utility of a hyper accurate model such as this would be a huge benefit in surgical planning applications. It would
allow surgeons to gain an idea of how changing a muscle insertion point, or lengthening a muscle could affect joint motion or force generation within a muscle.
7. Conclusion

This work provides the framework for a Lagrangian based model of the upper extremity that includes four segments and 13 degrees of freedom. The model presented in this work provides the foundation for future modeling. Using the approaches discussed in this work it is possible to create models of the lower extremity, or hyper accurate models of a singular joint and its surrounding soft tissue structures.

The model was proven to be valid in a number of different applications and is capable of being applied to baseball pitching mechanics which are some of the most complex and unnatural motions that the upper extremity performs. The results of this work illustrate the utility and accuracy of the model, and provide a number of examples where this model provides information that is similar to that of currently validated models, thereby helps to proving its validity. The results also illustrated this model has the ability to provide a great deal of information that is not currently available in the pitching literature concerning soft tissue structures. It also allows for the creation of patient specific models that can be used to treat or address problems in a pitcher’s mechanics.

The results of this model prove the utility of the Lagrangian approach in a biomechanical setting. The Lagrangian approach is often overlooked in favor of the more direct Newtonian approach which relies on computing power to solve for the variables of interest. However, the use of the Lagrangian model was able to provide a more adaptable model that is far more useful to the understanding of mechanisms of injury in baseball pitchers, and can be easily extended to better understand how modifications to input parameters affect the models outputs. This modeling approach provides a much deeper understanding of the relationship between variables allowing both clinicians and researchers the flexibility to explore scenarios that could never be tested within the confines of a laboratory.
References


Appendix A: Equations of Motion

This appendix contains the equations of motion governing the Glenohumeral and elbow joints.

Variable Dictionary

**Mass variables:**
- \(m_s\): mass of upper arm segment
- \(m_l\): mass of lower arm segment
- \(m_w\): mass of hand segment

**Length Variables:**
- \(L_1\): Length of Thoracic segment
- \(L_2\): Radius of Thoracic segment
- \(L_3\): Radius of upper arm segment
- \(L_4\): Length of upper arm segment
- \(L_5\): Length of lower arm segment
- \(L_6\): Radius of lower arm segment
- \(L_7\): Length of hand segment
- \(L_8\): Radius of hand segment
- \(L_{C}\): Length of the clavicle
- \(L_{S}\): Distance between base of the scapula and AC joint
- \(L_{B}\): Length of Biceps
- \(L_{T}\): Length of Triceps
- \(L_{D}\): Length of Deltoids
- \(L_{I}\): Length of Infraspinatus
- \(L_{P}\): Length of Pectorialis Major
- \(E_{B}\): Resting Length of Biceps
- \(E_{T}\): Resting Length of Triceps
- \(E_{D}\): Resting Length of Deltoids
- \(E_{I}\): Resting Length of Infraspinatus
- \(E_{P}\): Resting Length of Pectorialis Major

\[L_{S3} = \frac{1}{3}L_{S}\]

\[\beta = \frac{1}{12}(3L_3^2 + L_4^2)\]

\[\zeta = \frac{1}{12}(3L_6^2 + L_5^2)\]

\[\lambda = \frac{1}{12}(3L_8^2 + L_7^2)\]

**Constants:**
- \(K_B\): Spring constant for Biceps
- \(K_T\): Spring constant for Triceps
- \(K_D\): Spring constant for Deltoids
- \(K_P\): Spring constant for Pectorialis Major
- \(K_I\): Spring constant for Infraspinatus
- \(D_B\): Dampening constant for Biceps
Dt: Dampening constant for Triceps
Dd: Dampening constant for Deltoids
Dp: Dampening constant for Pectorialis Major
Di: Dampening constant for Infraspinatus
αb: Activation constant for Biceps
αt: Activation constant for Triceps
αd: Activation constant for Deltoids
αp: Activation constant for Pectorialis Major
αi: Activation constant for Infraspinatus

Angles:
A: $\cos \theta_T$ : Thoracic coronal plane angle
B: $\cos \phi_T$ : Thoracic sagittal plane angle
C: $\cos \psi_T$ : Thoracic transverse plane angle
D: $\sin \theta_T$ : Thoracic coronal plane angle
E: $\sin \phi_T$ : Thoracic sagittal plane angle
F: $\sin \psi_T$ : Thoracic transverse plane angle
X: Linear displacement of thorax in x direction
Y: Linear displacement of thorax in y direction
Z: Linear displacement of thorax in z direction

G: $\cos \theta_G$ : Glenohumeral coronal plane angle
H: $\cos \phi_G$ : Glenohumeral sagittal plane angle
I: $\cos \psi_G$ : Glenohumeral transverse plane angle
J: $\sin \theta_G$ : Glenohumeral coronal plane angle
K: $\sin \phi_G$ : Glenohumeral sagittal plane angle
L: $\sin \psi_G$ : Glenohumeral transverse plane angle

M: $\cos \phi_L$ : Elbow sagittal plane angle
N: $\sin \phi_L$ : Elbow sagittal plane angle

O: $\cos \theta_W$ : Wrist coronal plane angle
P: $\cos \phi_W$ : Wrist sagittal plane angle
Q: $\cos \psi_W$ : Forearm rotation angle
R: $\sin \theta_W$ : Wrist coronal plane angle
S: $\sin \phi_W$ : Wrist sagittal plane angle
T: $\sin \psi_W$ : Forearm rotation angle

Generic Variables:
. Those variables with a single dot indicate angular or linear velocity
.. Those variables with a double dot indicate angular or linear acceleration

Moments:
$M_b$: Biceps Moment
$M_d$: Deltoid Moment
$M_i$: Infraspinatus Moment
$M_p$: Pectorialis Moment
$M_t$: Triceps Moment
**Generalized Forces:**

\[ Q_{\theta s} = M_D - D\ddot{\alpha}D \left( L_D - \frac{L_C}{L_C - 0.04} G + \dot{\theta}_s \frac{L_C}{L_C^2 - 0.04^2} \dot{G}^2 \right) \]

\[ Q_{\phi s} = M_p + M_I - D\ddot{\alpha}I \left( L_I \frac{L_S^3}{L_S - 0.03} H + \dot{\phi}_s \frac{L_S^2}{L_S^2 - 0.03^2} \dot{H}^2 \right) - Dp\alpha \left( L_P \frac{L_S}{L_S - 0.06} H + \dot{\phi}_s \frac{L_S^2}{L_S^2 - 0.06^2} \dot{H}^2 \right) \]

\[ Q_{\psi s} = M_p + M_D - D\ddot{\alpha}D \left( L_D - \frac{L_C}{L_C - 0.04} I + \psi_s \frac{L_C^2}{L_C^2 - 0.04^2} \dot{I}^2 \right) - Dp\alpha \left( L_P \frac{L_S}{L_S - 0.06} I + \psi_s \frac{L_S^2}{L_S^2 - 0.06^2} \dot{I}^2 \right) \]

\[ Q_{\phi L} = M_B + M_T - D\ddot{\alpha}B \left( L_B \frac{L_S}{L_S - 0.02} M + \dot{\phi}_L \frac{L_S^2}{L_S^2 - 0.02^2} \dot{M}^2 \right) - D\tau \tau \left( L_T \frac{L_S}{L_S - 0.02} M + \dot{\phi}_L \frac{L_S^2}{L_S^2 - 0.02^2} \dot{M}^2 \right) \]

**Potential Functions:**

\[ V_{\theta s} = \frac{5}{2} m_s gL \dot{A} \dot{G} - KD(E_D G - L_D) \]

\[ V_{\phi s} = -K(P_E H - L_P) - KD(E_D H - L_D) \]

\[ V_{\psi s} = -KI(E_I L - L_I) - K(P_E I - L_P) \]

\[ V_{\phi L} = \frac{3}{2} m_L gL \dot{S} \dot{N} - KB(E_B M - L_B) - KT(E_T M - L_T) \]
Governing equation for $\theta_s$:

$$Q_{\theta s} = MS + ML + MW + V_{\theta s}$$

Where $Q_{\theta s}$ and $V_{\theta s}$ were previously defined, and

**MS is:**

$$m_s \left[ \frac{1}{2} \left( \frac{1 \cdot 12}{2} \beta \ddot{s} + L4 \dddot{s} - L4 \dot{s} \right) + LAL1 \left( \dot{\phi}_t HI + \dot{\phi}_t (-\dot{s} HL - \dot{s} IK) + \ddot{\theta}_t (-GI + HI - JKL) + \dot{s} \theta_t (IJ - GKL + GL - JKI) + \phi_t \dot{\theta}_t (-GK - JHL) \right) + LAL2 \left( -\ddot{s} \dot{\theta}_t HI + \ddot{s} \dot{\theta}_t (\ddot{s} HL + \dot{s} IK) - \dot{\theta}_t \dot{s} \dot{H}J \right) + L4 \left( \dddot{x} AC \dot{H}I + \ddot{x} BCGI + \dddot{x} ABGH + \ddot{x} (AC(-\dot{s} HL - \dot{s} IK) + HI(-\dot{\theta}_t DC - \dot{\theta}_t AF)) + \dddot{y} (\dot{s} BC(-GI + JI + GKL) - GI(-\dot{s} BF - \dot{s} CE)) + \ddot{z} (AC(-\dot{s} HJ - \dot{s} G K) + GH(-\dot{\theta}_t AE - \dot{x} BD)) \right) \right] - 2L1L2 \left( \dot{\theta}_t \dot{\phi}_t (JHI - BCGK) + \dot{\theta}_t^2 \left( -\frac{1}{2} G J I^2 + \frac{1}{2} G J K^2 L^2 + G^2 I L + J^2 K^2 I L \right) \right) - 2L2^2 \left( \dot{\theta}_t \dot{\phi}_t G^2 I L - \dot{\theta}_t G J I^2 \right) - 2L1L2 \left( \dot{\theta}_t (-\dot{\phi}_t H^2 I - \dot{\theta}_t J H I^2 + \dot{\phi}_t GHI KL) + \dot{\theta}_t^2 (GJ HI + H^2 J KL - G^2 I KL) \right) - 2L1 \left( \dot{\phi}_t (\ddot{y} BC(-JHI^2 + GHI KL - z ACJ^2 I) + \dot{\theta}_t (AC(DEF J H^2 - GHI KL) + AC(JI - GHI KL + GHI J K^2 L^2) + \ddot{y} BC(J^2 K^2 L + H^2 J KL) + \ddot{y} BC(G HI J - G^2 H K L + H^2 L K) - 2 \ddot{y} (\ddot{y} M(ABC + BCDEF) - BCDEF HI^2) + \ddot{y} B C^2 (J^2 K^2 L - G^2 I K L + G^2 J K^2 - I J^2 K L - G J H I) - 2 \ddot{y} (\ddot{x} J H I^2 A^2 BC + ACDEF - \frac{1}{2} \dddot{z} A B^2 B^2 J^2) \right)$$

**ML is:**

$$m_L \left[ L2L5 \dot{\phi}_t J H M - L1L5 \left( \dot{\phi}_t M (JI - GKL) - L5 \left( \dot{\phi}_t M (\ddot{y} BC(-JI + GKL) - z ACJ I) \right) \right) - 2L1L4 \left( \frac{1}{2} \ddot{z} DEFHI + \frac{1}{2} z \left( -DEF(\dot{\phi}_t K I + \ddot{s} H L) + HI(\dot{\theta}_t AEF + \ddot{s} D B F + \ddot{y} D E C) \right) + \dot{\theta}_t M^2 (-GI - JKL) + \phi_t H I M + \ddot{\theta}_t M (\ddot{s} (G L + J I M - \ddot{s} H L) - 2 N (G I + J K L) - \dot{s} J H M) + \ddot{\theta}_t (\dot{\phi}_t K I M - \ddot{s} H I N) \right) + 2L4^2 \left( \dddot{s} M^2 - \dddot{s} M (2 N M + M - N) \right) + 2L2L4 \left( \ddot{y} H I M + \ddot{\theta}_t G I M^2 + \ddot{y} \ddot{\theta}_t M (\ddot{s} H L M + \dddot{\phi}_t I K M + \dddot{\phi}_t H I N) + \ddot{\theta}_t (\ddot{s} G L - G I M N) \right) + 2L4 \left( \frac{1}{2} M (\ddot{x} A C H I + \ddot{y} B C G I M + \ddot{z} A B G I M) + \ddot{x} \left( A C(\ddot{s} H L M - \ddot{s} K I M - \ddot{s} H I N) + G I M (-\ddot{x} D C - \dddot{\theta}_t F A) \right) + \dddot{y} (\ddot{x} S B C M(-GL M - J I + G K L) - M^2 (\ddot{s} B C G K - \dddot{\phi}_t C E G I - \ddot{\theta}_t B F G I) - 2 B C G I M N) + \ddot{x} \left( M (-\ddot{s} A C J H - \ddot{\phi}_t A C G L M + \dddot{\phi}_t A E G H M - \ddot{\theta}_t G I M - 2 A C G I) \right) + \ddot{\theta}_t \ddot{s} J H M \right) - 2L1L2 \left( \dddot{\theta}_t M^2 (G J H I + H K L) \ddot{s} \ddot{\theta}_t M (G H I K L - J H I^2) - \dddot{\phi}_t \dddot{\phi}_t M J H I^2 \right) - 2L1 \left( \dddot{\theta}_t \left( \ddot{x} (M(A C J H I^2 + D E F J H I^2 - 2 A C G H I K L - 2 D E F G H I K L) + \ddot{y} (B C M^2 (G J I^2 - G J K^2 L^2) - B C I K L) + \ddot{z} A B M^2 (G J H I + J H)) \right) - 2L2 \left( \dddot{s} M \left( -\ddot{s} G H I K L + \ddot{s} G H I^2 - \frac{1}{2} \dddot{\theta}_t G J H I^2 - M - \dddot{x} (A C J H I^2 + D E F J H I^2) - \dddot{y} B C (G J H I M + H K L M) - \dddot{z} D E F G J H I^2 M \right) - 2 \dddot{y} \left( \ddot{x} M (A B C J H I^2 - A B C^2 K L G H I + \right) \right]$$

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\[BCDEF(-JH^2 + GHIKL)) + \dot{z}AB^2 CM^2(-GJHI + HKL) - \frac{1}{2} \dot{y}B^2 C^2 GJ^2 M^2 - 2\dot{z}(xJH^2 IM(-A^2 BC - ABDE) + \frac{1}{2} \dot{z}A^2 B^2 GJH^2 M^2)\]

and MW is:

\[m_w\left[-2L7(L\dot{\theta}_t JHMPQ(\phi_w - \theta_w)) - L2L7(L\dot{\theta}_t JIMOQ(\phi_w + \theta_w)) - L7(yBCMPQ(\phi_w GKL + \dot{\theta}_w G1 - \dot{\theta}_w GKL)) + \dot{z}JIMOQ(-\phi_w AC - \dot{\theta}_w AB)\right] -
\[2L5(\phi_t yACMPQRST(GKL - JI) - 2L5L2(\phi_t \dot{\theta}_t JHMOQRST)) + 2L1L4(\dot{\theta}_t HI(MOQ + RST) + \dot{\theta}_t OP^2 QM(-GI - JKLM) + \dot{\phi}_t (\psi_s MOQ(KI - HL) + \psi_s HLOPQRT + \dot{\phi}_w HI(-MQR - QR^2 ST + O^2 PST) - \psi_w HI(OTM + OQ^2 RT) - \dot{\phi}_t HIOQN + \dot{\psi}_w OQRST(-KI + H^2 IRST)) +
\[\dot{\phi}_t (\dot{\theta}_s (GKM^2 PQ - GKL^2 O^2 PQ - JIO^2 PQM + GKL^2 O^2 PQ) + \phi_w (GKM^2 PQ - JLM^2 O^2 PQ) + \dot{\phi}_w GIMO^2 SQ + \dot{\phi}_w GHSQ^2 PQ +
\[\psi_w (GIMO^2 PT + JKLM^2 OP^2 Q) + 2(GHMRQP + JKLMNOP^2 Q + ORPQMJKL) + \dot{\theta}_w (M^2 RST + RP^2 Q) + \dot{\psi}_w (GKM^2 OP^2 Q + JHMOP^2 Q - 2GKM^2 OQ^2 PQ) + 2GKM^2 OQ^2 PQ) +
\[2LAL2(\dot{\theta}_t GHM^2 O^2 P^2 + \dot{\theta}_t (M^2 O^2 P^2(-\dot{\theta}_t JH - \dot{\phi}_s GL) - 2GH(M^2 ORP^2 + M^2 O^2 PS + M^2 O^2 Q^2) - JHM(\dot{\psi}_s O^2 PQ + \dot{\psi}_s OQRST + O^2 PQ^2) )) + LAL7(OMP(\dot{\phi}_w - \dot{\theta}_w) + \dot{\phi}_w (-\phi_w OSM - \dot{\theta}_w RPM -
\[\dot{\phi}_t OPL) + \dot{\theta}_w (-\phi_w OSM - \dot{\theta}_w RPM - \dot{\phi}_t OPL)) + 2LALS(\dot{\phi}_t MOPRST + \phi_r \dot{\theta}_w (PR^2 STM + O^2 PSTM) - \dot{\phi}_r NOPRST + \phi_r (-MORS^2 T + GI^2 MRT) + \dot{\psi}_w MOPQRT) + 2L^2 (\dot{\psi}_s (1 - OQMRST) + \dot{\phi}_t MO^2 P^2 M + \dot{\psi}_s (\dot{\phi}_w M(PR^2 ST - O^2 PST) + \dot{\phi}_w M(OSR^2 T - OQ^2 RS) + \phi_r OPRSTN - \dot{\psi}_w OPRQST) - 2\dot{\phi}_t (OP^2 M^2 + O^2 PM^2 + O^2 P^2 MN) + 2L4(\dot{x}(DEFHIM(1 + O^2 PQ) +
\[ACHIM(O^2 PQ + OQRST)) + \dot{y}BCM^2(GIOP^2 Q + JKL^2 OPQ + JKL) + \dot{z}ABG^2 M^2 O^2 P^2 +
\[\dot{x}(\dot{\psi}_s HIM(-DEF - O^2 PQ(AC + DEF)) + \dot{\phi}_s KIM(-DEF - O^2 PQ(AC + DEF)) - \dot{\phi}_t HIM(-DEF - ACO^2 PQ) + \dot{\theta}_t HIM(AEF - CD^2 Q^2 + DCPQST + AEFO^2 PQ) + \phi_t DBFHIM(1 + O^2 PQ) +
\[\dot{\psi}_t HIM(GIOPRST + DEC(1 + O^2 PQ)) + \phi_r HIM(-ACO^2 SQ + ACORS^2 T - ACQ^2 PRS - DEF^2 PQ) + \dot{\psi}_w HIM(AC(-O^2 PT - OPRQ) - DEFO^2 PT) - 2HIMORPQ(AC + DEF) +
\[\dot{\theta}_w ACHIM(PR^2 ST - O^2 PST) + \dot{y}(\dot{\psi}_s BCM(OP^2 Q(-JH + GKL) + PQRST(-JL + GKL) +
\[JKIM(1 + O^2 PQ) + GLMOP^2 Q) + \dot{\theta}_w BCGKLM^2 +
\[\phi_t ECM^2(-JKL(1 + O^2 PQ) - \dot{\psi}_t BFM^2(-DEF(1 + O^2 PQ) - GIO^2 PQ) - 2BC(-JKL(MN +
\[M^2 OP^2 Q + NOP^2 Q) - GI(M^2 ORPQ + NOP^2 Q) - \dot{\theta}_w BCM^2 RP^2 Q(GI + JKL) +
\[\psi_t BCM^2 OP^2 T(-GI - JKL) +
\[\dot{z}(\dot{\psi}_s ABJIM(O^2 PQ - OQRST) - \dot{\psi}_t ACGL^2 M^2 O^2 P^2 - \dot{\theta}_t DBGIM^2 O^2 P^2 - \phi_t AEGIM^2 O^2 P^2) -
\[2ACGI(ORP^2 M^2 - M^2 PS - MNO^2 P^2) - 2L1L2(\dot{\theta}_t (\dot{\phi}_s M(-JH^2 IO^2 PQ - JHIORST) +
\[\dot{\phi}_s M(JH^2 IPQRST + GHIKLPQRST - JH^2 IPO^2 Q) + \dot{\theta}_t M^2 O^2 PQ(GJH^2 - HKL) -
\]
\[2L^2 \left( \dot{\psi}_t \dot{\theta}_t \mathcal{M}(JH^21O^2PQ - GJI^2OPRST) - \frac{1}{2} \dot{\theta}_t^2 \mathcal{M}(GJH^2M^2O^2P^2) - 2L^2 \left( \dot{x} \left( \dot{\psi}_t \mathcal{A}CHK^1L^2Q^2O^2 + \dot{\theta}_t \mathcal{M}(AC(-JH^21O^2PQ - JHIOQRST) + DEF(JH^21O^2PQ - JKI^2O^2PQ)) \right) + \dot{y} \left( \dot{\psi}_t \mathcal{B}CM(-GKLMO^2PQ + PQRST(JHI^2 - GHIKL)) + \dot{\theta}_t \mathcal{B}CMOP^2Q(-GJHIM + HKL) \right) + \dot{z} \left( \dot{\psi}_t \dot{AB}JI^2M(O^2PQ - OQRST) - \dot{\theta}_t \dot{AB}JHI^2M^2O^2P^2 \right) \right) \right) - 2L^1 \left( \dot{x} \dot{\theta}_t \mathcal{M}(DEF(JH^21O^2PQ + JH^2IPQRST - GKLOPQ^2 - GHIKLSPQRST) + ACOPQ^2(-GHIKL + GIGKL)) + \dot{y} \left( \dot{\psi}_t \mathcal{B}CM \left( (-JHI^2O^2P^2Q^2 + GHIKL^2P^2Q^2 - PQRST(JHI^2 - GHIKL) + \dot{\theta}_t \mathcal{B}CM^2P^2Q^2(GJHI - GJK^2L^2 - HKL - KIL) \right) + \dot{z} \left( -\dot{\psi}_t \mathcal{A}CJIMOQ^2RST + \dot{\theta}_t \mathcal{A}BM^2(GJHIOQ^2P^2 - O^2P^2) \right) \right) \right) - 2\dot{y} \left( \dot{x} \mathcal{M}(ABC^2OP^2(-JHI + GKL) + BCPQRST(JHI^2 - GHIKL) + BCDEFOPQ^2(JHI^2 - GHIKL) - BCDEFJHIHIRST) + \dot{y}B^2C^2M^2P^2Q^2 \left( (-GJI^2 + GJL + \frac{1}{2}GJK^2L^2) + \dot{z} \left( AB^2C^2M^2OP^2Q(-GJI^2 - ILK) \right) \right) - 2\dot{z} \left( \dot{x} \mathcal{M}(-ABCJI^2O^2PQ - ACJI^2OQ + A^2BCJI^2OQRST - ACDEFJIO^2PQ) - \frac{1}{2} \dot{z}A^2B^2GJI^2M^2O^2P^2 \right) \right) \]
Governing equation for $\phi_s$

$$Q_{\phi_s} = MS + ML + MW + V_{\phi_s}$$

Where $Q_{\phi_s}$ and $V_{\phi_s}$ were previously defined, and

$MS$ is:

$$m_s \left[ \frac{1}{2} L^2 \dot{\phi}_s^2 - L 4 L_1 \left( - \ddot{\phi}_s \left( IK(\dot{\phi}_t - \dot{\psi}_t) + \theta_t (GK + JHL) \right) + \dot{\psi}_s (\dot{\phi}_t KI + \dot{\theta}_t IJI) \right) - L 2 L_4 \left( \dot{\psi}_s (\dot{\phi}_t KI + \dot{\theta}_t G) \right) - L_4 \left( \dot{\phi}_t (x I (AC - DEF) + z AGLB + y BCJHL) + \dot{\psi}_s (\dot{x} I (AC + DEF) + z ACGK + y DEFIKI) \right) - 2 L^2_1 \left( \frac{1}{2} \dot{\phi}_t HKL^2 + \dot{\theta}_t (\dot{\phi}_t (GK^2 - BCJHL) + \dot{\theta}_t GHIJL + \frac{1}{2} J^2 HKL^2) \right) \right] - 2 L^2_2 \left( - \frac{1}{2} \dot{\psi}_t HKL^2 - \dot{\theta}_t HKG^2 \left( \dot{\psi}_t - \frac{1}{2} \dot{\theta}_t \right) \right) - 2 L_1 L_2 \left( \dot{\psi}_t \left( \dot{\phi}_t HK (I^2 - GI) + \dot{\theta}_t (-GKH^2 - JHL) \right) + \dot{\theta}_t^2 (G^2 IL + GJL^2) \right) - 2 L_1 \left( \dot{\theta}_t \left( x (AC (GK^2 + IL) - J^2 HKL^2) + DEF (GK^2 + IL) \right) - y BC (GHIJL + GHIKL) + z ABG^2 KI + \dot{\phi}_t (x (H^2 ILAC - DEFKL^2) + y BC (-GK^2 - JIL) - z ABGHKLI) \right) - 2 L_2 \left( \dot{\phi}_t (x HK (AC + DEF) + y BC (-GK^2 + JIL) + z ABGHKLI) + \dot{\theta}_t (x GHIK (AC - DEF) + y BC (-G^2 KI - GJL) + z ABG^2 HK) \right) - 2 \dot{x} \left( \dot{\psi}_t \left( GHI^2 (-AB - x BCDEF) + IJL (-AC^2 - BCDEF) \right) + \dot{\psi}_t^2 C (-GJL + \frac{1}{2} J^2 HKL^2) \right) - \dot{\psi}_t \left( \dot{\phi}_t (GK^2 - IJL) + DEF (GK^2 + IJL) \right) + y B^2 C^2 (-GJL + \frac{1}{2} J^2 HKL^2) + \dot{\psi}_t (x GHIK (AC^2 BC - ACDEF) - \frac{1}{2} z AB^2 C^2 G^2 HK) \right]$$

$ML$ is:

$$-m_L \left[ L \left( \dot{\phi}_L (\dot{\phi}_t KI + \dot{\theta}_t JHL) \right) + L_5 \left( \dot{\phi}_L (x IK (-AC - DEF)) + M (y BCJHL - z ACGK) \right) + 2 L_1 L_4 \left( \dot{\theta}_t M (\dot{\phi}_t KI - x HL) + \dot{\psi}_s (\dot{\phi}_s KI + \dot{\theta}_s JHL) \right) + 2 L_2 L_4 \left( \dot{\psi}_s M (\dot{\phi}_s KI + \dot{\theta}_s GKM) + \dot{\psi}_s (-\dot{\psi}_s KI + \dot{\theta}_s GKM) \right) + L_4 \left( \dot{\theta}_s M (\dot{\phi}_s (AC - DEF) - z ACGL) + \dot{\psi}_s (\dot{\theta}_s DEFIKI - y BCJHL + \dot{\psi}_s ACGKM) \right) + 2 L_2^2 \left( \dot{\theta}_t M (\dot{\phi}_s KI - \dot{\theta}_t G^2 HK + \dot{\psi}_t GHIK) \right) - \frac{1}{2} \dot{\psi}_t^2 H^2 KI^2 \right) + 2 L_1 \left( \dot{\theta}_t M (\dot{\phi}_t G^2 KI + x (AC (GK^2 + G^2 KI) + JIL) + DEF (GK^2 + IJL) \right) + \dot{\psi}_s (\dot{\theta}_s GHIJLM + GKM + J^2 KL^2) \right) + \dot{\psi}_s (x HK^2 (AC - DEF) + y BC (GK^2 + JHL) - z ACGK) \right) + 2 L_2 \left( \dot{\phi}_t (x (ACH^2 KI^2 (1 + N) + DEFKL^2 (1 + N)) + y ACM (GK^2 + JIL) + z ACHKLM) + \dot{\theta}_t (x M (ACGHK - DEFK) + z M^2 BC (-G^2 KI - GJL) - z DEHKG^2 M^2) \right] + x^2 K^2 (A^2 C^2 - D^2 E^2 F^2 - 2 ACDEF) + 2 \dot{\psi}_t (x M (-A^2 BCGLK^2 - AB^2 CG^2 KIM - BCDEF G^2 KI - BCDEF KL) + y B^2 C^2 M^2 (GHIJK - \frac{1}{2} HK^2 L^2) - z AB^2 CGJLM^2) + 2 \dot{\phi}_t (x M (-A^2 BCHKGI - ACDEF G^2 KI) - \frac{1}{2} z A^2 B^2 G^2 HKM^2) \right]$$
And \( M_w \) is:

\[
-m_w \left[ L1L7 \left( \dot{\phi}_w \left( -2\dot{\phi}_t \psi_1 GHI L \right) + \dot{\theta}_t \left( \dot{\psi}_t KI (OQ - RST) - \dot{\theta}_t M P Q (GK + JHL) \right) \right) + L2L7 \left( \dot{\phi}_w \psi_1 KI (OQ + RST) + \dot{\theta}_w \left( -\dot{\psi}_t K I (RST - OQ) + \theta_g G K M P Q \right) \right) + \\
L7 \left( \dot{\phi}_w (\dot{x} K I (A C (O Q + RST) - D E F O Q R S T) + \dot{\gamma} B C J H L M P Q) + \dot{\theta}_w (\dot{x} K (A C (O Q + RST) + D E F O Q R S T) - \dot{\gamma} B C J H L M P Q) \right) + 2 L 5 L 1 \left( \dot{\phi}_t \left( \dot{\psi}_t H I (O Q R S T - R^2 S^2 T^2) + \dot{\theta}_t O Q R S T (A C K I - J H L M) \right) \right) + 2 L 5 L 2 \left( \dot{\phi}_t (\dot{\psi}_t I (K O Q R S T + K R^2 S^2 T^2) - \dot{\theta}_t G K M O Q R S T) \right) + \\
2 L 5 \left( \dot{\phi}_t \dot{\psi}_s H I O Q (2 O Q + 2 R S T) + \dot{\theta}_s \dot{\psi}_s K I R S T (O Q + R S T) - \dot{\theta}_t H I M (O^2 P Q - O P R S T) \right) + 2 L 2 L 4 \left( \dot{\psi}_s K I (-2 O^2 Q^2 + 2 O Q R S T M^2 + O Q R S T) + \dot{\theta}_s \left( \dot{\psi}_s G K M O^2 P Q (1 + R S T) + \dot{\theta}_s G K M O^2 P Q \right) + 2 L 4 \left( \dot{\psi}_s \dot{x} K (O Q R S T (A C + D E F) + A C R^2 S^2 T^2) - \dot{\gamma} B C J H L M (P Q R S T - O P Q^2) \right) + \dot{\theta}_s \dot{x} K (A C (O Q R S T + A C R^2 S^2 T^2) + \dot{\gamma} B C J H L M (P Q R S T - O P Q^2) + D E F R S T (R S T - O^2 P Q) - \dot{\gamma} A C J H L M (O^2 P Q^2) \right) + 2 L 1 L 2 \left( \dot{\phi}_t \left( I^2 H K - \frac{1}{2} O^2 Q^2 - O Q R S T \right) + G H K I M O P Q^2 - \frac{1}{2} H K I^2 R^2 S^2 T^2 \right) + 2 L 2 \left( \dot{\phi}_t \dot{\theta}_t (J H L M O P Q^2 + G H K I M P Q R S T - J I L O Q R S T + \frac{1}{2} \dot{\theta}_t^2 M^2 P^2 Q^2 (-G^2 H K + J H L) \right) + 2 L 2 \left( \dot{\psi}_t H K I^2 \left( \frac{1}{2} O^2 Q^2 - O Q R S T \right) + \dot{\theta}_t M^2 \left( \dot{\psi}_t G K I O^2 P Q - \dot{\theta}_g G H K O^2 P Q \right) \right) + 2 L 2 \left( \dot{\psi}_t \left( H K I^2 (O^2 Q^2 + R^2 S^2 T^2) + 2 H K I M G K H K I O^2 P Q (-1 - R S T) + \dot{\psi}_t \dot{\theta}_t (G H I K P S T - H^2 J I M O P Q^2 - G H K I M P Q R S T - G H K I M O P Q^2) + \dot{\theta}_t^2 G J I L^2 O^2 P Q \right) + 2 L 1 \left( \dot{\phi}_t \left( \dot{x} (A C (H K I^2 O^2 Q^2 + K I O R S T - H K I^2 Q R S T + H K I^2 R^2 S^2 T^2) + D E F (-2 H K I^2 O^2 Q^2 + 2 H K I^2 O Q R S T - H K I^2 R^2 S^2 T^2) \right) - \dot{\gamma} B C M (I J L O P Q^2 + P Q R S T (K G I^2 + I G L) - z A B G H I J L M^2 O^2 P Q) + \dot{\theta}_t \dot{x} M (A C (K I O P Q^2 - G H K I P Q R S T + G H K I O P Q^2 + J H L P Q R S T) - z A B G H I J L M^2 O^2 Q^2) \right) + \\
2 L 2 \left( \dot{\phi}_t \left( \dot{x} (A C (G K I^2 O^2 Q^2 - J I O Q R S T - H K R^2 S^2 T^2 + M N I^2 R^2 S^2 T^2 + M N I^2 Q R S T) + D E F (H K I^2 O^2 Q^2 + 1 + 2 H K I^2 O R S T + H I^2 O Q R S T^2 T^2) \right) + \dot{\theta}_t \dot{x} \left( D E F (G H I^2 Q R S T - G H K I M O^2 P Q - G H K I M O^2 Q R S T) + A C (-G H K I M O^2 P Q - G H K I M O^2 Q R S T) - \dot{\gamma} M (B C O^2 P Q (G^2 H K M - G J I) - z A B G K L M O^2 P^2 \right) + 2 \dot{x}^2 \left( D E F S Q (A C O^2 Q^2 + O Q R S T - \frac{1}{2} O Q R S T - A C R^2 S^2 T^2 + \frac{1}{2} D E F O P Q^2 \right) - \frac{1}{2} A^2 G^2 H K I^2 M O P^2 Q^2 \right) + 2 \dot{\gamma} \left( \dot{x} (B C D E F M O P^2 Q^2 (-G K I^2 - J L I) + \dot{\gamma} B^2 C^2 M^2 P^2 Q^2 (G J H I + \frac{1}{2} J^2 H K I^2) + z A B^2 C H I J L M^2 O P^2 Q \right) + 2 z (\dot{x} A^2 B C G K I^2 (M O P^2 Q + N O Q R S T)) \right]
\]
**Governing Equation for \( \psi_s \)**

\[ Q_{\psi_s} = MS + ML + MW + V_{\psi_s} \]

Where \( V_{\psi_s} \) and \( Q_{\psi_s} \) were previously defined, and

**MS is:**

\[
ms \left[ \frac{1}{2} \left( \dot{\psi}_s \left( \frac{1}{6} \beta - LA^2 \right) - LA \ddot{\psi}_s \right) + LAL1 \left( -\ddot{\phi}_t HI + \ddot{\theta}_t (AC + DEF) + \dot{\phi}_t \left( \phi_s KL + HL (\psi_s + \dot{\theta}_s) \right) + \dot{\theta}_t (\psi_s (HL - JKI + DEC) + \theta_s (AEF - GL + JKI) + \phi_s DBF + \psi_s DEC) - \ddot{\theta}_s \psi_s HL \right) + LAL2 (\ddot{\phi}_t HI - \ddot{\theta}_s GL + \phi_s (\ddot{\theta}_t LG - \psi_s KL) + \ddot{\theta}_t \dot{\theta}_s JI) + LA \left( \dot{x} \left( HI \left( \dot{\theta}_s (JI - AEF) - \dot{\phi}_s JHL + \dot{\psi}_s (AF - DEC) + \phi_s (DEF + JHL) + \psi_s (DEC + DEF) \right) \right) + \ddot{\psi}_s (GI - JKL) + \ddot{\psi}_s HI (GI + JKL) + \ddot{\theta}_s HI (2JI - GKL) - \ddot{\phi}_s BCJHL + \ddot{\zeta} (GL (\ddot{\theta}_t DB + \phi_t AE) + AB (\dot{\theta}_t KH + \phi_s GKL)) + \ddot{x} HI (-AC - DEF) + \ddot{\gamma} (BC (GI - JKL) + \ddot{\zeta} ACGH) - 2L1^2 \left( \frac{1}{2} (\ddot{\phi}_t^2 ILH^2 - \ddot{\phi}_s (G^2 IL + J^2 K^2 IL)) + \ddot{\theta}_s \dot{\phi}_t (GHL - BCJKI) - \ddot{\theta}_s^2 GJK \right) - 2L2^2 \left( \ddot{\phi}_t (-\psi_s H^2 IL - \dot{\theta}_t G^2 H^2 L) - \ddot{\phi}_t \dot{\psi}_s (GHI + JKL) + \ddot{\theta}_s^2 (G^2 HL - GHJ) \right) - 2L1 \left( \dot{x} \left( \ddot{\phi}_t H^2 IL (-AC - DEF) + \ddot{\theta}_t (AC (GHI + HL) + DEF GHHL - BCJKI^2 IL) + \ddot{\gamma} (\ddot{\psi}_t (BC (GHI - HL) + \ddot{\theta}_t (AIC (G^2 IL + 2GJH + DEFJIK) + \ddot{\zeta} AB \left( -\ddot{\phi}_t GH^2 IL + \ddot{\phi}_s (G^2 HL + GHJ) \right) \right) - \ddot{\gamma} (\ddot{\phi}_s BC (GHI + HL) + \ddot{\theta}_t (-HIK + GHIJL) + \ddot{\theta}_t ACGH^2 IL) - 2\ddot{x}^2 \left( \frac{1}{2} H^2 IL (A^2 C^2 - D^2 E^2 F^2 - 2ACDEF) \right) - 2\ddot{\gamma} (\ddot{x} (ABC^2 (ABIL - HL) + BCDEF (GHI + HL)) + \ddot{\gamma} \phi^2 \psi^2 \left( -\frac{1}{2} G^2 IL - GJL - \frac{1}{2} J^2 K^2 IL \right) + \ddot{\gamma} AB^2 C^2 (GHIJ) - \ddot{\gamma} AB^2 C^2 (GHIJ) \right) \right] \]

**ML is:**

\[
mL \left[ -L1L5 \left( \ddot{\phi}_t (\psi_t M (GL - JKI) - \psi_t HL) \right) - L2L5 (\dot{\phi}_t \dot{\psi}_t GL) - L5 \left( \dot{\phi}_t (\ddot{x} HL (-AC - DEF) + \ddot{\gamma} BCM (-GL + JKI)) \right) + 2L4^2 (\ddot{\theta}_s M + \psi_s + \dot{\theta}_s \phi_L N) + 2L1L4 \left( \ddot{\theta}_t M (GI + JKL) - \dot{\phi}_t HI + \ddot{\theta}_t \dot{\theta}_s M (JI + GI + KLM + (-GL + JKI)) + \dot{\theta}_t (\ddot{\phi}_s JHL + \dot{\phi}_s GL + \phi_L N (JKL - GI)) + \phi_t (\ddot{\phi}_s IK + \dot{\phi}_s HLM) \right) + 2L4L2 (\ddot{x} HI - \ddot{\phi}_t GIM + \dot{\theta}_s (\ddot{\phi}_s HLM + \ddot{\theta}_s IK) + \ddot{\phi}_t (\ddot{\phi}_s GL + \phi_L GIN) - \ddot{\phi}_s \dot{\phi}_s HIK) + L4 \left( \ddot{x} \left( (AC + DEF) (\ddot{\phi}_s KL + \ddot{\psi}_s HL) + (\ddot{\phi}_t HI + \ddot{\phi}_t HI) (DC - AEF) - \ddot{\phi}_t DBFHI + 2ACHL (M - 1) \right) + \ddot{\gamma} (JKL (\ddot{\phi}_t EC + \psi_B BF) + BCM (-\ddot{\theta}_s GKL - \ddot{\phi}_s JHL) - ... \right) \right] \]

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\[ \dot{\psi}_s (IK) + \dot{\phi}_L B C J K L N + 2 B C M^2 \left( \dot{\theta}_s (KI) + G L (-\dot{\theta}_s - \dot{\psi}_s) \right) + \ddot{x} H I (A C - D E F) + \ddot{y} (-2 A B + J K L M) \] 
\[ - 2 L L 2 \left( \frac{1}{2} \dot{\theta}_s^2 M (G^2 IL - J^2 K^2 IL) - \dot{\theta}_s G J K M^2 - \frac{1}{2} \dot{\phi}_L^2 H^2 IL \right) - 2 L^2 \left( \frac{1}{2} \dot{\phi}_L^2 H^2 IL + \dot{\theta}_s \dot{\psi}_L G H^2 L M \right) - 2 L L L 2 \left( \dot{\theta}_s \dot{\psi}_L M (-2 G H I L - H J L) + \dot{\theta}_s^2 M (G^2 H L + G H J L) + \dot{\phi}_L M (\dot{\psi}_L H^2 IL - \dot{\theta}_s G H^2 L) \right) - 2 L \left( \dot{\theta}_s \left( \dot{\phi}_L G H I M + \dot{x} M (AC - D E F) (G H I L + H J L) \right) + \dot{y} B C M^2 (G^2 IL + G J L + G J K - J^2 K^2 IL) - \dot{z} G^2 H L A B M^2 \right) + \dot{\phi}_L (-\dot{x} A C H^2 IL + \dot{y} B C M (G K L + J K I) - \dot{z} A C G H^2 L M) \] 
\[ + 2 L 2 \left( \dot{\theta}_s \left( \dot{\phi}_L H J K M + \dot{x} M G H I L (-A C - D E F) + \dot{y} B C M^2 (G^2 H I + G J L) \right) + \dot{\psi}_L (\dot{x} H^2 IL (A C + D E F) + \dot{y} B C M (G H I + G J K) + \dot{z} A C G H^2 L M) \right) - 2 \dot{x} \left( \frac{1}{2} H^2 IL (A^2 C^2 + D^2 E^2 F^2 - 2 A C D E F) \right) \] 
\[ - 2 \dot{y} \left( \dot{x} (A B C^2 M (-G H I L - H J K) + D E F B C M (-G H I L - J K I L)) + \dot{y} \left( \frac{1}{2} B^2 C^2 M^2 (G^2 IL - 2 G J L) + \frac{1}{2} B^2 C^2 J^2 K^2 IL \right) + \dot{z} A B^2 C M (G^2 H L + G H J L) \right) - 2 \dot{z} \dot{x} G^2 H L M (A^2 B C - B C D E F) \] 

And MW is:

\[ m_i \left[ -L 1 L 4 \left( \dot{\phi}_L \dot{\phi}_L (H L (-O Q - R S T)) + \dot{\theta}_s M P Q (-\dot{\phi}_L J K I + \dot{\theta}_s (J K I - G L)) + \dot{\theta}_s H L (\dot{\theta}_s Q O + \dot{\phi}_L R S T) \right) \right] - L 2 L 7 \left( (\dot{\theta}_s \dot{\phi}_L H L + \dot{\theta}_w \dot{\psi}_L H L) (O Q - R S T) \right) - L 7 \left( \dot{\phi}_L (x \dot{H} L (A C (O Q + R S T) - D E F (O Q + R S T)) + M (\dot{y} B C J K I P Q - \dot{z} A C G I O P) + \dot{\theta}_w (\dot{x} H L (A C (O Q - R S T) + D E F O Q R S T) + \dot{y} B C M P Q (G L - J K I) - \dot{z} A B G I M O P) \right) - 2 L 5 L 4 \left( \dot{\phi}_L R S T (-\dot{\phi}_L I L R S T + \dot{\theta}_w M P Q (G L - J K I)) \right) \] 
\[ + 2 L 5 L 2 \left( \dot{\phi}_L H L R S T (\dot{\theta}_w (O Q + R S T) - \dot{\phi}_w (O Q + R S T) + \dot{\phi}_L R S T (-1 - R S T) - \dot{\phi}_L (\dot{\theta}_w O S T + \dot{\phi}_w R S T + \dot{\phi}_w R S Q + 2 (O R S^2 T^2 + R^2 P S^2 T^2 + R^2 S^2 Q T)) \right) - 2 L 5 \left( \dot{\phi}_L (x H L R S T (A C R S T + D E F (-O Q - R S T)) + \dot{y} B C M P Q R S T (-G L + J K I) - \dot{z} A B G L M O Q R S T) \right) \] 
\[ + L 4 L 7 \left( \dot{\phi}_w (O Q - R S T) - \dot{\psi}_w (R Q - O S T + R P T) + \dot{\psi}_w (O T - R S Q) - \dot{\phi}_w R P T) + \dot{\theta}_w (-\dot{R} Q + P S T + \dot{\phi}_L (O T - R S Q) + \dot{\psi}_w (O T - R S Q)) \right) \] 
\[ + 2 L 4 \left( \dot{\phi}_s (O^2 Q^2 + O Q R S T + R^2 S^2 T^2) + \dot{\phi}_s M (-O^2 P Q - P Q R S T) + \dot{\psi}_s \left( \dot{\theta}_w (-Q R^2 S T + O^2 Q S T - M O^2 Q S T + M Q R^2 S T) + \dot{\psi}_w (O R S T^2 + O^2 Q S R + M O R S T^2 - M Q R^2 S T) \right) + \dot{\phi}_w O P Q R T (1 - M) + 2 (O R Q^2 - O^2 Q T + R^2 O S T^2 + R^2 S^2 T^2 + O R S^2 T^2) + \dot{\phi}_s \left( \dot{\phi}_L O^2 Q P + O Q R S T + M (2 O R P Q - \dot{\phi}_w O^2 S Q + \dot{\psi}_w O^2 P T) \right) \right) + 2 L 4 L 1 \left( \dot{\phi}_L H I (O^2 Q^2 - 2 O Q R S T - H I R^2 S^2 T^2) + \dot{\theta}_s ((O P Q^2 + P Q R S T) (G H + J K L)) + \dot{\phi}_s (\dot{\phi}_s K I (O^2 Q^2 - R^2 S^2 T^2) + \dot{\phi}_s H L (M O^2 P Q + O Q R S T) + \psi_s H L O^2 Q^2 + 2 \dot{\theta}_w H I (Q R^2 S T - O^2 Q S T) + 2 \dot{\phi}_w H I O P Q R T + \dot{\psi}_w H I (2 O Q^2 R S T + 2 H I (O R Q^2 + O^2 Q T - O R S^2 T^2) - 214 \right) \]
\[ R^2PST^2 - R^2S^2QT^2 \] + \[ \dot{\theta}_p M((OPQ^2 - GL + JKI - JI + GKL) + PQRST(JI + GKL)) + \phi_p M((OPQ^2 - GL + JHL) + PQRST(GK + JHL)) + \phi_p N((OPQ^2 + PQRST) - (GL - JKL)) + \phi_w M((OPQST - RPQ^2(GH + JKL)) + \phi_w M((QRTS - 2OSQ^2)(GI + JKL)) + \psi_w M((PRST^2 + OQ^2RS)(GI + JKL)) + 2MOPQT(-GI - JKL)) + 2L4L2\left(\psi_t HI(O^2Q^2 + M^2OQRS + OQRST) + \dot{\psi}_w HI(M^2 - PRST^2 + OQ^2RS) + \phi_s KI(Q^2O^2 + M^2OQRS - OQRST) + \dot{\psi}_w HI(M^2 - PRST^2 + OQ^2RS) + OQRST + \phi_w HI(M^2OPQT + OP^2RT - ORS^2T) + \dot{\psi}_w HI(ORS^2 - M^2(ORST^2 + ORT)) - 2HI(ORQ^2 + HIO^2QT - MNOQRST)\right) + 2L4\left(\dot{x}(\dot{\theta}_p HIO^2Q^2(DC - AE)) + \dot{\psi}_t ECM(GIOPQ^2 + JKLOPQ^2 + JKLORST + GIPQRST + \psi_t BFM(JKLOPQ^2 + GIPQRST - JKLORST) + \dot{\phi}_t JIMOPQ^2 - JKLMPQST + GLM^2OP^2Q - JKIOPQ^2) + \phi_t BCJHLM(-OPQ^2 - OQRST) + \psi_t BCM(-JKIOPQ^2 + LGPQRST - JKIOPQST + JKIOPQ^2 - GLRST + JKIRST) + \dot{\phi}_t BCF(GIOPQ^2 - JKLOPQ^2 + PQRST(GI + JKL)) + \phi_w BCM((GI + JKL)(RPQ^2 - OQRST)) + \phi_w BCM(JKL(OSQ^2 - P^2QRT + QRS^2T) + GI(OSQ^2 + QRS^2T - P^2QRT) + \psi_w BCM(-GIPO^2RS + JKLQPSRT + JKLOPQ^2RS + GIPO^2RS) + 2BCMOPQ(GI + JKL))\right) + \dot{z}(\dot{\theta}_p GIM(BDMO^2PQ + DCRST) + \dot{\phi}_t AFGIMO^2PQ + \psi_t AFGIMRST + \dot{\phi}_t BCJIM(O^2PQ + RST + O^2Q^2) + \phi_t ABCGIN(O^2PQ + RST) - \psi_w ABGIMOST + \phi_w AFGIMO^2S^2Q^2 - RPT) + \psi_w ABGIM(O^2PT - RSQ) + 2ABGIMORPQ) + \dot{x}(ACHI(-O^2Q^2 - R^2S^2T^2) + ACGHOPRST + DEFTHI(-O^2Q^2 - RST)) + \psi_w BCM(-JKILOPQ^2 - GIOPQRST - JKLORST) + \dot{z}ABGIM(-RST - PQO^2) - 2L1^2\left(\dot{\phi}_t H^2ILOQ\left(-\frac{1}{2}OQ - RST\right) + \dot{\psi}_t \left(-\frac{1}{2}H^2ILR^2S^2T^2\right) + \dot{\psi}_t M(G^2H^2LOPQ^2 + HJKOPQ^2 - GHILPQST + HJKPQST + GH^2LOPQST + GH^2LPQRST)\right) + \dot{\theta}_t MP^2Q^2(G^2JK + HJKL)\right) - 2L2^2\left(\psi_t H^2I\left(-O^2Q^2 - OQRST - \frac{1}{2}R^2S^2T^2\right) + \phi_t G^2H^2LHMO^2PQ + OQRST\right) - 2L1L2\left(\psi_t \phi_t H^2ILO^2Q^2 + 2OQRST + R^2S^2T^2) + \phi_t G^2H^2LHMO^2PQ - \psi_t G^2H^2LHMO^2PQ - \right)
\[\begin{align*}
&\dot{P} + \dot{Q} = \dot{O} + \dot{R} + \dot{S} + \dot{T} + \dot{M} + \dot{I} + \dot{L} + \dot{P} + \dot{Q} + \dot{R} + \dot{S} + \dot{T} + \dot{O} + \dot{Q}
\end{align*}\]
Governing Equation for $\phi_L$:

$$Q_{\phi_L} = ML + MW + V_{\phi_L}$$

Where $Q_{\phi_L}$ and $V_{\phi_L}$ were previously defined

ML is:

$$m_L \left[ \frac{1}{2} \ddot{\phi}_L \left( \frac{1}{3} \xi + L_2^2 \right) + 5L_4 \left( \ddot{\phi}_s M - \dot{\phi}_s M \right) + 5L_1 \left( -\ddot{\phi}_s M (G_L + JKL) + \dot{\phi}_s (G_L + HIN) + \dot{\phi}_s (J1 - GKL) + \dot{\phi}_s (GL - JKI) - \dot{\phi}_s JHL + \dot{\phi}_s (\phi_L HI - M - \dot{\phi}_s K - \dot{\phi}_s HL) \right) + 5L_2 \left( -\ddot{\phi}_s HI + \dot{\phi}_s GHM + \dot{\phi}_s (\dot{\phi}_s K + \dot{\phi}_s HL) + \dot{\phi}_s (\dot{\phi}_s JH - \dot{\phi}_s GKL) \right) - 2L_4 \left( \ddot{\phi}_s N \left( \frac{1}{2} \dot{\phi}_s M + \dot{\phi}_s S \right) \right) - 2L_4 \left( \ddot{\phi}_s \dot{\phi}_s (AC + DEF (MN - 1)) - \ddot{\phi}_s \dot{\phi}_s BCN (G_L + JKL) - \ddot{\phi}_s \dot{\phi}_s ACGHMN + \dot{\phi}_s \left( \dot{\phi}_s M + GIN (BC\dot{\phi}_s + DEF \dot{\phi}_s) \right) \right) - 2L_1 \left( \ddot{\phi}_s^2 MN \left( \frac{1}{2} G_L^2 + GKL + \frac{1}{2} J^2 K_2^2 L^2 \right) \right) - 2L_2 \left( \ddot{\phi}_s N (-\dot{\phi}_s HJKL + \ddot{\phi}_s \dot{\phi}_s G^2 L^2 + \ddot{\phi}_s \dot{\phi}_s H^2 I^2 M) - 2L_1 \left( \ddot{\phi}_s N \left( \ddot{\phi}_s G^2 H^2 I - \ddot{\phi}_s G^2 JKL - \ddot{\phi}_s H^2 I^2 L^2 \right) \right) - 2L_2 \left( \ddot{\phi}_s N \left( \ddot{\phi}_s H^2 I^2 + HJKL + \ddot{\phi}_s G^2 H^2 I + \ddot{\phi}_s G^2 JKL + GIJKL + J^2 K_2^2 L^2 + 2M\dot{\phi}_s ABG^2 H^2 I \right) \right) + \ddot{\phi}_s N \left( \ddot{\phi}_s G^2 H^2 + JKL - \ddot{\phi}_s ACGH^2 I \right) + L_5 \dot{\phi}_s H (AC + DEF) + L_5 \dot{\phi}_s M \left( AC - \dot{\phi}_s G^2 L + JKL \right) + L_5 \dot{\phi}_s AG + L_5 \dot{\phi}_s \left( \dot{\phi}_s HI (AC - DEF + \dot{\phi}_s EFK + \ddot{\phi}_s AFH + \ddot{\phi}_s DECH) - \ddot{\phi}_s HI (DC + CE) + \ddot{\phi}_s DBF + \ddot{\phi}_s H (AC + DEF) \right) + L_5 \dot{\phi}_s BC (GL + JKI + JK) + \ddot{\phi}_s BC (IJ + GKL) + \ddot{\phi}_s CEM (G - JKL) + \ddot{\phi}_s BFM (HI - JKL) + \ddot{\phi}_s BC (HL + LM) + L_5 \dot{\phi}_s AEG - \dot{\phi}_s B + \dot{\phi}_s ABJ - 2L_4 \left( \ddot{\phi}_s N (ABC G^2 H^2 + ABC^2 H^2 JKL + BCDEF G^2 H^2) + \ddot{\phi}_s B^2 C^2 MN \left( \frac{1}{2} J^2 K_2^2 L^2 - G^2 K_2^2 L^2 \right) \right) - 2L_5 \left( \ddot{\phi}_s \dot{\phi}_s (A^2 BC + BCDEF) + L_5 \dot{\phi}_s MNAB^2 C (-G^2 HI - GKL) - \frac{1}{2} \dot{\phi}_s MNAB^2 C (-G^2 HI - GKL) \right) \right]$$

Mw is:

$$m_w \left[ L_5 L_7 \left( \ddot{\phi}_w \dot{\phi}_w \ddot{\phi}_w OST - \dot{\phi}_w RPT - \dot{\phi}_w RSQ \right) + 2L_5 \left( \ddot{\phi}_w RST + \dot{\phi}_w \ddot{\phi}_w OST + \dot{\phi}_w RPT + \dot{\phi}_w RSQ \right) + 2L_5 \left( \ddot{\phi}_w (RST + R^2 S^2 T^2 + ORS^2 T^2) + \ddot{\phi}_w MQQRST + \ddot{\phi}_w (QR^2 ST - O^2 QST) + \ddot{\phi}_w (QRST^2 - O^2 QRT) - \ddot{\phi}_w (POQRT - 2R^2 PST^2 - GRT^2 T^2) - \ddot{\phi}_w (PR^2 ST + MQ^2 ST) + \ddot{\phi}_w MQPQRS + \dot{\phi}_w M (-ORS^2 T + O^2 QRT) \right) + 2L_5 \left( \ddot{\phi}_w RST (HIOQ + HI + GIMOQ) - \right)$$
\[
G I O P^2 Q) - \theta_t(G^2 H^2 M O P^2 Q - G H J K L M O P^2 Q) - 2L1N \left( \phi_t(\delta AC(HI(OPQ^2 - P Q R S T + JK LO P Q^2 + JK L O Q R S T) + \delta DEF R S T(GH^2 IO Q + HIJK LO Q + HIJK L P Q)) + \gamma B C P^2 Q^2 M(G^2 HI + J K L + G I J K L) + \gamma A C M G I J K L O P Q^2 + G H I^2 O Q R S T) \right) - 2L2N(\psi_t(\gamma AC(H I M O^2 P Q - J K L M O^2 P Q + G H I^2 P Q R S T + H I J K L O Q R S T) + \gamma A C G H I^2 (O^2 P Q + O P Q R S T)) + \theta_t(\gamma D E F H^2 G I O P^2 Q - \gamma B C M O P^2 Q(G^2 HI + H I J K L) + \gamma A C G^2 H I M O^2 Q^2) - 2\gamma^2 D E F H^2 I^2 N O R S T - 2\gamma N \left( \delta(AB^2 C G H I O^2 P Q - AB^2 C H I J K L O Q R S T + A B C^2 G H I^2 P Q R S T + A B C^2 H I I J K L P Q R S T - D E F (B C H I O P Q^2 + H I J K L O P Q^2 + A C G H I^2 P Q R S T - A B H H I J K L O Q R S T)) - \gamma M B^2 C^2 \left( \frac{1}{2} G^2 I^2 O^2 Q^2 + G I J K L P^2 Q^2 + \frac{1}{2} \gamma J^2 K^2 L^2 P^2 Q^2 \right) - \gamma M O P Q^2 (A C G^2 I^2 + A C^2 G I J K L) \right) - 2\gamma N \left( \delta(A^2 B C G H I^2 O^2 P Q + A^2 B C G H I^2 O Q - GI(A C D E F O^2 P Q + D E F O Q R S T + A C P Q)) - \frac{1}{2} \gamma A^2 B^2 G^2 I^2 M O^2 Q^2 \right)
\]
Appendix B: Marker Set

This appendix contains the information for the complete pitching marker set and placement.

The marker set consists of 38 markers placed on the pitcher and an additional 2 markers placed on the ball (not pictured above).

The head markers:
LFHD= left front head
RFHD= right front head
LBHD= left back head
RBHD= right back head
The head markers are placed on a head band and the markers are placed at the “corners of the skull”. To ensure that the head is not misshaped in the animation the markers should be placed as equally as possible.

**The Trunk Markers:**

C7= 7\textsuperscript{th} cervical vertebra

T10= 10\textsuperscript{th} thoracic vertebra

STRN= sternum- marker is placed at the Xiphoid process

CLAV= clavicles- marker is placed at the sternal notch

**Non-Pitching Arm markers (based on Right handed diagram above):**

LSHO= left shoulder- marker is placed at the AC joint

LUPA=Left asymmetry marker- placed around the mid-point of the humerus- marker not required

LELB= Left elbow- Lateral condyle

LWRA= Thumb side wrist- thumb side styloid

LWRB= Pinkie side wrist- 5\textsuperscript{th} digit side styloid

LFIN= Left finger- marker placed on either the 3\textsuperscript{rd} digit or glove depending on how glove is worn

**Pitching arm shoulder (based on Right handed diagram above):**

RSAC-marker placed on the AC joint- this marker is removed after ROM trial
RSAA-marker placed on the most lateral portion of the scapular spine

RSPC- marker placed on the corcoid

**Pitching Arm (based on Right handed diagram above):**

RELB= Right lateral elbow- Lateral condyle

REMP=Right medial elbow- medial condyle

RLOA=right asymmetry marker- placed around mid point of forearm- marker not required

RWRA= Thumb side wrist- thumb side styloid

RWRB= Pinkie side wrist- 5th digit side styloid

RFIN= Right finger- marker placed on near the 3rd digit below the knuckle

**The Pelvic markers:**

LPSI= left posterior superior iliac spine

RPSI= right posterior superior iliac spine

LASI= left anterior superior iliac spine

RASI= right anterior superior iliac spine

**Lower Extremity Markers:**

_THI=Right and left thigh- marker is the thigh wand

_KNE=Right and left knee- marker placed on the lateral femoral condyle

_TIB= Right and left tibia- marker is the shank wand

_ANK=Right and left ankle- maker placed on the lateral malleoli

_TOE= Right and left toe- maker placed on the toe of shoe distal upper portion of the foot

_HEE= Right and left heel- marker placed on the heel of the shoe