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Holographic Analysis of Tooth Displacement Resulting From Known Axial Loads

Thomas W. Every

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Resulting from Known Axial Loads

Thomas W. Every
1979
Holographic Analysis of Tooth Displacement
Resulting from Known Axial Loads

Thomas W. Every, M. Dent. Sc.
The University of Connecticut, 1979

The purpose of this study was to determine the relationship of the rate of tooth displacement and the magnitude of tooth displacement due to a known extrusive force applied along the long axis of the tooth.

An orthodontic band with an attached lingual hook and a labial reference wire was cemented to a human maxillary incisor. The hook and a pulley system was used for loading at a force level of 50, 100, 200 and 300 grams. Lateral and occlusal radiographs were used to determine the tooth geometry and the position of the reference wire with respect to the long axis of the tooth. The reference wire was used to insure that the loading would lie as closely as possible to the root centroid of the tooth. Tooth motion was recorded from the instant of loading using double-exposure holograms taken at known time intervals over a period of two minutes. Holograms were reconstructed, photographed, photographs digitized and displacements and rotation calculated.
Based on the experimental results from holographic analysis of tooth displacement using axial loads, the relationship between force magnitudes, time and displacement was developed. Analysis of the data indicated that the crown of the incisor translated less than movement observed for loadings normal to the tooth using identical force levels. Furthermore, the results indicate that velocity is greatest initially, decreasing non-linearly with respect to time. The center of rotation, in all cases, was found in a plane parallel to the root centroid of the tooth. The center of rotation remained relatively constant over the entire loading. Force magnitudes and time were shown to be significant parameters in predicting axial translation of the teeth.
HOLOGRAPHIC ANALYSIS OF TOOTH DISPLACEMENT
RESULTING FROM KNOWN AXIAL LOADS

Presented by
Thomas W. Every, B.S., D.D.S.
ACKNOWLEDGEMENTS

The author wishes to express sincere appreciation to Dr. Charles J. Burstone for his guidance and counsel during this study. His help and knowledge proved to be invaluable.

The author is greatly indebted to Dr. Ryszard J. Pryputniewicz for the enormous help, encouragement and generous use of his expert assistance which made this study possible.

Acknowledgement is hereby extended to Dr. A. Jon Goldberg and Dr. Bruce Goldin for their aid in preparation of this thesis.

A special thanks is extended to Eugene Pryputniewicz for his most valuable technical assistance.

The author is forever indebted to his wife, Valerie, for her typing and proofreading of this manuscript. Her many personal sacrifices and her sustained understanding gave this effort a special meaning. It is to Valerie that this thesis is dedicated.
This project was supported in part by the National Institute of Health Grant No. DE 03545 and the Connecticut Research Foundation Grant No. 062.
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HOLOGRAPHIC ANALYSIS OF TOOTH DISPLACEMENT
RESULTING FROM KNOWN AXIAL LOADS

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A Thesis Submitted in Partial Fulfillment of the Requirements
for the Master of Dental Science Degree
at
The University of Connecticut
1979
CHAPTER 1

INTRODUCTION

One of the major mechanisms for predicting tooth movement is precise measurement of tooth displacement due to an applied force system. Investigation of tooth displacement as a function of an applied force system represents a basic approach to understanding the biological response involved in tooth movement.

In this thesis, a new non-invasive method that will allow precise and accurate measurement of three-dimensional displacements of teeth was utilized. This method, based on the principles of holography, is capable of determining tooth translations and rotations independent of rigid-body motion.

Translations and rotations of human maxillary central incisors were measured over a period of two minutes, with forces of variable but controlled magnitude and direction acting upon the crown of the tooth.

The time rate of crown displacement (linear velocity) was computed and plotted against the time measured from the instant of application of the force. These results were interpreted by integrating the area under the time rate of change curves. The velocity curves were interpreted to obtain total displacements and rotations. The total displacements and rotations were, in turn, used to compute the centers of rotation representing the type of tooth motion achieved during loading.
Based on the experimental results, the relationship between the force magnitude and rate of displacement was developed. The holographically determined tooth displacements were compared with some existing experimental results obtained by various researchers.

The results in this study will give significant insights into the type of tooth movement achieved and the biological response due to a known axial force system. This will aid in an understanding of the relationship between stress-strain distributions in the periodontal ligament and the biological functions involved in tooth movement.
CHAPTER 2

REVIEW OF LITERATURE

Accurate measurement of tooth displacement has become an important pathway for investigation of the biological response involved in tooth movement. Predictable tooth movement due to an applied force system requires a precise knowledge of tooth displacement. Knowledge of this relationship offers valuable insight into prediction of tooth movement, description of the type of tooth movement achieved, and estimation of stress in the periodontal ligament.¹

Burstone has stated that stress-strain studies offer the best opportunity to understand the biological response of the structures supporting the tooth.² Unfortunately, because of the nature of these structures, direct measurement of stress is limited.

Stress patterns in the periodontal ligament due to an applied force system produce strain. If this strain is large enough, tooth movement is observed.³ Displacement data is useful in the measurement of strain, which in turn, is important in an attempt at understanding stress.⁴ In the future, if clues to the constitutive behavior of the periodontal ligament are obtained through experimental analysis, tooth displacement data could be utilized to compute strain and attempt to estimate stress distributions at the interface of the periodontal ligament.
The need to correlate force application with tooth movement as a method for understanding the biological system involved in tooth movement has been pointed out by Burstone. Past studies have shown that accurate measurement of tooth displacement is a critical factor in evaluation of bone loss in periodontal disease, the tooth response to traumatic occlusion and tooth response to orthodontic and prosthetic appliances. Muhleman has emphasized the importance of tooth displacement studies and the considerable interest in this area, specifically as clinical diagnostic parameters of integrity, functional state, and disease of the periodontium.

In orthodontic treatment assumptions have been made such as stress-strain distributions in the periodontal ligament, the type of tooth movement achieved, and the biological response due to an applied force system. However, there is little reliable experimental data available concerning tooth displacement caused by a known load. Precision measurement of tooth movement caused by external forces represents a basic problem in orthodontics.

Previous studies of tooth displacement were seriously restricted by the limitations of the measuring devices used. Since the maximum tooth deflections are small, less than 0.5 mm (500 μm), inertial and deflection errors of a measuring device would place several limitations upon the accuracy of results. Also, because tooth movement occurs in three dimensions, the measuring system employed must measure displacement in all
three directions. A complete description of tooth movement requires knowledge of three components of displacement and three rotations about the axes of a coordinate system. Attempts to solve the problem of accurate measurement of tooth displacement have been restricted because of the physical limitations of the instrumentation employed. There has been a great deal of research concerning tooth movement with a variety of measurement systems. Many improvements in techniques and instrumentation over earlier methods have permitted more precise measurement of tooth displacement. However, not only the accuracy, but the ability to measure three-dimensional displacements must be carefully considered.

Muhleman's studies resulted in the development of a high precision mechanical gauge to measure horizontal tooth mobility. This device consisted of an intraorally attached dial indicator to determine the amount of crown movement produced by static forces. These experimental results were expressed as tooth mobility curves and rotation centers of teeth. Muhleman's measuring device registered displacements as small as 0.005 mm (5 µm). However, these measurements were restricted to only one direction and the overall sensitivity of the method was reduced due to repositioning errors.

O'Leary and Rudd utilized a measurement device similar to Muhleman's to investigate change in lateral mobility. A modification of this device was used by Noble and Martin to study the relationship of the degree of tooth mobility to
the position of occlusal interferences. Again this device was restricted to measurement in only one direction but with an increased accuracy of .0025 mm (2.5 μm).

Also, there were measuring techniques such as the one used by Lear and MacKay which employed electronic equipment with an intraoral splint supporting a solenoid and linear voltage displacement transducer to study horizontal movement of teeth. A somewhat different electronic system was used by Daly et al. to test torsional loads. A torque loading device was used with gauges attached to the tooth and to a cast maxillary clutch.

Although lateral tooth movements have been measured by Muhleman and other researchers, there are few devices which have been described that are capable of measuring the small physiological axial tooth movements. This fact was highlighted by Muhleman when he stressed that previous investigations have been more successful in measuring horizontal tooth displacement because vertical displacement was found to be much smaller than horizontal displacement.

Several investigators have attempted to study vertical tooth movement. Parfitt employed a coil with a movable core to study vertical tooth displacement. He obtained continuous time movement recordings with a wide range of forces. Picton measured axial tooth movement by means of transducers of movement incorporating resistance-strain gauges attached to the teeth. One end of a gauge was attached to a single
tooth and the other through a spring to the adjacent teeth.

Regardless of the type of recording equipment, whether it was a mechanical measuring gauge or electronic equipment system; these investigations employed invasive, intraoral devices which placed severe limitations upon the accuracy of their results.

Bowley et al. has stated that research concerning tooth displacement has been limited in attempts to attain reliable data because most of the experiments:

1. Employed forces that produced three-dimensional displacements of teeth, yet the tooth displacement was measured along one axis only.

2. Produced a three-dimensional tooth displacement which resulted from an applied force with three components, yet the force if monitored at all was measured in one axis only. Since the tooth displacement was measured along one axis only, there was no way of calculating the three-dimensional components of the applied force.

3. Employed a force system whose magnitude changed with any deflection, yet often the force magnitude was not measured as the tooth deflected.

4. Used deflection and force measuring systems whose accuracy was suspect because of mechanical inertia and deflection in the measuring instruments themselves.
In order to overcome the shortcomings of previous investigators a new method was developed using laser holography which permits more precise measurement of tooth displacement. Through the use of laser holography it is now possible to gather qualitative as well as quantitative data which minimizes many of the deficiencies mentioned. With laser holography small tooth displacements in three-dimensions are detected with great accuracy and in a non-invasive manner. This technique has been used successfully to measure tooth displacement by Burstone et al. with an accuracy of .05 μm in three-dimensional space.
CHAPTER 3

GENERAL OBJECTIVES

The objective of this study was to determine the relationship between the rate of tooth displacement and the magnitude of tooth displacement due to forces applied in an extrusive direction parallel to the long axis of the tooth. Actual tooth motion due to an axial load was described in terms of centers of rotation. Location of the center of rotation of a tooth as a function of an applied force is a particular method of representing tooth movement which has meaning to the clinician. Description of tooth motions in terms of centers of rotation requires accurate measurement of tooth displacement in three-dimensional space.

In order to meet the objectives of this experimental study of tooth displacement, a laser holography technique was utilized. This new non-invasive method allowed accurate measurement of three-dimensional displacements of teeth. Experimental results were correlated with the characteristics of subject tooth geometry, particularly with respect to the root dimensions. The holographic determined direction and magnitude of tooth displacements were used to compute the instantaneous centers of rotation.

Based on the objectives of this study, the experimental results from holographic analysis of tooth displacement due to
axial loads were utilized to develop the relationships between force magnitudes, time and displacements. The intent of this study was to gain insights into the type of tooth movement achieved and the biological response due to a known axial force system. This would in turn aid in an understanding of the relationship between stress-strain distributions in the periodontal ligament and the biological functions involved in tooth movement.
Holographic analysis of tooth displacement resulting from known axial loads was performed in vivo on four subjects; three males and one female of ages ranging from age 20 to 27. An orthodontic band was cemented to a maxillary central incisor for each subject. Impressions, photographs and periodontal measurements were taken. The impressions for study models were taken both before and after the orthodontic band was cemented to the maxillary central incisor. Attached to the orthodontic band was a lingual hook and a labial tube. In order to produce tooth displacement, a force system was applied to the tooth using a pulley system. A nylon string was attached to the tooth by way of the lingual hook on the band, then the line was threaded through the pulley system where loads of 50, 100, 200 and 300 grams were applied. Loading in this manner insured constancy of the magnitude of the extrusive force (see Figure 1).

In order to insure constancy of direction of the extrusive force, the labial tube on the band was used for placement of a reference wire bent to a known three-dimensional shape. Figure 2 indicates the type of reference wire used during loadings. The wire extended 20 mm perpendicular to the long axis of the
tooth. A second segment of the wire, 15 mm long, was bent at right angles to the 20 mm segment and was made parallel to the long axis of the tooth. Occlusal and lateral radiographs were used to determine the relative position of the reference wire with respect to the long axis and mesial-distal axis of the tooth. Figure 3 displays a tracing of a lateral radiograph which demonstrates the position of a reference wire relative to the tooth. The radiographs, Figures 3 and 4, in conjunction with the study models were to determine tooth geometry. It was essential that the tooth be acted upon by a force with a line of action parallel to the long axis of the tooth. This line of action was determined with respect to the reference wire. Before loading the tooth, the nylong string (which determined the line of action of the force), was positioned by adjusting the pulley so that the nylon string was parallel to the long axis of the tooth as determined with respect to the reference wire. Knowledge of the loading geometry and the tooth position was essential in order to meaningfully interpret the experiment results.

The maxillary central incisor was loaded in an axial (parallel to the long axis of the tooth) direction. As previously described, occlusal and lateral radiographs were used to determine the relationship between the rectangular wire and the tooth geometry. The loading geometry that was employed is demonstrated in Figures 1 and 5. As seen in these figures,
a load activates a microswitch which fires the laser at a predetermined load level. Based on previous force level studies of tissue response to axial force systems, loads of 50, 100, 200 and 300 grams were investigated.\textsuperscript{23,24} Double-exposure holograms were taken over a time period of two minutes from the instant of application of the force system, with an interval of 30 seconds between each double-exposure. The total displacement was then determined.

This study employed a double-exposure method of hologram interferometry. In this method, two separate exposures of the tooth were made on the same recording medium. Illumination for the recording of holograms was provided by a pulsed ruby laser. As shown in Figure 6, the laser beam was divided into two parts by means of a beam splitter. One of these parts reflected from the beam splitter was directed by mirrors and expanded by a negative lens to illuminate the holographic plate. This was a reference beam against which the modulated object beam was recorded. The object beam was a part of the laser output that went directly through the beam splitter. It was expanded and directed to illuminate the subject. The portion of the object beam reflected by the subject was intercepted by the holographic plate where it interfered with the object beam. This interference was recorded within the emulsion of the holographic plate.

As mentioned above, double-exposure holography involves two separate exposures recording the initial and final positions
of the patient's tooth on the same hologram. The time delay between the first and the second exposure was 450 msec. During this time delay, the tooth was being displaced by a known extrusive force. Upon reconstruction of the hologram, two images are formed faithfully representing tooth position before and after displacement. Since these images were reconstructed in coherent (laser) light, they interfered with each other. As a result of this interference, the tooth was covered by a set of alternating bright and dark lines called fringes. These fringes were a direct measure of the change in tooth position which occurred between two exposures. This information was then interpreted to obtain the displacements and rotations.

In order to improve the accuracy of the experimental results and to aid data reduction, a specially designed miniature tetrahedron of known geometry was cemented to the maxillary central incisor. The tetrahedron was photographed from the holograms and traced along with the fringe lines present due to the tooth displacement. The holograms were observed and photographed from several different directions. The number of observations was dependent upon the complexity of the fringe patterns. Using the system shown in Figure 7, photographs were digitized and displacements and rotations calculated using the computer program developed by Pryputniewicz. Figure 8 shows the tetrahedron on the computer
screen after the fringe lines (the dashed lines on the figure) have been digitized.

During the interval between exposures, gross head motions were minimized by using ear rods and a nasal support as shown in Figures 9 and 10. It was the short duration of the pulse which was responsible for essentially "freezing" the head motion. However, certain motions of the head will still take place causing tooth motion due to the extrusive loading to be superimposed with the rigid-body motion of the patient's head. Thereby causing the motions of the head to be included in the information yielded by the fringe patterns. The rigid-body motion must be measured and subtracted from the displacement and rotation determinations. The rigid-body motion is evaluated by means of parameters obtained using fringe patterns on three teeth not effected by the applied load. This allows subtraction of rigid-body motion of the patient from the total movement recorded on the tooth which receives the extrusive loading.

Using the displacements and rotations obtained following correction for rigid-body motion, the velocity curves were computed and plotted against the time measured from the instant of application of the force. The velocity curves were interpreted to obtain total displacements and rotations which yields curves of total tooth displacement and rotation. The total displacements and rotations were, in turn, used to compute the centers of rotation representing the motion of the tooth.
Holographic analysis of tooth displacement was performed on four subjects at load levels of 50, 100 and 300 grams and on one subject at 200 grams. All subjects were loaded in the same manner as described in Chapter 4 to insure constancy in the magnitude and direction of the extrusive force. As shown in Figure II, tooth motions, $\Delta d$, were recorded every 30 seconds for a time period of two minutes with a double-exposure time delay $\Delta t$ of 450 msec as measured from the instant of force application.

The graph in Figure II is a hypothetical time displacement curve for a tooth. During a time interval, $\Delta t$, the tooth is displaced a distance, $\Delta d$. It is this displacement $\Delta d$, which results in the fringe lines used for holographic analysis. As shown schematically in Figure II, the magnitude of the displacement varies with the velocity of the curve. In other words, at times 90 and 120 seconds where the velocity is much slower, the displacements are very small. In order to measure displacement, $\Delta d$, at very low velocities, the pulse separation of a double-exposure, $\Delta t$, would need to be longer than 450 msec. By increasing $\Delta t$, the tooth displaces a greater distance and can therefore be recorded. However,
increasing the time of the pulse separation introduces rigid-body motions of the head which are detrimental to the interpretation of double-exposure holograms.

The pulse separation must be chosen in such a way as to minimize rigid-body motions. This means that tooth displacements of a very low velocity can not be quantitatively analyzed with this technique.

At 50 and 100 gram loads, the tooth displacements were of a very low velocity which could not be quantitatively analyzed with this technique. Extending the time interval past 450 msec was tested, but resulted in excessive rigid-body motion detrimental to the analysis of the holograms. One way to overcome this problem is to use sandwich holography which allows for compensation of rigid-body motions which can be four orders of magnitude higher then the measured tooth motion.  

Because the displacements at 50 and 100 grams could not be quantified using this holographic technique, the experimental results presented in this thesis were limited to tooth loadings of 200 and 300 grams. With a pulse separation of 450 msec of double-exposure holograms, tooth motions were measured for a period of about two minutes from the instant of tooth loading.

Analysis of double-exposure holograms at load levels of 200 and 300 grams did result in measurable tooth displacement, with corresponding fringe shifts. In each loading, the tooth
when acted upon by an extrusive force translated continuously over the two minute time interval. As shown in Figure 11, a double-exposure hologram was taken at times of 1, 30, 60, 90 and 120 seconds. For a constant load to the tooth and a 450 msec pulse separation between exposures, the tooth moved a distance \( \Delta d \). This distance was accurately determined from the analysis of holograms as described in Chapter 4. The ratio of this distance, \( \Delta d \), to the time delay, \( \Delta t \), during the double-exposure gives the rate of change, \( \Delta d/\Delta t \), of crown displacement (linear velocity).

As displayed in Figures 12 thru 40 the results were presented using a Cartesian coordinate system in the following way: positive x-axis pointing in the mesial-distal direction, positive y-axis pointing in the incisal-gingival direction parallel to the long axis of the tooth, and the z-axis pointing in the lingual-labial direction normal to the long axis.

The linear velocity of displacement due to 200 and 300 gram loads was plotted as a function of time and delayed in Figure 12 thru 16. The linear velocity was found to vary non-linearly from a maximum of 4.3 \( \mu \text{m/sec} \) to a minimum of .3 \( \mu \text{m/sec} \) along the incisal direction. The graphs plot the y components of linear velocity. The x and z components were negligible. In each graph the highest displacement velocity was experienced in the early part of the
displacement cycle with the tooth slowing down yet still moving during the remaining portion of the loading. The peak velocity during the 300 gram loading varied from 4.3 μm/sec to 2.9 μm/sec. This variation in velocity curves might be expected because of physiological and morphological differences from subject to subject. For example, root geometry may have an influence on tooth movement due to a force system.

As seen in Figure 15, subject III displayed the highest velocity peak. As noted in Table I, subject III exhibits the shortest root and smallest cross-section. Direct measurement of root dimensions was not possible for comparison with experimental results. However, lateral and occlusal radiographs were used to estimate the dimensions of the root alveolar geometry. These measurements, using the known dimensions of the reference wire present on each radiograph, were corrected for magnification and displayed in Table I. Future research is needed to determine a reliable technique for determining tooth geometry in three dimensions. For purposes of this study only comparisons as to the longer or shorter root length and larger or smaller cross-section, were made due to the inability of measuring root dimensions directly.

The linear velocity curves for the same subject at 200 and 300 gram levels allows for comparisons of results exhibiting similar constitutive characteristics and the same root geometry. Figures 15 and 16 show that the peak velocity during the 300 gram load was over 2 μm/sec greater than that
of the 200 gram load. The reason for the differences in the velocity curves is that a given force system applied, results in a stress distribution in the supporting areas of the tooth. The stress results in strain or tooth displacement which is a function of the constitutive properties of the supporting structures of the tooth. Therefore when the load levels were varied, thereby, varying the stress distribution, the resultant tooth displacements were influenced. That is, the higher force was responsible for higher linear velocities.

The ratio of $\Delta \theta$, the change in angular position of the tooth during a pulse separation, $\Delta t$, gives the rate of change, $\Delta \theta/\Delta t$, of crown rotation (angular velocity). The angular velocity of rotation due to 200 and 300 gram loads was plotted as a function of time and displayed in Figures 17 thru 21. From these figures it is seen that the angular velocity varied non-linearly from 155 $\mu$rad/sec to 12 $\mu$rad/sec. The tooth in each case rotated primarily in the counterclockwise direction with respect to the $x$-axis of the tooth. The rotations with respect to the $y$ and $z$ axes were negligible. Comparisons of 200 gram and 300 gram loadings show that the angular velocity with respect to the $x$-axis was initially larger for the 300 gram force. For each loading, angular velocity curves demonstrated that the tooth was still moving after two minutes although at a much slower rate.

The absolute translations and rotations during the extrusive loadings were determined by integrating the area
under the linear and angular velocity curves. Figure 22 thru 25 give the absolute displacements of the maxillary central incisor of each of the four subjects. The translations at the end of the two minute time interval varied from 42 to 61 μm in the y-direction depending upon the load level and upon the subjects root length. The shorter the root the greater the displacement. Displacements along the x and z axes were negligible. Figure 25 displays the translation curves at 200 and 300 grams for subject III, with the higher force magnitude responsible for a larger initial tooth motion.

The absolute rotations of the maxillary central incisor as shown in Figures 26 thru 29 indicate that minimal rotation occurred with respect to the x-axis of the tooth. The rotations with respect to the y and z axes were negligible. However, the load was applied lingual to the root centroid, creating a moment which rotated the tooth with respect to the x-axis. The distance of the moment arm, lingual to the root centroid for subjects 106, 109, 110 and 111 was 1.4 mm, 3.2 mm, 2.4 mm and 3.6 mm respectively. Subject III had the largest moment and as seen in Figures 26 thru 29 displayed the largest rotation. But, because the load was applied parallel to the long axis, the maximum rotation recorded was only 1.6 mrad in a counterclockwise direction with respect to the x-axis. Table I shows that subject III displayed
the shortest root and smallest cross-section and as seen in Figure 29 displayed the largest rotation. Also Figure 29 shows a larger rotation during the 300 gram loading when compared to the 200 gram loading. As with the translation curves, the rotations were found to be inversely proportional to the dimensions of the root and directly proportional to the magnitude of the force.

From the absolute translation and rotation curves, the sequential motions of the tooth were determined. Figure 30 thru 33 show that the greatest displacement experienced by the tooth was in the early part of the displacement cycle. Figures 34 thru 37 display similar characteristics for the rotation curves. Examination of the sequential translation and rotation curves indicate that at the time of the last exposure at 120 seconds, the teeth still continue to displace although at much lower levels.

The results from the displacement curves were used to compute centers of rotation. The y-component of the center of rotation was found to be relatively constant over the entire loading. The two dimensional centers of rotation as a function of time are presented in Figure 38. The figure shows that the z-component of the center of rotation were shifting position with respect to time. This can be related to the sequential displacement results which were found to be decreasing with respect to time. With a changing displacement or strain, the stress distribution
was changing. The centers of rotation were found to be shifting over the two minute loading because they are a function of the stress distribution.

The center of rotation was found to lie in a plane parallel with the root centroid in the y-direction and to lie within a range of 250 to 450 mm in the z-direction. This range from 250 to 450 mm could be related to the moment created due to loading lingual to the root centroid. Using the centroid of a paraboloid of revolution as a model, the root centroid was located at 0.33 of the root length as measured apically from the alveolar crest and located at the mid-section of the root along the long axis of the tooth. When comparing Table I and II it is seen that the location of centers of rotation was dependent on the subjects root length; the longer the root the further apical the root centroid. Figure 39 shows that the centers of rotation determined for tooth motion parameters at 120 seconds after the instant of tooth loading lies in a plane through the root centroid of each subject.

Table IV describes the location of the center of rotation with respect to the root centroid for the 200 and 300 gram loadings for subject III. At both the 200 and 300 gram force level, the center of rotation was found to lie very close to the y-component of the
root centroid. However, the distance from the z-component was found to vary as was shown in Figure 40. The differences in position of the centers of rotation would be expected due to the difference in force levels and resultant stress distributions.

As can be seen from Tables III and V, the y-coordinate of the center of rotation was found to lie as close as .2 mm to the position of the root centroid. The z-coordinate was as large as 437 mm. The relative position of the center of rotation therefore could be described as being at a point approaching infinity in the z-direction from the tooth and in a plane parallel with the root centroid in the y-direction.

Tooth motion due to an extrusive load was described in terms of centers of rotation with respect to a coordinate system located on the labial surface of the incisor and at the center of the upper edge of the base of the tetrahedron and by measuring the magnitude of the rotation around the center of rotation as displayed in Figure 41. Table III summarizes the cumulative tooth motion at 300 grams. The y and z coordinates of the center of rotation and the cumulative displacements, Δyz, are used to calculate the magnitude of rotation, φ, with respect to the center of rotation. The cumulative rotation (angle φ) values are plotted in Figure 42. From the graph and Table I it appears that the magnitude of rotation was inversely proportional to the length and cross-section of the root.
Table V shows the cumulative tooth motion for subject III at 200 and 300 gram loads. From the table it was seen that the magnitude of displacement and position of centers of rotation were affected by the difference in the force levels. Also in Figure 43, the magnitude of rotation with respect to the center of rotation was found to be a function of force magnitude. Examination of these figures reveals that the velocity of motion of the teeth was decreasing non-linearly with time. It also points out that the magnitude of rotation around the center of rotation was decreasing non-linearly.

The non-linear nature of the results presented in this study could possibly be represented by a viscoelastic model. The constitutive behavior of the periodontal ligament is related to the composition and orientation of the components of its connective tissue framework and their interaction. The periodontal ligament is a supporting tissue surrounding the tooth. Its components include fibrous, cellular, extracellular, and vascular elements. The non-linear response of the results might be attributed to the cellular, extracellular and vascular elements of the periodontal ligament. However, each of the physiological components are contributing to the spectrum of relaxation time resulting in a complex viscoelastic system.

During the early part of the displacement cycle, the tooth exhibited more pronounced movement which gradually
decreased with time. This overall response was characteristic of viscoelastic elements. As the velocity of motion decreased, the viscous components may have been approaching equilibrium. As the hydraulic effect of the viscous elements stabilized, the slower velocity of the tooth may be more related to the constitutive characteristics of the periodontal fibers. However, the explanations of the mechanism of support requires additional investigation to isolate the roles of both the viscous elements and the periodontal fibers. Future studies of the tooth motion following loading could evaluate the viscous forces acting within the support system which result in the subsequent restoration of the tooth to its equilibrium position. Predictive mathematical models need to be determined to estimate the constitutive components involved in tooth support, and thereby allow complete characterization of the viscoelastic behavior. The displacement data from this study could be used as boundary conditions to finite element analysis for development of mathematical models to estimate stress in the periodontal ligament.
CHAPTER 6

SUMMARY AND CONCLUSIONS

This study indicates that double-exposure holography offers accuracy and precision in quantifying the effects of time and force magnitude of tooth motion. It has been demonstrated that double-exposure holography is an improvement in previous techniques. The results indicate that with a pulse separation of 450 msec, tooth motions due to a force of at least 200 grams can be measured for a period of about two minutes from the instant of tooth loading.

In each loading, the tooth when acted upon by the extrusive loading moved continuously over the two minute time interval. The tooth displacement was accurately determined from the analysis of double-exposure holograms. Analysis of the holograms during extrusive loadings resulted in velocity curves which varied with time and force magnitude decreasing non-linearly. For each loading the velocity curves demonstrated that the tooth was still moving after two minutes although at a much slower rate.

Small rotations were observed with respect to all coordinate axes, however, the tooth movement was primarily translatory. The largest displacement was found to lie in the incisal-gingival direction. The displacements with respect to the remaining directions were negligible. Also,
the greatest displacement experienced by the tooth was shown to be in the early part of the displacement cycle. Both translation and rotation were found to be inversely proportional to the dimensions of the root and directly proportional to the magnitude of the force.

The y component of the center of rotation during extrusive loading was shown to remain relatively constant over the entire loading. Whereas, the z component of the center of rotation was found to lie within a range of 250 to 450 mm. This range might have been related to the moment created due to loading lingual to the root centroid. The magnitude of rotation around the center of rotation was found to be a function of force magnitude and was shown to be decreasing non-linearly with respect to time.

Comparison of translatory results from this study with those of Muhleman 18, Parfitt 17, and Picton 19 agree conceptually but quantitatively comparisons of axial loadings were not possible because of limitations of previous measurement devices. These investigators employed invasive, intra-oral devices which were not capable of monitoring three-dimensional displacements simultaneously in three directions. Also, in this study, the tooth was shown to still be moving at the end of the loading interval which was not the case in these previous studies.

Data in three dimensions using laser holography was available for horizontal loading. 1 In comparing magnitudes of
translatory movement using identical force levels and time intervals, the axial displacements were consistently less than those observed normal to the long axis. Also, the velocity of motion was less than motion observed for loading normal to the long axis of the tooth. However, in each case, the results indicated the velocity was greatest initially, decreasing non-linearly with respect to time. The translation and rotations were found to be inversely proportional to the length of the root of the tooth, which was consistent with previous findings by Burstone.26

Future research is needed in the area of long term studies of tooth movement. Information from this study concerning primary centers of rotation will be useful in studying long term secondary centers of rotation. Also, there is a need for both short and long term studies in the area of relapse or relaxation curves of teeth. Burstone has indicated further useful investigations in the area of tooth movement which include clinical, cellular and stress-strain studies.2

In the future when more information on the constitutive behavior of the periodontal ligament is obtained through experimental analysis, tooth displacement data could be utilized to compute strain and attempt to estimate stress distributions at the interface of the periodontal ligament. The results of this study give significant insight into the
type of tooth movement achieved due to a known force system. Force magnitudes and time were shown to be important parameters in predicting axial translations of teeth.


The loading geometry from the lateral view.
FIGURE 2
Tracing of Lateral Radiograph.

FIGURE 3
Tracing of Occlusal Radiograph.

FIGURE 4
The loading geometry from the occlusal view

FIGURE 5
Schematic representation of a pulsed ruby laser holographic system.

FIGURE 6
Figure 11

Time, Seconds

Displacement, Micrometers
LINEAR VELOCITY VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 106-5-8

\[ \frac{\Delta y}{\Delta t}, \text{MICROMETERS/SECOND} \]

LONG AXIS
HOOK
FORCE
TETRAHEDRON

TIME, SECONDS

FIGURE 12
LINEAR VELOCITY VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR
300 GRAM EXTRUSIVE LOADING FOR SUBJECT 109-1-4

FIGURE 13
LINAR VELOCITY VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR
300 GRAM EXTRUSIVE LOADING FOR SUBJECT 110-43-46

Figure 14

TIME, SECONDS
0 30 60 90 120

LINEAR VELOCITY, MICROMETERS/SECOND

LONG AXIS
HOOIK
FORCE
TETRAHEDRON

47
LINEAR VELOCITY VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 111-17-20

LONG AXIS

HOOK

FORCE

TETRAHEDRON

\frac{\Delta y}{\Delta t}\text{, MICROMETERS/SECOND}

FIGURE 15

TIME, SECONDS
LINEAR VELOCITY VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

200 GRAM EXTRUSIVE LOADING FOR SUBJECT 111-1-4

FIGURE 16
ANGULAR VELOCITY VERSUS TIME FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 106-5-8

\[ \Delta \theta / \Delta t \]

ANGULAR VELOCITY, MICRO RADIANS/SECOND

TIME, SECONDS

FIGURE 17
ANGULAR VELOCITY VERSUS TIME FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 109-1-4

Figure 18

Time, Seconds

Angular Velocity, Microradians/Second
ANGULAR VELOCITY VERSUS TIME FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 110-43-46
ANGULAR VELOCITY VERSUS TIME FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 111-17-20

\[ \Delta \theta_x / \Delta t \]

ANGULAR VELOCITY, MICRORADIAN/SECOND

\[ \Delta \theta_y \]

\[ \Delta \theta_z \]

LONG AXIS

HOLE

TETRAHEDRON

FORCE

TIME, SECONDS

FIGURE 20
ANGULAR VELOCITY VERSUS TIME FOR MAXILLARY CENTRAL INCISOR

200 GRAM EXTRUSIVE LOADING FOR SUBJECT 111-1-4

\[
\frac{\Delta \theta_x}{\Delta t}, \text{ MICORORADIIANS/SECOND}
\]

\[
\begin{array}{c}
\text{TIME, SECONDS} \\
30 \quad 60 \quad 90 \quad 120
\end{array}
\]

FIGURE 21
Figure 22: Total Translation versus Time Curve for Maxillary Central Incisor

300 Gram Extrusive Loading for Subject 106-5-8
TOTAL TRANSLATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 109-1-4

TRANSLATION, MICROMETERS

TIME, SECONDS

FIGURE 23
TOTAL TRANSLATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 110-43-40

TIME, SECONDS

TRANSLATION, MICROMETERS

FIGURE 24
TOTAL TRANSLATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

SUBJECT NO. 111

- FORCE = 200 GRAMS
- FORCE = 300 GRAMS

$dy$ = TRANSLATION, MICROMETERS

TIME, SECONDS

FIGURE 25
TOTAL ROTATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 106-5-8

TIME, SECONDS

90

60

30

0

0.0

0.5

1.0

1.5

Rotation, Milliradians

Figure 26
TOTAL ROTATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 110-43-40

FIGURE 28
TOTAL ROTATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

EXTRUSIVE LOADING FOR SUBJECT 111

- 300 GRAMS
- 200 GRAMS

\[ \theta \]

TIME, SECONDS

FIGURE 29
SEQUENTIAL TRANSLATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 106-5-8

\[ d_y (\text{SEQ UENTIAL}) \]

\[ \text{TRANSLATION, MICROMETERS} \]

-50 -40 -30 -20 -10 0 10 20 30 40 50

\[ \text{TIME, SECONDS} \]

\[ \text{FIGURE 30} \]

Long Axis

Hook

Force

Tetrahedron

\[ y \]

\[ x \]

\[ z \]
SEQUENTIAL TRANSLATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 109-1-4

\[ dy \text{ (SEQUENTIAL)} \]

\[ \text{TRANSLATION, MICROMETERS} \]

\[ \text{TIME, SECONDS} \]

FIGURE 31
SEQUENTIAL TRANSLATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 110-43-46

Figure 32
SEQUENTIAL TRANSLATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 111-17-20

\[ d_y \text{ (SEQUENTIAL, MICROMETERS)} \]

\[ \begin{align*}
& \text{TIME, SECONDS} \\
& \text{FIGURE 33}
\end{align*} \]
SEQUENTIAL ROTATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 106-5-8

$\theta_x$ (SEQUENTIAL)

ROTATION, MILLIRADIANS

TIME, SECONDS

FIGURE 34
SEQUENTIAL ROTATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 109-1-4

\[ \theta_x (\text{SEQUENTIAL}) \]

\[ \theta_x \text{ (ROTATION, MILLIRADIAN)} \]

TIME, SECONDS

FIGURE 35
SEQUENTIAL ROTATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 110-43-46

$\theta_x$ (SEQUENTIAL)

ROTATION, MILLIRADIANS

-1.5

-1.0

-0.5

0.0

TIME, SECONDS

30 60 90 120

FIGURE 36
SEQUENTIAL ROTATION VERSUS TIME CURVE FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING FOR SUBJECT 111-17-20

\( \theta_x \) (SEQUENTIAL)

\( \theta_y \)

\( \theta_z \)

TIME, SECONDS

FIGURE 37
CUMULATIVE CENTERS OF ROTATION FOR MAXILLARY CENTRAL INCISOR

300 GRAM EXTRUSIVE LOADING

DISTANCE FROM THE ROOT CENTROID TO CENTER OF ROTATION, MILLIMETERS

TIME, SECONDS

FIGURE 38
LOCATION OF CENTER OF ROTATION
300 GRAM EXTRUSIVE LOADING

Y-COORDINATE, MILLIMETERS

Z-COORDINATE, MILLIMETERS

HOOK
FORCE

106 109 110 111
SUBJECT

CENTROID

CENTER OF ROTATION

FIGURE 39
CUMULATIVE CENTERS OF ROTATION FOR MAXILLARY CENTRAL INCISOR

EXTRUSIVE LOADING SUBJECTED 111

300 GRAMS
200 GRAMS

TIME, SECONDS

500
400
300
200
100
0

Z - COORDINATE

DISTANCE FROM THE ROOT CENTROID TO CENTER OF ROTATION, MILLIMETERS

FIGURE 40
CUMULATIVE
MAGNITUDE OF ROTATION WITH RESPECT TO CENTER OF ROTATION
EXTRUSIVE LOADING SUBJECT 111

300 GRAMS

200 GRAMS

MAGNITUDE OF ROTATION µRAD

FORCE

TIME, SECONDS

FIGURE 43
<table>
<thead>
<tr>
<th>Subject</th>
<th>Length From Alveolar Crest To Apex</th>
<th>Width At The Level Of Alveolar Crest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Labial(mm) Lingual(mm)</td>
<td>Labial-Lingual(mm) Mesial-Distal(mm)</td>
</tr>
<tr>
<td>106</td>
<td>14.5 14.7</td>
<td>6.5 8.0</td>
</tr>
<tr>
<td>109</td>
<td>13.6 13.6</td>
<td>6.1 6.9</td>
</tr>
<tr>
<td>110</td>
<td>14.0 14.2</td>
<td>6.5 7.4</td>
</tr>
<tr>
<td>111</td>
<td>12.5 12.7</td>
<td>6.0 6.8</td>
</tr>
</tbody>
</table>
### TABLE II

LOCATION OF CENTER OF ROTATION WITH RESPECT TO ROOT CENTROID

<table>
<thead>
<tr>
<th>Subject Number</th>
<th>Force (gm)</th>
<th>Location of Root Centroid* (mm)</th>
<th>Experimental Center of Rotation** (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\begin{array}{c} y \ z \end{array}$</td>
<td>$\begin{array}{c} y \ z \end{array}$</td>
</tr>
<tr>
<td>106</td>
<td>300</td>
<td>14.3, -1.88</td>
<td>0.25, 437.95</td>
</tr>
<tr>
<td>109</td>
<td>300</td>
<td>13.2, -1.73</td>
<td>0.34, 399.48</td>
</tr>
<tr>
<td>110</td>
<td>300</td>
<td>12.9, -3.10</td>
<td>0.20, 284.35</td>
</tr>
<tr>
<td>111</td>
<td>300</td>
<td>12.1, -1.19</td>
<td>0.30, 373.48</td>
</tr>
</tbody>
</table>

*The distance is determined from the coordinate system at the base of the tetrahedron.

**The distance is determined from the root centroid at 0.33 of the root length apical to the alveolar crest.

x-direction displacements were negligible.
### TABLE III

TOOTH MOTION - CUMULATIVE
300 GRAM EXTRUSIVE LOADING

<table>
<thead>
<tr>
<th>Subject</th>
<th>Time (sec)</th>
<th>Location of Center of Rotation*</th>
<th>Cumulative Displacement $\Delta yz$(mm)</th>
<th>Magnitude of Rotation With Respect to Center of Rotation $\phi$(μRad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>106</td>
<td>30</td>
<td>14.460</td>
<td>449.89</td>
<td>.032</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>14.500</td>
<td>441.36</td>
<td>.041</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>14.540</td>
<td>445.74</td>
<td>.047</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>14.550</td>
<td>435.74</td>
<td>.050</td>
</tr>
<tr>
<td>109</td>
<td>30</td>
<td>13.366</td>
<td>306.88</td>
<td>.036</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>13.415</td>
<td>379.41</td>
<td>.047</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>13.456</td>
<td>429.16</td>
<td>.053</td>
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<tr>
<td></td>
<td>120</td>
<td>13.532</td>
<td>397.75</td>
<td>.057</td>
</tr>
<tr>
<td>110</td>
<td>30</td>
<td>11.361</td>
<td>317.96</td>
<td>.029</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>13.044</td>
<td>287.99</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>13.051</td>
<td>280.24</td>
<td>.043</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>13.064</td>
<td>281.25</td>
<td>.045</td>
</tr>
<tr>
<td>111</td>
<td>30</td>
<td>12.260</td>
<td>361.24</td>
<td>.040</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>12.326</td>
<td>372.93</td>
<td>.054</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>12.359</td>
<td>385.91</td>
<td>.059</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>12.364</td>
<td>372.29</td>
<td>.062</td>
</tr>
</tbody>
</table>

*The distance is determined from the coordinate system at the base of the tetrahedron.

x-direction displacements were negligible.
TABLE IV
LOCATION OF CENTER OF ROTATION
WITH RESPECT TO ROOT CENTROID

<table>
<thead>
<tr>
<th>Subject Number</th>
<th>Force(gm)</th>
<th>Location of Root Centroid(^*) (mm)</th>
<th>Experimental Center of Rotation(^**) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>111</td>
<td>300</td>
<td>12.1</td>
<td>-1.19</td>
</tr>
<tr>
<td>111</td>
<td>200</td>
<td>12.1</td>
<td>-1.19</td>
</tr>
</tbody>
</table>

\(^*\)The distance is determined from the coordinate system at the base of the tetrahedron.

\(^**\)The distance is determined from the root centroid at 0.33 of the root length apical to the alveolar crest.

x-direction displacements were negligible.
<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Location of Center of Rotation*</th>
<th>Cumulative Displacement Δxyz (mm)</th>
<th>Magnitude of Rotation With Respect To Center of Rotation φ (Rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>30</td>
<td>12.007</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>12.555</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>14.115</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>12.269</td>
<td>0.042</td>
</tr>
<tr>
<td>300</td>
<td>30</td>
<td>12.260</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>12.326</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>12.359</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>12.360</td>
<td>0.062</td>
</tr>
</tbody>
</table>

*The distance is determined from the coordinate system at the base of the tetrahedron. x-direction displacements were negligible.