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A General Schema for Optimal Monetary Policymaking: Objectives and Rules

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Abstract

This paper examines four equivalent methods of optimal monetary policymaking, committing to the social loss function, using discretion with the central bank long-run and short-run loss functions, and following monetary policy rules. All lead to optimal economic performance. The same performance emerges from these different policymaking methods because the central bank actually follows the same (similar) policy rules. These objectives (the social loss function, the central bank long-run and short-run loss functions) and monetary policy rules imply a complete regime for optimal policy making. The central bank long-run and short-run loss functions that produce the optimal policy with discretion differ from the social loss function. Moreover, the optimal policy rule emerges from the optimization of these different central bank loss functions.

1.   Introduction

In this paper, we consider the design of monetary policy – either a central bank objective function or a central bank policy rule – to optimize social welfare. We assume that the social welfare function corresponds to the representative household's utility function, which captures the final objectives of monetary policy. As a practical matter, however, the central bank should not adopt the social welfare function as its "general targeting rule" (i.e., objective function) under discretion.[1] That is, if the social welfare function incorporates the target values for the target variables that prove mutually inconsistent with the structure of the economy, then optimizing the social welfare function generates time inconsistency under rational expectations. Rather, the central bank's general targeting rule must incorporate target values that prove mutually consistent with the structure of the economy. If so, then optimal policy that optimizes the social welfare function will prove consistent as well.

The central bank's general targeting rule (i.e., objective function) implies a "specific targeting rule" (i.e., policy rule), which also ensures that optimal policy proves consistent.[2] We find, however, that an infinite number of general targeting rules associate with a specific targeting rule. Once again, the specific targeting rule (i.e., central bank policy rule) ensures that we optimize the social welfare function and that we adopt an optimal and consistent policy.

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[1] Svensson (2003) defines a general targeting rule as incorporating "the objectives to be achieved, for instance, by listing the target variables, the targets (target levels) for those variables, and the (explicit or implicit) loss function to be minimized." (p. 429). Moreover, Svensson (2003) argues "general targeting rules essentially specify operational objectives for monetary policy …" (p. 430).

[2] Svensson (2003) defines a specific targeting rule "specifies conditions for the target variables (or forecasts of the target variables), for instance, like the … rule of thumb of the Bank of England and the Riksbank." (p. 429). Moreover, Svensson (2003) argues "specific targeting rules essentially specify operational Euler conditions for monetary policy." (p. 430).

Rogoff (1985), Walsh (1995, 2003), Svensson (1997b), and others adopt the notion that the central bank with discretionary policy employs an objective function (i.e., general targeting rule) differing from the social welfare function to ensure optimal policy proves consistent. For instance, Walsh (2003, p. 276) states, "I have assumed the relevant policy regime is one of discretion, and the problem faced in designing policy is to assign a loss function to the central bank." Similarly, Svensson (1999, p. 636) states "Both money-growth targeting and nominal-GDP targeting are interpreted as intermediate-targeting rules, that is, the assignment or adoption of an intermediate loss function with money growth or nominal GDP as a target variable. Since the purpose of an intermediate-targeting strategy is to fulfill some final loss function, the performance of the intermediate-targeting rule must be evaluated according to that final loss function rather than to the intermediate loss function." In our context, the intermediate loss function refers to the central bank objective function (i.e., general targeting rule), which takes the same form as the social loss function but with different parameters. Yuan and Miller (2007a, 2006) and Yuan, Miller and Chen (2007b) argue that the social welfare criterion (loss function) proves inappropriate for the direct central bank objective function in monetary policymaking, because the target levels in the social loss function are inconsistent with each other and the central bank under discretion faces a dilemma if delegated the social loss function. This paper extends Yuan, Miller and Chen (2007b) with static Barro-Gordon model by using a dynamic backward-looking model with employment persistence.

In sum, we explore the delegation of central bank objective functions and policy rules to produce optimal and consistent policy outcomes. The rest of this introduction provides

background information on these issues.

Central Bank Intertemporal (long-run) Loss Function

Designing a scientific and direct objective for monetary policy is one of the main tasks of this paper. We will show that the equilibrium (i.e., discretion or consistent) policy under the designed and delegated central bank objective function replicates the optimal policy under the social welfare function with commitment. That is, the consistent policy under the designed and delegated objective function proves optimal under the social welfare criterion. So designing the direct central bank objective function for monetary policy serves as a means, not an end in itself, to optimizing the social welfare.

The designed and delegated monetary policy (central bank) objective function possesses a straightforward interpretation. It exhibits four characteristics (properties). First, the equilibrium (consistent) policy under the designed and delegated objective function replicates the optimal policy under the social welfare function with commitment. Thus, consistency and optimality reconcile under the designed and delegated objective function. Second, the target levels in the designed and delegated objective function prove moderate, in that they are attainable, on average, each period. Thus, the central bank can easily earn credibility and accountability. Third, the target level of employment (output) equals the natural (potential) one. This well-known outcome requires that the central bank not adopt an output bias in its direct objective function. Moreover, as Svensson (2002) argues "There is general agreement that inflation-targeting central banks do normally not have overambitious output targets, that is, exceeding potential output." (p. 774). Our paper arrives at this result from a different direction. Fourth, the relative weight placed on the two target variables --

inflation and output (employment) -- reflects the social preference, as well as the economic structure. These four properties exhibit robustness because they also hold in Yuan and Miller (2007a, 2006) and Yuan, Miller and Chen (2007b).

> Svensson (2003) writes:
>
> What are the problems with a commitment to a general targeting rule? One problem is that the objectives may still not be sufficiently well specified not to be open to interpretation. For instance, the relative weight on output-gap stabilization in flexible inflation targeting… is not directly specified by any inflation-targeting central bank…
> A second potential problem… is the potential consequences of the discretionary optimization under a commitment to a general targeting rule, more precisely that such discretionary optimization is not fully optimal in a situation with forward-looking variables.[3] (p. 454.)

Our paper to some extent solves the above two problems in theory -- how to design a direct central bank objective function for monetary policy and how to implement optimal discretionary (consistent) policy.

In addition, similarity exists between our findings and those of Clarida, Gali and Gertler (1999) "The solution under commitment in this case perfectly resembles the solution that would obtain for a central bank with discretion that assigned to inflation a higher cost than the true social cost" (p. 1681) What differs? They place more weight on inflation variability; we put more weight on employment (output) gap variability. The difference results from different models. Intuitively, with employment persistence, any employment gap not eliminated today persists into the future and, thus, induces more loss. To reduce loss, we place more weight on employment.

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[3] The backward-looking models in Svensson (1997a, 2003) actually conform to dynamic programming problems, not games. Inconsistency issues of optimal policy do not exist in these backward-looking models, but do exist in the forward-looking model of Svensson (2003). Though our model is backward-looking with employment persistence, the inconsistency issue of optimal policy exists. We use the backward-looking model of Svensson (1997b) in our paper to illustrate our ideas. Using a forward-looking model makes our ideas less transparent because of more complicated mathematics.

Monetary Policy Rules

The property of optimal and consistent policy proves a good outcome. But, why? In contemplating the answer, we find that the reason proves surprisingly simple. That is, the first-order condition of the value function under the designed and delegated central bank objective function with discretion exactly equals the first-order condition of the value function under the social welfare function with commitment. As a result, discretionary policy under the designed and delegated objective function replicates the commitment policy under the social objective function. From this view, monetary policy rules appear more basic than the objectives functions. We also examine such policy rules in this paper.

Different meanings for monetary policy rules exist in literature. When rules appear in the phrase "rules versus discretion", rules mean commitment. That is, McCallum (2004) states, for example, that "monetary policy is conducted … in a 'rule-based' manner that views policy as an ongoing process" (p. 367), rather than on a period-by-period basis. When rules appear in "Taylor rules" or "McCallum rules," rules mean instrument rules in response to current economic conditions. Svensson (2003) argues, "the concept of monetary-policy rules should be broadened beyond the narrow instrument rules and also include targeting rules." (p. 466). We define a monetary policy rule as a central bank's behavior equation, which equals an explicit or implicit function of instruments or target variables in relation with predetermined variables and structural shocks. So, we differentiate between a "rule" versus an "objective", as does Cecchetti (2000). In addition, we also assume with Cecchetti (2000) that a rule responds to economic variables as well as demand and supply shocks, if the rule

performs at its best.[4] Moreover, monetary policy rules in our definition prove broader than instrument rules. They can reflect specific targeting rules or instrument rules in Svensson's terms, depending on the assumptions and the economic structure that we use. If the economic structure involves only an aggregate supply function and the central bank directly controls a target variable, then the optimal monetary policy rule is a specific targeting rule, which includes target variables, predetermined variables, and structural shocks. If the economic structure involves both aggregate demand and aggregate supply functions and the central bank directly controls an instrument (not a target variable), then a monetary policy rule is an instrument rule, which includes target variables, predetermined variables, structural shocks, as well as the instrument variable.

This paper identifies four ways to obtain optimal policy rules -- derived from the first-order condition of the value function of the social welfare function under commitment, derived from the first-order condition of the value function of the central bank long-run and short-run objective functions under discretion, and derived from optimizing the social welfare function using our definition of a monetary policy rule. To concentrate on the main issues, we consider only specific targeting rules. We can easily obtain optimal instrument rules, however, by combining specific targeting rules with the aggregate demand function.

Given our definitions of monetary policy objectives and rules, our paper concludes that monetary policy objectives and rules can theoretically play the same role in monetary policymaking. Specifically, the four optimal policymaking methods -- commitment to the social welfare objective, discretion and the designed and delegated long-run and short-run

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[4] Bernanke and Mishkin (1997) and Svensson (1997a, 2002, 2003) apparently regard monetary policy rules as mechanical responses to current and forecast variables, excluding responses to shocks.

central bank objective functions,[5] and just follow the designed and delegated policy rule -- yield equivalent outcomes. They all produce the same optimal and consistent outcomes. This conclusion seemingly contradicts Svensson's (1997a) argument "Commitment to 'target rules' may be better than commitment to 'instrument rules'." (p. 1111).[6] The contradiction, however, occurs because of different assumptions concerning knowledge about the economic structure as well as a difference in the understanding of the rules. No essential conflicts exist in the debate on whether targeting rules prove superior to instrument rules. With imperfect knowledge about the economic structure, targeting rules (general targeting rules and specific targeting rules) may dominate instrument rules. More specifically, with imperfect knowledge about the aggregate supply function, general targeting rules may dominate specific targeting rules. With perfect knowledge about the aggregate supply function, however, using general targeting rules or specific targeting rules makes no difference. With perfect knowledge about the aggregate supply function and imperfect knowledge about the aggregate demand function, specific targeting rules may dominate instrument rules. Still, with perfect knowledge about the economic structure (aggregate supply and demand functions), general targeting rules, specific targeting rules, and instrument rules prove essentially the same.

Assuming imperfect knowledge about the economic structure is more realistic and practical. Since our theoretical paper only addresses the issues of the consistency and optimality of monetary policy (not the practical implementation of monetary policy, such as in Svensson, 1997a), we assume perfect knowledge about the economic structure by the

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[5] We discuss the central bank short-run objective function in the next subsection.

[6] As noted before, Svensson's (2003) target rules include general targeting rules and specific targeting rules. General targeting rules specify an operational objective for monetary policy with a commitment to that objective. Monetary policy rules in our context prove broader than, and thus include, instrument rules.

central bank and the public. Under this assumption and our definition of monetary policy rules, it makes no difference whether the central bank operates monetary policy by optimizing policy objectives[7] or by following policy rules.

Central-Bank Period (Short-Run) Loss Function

In this paper, we also consider myopic policy. Generally an equilibrium of an infinite-period dynamic game requires strong assumptions, including that all players possess high intelligence and make no mistakes. Accordingly, we assume that a boundedly rational central bank operates policy myopically, minimizing only the current period loss. We still hope, however, that the myopic equilibrium policy replicates the optimal policy.

For convenience, we define the short-run objective function, where a bounded-rational central bank optimizes the period objective, and the long-run objective function, where an unbounded central bank optimizes the intertemporal objective. We find that the optimal short-run objective in each period equals the period objective of the designed long-run objective. We interpret this result roughly as follows. Recall that one characteristic of the designed and delegated long-run objective function is that target levels are realized each period, on average. That is, the central bank minimizes the loss each period at zero, resulting in the optimization of the intertemporal objective function. In other words, as long as the central bank currently minimizes each period's loss function of the intertemporal objective, the myopic equilibrium policies of all periods replicates the optimal policy path. In other words, minimizing the intertemporal loss function also minimizes the period loss function. In short, no intertemporal loss substitution occurs. Similarly, the first-order

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[7] Optimizing policy objectives means either optimizing the social welfare objective with commitment, or optimizing the direct central bank objective function with discretion.

condition of the optimal short-run objective function replicates the optimal policy rule.

We organize the paper as follows. Section 2 presents the model and its commitment (optimal) and discretion (consistent) policy. Consistent policy does not prove optimal. Section 3 designs the central-bank intertemporal (long-run) loss function. We find the discretionary policy under the designed and delegated central-bank intertemporal (long-run) loss function replicates optimal policy and the loss function possesses a straightforward interpretation. Discretionary policy under the designed and delegated central-bank loss function replicates the optimal policy because the first-order conditions of their value functions (the designed and delegated central-bank intertemporal loss function and the social intertemporal loss function) prove identical. As a result, Section 4 studies monetary policy rules, providing three ways of designing optimal monetary policy rules. Section 5 designs the central-bank period (short-run) loss function. We obtain intuitive results. The designed and delegated central-bank period loss function coincides with the period loss function of the designed and delegated central-bank intertemporal loss function, implying that the first-order condition of the designed and delegated central-bank period loss function also replicates the optimal policy rule. Section 6 concludes.

2.   Optimal and Consistent Policy in a Simple Model

The model follows the analysis of Svensson (1997b).[8] Society minimizes the following intertemporal (long-run) loss function:

(1) $$E_0 \left(\sum_{t=1}^{\infty} \beta^{t-1} L_t \right),$$

where the discount factor equals β, $0 < \beta < 1$, E equals the mathematical expectations

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[8] See footnote 3 for the reasons that we use the model in Svensson (1997b).

operator, and the period (short-run) loss function equals L_t. The period loss function equals the following:

(2) $$L_t \equiv L\left(\pi_t, \ell_t; \pi^*, \ell^*, \lambda\right) = \frac{1}{2}\left[\left(\pi_t - \pi^*\right)^2 + \lambda\left(\ell_t - \ell^*\right)^2\right],$$

where π equals the inflation rate, ℓ equals the natural logarithm of the employment rate (i.e., the share of employment in full employment), starred values identify society's target values for the inflation and employment rates, and λ measures the relative importance of employment and inflation deviations from their target rates.

The economic structure equals the following two-equation system, which incorporates employment persistence:

(3) $$\ell_t = \rho \ell_{t-1} + \alpha\left(\pi_t - \pi_t^e\right) + \varepsilon_t \quad \text{and}$$

(4) $$\pi_t^e = E_{t-1}\left(\pi_t\right),$$

where ε_t equals the white noise random shock with variance equal to σ^2. Note that the steady-state value of ℓ equals zero, since at full employment, the employment rate equals one and its natural logarithm equals zero. Thus, the case without persistence (i.e., $\rho = 0$) corresponds to the standard Barro-Gordon (1983) model.

Optimal Policy (Benchmark)

Assume that the government directly controls the central bank and that the government can commit to a state-contingent rule on the inflation rate. The Bellman equation for determining the optimal policy and outcomes from the optimization equals the following:

(5) $$V^*(\ell_{t-1}) = \min_{\pi_t, \pi_t^e} E_{t-1}\left\{\frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda(\ell_t - \ell^*)^2\right] + \beta V^*(\ell_t)\right\}.$$

We minimize this equation subject to the economic structure given in equations (3) and (4).[9]

The solution for $V^*(\ell_{t-1})$ must equal a quadratic form, since we minimize the quadratic objective function subject to linear constraints. Thus, the hypothesized solution equals the following equation:

(6) $$V^*\left(\ell_t\right) = \gamma_0^* + \gamma_1^*\ell_t + \frac{1}{2}\gamma_2^*\ell_t^2,$$

where we need to determine the unknown coefficients in equation (6). The solution equals the following:

(7) $$\gamma_1^* = -\frac{\lambda\rho\ell^*}{1-\beta\rho} \quad \text{and} \quad \gamma_2^* = \frac{\lambda\rho^2}{1-\beta\rho^2}.$$

The solution for the optimal policy produces the following:

(8) $$\pi_t^{optimal} = \pi^* - b^*\varepsilon_t,$$

where $b^* = \dfrac{\lambda\alpha}{1-\beta\rho^2 + \lambda\alpha^2}$. The optimal employment rate equals the following:

(9) $$\ell_t^{optimal} = \rho\ell_{t-1} + \frac{1-\beta\rho^2}{1-\beta\rho^2 + \lambda\alpha^2}\varepsilon_t.$$

Consistent (Discretionary) Policy

Now, assume that the government still directly controls the central bank, but it cannot commit to a state-contingent rule on the inflation rate. As such, the decision problem of the government takes the expected inflation rate as a given. That is, no longer does the government internalize the effects of its decisions on the expected inflation rate.

Carrying out the optimization produces the following consistent inflation and employment rates:[10]

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[9] Lockwood et al. (1995) and Svensson (1997b) provide more details of the derivation.

[10] Svensson (1997b) also reports two additional existence conditions in his Appendix.

(10) $\pi_t^{discretion} = a - b\varepsilon_t - c\ell_{t-1}$ and

(11) $\ell_t^{discretion} = \rho\ell_{t-1} + (1-\alpha b)\varepsilon_t$,

where

$$a = \pi^* + \frac{\lambda\alpha\ell^*}{1-\beta\rho-\beta\alpha c}, \quad b = \frac{\lambda\alpha + \beta\alpha c^2}{1-\beta\rho^2 + \alpha\left(\lambda\alpha + \beta\alpha c^2\right)}, \text{ and}$$

$$c = \frac{1}{2\alpha\beta\rho}\left[1-\beta\rho^2 - \sqrt{\left(1-\beta\rho^2\right)^2 - 4\lambda\alpha^2\beta\rho^2}\right].$$

Comparing equations (8) and (10), the inflation bias in the consistent inflation rate under discretion equals the following:

(12) $\pi_t^{discretion} - \pi_t^{optimal} = a - \pi^* - c\ell_{t-1} - (b-b^*)\varepsilon_t$.

The inflation bias includes an *average* inflation bias ($a-\pi^*$), a *state-contingent* inflation bias ($-c\ell_{t-1}$), and a *stabilization* inflation bias $[-(b-b^*)\varepsilon_t]$.

In sum, consistent policy does not prove optimal.

3   Design of Central Bank Long-Run Loss Functions

We show in the prior section that discretion produces a consistent, but non-optimal, policy. That finding implicitly assumes that the central bank adopts the social loss function as its own loss function. Our paper first considers delegating a loss function to the central bank that differs from the social loss function, but that delivers the optimal outcomes when the central bank adopts a consistent policy based on the delegated loss function. That is, can we find a loss function that when delegated to the central bank yields optimal outcomes?

When the central bank minimizes the intertemporal expected loss from the current and all future periods, we call that objective function the long-run central-bank loss function. This Section examines this problem. Correspondingly, when the central bank only minimizes the

current-period expected loss, we call that objective function the short-run central-bank loss function. Section 5 considers the current-period expected-loss minimization problem.

Equations (1) and (2) represent the long-run (intertemporal) social loss function and its short-run (period) component. The proposed delegated central bank short-run (period) loss function equals the following expression:

(13) $\qquad L_t^b = \frac{1}{2}\left[\left(\pi_t - \pi_t^b\right)^2 + \lambda^b\left(\ell_t - \ell_t^b\right)^2\right],$

where π_t^b and ℓ_t^b equal state-contingent targets (i.e., $\pi_t^b = g_0 + g_1\ell_{t-1}$ and $\ell_t^b = h_0 + h_1\ell_{t-1}$). That is, the proposed delegated short-run central bank loss function mirrors the short-run social loss function, but with potentially different parameters. Moreover, the proposed short-run loss function allows state-continent targets to reflect the persistence of employment in the economy.

Based on the discussion in the Introduction that central-bank loss function serves as a means to the end—minimization of the social long-run loss function, we design the central bank loss function through the following model:

(14) $\qquad \min_{g_0,g_1,h_0,h_1,\lambda^b} E_0\left\{\sum_{t=1}^{\infty}\beta^{t-1}\cdot\frac{1}{2}\left[\left(\pi_t - \pi^*\right)^2 + \lambda\left(\ell_t - \ell^*\right)^2\right]\right\}$

$$s.t.\begin{cases}\min_{\{\pi_t\}_{t=1}^{\infty}} E_0\left\{\sum_{t=1}^{\infty}\beta^{t-1}\cdot\frac{1}{2}\left[\left(\pi_t - g_0 - g_1\ell_{t-1}\right)^2 + \lambda^b\left(\ell_t - h_0 - h_1\ell_{t-1}\right)^2\right]\right\}\\ s.t.\begin{cases}\ell_t = \rho\ell_{t-1} + \alpha\left(\pi_t - \pi_t^e\right) + \varepsilon_t\\ \pi_t^e = E_{t-1}\left(\pi_t\right)\end{cases}\end{cases}.$$

We solve this model in two steps and obtain an infinite number of optimal central bank loss functions. With more assumptions, we pin down the unique reasonable central bank loss function from the infinite number of optimal candidates. Specifically, Step I solves the following partial model:

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$$\min_{\{\pi_t\}_{t=1}^{\infty}} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \Big[(\pi_t - g_0 - g_1 \ell_{t-1})^2 + \lambda^b (\ell_t - h_0 - h_1 \ell_{t-1})^2 \Big] \right\}$$

(15)

$$s.t. \begin{cases} \ell_t = \rho \ell_{t-1} + \alpha (\pi_t - \pi_t^e) + \varepsilon_t \\ \pi_t^e = E_{t-1}(\pi_t) \end{cases}$$

That is, Step I proposes a class of loss functions, involving parameters that define the precise function chosen from the class of functions, and derives a consistent policy when the central bank minimizes that class of loss functions subject to the economic structure. Clearly, the consistent policy will depend on the parameters that define the precise loss function chosen from the class of loss functions. The consistent (equilibrium) inflation and employment rates equal:[11]

(16) $\pi_t = \Big[g_0 + \alpha \big(\lambda^b h_0 - \beta \gamma_1 \big) \Big] + \Big[\big(g_1 + \lambda^b \alpha h_1 \big) - \alpha \rho \big(\lambda^b + \beta \gamma_2 \big) \Big] \ell_{t-1} - \dfrac{(1-q)}{\alpha} \varepsilon_t$ and

(17) $\ell_t = \rho \ell_{t-1} + q \varepsilon_t$,

where $q = 1 / [1 + \alpha^2 (\lambda^b + \beta \gamma_2)]$,

(18) $\gamma_2 = \Big[\lambda^b \alpha h_1 - \alpha \rho \big(\lambda^b + \beta \gamma_2 \big) \Big]^2 + \lambda^b (h_1 - \rho)^2 + \beta \gamma_2 \rho^2$ or $\gamma_2 \equiv \gamma_2 \big(\lambda^b, h_1 \big)$, and

(19) $\gamma_1 = \alpha \big(\lambda^b h_0 - \beta \gamma_1 \big) \Big[\lambda^b \alpha h_1 - \alpha \rho \big(\lambda^b + \beta \gamma_2 \big) \Big] + \lambda^b h_0 (h_1 - \rho) + \beta \gamma_1 \rho$ or

$\gamma_1 \equiv \gamma_1 \big(\lambda^b, h_0, h_1 \big).$[12]

Given the solution for the consistent or equilibrium outcomes for inflation and employment that depend on the parameters of the central bank loss function (i.e., g_0, g_1, h_0, h_1, and λ^b), Step II choose values for those parameters to minimize the expected intertemporal social loss. The analytical problem equals the following:

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[11] Equations (16) and (17) equal, respectively, equations (A-8) and (A-9) in Appendix A.

[12] Equations (18) and (19) equal, respectively, equations (A-10) and (A-11) in Appendix A.

$$\min_{g_0, g_1, h_0, h_1, \lambda^b} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\}$$

(20)

$$s.t. \begin{cases} \pi_t = \left[g_0 + \alpha \left(\lambda^b h_0 - \beta \gamma_1 \right) \right] + \left[\left(g_1 + \lambda^b \alpha h_1 \right) - \alpha \rho \left(\lambda^b + \beta \gamma_2 \right) \right] \ell_{t-1} - \frac{(1-q)}{\alpha} \varepsilon_t \\ \ell_t = \rho \ell_{t-1} + q \varepsilon_t \end{cases}$$

The problem yields the following first-order conditions:[13]

(21) $\qquad g_0 + \alpha \left(\lambda^b h_0 - \beta \gamma_1 \right) - \pi^* = 0$,

(22) $\qquad \left(g_1 + \lambda^b \alpha h_1 \right) - \alpha \rho \left(\lambda^b + \beta \gamma_2 \right) = 0$, and

(23) $\qquad q \left\{ = \dfrac{1}{1 + \alpha^2 (\lambda^b + \beta \gamma_2)} \right\} = \dfrac{1 - \beta \rho^2}{1 - \beta \rho^2 + \lambda \alpha^2}$.

By using equations (21) to (23), the consistent policy outcomes for the inflation and employment rates (16) and (17) equal:

(24) $\qquad \pi_t = \pi^* - \dfrac{\lambda \alpha}{1 - \beta \rho^2 + \lambda \alpha^2} \varepsilon_t$ and

(25) $\qquad \ell_t = \rho \ell_{t-1} + \dfrac{1 - \beta \rho^2}{1 - \beta \rho^2 + \lambda \alpha^2} \varepsilon_t$,

which equal their optimal outcomes. See equations (8) and (9).

Generally, model (14) leads to an infinite set of possible central bank loss functions, since the class of loss functions contains more than the minimum number of parameters needed to lead to a solution of the minimization. Specifically, as long as the 7 parameters, g_0, g_1, h_0, h_1, λ^b, γ_1, and γ_2, of the central bank loss function, satisfy equations (18), (19), (21), (22), and (23), the consistent policy will prove optimal.

Among the infinite number of optimal candidates for the central bank loss functions, does one appear more reasonable? Yes. We argue that the chosen parameters should also minimize the designed and delegated central bank loss function itself. This idea pins down a

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[13] Equations (21), (22) and (23) equal, respectively, equations (A-16), (A-17) and (A-18) in Appendix A.

unique central bank loss function. The problem equals the   following:

(26)
$$\min_{g_0,g_1,h_0,h_1,\lambda^b} \frac{1}{2} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \left[\pi_t - \left(g_0 + g_1 \ell_{t-1} \right) \right]^2 + \lambda^b \left[\ell_t - \left(h_0 + h_1 \ell_{t-1} \right) \right]^2 \right\} \right\}$$

$$s.t. \begin{cases} \pi_t = \pi^* - \dfrac{\lambda \alpha}{1 - \beta \rho^2 + \lambda \alpha^2} \varepsilon_t \\[4mm] \ell_t = \rho \ell_{t-1} + \dfrac{1 - \beta \rho^2}{1 - \beta \rho^2 + \lambda \alpha^2} \varepsilon_t \end{cases}.$$

The optimization yields the following solution:[14]

(27)
$$g_0 = \pi^*, \, g_1 = 0, h_0 = 0 \text{ and } h_1 = \rho.$$

Equivalently,

(28)
$$\pi_t^b = \pi^* \text{ and } \ell_t^b = \rho \ell_{t-1}.$$

Moreover, if ρ equals zero, then ℓ_t^b equals zero or full employment, given that h_0 equals zero.

Viewing the problem somewhat differently, but leading to the same conclusion, modern central banks must account for their actions. How can we make central banks accountable? We do so by delegating achievable target levels. The central bank, constrained by the economic structure, will face a dilemma if it cannot achieve the delegated target levels. That is, we assume the delegated target levels are averagely attainable:

(29)
$$\pi_t^b = E_{t-1}\left(\pi_t \right) \text{ and } \ell_t^b = E_{t-1}\left(\ell_t \right).$$

Thus, since $E_{t-1}\left(\pi_t \right) = \pi^*$ and $E_{t-1}\left(\ell_t \right) = \rho \ell_{t-1}$ from equations (24) and (25) hold for each of the optimal central bank loss functions, equation (28) also holds.

Finally, we determine the optimal central bank long-run loss function as follows:[15]

(30)
$$L_t^b = \frac{1}{2} \left[\left(\pi_t - \pi_t^b \right)^2 + \lambda^b \left(\ell_t - \ell_t^b \right)^2 \right],$$

where $\pi_t^b = \pi^*, \ell_t^b = \rho \ell_{t-1}$, and $\lambda^b = [\lambda/(1 - \beta \rho^2)]$.

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[14] Equation (27) equals equation (A-25) in Appendix A.

[15] See Appendix A for further details.

Three observations emerge from these findings. First, the parameter values that minimize both the social and central banker loss functions (i.e., the unique solutions) simultaneously imply that the targets in the central banker loss function prove rational, in the sense that the expected inflation and employment rates equal the central banker targets. That is,

(31) $\qquad E_{t-1}(\pi_t) = \pi_t^b$ and $E_{t-1}(\ell_t) = \ell_t^b$.

In addition, the result of $\ell_t^b = \rho \ell_{t-1}$ means that the optimal employment target should equal the potential employment level each period. This result proves consistent with "general agreement that inflation-targeting central banks do normally not have overambitious output targets, that is, exceeding potential output (Svensson, 2002, p. 774)", and also conforms to Blinder's intuition "I can assure you that it would not surprise my central banker friends to learn that economic theories that model them as seeking to drive unemployment below the natural rate imply that their policies are too inflationary. They would no doubt reply, 'Of course that would be inflationary. That's why we do not do it.'(Blinder, 1998, p. 42-43)"

Second, Rogoff's (1985) conservative central banker proposal (i.e., $\lambda^b < \lambda$) proves inconsistent with our finding. To compare with Rogoff's model, which does not incorporate employment persistence, we set $\rho = 0$. Then, $\lambda^b = \lambda$.[16] With employment persistence, we find that a less-conservative central banker than society proves optimal, since

(32) $\qquad \lambda^b > \lambda$.

Intuitively, employment persistence in our model means that any employment gap not eliminated today persists into future, and thus induces loss. To reduce loss, more weight goes

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[16] Yuan, Miller and Chen (2007b) discuss the result of $\lambda^b = \lambda$.

on the employment target.

Finally, our specification that achieves the optimal outcomes involves delegating to the central bank an inflation target equal to the social inflation target (i.e., $\pi^b = \pi^*$), a state-contingent employment target equal to the short-run natural rate of employment (i.e., $\ell^b = \rho \ell_{t-1}$), and an employment weight in the central bank short-run (period) loss function greater than the social weight (i.e., weight-liberal central bank, $\lambda^b = [\lambda/(1-\beta\rho^2)] > \lambda$). This offers a solution that differs from that proposed in Svensson (1997b).

4.   Design of Monetary Policy Rules

Until now, we present two methods of implementing an optimal monetary policy -- commitment to the social loss function and discretion to the designed and delegated central bank loss function. The two methods achieve the same optimal outcomes, since the first-order condition of the value function of the social loss function with commitment equals the first-order condition of the value function of the designed and delegated central bank loss function with discretion. That is, although it appears that policy operates in different ways, the central bank actually follows the same behavioral equation, inevitably resulting in the same economic performance. As a result, monetary policy rules can theoretically play the same role in policymaking as monetary policy objectives.

As defined in the Introduction, a monetary policy rule specifies a behavioral equation for the central bank. It can include predetermined variables, target variables, instrument variables, as well as structural shocks. No instrument variables appear in our monetary policy rules because we assume that the central bank directly controls the inflation rate, the target variable. So we study only 'specific targeting rules' in Svensson's terminology. We can easily

derive instrument rules, however, from specific targeting rules and an aggregate demand function.

We now present three methods of deriving optimal monetary policy rules. Two reflect the first-order conditions mentioned above. The third method mirrors the method of designing and delegating a central bank loss function, by choosing parameters from policy rules that minimize the social loss. Actually, the third method frequently appears in the literature.

First-Order Condition of Commitment to the Social Loss Function

Repeat the Bellman equation (5) as follows:

(5) $$V^*(\ell_{t-1}) = \min_{\pi_t, \pi_t^e} E_{t-1} \left\{ \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda(\ell_t - \ell^*)^2 \right] + \beta V^*(\ell_t) \right\}.$$

The first-order condition equals:

(33) $$\left(\pi_t - \pi^* \right) + \lambda\alpha\left(\ell_t - \ell^* \right) + \alpha\beta V_\ell^*(\ell_t) - E_{t-1}\left[\lambda\alpha\left(\ell_t - \ell^* \right) + \alpha\beta V_\ell^*(\ell_t) \right] = 0.$$

Substituting $V_\ell^*(\ell_t) = \gamma_1^* + \gamma_2^*\ell_t$, $E_{t-1}(\ell_t) = \rho\ell_{t-1}$ and $\gamma_2^* = \dfrac{\lambda\rho^2}{1 - \beta\rho^2}$ into equation (33) eventually reduces to:

(34) $$\left(\pi_t - \pi^* \right) + \frac{\lambda\alpha}{1 - \beta\rho^2}\left(\ell_t - \rho\ell_{t-1} \right) = 0.$$

This defines the specific targeting rule.

To express the policy rule as an explicit function of predetermined variables (π_t^e and ℓ_{t-1}) and structural shocks (ε_t) substitute $\ell_t = \rho\ell_{t-1} + \alpha\left(\pi_t - \pi_t^e \right) + \varepsilon_t$ into equation (34) and rearrange terms to give:

(35) $$\pi_t = \frac{1 - \beta\rho^2}{1 - \beta\rho^2 + \lambda\alpha^2}\pi^* + \frac{\lambda\alpha^2}{1 - \beta\rho^2 + \lambda\alpha^2}\pi_t^e - \frac{\lambda\alpha}{1 - \beta\rho^2 + \lambda\alpha^2}\varepsilon_t.$$

This defines the optimal monetary policy rule.

First-Order Condition of Discretion with the Designed and Delegated Central Bank Loss

Function

The Bellman equation for this problem equals the following:[17]

(36)
$$V\left(\ell_{t-1}\right) = E_{t-1} \min_{\pi_t}\left\{L_t^b + \beta V\left(\ell_t\right)\right\},$$

where $L_t^b = \dfrac{1}{2}\left[\left(\pi_t - \pi^*\right)^2 + \lambda^b\left(\ell_t - \rho\ell_{t-1}\right)^2\right]$.

The first-order condition equals:

(37)
$$\left(\pi_t - \pi^*\right) + \lambda^b\alpha\left(\ell_t - \rho\ell_{t-1}\right) + \alpha\beta V_\ell\left(\ell_t\right) = 0.$$

Note that $V_\ell\left(\ell_t\right) = \gamma_1 + \gamma_2\ell_t$. Also, Appendix A demonstrates that $\gamma_1 = \gamma_2 = 0$.[18] Thus,

equation (37) reduces to:

(38)
$$\left(\pi_t - \pi^*\right) + \lambda^b\left(\ell_t - \rho\ell_{t-1}\right) = 0,$$

where $\lambda^b = [\lambda/(1-\beta\rho^2)]$. Policy rule (38) replicates the rule (34).

Policy Rules that Minimize the Social Loss Function

The method of designing policy rules in this subsection is actually used frequently in the

existing literature (e.g., Clarida, Gali and Gertler 1999). Results, however, depend on the

correct definition of monetary policy rules. As mentioned in the Introduction, monetary

policy rules in our context imply that the control target variable (π_t) depends on

predetermined variables (π_t^e and ℓ_{t-1}) and structural shocks (ε_t) as follows:

(39)
$$\pi_t = a + b\pi_t^e + c\ell_{t-1} + d\varepsilon_t.$$

The central bank just follows the delegated policy rule (24). Thus, the model used to design

the optimal policy rule equals the following:

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[17] See Appendix A, equation (A-4).

[18] See Appendix A, equations (A-26) and the related discussion..

$$\min_{a,b,c,d} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\}$$

(40)
$$s.t. \begin{cases} \pi_t = a + b\pi_t^e + c\ell_{t-1} + d\varepsilon_t \\ \ell_t = \rho\ell_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + \varepsilon_t \\ \pi_t^e = E_{t-1} \left(\pi_t \right) \end{cases} .$$

The design of an optimal policy rule proceeds in two steps. Step I derives the equilibrium outcomes, given the policy rule. Step II chooses the parameters from the policy rule that minimize the social loss function. The optimal policy equal:[19]

(41)
$$\pi_t = (1-b)\pi^* + b\pi_t^e - \frac{\lambda\alpha}{1 - \beta\rho^2 + \lambda\alpha^2} \varepsilon_t,$$

where $0 \le b < 1$. The equilibrium inflation and employment rates equal the following:

(42)
$$\pi_t = \pi^* - \frac{\lambda\alpha}{1 - \beta\rho^2 + \lambda\alpha^2} \varepsilon_t \quad \text{and}$$

(43)
$$\ell_t = \rho\ell_{t-1} + \frac{1 - \beta\rho^2}{1 - \beta\rho^2 + \lambda\alpha^2} \varepsilon_t,$$

which equal the optimal outcomes. See equations (8) and (9).

Three characteristics of policy rule (41) emerge. First, it proves optimal because the equilibrium outcomes remain optimal as long as the central bank follows the rule. Second, the shock coefficient equals that of rule (35). That is, a unique way exists to respond optimally to the supply shock. Third, a class of optimal policy rules exists, which include the special case of rule (35). Without considering shocks, any inflation rate can do as long as it equals the weighted average of social target value and the private sector's inflation expectation, or in the simplest case, the inflation rate equals the social target $\pi_t = \pi^*$ $(b = 0, \varepsilon_t = 0)$.

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[19] See Appendix B for details.

5.   Design of Central Bank Short-Run Loss Functions

From our prior discussion, we know that the central bank can implement optimal policy by committing to the social intertemporal loss function, using discretion to the designed and delegated central bank intertemporal loss function, or just following the delegated optimal policy rule.

Supporting the equilibrium of an infinite-period dynamic game requires strong assumptions and making no mistakes, including extremely intelligent players. Shubik (1998) argues "…it can be proved that chess is an 'inessential game,' i.e., if one could do all the calculations there would be no reason to play chess as each side would have an optimal strategy (Zermelo, 1912)." (p. 3). Accordingly, we assume in this section that the central bank possesses bounded rationality and only minimizes the current period's central bank loss function. That is, the central bank implements its policy myopically. We will determine whether the central bank with bounded rationality can replicate optimal policy.

We assume that the central bank short-run (period) loss function equals the following relationship $L_t^b = \frac{1}{2}\left[\left(\pi_t - \pi_t^b\right)^2 + \lambda^b\left(\ell_t - \ell_t^b\right)^2\right]$, where the central bank uses state-contingent inflation and employment rate targets (i.e., $\pi^b = g_0 + g_1\ell_{t-1}$ and $\ell^b = h_0 + h_1\ell_{t-1}$). Since we design (choose) the parameters of the central bank loss function, we do not optimize myopically, even though the central banker does. That is, we minimize the infinite horizon social loss function with the knowledge that the central banker, who actually implements policy, only optimizes myopically. That is, the optimizing problem is expressed as follows:

$$\min_{g_0,g_1,h_0,h_1,\lambda^b} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\}$$

(44)
$$s.t. \begin{cases} \min_{\pi_t} \dfrac{1}{2} \left[\left(\pi_t - g_0 - g_1 \ell_{t-1} \right)^2 + \lambda^b \left(\ell_t - h_0 - h_1 \ell_{t-1} \right)^2 \right]. \\ s.t. \begin{cases} \ell_t = \rho \ell_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + \varepsilon_t \\ \pi_t^e = E_{t-1}\left(\pi_t \right) \end{cases} \end{cases}$$

As before, we solve this model in two steps and obtain an infinite number of optimal central bank period loss functions. With additional assumptions, we pin down the unique reasonable central bank period loss function from the infinite number of optimal candidates. Specifically, Step I computes the consistent or equilibrium outcomes for the inflation and employment rates, given that the central bank makes policy myopically from the central bank period loss function. Step II chooses the parameters, g_0, g_1, h_0, h_1, and λ^b, for the central bank loss function to minimize the expected social loss. Once again, an infinite combination of parameters exits that minimizes the expected social loss. In order to determine a unique solution, we choose the same parameter set as in Step II to minimize the central bank loss function.

Through the above steps, we determine the optimal central bank short-run loss function as follows:[20]

(45)
$$L_t^b = \frac{1}{2} \left[\left(\pi_t - \pi_t^b \right)^2 + \lambda^b \left(\ell_t - \ell_t^b \right)^2 \right],$$

where $\pi_t^b = \pi^*, \ell_t^b = \rho \ell_{t-1}$, and $\lambda^b = [\lambda /(1 - \beta \rho^2)]$. This outcome proves identical to the long-run central bank period loss function in equation (30).

Now, we can guess that the first-order condition of (45) must equal equation (34) or (38), because monetary policy under the short-run loss function (45) replicates optimal policy.

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[20] See Appendix C for details.

Obviously, the first-order condition of (45) equals:

(46) $$\left(\pi_t - \pi^*\right) + \lambda^b \left(\ell_t - \rho \ell_{t-1}\right) = 0,$$

where $\lambda^b = [\lambda/(1-\beta\rho^2)]$. Policy rule (46) replicates the rule (34) or (38).

Why does the central bank short-run loss function coincide with the period loss function of the long-run loss function? Notice from Appendix A, equation (A-26) that $\gamma_1 = \gamma_2 = 0$. This means that the central bank minimizes the intertemporal loss at zero with $\varepsilon_t = 0$ for all t, implying that the minimization of each period's loss occurs at zero, too. Conversely, if the central bank minimizes each period's loss at zero, then the minimization of the intertemporal loss also occurs at zero.

6.   Conclusion

This paper presents four ways of optimal policymaking, committing to the social loss function, using discretion and the central bank long-run and short-run loss functions, and following monetary policy rules. They all lead to optimal economic performance. The same performance emerges from these different policymaking methods because the central bank actually follows the same (similar) policy rules. Based on the results, we conclude that what matters in optimal policymaking is the way of policymaking.[21]

Specifically, the optimal policy benchmark comes from committing to the intertemporal social loss function subject to the economic structure. Then the benchmark optimal policy provides the goal for designing the central bank long-run and short-run loss functions, as well as the optimal policy rule. In short, the designed central bank long-run and short-run loss functions, as well as the optimal policy rule, emerge from optimizing the

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[21] We assume a perfect-information and, thus, the public knows how the central bank implements its policy.

intertemporal social loss function and the economic structure.

We reconcile consistent and optimal outcomes in our model structure. In our paper, the implementation of optimal monetary policy relies on perfect knowledge of the economic structure. The effects of economic model uncertainty on policymaking lie beyond our scope.

What implications emerge from these theoretical results for monetary policy? The three objectives (the social welfare criterion, central-bank long-run and short-run objectives) and optimal policy rules together constitute a regime for optimal policy making. The social intertemporal welfare function informs the public about the final objective of monetary policy. It provides the ultimate objective for monetary policy. The central bank long-run and short-run objective functions provide the means for achieving the ultimate social welfare objective. The public can understand the intermediate objectives of monetary policy, that is, how the central bank optimizes its objective function to achieve the social welfare optimum. In addition, monetary policy gains credibility and accountability with the intermediate and attainable objectives. The policy rules make monetary policy operational for the central bank. In other words, the social welfare criterion establishes the ultimate objective, central-bank long-run and short-run objectives provide the means to the end, and policy rules provide an operational short cut.

In conclusion, we advocate assigning objectives, not rules, for the central bank. First, we argue that "a central bank should have instrument independence, but should not have goal independence." (Fischer, 1995, p. 202). Second, the operations of the central bank becomes clear with explicit objectives. Specifically, the social welfare criterion and the central bank long-run and short-run objectives, as understood by the public, increases policy credibility

and accountability. "Specifying explicit objectives, together with operational independence and effective accountability structures is rightly considered essential in an effective monetary-policy setup" (Svensson, 2003, p. 454) Third, competent central bankers do not need instructions on how to operate monetary policy, if delegated clear objectives. With instrument independence, central bankers can optimize the delegated objectives. Finally, with explicit policy objectives, the public can easily understand monetary policy and, thus, make good choices. The public must infer, which they can do, the implied policy objectives, if monetary policy rules are delegated to the central bank

Appendix A   Design of Central Bank Long-Run Loss Functions

The model for determining the long-run central bank loss function is given as follows:

(A-1)

$$\min_{g_0,g_1,h_0,h_1,\lambda^b} E_0\left\{\sum_{t=1}^{\infty}\beta^{t-1}\cdot\frac{1}{2}\Big[\big(\pi_t-\pi^*\big)^2+\lambda\big(\ell_t-\ell^*\big)^2\Big]\right\}$$

$$s.t.\left\{ \min_{\{\pi_t\}_{t=1}^{\infty}} E_0\left\{\sum_{t=1}^{\infty}\beta^{t-1}\cdot\frac{1}{2}\Big[\big(\pi_t-g_0-g_1\ell_{t-1}\big)^2+\lambda^b\big(\ell_t-h_0-h_1\ell_{t-1}\big)^2\Big]\right\}. \atop s.t.\left\{\ell_t=\rho\ell_{t-1}+\alpha\big(\pi_t-\pi_t^e\big)+\varepsilon_t \atop \pi_t^e=E_{t-1}\big(\pi_t\big)\right.\right.$$

As noted in Section 3, we solve this model in two steps and obtain an infinite number of optimal central bank loss functions. With additional assumptions, we pin down the unique reasonable central bank loss function from the infinite number of optimal candidates. We denote this last step as Step III. Specifically, Step I computes the consistent, or equilibrium, policy when given the following short-run central bank loss function:

(A-2) $$L_t^b=\frac{1}{2}\left\{\Big[\pi_t-\big(g_0+g_1\ell_{t-1}\big)\Big]^2+\lambda^b\Big[\ell_t-\big(h_0+h_1\ell_{t-1}\big)\Big]^2\right\}.$$

The consistent, or equilibrium, policy comes from minimizing the long-run discounted central bank loss function by choosing the path for the inflation rate (i.e., $\{\pi_t\}_{t=1}^{\infty}$). Step II minimizes the long-run expected social loss function by choosing the parameters, g_0, g_1, h_0, h_1, and λ^b, which come from the short-run central-bank loss function. Finally, Step III pins down the precise central-bank loss function by choosing the parameters, g_0, g_1, h_0, h_1, and λ^b, to minimize the long-run central bank loss function.

Step I: Derivation of Consistent Policy, Given the Central Bank Loss Function

We first minimize the long-run central-bank loss function by choosing the path of the inflation rate. The following specifies the optimization problem:

$$(A\text{-}3) \quad \min_{\{\pi_t\}_{t=1}^{\infty}} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \left[\left(\pi_t - g_0 - g_1 \ell_{t-1} \right)^2 + \lambda^b \left(\ell_t - h_0 - h_1 \ell_{t-1} \right)^2 \right] \right\}$$

$$s.t. \begin{cases} \ell_t = \rho \ell_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + \varepsilon_t \\ \pi_t^e = E_{t-1} \left(\pi_t \right) \end{cases}$$

The Bellman equation for determining the optimal policy and outcomes from the optimization is given as follows:

$$(A\text{-}4) \quad V \left(\ell_{t-1} \right) = E_{t-1} \min_{\pi_t} \left\{ L_t^b + \beta V \left(\ell_t \right) \right\},$$

where L_t^b is given in equation (A-2). We minimize this relationship subject to the structure of the economy, given in equations (3) and (4).

The solution for $V(\ell_{t-1})$ must equal a quadratic form, since we minimize a quadratic objective function with linear constraints. Thus, the hypothesized solution equals the following equation:

$$(A\text{-}5) \quad V \left(\ell_t \right) = \gamma_0 + \gamma_1 \ell_t + \frac{1}{2} \gamma_2 \ell_t^2,$$

where we need to determine the unknown coefficients in equation (A-5).

The first-order condition for the minimization of equation (A-4) yields the following:

$$(A\text{-}6) \quad \pi_t - \left(g_0 + g_1 \ell_{t-1} \right) + \lambda^b \alpha \left[\ell_t - \left(h_0 + h_1 \ell_{t-1} \right) \right] + \alpha \beta \left(\gamma_1 + \gamma_2 \ell_t \right) = 0.$$

Rearranging the terms and substituting $\ell_t = \rho \ell_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + \varepsilon_t$ into the above equation produces the following:

$$(A\text{-}7) \quad \begin{aligned} &\pi_t - \left(g_0 + \lambda^b \alpha h_0 - \alpha \beta \gamma_1 \right) - \left(g_1 + \lambda^b \alpha h_1 \right) \ell_{t-1} + \alpha \left(\lambda^b + \beta \gamma_2 \right) \rho \ell_{t-1} \\ &+ \alpha^2 \left(\lambda^b + \beta \gamma_2 \right) \left(\pi_t - \pi_t^e \right) + \alpha \left(\lambda^b + \beta \gamma_2 \right) \varepsilon_t = 0. \end{aligned}$$

Finding the expected value of equation (A-7) and solving for the solutions for the inflation rate and employment gives the following results:

$$(A\text{-}8) \quad \pi_t = \left[g_0 + \alpha \left(\lambda^b h_0 - \beta \gamma_1 \right) \right] + \left[\left(g_1 + \lambda^b \alpha h_1 \right) - \alpha \rho \left(\lambda^b + \beta \gamma_2 \right) \right] \ell_{t-1} - \frac{(1-q)}{\alpha} \varepsilon_t \quad \text{and}$$

(A-9) $\qquad \ell_t = \rho\ell_{t-1} + q\varepsilon_t,$

where $q = q(\lambda^b, \gamma_2) = \left\{ 1/[1 + \alpha^2(\lambda^b + \beta\gamma_2)] \right\}.$

Now, compute $V(\ell_{t-1}) = E_{t-1}\min_{\pi_t}\left\{ L_t^b + \beta V(\ell_t) \right\}$ with the solutions for π_t and ℓ_t in

equations (A-8) and (A-9) and compare the coefficients with equation (A-5). Thus,

(A-10) $\qquad \gamma_2 = \left[\lambda^b\alpha h_1 - \alpha\rho(\lambda^b + \beta\gamma_2) \right]^2 + \lambda^b(h_1 - \rho)^2 + \beta\gamma_2\rho^2$ or $\gamma_2 \equiv \gamma_2(\lambda^b, h_1),$ and

(A-11) $\qquad \gamma_1 = \alpha\left(\lambda^b h_0 - \beta\gamma_1\right)\left[\lambda^b\alpha h_1 - \alpha\rho(\lambda^b + \beta\gamma_2) \right] + \lambda^b h_0(h_1 - \rho) + \beta\gamma_1\rho$ or

$$\gamma_1 \equiv \gamma_1(\lambda^b, h_0, h_1).$$

In sum, the consistent or equilibrium outcomes for the inflation rate and employment

appear in equations (A-8) and (A-9) with $q = q(\lambda^b, \gamma_2)$, $\gamma_2 \equiv \gamma_2(\lambda^b, h_1)$ and

$\gamma_1 \equiv \gamma_1(\lambda^b, h_0, h_1).$

Step II: *Determining the Central Bank Loss Function that Minimizes the Expected*

                Social Loss

Given our solution for the consistent or equilibrium outcomes for inflation and employment

that depend on the parameters of the central bank loss function (i.e., g_0, g_1, h_0, h_1, and λ^b),

we now choose values for those parameters to minimize the expected intertemporal social

loss. The analytical problem equals the following:

$$\min_{g_0,g_1,h_0,h_1,\lambda^b} E_0\left\{ \sum_{t=1}^{\infty}\beta^{t-1}\cdot\frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda(\ell_t - \ell^*)^2 \right] \right\}$$

(A-12)

$$s.t.\begin{cases} \pi_t = \left[g_0 + \alpha(\lambda^b h_0 - \beta\gamma_1) \right] + \left[(g_1 + \lambda^b\alpha h_1) - \alpha\rho(\lambda^b + \beta\gamma_2) \right]\ell_{t-1} - \dfrac{(1-q)}{\alpha}\varepsilon_t \\ \ell_t = \rho\ell_{t-1} + q\varepsilon_t \end{cases}$$

To solve this problem requires the recursive substitution for ℓ_{t-1} back to ℓ_0 in the

consistent outcomes in equations (A-8) and (A-9) and then substituting the solutions into the

social intertemporal welfare (loss) function. Carrying out the algebra leads to the following

solution for the expected social loss:

(A-13) $$L \equiv E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\} \equiv L(\mathrm{I}) + L(\mathrm{II}),$$

where

(A-14)
$$
\begin{aligned}
L(\mathrm{I}) &\equiv \frac{1}{(1-\beta)} \left[g_0 + \alpha \left(\lambda^b h_0 - \beta \gamma_1 \right) - \pi^* \right]^2 \\
&+ \frac{1}{(1-\beta\rho^2)} \left[\left(g_1 + \lambda^b \alpha h_1 \right) - \alpha\rho \left(\lambda^b + \beta\gamma_2 \right) \right]^2 \ell_0^2 \\
&+ \frac{2}{(1-\beta\rho)} \left[g_0 + \alpha \left(\lambda^b h_0 - \beta\gamma_1 \right) - \pi^* \right] \left[\left(g_1 + \lambda^b \alpha h_1 \right) - \alpha\rho \left(\lambda^b + \beta\gamma_2 \right) \right] \ell_0 \\
&+ \frac{\lambda \left(\ell^* \right)^2}{(1-\beta)} - \frac{2\lambda\rho\ell^*}{(1-\beta\rho)} \ell_0 + \frac{\lambda\rho^2}{(1-\beta\rho^2)} \ell_0^2
\end{aligned}
$$

and

(A-15)
$$
\begin{aligned}
L(\mathrm{II}) &\equiv q^2 \left[\left(g_1 + \lambda^b \alpha h_1 \right) - \alpha\rho \left(\lambda^b + \beta\gamma_2 \right) \right]^2 \frac{\beta}{(1-\beta)(1-\beta\rho^2)} \sigma^2 \\
&+ \frac{1}{(1-\beta)} \left(\frac{1-q}{\alpha} \right)^2 \sigma^2 + \frac{\lambda q^2}{(1-\beta)(1-\beta\rho^2)} \sigma^2
\end{aligned}
$$

$L(\mathrm{I})$ and $L(\mathrm{II})$ equal the components of the social welfare (loss) function that incorporates non-stochastic and stochastic terms, respectively. Choosing the values for the following parameters, g_0, g_1, h_0, h_1, and λ^b, to minimize the expected social loss yields the following conditions:

(A-16) $$g_0 + \alpha \left(\lambda^b h_0 - \beta\gamma_1 \right) - \pi^* = 0,$$

(A-17) $$\left(g_1 + \lambda^b \alpha h_1 \right) - \alpha\rho \left(\lambda^b + \beta\gamma_2 \right) = 0, \text{ and}$$

(A-18) $$q \left\{ = \frac{1}{1 + \alpha^2 \left(\lambda^b + \beta\gamma_2 \right)} \right\} = \frac{1 - \beta\rho^2}{1 - \beta\rho^2 + \lambda\alpha^2}.$$

As long as the 7 parameters, g_0, g_1, h_0, h_1, λ^b, γ_1, and γ_2, of the central bank loss function satisfy equations (A-10), (A-11), (A-16), (A-17), and (A-18), the consistent policy

will prove optimal. Note also that by using equations (A-16) and (A-17), we can rewrite equations (A-10) and (A-11) as follows:

(A-19) $\gamma_2 = [g_1]^2 + \lambda^b (h_1 - \rho)^2 + \beta\gamma_2\rho^2$ or $\gamma_2 = \dfrac{[g_1]^2 + \lambda^b (h_1 - \rho)^2}{1 - \beta\rho^2}$, and

(A-20) $\gamma_1 = (g_0 - \pi^*)[g_1] + \lambda^b h_0 (h_1 - \rho) + \beta\gamma_1\rho$ or $\gamma_1 = \dfrac{(g_0 - \pi^*)[g_1] + \lambda^b h_0 (h_1 - \rho)}{1 - \beta\rho}$.

By using equations (A-16) to (A-18), the consistent policy outcomes for the inflation and employment rates (A-8) and (A-9) equal:

(A-21) $\pi_t = \pi^* - \dfrac{\lambda\alpha}{1 - \beta\rho^2 + \lambda\alpha^2}\varepsilon_t$ and

(A-22) $\ell_t = \rho\ell_{t-1} + \dfrac{1 - \beta\rho^2}{1 - \beta\rho^2 + \lambda\alpha^2}\varepsilon_t$,

which equal their optimal outcomes. See equations (8) and (9).

Step III: Determining the Central Bank Loss Function that Minimizes the Its Expected Loss

An infinite number of solutions for the parameters g_0, g_1, h_0, h_1, λ^b, γ_1, and γ_2 satisfy equations (A-16) to (A-20) and minimize the social loss function. Only one set of those parameter values, however, will also minimize the central bank loss function. That is, we want to choose parameter values for g_0, g_1, h_0, h_1, and λ^b to solve the following problem:

(A-23)
$$\min_{g_0, g_1, h_0, h_1, \lambda^b} \frac{1}{2} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \left[\pi_t - (g_0 + g_1\ell_{t-1}) \right]^2 + \lambda^b \left[\ell_t - (h_0 + h_1\ell_{t-1}) \right]^2 \right\} \right\}$$
$$s.t. \begin{cases} \pi_t = \pi^* - \dfrac{\lambda\alpha}{1 - \beta\rho^2 + \lambda\alpha^2}\varepsilon_t \\ \ell_t = \rho\ell_{t-1} + \dfrac{1 - \beta\rho^2}{1 - \beta\rho^2 + \lambda\alpha^2}\varepsilon_t \end{cases}.$$

The solution of this problem first requires the recursive substitution for ℓ_{t-1} back to ℓ_0 into the employment rate equation. Then substitute the values of the optimal inflation and

employment rates into the central bank loss function and calculate the value of the expected

central bank loss as follows:

$$L^b \equiv E_0 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \left[\pi_t - \left(g_0 + g_1 \ell_{t-1} \right) \right]^2 + \lambda^b \left[\ell_t - \left(h_0 + h_1 \ell_{t-1} \right) \right]^2 \right\}$$

(A-24)

,

$$\equiv L^b \left(\text{I} \right) + L^b \left(\text{II} \right)$$

where

$$L^b \left(\text{I} \right) \equiv \frac{1}{\left(1-\beta\right)} \left(g_0 - \pi^* \right)^2 + \frac{1}{\left(1-\beta\rho^2\right)} \left(g_1 \ell_0 \right)^2 + \frac{2}{\left(1-\beta\rho\right)} \left(g_0 - \pi^* \right) g_1 \ell_0$$

$$+ \lambda^b \frac{1}{\left(1-\beta\right)} h_0^2 + \lambda^b \frac{1}{\left(1-\beta\rho^2\right)} \left(h_1 - \rho \right)^2 \ell_0^2 + \lambda^b \frac{2}{\left(1-\beta\rho\right)} h_0 \left(h_1 - \rho \right) \ell_0$$

and

$$L^b \left(\text{II} \right) \equiv \left(g_1 \frac{1-\beta\rho^2}{1-\beta\rho^2 + \lambda\alpha^2} \right)^2 \frac{\beta}{\left(1-\beta\right)\left(1-\beta\rho^2\right)} \sigma^2 + \frac{1}{\left(1-\beta\right)} \left(\frac{\lambda\alpha}{1-\beta\rho^2 + \lambda\alpha^2} \right)^2 \sigma^2$$

$$+ \lambda^b \left(h_1 - \rho \right)^2 \left(\frac{1-\beta\rho^2}{1-\beta\rho^2 + \lambda\alpha^2} \right)^2 \frac{\beta}{\left(1-\beta\right)\left(1-\beta\rho^2\right)} \sigma^2 + \lambda^b \frac{1}{\left(1-\beta\right)} \sigma^2$$

L^b(I) and L^b(II) equal the components of the social welfare (loss) function that

incorporates non-stochastic and stochastic terms, respectively. Choosing the values for the

following parameters, g_0, g_1, h_0, h_1, and λ^b, to minimize the expected central bank loss

yields the following results:

(A-25) $g_0 = \pi^*$, $g_1 = 0$, $h_0 = 0$ and $h_1 = \rho$.

Substituting these conditions into equations (A-19) and (A-20) yields:

(A-26) $\gamma_1 = \gamma_2 = 0$.

Using equation (A-18) and the definition of q that follows equation (A-9) produces:

(A-27) $\lambda^b = \left[\lambda / \left(1 - \beta\rho^2\right) \right]$.

Finally we determine the optimal central bank long-run loss function as follows:

(A-28) $L_t^b = \frac{1}{2} \left[\left(\pi_t - \pi_t^b \right)^2 + \lambda^b \left(\ell_t - \ell_t^b \right)^2 \right]$,

where $\pi_t^b = \pi^*$, $\ell_t^b = \rho \ell_{t-1}$, and $\lambda^b = \left[\lambda / \left(1 - \beta\rho^2\right) \right]$.

Appendix B   Design of Monetary Policy Rules

The model for determining the monetary policy rule equals the following:

$$\min_{a,b,c,d} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\}$$

(B-1)
$$s.t. \begin{cases} \pi_t = a + b\pi_t^e + c\ell_{t-1} + d\varepsilon_t \\ \ell_t = \rho\ell_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + \varepsilon_t \\ \pi_t^e = E_{t-1} \left(\pi_t \right) \end{cases}.$$

Solve the model in two steps. Step I computes the equilibrium policy under the given policy

rule, $\pi_t = a + b\pi_t^e + c\ell_{t-1} + d\varepsilon_t$. Step II chooses the parameters, a, b, c and d for the policy

rule to minimize the expected total social loss.

Step I: Determination of Consistent Policy, Given the Policy Rule

When the central bank just follows the rule in equation (39) and interacts with the private

sector, the problem equals:

(B-2)
$$\begin{cases} \pi_t = a + b\pi_t^e + c\ell_{t-1} + d\varepsilon_t \\ \ell_t = \rho\ell_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + \varepsilon_t \\ \pi_t^e = E_{t-1} \left(\pi_t \right) \end{cases}.$$

Solving for the equilibrium outcomes for the inflation and employment rates yields the

following:

(B-3) $\pi_t = \dfrac{a}{1-b} + \dfrac{c}{1-b} \ell_{t-1} + d\varepsilon_t$ and

(B-4) $\ell_t = \rho\ell_{t-1} + \left(1 + \alpha d \right)\varepsilon_t$.

Step II: Determining the Policy Rule that Minimizes the Expected Social Loss

Now, choose the parameters a, b, c, and d to minimize the expected total social loss, given the

equilibrium outcomes. That is, solve the following problem:

$$\min_{a,b,c,d} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\}$$

(B-5)
$$s.t. \begin{cases} \pi_t = \dfrac{a}{1-b} + \dfrac{c}{1-b} \ell_{t-1} + d\varepsilon_t \\ \ell_t = \rho \ell_{t-1} + \left(1 + \alpha d\right) \varepsilon_t \end{cases}$$

We need first to substitute recursively into the equilibrium outcomes for the inflation

and employment rates for ℓ_{t-1} back to ℓ_0 and then calculate the expected value of the

social loss function as follows:

(B-6) $$L \equiv E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\} \equiv L(\mathrm{I}) + L(\mathrm{II}),$$

where

$$L(\mathrm{I}) \equiv \frac{1}{1-\beta} \left(\frac{a}{1-b} - \pi^* \right)^2 + \frac{1}{1-\beta\rho^2} \left(\frac{c}{1-b} \right)^2 \ell_0^2 + 2 \frac{1}{1-\beta\rho} \left(\frac{a}{1-b} - \pi^* \right) \frac{c}{1-b} \ell_0$$

(B-7)
$$+ \frac{\lambda \left(\ell^* \right)^2}{1-\beta} - \frac{2\lambda\rho\ell^*}{1-\beta\rho} \ell_0 + \frac{\lambda\rho^2}{1-\beta\rho^2} \ell_0^2$$

and

$$L(\mathrm{II}) \equiv \left(\frac{c}{1-b} \right)^2 \left(1 + \alpha d\right)^2 \frac{\beta}{(1-\beta)(1-\beta\rho^2)} \sigma^2 + \lambda \left(1 + \alpha d\right)^2 \frac{1}{(1-\beta)(1-\beta\rho^2)} \sigma^2$$

(B-8)
$$+ \frac{1}{1-\beta} d^2 \sigma^2.$$

$L(\mathrm{I})$ and $L(\mathrm{II})$ equal the components of the social welfare (loss) function that

incorporate non-stochastic and stochastic terms, respectively. Choosing the values for the

following parameters, $a, b, c, and\ d$, to minimize the expected social loss yields the

following conditions:

(B-9) $$\frac{a}{(1-b)} = \pi^*,$$

(B-10) $$\frac{c}{(1-b)} = 0,$$

(B-11) $$d = -\frac{\lambda\alpha}{1-\beta\rho^2 + \lambda\alpha^2}$$

As a result, the optimal policy equals the following:

(B-12) $$\pi_t = (1-b)\pi^* + b\pi_t^e - \frac{\lambda\alpha}{1-\beta\rho^2 + \lambda\alpha^2}\varepsilon_t,$$

where $0 \le b < 1$. The equilibrium inflation and employment rates equal the following:

(B-13) $$\pi_t = \pi^* - \frac{\lambda\alpha}{1-\beta\rho^2 + \lambda\alpha^2}\varepsilon_t \quad \text{and}$$

(B-14) $$\ell_t = \rho\ell_{t-1} + \frac{1-\beta\rho^2}{1-\beta\rho^2 + \lambda\alpha^2}\varepsilon_t,$$

which equal the optimal outcomes. See equations (8) and (9).

Appendix C   Design of Central Bank Short-Run Loss Functions

The optimizing problem equals the following system:

(C-1)
$$\min_{g_0,g_1,h_0,h_1,\lambda^b} E_0\left\{\sum_{t=1}^{\infty}\beta^{t-1}\cdot\frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda(\ell_t - \ell^*)^2\right]\right\}$$
$$s.t.\begin{cases}\min_{\pi_t}\frac{1}{2}\left[(\pi_t - g_0 - g_1\ell_{t-1})^2 + \lambda^b(\ell_t - h_0 - h_1\ell_{t-1})^2\right].\\ s.t.\begin{cases}\ell_t = \rho\ell_{t-1} + \alpha(\pi_t - \pi_t^e) + \varepsilon_t\\ \pi_t^e = E_{t-1}(\pi_t)\end{cases}\end{cases}$$

Step I: Derivation of Consistent Policy, Given the Central Bank Loss Function

To determine the consistent or equilibrium outcomes, we solve the following problem:

(C-2)
$$\min_{\pi_t}\frac{1}{2}\left[(\pi_t - g_0 - g_1\ell_{t-1})^2 + \lambda^b(\ell_t - h_0 - h_1\ell_{t-1})^2\right]$$
$$s.t.\begin{cases}\ell_t = \rho\ell_{t-1} + \alpha(\pi_t - \pi_t^e) + \varepsilon_t\\ \pi_t^e = E_{t-1}(\pi_t)\end{cases}$$

The equilibrium inflation and employment outcomes equal the following:

(C-3) $$\pi_t = g_0 + \lambda^b\alpha h_0 + (g_1 + \lambda^b\alpha h_1 - \lambda^b\alpha\rho)\ell_{t-1} - \lambda^b\alpha q\varepsilon_t \quad \text{and}$$

(C-4) $$\ell_t = \rho\ell_{t-1} + q\varepsilon_t,$$

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where $q \equiv [1/(1+\lambda^b \alpha^2)]$.

Step II: *Determining the Central Bank Loss Function that Minimizes the Expected Social Loss*

Now, we choose the parameters, g_0, g_1, h_0, h_1, and λ^b, to minimize the expected total social loss. The problem equals the following system:

(C-5)
$$\min_{g_0, g_1, h_0, h_1, \lambda^b} E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \cdot \frac{1}{2} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\}$$
$$s.t. \begin{cases} \pi_t = g_0 + \lambda^b \alpha h_0 + \left(g_1 + \lambda^b \alpha h_1 - \lambda^b \alpha \rho \right) \ell_{t-1} - \lambda^b \alpha q \varepsilon_t \\ \ell_t = \rho \ell_{t-1} + q \varepsilon_t \end{cases}$$

We need first to substitute recursively into the equilibrium outcomes for the inflation and employment rates for ℓ_{t-1} back to ℓ_0 and then calculate the expected value of the social loss function as follows:

(C-6)
$$L \equiv E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left[\left(\pi_t - \pi^* \right)^2 + \lambda \left(\ell_t - \ell^* \right)^2 \right] \right\} \equiv L(\mathrm{I}) + L(\mathrm{II}),$$

where

(C-7)
$$L(\mathrm{I}) = \frac{1}{(1-\beta)} \left(g_0 + \lambda^b \alpha h_0 - \pi^* \right)^2 + \frac{1}{(1-\beta\rho^2)} \left(g_1 + \lambda^b \alpha h_1 - \lambda^b \alpha \rho \right)^2 \ell_0^2$$
$$+ \frac{2}{(1-\beta\rho)} \left(g_0 + \lambda^b \alpha h_0 - \pi^* \right) \left(g_1 + \lambda^b \alpha h_1 - \lambda^b \alpha \rho \right) \ell_0 + \frac{\lambda \left(\ell^* \right)^2}{(1-\beta)}$$
$$- \frac{2\lambda\rho\ell^*}{(1-\beta\rho)} \ell_0 + \frac{\lambda\rho^2}{(1-\beta\rho^2)} \ell_0^2$$

and

(C-8)
$$L(\mathrm{II}) = q^2 \left(g_1 + \lambda^b \alpha h_1 - \lambda^b \alpha \rho \right)^2 \frac{\beta}{(1-\beta)(1-\beta\rho^2)} \sigma^2 + \frac{1}{(1-\beta)} \left(\frac{1-q}{\alpha} \right)^2 \sigma^2$$
$$+ \frac{\lambda q^2}{(1-\beta)(1-\beta\rho^2)} \sigma^2 .$$

L(I) and *L*(II) equal the components of the social welfare (loss) function that

incorporate non-stochastic and stochastic terms, respectively. Choosing the values for the following parameters, g_0, g_1, h_0, h_1, and λ^b, to minimize the expected social loss yields the following conditions:

(C-9) $g_0 + \lambda^b \alpha h_0 - \pi^* = 0$,

(C-10) $g_1 + \lambda^b \alpha h_1 - \lambda^b \alpha \rho = 0$, and

(C-11) $q\left\{ = \dfrac{1}{1 - \lambda^b \alpha^2} \right\} = \dfrac{1 - \beta \rho^2}{1 - \beta \rho^2 + \lambda \alpha^2} \quad \Rightarrow \quad \lambda^b = \dfrac{\lambda}{1 - \beta \rho^2}$.

As a result, the equilibrium inflation and employment outcomes equal the following:

(C-12) $\pi_t = \pi^* - \dfrac{\lambda \alpha}{1 - \beta \rho^2 + \lambda \alpha^2} \varepsilon_t$ and

(C-13) $\ell_t = \rho \ell_{t-1} + \dfrac{1 - \beta \rho^2}{1 - \beta \rho^2 + \lambda \alpha^2} \varepsilon_t$,

which equal the optimal outcomes. See equations (8) and (9).

As long as the four parameters, g_0, g_1, h_0, and h_1,[22] of the central bank loss function satisfy equations (C-9) and (C-10), the consistent policy under the myopic central bank proves optimal.

Step III: Determining the Central Bank Loss Function that Minimizes Its Expected Loss

An infinite number of solutions for the parameters g_0, g_1, h_0, and h_1 satisfy equations (C-9) and (C-10) and minimize the expected social loss. Only one set of those parameter values will also minimize the central bank short-run (period) loss function, where the central bank operates myopically. That is, we want to choose parameter values for g_0, g_1, h_0, and h_1 to solve the following problem:

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[22] Equation (C-11) pins down the value of λ^b.

$$\min_{g_0,g_1,h_0,h_1} \frac{1}{2} E_{t-1}\left\{\left[\pi_t - \left(g_0 + g_1\ell_{t-1}\right)\right]^2 + \lambda^b\left[\ell_t - \left(h_0 + h_1\ell_{t-1}\right)\right]^2\right\}$$

(C-14)
$$s.t.\begin{cases} \pi_t = \pi^* - \dfrac{\lambda\alpha}{1-\beta\rho^2+\lambda\alpha^2}\varepsilon_t \\[2mm] \ell_t = \rho\ell_{t-1} + \dfrac{1-\beta\rho^2}{1-\beta\rho^2+\lambda\alpha^2}\varepsilon_t \end{cases}.$$

Since the central bank operates myopically, we do not need to recursively substitute for ℓ_{t-1} back to ℓ_0. Thus, we merely substitute the consistent or equilibrium outcomes into the central bank loss function and minimize with respect to g_0, g_1, h_0, and h_1. The expected value of the central bank loss function with the consistent or equilibrium outcomes reduces to the following:

(C-15)
$$E_{t-1}\left(L_t^b\right) = \frac{1}{2}\left(g_0 - \pi^* + g_1\ell_{t-1}\right)^2 + \frac{1}{2}\left(\frac{1-q}{\alpha}\right)^2\sigma^2 + \lambda^b\frac{1}{2}\left[h_0 + \left(h_1 - \rho\right)\ell_{t-1}\right]^2$$
$$+ \lambda^b\frac{1}{2}q^2\sigma^2.$$

Choosing the parameters g_0, g_1, h_0, and h_1 to minimize the expected value of the central bank loss function produces the following results:

(C-16) $g_0 - \pi^* + g_1\ell_{t-1} = 0$ and

(C-17) $h_0 + \left(h_1 - \rho\right)\ell_{t-1} = 0$.

To make the inflation target in the central bank loss function independent of the state variable ℓ_{t-1}, a reasonable assumption, we set the parameter values as follows:

(C-18) $g_0 = \pi^*, g_1 = 0, h_0 = 0$ and $h_1 = \rho$.

Finally, this produces the optimal central bank loss function as follows:

(C-19) $L_t^b = \frac{1}{2}\left[\left(\pi_t - \pi_t^b\right)^2 + \lambda^b\left(\ell_t - \ell_t^b\right)^2\right]$,

where $\pi_t^b = \pi^*, \ell_t^b = \rho\ell_{t-1}$, and $\lambda^b = \left[\lambda/(1-\beta\rho^2)\right]$. This loss function matches the loss function derived for the long-run intertemporal optimizing central bank. See equation (30).

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