Three Essays on the Property Rights Theory of the Firm

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Leshui He, Ph.D.
University of Connecticut, 2013

ABSTRACT

My dissertation research focuses on the efficiency of various governance structures using the basic framework of the Grossman-Hart-Moore (GHM) property rights model. The first two chapters expand the GHM framework to describe a richer spectrum of governance structures—including not only fully integrated firms and fully disintegrated market transactions but also asset-less firms and exclusive dealing between firms. The general framework combines the GHM model with a model of bargaining control rights, yielding an allocation of ownership rights that may differ from what the GHM model implies. The results are related to the general principles of employment law. The third chapter offers a formal economic theory that analyzes the differences between the subsidiary and the division as alternative governance structures of an internal business unit. Subsidiaries and divisions are widely observed as alternative governance structures, and one does not seem to completely dominate the other. Formal economic theory is almost silent on the topic of subsidiaries.
Three Essays on the Property Rights Theory of the Firm

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M.A. Economics, University of Connecticut, 2012
B.S. Business Administration, Beihang University, 2006

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Submitted in Partial Fulfillment of the
Requirements for the Degree of
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Leshui He

2013
Three Essays on the Property Rights Theory of the Firm

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2013
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Chapter 1

Beyond Asset Ownership: Employment and Asset-less Firms in a Property-Rights Theory of the Firm

1.1 Introduction

Although most firms own alienable assets, many firms do not. Professional-services firms such as law firms, accounting firms, consulting firms, design firms and many health care providers own few if any alienable assets. Instead, as Holmström and Roberts (1998) and others have observed, such firms rely on inalienable human assets that inhere in and move with the firm’s employees. On the other hand, a key feature of the duly celebrated Grossman-Hart-Moore (GHM) theory of the firm (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995) is the role of alienable assets in explaining the boundaries of the firm. How then to explain asset-less firms?
This paper approaches the problem by embedding the GHM model within a larger theoretical framework that can describe a richer spectrum of governance structures—including not only fully integrated firms and fully disintegrated market transactions, but also asset-less firms and exclusive dealing between firms. This larger framework combines the GHM model of property rights with a model of *bargaining control rights*. Different from the residual rights of control over alienable assets that is endowed by asset ownership, bargaining control rights are *institutional restrictions* designed and controlled by the upper level of the economic organization imposed to limit the freedom to bargain of the lower-level parties. We find that the optimal governance structure often involves not only allocating property rights, as in GHM, but also restricting bargaining rights for some players. In some cases, we also find that the optimal allocation of property rights differs from what the GHM model implies.

When we interpret the model at the level of individuals as opposed to the level of business units, the paper shows that it can be efficient to prohibit employees within one firm from side-contracting with each other. Furthermore, the model shows that preventing other firms from side-contracting with one firm’s employees could also improve efficiency. These results are consistent with what we observe in employment law. An important benefit of our approach is a clear interpretation of the employment relationship.

The GHM approach is close to silent on employment issues. For example, consider a model with three parties and two assets, and suppose that the GHM analysis prescribes non-integrated asset ownership as the optimal governance structure. Who, then, does the third party (the one without an asset) work for, if anyone? This paper enriches the GHM approach so as to answer this question.

The model assumes that all parties are free to bargain with all other parties,
unless a party has endogenously restricted bargaining rights resulting from the *ex ante* institutional design. We interpret a party with unrestricted bargaining rights as the owner of a firm (although that firm might consist of only that party, in the case of self employment). If this party controls the bargaining rights of any other parties, then the controlling party is the boss and the controlled parties are subordinates, such as employees and divisions. That is, the bosses of the firms, are free to bargain with their own subordinates. And the bosses are free to bargain with any other bosses. However, the bosses of firms have bargaining control rights over their subordinates, who are endogenously restricted to bargaining with only their employer.

Many observations about the business firm fit the characteristics of bargaining control rights. When it comes to bargaining over decisions, the owner of the firm bargains for the firm *as a whole*.

1 She bargains, representing her employees, against other business firms and customers. And she also bargains against her own employees, representing the outside contractual relationships with other firms and customers. The subordinates have very limited rights to bargain with anyone other than their bosses. For simplicity, in this model, the boss can restrict its employees to bargain *only* with the firm itself.

To illustrate the model another way, the boss can block direct bargaining among the employees themselves as well as bargaining between her employee and any outside party in the transaction. For example, a grocer cannot deal with his favorite customer if he does not work for the supermarket anymore. And the customer of the supermar-

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1 We quote from Holmström (1999): “One possible explanation is that ownership strengthens the firm’s bargaining power vis-a-vis outsiders. Suppliers and other outsiders will have to deal with the firm as a unit rather than as individual members... The general point though is that institutional affiliation, and not just asset allocation, can significantly influence the nature of bargaining.”

2 All qualitative results of the model still hold when this modeling assumption is relaxed. What the model needs is that the boss can at least block some bargaining between the subordinate and any third party. See more discussion on robustness in Section 1.6.
ket cannot obtain services from her favorite grocer without shopping at the market he works for, which she might dislike. As another example, non-compete clauses in employment contracts are *ex ante* voluntarily engaged restriction over *ex post* bargaining freedom. They are a reinforced and explicit form of bargaining control. Although non-compete clauses present issues regarding enforcement, they are still frequently observed in employment contracts between the firm and its critical employees. For example, Kaplan and Strömberg (2003) document that it is common—more than 70% of contracts in their sample—for venture capital firms to use non-complete clauses.

Why do we interpret those unrestricted parties as bosses and those restricted as employees? There are at least two factors that give the firm the advantage of bargaining control rights over employees, divisions and other internal entities. First, firms are legal persons in business contracts, whereas employees or divisions are not (Iacobucci and Triantis, 2007; Hansmann and Kraakman, 2000). With very few exceptions, all employees bargain with their employer over their employment contracts. In stark contrast, most employees do not participate directly in bargaining with other employees and with other outsiders. When they do, they bargain on behalf of their employer firm for the contract, not on behalf of themselves.

Second, it is a stylized fact that side contracts between employees within a firm or between an employee and other outsiders are rarely permitted in firms. Employees are forbidden, and rarely observed, to formally side-contract among themselves, such as to game the incentive systems of their employer. First, although employees are free to leave the firm, firms tend to implement the bargaining control rights by committing not to frequently renegotiate their employment contracts. Second, according to the employment laws, employees have a fiduciary duty to act in the best interest of their employer. So side-contracting among employees or between an employee and an
outside party also tends to violate this legal restriction.

Bargaining control rights are not exclusive to the hierarchical structure within a firm. When we interpret the parties in the model at the level of business units, the parties whose bargaining rights are restricted are interpreted differently depending on their ownership of assets. If they do not own any asset, they are interpreted as internal business units within a firm, such as divisions or subsidiaries. If they own assets, then they are interpreted as firms under exclusive dealing contract with those firms who have bargaining control over them.

Similar to our modeling assumption, Segal and Whinston (2000) also consider bargaining control rights as designed instruments to govern transactions. Focusing their interpretation at the business unit level, Segal and Whinston (2000) characterize exclusive contracts as restricted bargaining rights between a seller-buyer relationship. The current model shares the common characteristic with their work in that we both emphasize the role of bargaining rights as a special instrument in the governance structure different from regular asset ownership. But this paper departs from theirs in two aspects. First, we consider the effect of bargaining control rights simultaneously with that of asset ownership, whereas they focus on studying bargaining control rights given fixed asset ownership structure. Segal and Whinston (2000) discuss the conditions under which exclusive dealing is more efficient than non-integration. In particular, they found that if one trading party’s investment has very high marginal product, it is efficient for her to control the other firm through an exclusive dealing contract. However, they do not explore whether exclusive dealing can still be efficient if this firm can simply integrate the other. In other words, can exclusive dealing be more efficient than both non-integration and integration? My paper explores this question by considering bargaining control rights together with allocation of asset
ownership. My model shows that exclusive dealing can indeed be more efficient than both integration and non-integration. Second, we generalize their interpretation of bargaining control rights beyond the exclusive dealing contracts to associate with the boss-subordinate relationship, which consequently provides an interpretation of asset-less firms. To some extent, one can also see the current paper as a generalization of Segal and Whinston (2000) that applies to the boundaries of the firm problem with asset allocation.

The paper proceeds as follows. Section 1.2 reviews some of the most related literature to highlight the paper’s contributions. Section 1.3 describes the setup of the model as well as the rules of interpretation under the three-party case. Section 1.4 provides an example to highlight the most important findings of the model. Section 1.5 provides an analysis of the three-party model and offers propositions that explain the observed patterns in the example. Section 1.6 concludes.

### 1.2 Related Literature

Our model shares the spirit of the subeconomy theory of the firm (Holmström and Milgrom, 1991; Holmström, 1999). In their works, the firm can use various incentive instruments for their employees to selectively isolate those employees from undesirable activities. In Holmström and Milgrom (1991), the principal can choose a set of allowable tasks for the agent. In Holmström (1999), the firm can “regulate trade within a firm” as a subeconomy in the sense that the principle is able to set rules over different activities of its employees, such as working from home. We do not study

---

3Because the key ingredient of the bargaining control rights is the ability of one party to bargain with a third party without going through the second one, the model operates with at least three parties.
the problem with a contracting approach, nor do we emphasize the information or measurement problem in organizations as they do. Instead, we analyze a structure that allows the firm to isolate outsiders and its employees from each other.

Rajan and Zingales (1998) is also a theory of the boundaries of the firm that does not rely on the ownership of assets and that sees the firm as a hierarchical structure. Assuming that the owner of the firm is fixed, Rajan and Zingales (1998) focus on the allocation of \textit{ex ante} contractible access to the productive resource controlled by the owner. Those agents granted access become employees of the firm and those who do not have access are interpreted as outsiders. The present paper is different in several respects. First, I emphasize different characteristics of the firm. The model emphasizes the ability for the firm to bargain as a whole vis-à-vis different parties, not the right to grant or deny the access to the resources that are under the firm’s control. Second, in their model, the identity of the party who controls the firm, as well as the ownership of the critical productive asset, are exogenous and fixed. By contrast, one of the major purposes of this model is precisely to answer these two questions: who should control the firm and who should own which assets? The answers to these two questions are the core endogenous results of the model. Third, their original model has only one focal firm, i.e., the firm except for the possible outside contractors. By contrast, the present model allows the number of firms involved in the transaction to be a fully endogenous choice; with a model of more than three parties, we can have multiple firms with subordinates. Although a simple extension of their model with multiple critical assets can also model an environment with multiple firms involved in the transaction, this feature is always exogenously fixed at the number of parties who control the critical assets. Fourth, We interpret the hierarchical structure differently. Their work interprets the party who gives out access as the boss, those who receive
access as the subordinates, and those who do not receive access as the outsiders. This model interprets those who can freely bargain as the bosses, those who cannot freely bargain as the subordinates.

There have been studies of the GHM model with alternative bargaining solutions. Most importantly, de Meza and Lockwood (1998) consider alternating-offer bargaining in place of the Shapley value used in GHM.\(^4\) The main purpose of their paper is to evaluate the robustness of the results in GHM when the model adopts a different bargaining solution. Instead of replacing the bargaining solution of Shapley value in GHM, our paper adopts a more general bargaining game which makes GHM a special case in our framework. And, more importantly, we use the generalized bargaining network to model an additional governance structure other than asset ownership. For this reason, our model is more closely related to Segal and Whinston (2000) than to de Meza and Lockwood (1998).

de Fontenay and Gans (2005) and Kranton and Minehart (2000) are similar to this paper in that they both study vertical integration and networks. de Fontenay and Gans (2005) adopt the GHM framework to compare outcomes under upstream competition and monopoly. Both de Fontenay and Gans (2005) and the current paper study integrations and both involve endogenous incomplete bargaining networks. The main difference is that I focus on analyzing governance structures with asset allocation in one given transaction that involves at least three parties. Whereas they study governance structures involving pairs of upstream and downstream parties across multiple such pairwise transactions without asset allocation. Most importantly, the network in our model represents status in the hierarchy, i.e. whether a party is free to

\(^4\)The generalized Nash bargaining solution with equal bargaining power under the two-party case is a special case of the Shapley value.
bargain in the market as a firm or is restricted to bargain as a subordinate. However, in de Fontenav and Gans (2005), the network represents the various transaction flows across different pairs of upstream and downstream players.

Kranton and Minehart (2000) studies the tradeoff between a vertically integrated transaction versus a network of supplier relationships in an environment with specialization and individual demand shocks. Their network is different from mine in that it describes a supply structure involving, mostly, one buyer and multiple competing suppliers with uncertainty, whereas my network describes a chain of jointly producing parties without competition or uncertainty.

Our work is the first formal model that study asset-less firms and exclusive dealing contracts side-by-side with classical integrated and non-integrated firms in economic theory of the firm. Other economic theories of the asset-less firms, such as Dow (1993), offer specialized models of this particular type of organization and do not consider integration between firms. Hansmann (1988) offers a conceptual framework to study a broad scope of various firm structures, but it does not consider asset ownership.

1.3 A Model of Three Parties

In this section, we introduce the modeling framework with a three-party model. It illustrates all the key ingredients of the general model and delivers most (but not all) of the results.  

See Section 2.1 the setup and results of a general model with any number of players and any number of assets.
1.3.1 Economic Environment

We consider a transaction involving three parties, \( N = \{1, 2, 3\} \), who jointly produce a final product or service. To govern their joint transaction, they agree on a *governance structure*, \( g = (A, B) \), including the *asset ownership*, \( A \), and the *bargaining control rights*, \( B \).

To obtain the value of the final output, these three parties need access to a finite number of alienable assets, \( M = \{m, m_2\} \). The assets are alienable in the sense that their ownership can be transferred between different parties. We use a mapping \( A(S) \) from the set of subsets of \( N \) to the set of subsets of \( M \) to denote the assets owned by any coalition of players \( S \in N \). We defer the specification the bargaining control rights, \( B \), to the later part of this section.

Investment

Each party \( i \) makes *ex ante* non-contractible human-capital investment \( e_i \) with private cost \( \Psi_i(e_i) \). We assume standard properties of the cost function, i.e. \( \Psi_i(e_i) \) is continuous, twice differentiable, increasing and convex in \( e_i \). The investments happen *ex ante* in the sense that the state of the world has not fully realized at the point of investment. They are non-contractible by the assumption that the investments are so complicated that they cannot be specified in a contract, nor can they be verified by any outside party, such as the court.

Production

After the state of the world realizes, i.e. at the *ex post* stage, the three parties can make decisions over the usage of the assets. These three parties can potentially
produce in different coalitions among themselves. Specifically, any coalition $S \subseteq N$ can produce a value $v_S$. For instance, 1 and 2 might decide to produce together without 3, which will generate a value of $v_{12}$. For these three parties, there are seven production possibilities in total, including $v_{123}, v_{12}, v_{13}, v_{23}, v_1, v_2$ and $v_3$.

The value that any coalition $S$ can produce, $v_S(e, A)$ is determined jointly by the vector of ex ante investments $e$ and the asset owned by players in $S$. It is important to remark that the production function $v_S(e, A)$ may depends on investment of parties who are not in $S$. This feature is called cross-investment, in the sense that one party’s investment also benefit other parties’ productions. As an example of cross-investment, a firm’s investment in R&D is likely to accumulate valuable experiences for the engineers and scientists. If these experiences are not entirely specific to the investor firm, then these investments increase the value of production for the engineers and scientists even if they do not work with the investor firm.\textsuperscript{6} Our analysis in later sections shows that cross-investment is critical for the bargaining control rights to be efficient.

Following Hart and Moore (1990), we assume the following properties for the value functions $v_S(e, A)$. (i) Given asset allocation $A$, $v_S(e, A)$ is non-decreasing, continuous, twice differentiable and concave in $e_i$, for any $i \in N$. Moreover, an empty coalition produces nothing, $v_\emptyset(e, A) = 0$. (ii) Assets are complementary to the investments. That is $\frac{\partial v_S(e, A')}{\partial e_i} < \frac{\partial v_S(e, A)}{\partial e_i}$ if $A'(S) \subset A(S)$ (iii) The investments are weak strategic complements, i.e. $\frac{\partial^2 v_S(e, A)}{\partial e_i \partial e_j} \geq 0$ for $i \neq j$. (iv) To make sure the problem
\textsuperscript{6}These following two examples are provided in Che and Hausch (1999): Nishiguchi (1994) p.138 reports that suppliers “send engineers to work with automakers in design and production. They play innovative roles in ... gathering information about the automakers’ long-term product strategies.”

After Honda chose Donnelly Corporation as its sole supplier of mirrors for its U.S.-manufactured cars, “Honda sent engineers swarming over the two Donnelly plants, scrutinizing the operations for kinks in the flow. Honda hopes Donnelly will reduce costs about 2% a year, with the two companies splitting the savings” (Magnet, 1994).
is interesting, we assume that, other things equal, the value of production is super-
additive. That is, any two coalitions produce a smaller total value than they could if they were producing as a joint coalition.\(^7\) Specifically, given investment level \(e\), \(v_{S'}(e, A) + v_{S\backslash S'}(e, A) < v_S(e, A)\) for any \(S' \subset S\). To economize on notation, whenever the investment level \(e\) and asset ownership \(A\) is fixed, we write \(v_S = v_S(e, A)\).

As a result of the bargaining structure we adopt, the players always reaches \textit{ex post} efficient renegotiation result.\(^8\) Therefore under assumption (iv), only the grand-
coalition production \(v_{123}\) will be produced at the final stage. However, each party can use other production possibilities \(v_{S}\) as outside options to deviate a bigger share of the total payoff \(v_{123}\) toward herself during the bargaining.

**Bargaining with Incomplete Networks**

We apply the Myerson-Shapley value (Myerson, 1977), or Myerson value, to charac-
terize the payoff for each party from the joint production. Myerson shows that this solution generalizes the Shapley value to bargaining in incomplete networks, in two senses: (i) the Myerson value equals the Shapley value when the bargaining network is complete; and (ii) the Myerson value is the unique solution satisfying axioms akin to those that produce the Shapley value.

In terms of rights to bargain, we require each party to be one of two types. A party is either \textit{restricted to bargain}—she is restricted to bargain with one and only

\(^7\)This is a somewhat restrictive assumption. If it does not hold, there is no benefit for these parties to produce together, so the problem is no longer interesting. In fact, this is an maintained assumption in almost the entire literature of property rights theory.

\(^8\)Grossman and Hart (1986) assumes Nash bargaining solution, which delivers efficient bargaining \textit{ex post}. Here in this model, we adopt the Myerson value which allows for incomplete bargaining networks. But since the network is always connected under the grand coalition (a result of the way we construct the network, see Section 2.1), the \textit{ex post} bargaining is still always efficient.
one other party. Or the party is free to bargain—she can bargain with the other two parties.\footnote{In a model with more than three parties, we require that the free-to-bargain party needs to be able to bargain with at least two parties, and, moreover, all free-to-bargain parties are able to bargain with each and everyone of themselves. Within a three-party model, it is equivalent to the general definition to be able to bargain with the other two parties.}

The requirement that each party has to be restricted to bargain or free to bargain implies that the bargaining networks that we consider have to be connected.\footnote{See Section 2.1 for the proof in an $N$-party model.} We use $i : j$ to denote the bargaining link between any two parties $i$ and $j$. A bargaining network is a set of bargaining links. For three parties, there are four possible connected bargaining networks (Table 1.1). There is one complete network, $B_c = \{1 : 2, 1 : 3, 2 : 3\}$, in which each party is free to bargain, they can form any coalitions to jointly produce. And there are three incomplete networks, $B_i = \{i : j, i : k\}$ for $i, j, k \in N$ and $i \neq j \neq k$. In these networks, party $i$ is the only “connecting” party who can bargain with the other two. In a model with only three parties, we will sometimes refer to party $i$ as the nexus because of $i$’s central position in the network. We will also say party $i$ has bargaining control over party $j$ if $j$ is restricted to bargain with $i$. The key implication of the incomplete bargaining network is ruling out coalition between $j$ and $k$. In $B_i$, $j$ and $k$ cannot bargain with each other without $i$. So $j$ and $k$ are not able to form coalition to produce $v_{jk}$ together without the participation of party $i$.

This following definition is the key for us to model the incomplete bargaining
network. We define

\[ v^B_S = \begin{cases} 
   v_i + v_j & \text{if } S = \{i, j\} \text{ and } B = B_k \\
   v_S & \text{otherwise}
\end{cases} \quad (1.1) \]

When the two parties \( S = \{i, j\} \) cannot bargain directly in the network, \( v_S \) is replaced by the sum of values produced by finer partitions of \( S \). Under network \( B_k \), because \( i : j \notin B_k \), \( i \) and \( j \) cannot cooperate with each other without \( i \). Consequently, instead of cooperatively producing \( v_{ij} \), they can only produce separately and obtain \( v_i + v_j \).\(^{11}\)

Using this notation, the bargaining payoff of party \( i \) is defined by the Myerson value as

\[ Y^B_i(v_S) = \phi_i(v^B) = \sum_{S \subseteq N} p(S)\{v^B_S - v^B_{S \setminus \{i\}}\}, \quad (1.2) \]

where \( v_S \) stands for the vector of the production functions of all the possible coalitions \( S \subseteq N \); \( \phi_i \) is the Shapley value operator; \( p(S) = \frac{(\binom{|N| - |S|}{|S|})!|S|!}{|N|!} \) and \( |N|, |S| \) are the number of elements in the set \( N \) and \( S \), respectively. Under complete network \( B_c \), the bargaining payoff reduces to the original Shapley value payoff as is used in Hart and

\(^{11}\)The notation \( v^B \) is in fact a characteristic function game, see, for example, Myerson (1977). The way we define it here is its special form applied to the three-party case under connected networks.
Moore (1990). This feature allows us to compare the implications of the incomplete bargaining network with the benchmark of their original model.

**Governance Structure**

The governance structure is a double \( g = (A, B) \). Given asset allocation \( A \) and *ex ante* investments \( e \) fixed, we can characterize the bargaining payoff \( Y_i \) for any party \( i \in \{1, 2, 3\} \) under the three different networks \( B_c, B_i, B_j \) as

\[
Y_{i}^{c}(v_S) = \frac{1}{3}v_{ijk} + \frac{1}{6}v_{ij} + \frac{1}{6}v_{ik} + \frac{1}{3}v_i - \frac{1}{3}v_{jk} - \frac{1}{6}v_j - \frac{1}{6}v_k; \tag{1.3}
\]

\[
Y_{i}^{j}(v_S) = \frac{1}{3}v_{ijk} + \frac{1}{6}v_{ij} + \frac{1}{6}v_{ik} + \frac{1}{3}v_i - \frac{1}{2}v_j - \frac{1}{2}v_k; \tag{1.4}
\]

\[
Y_{i}^{j}(v_S) = \frac{1}{3}v_{ijk} + \frac{1}{6}v_{ij} + \frac{1}{2}v_i - \frac{1}{3}v_{jk} - \frac{1}{6}v_j; \tag{1.5}
\]

respectively.\(^\text{12}\)

We take a close look at these three equations to explain why the current model can be viewed as a combination of GHM with Segal and Whinston (2000) by jointly considering effects of both \( A \) and \( B \).

Asset ownership \( A \) directly determines the value produced by each given coalition \( S \), but it has no effect on which production possibilities are available. The standard GHM model uses equation (1.3) (and its counterparts if the number of parties is different from three) to characterize the payoffs. Assuming all the production possibilities \( v_S \) are available (i.e. fixing equation (1.3)), GHM explores effects on the investment levels \( e \) when \( A \) changes all the \( v_S \).

\(^\text{12}\)By the efficiency property of Myerson value and Shapley value, in a given network \( B \), the sum of the payoffs to all parties equal to the final value that is produced, \( v_{ijk} \). It can be readily checked that \( \sum_{i \in \{1, 2, 3\}} Y_{i}^{b}(v_S) = v_{123} \) for \( b = c, i, j \).
The bargaining network $B$ has no direct effect on the values produced by each coalition. But it determines whether a particular coalition is able to pursue joint production. For example, $v_{23}$ can be produced under $B_c, B_2, B_3$, but not under $B_1$. $B$ determines which payoff function among equations (1.3) through (1.5) determines party $i$’s bargaining payoff. Segal and Whinston (2000) can be viewed as a model analyzing the effects of the bargaining network $B$ by comparing equation (1.3) with equations (1.4) and (1.5), assuming $A$ is fixed.

**Timing**

It is useful to summarize the timing of the stage game described so far. The timing of this model is almost identical to that of the GHM model, with the only change that the governance structure is now enriched with a second dimension: bargaining networks.

At $t = 0$, information is symmetric. All parties agree on a governance structure $g = (A, B)$. At $t = 1$, parties make *ex ante* non-contractible relationship-specific investments. The investments are observable to all parties, but non-verifiable to all outside parties. At $t = 1.5$, state of the world realizes, and is common knowledge to all parties. At $t = 2$, parties engage in *ex post* efficient bargaining based on the governance structure $g = (A, B)$. Finally, at $t = 3$, the transaction is carried out and the final value is produced and divided by the parties according to the Myerson bargaining solution.
Similar to almost all other property rights models, the only inefficiency in this model rises from the *ex ante* investment stage. Because parties maximize their individual bargaining returns instead of the joint return of the entire transaction, the presence of such externality biases their investment levels away from the first-best. The governance structure affects the efficiency of the transaction because the *ex ante* agreed governance structure determines the outcome of the *ex post* bargaining return for each individual, and thus it in turn governs each parties’ investment decision *ex ante*. The most efficient governance structure is the one associated with the investments that delivers highest level of final product net of the costs of investments.

**An Example of Six Governance Structures**

In the remaining part of this section, we present the model in its simplest form by focusing on a limited types of asset ownership and bargaining networks. Without losing much generality, these simplifications allow us to focus on several representative governance structures by ruling out many economically identical ones. Neither the modeling framework nor the propositions that follow in the analysis section hinge on these restrictions. We only put them in place to help demonstrate the key features of the model.

In this example, we suppose that parties 2 and 3 are identical in production technologies and costs. This assumption rules out all the governance structures where party 3 owns either asset(s) or has bargaining control, because these structures are economically identical to the ones where party 2 is in the same position.

In terms of asset ownership, $A$, we choose to follow the tradition of most applications of the GHM models to focus on the two cases that are most closely related
to empirical works: the integrated asset ownership case, in which the assets are collectively owned and the non-integrated asset ownership case, in which the assets are separately owned. To evaluate these two cases, we assume that there are only two productive alienable assets, $m$ and $m_2$. As a normalization, we shall always assign ownership of $m_2$ to party 2 but choose between allocating ownership of $m$ to either party 1 or party 2. We will then denote these two cases by $A = A_N$ for non-integrated asset ownership, i.e. if 1 owns $m$. And we denote $A = A_I$ for integrated asset ownership, i.e. if 2 owns $m$.$^{13}$

Without loss of generality, our model only considers connected bargaining networks. Because the bargaining control rights are institutional restrictions on the ability to bargain, rather than technological difficulties that fundamentally block communication among parties, the three parties can always eventually reach agreements together. As 2 and 3 are identical, we will rule out $B_3$ and only consider three possible candidates for the optimal bargaining network: the original GHM complete bargaining network $B_c$ and the incomplete bargaining networks $B_1$ and $B_2$, in which party 1 or party 2 has bargaining control rights, respectively.

The simplest model is thus a choice over 6 candidate governance structures, $g \in \{(A_N, A_I) \times \{B_c, B_1, B_2\}\}$. And they are presented graphically in Table 1.2. In these graphs, the dashed lines represents the bargaining links, which indicates the ability for any two parties to bargain with each other.

$^{13}$These assumptions reduce the problem of choosing the correspondence $A$ to a binary choice. Formally, in this case, $A \in \{A_N, A_I\}$, where $A_N(\{1\}) = \{m\}, A_N(\{2\}) = \{m_2\}$, and $A_I(\{1\}) = \{\emptyset\}, A_I(\{2\}) = \{m, m_2\}$.
1.3.2 Interpreting Six Candidate Governance Structures

We dedicate this subsection to explain our interpretations of the two dimensional governance structures. The first part introduces our interpretation rules. The second part makes specific interpretations for the six candidate governance structures introduced in the previous example.

**General Interpretation Rules**

We interpret any party who is free to bargain as a *boss*, regardless of whether she owns asset or not. And we interpret those parties who are restricted to bargain to a boss *and* do not own any asset as this boss’s *subordinates*. Furthermore, we interpret someone who is restricted to bargain and owns some asset(s) as a *boss* of a self-managed firm under exclusive dealing contract.

Since we can label any party in this model as either a boss or a subordinate, a natural interpretation of the business *firm* rises from the model without hinging on
the ownership of assets. That is, a *firm* is consisted of a boss and her subordinates, if she has any.

**Interpretation of the Example with Six Governance Structures**

We apply these interpretation rules to the six candidate governance structures from Table 1.2. In Table 1.3, we present the bargaining graphs on the top, and the interpretation graphs right below them. In the interpretation graphs, the vertical position represents our interpreted hierarchical structure. The bosses are placed on the top level, and outlined with thick and black circles. The subordinates are placed on the bottom level, and outlined with thin circles. We organize the rows in the table by the decreasing order of the number of firms involved in the transaction.

Under the complete bargaining network $B_c$, every party has freedom to bargain with everyone else, so all three parties are interpreted as bosses, with or without asset. Thus the two GHM cases on the top row of Table 1.3 are interpreted as three firms dealing in the market.

The next pair of cases under non-integrated asset ownership offers clearly identified employment relationship that we cannot always identify in the classical GHM framework. Under network $B_1$, when asset ownership is non-integrated, 2 is interpreted as an independent firm because she has ownership over asset $m_2$. 3 is seen as the subordinate of 1 because he cannot bargain freely with 2. So we interpret case

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14Whether 2 has an exclusive dealing contract with 1 is ambiguous in the three-party two-firm setting, as we do not explicitly see whether 2 has the right to freely bargain with a third firm. If an application has a specific setting, introducing a forth party as necessary could help ameliorate this ambiguity.
Table 1.3: Interpreting Six Candidate Governance Structures

\[ g = (A_N, B_1) \] as: the firm ran by boss 1 controlling asset \( m \) with subordinate 3 dealing with another firm 2 who controls asset \( m_2 \).
Similarly, $g = (A_N, B_2)$ is interpreted as a transaction involving two firms, each controlling one asset, dealing through the market. The only difference from the $g = (A_N, B_1)$ case is that party 3 is the subordinate of firm 2, instead of firm 1. This difference between these two cases cannot be formally modeled in a classical GHM model. This feature highlights a benefit of introducing bargaining control rights.

$g = (A_I, B_1)$ offers a case of an asset-less firm in the transaction. As the graph shows, party 1 is a boss with subordinate 3, dealing with another firm 2. In this case, the firm ran by 1 with subordinate 3 does not have control over any asset. All the assets needed for production is owned by party 2. We interpret this case as a asset-less firm dealing with another firm abundant with productive assets, such as a consulting firm providing services to a manufacturer.

$g = (A_I, B_2)$ describes a classical firm in the sense that the owner of the firm is also the owner of all the assets. This case thus represents a fully vertically integrated transaction.

### 1.4 A Parametrized Example

In this section, we introduce some specific parametric assumptions to build an example demonstrating that the incomplete bargaining networks, i.e. having bargaining control rights, can be more efficient than the complete bargaining networks (classical GHM). Furthermore, we will observe a surprising result that after introducing the bargaining control rights as a part of the governance structure design, the optimal asset ownership can be different from what is predicted in the classical GHM model. In other words, the choice of optimal asset ownership $A^*$ chosen as the jointly opti-
mal governance structure $g^{**} = (A^{**}, B^{**}) \in \{A_N, A_I\} \times \{B_c, B_1, B_2\}$ can be different from the optimal asset ownership $g^* = A^* \in \{A_N, A_I\}$ fixing bargaining network $B_c$. Finally, in some situations, we will be able to see that, as one party’s investment becomes more and more important relative to others’, the ownership of the same asset is transferred for multiple times between the same dyad of parties. This result is in stark contrast to the standard property rights models where the party who makes more important investment tends to own more assets.

In the following sections, we will very often compare a governance structure with incomplete bargaining network, say $g'$, with one that has a complete network, say $g$. In these comparisons, we will discuss it as if the governance structure changed from $g$ to $g'$. To put it another way, in the thought experiments, we will pretend as if the party who has bargaining control under $g'$ acquired the bargaining control rights over her subordinate. Therefore we will refer to the boss in $g'$ with bargaining control rights as the integrating party, and refer to the subordinate as the integrated party.

1.4.1 Model Setup

Specific Parametrization of Production Functions

We follow Whinston (2003)’s linear-quadratic setup to formulate the model. Each party $i$ makes ex ante non-contractible relationship-specific investment $e_i$.

We assume that the parties’ investments have two potential benefits, it has a self-investment aspect and a cross-investment aspect. Self-investments means that the investments benefit the productions in which the investor participates. On the contrary, cross-investments means that investments benefit the productions that the investor is not a part of. For example, if Apple Inc. invests in improving its iphone’s
compatibility with Google Inc.’s map application, it is likely to not only benefit Apple, but also benefit Google by attracting more users who contributes usage data. Consequently, the effect of the usage data may spillover to Google’s own mobile devices.

We assume the three parties make investments at private costs with a quadratic form $\Psi_i(e_i) = \frac{e_i^2}{2}$. The production functions for the seven possible coalitions are assumed to take a linear form as follows.

\[
\begin{align*}
    v_{123}(e, A) &= \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 \\
    v_3(e, A) &= \beta_{cross} e_1 + \beta_{cross} e_2 + e_3 \\
    v_1(e, A) &= (\Omega_1 m + (1 - \Omega_1))(e_1 + \beta_{cross} e_2 + \beta_{cross} e_3) \\
    v_2(e, A) &= (\Omega_1 + (1 - \Omega_1)m)(\beta_{cross} e_1 + e_2 + \beta_{cross} e_3) \\
    v_{12}(e, A) &= m(k_s e_1 + k_s e_2 + \beta_{cross} k_c e_3) \\
    v_{13}(e, A) &= (\Omega_1 m + (1 - \Omega_1))(k_s e_1 + \beta_{cross} k_c e_2 + k_s e_3) \\
    v_{23}(e, A) &= (\Omega_1 + (1 - \Omega_1)m)(\beta_{cross} k_c e_1 + k_s e_2 + k_s e_3)
\end{align*}
\]

From the top down, in these equations, $\alpha_1$ (\overline{\pi}) is the marginal product of party 1’s (party 2 and 3’s) investment in the final production. The higher $\alpha_1$ is relative to $\overline{\pi}$, the more important is party 1’s investment.

$\beta_{cross}$ is an indicator variable controlling whether there is cross investment. If $\beta_{cross} = 0$, party $i$’s investment does not have an effect on the productions that she does not participate in.

$\Omega_1$ is the indicator variable controlling whether party 1 owns the asset $m$. $\Omega_1 = 1$ if $A = A_N$, and $\Omega_1 = 0$ if $A = A_I$. 
$m$ is the multiplicative effect of owning the alienable asset $m$. We assume $m > 1$, so that the asset is always productive. If the asset is under control of party $i$, then the marginal product of all the productions that $i$ participates in is multiplied by $m$.

$k_s$ is the marginal product of self-investment in joint production of the investing party and any other party; whereas $k_c$ is the marginal product of cross-investment in joint production of the other two parties. We assume $k_s, k_c > 2$ so the investments are more productive in bigger coalitions.

**Investment Choices Given $g = (A, B)$**

At the *ex ante* stage, each party $i$ chooses non-contractible investment $e_i$ at private cost $\Psi_i(e_i)$ to maximize her own bargaining payoff $Y_i$. The network $B_c$, $B_i$ or $B_j$ determines which equation in (1.3) to (1.5) is party $i$’s bargaining payoff. The asset ownership $A$ determines the values of productions by entering into the seven production functions $v_S$ for $S \subseteq \{1, 2, 3\}$.

The equilibrium choice of $e_i$ under governance structure $g = (A, B)$ is characterized by

$$e_i^g = \arg \max_{e_i} \left\{ Y_i^B(v_S(e_i, A)) - \frac{e_i^2}{2} \right\}.$$

The social surplus from the transaction under this governance structure $g$ is thus given by

$$\pi^g = Y_i^B(v_S(e_i^g, A)) - \frac{(e_i^g)^2}{2}.$$

The most efficient governance structure is the one among all others that generates
the highest level of social surplus.

1.4.2 “Horse Races” Among Six Governance Structures

In the remaining part of this section, we compare the efficiency of the six governance structures in Table 1.3. We will show that, in this example, only when some party’s investment has a cross-investment aspect, having bargaining control rights can be more efficient than using complete bargaining networks. Moreover, in some cases, after introducing the the incomplete bargaining network, the optimal asset ownership prediction can be different from the GHM result.

To demonstrate these findings, we discuss three sets of “horse races”. In Case I, every party’s investment only has a self-investment aspect ($\beta_{cross} = 0$), we call it no-cross-investment case. Complete bargaining network is always more efficient. In Case II, we allow for the cross-investment aspect in production functions ($\beta_{cross} = 1$). Incomplete bargaining networks can be more efficient than complete bargaining networks, but the optimal asset allocation predictions remain the same as in GHM. In Case III, we allow the marginal products of cross- and self-investments to be different ($k_{cross} \neq k_{self}$), the optimal asset allocation predictions are different from the GHM predictions.

We choose to fix values for some variables and directly demonstrate the results with figures reporting the optimal governance structure under different parameter values. In what follows, we fix $m = 2$, $\alpha = 20$. We let $\beta_{cross}, \alpha_1, k_s$ and $k_c$ vary as choice variables and report the optimal governance structures.\footnote{$\Omega_1$ is not an exogenous choice variable, because it is determined endogenously by asset ownership $A$.}
Case I: No Cross-investment

We say there is *no cross-investment* if no party’s investment has a marginal benefit in productions that she is not a part of. In the first case, we consider the situation where there is no cross-investment, i.e. $\beta_{\text{cross}} = 0$ in the production functions. At the same time, we impose the restriction that marginal products of cross- and self-investments to be the same ($k_{\text{cross}} = k_{\text{self}}$). The most efficient governance structures under different parameter values are reported in Figure 1.1.

Figure 1.1a reports the optimal governance structures in the classical GHM framework, where everyone has freedom to bargain. The choices of governance structure is between non-integrated asset ownership versus integrated asset ownership. In comparison, Figure 1.1b reports the optimal governance structure under the same parameter settings when all six governance structures are in the horse race. Both graphs share identical horizontal and vertical axis. The horizontal axis, $\alpha_1$, is the relative importance of party 1’s investment. Party 1’s investment is more important than that of 2 and 3 if $\alpha_1$ is greater than $\overline{\alpha}$. The vertical axis, $k = k_c = k_s$, is set to be the value of the marginal benefit of investments in sub-coalitional productions for cross- and self-investments.

Figure 1.1a predicts that assets should be owned by the party who makes more important investments. When party 1’s investment is less important than that of party 2, it is more efficient for party 2 to own asset $m$. But once 1’s investment is more important than 2’s investment, it is optimal to assign ownership of asset $m$ to party 1. Figure 1.1b reports that, if there is no cross-investment, it is not efficient to have bargaining control rights. In other words, $B_1$ and $B_2$ are never more efficient
(a) $(A_N, B_c)$ vs. $(A_I, B_c)$

(b) All Six Gov. Structures

**Figure 1.1:** Optimal Governance Structures without Cross-investment
than $B_c$. And the asset allocation predictions remain the same as that of GHM.

**Case II: with Cross-investment, $k_c = k_s$**

In this case, we explore the alternative that there is cross-investment ($\beta_{cross} = 1$), while keeping $k_{cross} = k_{self}$. The results are reported in Figure 1.2. The format of Figure 1.2 is identical to that of Figure 1.1.

The predictions under GHM is identical to the previous case—allocating asset to the party who makes the most important investment (Figure 1.2a). But the optimal governance structures have a much richer pattern when we introduce bargaining control rights (Figure 1.2b).

Four observations emerge in this Figure. First, governance structures with bargaining control rights can be the most efficient sometimes. This shows that restricting bargaining rights can improve efficiency besides allocation of asset ownership.

Second, in this case, the model predicts identical optimal asset ownership as the classical GHM. That is, it is optimal to allocate $m$ to party 2 to the left of the vertical dashed line in the middle ($\alpha$, where party 1’s investment is less important, and to allocate $m$ to 1 to the right. This pattern of optimal asset ownership is consistent with the predictions of the classical property-rights theories.

Third, the boundary that determines which bargaining network is most efficient is not vertical or horizontal. This pattern reflects the interaction between the two instruments in governance structures.

The fourth observation is that we see a series of changes in the optimal governance structure. If we fix $k$ and move from left to right, as party 1’s investments becomes

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16This result hinges on the assumption that $k_s > 1$, i.e. the investment is always self-investment superadditive at the margin. See Section 1.5 for more details.
Figure 1.2: Optimal Governance Structures with Cross-investments
more important, it is efficient for her to own more assets, and to have more bargaining rights. The optimal governance structure changes as party 1’s investment becomes more and more important. When party 1’s investment is very unimportant (left of Figure 1.2b), \((A_I, B_2)\) wins. It is efficient to give party 2 all the asset ownership and the bargaining control over 1, i.e. 2 integrating 1 to work as a subordinate. As 1 becomes more important, \((A_I, B_c)\) is the most efficient. That is to give party 1 bargaining freedom and let her participate in the transaction as an independent firm. As 1 becomes even more important but not more so than 2, it can be efficient to choose \((A_I, B_1)\). That is to let 1 have bargaining control over 3 and deal with 2, who controls all the assets. This is the case in which party 1 runs an asset-less firm, such as a professional services firm, and deals with firm 2 that controls both productive assets, such as a manufacturing firm. As soon as party 1’s investment becomes more important than 2’s, \((A_N, B_2)\) wins. The asset ownership shifts across the vertical line of \(\alpha\). But in order to balance 2’s investment incentives, it is efficient to let 2 having bargaining control over party 3. When 1’s investment gets even more important, case \((A_I, B_c)\) wins. It is efficient to give 1 and 3 their freedom to bargain with each other. And, finally, case \((A_I, B_1)\) wins. Giving 1 both the bargaining control and the asset ownership is optimal when 1 is much more important than 2.\(^{17}\)

**Case III: with Cross-investment, But \(k_c \neq k_s\)**

In the two previous examples, we set the marginal product of investments on sub-coalitional productions, \(k_s\) and \(k_c\) to be the same. In this case, we make the distinction between the cross-investment aspect and self-investment aspect of the marginal ben-

\(^{17}\)In this example, we do not have a result that party 1 owns both assets because of the restriction that the ownership of \(m_2\) is always controlled by party 2.
efits in sub-coalitional productions. We explore the optimal governance structure choice when $k_c \neq k_s$. As will be discussed extensively in the next section, other things the same, the greater $k_c$ is, the greater the benefit is to have bargaining control rights. But the greater is $k_s$, the greater the cost is to use bargaining control rights. Whether incomplete bargaining network can be more efficient than the complete bargaining network is essentially a tradeoff between these two aspects. So we should expect to see the incomplete bargaining networks, $B_1, B_2$, being more likely to win if $k_c$ is relatively large comparing to $k_s$, and $B_c$ more likely to be efficient if the opposite holds. Figure 1.3 reports the result.

We highlight three observations in this case. First, the incomplete bargaining networks tend to be efficient when $k_c$ is relatively large comparing to $k_s$. When
$k_s < k_c$, the benefit of having bargaining control rights tends to overweight its cost. The two GHM governance structures are dominated towards the bottom part of Figure 1.3b. As $k_s$ gets closer to the magnitude of $k_c$ and goes above, the structures using bargaining control rights start to lose to GHM.

Second, this model predicts that, once we introduce bargaining control rights, the optimal asset ownership can be different from what is predicted in GHM. In Figure 1.3b, the optimal governance structures are not all integrated asset ownership to the left of $\bar{\alpha}$ and all non-integrated asset ownership to the right. This indicates that it can be efficient for party 1 to control the asset even though her investment is not as important as 2’s. The intuition for this case is the following. When the benefit of using bargaining control is relatively large comparing to its cost, having bargaining control can more effectively motivate investment. In this case, bargaining control rights become a more effective instrument than asset ownership. The party who makes relatively more important investment should have the bargaining control rights. So as party 1’s investment gets important but not more so than 2, it is efficient to have her run an independent firm with asset (case $(A_N, B_2)$), rather than making her control a firm with the subordinate (case $(A_I, B_1)$). This pattern is in stark contrast to what is predicted in the previous case where $k_c = k_s$. In fact, in the lower part of Figure 1.3b, when 1’s investment is less important than 2’s, 2 always has bargaining control rights over 1. And it is always efficient for 1 to hold bargaining control rights over 2 once 1’s investment becomes more important.

Third, fixing $k_s$ and moving from the left to the right, as $\alpha_1$ increases, there are multiple rounds of transfers of asset ownership. When $\alpha_1$ is very small, the asset $m$ is controlled by party 2. As $\alpha_1$ gets greater and approaches $\bar{\alpha} = \alpha_2 = \alpha_3$, it is efficient for 1 to control the asset. We see another round of transfer of asset ownership once
\( \alpha_1 \) becomes greater than \( \overline{\alpha} \). When \( \alpha_1 \) crosses the vertical dashed line of \( \overline{\alpha} \), the asset ownership changes back to party 2, then changes back again to party 1 as \( \alpha_1 \) gets very large relative to \( \overline{\alpha} \). This pattern is not consistent with the classical prediction that assets should be owned by the party who makes more important investments (Hart and Moore, 1990). In fact, in the presence of bargaining control rights, their prediction does not always generalize. This result provides one possible explanation to the fact that, in many transactions, parties who provides very valuable services owns asset-less firms.

To briefly summarize the findings in this section, among the observations, two stand out as the most interesting. First, the model shows that with cross-investment, introducing bargaining control rights as instruments in the governance structure can further improve the efficiency of transactions in addition to using allocation of asset ownership. Second, the model can predict different optimal asset ownership as GHM does.

### 1.5 Analysis of the Model of Three Parties

After observing some of the interesting features in the previous section, we devote this section to rigorous analysis of the effects of incomplete bargaining networks in the presence of asset ownership allocation. The different propositions provide the general intuitions behind the patterns we observe previously in the example. We offer discussions of the propositions regarding their interpretations relating to the vertical integration of transactions. All proofs of the propositions are omitted and included in the Appendix 1.A.
We analyze the model backwards. First, we analyze the bargaining payoffs at the *ex post* stage under different governance structures. Then we move on to study how different bargaining payoffs affect the three parties’ *ex ante* investment incentives. From the associated investment incentives, we are able to draw some general conclusions regarding the choice of the optimal governance structure.

### 1.5.1 *ex post* Bargaining Payoffs

Having characterized the bargaining payoffs for the three-party case under different governance structures in equations (1.3) through (1.5), we start by analyzing observations from them.

By subtracting the three equations from each other, we have

\[ Y_i - Y_c = \frac{1}{3}(v_{jk} - v_j - v_k); \tag{1.6} \]
\[ Y_j - Y_i = -\frac{1}{6}(v_{ik} - v_i - v_k). \tag{1.7} \]

By the assumption that the production is superadditive, i.e. \( v_{ij} > v_i + v_j, \forall i, j = 1, 2, 3 \), we have the following result.

**Remark 1.1.** Given fixed *ex ante* investment levels and fixed asset allocation, bargaining control rights provide extra bargaining payoff. Specifically, \( Y^c_i > Y^c_i > Y^c_i. \tag{1.8} \)

Intuitively, party \( i \) obtains a higher payoff under \( B_i \) because, comparing to \( B_c \), she is no longer jointly threatened by \( k \) and \( j \) together. Party \( i \) is able to prevents

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\(^{18}\)In terms of the timing of the model, this result confirms that the bargaining control over other party is “sub-game perfect”. That is, once a party obtains bargaining control from the agreed governance structure, she will *not* give up the control right in the *ex post* bargaining stage to let the other two parties freely bargain with each other.
j and k from bargaining with each other to form a contract without her. In reality, an employee is unable to reach a side-contract with an outside firm or with another employee at the same firm. Thus they are unable to jointly make a credible threat against the employer firm for a more favorable term in their respective contracts. As a consequence, j and k’s bargaining payoffs are lower comparing to those under $B_c$.

In all the incomplete bargaining networks, the control rights over other parties’ ability to bargain diverts a greater share of final value from those who lost the bargaining rights to the party who obtains bargaining control.

By observation from equations (1.3) through (1.5), the following proposition becomes obvious.

**Proposition 1.1.** Comparing to all other cases in which party $j$ is free to bargain, if some party $i$ has bargaining control rights over party $j$, then we have (i. **Insulation Effect**) the outside option $v_{jk}$ between $j$ and the party other than $i$ is insulated from every parties’ bargaining payoff. Specifically, for any $k \neq i$, $\frac{\partial Y^i}{\partial v_{jk}} \neq 0, \forall l = 1, 2, 3$ for $b \neq i$. But $\frac{\partial Y^i}{\partial v_{jk}} = 0, \forall l = 1, 2, 3$. (ii. **Concentration Effect**) the individual outside options $v_j$ and $v_k$ have higher weight in every parties’ bargaining payoff. Specifically, for any $k \neq i$, $|\frac{\partial Y^i}{\partial v_j}| > |\frac{\partial Y^c}{\partial v_j}|$ and $|\frac{\partial Y^i}{\partial v_k}| > |\frac{\partial Y^c}{\partial v_k}|, \forall l = 1, 2, 3$.

These effects follow directly from the way we defined the incomplete bargaining networks. The intuition is that if party $j$ can only bargain directly with party $i$, no one other than $i$ is able to form an agreement with $j$ without involving $i$. Consequently, $v_{jk}$ is no longer a credible threat for either $j$ or $k$ against $i$. As a result, $j$ and $k$ will have no incentive to invest *ex ante* in $v_{jk}$. The benefit of this effect is that if party $i$’s investment has an cross-investment aspect that also benefits $v_{jk}$, she will have greater incentive to invest. Because she need not be concerned about increasing $v_{jk}$.
that will turn into a potential threat against her own payoff. More specific discussions regarding the influence of this property will continue in our analysis about the \textit{ex ante} stage investments.

Following our interpretation of the bargaining control rights as a hierarchical structure, the proposition says that integration of party \( j \) by party \( i \) fundamentally changes the payoff structure of every party. Besides parties \( i \) and \( j \), this effect influences all parties involved in the transaction, including, in this case, firm \( k \).\footnote{In a three-party model, one might argue that in \( B_{ij} \), \( j \) and \( k \) simultaneously lose their bargaining rights to party \( i \). So it seems too strong to make the point that the insulation effect also affects those parties who are not integrated. However, we show that the insulation effect indeed generalizes to a model with any number of parties. Following the integration of any party, all outside options that involves joint production with this party are insulated from all parties’ payoffs. Specifically, in any network \( B \) that \( j \) can only bargain with \( i \), \( \frac{\partial Y_i}{\partial v_S} = 0 \), for all parties \( l \) and \textit{all} coalitions \( S \) such that \( S \not\ni i \) and \( S \ni j \). For the specific statement and proof, see Proposition 2.4.}

The insulation effect describes the benefit of bargaining control rights. By removing some potential outside options from all the parties involved in the transaction, it can help align the interests of some parties with the social interest, \( v_{123} \).

Unsurprisingly, the bargaining control rights comes with a cost as well. The concentration effect highlights the cost side of limited bargaining rights. A comparison between equations (1.6) and (1.7) highlights that restriction in bargaining rights only shifts parties’ interests from pursuing a joint sub-coalitional outside option to pursuing individual outside options.\footnote{However, it offers an efficiency improving opportunity if putting more concerns over the individual outside option, in place of the joint sub-coalitional outside options, improves the productive investment incentives or reduces the wasteful investment incentives. See Holmström and Milgrom (1991); Gibbons (2005).} The efficiency of using bargaining control rights depends on the tradeoff between lighter weights spread on more outside options and heavier weights condensed on less smaller-scale outside options.

If we interpret Proposition 1.1 in the context of vertical integration, it says that as a result of integration, by which we mean obtaining control over another party’s
bargaining rights, the incentives of all the parties involved in the transaction become more focused. On one hand, they are more focused in the sense that they care about less types of outside options (the insulation effect). One the other hand, they are more focused because they care more about some particular smaller-scale outside options (the concentration effect).

This model predicts that integration of one other firm fundamentally changes outside options for all transaction-related parties. Integration protects the integrating firm from joint hold-up threats that involves the integrated party. And integration removes all other, integrated or not-yet-integrated, parties’ incentives to invest toward these sub-coalitional outside options. However, as its downside, it creates more narrow minded parties who puts a heavier weight on their own outside opportunities.

### Bargaining Payoffs under Different Asset Ownership

Previously we have only discussed the bargaining payoffs given a fixed asset ownership structure. In this part of the section, we analyze the interactions between asset ownership and bargaining control rights.

Consider two otherwise identical asset allocation rules, $\overline{A}$ and $\overline{A}$, except that one asset is assigned differently. Recall that the asset ownership affects the ex post bargaining payoffs through the production functions, $v_S(e, A)$. We can obtain the bargaining payoff for party $i$ under governance structure $g = (A, B)$ for $A \in \{\overline{A}, \overline{A}\}$ and $B \in \{B_c, B_1, B_2, B_3\}$ as

$$Y_i^b = Y_i^{b|A=\overline{A}},$$

$$Y_i^\overline{b} = Y_i^{\overline{b}|A=\overline{A}},$$
where $Y^b_i$ is given in equations (1.3) through (1.5).

Let us define the following operation $\Delta(v_S(e)) = v_S(e, \overline{A}) - v_S(e, A)$ as the difference in the production value $v_S$ under the two asset ownership structures for coalition $S$. In a similar form as equations (1.6) and (1.7), we have

$$Y_i^c - Y_i = Y_i^c - Y_i^{c} + \frac{1}{3}\Delta(v_{jk} - v_j - v_k);$$

(1.8)

$$Y_i^j - Y_i = Y_i^j - Y_i^c - \frac{1}{6}\Delta(v_{ik} - v_i - v_k).$$

(1.9)

The following result follows immediately from these two equations.

Proposition 1.2. The change of asset ownership can have different effects on payoffs under different bargaining networks. Specifically, there is difference in payoffs across different networks if the asset ownership changes the superadditivity in sub-coalitional cooperation, i.e. $\Delta(v_{jk} - v_j - v_k) \neq 0$.

Proposition 1.2 offers the interaction between the two dimensions of the seemingly independent governance structures. It says that the effect of the asset ownership can vary across different allocations of bargaining control rights.

With our interpretation, Proposition 1.2 predicts that the transfer of ownership over the same asset between the same pair of parties can cause different changes in payoff distribution. The amount of payoff each party can gain or lose from the transfer can depend on the level of integration in the transaction. Suppose there are two cases, in the first, $i$ and $j$ are both free to bargain and controls no other party; whereas in the second case, $i$ has bargaining control over some other party $k$. Then the ex post rent distribution can differ in these two cases following a transfer of the same asset from $i$ to $j$.\textsuperscript{21}

\textsuperscript{21}With more than three parties, we can possibly identify a firm under exclusive dealing restrictions
To summarize our analysis up to now, bargaining control rights diverts a greater bargaining payoff from those parties who become restricted to bargain toward those who have control. This shift removes all the outside options of joint productions that involve the integrated parties. It shifts the parties’ interests to focus more heavily on outside options involving less parties. The asset ownership and the allocation of bargaining control rights can interact with each other. The ex post benefit or loss from obtaining the ownership of the same asset from the same party may differ depending on the bargaining control rights. The answer regarding whether restricting bargaining rights can improve efficiency, however, depends on the specific nature of investments. The following subsection studies these implications in further detail.

1.5.2 ex ante Investment Incentives

In the ex ante stage, each party $i$ chooses her non-contractible relationship-specific investment level $e_i$ at private cost $\Psi_i(e_i)$ to maximize her future bargaining payoff given the agreed upon governance structure. In this section, we analyze different investment incentives under different governance structures. And consequently, we are able to draw some implications from the model regarding the efficiency of the respective structures.

First-best Benchmark

Before specifying the ex ante investment problem under any specific governance structure, we will analyze the first-best investment level as a benchmark.

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in the model. A generalization of Proposition 1.2 then implies that the payoff changes following a transfer of the same asset between a firm restricted by exclusive dealing and another firm can be different should the restricted party were an independent firm.
The first-best level of investment $e_i^{FB}$ is the choice of $e_i$ that maximizes the final value of production $v_{123}(e, A)$ given the costs $\Psi_i(e_i)$ for all parties. It is characterized by

$$\frac{\partial v_{123}(e_i \{m, m_2\})}{\partial e_i} = \Psi'_i(e_i). \quad (1.10)$$

**Investments Given Fixed Asset Ownership**

We first characterize the *ex ante* investment levels, $e_i^{A, B_c}, e_i^{A, B_1}$ and $e_i^{A, B_2}$ under the three different bargaining networks given fixed asset ownership $A$.\(^{22}\)

Party $i$ obtains her associated payoff $Y_i$ under the particular bargaining network. Under $B_c$, party $i$ will obtain $Y_i^c$ *ex post*, so $e_i^{A, B_c}$ is characterized by

$$\frac{\partial Y_i^c(v_S)}{\partial e_i} = \Psi'_i(e_i). \quad (1.11)$$

where $Y_i^c(v_S)$ is given in equation (1.3), and each $v_S$ in vector $v_S$ is a function of both investment level $e$ and asset allocation rule $A$.

Similarly, $e_i^{A, B_1}$ and $e_i^{A, B_2}$ are characterized by $\frac{\partial Y_i^c(v_S(e, A))}{\partial e_i} = \Psi'_i(e_i)$, respectively. But we can utilize equations (1.6) and (1.7) to rewrite them as

$$\frac{\partial Y_i^c(v_S)}{\partial e_i} + \frac{1}{3} \frac{\partial (v_{jk} - v_j - v_k)}{\partial e_i} = \Psi'_i(e_i). \quad (1.12)$$

$$\frac{\partial Y_i^c(v_S)}{\partial e_i} - \frac{1}{6} \frac{\partial (v_{ik} - v_i - v_k)}{\partial e_i} = \Psi'_i(e_i). \quad (1.13)$$

**Assumption 1.1.** We assume that the marginal product of each party $i$’s invest-
ment $e_i$ is strictly lower in the sub-coalitional productions comparing to that in the production of the grand coalition, i.e. $\frac{\partial v_S}{\partial e_i} < \frac{\partial v_N}{\partial e_i}$, $\forall S \subset N$.

**Proposition 1.3.** Under assumption 1.1, there is always under-investment in any bargaining network $B_c$, $B_i$ and $B_j$. That is $e_i^{A,B} < e_i^{FB}$ for any $i \in \{1, 2, 3\}$ and any $B \in \{B_c, B_i, B_j\}$.

**Proposition 1.4.** If any governance structure $g$ induces a higher investment vector $e^g$ than the alternative $g'$ does, then $g$ is more efficient than $g'$. That is $v_{123}(e^g, \{m, m_2\}) - \sum_i \Psi_i(e^g_i) \geq v_{123}(e^{g'}, \{m, m_2\}) - \sum_i \Psi_i(e^{g'}_i)$ if $e^g \geq e^{g'}$.

Having laid the ground for evaluating the relative efficiencies of different governance structures, we move on to compare the complete bargaining network $B_c$ with the incomplete bargaining networks $B_i$.

At this point, it is convenient for what follows to introduce some definitions.

**Definition.** We say there is cross investment for $e_i$ if for any $S \ni i$, $\frac{\partial v_S}{\partial e_i} > 0$.

**Definition.** We say the investment $e_i$ is cross-investment superadditive at the margin (CSM) with respect to coalition $S$ if for coalition $S \ni i$ and $S' \subset S$, $\frac{\partial v_S}{\partial e_i} > \frac{\partial v_{S'}}{\partial e_i} + \frac{\partial v_{S \setminus S'}}{\partial e_i}$

We say the investment $e_i$ is cross-investment superadditive at the margin if $e_i$ is cross-investment superadditive at the margin with respect to all coalitions.

One sufficient condition for investments to satisfy CSM is if the nature of the investment is (i) non-specific to the investor ($\frac{\partial v_S}{\partial e_i} > 0$ for some $S \ni i$), such as

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23Assumption 1.1 is in place so we can anchor the relative relationship between the first-best and second-best investment levels. We do not think the assumption is substantive as long as the sign of the inequality is consistently positive or negative. The sign can be understood as the direction we choose to interpret the nature of the investment.

24Proposition 1.3 and Proposition 1.4 together are the counterparts of Proposition 1 in Hart and Moore (1990).

25This definition of cross investment is also introduced in Whinston (2003).
investment in capabilities, knowledge, process or routine that benefits other parties, but (ii) generates more marginal benefits when other parties jointly participate with their resources \( \frac{\partial v_{jk}}{\partial e_i} > \frac{\partial v_{j}}{\partial e_i} + \frac{\partial v_{k}}{\partial e_i} \). One such example is investment in workers’ skills to operate a information system that are not specific to the investor but specific to, say, the supplier company of the investor. For another instance, investment in a complicated early-stage R&D project that requires joint work of designing specialists and marketing specialists.

**Definition.** We say the investment \( e_i \) is **self-investment superadditive at the margin** (SSM) with respect to coalition \( S \) if for coalition \( S \ni i \) and \( S' \subset S \), \( \frac{\partial v_{S}}{\partial e_i} > \frac{\partial v_{S'}}{\partial e_i} + \frac{\partial v_{S \setminus S'}}{\partial e_i} \). We say the investment \( e_i \) is **self-investment superadditive at the margin** if \( e_i \) is self-investment superadditive at the margin with respect to all coalitions.

One sufficient condition for investments to satisfy SSM is if the nature of the investment is specific to the investor \( \frac{\partial v_{j}}{\partial e_i} = 0 \), such as investment in assets that’s currently under control, but complementary to other parties’ existing resources \( \frac{\partial v_{ij}}{\partial e_i} > \frac{\partial v_{i}}{\partial e_i} \). For example, investment in firm-specific human capital.

Some investment can be both SSM and CSM. For example, investment in knowledge \( \frac{\partial v_{ij}}{\partial e_i} > 0 \) that is specific to the particular transaction \( \frac{\partial v_{i}}{\partial e_i} = 0 \), but not specific to the investor \( \frac{\partial v_{ij}}{\partial e_i} > 0 \).

Moving on to the analysis, equations (1.12) and (1.13) provides two interesting observations regarding the effect of bargaining control rights on the investment incentives.

First, comparing to the complete bargaining network case, obtaining bargaining control over another party only increases the marginal benefit of this party’s investment if and only if her investment is CSM. This is shown by the second term in
Second, comparing to the complete bargaining network case, losing bargaining rights to some other party \( j \) reduces the marginal benefit of this party’s investment if and only if her investment is \( SSM \), which is shown by the second term in equation (1.13), \( \frac{\partial (v_{jk} - v_j - v_k)}{\partial e_i} \).

Although the first-order effects of bargaining control rights is clear, the net effect on the equilibrium investment levels are ambiguous in general conditions due to second-order interactions in parties’ investments. The following Remark summarizes these “asymmetric” first-order effects under a special environment.

**Definition.** We say the investments of any two parties \( i \) and \( j \) are *technologically independent* if their investments has no effect on each other’s marginal product, i.e.

\[
\frac{\partial v_j^2}{\partial e_i \partial e_j} = 0, \forall S.
\]

**Remark 1.2.** If all parties’ investments are technologically independent, then comparing to the baseline of complete bargaining network, suppose party \( i \) obtains bargaining control rights over party \( j \), \( e_i \) increases after the fact if and only if it is CSM with respect to coalition \( jk \); \( e_j \) and \( e_k \) decreases after the fact if and only if they are SSM with respect to coalition \( jk \).

The following remark is a counterpart of the previous one presented in a comparative-static manner.

**Remark 1.3.** Comparing to the baseline of complete bargaining network, suppose party \( i \) obtains bargaining control over party \( j \), (i) if only \( i \) makes investment, then the change is more efficient if and only if \( e_i \) is CSM with respect to coalition \( jk \); (ii) if only \( j \) (or \( k \)) makes investment, then the change is less efficient if and only if \( e_j \) (or \( e_k \)) is SSM with respect to coalition \( jk \).
Remark 1.3 provides the basis for a thought experiment under the general environment where every party makes investments. The efficiency of having bargaining control rights depends on whether the increased investment incentives by alleviating investor’s concern in cross-investment can overweight the reduced investment incentives due to restricted outside options.\textsuperscript{26}

Indeed, remark 1.3 is the counterpart of the result in Hart and Moore (1990) regarding the optimal governance structure if only one party makes investment. GHM predicts that if only one party makes investment, she should own all the assets as long as her investments are complementary with the assets. Our model predicts that the only investor should obtain bargaining control rights over others if and only if her investment supports other parties’ cooperation without her.

The following proposition outlines the tradeoff in an extreme case without assuming technological independence in investments.

**Proposition 1.5.** If there is no CSM, and every parties’ investments are SSM with respect to all coalitions $S \subseteq \{1, 2, 3\}$, then it is never efficient to have bargaining control rights, i.e. $B_c$ is always more efficient.

**Corollary 1.1.** If there is no cross investment, then under Assumption 1.1, it is never efficient to have bargaining control rights, i.e. $B_c$ is always more efficient.

We interpret Proposition 1.5 and Corollary 1.1 in the backward order.

Indeed, Corollary 1.1 is a very strong result based on a simple, although not necessarily weak, assumption. The environment without cross-investment corresponds

\textsuperscript{26}Reducing self-interested investments need not be efficiency reducing, we have this result because there is always under investment. This is not the case, if the investment is purely rent-seeking without being productive. But the predictions for the latter situation can be easily induced from our results with minimal differences in the signs. This case can be readily studied by a straightforward extension of the current framework with a multi-tasking agent model.
to a situation where the effects of every party’s investment is well-contained in the productions that she is a part of. Loosely speaking, this property describes a world without externality. If we follow our interpretation that bargaining control rights is a hierarchy in the firm, we can read Corollary 1.1 as saying that if there is no externality, there should not be vertically integrated firms in the transaction. In this situation, market transaction, $B_c$, is the most efficient governance structure. In other words, by stating that a hierarchical structure is inefficient without externality, Corollary 1.1 implies that the firm is an institution that helps reducing certain externalities among those parties involved in a transaction.

Proposition 1.5 describes the specific type of externality on which integration has effect. Should the investment be CSM, integration would help motivate investment of the integrating party by protecting her from joint hold-up threats. But if her investment is not CSM, then replacing the joint hold-up threats with individual hold-up threats actually lowers her investment incentives. Proposition 1.5 says that integration into a hierarchical structure is never efficient if protecting the owner from larger-scope joint threats worsens her overall hold-up concerns, even though Proposition 1.1 shows the integrating party obtains a higher level of payoff.

As a comparison to Proposition 1.5, we provide the following result, which is an opposite result that describes an extreme condition in which it is always efficient to use bargaining control rights.

**Proposition 1.6.** *If all parties’ investments are only SSM with respect to coalitions that include party i, and suppose party i’s investment is weakly CSM with respect to other coalitions, then it is always optimal for i to have bargaining control rights over*
To interpret, loosely speaking, Proposition 1.6 says that if every parties’ investments are only “complementary” to one party, then this party should be the boss of everyone. In other words, all parties should be integrated into the same firm that is controlled by this party who is complementary to every one’s investments.

We can relate the main results in this section to the classical Coasean tradeoff between the cost to use the market and the cost to use fiat. In this model, the cost of using the market is exposing the integrating party to potential joint hold-up by others. Integration can help protect investment incentives by reducing the externality from her investments and replacing it with several individual level hold-up threats. Integration would help in this case only if the investment is productive to other parties’ productions and helpful for other parties’ cooperation. But it comes with the cost of lowering the investment incentives for the integrated party due to a worse agency problem. Moreover, our model highlights that integration also worsens the agency problem for all other parties involved in the transaction.

Most interestingly, although the benefit of integration is rooted in externality, the cost of integration is not. All these parties’ investment incentives tend to be lower because they are restricted to work with their boss, which in turn restricts their outside options.

**Investments under Different Asset Ownership**

From observations of equations (1.8) and (1.9), we find that the effect of a given asset ownership change over the marginal benefit of the _ex ante_ investments can vary.

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27By “weakly CSM”, we refer to a condition \( \frac{\partial v_{SI}}{\partial e_i} \geq \frac{\partial v_{S1\ast}}{\partial e_i} + \frac{\partial v_{S1\ast}}{\partial e_i} \), which need not necessarily hold in its strict form.
depending on the allocation of bargaining rights.

The following remark compares the “likelihood” of an asset being owned by one party rather than another in a fixed dyad under different bargaining networks. Taking derivatives of equations (1.8) and (1.9) with respect to the ex ante investments yields the following remark.

**Remark 1.4.** Under different bargaining networks, a given transfer of asset ownership between two parties can have different first-order effects on parties’ marginal benefit of investments. Specifically, compare the transfer of the asset $m$ from $j$ to $i$ under network $B_i$ and $B_c$. Suppose all other things equal. (i) If losing $m$ decreases (increases) the level of SSM for party $j$ and $k$, then the transfer is associated with less (more) of a drop in $e_j$ and $e_k$ under $B_i$ than under $B_c$. (ii) If gaining $m$ increases (decreases) the level of CSM for party $i$, then the transfer is associated with more (less) of an increase in $e_i$ under $B_i$ than under $B_c$.

Roughly speaking, Remark 1.4 states the conditions which increase the likelihood that bargaining control rights and ownership of assets are allocated to the same party. In other words, given it is efficient for a party to have bargaining control rights, it might be more likely for her to have asset ownership in the optimal governance structure.

For example, if the ownership of an asset plays an important role in the cooperation between $j$ and $k$ (decreases the level of SSM for party $j$ and $k$), then after party $i$ obtains bargaining control over one of $j$ or $k$ (under $B_i$), this asset is more likely to be owned by party $i$, instead of one of $j$ or $k$. In this case, bargaining control rights and ownership of assets are likely to be jointly owned.

The logic of Remark 1.4 provides the intuition behind the pattern in Section 1.4.
Case III. In fact, in Section 1.4 Case III, bargaining control rights and asset ownership are likely to be owned together. Because as $k_s$ increases, the SSM decreases when someone loses the asset $m$.\footnote{In the example, the SSM for party $j$ and $k$ when they own $m$ is $k_{self} - 2$, the SSM when $i$ owns $m$ is $mk_{self} - m - 1$. Subtracting one term from another, the change in SSM before and after losing the asset to be $(m - 1)(1 - k_{self})$, which is decreasing in $k_{self}$. The change in SSM for party 1 and 3 before and after losing the asset to 2 is exactly the same.} This is why, in the lower part of the figure (b), the two boundary lines demarcating the shift of assets between 2 and 1 (lines separating blue from red, except for the middle line) tilt toward the center. In the south-west part, given the it is optimal for party 2 to have bargaining control rights, fix the importance of party 1’s investment, $\alpha_1$, as $k_s$ increases, it is more likely for 2 to own the asset. Similarly, in the south-east part, given that 1 has bargaining control is optimal, fix $\alpha_1$, it is more likely for party 1 to own the asset as $k_s$ increases.

1.6 Concluding Remarks

Main Results

This paper studies the effect of an endogenous institutional restriction that limits some parties’ ability to bargain freely with each other when asset ownership is also in place as an organizational design instrument. In this particular model, we embed this idea in the framework of the property-rights theory of the firm to evaluate whether introducing such restrictions in bargaining rights can improve efficiency in addition to using allocation of property rights over assets. Our main finding is that, when there is cross-investment, restricting some parties’ ability to bargain with others in the transaction can improve efficiency in addition to using asset ownership. Furthermore, the predicted optimal asset allocation can differ from the result prescribed in classical
property-rights model without restriction in bargaining rights.

Other results from the model include: (i) Restricting bargaining rights insulates some of the outside options from all parties’ objectives, but replaces them with smaller-scale outside options. (ii) Bargaining control rights and asset ownership can interact with each other. (iii) Under mild assumptions, cross-investment is a necessary condition for the efficiency of restricting any party’s bargaining rights. (iv) In the presence of cross-investment, it tends to be optimal to allocate bargaining control rights to the party who makes important non-contractible investments. (v) When one party obtains or loses bargaining control rights of some party, it does not affect the investment incentives for those parties who are already under bargaining control of the first party (See Chapter 2 for more details).

**Interpretation and Discussion**

We interpret this modeling framework to match many observed governance structures in the real world. We claim that the bargaining control rights resembles the vertical hierarchical structure in a business firm. This interpretation and our model together offer a theory of the boundary of the firm without relying on the asset ownership. This feature allows us to expand the scope of the traditional theories of the firm to understand asset-less firms and employment relationships. The model suggests that all these different forms of governance structures can be rationalized within the same framework. The answer regarding the optimal choice of governance structure depends on the specific characteristics of the industry and technology.

The efficiency of the rich set of governance structures under different scenarios helps us rationalize the real life counterparts of these structures, such as asset-less
firms and non-compete contracts.

Using our interpretation of the model, this paper makes the following predictions. (i) Asset-less firms are efficient governance structures adapted to different economic environments. (ii) Integration insulates the firm's subordinates from contractual externalities in the market, and it also attenuates the externalities for outside firms and their subordinates; but at the possible cost of worsening the motivation problems. (iii) Under some conditions, the firm that has bargaining control rights tends to own all the productive alienable assets. (iv) Under mild assumptions, cross-investment is a necessary condition for the firm to be a more efficient governance structure than the market. (v) In the presence of cross-investment, the party who makes important non-contractible investments should control the firm. (vi) Establishing control over another firm through exclusive dealing or integrating existing independent contractors does not affect the investment incentives for existing subordinates of the integrating firm.

It is worth noting that the insulation effect and concentration effect from the model together resembles the spirit of the subeconomy view of the firm (Holmström, 1999). The model suggests that the firm is an institution that reduces externalities and trade it off with motivation problems. This is the case in this model because the firm isolates its subordinates from outside options involving external parties in the transaction. On one hand, this isolation can possibly better align the incentives of the subordinates and the external parties with those of the boss of the firm to protect the investment incentives of the boss. But, as its cost, the isolation dulls the motivation of all other parties.
Robustness of the Results

In reality, firms very rarely have the full control over subordinates’ bargaining power to totally block bargaining between the outsiders and subordinates. For instance, different states in the U.S. treats non-compete clauses very differently in court Garmaise (2011). However, given the reasons we have discussed in the introduction, it is likely that the real-world firms have significant control over their subordinates’ bargaining rights. Thus the insights of the model remains because the reality seems to lie somewhere between the two extremes.

The qualitative implications of our analysis holds true even if the firm has imperfect bargaining control rights. To see this point, consider a straightforward extension of our model. The firms are assigned with an exogenous value describing the intensity of bargaining control rights, which may be determined by the local institutions, such as enforcement of non-compete clauses. Let the intensity, \( q \) be a value between 0 and 1. Then the bargaining payoff for each party is modeled by a linear combination of the payoff under complete network and the payoff under the corresponding incomplete network, such as \((1 - q)Y^c_i + qY^i_i\). In this model, all the qualitative implications would be identical to the insights of the current model.

Future Directions

The current modeling framework has the potential to be extended to study the difference among independent firms, subsidiaries and divisions. In non-wholly owned subsidiaries, each parent firm may not have residual rights of control over the assets that are legally owned the subsidiary. Classical GHM model does not have enough details in governance structures to distinguish an independent firm from a subsidiary
that owns assets. However, our model sketches one aspect that differentiates the subsidiaries from independent firms—bargaining control rights. The non-wholly owned subsidiary can be modeled as a party who owns assets but under bargaining control of its parent firm. In this aspect, this paper provides an elementary model that can potentially contribute to a more sophisticated model to study the differences among independent firms, subsidiaries and divisions.

Although left unmodeled, our results provide a hint that incentives provided within the firm can never, and should not, resemble those at the market. This idea echos previous models such as Baker et al. (2002), but holds by a different logic in this paper. In our model, even with the same asset ownership allocation profile, every party has very different objectives regarding the outside options when some parties are restricted to bargain comparing to the alternative case in which every party is free to bargain. Therefore, the same incentive contract between independent firms would perform differently if it were used within a firm versus between two firms.

To maintain the generality of our modeling framework, we chose not to impose much specific institutional or technological details in this paper. As a consequence, this paper does not extensively discuss any specific governance structure, such as the asset-less firms. In future works, it will be fruitful to apply modeling framework of this paper to more specific settings with more institutional details.

As a restriction, this paper starts with the assumption that firms are able to control the bargaining rights of its subordinates without going into the microeconomic details regarding how the employment contract, or ownership of the firm translates into the control of bargaining rights. We suspect that one important channel that links the two ends lies in specialization through job assignments. Microfounding any possible channel that links the ownership of the firm to the bargaining control rights may
provide more insights about the theory of the firm.
1.A Omitted Proofs for Propositions in Section 1.3

Proposition 1.3. Under assumption 1.1, there is always under-investment in any bargaining network $B_c, B_i$ and $B_j$. That is $e_i^{A,B} < e_i^{FB}$ for any $i \in \{1,2,3\}$ and any $B \in \{B_c, B_i, B_j\}$.

**Proof.** For any coalition $S$ such that $S \in \{1,2,3\}$ and $S \ni i$, assumption 1.1 guarantees that $\frac{\partial v_{123}(e_i, \{m, m_2\})}{\partial e_i} > \frac{\partial v_{s}(e_i, \{m, m_2\})}{\partial e_i}$. Furthermore, by the assumption that assets are complementary to investments, under any asset ownership $A$, $\frac{\partial v_{s}(e_i, \{m, m_2\})}{\partial e_i} \geq \frac{\partial v_{s}(e, A)}{\partial e_i}$ because $A(S) \subseteq \{m, m_2\}$. So $\frac{\partial v_{123}(e_i, \{m, m_2\})}{\partial e_i} > \frac{\partial v_{s}(e, A)}{\partial e_i}$.

Then for bargaining payoffs under $B_c$, by equation (1.3), we have $\frac{\partial y_{123}^c}{\partial e_i} < \frac{\partial y_{123}^f}{\partial e_i} < \frac{\partial v_{123}(e_i, \{m, m_2\})}{\partial e_i} < \frac{\partial v_{123}(e_i, \{m, m_2\})}{\partial e_i}$. Since $v_{123}$ is increasing in investments $e_i$ for any production $e_i$, therefore by equations (1.10) and (1.11), $e_i^{A,B_c} < e_i^{FB}$ for $i = 1,2,3$. Similar reasoning also applies to $e_i^{A,B_1}$ and $e_i^{A,B_2}$. \(\square\)

Proposition 1.4. If any governance structure $g$ induces a higher investment vector $e^g$ than the alternative $g'$ does, then $g$ is more efficient than $g'$. That is $v_{123}(e^g, \{m, m_2\}) - \sum \Psi_i(e^g) > v_{123}(e^{g'}, \{m, m_2\}) - \sum \Psi_i(e^{g'})$ if $e^g \geq e^{g'}$.

**Proof.** By Proposition 1.3 and the assumption that $v_{123}$ is non-decreasing in investments, an increase in investment vector $e$ increases the social surplus. The result then follows for $e^g > e^{g'}$. \(\square\)

Proposition 1.5. If there is no cross-investment superadditivity at the margin, it is never efficient to have bargaining control rights. $B_c$ is always more efficient.

**Proof.** Suppose $i$ obtains bargaining control, i.e. the new governance structure is under network $B_i$. Then by equation (1.13) and self-investment superadditivity at the margin, $e_i^{A,B_i} < e_i^{A,B_c}$ for any party $j \neq i$ who does not gain bargaining control.

But by equation (1.12), if there is no cross-investment superadditivity at the margin, $e_i^{A,B_i} \leq e_i^{A,B_c}$ for the party who obtains bargaining control rights. Further by complementary assumption $\frac{\partial v_{123}}{\partial e_i \partial e_j} > 0$, $B_c$ induces at least as high investments as in $B_i$ even for the party $i$ who gains bargaining control.

Thus by Proposition 1.4, $B_c$ is always more efficient than any incomplete network $B_i$. \(\square\)

Proposition 1.6. If all parties’ investments are only SSM with respect to coalitions that include party $i$, and suppose party $i$’s investment is weakly CSM with respect to other coalitions, then it is always optimal for $i$ to have bargaining control rights over others.

**Proof.** The result follows from Remark 1.2. If all parties’ investments are only SSM with respect to coalitions that include party $i$, then comparing to $B_c$, under network $B_i$, except for party $i$, no party’s marginal benefit of investment is lower.
And because party \( i \)'s investment is weakly CSM with respect to other coalitions, party \( i \)'s marginal benefit of investment is at least as high under \( B_i \) than under \( B_c \). Therefore \( B_i \) necessarily induces a higher investment vector than \( B_c \). So the result follows by Proposition 1.4. □
Chapter 2

Extensions of the Basic Framework: $n$ Parties

This chapter is devoted to extensions of the basic framework established in the previous chapter. The n-party model provides new insights regarding the effect of bargaining control rights on the bargaining payoffs for different parties under a richer environment. Most importantly, we show that integration of a firm in terms of bargaining control rights has no effect on the bargaining payoff or the marginal product of investment for the existing subordinates of the firm. In other words, the firm insulates both level effects and marginal effects from further integration or dis-integration for its subordinates.

The main propositions under the 3-party model generalize to the n-party case with one mild additional assumption, but since the results are basically restating the previous propositions, we leave the statements and the proofs in Appendix 2.A.
2.1 Setup of the Model

Let there be a finite set of risk neutral players \( N = \{1, 2, \ldots, n\} \) and a finite set of alienable assets \( M = \{m_1, m_2, \ldots, m_N\} \). There is a contractual network \( B \) connecting all parties in \( N \). For given coalition \( S \subseteq N \), we denote the asset allocation rule by \( A(S) \rightarrow M \), which assigns each asset to certain party. Each party makes \textit{ex ante} noncontractible, relationship-specific investments \( e = \{e_1, e_2, \ldots, e_n\} \) at private cost \( \Psi_i(e_i) \). The production function, or characteristic function of the coalitional form game, is a function of the coalition \( S \), the asset allocation rule by \( A \) and the \textit{ex ante} noncontractible investments \( x \) of different players, formally we write \( v_S(e, A) \in \mathbb{R} \).

The governance structure in the general model is a two-dimensional object \( g = (A, B) \), including the asset allocation rule \( A \), and the bargaining network structure \( B \).

The timing of the stage game and the assumptions on \( \Psi_i \) and \( v_S \) are exactly the same as in Section 1.3.

**Bargaining Networks**

According to terminologies and notations of Myerson (1977), a \textit{link} is an unordered set \( \{i : j\} \) for \( i, j \in N \). And a \textit{network (graph)} on the players \( N \) is a set of links, such as \( B = \{1 : 2, 2 : 3, 3 : 4, 1 : 4\} \) for \( 1, 2, 3, 4 \in N \). Let \( B_c \) be the complete network that contains all links between any two parties in \( N \), i.e. \( B_c = \{i : j | i, j \in N, i \neq j\} \).

**Definition.** A party, \( i \), is \textit{restricted to bargain under a network} \( B \) if she is connected with one and only one party, \( j \), under network \( B \), i.e. \( i : j \in B \), and \( i : k \notin B, \forall k \neq j \). And we denote the set of all the restricted-to-bargain parties under network \( B \) by
Definition. We say a party, $i$, is free to bargain under a network $B$ if she can bargain with at least two parties, and she can bargain with any other party who is not restricted to bargain. Specifically, we define the set of parties who are free to bargain under bargaining network $B$ as $F_B = \{i|i : j, i : k \in B \text{ for some } j \neq k, \text{ and } i : j \in B \text{ for any } j \in N \setminus R_B\}$.

In this model, we are interested in two types of parties. One that behaves like a firm, who acts as nexus of contracts and is able to form employment contracts with its employees, as well as forming business contracts directly with any other firms. The other type of party, however, behaves like subordinates in the firm, such as employees, divisions or subsidiaries. They are usually disciplined by the contract with their employers or headquarters. The subordinates are incapable to bargain and form contracts directly with outside suppliers, downstream customers or even other employees while still working for their employer. Their role in the transaction is governed by a vertical relationship closely related with their firms. But they do not directly involve in contracts with outside parties or with each other.

We require the parties to be either restricted to bargain, or be free to bargain. The restricted-to-bargain parties should be able to bargain with only one party. And this party should be free-to-bargain, since she represents the subordinates’ boss. Furthermore, the free-to-bargain parties should be able to bargain with anyone among themselves. We define the set of bargaining networks as $\mathcal{B} = \{B|i \in R_B \text{ or } i \in F_B, \text{ and if } i \in R_B, j \in F_B \text{ for } i : j \in B\}$.

By definition, $R_B$ and $F_B$ are mutually exclusive. Thus our definition of $\mathcal{B}$ immediately implies that for any bargaining network $B \in \mathcal{B}$, the two sets $R_B$ and $F_B$ form
a partition of $N$. Furthermore, under this definition, all networks in $\mathcal{B}$ need to be connected.

**Definition.** A network $B$ is *connected* if for any $i, j \in N$, there exists a path $\{i : k_1, k_1 : k_2, k_2 : k_3, \ldots, k_p : j\} \subseteq B$ linking $i$ and $j$ in $B$ for some $k_1, \ldots, k_p \in N$.

**Lemma 2.1.** Any network $B \in \mathcal{B}$ is connected.

Thus for any network $B$, we can uniquely define a function $f_B(i) : R_B \to F_B$ for all $i \in R_B$ to identify the free-to-bargain party that is uniquely linked with the restricted-to-bargain party $i$. The definition of $R_B$ requires, $i \in R_B$, must be associated with one and only one free-to-bargain party, $j$.

**Definition.** We say $j$ has *bargaining control over* $i$ under network $B$ if $i \in R_B$ and $f_B(i) = j$.

**Lemma 2.2.** Given the players $N$, a free-to-bargain set $F \subset N$ and a mapping $f : N \setminus F \to F$ uniquely defines a network $B \in \mathcal{B}$.

By Lemma 2.2, we can uniquely refer to a bargaining network $B \in \mathcal{B}$ as $B(F, f(\cdot))$, where the restricted to bargain set of parties under network $B$ is $R_B = N \setminus F$, whose unique link to the rest of the network is identified by $f(\cdot)$.

Because $F$ and $f$ uniquely define $B(F_B, f_B)$, for any incomplete network, i.e. $B \in \mathcal{B}$ such that $R_B \neq \emptyset$, and the complete network, i.e. $B_c$ such that $R(B_c) = \emptyset$, it is obvious that we can convert $B$ to $B_c$ in finite steps by moving one party from $R_B$ to $F_B$ at a time. And we can also convert from $B_c$ to $B$ by moving parties from $F$ to $R$ and setting $f$ correspondingly. Therefore, any two networks $B_1 \neq B_2 \in \mathcal{B}$ can be converted to each other.\footnote{We can convert $B_1$ to $B_c$ and convert $B_c$ to $B_2$. Each step only involves moving one party between $F$ and $R$, and set $f$.} The basic step of the change between two different
networks $B_1$ and $B_2$ is to move one party from $F$ to $R$ or from $R$ to $F$, and to set the corresponding function $f$.

**Interpretation: Definition of the Firm**

When we jointly allocate the bargaining network $B$ and the asset allocation $A$, we can clearly define the boundaries of the firm from the governance structure $g = (A, B)$.

**Definition.** Any free-to-bargain party $i \in F_B$ is the boss of a firm $FM_i$, independent of whether $i$ owns any assets.

**Definition.** Any restricted-to-bargain party $j \in R_B$ is a subordinate of the firm controlled by $f(j) \in F_B$. In other words, $f(j) \in F_B$ is the boss of $j \in R_B$.

Denote the set of firms by $\{FM_1, FM_2, \ldots, FM_n\}$. The following lemma shows that there is no party who belongs to two firms, and there is no party who is left out of any firm either.

**Lemma 2.3.** $\{FM_1, FM_2, \ldots, FM_n\}$ partitions $N$.

**An Example of Five Parties with Subsidiary**

In Table 2.1, we provide an example with five parties. Unlike the three party case, in this example, we can clearly identify the subsidiary, who is restricted to bargain but owns asset. The first row involves four firms in the transaction, while the second row involves only three firms.
Partitions of a Coalition by Network and Bargaining under the Incomplete Network

At this point, we detour slightly to formally introduce the Myerson value definition under a general $N$ party environment.

**Definition.** Suppose for any coalition $S \subseteq N$, the network $B \subset B_c$ contains the a path linking $i$ and $j$ and stays within $S$, such as $\{i : k_1, k_1 : k_2, k_2 : k_3, \ldots, k_n : j\} \subseteq B$ for $i, j, k_1, \ldots, k_n \in S$, then we say $i$ and $j$ are connected in $S$ under $B$.

By connectedness, the network $B$ uniquely partitions the coalition $S$ into groups of connected players. We denote the partition $S/B = \{\{i\}|i$ and $j$ are connected in $S$ under $B\}|j \in S\}$. For example, if $N = \{1, 2, 3\}$ and $B_1 = \{1 : 2, 1 : 3\}$, then $N/B_1 = \{\{1, 2, 3\}\}$ because everyone is connected in $N$, but $\{2, 3\}/B_1 = \{\{2\}, \{3\}\}$ because 2 and 3 are not connected without player 1. But instead, for $\tilde{B}_c = \{1 : 2, 1 : 3, 2 : 3\}$, $\{2, 3\}/\tilde{B}_c = \{\{2, 3\}\}$ because without 1, 2 and 3 can still maintain a coalition under network $B_c$. 
We define the following operation $v/B$ as

$$v/B_S = \sum_{T \subseteq S/B} v_T.$$  

(2.1.1)

The Myerson value then defines the coalitional bargaining return as in Equation (1.2). When the network is complete, i.e. $B = B_c$, the Myerson value corresponds with the Shapley value. Therefore in this model, the organization with the complete bargaining network is exactly the same as Hart and Moore (1990).

### 2.2 Generalized Results

Our analysis shows that all the key insights obtained in the 3-party model generalizes to the $n$-party model under very mild conditions. Two new observations present themselves in the model with more than three-parties. In Proposition 2.2, we learn that an integration in terms of the bargaining control over some third party affects the bargaining payoffs of the freshly integrated party, the integrating party and all other parties outside of the firm, but does not affect the previous subordinates of the firm. Furthermore, Corollary 2.1 shows that the marginal investment incentives of the previous subordinates of the firm is also unaffected by the integrations or dis-integrations of this firm in terms of bargaining control. We will leave all proofs for the propositions in the appendix.

The Myerson-Shapley value is the unique bargaining rule if the allocation rule $Y$ is fair, i.e. $Y_i(B) - Y_i(B \setminus \{i : j\}) = Y_j(B) - Y_j(B \setminus \{i : j\}), \forall B \in B, \forall i : j \in B$ (Myerson, 1977). The fairness of property requires a notion of equal bargaining power among all parties. In other words, when a contract is established (broken), the benefit or loss (loss or benefit) is equally shared by the two parties involved in the relationship. Note that this assumption does not necessarily require a positive gain from the bargaining relation.
Bargaining Payoffs

We denote the bargaining return for party $i$ under network $B$ as $Y_{i}^{B}$. Furthermore, for coalition $S \subseteq N$ and network $B \in \mathcal{B}$, we denote the set that includes all the free-to-bargain parties in $S$ and their associated restricted-to-bargain parties in $S$ by $T_{B}(S)$. Specifically, $T_{B}(S) = \{i| i \in F_{B} \cap S \text{ or } f(i) \in S \text{ for } i \in S\}$. Notice that, from $S$, $T_{B}(S)$ filters out all the restricted-to-bargain parties who are disconnected with others in $S$, i.e. $S \setminus T_{B}(S) = \{i| f(i) \notin S \text{ for } i \in S\}$.

We also introduce another notation $R_{B}^{i}(S) = \{k| f_{B}(k) = i \text{ and } k \in S\}$ as the set of parties that are under bargaining control of party $i$ in coalition $S$ under network $B$.

The following Proposition characterizes the bargaining payoff for any party $i$ under any bargaining network $B \in \mathcal{B}$ with production function $v_{S}$.

**Proposition 2.1.** Each party’s bargaining payoff under network $B \in \mathcal{B}$ is given by

$$Y_{i}^{B}(v_{S}) = \begin{cases} \sum_{S \ni i} p(S) \left[ v_{T_{B}(S)} - v_{T_{B}(S) \setminus \{i\}} \setminus R_{B}^{i}(S) - \sum_{k \in R_{B}^{i}(S)} v_{k} \right] & \text{if } i \in F_{B} \\ \sum_{S \ni f_{B}(i)} p(S) v_{i} + \sum_{S \ni f_{B}(i)} p(S) \left[ v_{T_{B}(S)} - v_{T_{B}(S) \setminus \{i\}} \right] & \text{if } i \in R_{B} \end{cases} \quad (2.2.1)$$

**Change in the Bargaining Payoffs Following a Change in the Bargaining Network**

In order to simplify the statement of the following proposition, we introduce the following assumption, we will be explicit whenever it is needed for our result.

**Assumption 2.1.** The production function $v_{S}$ is convex with respect to the size of the coalition. That is, fix $e$ and $A$, for any party $i$, and any coalitions $S' \subset S$ such that $i \in S'$, $v_{S} - v_{S \setminus \{i\}} > v_{S'} - v_{S' \setminus \{i\}}$. 

Assumption 2.1 states that the marginal contribution of a given member increases in the size of the group that she is cooperating with.

The following proposition generalizes Proposition 1.1 to consider the payoff changes when some party $i$ loses bargaining control to party $j$.

**Proposition 2.2.** For any bargaining network $B \in \mathcal{B}$ that has a party $i$ who is free-to-bargain but controls no other party, i.e. $i \in F_B$ and $R_B^i(N) = \emptyset$. Let there be another network $\tilde{B}$ that is identical to $B$ except that party $i$ is restricted to bargain with party $\tilde{i}$, i.e. $\tilde{B} = B \setminus \cup_{k \neq \tilde{i}} \{i : k\}$. Then we have

$$Y_{\tilde{i}}^{\tilde{B}}(v_S) - Y_i^{B}(v_S) \geq 0 \quad \text{for } \tilde{i} = f_{\tilde{B}}(i)$$

$$Y_{i}^{\tilde{B}}(v_S) - Y_i^{B}(v_S) \leq 0 \quad \text{for } i$$

$$Y_{\tilde{j}}^{\tilde{B}}(v_S) - Y_j^{B}(v_S) \leq 0 \quad \text{for any } \tilde{j} \in F_B \text{ and } \tilde{j} \neq f_{\tilde{B}}(i) \text{ if Assumption 2.1 holds}$$

$$Y_{j}^{\tilde{B}}(v_S) - Y_j^{B}(v_S) \leq 0 \quad \text{for any } j \in R_B \text{ and } f_B(j) \neq f_{\tilde{B}}(i) \text{ if Assumption 2.1 holds}$$

$$Y_{i'}^{\tilde{B}}(v_S) - Y_i^{B}(v_S) = 0 \quad \text{for any } i' \in R_B \text{ and } f_B(i') = f_{\tilde{B}}(i)$$

**Corollary 2.1.** $|\frac{\partial Y_{\tilde{i}}^{\tilde{B}}}{\partial e_{i'}}| = |\frac{\partial Y_i^{B}}{\partial e_{i'}}|$ for any party $i'$ such that $f_B(i') = \tilde{i}$.

Proposition 2.2 generalizes Proposition 1.1. \(^3\)

More importantly, the proposition describes the changes in the bargaining returns associated with every party in the network when one party obtains bargaining control rights over another party. Since any network in $\mathcal{B}$ can be constructed from another one by finite number of moves which shifts one party between the restricted-to-bargain set $R$ and the free-to-bargain set $F$, Proposition 2.2 can help us predict the changes.

\(^3\)It confirms that the once some party obtains bargaining control over another party, it is at her best interest to enforce the restriction in bargaining *ex post*. In other words, bargaining control rights is sub-game perfect.
in bargaining returns when the bargaining network changes.

For example, suppose under network $B_1$, party $k$ is under bargaining control of party $i$. We further suppose that network $B_2$ has the identical structure as $B_1$ except that, in $B_2$, $k$ is under bargaining control of party $j$. Given \textit{ex ante} investment level fixed, Proposition 2.2 can help us understand the absolute payoff changes as a consequence of such a change in the bargaining network from, say, $B_1$ to $B_2$.

We can decompose the change from $B_1$ to $B_2$ into two steps. Suppose there is a third bargaining network $B_3$ which is identical to $B_1$ and $B_2$, except that party $k$ is free to bargain. Then the change from $B_1$ to $B_2$ can be broken down to a two-step change from $B_1$ to $B_3$, then from $B_3$ to $B_2$. Proposition 2.2 offers payoff changes for each party in the network in each of these two steps.

From $B_1$ to $B_3$, $k$ obtains freedom to bargain. The payoff of his boss under $B_1$, party $i$, decreases. The payoffs of all other restricted-to-bargain parties under party $i$ remain the same. And the payoff of all other parties, including $k$, increases. From $B_3$ to $B_2$, party $j$ obtains bargaining control over party $k$. Party $j$’s payoff increases. The payoffs of all other restricted-to-bargain parties under party $j$ remain the same. Party $i$, along with all other parties, including $k$, obtains lower payoffs. As a net result, party $i$’s payoff decreases, so does all restricted-to-bargain parties under $i$ except for $k$. Party $j$’s payoff increases, so does all restricted-to-bargain parties under $j$ except for $k$. Party $k$ and all other parties’ payoff changes are ambiguous.

In terms of its interpretation, Proposition 2.2 says that when a subordinate, either an employee or a division, of firm $i$ is integrated by firm $j$, firm $i$’s ex post bargaining payoff decreases, including that for both its boss and subordinates. On the contrary, firm $j$’s ex post bargaining payoff increases for both its boss and subordinates. The effect in payoff for the recently integrated party and all other firms involved in the
transaction remain ambiguous.

Following our interpretation, Corollary 2.1 says that, in terms of bargaining control rights, any integration or dis-integration for a firm of another free-to-bargain party does not affect its existing subordinates’ first-order incentives. This result is very strong and robust, and it resembles the idea similar to Holmström (1999) that the firm is a subeconomy like an island that insulates the outside market from its inside incentive systems. For instance, in Table 2.1, party 3’s bargaining return and investment incentives remain unchanged before and after the integration of party 1 by party 2 in the two respective columns.4

As a comparison to Corollary 2.1, it requires a much stronger condition for a change in the asset ownership to have a similar “neutral” impact on the existing subordinates. Suppose instead that party \( j \), who has bargaining control over \( k \), integrated an asset from any other party \( i \). In this scenario, we have the following proposition.

**Proposition 2.3.** Suppose party \( i' \) is under bargaining control of party \( i \), then compare two almost identical governance structures, \( g_i \) and \( g_j \), that are otherwise the same, except that asset \( m \) is owned by \( i \) in \( g_i = (A_i, B) \) but owned by \( j \) in \( g_j = (A_j, B) \). Then the bargaining payoffs, thus the first-order investment incentives, for party \( i' \) under \( g_i \) and \( g_j \) are identical if and only if

\[
\left( v_{T_i}^A - v_{T_i}^{A_i}(S) \right) - \left( v_{T_j}^A - v_{T_j}^{A_j}(S) \right) = 0
\]

for all \( S \ni i' \), \( S \ni i \), \( S \ni j \), where \( v_S^A \) is short for \( v_S(e, A) \) given \( e \) fixed.

Roughly speaking, in order for the existing subordinates’ payoff remain constant

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4As a caveat, Corollary 2.1 is *not* arguing that after the integration, the existing subordinates’ *investment levels* remain constant, although their investment incentives do. Their investment levels may change because the second-order effects from other parties’ investment levels will likely influence the subordinates’ equilibrium choice of investment, although their own objective payoffs remain the same.
following an acquisition of an asset by his boss, the subordinates’ contribution to all productions with his boss should remain the same, with or without the asset. In other words, the statement is true if the subordinates’ participation is not complementary to the asset. This is a much stronger condition comparing to Proposition 2.2 and Corollary 2.1, which holds true without any additional assumptions for the existing subordinates, \( i' \), of the integrating party.
2.A Omitted Proofs for Propositions in Section 2.1

Lemma 2.1. Any network $B \in \mathcal{B}$ is connected.

Proof. We will prove that any two parties $i, j \in N$ are connected under any given network $B \in \mathcal{B}$.

If $i, j \in F_B$, $i$ and $j$ are connected by definition of $F_B$. If $i \in R_B$ and $j \in F_B$, then $i : \tilde{i} \in B$ for some $\tilde{i} \in F_B$ by definition of $R_B$. But we also have $\tilde{i} : j \in B$ because $\tilde{i}, j \in F_B$. Thus $i$ and $j$ are connected.

Otherwise if $i, j \in R_B$, then $i : \tilde{i} \in B$ and $j : \tilde{j} \in B$ for some $\tilde{i}, \tilde{j} \in F_B$. But it must be either $\tilde{i} = \tilde{j}$ or $\tilde{i} : \tilde{j} \in B$. So $i$ and $j$ are connected.

Lemma 2.2. Given the players $N$, a free-to-bargain set $F \subset N$ and a mapping $f : N \setminus F \to F$ uniquely defines a network $B \in \mathcal{B}$.

Proof. Suppose $F$ and $f(\cdot)$ define both $B$ and $\tilde{B} \in \mathcal{B}$ s.t. $B \neq \tilde{B}$. Then for $F = F_B = F_{\tilde{B}}$ and $f = f_B = \tilde{f}_B$, there must exist a link $i : \tilde{i}$ in one of the networks $B$ or $\tilde{B}$, but not in the other, for some $i \in N$. With out loss of generality, we suppose $i : \tilde{i} \in B$ but $i : \tilde{i} \notin \tilde{B}$.

Since $i : \tilde{i} \neq \tilde{B}$, either $i$ or $\tilde{i}$ is not in $F$ by the definition of $F$. Without loss of generality, let $i \in F$ and $\tilde{i} \in N \setminus F = R$. Since $i : \tilde{i} \neq \tilde{B}$, $i \neq f(\tilde{i})$.

But since $F$ and $f(\cdot)$ also defines $B$, for $\tilde{i} \in N \setminus F, i : \tilde{i} \in B$ implies that $i = f(\tilde{i})$ by definition of $R$. Thus it must be that $f_B = f_{\tilde{B}}$, therefore we reach a contradiction.

Lemma 2.3. $\{FM_1, FM_2, \ldots, FM_n\}$ partitions $N$.

Proof. First, we show that any party $i \in N$ is in a firm.

Sets $F_B$ and $R_B$ partitions $N$ by definition. Suppose party $i \in F$. Then $i$ is a firm. Suppose, instead, $i \in R$, then $i$ is the subordinate for firm $f_B(i) \in F$. The above categorization exhausts $N$, thus all parties in $N$ is in a firm.

Next, we show that no $i \in N$ belongs to two firms.

Suppose $i \in FM_1$ and $i \in FM_2$ for $FM_1 \neq FM_2$. By definition of $\mathcal{B}$ and the definition of firms, $i$ cannot be a subordinate for both firms. And $i$ cannot be a subordinate for one firm and the boss the other because $R_B$ and $F_B$ partition $N$. Moreover, by definition of the boss, a party cannot be the boss for two firms. This concludes the proof.

Proposition 2.1. Each party’s bargaining payoff under network $B \in \mathcal{B}$ is given by

$$Y_i^B(v_S) = \begin{cases} \sum_{S_{\beta i}} p(S)[v_{T_B}(S) - v_{T_B(S) \setminus \{i\}} R_B(S) - \sum_{k \in R_B} R_B(S) v_k] & \text{if } i \in F_B \\ \sum_{S_{\beta f_B(i)}} p(S)v_i + \sum_{S_{\beta f_B(i)}} p(S)[v_{T_B}(S) - v_{T_B(S) \setminus \{i\}}] & \text{if } i \in R_B \end{cases}$$

Before proving the Propositions, it is convenient to prove some lemmas first.

Lemma 2.4. $T_B(S)$ is the only element in $S/B$ that contains more than one party.
Proof. Suppose there exists $T'_B(S) \cap T_B(S) = \emptyset$ such that $i, j \in T'_B(S)$ for some $i \neq j$.

Because $T'_B(S) \in S/B$, by definition, $i$ and $j$ are connected in $S$ under $B$. Thus there must be a link $\{i : k_1, k_1 : k_2, ..., k_n : j\} \subseteq B$ for $i, j, k_1, ..., k_n \in S$. By definition of $B$, it cannot be the case that none of them is in $F$ while being connected to each other. But suppose any one of them is in $F$, $T'_B(S) \cap T_B(S) \neq \emptyset$, we reach a contradiction. \hfill \Box

Lemma 2.5. For all $S \subseteq N$, we have

$$v^B_S = v_{T_B(S)} + \sum_{k \not\in T_B(S)} v_k.$$  

(2.A.1)

Proof. By Lemma 2.4, all $k \not\in T_B(S)$ are singleton components containing only one party. For any $S \subseteq N$, $S/B$ contains only one connected non-singleton component $T_B(S)$ and a group of other unconnected singleton components. Then the result follows by the definition of $v^B_S$.

Lemma 2.6. For all $S \subseteq N$, we have

$$v^B_{S\{i\}} = \begin{cases} v^B_S - v_i & \text{if } i \not\in T_B(S) \\ v_{T_B(S)\{i\}\setminus R^B_i} + \sum_{k \in S\{i\}\setminus f_B(k) \not\in S} v_k + \sum_{k \in f_B(k) \not\in T_B(S)} v_k & \text{if } i \in T_B(S) \end{cases}$$  

(2.A.2)

Proof. Lemma 2.5 helps unpack $v^B_S$ into the form of $v_S$. We can also apply Lemma 2.5 again to unpack $v^B_{S\{i\}}$.

By Lemma 2.5,

$$v^B_{S\{i\}} = v_{T_B(S)\{i\}} + \sum_{k \not\in T_B(S)\{i\}} v_k.$$

Furthermore, by definition of $T_B(S)$,

$$T_B(S\{i\}) = \begin{cases} T_B(S) & \text{if } i \not\in T_B(S), \\ T_B(S)\{i\}\setminus R^B_i & \text{if } i \in T_B(S). \end{cases}$$

Therefore, if $i \not\in T_B(S)$,

$$v^B_{S\{i\}} = v_{T_B(S)} + \sum_{k \in S\{i\}\setminus T_B(S)} v_k = v_{T_B(S)} + \sum_{k \not\in T_B(S)} v_k - v_i = v^B_S - v_i.$$
Proof of Proposition 2.1

Proof. By definition of the Myerson value, to specify the bargaining payoff $Y^B_i(v_S)$, we need to specify the term $v^B_S - v^B_{S \setminus \{i\}}$.

Subtract Equation (2.A.2) from Equation (2.A.1), we have

$$v^B_S - v^B_{S \setminus \{i\}} = \begin{cases} v_i & \text{if } i \notin T_B(S), \\ v_T(B) - v_{T_B(S) \setminus \{i\}} - \sum_{k \in T_B(S) \setminus \{i\} \setminus R^i_B(S)} v_k & \text{if } i \in T_B(S). \end{cases}$$  \hfill (2.A.3)

Plug Equation (2.A.3) into the definition of Myerson value, we have

$$Y^B_i(v_S) = \sum_{S \ni i, T_B(S) \ni i} p(S)v_i + \sum_{S \ni i, T_B(S) \setminus i} p(S)\left\{v_T(B) - v_{T_B(S) \setminus \{i\}} - \sum_{k \in R^i_B(S)} v_k\right\}.$$  \hfill (2.A.4)

When $i$ is free to bargain, the first term in Equation (2.A.4) drops out, which yields the payoff for any free-to-bargain party. When $i$ is restricted to bargain with a given party, the first term in Equation (2.A.4) remains. And the second term in Equation (2.A.4) reduces to $v_T(B) - v_{T_B(S) \setminus \{i\}}$ because $R^i_B(S) = \emptyset$ when $i$ is restricted to bargain. Therefore we have

$$Y^B_i(v_S) = \begin{cases} \sum_{S \ni i} p(S)[v_T(B) - v_{T_B(S) \setminus \{i\}} - \sum_{k \in R^i_B(S)} v_k] & \text{if } i \in F_B \\ \sum_{S \ni i} p(S)v_i + \sum_{S \ni i, f_B(i)} p(S)v_T(B) - v_{T_B(S) \setminus \{i\}} & \text{if } i \in R_B \end{cases}$$

\hfill \Box

**Proposition 2.2.** For any bargaining network $B \in \mathcal{B}$ that has a party $i$ who is free-to-bargain but controls no other party, i.e. $i \in F_B$ and $R^i_B(N) = \emptyset$. Let there be another network $\tilde{B}$ that is identical to $B$ except that party $i$ is restricted to bargain with party $\tilde{i}$,
i.e. \( \tilde{B} = B \setminus \cup_{k \neq i} \{ i : k \} \). Then we have

\[
\begin{align*}
Y_i^\tilde{B}(v_S) - Y_i^B(v_S) & \geq 0 \quad \text{for } \tilde{i} = f_B(i) \\
Y_i^\tilde{B}(v_S) - Y_i^B(v_S) & \leq 0 \quad \text{for } i \\
Y_j^\tilde{B}(v_S) - Y_j^B(v_S) & \leq 0 \quad \text{for any } j \in F_B \text{ and } \tilde{j} \neq f_B(i) \\
Y_j^\tilde{B}(v_S) - Y_j^B(v_S) & \leq 0 \quad \text{for any } j \in R_B \text{ and } f_B(j) \neq f_B(i) \\
Y_i^\tilde{B}(v_S) - Y_i^B(v_S) & = 0 \quad \text{for any } i' \in R_B \text{ and } f_B(i') = f_B(i)
\end{align*}
\]

**Lemma 2.7.** Given \( B \) and \( \tilde{B} \) defined in Proposition 2.2, for any \( S \subseteq N \) and \( \tilde{i} = f_B(i) \),

\[
T_{\tilde{B}}(S) = \begin{cases} 
T_B(S) & \text{if } i \notin S \text{ or if } \tilde{i} \in S \\
T_B(S) \setminus \{ i \} & \text{if } i \in S \text{ but } \tilde{i} \notin S
\end{cases}
\]

**Proof.** First of all, for any \( j \neq i, j \in F_B \) if and only if \( j \in F_B \) and \( j \in R_B \) if and only if \( j \in R_B \) with \( f_B(j) = f_B(j) \). So \( j \in T_B(S) \) if and only if \( j \in T_{\tilde{B}}(S) \) for any \( j \neq i \) and any \( S \subseteq N \).

Thus if \( i \notin S \), then \( \forall j \in S, j \in T_B(S) \) if and only if \( j \in T_{\tilde{B}}(S) \). So \( T_{\tilde{B}}(S) = T_B(S) \) if \( i \notin S \).

If \( i \in S \) and \( \tilde{i} \in S \), then \( i \in T_{\tilde{B}}(S) \) if \( i \in S \), and \( i \notin T_{\tilde{B}}(S) \) if \( i \notin S \). Under network \( B \), we also have \( i \in T_B(S) \) if and only if \( i \in S \) because \( i \in F_B \). Thus \( T_{\tilde{B}}(S) = T_B(S) \) if \( i \in S \) and \( \tilde{i} \in S \).

Otherwise if \( i \in S \) and \( \tilde{i} \notin S \), then \( i \in T_B(S) \) because \( i \in F_B \), but \( i \notin T_{\tilde{B}}(S) \) since \( \tilde{i} \notin S \). Yet as is shown, for all other \( j \neq i, j \in T_B(S) \) if and only if \( j \in T_{\tilde{B}}(S) \). So \( T_{\tilde{B}}(S) = T_B(S) \setminus \{ i \} \). \qed

**Proof of Proposition 2.2.**

**Proof.** We will use Lemma 2.7 repeatedly in the following calculations to help us simplify the expressions.

For party \( i \), who becomes restricted to bargain under party \( \tilde{i} \), we have, by Proposition 2.1,

\[
Y_i^\tilde{B}(v_S) - Y_i^B(v_S) = \sum_{S_{\tilde{i}i}} p(S)v_i + \sum_{S_{\tilde{i}i}} p(S)[v_{t_B}(S) - v_{T_B}(S) \setminus \{ i \}] \\
- \sum_{S_{\tilde{i}i}} p(S)[v_{T_B}(S) - v_{T_B}(S) \setminus \{ i \}] \\
= \sum_{S_{\tilde{i}i}} p(S)v_i + \sum_{S_{\tilde{i}i}} p(S)[v_{T_B}(S) - v_{T_B}(S) \setminus \{ i \}] + \sum_{S_{\tilde{i}i}} p(S)[v_{T_B}(S) - v_{T_B}(S) \setminus \{ i \}] \\
= - \sum_{S_{\tilde{i}i}} p(S)[v_{T_B}(S) - v_{T_B}(S) \setminus \{ i \} - v_i].
\]

(2.2.5)
The second step is by Lemma 2.7. So \( Y_i^B(v_S) - Y_i^B(v_S) \leq 0 \) by the assumption that production functions \( v_S \) are superadditive.

For party \( \tilde{i} \), who obtains bargaining control over party \( i \), by definition,

\[
\tilde{R}_B^i(S) = \begin{cases} 
\tilde{R}_B^i(S) \cup \{i\} & \text{if } S \ni i \\
R_B^i(S) & \text{if } S \not\ni i
\end{cases}
\]

Therefore, we have, again by Proposition 2.1,

\[
Y_i^B(v_S) - Y_i^B(v_S) = \sum_{S \ni i} p(S) \left[ v_{TB}(S) - v_{TB}(S) \setminus \{i\} \setminus \tilde{R}_B^i(S) - \sum_{k \in \tilde{R}_B^i(S)} v_k \right] \\
- \sum_{S \not\ni i} p(S) \left[ v_{TB}(S) - v_{TB}(S) \setminus \{i\} \setminus \tilde{R}_B^i(S) - \sum_{k \in \tilde{R}_B^i(S)} v_k \right] \\
= \sum_{S \ni i} p(S) \left[ v_{TB}(S) - v_{TB}(S) \setminus \{i\} \setminus \tilde{R}_B^i(S) - \sum_{k \in \tilde{R}_B^i(S)} v_k \right] \\
+ \sum_{S \not\ni i} p(S) \left[ v_{TB}(S) - v_{TB}(S) \setminus \{i\} \setminus \tilde{R}_B^i(S) \setminus \{i\} - \sum_{k \in \tilde{R}_B^i(S)} v_k - v_i \right] \\
- \sum_{S \not\ni i} p(S) \left[ v_{TB}(S) - v_{TB}(S) \setminus \{i\} \setminus \tilde{R}_B^i(S) - \sum_{k \in \tilde{R}_B^i(S)} v_k \right] \\
= \sum_{S \ni i} p(S) \left[ v_{TB}(S) \setminus \{i\} \setminus \tilde{R}_B^i(S) - v_{TB}(S) \setminus \{i\} \setminus \tilde{R}_B^i(S) \setminus \{i\} - v_i \right]. \quad (2.A.6)
\]

Again, by the assumption that production functions \( v_S \) are superadditive, \( Y_i^B(v_S) - Y_i^B(v_S) \geq 0 \).

For any other free-to-bargain party \( \tilde{j} \in F_B \) who does not gain bargaining control over
party \( i, \tilde{j} \neq f_{\tilde{B}}(i) \), we have \( R^1_{\tilde{B}}(S) = R^1_{\tilde{B}}(S), \forall S \). So we have

\[
Y^B_{\tilde{j}}(v_S) - Y^B_j(v_S) = \sum_{S_{\tilde{j}} \notin S} p(S) \left[ v_{T_{\tilde{B}}(S)} - v_{T_{\tilde{B}}(S) \setminus \{j\}} \right] + \sum_{S_{\tilde{j}} \subseteq S} p(S) \left[ v_{T_{\tilde{B}}(S) \setminus \{j\}} - v_{T_{\tilde{B}}(S) \setminus \{j\}} \right] - \sum_{S_{\tilde{j}} \subseteq S} p(S) \left[ v_{T_{\tilde{B}}(S) \setminus \{j\}} - v_{T_{\tilde{B}}(S) \setminus \{j\}} \right] \]

(2.A.7)

By Assumption 2.1, the production function is convex in participation. Party \( i \)'s marginal contribution is greater in a larger coalition. Thus \( Y^B_{\tilde{j}}(v_S) - Y^B_j(v_S) \leq 0 \).

For any restricted-to-bargain party \( j \) under any party other than \( i \), i.e. \( f_B(j) = f_{\tilde{B}}(j) = \tilde{j} \neq f_{\tilde{B}}(i) = \tilde{i} \), we have

\[
Y^B_{\tilde{j}}(v_S) - Y^B_j(v_S) = \sum_{S_{\tilde{j}} \notin S} p(S) v_j + \sum_{S_{\tilde{j}} \subseteq S} p(S) \left[ v_{T_{\tilde{B}}(S) \setminus \{j\}} \right] - \sum_{S_{\tilde{j}} \subseteq S} p(S) \left[ v_{T_{\tilde{B}}(S) \setminus \{j\}} \right] - \sum_{S_{\tilde{j}} \subseteq S} p(S) \left[ v_{T_{\tilde{B}}(S) \setminus \{j\}} \right] - \sum_{S_{\tilde{j}} \subseteq S} p(S) \left[ v_{T_{\tilde{B}}(S) \setminus \{j\}} \right] \]

(2.A.8)

Again, by Assumption 2.1, \( Y^B_{\tilde{j}}(v_S) - Y^B_j(v_S) \leq 0 \).

For any restricted-to-bargain party \( i' \) under party \( \tilde{i} \), i.e. \( f_B(i') = f_{\tilde{B}}(i') = f_{\tilde{B}}(i) = \tilde{i} \), we
have

\[ Y_{i'}^B (v_S) - Y_{i'}^B (v_S) = \sum_{S \neq S' \neq i} p(S) v_{i'} - \sum_{S \neq S' \neq i} p(S) \left[ v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}} \right] \\
- \sum_{S \neq S' \neq i} p(S) v_{i'} - \sum_{S \neq S' \neq i} p(S) \left[ v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}} \right] \\
= \sum_{S \neq S' \neq i} p(S) \left[ v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}} \right] - \sum_{S \neq S' \neq i} p(S) \left[ v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}} \right] \\
= 0 \quad (2.1) \]

Therefore, for any party \( i' \) who is already under bargaining control of party \( i \), when \( i \) obtains bargaining control over some other party \( i' \), \( i' \)'s bargaining payoff does not change. \( \square \)

**Proposition 2.3.** Suppose party \( i' \) is under bargaining control of party \( i \), then compare two almost identical governance structures, \( g_i \) and \( g_j \), that are otherwise the same, except that asset \( m \) is owned by \( i \) in \( g_i = (A_i, B) \) but owned by \( j \) in \( g_j = (A_j, B) \). Then the bargaining payoffs, thus the first-order investment incentives, for party \( i' \) under \( g_i \) and \( g_j \) are identical if and only if \( (v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}}) - (v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}}) = 0 \) for all \( S \neq i', S \neq i \), \( S \neq j \), where \( v_A^S \) is short for \( v_A(S, e, A) \) given \( e \) fixed.

**Proof.** Given network \( B \) such that \( i' \) is under bargaining control of party \( i \), and party \( j \) is free-to-bargain.

Let’s denote the production functions as \( v_i^S \) and \( v_j^S \) for asset allocations \( A_i \) and \( A_j \), respectively. Then we have for party \( i' \)'s payoff following an asset transfer from \( j \) to \( i \) as

\[ Y_{i'}^B (v_S) = \sum_{S \neq S' \neq i} p(S) v_{i'} \cdot \left[ v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}} \right] - \sum_{S \neq S' \neq i} p(S) v_{i'} \cdot \left[ v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}} \right] \\
= \sum_{S \neq S' \neq i} \left[ (v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}}) - (v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}}) \right] \\
= \sum_{S \neq S' \neq i} \left[ (v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}}) - (v_{T_B(S)} - v_{T_B(S) \setminus \{i'\}}) \right] \\
= 0 \quad (2.2) \]

The last step follows because if both \( i \) and \( j \) are in \( S \), then \( v_i^S = v_j^S \). \( \square \)
2.B Omitted Statements and Proofs for the General n-Party Model

Proposition 2.4. (Insulation Effect) Let $S_j$ be any non-singleton coalition that include $j$ but not $\tilde{j}$. Under any network $\tilde{B}$ such that $j \in R_{\tilde{B}}$, $f_{\tilde{B}}(j) = \tilde{j}$, we have $\frac{\partial Y_{\tilde{B}}}{\partial v_{S_j}} = 0$ for any party $i \in N$. Otherwise under any network $B$ such that $j \in F_B$, we have $\frac{\partial Y_B}{\partial v_{S_j}} \neq 0$ for any party $i \in N$.

Proof. By definition of $T_{\tilde{B}}(S)$, if $j \in R_{\tilde{B}}$, we have $j \notin T_{\tilde{B}}(S)$ for any non-singleton set $S \not\ni \tilde{j}$. In other words, $T_{\tilde{B}}(S)$ cannot be a non-singleton set that includes $j$. So there is no coalition $S$ that has a corresponding $S_j = T_{\tilde{B}}(S)$ that is non-singular, contains $j$ but not $\tilde{j}$. By Proposition 2.1, the bargaining payoff for any party $i$ does not include the term $v_{S_j}$. Thus $\frac{\partial Y_{\tilde{B}}}{\partial v_{S_j}} = 0$ for any $i \in N$.

Instead, if $j \in F_B$, we always have $j \in T_B(S_j)$ as long as $S_j \not\ni j$. Therefore for any $S \ni j$, we have $S_j = T_B(S)$ that is non-singleton, including $j$, and not including some other party $\tilde{j}$. Again, by Proposition 2.1, $v_{S_j} = v_{T_B(S)}$ shows up in the payoff function for party $i$. Furthermore, whenever $i \in S_j$, the weight on $v_{S_j} = v_{T_B(S)}$ is always positive, and otherwise, the weight is negative. Thus $\frac{\partial Y_B}{\partial v_{S_j}} \neq 0$. 

Proposition 2.5. (Concentration Effect) $\left| \frac{\partial Y_B}{\partial v_j} \right| > \left| \frac{\partial Y_{\tilde{B}}}{\partial v_j} \right|$ for any party $i$ such that $f_B(i) \neq \tilde{j}$. Moreover, let $S_{-j}$ be any coalition such that $S \ni j, S \not\ni \tilde{j}$. Then we have $\left| \frac{\partial Y_B}{\partial v_{S_{-j}}} \right| > \left| \frac{\partial Y_{\tilde{B}}}{\partial v_{S_{-j}}} \right|$ for any party $i$ such that $f_B(i) \neq \tilde{j}$.

Proof. The result follows directly taking derivatives from equations (2.A.5) to (2.A.9) with respect to $v_i$ and $v_S$ for $S \ni i, S \ni \tilde{i}$. 

Proposition 2.6. The shift of asset ownership can have different effects on payoffs under different bargaining networks.

Proof. Using operation $\Delta_{N-I}$, we can apply the same operations to equations (2.A.5) to (2.A.9), then a similar result to Proposition 1.2 follows. 

Proposition 2.7. Under any governance structure $g = (A, B)$, there is always under-investment. That is $e_{i}^{A,B} < e_{i}^{F_B}$ for any $i \in N$.

Proof. The first-best level of investment is characterized by $\frac{\partial v_N}{\partial e_i} = \Psi'_i(e_i)$. And the second-best investments are characterized by $\frac{\partial Y^g_{i}(e_A)}{\partial e_i} = \Psi'_i(e_i)$.
By definition of Myerson value

\[ Y^B_i = \sum_{S \ni i} p(S) \{ v^B_S - v^B_{S \setminus \{i\}} \} \]

\[ < \sum_{S \ni i} p(S) v^B_S \]

\[ < \sum_{S \ni i} p(S) v_S \]

\[ < \sum_{S \ni i} p(S) v_N, \]

where the last inequality holds by Assumption 1.1.

Thus \( \frac{\partial Y^A_e(e_i)}{\partial e_i} < \frac{\partial v_N}{\partial e_i} \), which implies that the second-best investment is strictly less than the first-best level. \( \square \)

**Proposition 2.8.** If there is no CSM, and every parties’ investments are SSM with respect to all coalitions \( S \subseteq N \), then it is never efficient to have bargaining control rights.

**Proof.** Suppose in network \( B_K \), there are \( K \) parties who are restricted to bargain. We can compare the network \( B_K \) with a similar network, \( B_{K-1} \), that is otherwise identical, but with only \( K-1 \) parties restricted to bargain. Without loss of generality, label this party as \( i \), then the payoff comparisons between these two networks for any party \( k, Y^B_k(v_S) - Y^B_{K-1}(v_S) \), are given by equations (2.A.5) to (2.A.9), depending on the bargaining rights of each party.

If there is no cross-investment superadditivity at the margin, it can be readily verified from equations (2.A.5) and (2.A.6) that \( \frac{\partial Y^B_k(v_S)}{\partial e_k} - \frac{\partial Y^B_{K-1}(v_S)}{\partial e_k} < 0 \) for party \( k = i \) and party \( k = \tilde{i} \).

We can rewrite equation (2.A.7) as

\[ Y^B_j(v_S) - Y^B_{\tilde{j}}(v_S) = \sum_{S \ni j, S \ni \tilde{j}} p(S) \left[ (v_T(S) - v_T(S \setminus \{i\})) - (v_T(S \setminus \{\tilde{j}\}) \setminus R^j_T(S) - v_T(S \setminus \{\tilde{j}\}) \setminus R^j_T(S \setminus \{i\}) \right] \]

\[ = \sum_{S \ni j, S \ni \tilde{j}} p(S) \left[ (v_T(S) - v_T(S \setminus \{i\}) - v_i) - (v_T(S \setminus \{\tilde{j}\}) \setminus R^j_T(S) - v_T(S \setminus \{\tilde{j}\}) \setminus R^j_T(S \setminus \{i\}) - v_i \right] \]

By self-investment superadditivity at the margin, the partial derivative of the first term in bracket with respect to \( e_j \) is positive. And since there is no cross-investment superadditivity at the margin, the the partial derivative of the second term in bracket with respect to \( e_j \) is negative. So overall, \( \frac{\partial Y^B_k(v_S)}{\partial e_k} - \frac{\partial Y^B_k(v_S)}{\partial e_k} < 0 \) for all free-to-bargain parties \( \tilde{j} \neq j \). Same logic applies to equation (2.A.8) and so the result also follows for all \( k \) such that \( f_{B_{K-1}} \neq \tilde{i} \).

By equation (2.A.9), \( \frac{\partial Y^B_k(v_S)}{\partial e_k} - \frac{\partial Y^B_k(v_S)}{\partial e_k} = 0 \) for all \( k \) such that \( f_{B_{K-1}} = \tilde{i} \).

Therefore, we have \( e^B_i < e^B_{K-1} \). Thus given asset allocation \( A \), bargaining network \( B_K \) is strictly less efficient than \( B_{K-1} \).
We can then repeat the same logic and iterate all the way through $K = 1$ and compare it with the complete bargaining network $B_c$. As a consequence, $B_c$ is strictly more efficient than any incomplete bargaining network.

\[ \square \]

**Corollary 2.2.** If there is no cross-investment, then under Assumption 1.1, it is never efficient to have bargaining control rights.

**Proof.** If there is no cross-investment, there cannot be cross-investment superadditivity at the margin. Then the result follows from Proposition 2.8.

\[ \square \]
Chapter 3

Subsidiaries vs. Divisions in a Property-Rights Theory of the Firm

3.1 Introduction

Subsidiaries and unincorporated divisions are alternative governance structures for large corporations and business groups to organize their internal business units. Both forms are widely observed, and one form does not seem to dominate another. Why is one form used as oppose to another is both a curious case to be solved for researchers and an important issue to be understood for strategic and policy purposes.

Subsidiaries are ubiquitous in modern corporations. Squire (2011) documents that “in 2010, the one hundred US public companies with the highest annual revenues reported an average of 245 major subsidiaries, with 114 as the median. Only five reported fewer than five major subsidiaries”. Alfaro and Charlton (2009) observe in
their data that, as of 1999, “2,248 entities that report to General Motors Corporation as their ‘global ultimate parent.’

Divisions are also widely adopted as the governance structure for business units. Hewlett-Packard has 58 divisions in 1992 versus 40 in 1980 (Beer and Rogers, 1995). The animation business in Disney is governed as a division, and Pixar was incorporated from the computer division from Lucasfilm (Alcacer et al., 2010).

The legal distinctions between the subsidiaries and the divisions are relatively clear. From the legal perspective, subsidiaries are distinct and independent legal entities from their parent firms. Whereas unincorporated divisions are integrated parts within the large firm’s corporate shell.¹

From the economics perspective, the differences between these two seemingly similar organizational forms are much more subtle and formal economic theory provides little guidance on the tradeoff between these two alternatives. In fact, given the fact that the parent firm has control over the business unit, the classical theories of the firm do not even capture the differences in the legal ownership of assets under subsidiaries and divisions. Under the scope of classical property-rights theory of the firm by Grossman and Hart (1986); Hart and Moore (1990); Hart (1995) (GHM), the owner of the business, either the parent firm of a subsidiary or the headquarter of a division, has residual rights of control over the assets to which the business unit

¹The classical literature on corporate law mainly emphasizes a credit monitoring theory (Posner, 1976; Hansmann and Kraakman, 2000; Hansmann et al., 2006). This theory argues that, because of the separate legal entity status, subsidiaries are protected from limited liabilities and entities shielding. Thus the creditors of a business unit is much easier to assess the financial conditions if the business unit were a subsidiary rather than a division. However, we choose not to emphasize this particular theory for two reasons. First, Squire (2011) points out that typical large corporations extensively uses debt guarantees to override liability barriers provided by the corporate shell. Second, the credit monitoring theory one-sidedly favors subsidiaries and therefore fail to explain the co-existence of divisions with subsidiaries.
has access. Therefore, although the legal ownership of assets differ for subsidiaries and divisions, in the GHM framework, subsidiaries and divisions are identical forms of organizations, whose parent or headquarter controls the (economic) ownership of assets that are used for production.

The current paper provides a formal economic theory that studies the subsidiaries-versus-divisions distinction from a different perspective—the transferability of business units. Instead of emphasizing the differences in the legal ownership of the assets, we build the model based on the legal foundation that, for its owner, a business is easier to be transferred as a subsidiary than as a division (Ayotte and Hansmann, 2012). We focus on studying the implications of this particular legal implication on the economic outcomes. The model utilizes a modified GHM framework without assets and focus on the inefficiencies in ex ante investment.

Although the model is built on the idea that subsidiaries are easier to be transferred, the main results are contrary to the modeling presumption that subsidiaries are setup to be sold. When the original owner’s investment is the primary concern for efficiency purposes, the model predicts that setting up a business as a subsidiary is more efficient than making it a division when (i) the chance of the business being taken over is low; (ii) investments by the owner of the business unit are non-specific with respect to the owner’s identity of the business unit. These two conditions are likely to hold when the business unit is a generic one. And the analysis shows that using division is more efficient if (i) the chance of the business being taken over is high;

\footnote{Ayotte and Hansmann (2012): “Theories of this sort (property rights theory of the firm)... have nothing to say, however, about why a corporation would create wholly-owned subsidiary corporations. Indeed, for the purposes of these theories there is no difference between, on the one hand, a single corporation with its various businesses operated as divisions and, on the other hand, the same businesses operated as separately incorporated but wholly-owned subsidiaries of the parent. In either case, the parent corporation controls all of the assets, so these configurations are equivalent from the perspective of the property rights theory.”}
(ii) investments by the owner of the business unit are specific. These two conditions are likely to hold when the business unit is a distinct one.

The paper is organized as the following. Section 3.2 explains the legal foundations regarding the distinct transferability of subsidiaries and divisions. Section 3.3 introduces the setup of the formal model. Section 3.4 analyzes the model and provide predictions of the model. Section 3.5 discusses the results and concludes.

### 3.2 Discussion of Modeling Assumptions

#### 3.2.1 Legal Foundations

Ayotte and Hansmann (2011, 2012) proposes a theory of legal entities as a bundle of reassignable contracts. This model relies on the concept of this theory to model divisions and subsidiaries as different organizational forms in a transaction.

Ayotte and Hansmann (2012) provides the following two facts. On one hand: “The Law of Bundled Assignability The law of contracts provides that a party’s rights and obligations under a contract may or may not be transferable (or, as we will somewhat loosely say, assignable) to a third party without the permission of the other party to the contract, depending on the subject of the contract and the circumstances.”

On the other hand: “If a business corporation (or other legal entity) is a party to a contract, a transfer of some or even all of the ownership shares in the corporation is not considered an assignment of the contract. The corporation, and not its shareholders, is considered to be a party to the contracts, and the corporation maintains its identity as its ownership changes. This rule is generally interpreted quite broadly.”

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^3Ayotte and Hansmann (2012): “If the counterparty to a contract with a corporation wishes
Ayotte and Hansmann (2012, 2011) highlights the tension between liquidity needs and opportunistic assignment of contracts. On one hand, the owner of a business (O) has liquidity needs, she might want to sell the business _ex post_. Suppose the business relies on one supplier (S) and one leasing company (L) who provide the space for the business. Were the contracts with S and L non-assignable, upon the time of takeover, O is likely to be held up by S and L. Expecting this, O is likely to make inefficiently low _ex ante_ investments, which result in inefficiency. On the other hand, should these two contracts individually assignable by O, S and L may be discouraged to make inefficiently low _ex ante_ relationship-specific investments.

To release this tension, Ayotte and Hansmann (2012, 2011) argue that contracts should be individually non-assignable, but assignable with the transfer of the ownership of the legal entity.

Notice, however, that in our example, S will not be discouraged to make relationship-specific investments as long as her individual contract can only be reassigned with the contract of L, as well as possibly some particular assets of O. In this case, it might be possible to achieve this “bundled assignability” without relying on the legal entity.

However, Ayotte and Hansmann (2012) argues that using the legal entity is likely to be the most simple and efficient way. Because “entities make it possible to identify a constantly changing bundle of assets and contracts”. If the assignability is achieved “without an entity by way of a general description of the bundle, ...in the event that the owner wishes to separate” the business “from another business she owns, or from her personal assets...(for example, because of shared facilities or overhead). The suppliers might argue, for example, that the definition of the bundle being sold is to limit the persons to whom ownership or control of the corporation can be sold, they must do this through specific language to that effect in the contract (a ‘change of control’ clause); a non-assignment clause will not suffice.”
under-inclusive.” This contractual incompleteness regarding the assignability again creates hold-up opportunities that deter *ex ante* investments.4

As a caveat, they also point out that, from the empirical point of view, (i) not all contracts are individually non-reassignable;5 (ii) a small proportion of contracts are not reassignable even with the transfer of the ownership of the legal entity; and (iii) although creating legal entity is likely to be the most convenient way, creating the legal entity is not the only way to create transferability for an individually non-assignable contract.

Using a sample of “287 lease and supply agreements from the SEC Edgar database between 2007 and 2009, filed as a ‘Material Contract’ (Exhibit 10)”, Ayotte and Hansmann (2012) shows that 95.5% of the contracts “explicitly impose restrictions on assignment of the contract on an individual basis”.

These 287 contracts can be divided into four groups. (i) 14.3% explicitly forbid transfer of the contract even with the transfer of the legal entity.6 (ii) 40.1% explicitly allow for transfer of the contracts *with and only with* the legal entity.7 (iii) 23% permit

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4Ayotte and Hansmann (2012): “One way is for the owner to sign personal contracts that are nonassignable, but that enumerate, in each contract, the other components of the bundle with which the counterparty wishes to be tied, allowing for assignment to occur only with all the other contracts in the defined bundle. For a business of any measurable complexity, however, this would be extremely costly and unreliable”. “Boeing uses 700 different suppliers to create one of its airplanes. Attempting to identify and bundle each of the 700 supply contracts with the 699 other contracts would be messy, labor-intensive, and potentially fraught with error and ambiguities in identification. Moreover, because suppliers change over time, each contract would need to anticipate these future contracts and identify them in some way before they come into existence. In short, this is unlikely to be a practical solution in most realistic cases.”

5Ayotte and Hansmann (2012): “The rights of a promisee under a simple contract for payment of a definite sum of money are, as a default rule of contract law, generally presumed assignable. In contrast, the rights of an employer to receive labor services from an employee are generally presumed non-assignable.”

6Ayotte and Hansmann (2012) Table 2.

7Ayotte and Hansmann (2012) Table 2 and Table 3 combined, 40.1% of entire sample is *entity bundles only* (Table 3: 63.5% in the 36.1%, which yields 40.1%), which means “assignment is permitted in the event of a merger, acquisition, or a sale of all or substantially all of the assets of the debtor party to the contract.”
the transfer of the contracts as a “bundle” that does not necessarily include the legal entity (among these 23%, 83.3% also refers to the legal entity, but not restricting only to the legal entity).^8 (iv) 22.6% of the sample do not explicitly forbid transfer of the contracts as a bundle.

### 3.2.2 Modeling Assumptions

In this framework, the parent firm of a subsidiary may transfer all of the contractual relations of the subsidiary as well as the assets under its legal ownership as a bundle to an alternative holding company without consent from all these related parties under contracts with the subsidiary. This is possible, because the transfer of all these contracts and assets follows from the transfer of legal ownership of the subsidiary from one parent firm to another. The transfer of the ownership only involves bargaining between the two parent firms.

In stark contrast, when the parent firm of a division wants to transfer its “ownership” of the division to another firm, it may not generally be able to reassign the contracts due to their individual non-assignability. For example, labor contracts are generally presumed nonassignable by the employer (Ayotte and Hansmann, 2011). Following this idea, in this model, upon the transfer of the “ownership” of this division, we assume that all of the contractually related parties of the division will participate in the bargaining over ownership transfer in addition to the current and future parents.

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^8 Ayotte and Hansmann (2012) Table 2 and Table 3 combined, 23% of the sample permit assignability as a non-entity bundle (Table 3: 36.5% in the 36.1%, which yields 23%). “A bundle is defined as a non-entity bundle if assignment is permitted with (a) includes specific asset(s) or contract(s) or uses a general definition without specifically reference the debtor entity, such as ‘business’ or ‘segment’.”
My paper loosely relates to Hellmann and Thiele (2012) because both papers consider governance structures that provide freedom to unilaterally dissolve a contract \textit{ex post}. But we differ both in the governance structure of interest and the emphasized source of inefficiency. Instead of studying subsidiaries and divisions, Hellmann and Thiele (2012) studies the tradeoff between integrated joint ownership and non-integration. In their model, under integration, both partners need to agree to dissolve the integrated firm in order for one of the partners to depart and pursue better opportunities. But under non-integration, each party is free to unilaterally terminate the cooperation if better opportunities present themselves. Regarding the model, the current paper differs from Hellmann and Thiele (2012) in the emphasized source of inefficiency. The current model utilizes the framework of property rights approach. I assume efficient \textit{ex post} renegotiation and analyzes the inefficiency in \textit{ex ante} investments. So the cooperation will never be inefficiently dissolved. But Hellmann and Thiele (2012) poses a \textit{ex post} wealth constraint and therefore rules out perfect renegotiations. Their core result relies on the feature that parties will inefficiently dissolve the cooperation.

\subsection*{3.3 Model Setup}

\textbf{Governance Structure}

There is a transaction that involves three entities, say, a major airline (M), a regional airline as a business unit under control of M and a airline food provider who is an independent firm (F) that supplies to the regional. We assume the food supplier is an independent firm. And we suppose the regional may be a subsidiary or a division
of the major. The governance structure is thus $g = \{D, S\}$.

**Investment and Production**

We assume only M makes two-dimensional *ex ante* non-contractible investments $e = (e_1, e_2)$ at strictly increasing and convex private cost $\Psi(e)$. And we assume the production functions $v_S$ for any coalition $S$ are increasing and weakly concave in the investments.

Let $v_S^i(e)$ denote the partial derivative of function $v_S(e)$ with respect to $i \in \{1, 2\}$, i.e. $v_S^i(e) = \frac{\partial v_S(e)}{\partial e_i}$. We make the following assumptions on the technology.

**Assumption 3.1.** $v_{MF}^1(e) > v_{MF}'(e); v_{MF}^2(e) < v_{MF}'(e); \frac{\partial v_S^i(e)}{\partial e_1, \partial e_2} = 0$ for any $S$; and $\frac{\partial \Psi^2(e)}{\partial e_1, \partial e_2} = 0$.

Intuitively, Assumption 3.1 boils down to saying that $e_1$ and $e_2$ are different types of investments. We will refer to $e_1$ as the “(incumbent) owner-specific investment” because of its higher marginal benefit on the “inside option”, $v_{MF}$, than that on the value of output after takeover, $v_{MF}'$. In other words, $e_1$ is more useful when the incumbent owner is part of the production. We refer to $e_2$ as the “cooperative investment” because $e_2$ creates more benefit for other parties than for the investor herself.\(^9\) The last part assumes the cross-partials between the two investments to be zero. This part can be called the “technological independence” assumption, which is a simplifying assumption that allows us to treat the first order conditions with respect to these two investments separately.

\(^9\)For a detailed discussion on cooperative investments, see Che and Hausch (1999).
ex post Bargaining: Interim Takeover Bargaining

At time $\tau = 1.5$, an alternative major airline $M'$ shows up with the interest to take over the business unit from $M$.

The $\tau = 2$ interim takeover bargaining depends on the governance structure of the business. If the business is a subsidiary of $M$, the takeover bargaining only involves negotiations between $M$ and $M'$. This is the case because all the business contracts related to the business unit, say the contract with $F$, is signed with the business unit as a legal entity. The transfer of the ownership of this entity from $M$ to $M'$ does not in itself constitute breach of the non-assignability of the contract.

If the business is a division, however, the takeover bargaining involves $M$ and $M'$, plus all parties that has related contracts with the to-be-takeover business. In our simplest case, it is a three-way bargaining involving $M$, $M'$ and $F$. Table 3.1 summarizes these relationships as networks.

![Diagram of contractual networks and interim takeover bargaining networks](image)

**Table 3.1:** Contractual Networks and Interim Takeover Bargaining Networks
ex post Bargaining: \( \tau = 3 \)

There are four sub-bargaining-games we need to consider at \( \tau = 3 \). For simplicity, we assume the following: (a) each of these three parties always has an individual outside option of 0; (b) equal bargaining power for all parties; and (c) the bargaining solutions are determined by Shapley value.

(i) No takeover: If no \( M' \) shows up (with probability \( 1 - \lambda \)), there is no bargaining over the takeover decision. The game proceeds as \( M \) owning the business. The contract between \( M \) and \( F \) is still effective. \( M \) and \( F \) bargain over the final value of production, \( v_{MF} \), at \( \tau = 3 \). This bargaining is the same regardless of whether the business is a subsidiary or a division.

If \( M' \) shows up (with probability \( \lambda \)), there are several sub-cases depending on the decisions made at the takeover bargaining at \( \tau = 2 \).

(ii) Unsuccessful takeover: If any one of \( M \) and \( M' \) disagrees to the takeover deal, \( M \) and \( F \) continues to deal and produce \( v_{MF} \), as before because \( M \) still have control over the business. What’s different from case (i) is that \( M' \) has shown up. We assume that although \( M' \) failed in taking over the business, \( M' \) and \( F \) may still find it profitable to reach a contract between themselves without the business. In this case, \( M' \) and \( F \) also bargains over a final value of production without the business, \( v_{nM'F} \), with superscript \( n \) stands for no takeover.

If both \( M \) and \( M' \) agrees to the takeover deal, the ownership of the business will be turned over to \( M' \). But the relationship between \( F \) and the business may or may not be transferred with the ownership of the business, depending on what the governance structure is and whether \( F \) agrees to the reassignment of contract.

(iii) Successful takeover without reassigning contract with \( F \). This case never hap-
pens under \( g = S \). If M and M’ agree to the takeover deal, but \( g = D \) and F rejects to reassign the contract from M to M’, then the business is turn over to M’, but M and F still maintains the original contract, operating without the business. We assume M’ and F may reach a separate contract through renegotiation for F to serve the business, which is now under control of M’, while still working with M. Similar to the previous case, we assume M and F creates no extra surplus dealing with each other after M’ takes over the business. But M’ and F can still reach a side agreement through renegotiation to produce a value of \( v_{M'F}^r \) where superscript \( r \) stands for no reassignment. We assume that \( v_{M'F}^r \) is only a fraction of \( v_{MF} \) due to the fact that F is still bonded by its contract with M.

(iv) Successful takeover with reassigning contract with F. If the business is set up as a subsidiary \( (g = S) \), the contract between F and the business is automatically transferred after M’ takes control. F has no decision on this matter. If, instead, the business is set up as a division \( (g = D) \), then the contract between F and the owner of the business, M, will only be transferred under F’s consent. In this case, M’ and F bargains over the value of the final product \( v_{M'F} \). We assume that M and F will not generate any value by keeping their contract once M lost the business, i.e. they can only jointly produce a value that is equal to the sum of their respective outside options. Otherwise, the reassignment would be unnecessary.\(^\text{10}\)

It is convenient to relate the partial value \( v_{M'F}^r \) to the full value \( v_{MF} \). Recall that \( v_{M'F}^r \) is the value M’ and F can produce with the business while F is still supplying to M. And \( v_{MF} \) is the full value they could produce with the business and without \(^\text{10}\)A relaxation of this assumption can be modeled with straightforward extension, which allows for M and F to produce a fraction of what they were able to produce together when the business is still under control of M. This extension, however, does not change our main qualitative results. See the following Section for details.
the burden of contract with M. $v_{M'F}^r$ should be bounded above by $v_{M'F}$, and bounded below by the sum of their individual outside options without cooperation, which is assumed to be 0. Thus we introduce an exogenous variable $\rho \in [0,1]$ and define $v_{M'F}^r = \rho v_{M'F}$. $\rho$ can be interpreted as the remainder in value of production that can be obtained by M' and F due to the burden of non-reassigned contract. This setting accounts for the cases that, for example, F may run into a binding capacity constraint when it serves both M and M'.\footnote{For example, with the non-reassigned contract between M and F, F still has obligations to supply the intermediate product to M, and M still has the contractual obligation to purchase the product. If F has exhausted its capacity to the production of supply for M, F would not be able to commit to supplying the business unit, which is under control of M'.} $\rho = 1$ corresponds to a case in which F faces no capacity constraint, or in which F simply do not deliver anything to M anymore. Whereas $\rho = 0$ corresponds to a case where F is completely lock-in by the contract with M and cannot renegotiate with M'.

Similarly, We also relate the partial value $v_{M'F}^n$ to the full value of production $v_{M'F}$. Recall that $v_{M'F}^n$ is the value M' and F can produce without the business while F is still supplying to M. So it is intuitive to assume $v_{M'F}^n$ to be bounded above by $v_{M'F}^r$, and bounded below by the sum of their individual outside options without cooperation, which is assumed to be 0. We introduce another exogenous variable $\eta$ and define $v_{M'F}^n = (\rho - \eta)v_{M'F}$, such that $\eta \in [0,1]$ and $\rho - \eta > 0$. Because $\eta$ is the further reduced share of production value from $v_{M'F}^r$, $\eta$ can be interpreted as the essentialness of the business for the value creation involving M' and F given that F is still supplying M.
ex post Takeover Bargaining: $\tau = 2$

We assume that $v_{MF}'$ is sufficiently high relative to $v_{MF}$ such that $v_{MF}' > v_{MF} + v_{MF}'$, so that M and M’ will always agreed on the takeover deal.\textsuperscript{12} Given that $\rho > 0$, we always have $v_{MF}' > v_{MF}'$, so given M and M’ agrees on the takeover, it is always optimal for F to agree to reassign the contract in renegotiation.

The $\tau = 2$ takeover bargaining bargains over each parties’ decision over whether agreeing to the takeover deal or whether endorse the reassignment of the contract in exchange for a payment. In our setup, once M’ shows up, it is always optimal for all three parties to agree to the takeover as well as reassignment of the supplying contract. To put it another way, once M’ shows up, the only subgame on the equilibrium path is the case (iv) defined above (successful takeover with reassigned contract). Case (ii) and (iii), however, serve as party M and F’s outside options.

Timing

The timing of the stage game is summarized in the following graph.

\textsuperscript{12}This assumption is not substantive because what matters for the analysis is the marginal product.
3.4 The Analysis

3.4.1 Bargaining Payoffs at $\tau = 3$

Case (i): No $M'$ Showing up

There is no take over if $M'$ does not show up for takeover. The governance structures $S$ and $D$ yield the same bargaining payoffs at $\tau = 3$. The bargaining payoff for $M$ without takeover, $Y_M^n$, is

$$Y_M^n = \frac{1}{2} [v_M + v_M - v_F] = \frac{1}{2} v_{MF}.$$
Case (ii): No Takeover (n)

If M' shows up, but either M or M' rejects the takeover, the governance structures S and D yield the same bargaining payoffs at $\tau = 3$. The bargaining payoff for M without takeover, is also $Y^n_M$ as if no M' shows up.

The bargaining payoff for M' without takeover, $Y^n_{M'}$, is

$$Y^n_{M'} = \frac{1}{2} (\rho^M F + \nu_{M'} - \nu F) = \frac{1}{2} (\rho - \eta) v_{M' F}.$$

The bargaining payoff for F without takeover, $Y^n_F$, is

$$Y^n_F = \frac{1}{2} (\nu_{M' F} + \nu_F - \nu_{M'}) + \frac{1}{2} (\nu_{M F} + \nu_F - \nu_M) = \frac{1}{2} (\rho - \eta) v_{M' F} + \frac{1}{2} v_{M F}.$$

Case (iii): Successful Takeover without Reassignment of Contract (r)

The three parties wind up in this case if M' shows up, both M and M' agree to the takeover, but $g = D$ and F rejects the reassignment of the contract. The bargaining payoff for M after the business is taken by M', is $Y^r_M = 0$.

The bargaining payoff for M' in this case, $Y^r_{M'}$, is

$$Y^r_{M'} = \frac{1}{2} (\nu_{M' F} + \nu_{M'} - \nu_F) = \frac{1}{2} \rho v_{M' F}.$$

The bargaining payoff for F without reassigning the contract, $Y^r_F$, is

$$Y^r_F = \frac{1}{2} (\nu_{M' F} + \nu_F - \nu_{M'}) = \frac{1}{2} \rho v_{M' F}.$$
Case (iv): Successful Takeover without Reassignment of Contract (t)

If both M and M' agrees under $g = S$ or all three of M, M' and F agrees under $g = D$, the takeover is successful with reassigned contract. The bargaining payoff for M after the business is taken by M', is $Y^t_M = 0$.

The bargaining payoff for M' in this case, $Y^t_{M'}$, is

$$Y^t_{M'} = \frac{1}{2} \left[ v_{M'F} + v_{M'} - v_F \right] = \frac{1}{2} v_{M'F}.\quad (3.4.1)$$

The bargaining payoff for F without reassigning the contract, $Y^t_F$, is

$$Y^t_F = \frac{1}{2} \left[ v_{rM'F} + v_F - v_{M'} \right] = \frac{1}{2} v_{M'F}.\quad (3.4.1)$$

3.4.2 Takeover Bargaining Payoffs at $\tau = 2$

Takeover bargaining under $g = S$

Under $g = S$, the takeover bargaining involves only M and M'. M demands a payment from M' that is no less than what she could have made rejecting the takeover offer. In the takeover bargaining, the an outside option of M is $Y^n_M$. However, M' is unwilling to offer anything more than $Y^t_{M'}$, which is the entire payoff that $M'$ could earn after the takeover. So M’s bargaining payoff in the takeover stage, $Y^{S,t}_M$, is

$$Y^{S,t}_M = \frac{1}{2} Y^n_M + \frac{1}{2} \left[ Y^t_{M'} - Y^n_{M'} \right] = \frac{1}{4} \left[ v_{MF} + (1 + \eta - \rho) v_{M'F} \right].$$

13If $Y^n_M > Y^t_{M'}$, the agreement cannot be reached because there is no gains from trade, and the game goes to the no-takeover case. Thus $\lambda$ can be interpreted as the probability that the project value $v_{MF}$ falls below the market level.
Takeover bargaining under \( g = D \)

Under \( g = D \), M and \( M' \) jointly determine whether the takeover deal is successful, while F determines whether the contract will be reassigned. By Shapley value, the payoff for M, \( Y^{D,t}_M \) is

\[
Y^{D,t}_M = \frac{1}{3} Y^n_M + \frac{1}{6} \left[ Y^n_M + Y^n_F - Y^n_F \right] + \frac{1}{6} \left[ Y^t_M + Y^t_M' - Y^n_M \right] + \frac{1}{3} \left[ Y^t_M + Y^t_M' + Y^t_F - Y^n_M' - Y^n_F \right]
\]

\[
= \frac{1}{6} \left[ v_{MF} + (4 + 5\eta - 4\rho)v_{M'F} \right].
\]

Comparing the bargaining solutions under the two governance structures provides the intuition regarding the different implications of takeover bargaining under \( g = D \) and \( g = S \). Checking the interim steps in equations (3.4.1) and (3.4.2), the sum of the first two terms in equation (3.4.2) yields the same result as the first term in equation (3.4.1), \( \frac{1}{2} Y^n_M \). The difference comes from the third and fourth terms in equation (3.4.2).

First, M obtains a lower bargaining payoff under \( g = D \) in the takeover stage because she has less control rights over the assignability of the contract with F. Under \( g = S \), M can unilaterally decide to transfer both the business unit and the associated contract with F without consent from F. But under \( g = D \), M faces the threat from F that he might not agree to reassign the contract, which lowers the value of the business unit after the takeover. This makes M’s decision regarding the transfer of the business unit less valuable under \( g = D \). This intuition is reflected by the third term in equation (3.4.2), \( Y^r_M + Y^r_M' - Y^n_M' \), which is the value of the business unit without the original contract with F. This value is presumably smaller than the value when the business unit is transferred with the the contract with F by default,
as in \( g = S \), which is \( Y_{M'}^{t} - Y_{M'}^{n} \).

Second, given our setup that \( F \) does not participate in the takeover bargaining under \( g = S \), \( M \) has a different advantage under \( g = D \) comparing to \( g = S \). That is, under \( g = D \), \( M \) can potentially appropriate more rents during the takeover bargaining from \( F \) by threatening to reject the takeover deal. This threat can potentially extract a share of future rents that \( F \) can obtain after the takeover, \( Y_{F}^{t} - Y_{F}^{n} \). This intuition is captured by comparing the second term in equation (3.4.1), \( Y_{M'}^{t} - Y_{M'}^{n} \), with the fourth term in equation (3.4.2), which equals to \( Y_{M'}^{t} - Y_{M'}^{n} + Y_{F}^{t} - Y_{F}^{n} \) because \( Y_{M}^{t} = 0 \).

### 3.4.3 Expected Payoff Comparison at \( \tau = 1 \): Subsidiaries vs. Divisions

The expected payoffs at \( \tau = 1 \) for \( M \) under \( g = S \) and \( g = D \) are

\[
Y_{M}^{S} = (1 - \lambda)Y_{M}^{n} + \lambda(Y_{M}^{S,t} + Y_{M}^{t}),
\]
\[
Y_{M}^{D} = (1 - \lambda)Y_{M}^{n} + \lambda(Y_{M}^{D,t} + Y_{M}^{t}).
\]

Subtracting \( Y_{M}^{S} \) from \( Y_{M}^{D} \), and substituting the bargaining payoffs with our previous characterizations, we have

\[
Y_{M}^{S} - Y_{M}^{D} = \frac{12}{12} \lambda \left[ 2v_{MF} - (1 + 2\eta - \rho)w_{M'F} \right]. \tag{3.4.3}
\]

This result shows that, comparing to \( g = D \), \( M \) always extracts more rents under

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14Under \( g = S \), \( M \) does not have this advantage because we assume that \( F \) does not participate in the takeover bargaining. One might argue that even under \( g = S \), \( M \) can still potentially reach out to \( F \) and threaten in the same way to extract rents. This would eliminate \( M \)'s advantage under \( g = D \) relative to \( g = S \). However, this alternative setting does not change our qualitative results.
$g = S$ in the takeover bargaining from her “insider option”, $v_{MF}$, that she produces with F. Moreover, comparing to $g = D$, M always extracts less rents under $g = S$ from the value produced after the business is taken over, $v_{MF}$.$^{15}$

### 3.4.4 Investment Levels

**Investments at $\tau = 1$: First-best Benchmark**

The total surplus is $s(e) = (1 - \lambda)v_{MF}(e) + \lambda v_{MF}'(e) - \Psi(e)$, so the first-best level of investment is defined by

$$(1 - \lambda)v_{MF}'(e^{FB}) + \lambda v_{MF}'(e^{FB}) = \Psi(e^{FB}), \tag{3.4.4}$$

where $v'(\cdot)$ denotes the partial derivative of the function $v$ with respect to its argument—investment $e$.

**Investments at $\tau = 1$: $g = S$ vs. $g = D$**

In equilibrium, M’s ex ante investment levels under governance structure $g = e^S$ and $e^D$ for $g = S, D$ respectively—are characterized by

$$Y_{g}^{M}(e^{g}) = \Psi'(e^{g}). \tag{3.4.5}$$

We can derive the difference in M’s marginal benefit of investment under the two

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$^{15}$Equation (3.4.3) shows that only if $\rho = 1$ and $\eta = 0$, M’ obtains the same share of output from $v_{MF}$. But given the assumption that $\rho < \eta$, this case never holds.
governance structures from Equation (3.4.3). That is

\[
Y^S_M(e) - Y^D_M(e) = \frac{1}{12} \lambda (2v_{MF}'(e) - (1 + 2\eta - \rho)v_{MF}'(e)).
\] (3.4.6)

**Proposition 3.1.** (i) \(e_1\) is always higher under subsidiary than under division, i.e. \(e_1^S > e_1^D\). (ii) \(e_2\) is higher under subsidiary than under division if \(e_2\) is sufficiently non-specific to the future value of output after takeover. Specifically, there exists a cutoff value \(\omega = \frac{1}{2} + \frac{\lambda}{2} < 1\), such that \(e_2\) is higher under subsidiary than under division if \(v_{MF}' > \omega\), and vice versa. That is \(e_2^S > e_2^D\) if \(\frac{v_{MF}^2}{v_{MF}'} > \omega\); and \(e_2^S < e_2^D\) if \(\frac{v_{MF}^2}{v_{MF}'} < \omega\).

**Proof.** From equation (3.4.6), we can write

\[
(Y^S_M(e) - \Psi(e)) - (Y^D_M(e) - \Psi(e)) = \frac{1}{12} \lambda (2v_{MF}'(e) - (1 + 2\eta - \rho)v_{MF}'(e)).
\] (3.4.7)

Notice that the two terms on the left-hand side are the respective first-order conditions under S and D. And given that the production functions \(v\) are concave in investments and \(\Psi\) is convex, the result follows by Assumption 3.1.

**Efficiency:** \(g = S\) vs. \(g = D\)

**Definition.** We say that there is under-investment (over-investment) in \(e_i\) under a governance structure \(g \in \{S, D\}\) if the equilibrium investment level, \(e^g\), is below (above) the first-best level, \(e^{FB}\).

**Lemma 3.1.** (i) There is always under-investment in \(e_2\) under both subsidiary and division. (ii) There is over-investment in \(e_1\) under subsidiary if and only if \(\frac{2}{3} < \lambda < 1\) and \(\frac{v_{MF}'}{v_{MF}^2} < \frac{3\lambda - 2}{\lambda(3 - \eta + \rho)}\), otherwise there is underinvestment. (iii) There is over-
investment in $e_1$ under division if and only if \( \frac{6}{7} < \lambda < 1 \) and \( \frac{v_1^{MF}}{v_M} < \frac{7\lambda - 6}{\lambda(8+5\eta-4\rho)} \), otherwise there is underinvestment.

Proof. See Appendix 3.A.

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Proposition 3.2. Setting up a business as a subsidiary is more efficient than making it a division if (i) the cooperative investment $e_2$ is sufficiently non-specific to the value of output after takeover; and one of the two following: (ii.a) the chance of the takeover opportunity is small, i.e. $\lambda$ is sufficiently small; or (ii.b) the chance of the takeover opportunity is large, i.e. $\lambda$ is large, but the owner-specific investment, $e_1$, is sufficiently non-specific to the investor herself.

Proof. By Lemma 3.1, there is always under-investment for $e_2$. When $\lambda < \min\{\frac{2}{3}, \frac{6}{7}\} = \frac{2}{3}$ or if $\frac{2}{3} < \lambda < 1$ but $\frac{v_1^{MF}}{v_M} > \max\{\frac{3\lambda - 2}{\lambda(3-\eta+\rho)}, \frac{7\lambda - 6}{\lambda(8+5\eta-4\rho)}\} = \frac{3\lambda - 2}{\lambda(3-\eta+\rho)}$, there is under-investment under both S and D for $e_1$.

By Proposition 3.1, $e_2^{S} > e_2^{D}$, when $\frac{v_2^{MF}}{v_M} > \underline{\omega}$. Furthermore, it is always true that $e_1^{S} > e_1^{D}$. Therefore, S is always more efficient than D when (i) $\frac{v_2^{MF}}{v_M} > \underline{\omega}$ and one of the two following: (ii.a) $\lambda < \frac{2}{3}$; or (ii.b) $\frac{2}{3} < \lambda < 1$ but $\frac{v_1^{MF}}{v_M} > \frac{3\lambda - 2}{\lambda(3-\eta+\rho)}$.

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Proposition 3.3. Setting up a business as a division is more efficient than making it a subsidiary if all three of the following conditions are satisfied (i) the cooperative investment, $e_2$, is sufficiently specific to the value of output after takeover; (ii) the chance of the takeover opportunity is high, i.e. $\lambda$ is sufficiently large; and (iii) the owner-specific investment, $e_1$, is sufficiently specific to the investor, i.e. $\frac{v_1^{MF}}{v_M}$ is sufficiently small.

Proof. The proof is similar to the previous proposition. By Lemma 3.1, there is always under-investment for $e_2$. Under conditions (ii) and (iii), when $\lambda > \max\{\frac{2}{3}, \frac{6}{7}\} = \frac{6}{7}$, and
\[
\frac{v_{1M'F}}{v_{1MF}} < \min\left(\frac{3\lambda-2}{\lambda(3-\eta+\rho)}, \frac{7\lambda-6}{\lambda(8+5\eta-4\rho)}\right) = \frac{7\lambda-6}{\lambda(8+5\eta-4\rho)},
\]
there is over-investment under both S and D for \( e_1 \).

By Proposition 3.1, \( e_2^S < e_2^D \), when \( \frac{v^{S'F}}{v_{MF}} < \overline{w} \). Furthermore, it is always true that \( e_1^S > e_1^D \). Therefore, D is always more efficient than S when (i) \( \frac{v^{S'F}}{v_{MF}} < \overline{w} \), (ii) \( \lambda > \frac{6}{7} \), and (iii) \( \frac{v^{S'F}}{v_{MF}} < \frac{7\lambda-6}{\lambda(8+5\eta-4\rho)} \).

Propositions 3.2 and Proposition 3.3 form a pair of surprising results regarding the probability of takeover, \( \lambda \). As the chance of the business being taken over increases, the model predicts that division is more efficient. This is by itself against the modeling assumption that firms use subsidiaries as oppose to divisions when the owner of the business (M in our model) has stronger intention to sell the business in the future. However, this pattern does correspond to the reality where subsidiaries have very stable relationships with their parent firms in that they tend not to be sold very frequently.

The intuition behind this result is subtle. Setting up the business as a subsidiary provides the owner with better bargaining position in the takeover bargaining stage. Specifically, the owner is able to extract a greater share of its potential output, \( v_{MF} \). This advantage attracts the owner to invest more \( \text{ex ante} \) toward this "inside option"—\( v_{MF} \)—rather than the value of output after takeover, \( v_{M'F} \). But when the chance of takeover happening increases (higher \( \lambda \)), high investment toward the "inside option" becomes wasteful and low investment toward the future product becomes inefficient. Thus subsidiary becomes less efficient than division.

Regarding the specificity of investment, the model predicts that the business that uses more specific investment—no matter it is owner-specific investments or cooperative investments that is specific to other parties—should be housed inside the firm.
Particularly in this case, it means using the division rather than the subsidiary. But this result comes out of the current model in a rather subtle manner. As is discussed above, the main benefit of subsidiary is that it provides better ex ante investment incentives for the owner toward the “inside option”. And since the owner is more focused on building the “inside option”, an cooperative investment that is very specific to the value of output after takeover is unattractive to the owner. This is the case because such specific cooperative investment has little benefit in boosting the “inside option”. Comparing to the case under division, under subsidiary, the owner tends to over-invest in “inside option” (i.e. owner-specific investment) and under-invest in value of output after takeover (i.e. cooperative investment). This pattern is particularly inefficient when the takeover deal is likely to happen. Because when a takeover is likely, the owner-specific investment is likely to be a waste for the total surplus, while the cooperative investment tends to be crucial for efficiency. Thus when investments are specific, divisions tend to be more efficient than subsidiaries.

We present a comparative statics result on the two parameters $\rho$ and $\eta$. We say a supplier $F$ is dedicated (non-dedicated) to the contract if $\rho$ is small (large). This is the case because $\rho$ measures the discount in the value $F$ is able to produce with the new owner $M'$ after takeover but without assigning the contract. If $F$ is dedicated to the original contract, it is likely that he will run into capacity constraints, which corresponds to a low value of $\rho$. In the opposite case, if $F$ is non-dedicated, it is unlikely for him to be constrained by the existing contract with $M$. Thus $F$ is likely to be able to renegotiate with $M'$ and produce a high value of output with very little discount, which corresponds to a high value of $\rho$.

Moreover, we say that the business unit is essential (inessential) for the transaction if $\eta$ is high (low). $\eta$ measures the further discount of the value produced by $F$
and $M'$ if $M'$ does not control the business unit given that $F$ still carries liability from a binding contract with $M$.\footnote{See definition of $Y_{n_{M'}}^{n}$ in the setup of the model.} If the business unit is very essential for the transaction between $F$ and $M'$, the discount should be large, which corresponds to a large value of $\eta$.

**Proposition 3.4.** All other things the same, the subsidiary is more likely to dominate the division if the supplier is non-dedicated (high $\rho$) or if the business is inessential (low $\eta$). The division is more likely to dominate the subsidiary if the supplier is dedicated (low $\rho$) or if the business is essential (high $\eta$), given that the chances of takeover happening is high enough.

**Proof.** From Propositions 3.2, we learned that: given the specificity of the investments $(v_{M,F}^{M'}$ or $v_{M,F}^{M'})$ fixed, the likelihood of subsidiaries being more efficient than divisions increases if $\omega = \frac{1 - 2\eta - \rho}{2}$ is lower and if $\frac{3\lambda - 2}{3(1 - \eta + \rho)}$ is lower given $\lambda > \frac{2}{3}$. But notice that both of these terms decreases in $\rho$ and increases in $\eta$. So need them to be smaller is equivalent to having a lower $\eta$ and a higher $\rho$. For $\lambda < \frac{2}{3}$, there is always over-investment in $e_1$ under both S and D, then we only need $e_2$ to be higher under S than under D. This boils down to a lower $\omega$, which also corresponds to a lower $\eta$ and a higher $\rho$. Thus we established the first part of the proposition.

Similarly, from Proposition 3.3, for divisions to be more efficient, we need $\omega$ to be large and $\frac{7\lambda - 6}{\lambda(8 - 5\eta + 4\rho)}$ to be large given that $\lambda > \frac{2}{7}$. These two conditions corresponds to a higher $\eta$ and a lower $\rho$. This holds because, again, these two terms are both increasing in $\eta$ and decreasing in $\rho$. \hfill $\square$

As of the essentialness of business for the potential buyer, $\eta$, our model predicts that subsidiary is efficient when the business is relatively inessential for the potential
buyer (higher $\eta$), and division should be used in the opposite case. Our model shows that, comparing to subsidiary, division puts the owner in a weaker position to extract the “inside option”, but provides the owner a better position to extract the additional surplus created after the buyer, $M'$, acquires the business. When the business is inessential, the additional surplus created from the takeover is small. Thus there is not much for the present owner to extract under division. In this case, the owner is reluctant to make as high level of *ex ante* investment toward future output. So divisions are less efficient than subsidiaries. When the business is relatively essential (lower $\eta$), however, the intuition works in the opposite direction. The owner has stronger incentives to invest toward future product under division, thus divisions tend to be more efficient than subsidiaries.

The degree of the supplier’s dedication, $\rho$, affects the result because of the role of the supplier in the takeover bargaining when the business unit is a division. If $g = D$, the supplier has the opportunity to threaten $M$ and $M'$ that he will not agree to reassign the contract. Dedication gives the supplier commitment power in not renegotiating with $M'$ after rejecting to reassign the contract. Thus the more dedicated $M$ is, the more powerful this threat will be, which actually helps the supplier extracting a greater share of $v_{M'F}$ from $M$ and $M'$. Under $g = S$, $M$ is immune from this threat. And furthermore, the dedication of the supplier becomes $M$’s advantage to bargain against $M'$ because $M$’s decision over whether to agree to takeover includes the reassignment of the supplying contract with $M$. 
3.5 Conclusion

This paper provides a formal economic theory of the subsidiary versus division. The idea of the model builds on the legal foundation that, for its owner, a business is easier to be transferred as a subsidiary than as a division. The model utilizes the framework of property-rights theory of the firm and focus on the inefficiencies in \textit{ex ante} investment. Although the model is built on the idea that subsidiary is easier to be transferred, the main result is contrary to the common intuition that subsidiary is setup to be sold. The model predicts that setting up a business as a subsidiary is more efficient than making it a division when (i) the cooperative investment by the owner is relatively non-specific; and one of the two following: (ii.a) the chances that the business will be taken over is low; or (ii.b) the owner-specific investment is relatively non-specific. And using division is more efficient if (i) the cooperative investment by the owner is relatively specific to the value that can be produced after the takeover; (ii) the chance of the business being taken over is high; and (iii) the owner-specific investment is relatively specific.

It is important to remark that Propositions 3.2 and Proposition 3.3 do not exhaust the entire parameter space. Without further assumptions about the production technology, there are cases in which we are unable to draw clear conclusions over which governance structure is more efficient. For example, as indicated by Lemma 3.1, it could be that there is under-investment in $e_1$ under S but over-investment in $e_1$ under D (e.g. when $\frac{2}{3} < \lambda < \frac{6}{7}$). Then we would not be able to draw clear conclusions over which structure is more efficient. Such cases, however, may be studied when more detailed institutional structures are added to the model.
3.A Omitted Proofs

**Lemma 3.1.** (i) There is always under-investment in $e_2$ under both subsidiary and division. (ii) There is over-investment in $e_1$ under subsidiary if and only if $\frac{2}{3} < \lambda < 1$ and $\frac{v_{MF}^1}{v_{MF}^2} \leq \frac{3\lambda-2}{\lambda(3-\eta+\rho)}$, otherwise there is underinvestment. (iii) There is over-investment in $e_1$ under division if and only if $\frac{6}{7} < \lambda < 1$ and $\frac{v_{MF}^1}{v_{MF}^2} \leq \frac{7\lambda-6}{\lambda(8+5\eta-4\rho)}$, otherwise there is underinvestment.

**Proof.** By our assumptions that $\Psi$ is strictly convex and the production functions are weakly concave, the equilibrium investments for the first-best, subsidiary case and division case are all unique. Comparing equations (3.4.4) with (3.4.5), because they all have the same marginal cost function, when a governance structure has a higher (lower) marginal benefit than the first-best for any given level of investments, there is over-investment (under-investment).

Furthermore, because our assumption that the cross-partial of the two investments are zero, we can separate the first-order conditions to evaluate $e_1$ and $e_2$ separately for any given governance structure.

Subtracting the left-hand-side of equation (3.4.5) from the left-hand-side of equation (3.4.4) for each governance structure, there is over-investment if the difference is negative, and positive-investment if the difference is positive. Denote the marginal benefit under subsidiary and division as $MB^s$ and $MB^d$, respectively. Then we have:

\begin{align}
(1 - \lambda)v_{MF}^i + \lambda v_{MF}' - MB^s &= \frac{1}{4} ((2 - 3\lambda)v_{MF} + \lambda(3 - \eta + \rho)v_{MF}') \quad (3.A.1) \\
(1 - \lambda)v_{MF}^i + \lambda v_{MF}' - MB^d &= \frac{1}{12} ((6 - 7\lambda)v_{MF} + \lambda(8 - 5\eta + 4\rho)v_{MF}') \quad (3.A.2)
\end{align}

in which Assumption 3.1 holds for $i = 1, 2$ respectively.

Requiring (3.A.1) to be negative (over-investment) boils down to $\frac{2}{3} < \lambda < 1$ and $0 \leq v_{MF}^i < (\frac{3\lambda-2}{\lambda(3-\eta+\rho)})v_{MF}'$. This can only be satisfied for $e_1$ since $v_{MF}^2 < v_{MF}^1$. So we always have under-investment in $e_2$ and over-investment in $e_1$ when $\frac{2}{3} < \lambda < 1$ and $0 \leq \frac{v_{MF}^i}{v_{MF}^2} \leq \frac{3\lambda-2}{\lambda(3-\eta+\rho)}$, and under-investment in $e_1$ otherwise.

Similarly, (3.A.2) yields the similar result that is provided in the statement. □
Bibliography


