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Abstract

The theory of adverse selection in insurance markets has been enormously influential among scholars, regulators, and the judiciary. But empirical support for adverse selection has been much less persuasive, and several recent studies have found little or no evidence of such selection in insurance markets. "Propitious" (advantageous) selection offers an alternative mechanism that is consistent with these empirical findings. Like adverse selection, the theory assumes that insureds have an informational advantage over insurers. However, propitious selection relies on the plausible assumption that risk aversion is negatively correlated with the riskiness or probability of loss across insureds - the more risk-averse are also the more careful, and hence are least likely to experience a loss. Theorists have recognized the possibility of equilibria in which highly risk averse insureds with a low probability of loss are willing to remain in the market, despite an actuarially unfair premium. But these conclusions derive from models with only two types of insureds. We use a simulation model that allows for flexible correlation between risk aversion and riskiness across a continuum of types, with plausible distributions of risk aversion and riskiness. We find that propitious selection alone can not preserve equilibrium in insurance markets. When insureds have moderate uncertainty about their own riskiness, however, equilibrium does become possible, albeit with considerable selection.

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I Introduction

Rothschild and Stiglitz’s (1976) model of adverse selection in insurance markets helped usher in the economics of asymmetric information and constitutes one of the triumphs of modern economic theory. More recently, however, a growing literature (for example, Chiappori and Salanié, 2000; Cawley and Philipson, 1999; Cardon and Hendel, 2001; Siegelman, 2004, provides a survey) suggests that empirical support for the importance of adverse selection is quite weak: while nobody denies that adverse selection can (and does) sometimes occur, it turns out that actual instances of any significance are surprisingly difficult to demonstrate. The absence of more widespread evidence for adverse selection constitutes a significant anomaly for a theory that seems so compelling when taken on its own terms.

The significance of adverse selection is of more than merely theoretical interest, however. As Siegelman (2004) demonstrates, the threat of adverse selection has colored judicial and regulatory decisions in several areas of insurance law and adjacent fields. A concern for adverse selection has led courts to permit practices that would ordinarily present serious anti-trust concerns (Ocean State Physicians Health Plan, 1988), has motivated Congress to permit restrictions on insureds’ ability to switch insurers (Rovner, 1988, p. 191), and has explicitly prompted several state legislatures to require insurers to provide mental health coverage (for example, 1973 Mass. Acts 1427 (West Supp. 1998)). Details of these and numerous other examples of adverse selection’s influence on various public policy debates can be found in Siegelman (2004).

Motivated by the lack of fit between theory and data, and building on recent theoretical insights, this paper simulates an alternative model of insurance markets, based on “propitious” or favorable selection and calibrated with plausible parameters from the literature. Hemenway (1990) coined the phrase “Propitious Selection” to describe this phenomenon, and since he has priority in time, we adopt his terminology. Under this theory, it is the best - rather than the worst - risks who find insurance to be most attractive, and this may
permit insurers to offer a pooled rate to both good and bad risks without fear that the good risks will drop out of the market. We conclude that propitious selection - taken by itself - is unlikely to explain the ability of insurance markets to continue functioning in the presence of asymmetric information favoring insureds. Based on a range of reasonable parameter values, our findings reveal that propitious selection can not prevent the unraveling of insurance markets. Our model does suggest, however, that giving insureds relatively modest uncertainty about their own probability of loss can stabilize insurance markets, even though insureds still know more about themselves than their insurers do.

A. What is Propitious Selection?

Recent scholarship (Hemenway, 1990; De Meza and Webb, 2001) has broached a theoretical explanation for the absence of significant adverse selection pressures, suggesting that insurance markets are characterized by a different selection mechanism than is contemplated in the Rothschild/Stiglitz model. The theory of “Propitious” or “Advantageous” selection posits that insureds differ not only in their probability of loss (as in Rothschild and Stiglitz), but also in their aversion to risk. Moreover, the theory assumes that riskiness and risk aversion are negatively correlated - risk averse individuals take more care, pose a lower risk of loss to their insurers, and value insurance coverage more than those who are less cautious and more prone to experience losses. If “belt-and-suspenders” types place such a high value on the reduction in risk provided by insurance that they are willing to pay more-than-actuarially-fair premiums to achieve the certainty that insurance provides, they may not drop out of insurance pools even when they are cross-subsidizing their riskier compatriots. This kind of selection, based on both risk aversion and riskiness, is favorable (“propitious”) for the insurer, since it means that the best risks prefer to buy insurance, rather than the worst.

There is abundant anecdotal evidence for the existence of propitious selection, and several empirical studies yield results that are consistent with this story. For instance, De Meza and
Webb (2001) observe that while 4.8 percent of all U.K. credit cards are reported lost each year, the loss rate for those cards that are insured against loss is only 2.7 percent. This suggests that it is the lowest, rather than the highest risks who buy insurance. Hemenway (1990) provides several other suggestive examples:

- Motorcyclists without helmets tend disproportionately to be uninsured, despite their obviously higher risk of needing expensive coverage (and even though insurers didn’t ask about motorcycle use);
- Drivers of rental cars who decline to purchase insurance are less likely to wear seatbelts;
- AAA members drive newer cars than average (and are thus presumably less likely to need free towing).

Davidoff and Welke (2005) find that reverse mortgage holders are less, rather than more, likely to move out of their homes as compared to conventional mortgagees. Their preferred explanation is that the more risk averse are both more likely to purchase insurance and to take independent precautions to prevent the occurrence of the insured-against event. A similar story is offered by Finkelstein and McGarry (2003) to explain why it is the relatively healthy - rather than the relatively sick - who preferentially purchase long term care insurance.

More recently, Fang, Keane and Silverman (2006) offer a more-nuanced empirical account of selection in the market for Medigap insurance. They conclude that although insurance purchasers are in fact more risk-averse and healthier than non-purchasers - as propitious selection models predict - the explanation for this correlation is not that risk-aversion leads to both precautionary behavior (and thus lower riskiness) and greater demand for insurance. Rather, they emphasize selection on factors other than risk aversion. Specifically, they focus on “cognitive ability,” which is empirically associated with both increased demand for insurance (because the more sophisticated are better able to understand the need for insurance) and better health status (because the more sophisticated take more precautions). Because
of a positive correlation between cognitive ability and risk-aversion, however, risk aversion and riskiness will be negatively correlated, even though this correlation does not in fact drive insurance demand.

Some empirical studies do find a positive correlation between riskiness and risk aversion. Cohen and Einav (2005), for example, use data on deductible choice and subsequent claiming behavior to draw inferences about underlying riskiness and risk aversion parameters. They infer a positive correlation between riskiness and risk aversion among those purchasing automobile insurance. However, they point out that this seems counter-intuitive, and suggest it may be explicable by confounding with income or because ostensibly more “careful” driving behavior may make one more accident-prone (e.g., by driving too slowly). Although some of their conclusions contradict the propitious selection model, they do find that “unobserved heterogeneity in risk preference is more important than heterogeneity in risk” (p. 37).

Of course, even in the presence of adverse selection, death spirals are far from inevitable; insurance markets with modest amounts of asymmetric information have apparently operated for a considerable time without degenerating. Siegelman (2004) provides evidence on this point. Cutler and Zeckhauser (1998) offer a compelling example of a death spiral in action.

Regardless of their specific conclusions, all these studies give the strong impression that, at least in some circumstances, a very different selection mechanism is operating in insurance markets from the one classically postulated in economics.

B. Risk Aversion Across Physical and Financial Risks

Technically, “risk aversion” is a term of art that should properly be used (at least by economists) only to describe utility functions characterized by a decreasing marginal utility of wealth. While the term is often used loosely to describe attitudes towards both physical
and financial risks, there is no a prior logical reason why the same utility functions that exhibit diminishing marginal utility of wealth should also show a preference for physical safety, job security, and so forth. In other words, there is essentially nothing in economic theory that explains the unlikeliness of a risk averse snow boarder or a risk-loving accountant. As long as someone holds a diversified portfolio of assets and buys actuarially fair insurance, economists properly deem him or her risk averse, even if he is a thrill-seeker in every non-financial context.

Several recent studies offer direct empirical support for the existence of a negative correlation between (physical) risk-taking and (financial) risk aversion. Measures of financial risk aversion from experimental and survey data generally correlate well with aversion to physical risks such as driving behavior, choice of occupation, and so on (Dohmen, et al, 2006; Guiso and Paiella, 2001; Barsky, et al, 1997; Cohen and Einav, 2005, are exceptional in that they infer a positive correlation between riskiness and risk-aversion). This psychologically plausible relationship between aversion to physical and financial risks underlies the propitious selection model. A different take on the problem is offered by De Donder and Hindriks (2006), who demonstrate that propitious selection, standing alone, may be insufficient to generate a negative relationship between riskiness and insurance purchases. When moral hazard is combined with propitious selection, they show that the resulting equilibrium will only be characterized by a negative correlation between risk and insurance demand under special circumstances.

Despite the wealth of empirical and anecdotal evidence, however, direct tests of propitious selection are extremely difficult to carry out, for several reasons. Insurance data are generally proprietary and are difficult to obtain in many settings. Combining insurance purchase data with direct information on risk preferences and risky behavior is more difficult still. Moreover, given the intractability of theoretical models of insurance markets with multiple sources of heterogeneity (Landsberger and Meilijson, 1999) simulation is an obvious research strategy. Chandler (2001) is the only other simulation study of insurance markets; his work
is an inspiration for this study, but does not consider the possibility of propitious selection.

II The Model

A. Demand for Insurance

Our strategy is to examine the possibility of a propitious selection equilibrium using a simulation model. We begin with potential insurance purchasers (consumers) who are heterogeneous in both their probability of loss and their risk aversion. Each agent \( i (i =1, ..., n) \) knows his type (his riskiness and his risk aversion) and chooses the amount of insurance purchased for a given premium per dollar of coverage so as to maximize a von Neumann-Morgenstern expected utility function:

\[
\max_{X_i} EU_i = p_i U_i(W - L - \pi X_i + X_i) + (1 - p_i) U_i(W - \pi X_i)
\]

s.t. \( 0 \leq X_i \leq L \)  

where \( \pi \) is the insurance premium per dollar of coverage (henceforth we’ll refer to it as premium), \( p_i \) is the probability of loss for the \( i^{th} \) individual and \( X_i \) is the insured amount for the \( i^{th} \) individual. Agents are identical with respect to their wealth, \( W \), and the amount of loss, \( L \), suffered in the case the insured event occurs. Further, the loss amount is independent of \( X_i \), i.e. we do not allow for moral hazard.

We assume throughout that \( 0 \leq X_i \leq L \). The first inequality is uncontroversial. The second arises from the fundamental indemnification principle in insurance law, under which no insurer is obligated to pay more than the size of the actual loss. See, e.g., Keeton (1971, p. 88):

“Insurance is aimed at reimbursement, but not more. The principle that insurance contracts shall be interpreted and enforced consistently with this objective of conferring a benefit no greater
in value than the loss suffered [is known as] . . . the principle of indemnification. . . . Any opportu-
nity for net gain to an insured through the receipt of insurance proceeds is inconsistent with the principle of indemnity.”

B. Parameterization

We assume that individuals are identical in terms of their wealth and possible loss if the unfavorable state of the world realizes. We do not allow for heterogeneity in wealth and loss for a variety of reasons. First, we would like to control for the wealth effect on our results. Second, we would like to bypass one of the caveats of expected utility theory. Theoretical work has pointed out to the limitations of expected utility theory in providing a realistic account of risk tolerance over moderate stakes (Arrow, 1971). Rabin (2000) shows that plausible risk aversion over moderate stakes implies implausibly high risk aversion over high stakes as the concavity of the utility function implies a very fast decline in the marginal utility of wealth. While these results pertain to an individual’s risk attitudes over different levels of lifetime wealth or different stakes, the implication for our model is that it is hard to justify using the same functional form for the utility of wealth for all individuals. However, we test the sensitivity of our results to the magnitude of the loss.

We use the lifetime wealth as a measure of wealth. Data from the Panel Study of Income Dynamics (PSID) for the latest available year, 2003, is used to compute the lifetime wealth per capita. PSID is a longitudinal survey of a representative sample of US families conducted annually since 1968 (biannually since 1997) by the University of Michigan. The sample consists of three independent samples in 2003: a cross-sectional national sample drawn by the Survey Research Center (SRC); a national sample of low-income families from the Survey of Economic Opportunity conducted by the US Census Bureau; and a sample of immigrant families added in 1997. As we are interested in a representative sample, we only use observations for the families that were sampled by the SRC in 1968. Our sample comprises of 5,158 families.
The measure of family wealth provided by the PSID is the sum of the net value of all asset holdings (assets minus liabilities). Assets include farm or business owned, cash assets, real estate other than home, stocks, motor vehicles owned, private annuities or IRAs, home equity, and other assets owned. The wealth per capita is obtained by dividing family wealth by the number of family members where adults are assigned a weight of 1 and children a weight of 0.5.

As the distribution of wealth is very skewed, we use the median as a proxy for the per capita wealth. The median wealth per capita in our sample is $31,333, implying a lifetime wealth of approximately $1 million for an interest rate of 3 percent.

We make several assumptions about the distribution of two key parameters: the coefficient of risk aversion, $r$, and the probability of loss, $p$. Although we determine these two parameters simultaneously, as described below, it is convenient to discuss them separately.

1. **Coefficient of Risk Aversion**

   We employ the widely used Constant Relative Risk Aversion (CRRA) utility function of the form (2),

   $U_i(W) = \begin{cases} 
   \frac{W^{1-r_i}}{1-r_i} & \text{for } r_i \neq 1 \\
   \ln(W) & \text{for } r_i = 1 
   \end{cases}$

   where $r_i$ is the Arrow-Pratt’s coefficient of constant relative risk aversion for individual $i$. On theoretical ground, Arrow argues that it should be around 1. Empirical results are not unanimous as to the mean, variance, and even range of the coefficient of (constant relative) risk aversion. For example, based on consumption and stock return data, Hansen and Singleton (1982) find that the CRRA is between .35 and 1 while based on life insurance data
We assume that $r$ is randomly distributed and follows a (generalized) beta distribution with parameters $\alpha = 2$ and $\beta = 7$. This yields an average coefficient of relative risk aversion in our sample of 2.2, and a standard deviation of 1.3. Beta distribution is chosen because first, it has a range bounded between 0 and 1 that can be rescaled and second, it can approximate any unimodel distribution which is bounded inside a range. Figure 1 plots the distribution of $r$’s in our simulation. Roughly 25 percent of our agents have coefficients greater than 3, while 3 percent have coefficients greater than 5.

These figures are in line with plausible estimates based on experimental studies. We rely on recent experimental work, especially Holt and Laury (2002) and Dohmen et al (2005), to suggest a plausible distribution for the coefficient of risk aversion in the population. Although Dohmen et al do not give precise summary statistics for the implied coefficients of risk aversion in their sample, their Figure 7 shows that the distribution is skewed, appears to have a mean of about 2, and has relatively few individuals with $r$’s as high as 5 or 6. Holt and Laury’s results for the treatment with the highest real payoffs reported in their paper suggest that 64 percent of population have a coefficient of relative risk aversion between 0.15 and 0.97 and less than 10 percent are risk seekers. However, they do not provide lower and upper bounds of the CRRA. We exclude risk seekers but our results are not sensitive to this restriction. We set the upper bound of the CRRA at 10 as Gollier (2001, p. 31) among many others points out that a CRRA coefficient in excess of 10 “seems totally foolish,” since it implies excessive conservatism relative to the behavior of most individuals.

2. **Probability of Loss**

It is even more difficult to obtain the distribution of the probability of loss from empirical studies. Actuarial studies are concerned not only with the likelihood that the insured event
will occur but also, with the magnitude of the loss. In addition, the likelihood that the insured event will occur varies across different insurance markets.

We assume that the probability of loss, \( p \), follows a beta distribution with \( \alpha = 2 \) and \( \beta = 18 \). The mean of \( p \) is 0.1 and the standard deviation is 0.065. The average probability of loss is consistent with the accident rate for homeowners insurance, which according to data of the Insurance Information Institute, 2003, quoted in Cutler & Zeckhauser (2004), is 9.3 percent. We use a slightly higher mean to account for the fact that accidents tend to be underreported particularly in cases where the damage does not exceed the deductible. However, we extensively test our results for sensitivity to the chosen distribution of \( p \).

The probability density function of \( p \) that we chose is shown on Figure 2. The distribution is skewed: roughly 40 percent of individuals have a \( p \) that is larger than 0.1. As with \( r \), each consumer receives a once-and-for-all realization of \( p \), drawn from this distribution, so that before the simulation begins, all consumers know all relevant parameters in their insurance purchase decision (the premium, \( \pi \), is described below).

3. Correlation Structure

The essence of the propitious selection model that is at heart of this exercise is a negative correlation between \( r \) and \( p \). However, there is no empirical evidence on their joint distribution and the marginal distributions of these variables and their correlation do not provide enough information to derive their joint probability density function (p.d.f.). Using copulas - functions that account for the dependencies among random variables - we can model the (exogenous) correlation between the CRRA parameter and the probability of loss without knowing their joint p.d.f. The procedure entails sampling from a joint standard normal distribution with the desired correlation structure to obtain marginal distributions for \( r \) and \( p \). We then transform these marginal distributions from standard normal to the forms described above. For example, to transform \( Y \sim N(0,1) \) into \( \hat{Y} \sim \text{beta distribution with} \)
parameters \((\alpha, \beta)\), we find \(y\) and \(\hat{y}\) such that \(\text{Prob}(Y \leq y) = \text{Prob}(\hat{Y} \leq \hat{y})\): that is,

\[
\int_{-\infty}^{y} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt = \int_{0}^{\hat{y}} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} t^{\alpha-1}(1 - t)^{\beta-1} dt
\]  

(3)

Since Pearson’s coefficient of correlation \((\rho)\) will not be invariant to the transformation of the joint normal distribution into the respective beta distributions for \(r\) and \(p\), we choose to use the non-parametric Kendall’s \(\tau\) as a measure of correlation, where \(\tau_{r,p} = f(\rho)\), and \(\rho = \tau_{r,p}\) for 0 and 1.

C. Supply of Insurance and Market Equilibrium

The supply side of the market consists of a single representative firm that has a zero cost of providing insurance and behaves competitively, setting premiums so as to earn a zero profit (if that is possible) in equilibrium. The market ceases to exist whenever there is no premium that yields zero profits for the insurer.

We recognize that this modeling strategy represents a departure from the traditional theoretical focus on the definition of strategic equilibria, some of which permit pooling in insurance markets with asymmetric information, as in Wilson (1977), Riley (1979), or Grossman (1979). Insurers do, in fact, use historical data to set premiums, however; and they probably make only very crude guesses as to how rivals will react to the introduction of new policies or changes in rates. Since our model permits only a single contract, a separating equilibrium a la Rothschild and Stiglitz is not possible. Our test is thus a crude one - can propitious selection prevent the market from unraveling in a death spiral, or can a single premium be sustained despite the heterogeneity of insureds?

The insurance company is risk-neutral and maximizes its expected profit, \(\Pi\):
\[ \Pi = \pi E[X] - E[pX] \]

\[ = \pi \int_{r_{\min}}^{r_{\max}} \int_0^1 \frac{X(p, r)}{G(r, \pi, W, L, X)} f(p, r) \, dp \, dr - \int_{r_{\min}}^{r_{\max}} \int_0^1 \frac{pX(p, r)}{G(r, \pi, W, L, X)} f(p, r) \, dp \, dr \]  

The first term in the above equation is an expression for the (stochastic) gross revenue of the insurer while the second term accounts for the (stochastic) gross payout when the insurance event occurs. The range of the coefficient of risk aversion is given by \((r_{\min}, r_{\max})\) and \(f(p, r)\) denotes the joint density function of the probability of loss and the coefficient of risk aversion. The lower bound of the probability of loss of those who purchase insurance is found where individuals are indifferent between purchasing insurance and staying uninsured. It is a function of the coefficient of risk aversion, wealth, possible loss, and the insured amount, \(G(r, \pi, W, L, X)\).

Given consumer preferences, the joint distribution of the probability of loss and the coefficient of risk aversion, the insurance premium per dollar of coverage that the insurer will charge in equilibrium, \(\pi^*\), is given by:

\[ \pi^* = \frac{\int_{r_{\min}}^{r_{\max}} \int_0^1 \frac{pX(p, r)}{G(r, \pi^*, W, L, X)} f(p, r) \, dp \, dr}{\int_{r_{\min}}^{r_{\max}} \int_0^1 \frac{X(p, r)}{G(r, \pi^*, W, L, X)} f(p, r) \, dp \, dr} \]  

The problem does not have a close-form solution. Even for the simplest case of full insurance contract and zero correlation between the probability of loss and the coefficient of risk aversion it is not possible to obtain an analytical solution:

\[ \pi^* = \frac{\int_{r_{\min}}^{r_{\max}} \int_0^1 \frac{pf(p, r)}{G(r, \pi^*, W, L)} \, dp \, dr}{\int_{r_{\min}}^{r_{\max}} \int_0^1 \frac{f(r, p)}{G(r, \pi^*, W, L)} \, dp \, dr} \]
Analytical solution of the problem (if possible at all) would require unrealistic simplifications of the joint probability density function of $p$ and $r$ as well as the utility function that would render any analytical results worthless.

However, Figure 3 provides an intuition into the solution of the problem when the insurer offers only a full insurance contract. The figure is a scatter plot of the probability of loss and the coefficient of risk aversion drawn from their joint probability density function described in the previous section. The green (dash-dotted) line is the insurance premium per dollar of coverage charged by the insurance company. “Bad” risks or individuals with probability of loss equal or greater than the per dollar premium charged by the insurer would purchase full insurance at the fair (or better) premium regardless of their coefficient of risk aversion. “Good” risks or individuals with a probability of loss lower than the per dollar premium would purchase full insurance as long as the premium does not exceed their certainty equivalent. On average, the insurer realizes a loss from bad risks, the region to the right of the premium.

The red (solid) line is the lower bound of the probability of loss of good risks who buy insurance at unfair premium. It is found where individuals are indifferent between purchasing insurance and staying uninsured for given premium, wealth, loss, and risk aversion. For the preferences that we assume, the lower bound of the probability of loss is given by:

$$p = \frac{(W - \pi L)^{1-r} - W^{1-r}}{(W - L)^{1-r} - W^{1-r}}$$

On average, the insurer realizes a profit from good risks, the region bound between the lower bound of the probability of loss and the premium. For the insurer to break even, there must be sufficiently many good risks in the region bound between the red and green lines to offset the loss realized from bad risks in the region to the right of the green line.
We assume that the insurer offers only a full insurance contract. Thus, individuals choose between buying full insurance and bearing the risk. However, our results are robust to this assumption.

D. Methodology

The basic algorithm for computing insurer profitability starts by assuming a very low premium, say $\pi_1 = 0.01$. Given that premium, the model calculates which insureds are willing to purchase insurance. This generates both revenues and payouts for the insurance company, and allows a computation of profits. The premium is then incremented by a small amount. At the new premium, $\pi_2$, a second profit computation is undertaken, based on the participation decisions of insureds at the new premium level. This traces out a function giving a unique profit level for every choice of premium. The insurance company then picks the premium that yields zero profit (consistent with the participation decisions of all individuals), if such a premium exists. If no such premium exists, then the market is unsustainable due to adverse selection. All computations were done in MATLAB.

III Results

A. No Correlation Between $r$ and $p$

Propitious selection operates - if at all - by way of a negative correlation between riskiness and risk-aversion. However, we begin by testing the most favorable case for the adverse selection/“Death Spiral” story, by assuming that $p$ and $r$ are uncorrelated. A graphical depiction of initial conditions on the demand side of the market is provided by Figure 4, which plots a 3-D histogram of the distribution of $r$ and $p$, assuming zero correlation between these two variables.
Market equilibrium - or rather, the lack thereof - is depicted in Figures 5 and 6. As can be seen in Figure 5, profits are negative for any premium the insurer chooses below 0.41. At that premium, however, essentially no insureds are in the market - Figure 6. Recall that the average probability of loss is only 10 percent, so only the most spectacularly risk-averse (and/or those with a very high probability of loss) would purchase insurance at a premium so high.

Figure 7 illustrates the selection process at work by graphing the average coefficient of risk aversion, $r$, among insurance purchasers as a function of the premium charged, $\pi$. Holding $p$ constant, an increase in the premium should cause the least risk averse to drop out of the market, so the average level of $r$ among remaining market participants should increase with the premium, as Figure 7 shows: the coefficient of risk aversion among those remaining in the market rises from just over 2, at the lowest premium, to just under 3 when $\pi = 0.4$.

Somewhat counter-intuitively, the cross-sectional standard deviation of $r$ does not change much as the premium increases, even though one would expect the selected group to be more homogenous. To see why this occurs, it is helpful to think about two groups of insureds for any given premium, as depicted in Figure 8. Group 1 consists of those for whom the premium is actuarially fair (or better). Group 2 contains those for whom the premium is actuarially unfavorable (but still preferable to going without any insurance at all). When the premium is very low, most insureds fall in group 1. As the premium rises, the proportion of all insureds who are in group 1 falls, while the proportion (and, for a time, the absolute number) in group 2 rises. The overall average risk aversion coefficient of those in the market is a weighted average of those in group 1 and group 2, with weights equal to the relative size of the two groups. Since group 2’s weight increases as the premium gets higher, and since this group has the higher average $r$, the average risk aversion of those in the market increases as well. However, since the standard deviation of $r$ within Groups 1 and 2 is roughly the same (both are roughly equal to the population standard deviation of $r$), higher premiums do not have much of an effect on the standard deviation of $r$. 

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If we look instead at the relationship between the probability of loss and the premium, we see the expected pattern, as Figure 9 illustrates. Raising the premium tends to drive out those with the lowest probability of loss (holding $r$ constant), increasingly leaving the market to those with the highest loss probability. Thus, average $p$ among market participants increases with the premium level.

In sum, our initial results confirm the old news that adverse selection can be a problem: there is no pooling equilibrium when insureds know their own riskiness and risk aversion but the insurer doesn’t. This is true even when there is heterogeneity in both riskiness and risk aversion, as long as the two are uncorrelated.

B. Negative Correlation Between $r$ and $p$

We are now in a position to consider the effects of allowing for correlation between $r$ and $p$, as the Propitious Selection story suggests. Does such correlation permit the existence of a zero profit equilibrium when none was possible without it? Figures 10-12 demonstrate that the answer is clearly “No.” Figure 10 plots the joint histogram of $r$ and $p$ for the extreme case of $\tau_{r,p} = -1$ - perfect negative correlation. A comparison with Figure 4 reveals that the distribution now looks significantly different. Nevertheless, Figures 11 and 12 show that there is still no zero-profit equilibrium - or indeed, any premium that generates positive profits - even when we require that the most risk-averse agents also have the lowest probability of loss. At a premium of 0.35, the market has essentially dried up, but profits are still negative.

Somewhat anomalously, the relationship between the premium and the average coefficient of risk aversion for those purchasing insurance now becomes negative, as demonstrated by Figure 13. That is, as the premium increases, the average risk aversion of those buying insurance actually falls. The explanation is that the probability of loss seems to dominate the risk aversion effect in generating insurance demand. When $r$ and $p$ are strongly negatively
correlated, it is predominantly the most risky (rather than the most risk-averse) who remain in the market as the premium increases. With $\tau_{r,p} = -1$, the stayers are of largely those with very high $p$'s and correspondingly low levels of risk-aversion. Since the departures are low-$p$, high-$r$, the average $r$ of those who remain actually falls.

Because of the dominance of the probability of loss, the relationship between premiums and loss probabilities remains as it was when $\tau_{r,p}$ was zero. An increase in the premium causes those with the lowest probability of loss to drop out of the market, as Figure 14 demonstrates. In sum, propitious selection does not appear to be strong enough to dominate the force of adverse selection when insureds know their own probability of loss.

D. Robustness

We tried a variety of specifications for the p.d.f. of the probability of loss. We picked up 17 data points at equal intervals for the mean and standard deviation of $p$ in the interval $[0.4;36]$ and we examined the results for all possible combinations of these two variables (note that some combinations cannot be generated with a beta distribution, e.g. low mean and high standard deviation).

Figure 17 shows the percentage of population purchasing insurance for different combinations of $p$ and $r$ and zero correlation. Zero values of the percentage of population buying insurance correspond to those combinations of $p$ and $r$ that could not be generated with beta distribution. The percentage of population purchasing insurance or the size of the market is negatively related to the variance of $p$ and positively related to its mean (this result holds true for negative correlation between $p$ and $r$ as well). In fact, the highest percent of market participation (35.5 percent) is achieved for the combination of the highest mean and the lowest standard deviation that we allow for: $\mu_p = 0.36$ and $\sigma_p = 0.04$ (alpha = 51.48 and beta = 91.52). When there is high market participation, the insurer is able to break even at premiums close to the average probability of loss.
At first glance it may appear that our results imply the existence of equilibria with considerable market participation even when $p$ and $r$ are uncorrelated. However, these equilibria are associated with distributions of $p$ which are nearly symmetric with high mean and low variability. The corresponding break-even premia are high as well. Such distributions do not seem to adequately describe individual riskiness as well as premia observed on insurance markets.

Once we allow for a negative correlation between $p$ and $r$, our results are very robust. Even for distributions of $p$ with high mean and low variability we obtain very low market participation (3.1 percent market participation is the highest that we obtain for the admissible regions of $p$ and $r$.

Interestingly, positive correlation between $p$ and $r$ induces more market participation than negative correlation (see Figure 18). When the correlation is negative the most risk averse individuals are clustered at low $p$ while when the correlation is positive bad risks are also very risk averse. For premiums below the average $p$ the insurer realizes a sizable loss from bad risks and there are too few good risks (even with negative correlation) to outweigh the loss. For premiums equal or greater than the average $p$ good risks when the correlation is positive are more risk averse than good risks when the correlation is negative and thus, more agents buy insurance at unfair premiums, which enables the insurer to break even at lower premiums (and induces higher market participation). However, these results are susceptible to the same criticism as the results obtained for zero correlation between $p$ and $r$.

Allowing for different means and variances of $r$ as well as a weaker correlation between $r$ and $p$ does not alter the results. Neither does allowing agents to purchase partial insurance (which is optimal for risk averse agents, whenever premiums are unfair) or allowing for risk-seeking agents.
Table 1: Sensitivity results for different losses

<table>
<thead>
<tr>
<th>Loss as % of W</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break-even premium</td>
<td>34</td>
<td>24</td>
<td>18</td>
<td>15</td>
<td>13</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>% of population buying insurance</td>
<td>1%</td>
<td>9%</td>
<td>28%</td>
<td>46%</td>
<td>61%</td>
<td>71%</td>
<td>79%</td>
<td>85%</td>
<td>90%</td>
<td>97%</td>
</tr>
</tbody>
</table>

However, the size of the loss relative to wealth does have an effect on the equilibrium (see Table 1). For very large losses, there is sufficient demand for insurance that the market can exhibit a zero profit equilibrium with significant market participation. But this is true regardless of the strength of the correlation between $r$ and $p$, so the result - while perhaps interesting - has nothing to say about propitious selection.

In conclusion, we tried numerous alternative specifications to check the robustness of our findings that propitious selection does not facilitate the existence of equilibrium. In none of them were the results qualitatively different. It does not appear that propitious selection - at least as we have modeled it - is capable of explaining the apparent absence of adverse selection uncovered in many recent studies of insurance markets.

In the next section, we use our model to suggest an alternative story that may offer more promise as an explanation of the empirical findings.

C. Errors in Estimating $p$

Models of adverse selection typically assume that insureds know their own probability of loss (while insurers know only the average loss rate for the pool of insureds). But in most
cases, this is unlikely to be an accurate description of reality. For example, an individual insured may know her driving style, while her insurance company may not. But there is a long cognitive distance between knowing one’s driving style and a precise estimate of one’s probability of having an accident. Svenson (1981), for instance, finds that 46.3 percent of drivers surveyed believed themselves to be among the top 20 percent safest drivers.

Indeed, it is plausible that an insurance company’s actuarial prediction of the likelihood of an accident (based on observable factors such as age, sex, and miles driven) is more accurate than an individual’s estimate, even when the insured has more detailed information about himself than his insurer has. Grove and Meehl (1996) survey 136 studies in which the judgments of trained “experts” were pitted against a simple statistical or actuarial model that was used to make predictions in a variety of contexts ranging from criminal recidivism to college performance. In all cases, the experts had access to at least as much information as the actuarial prediction model (often, more). The “model” was typically much cruder than a regression. Yet the experts essentially never outperformed the model, while the model predicted better than the experts in almost half of the cases. Of course, this will not always be true: an individual who knows that he has a fatal disease is an obvious position to select against his insurer, but this is an unusual scenario.

We therefore investigate a variation of the model above in which insureds use an estimate of the probability of loss, rather than the actual probability, in formulating their insurance purchase decisions. To do this, we simply give each insured $i$ a noisy but unbiased estimate of his own probability of loss, $p_i$:

$$\hat{p}_i = p_i + \varepsilon$$

We draw random numbers from a standard normal distribution. Then, for each agent we account for estimation errors by transforming the $\mathcal{N}(0,1)$ into a normal distribution with mean equal to $p_i$ and standard deviation equal to the desired estimation error. As the probability of loss cannot be negative, we then truncate the transformed distribution to belong to the interval $[0,1]$. 

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We then re-run the simulation, assuming zero correlation between \( r \) and \( p \) and replacing \( p_i \) by \( \hat{p}_i \) in (1). Results are described in Figures 15 and 16.

Taken together, these two figures suggest that introducing a relatively modest amount of noise or cognitive friction into insureds’ estimates of their own probability of loss can indeed stabilize the market. As the spread in the \( \varepsilon \) distribution increases, the insurer’s break-even premium falls, and the number of people in the market increases. (The technical explanation for this effect turns out to be surprisingly complicated, and is described more fully below.) With a standard deviation of 0.05 - one half the average probability of loss, and a variance of only 0.0025 - the insurer could break even at a premium of 20.5 percent (Figure 15), while still attracting nearly 15 percent of insureds. That is true even though this premium charged is twice the population average probability of loss. This is not an entirely happy scenario, however, since those buying insurance represent only a small percent of the population as a whole. The market is considerably diminished.

To get the break-even premium down to 12.5 percent, insureds’ estimates must be considerably more noisy. We require a standard deviation in \( \varepsilon \) of 0.12 (a variance of 0.014), which is nearly the same size as the average probability of loss in the population as a whole. This does, however, raise the number of insureds to about 9,000, or 45 percent of the total eligible population.

The explanation of why introducing errors in estimating \( p \) leads to a viable market is complicated because there is no clear-cut relationship between the premium charged by the insurance company and the probability of loss of those who purchase full insurance. For a given premium, \( \pi \), the introduction of errors in estimating \( p \) can lead to any one of seven alternative effects. We can classify these effects into three groups:

1. Errors may cause some agents for whom the premium is fair or better (\( p_i \geq \pi \)) to
underestimate their actual probability of loss.

a. They may not purchase full insurance, depending on their $r_i$. In this case, risky agents who would have bought full insurance if they knew their actual probability of loss leave the pool of insureds. The pool of insured improves (from the insurer’s perspective). The larger the variance in the estimation error, the more favorable this effect for the insurer.

b. They may still purchase full insurance if they are sufficiently risk averse. This results in no change in the pool of insureds.

c. Some agents for whom the premium is fair, may overestimate their actual probability of loss. This leaves no change in the pool of insureds.

2. Some agents for whom the premium is unfair may underestimate their probability of loss.

d. If they were purchasing full insurance at unfair premium, they may stay in the pool of insureds if they are sufficiently risk averse and/or the estimation error is low. Again, this causes no change in the pool.

e. If they were purchasing full insurance at an unfair premium, they may exit the market. This worsens the pool of insureds.

3. Finally, some agents for whom the premium is unfair may overestimate their probability of loss.

f. If they were not purchasing full insurance, they may do so, improving the pool of insureds.

g. If they were purchasing full insurance, there will be no change in the insurance pool.

In sum, two trends work in favor of the insurance company: agents with $p_i < \pi$ tend to purchase full insurance because they overestimate their actual probability of loss and thus
are willing to purchase full insurance at an actuarially unfair premium. And agents with $p_i > \pi$ do not purchase full insurance because they underestimate their riskiness and thus, do not purchase insurance at a fair or better premium. Both trends stabilize the market as they improve the pool of insureds: more prudent agents join the pool, while some risky agents exit the pool.

Our behavioral story relies on random errors in estimating $p$, errors that, in the end, have the net effect of diluting the informational asymmetry in favor of insureds, albeit through a more complicated mechanism than one might have expected. The behavioral story proposed by Fang, et al is somewhat related. Their stabilizing mechanism revolves around a three-way correlation among risk-aversion, riskiness and cognitive ability. While the cognitively-able also turn out to be risk-averse, the health/risk-aversion relationship is essentially epiphenomenal for insurance demand: it is not that the healthy are more risk averse that explains their greater insurance demand, but rather, that the healthy are more “wise,” and this wisdom leads to both better health and more willingness to pay for insurance.\footnote{With apologies to Benjamin Franklin, one might say that “to be wise makes one healthy (by being early to bed and early to rise), and wealthy. Or at least well-insured.”} Our model does not, of course, incorporate any such mechanism.

IV Conclusion

What have we learned from all this? The obvious conclusion is that propitious selection does not on its own seem capable of preventing the evaporation of insurance markets, given what we take to be plausible parameters for risk aversion, and the probability and size of loss. While this mechanism seemed to offer an compelling explanation for the apparent absence of adverse selection, the propitious selection story can not be verified by our simulation model. Of course, it is always possible that we’ve chosen inappropriate parameters, but our robustness checks lead us to be reasonably certain that our results are not merely a function of our initial assumptions. We are left with a puzzle: studies of insurance demand increasingly

suggest that something like propitious selection is at work, and survey/experimental data seem to confirm a crucial requirement of propitious selection - that more financially risk averse people are also more careful and have lower probabilities of loss. In spite of that, however, the theory seems not work.

The theoretical insights of De Donder and Hindriks (2006) - who show that propitious selection can only yield a viable insurance market when additional restrictions are imposed - offer one way to resolve this conflict. The empirical work of Fang, et al (2006) offers another possible resolution. A third possibility is that if we dilute the informational advantage that insureds have over their insurers, we can diminish the severity of adverse selection. Individual’s errors in estimating their own $p$ allow for the existence of a larger market, with lower premiums. But the degree of cognitive friction required to make an appreciable difference to market equilibrium is not trivial, and it is hard to say whether the errors needed to stabilize the market are of a plausible magnitude. There is some irony in the fact that cognitive errors lead to better outcomes by forestalling some of the adverse selection that would otherwise occur when insureds know “too much” about themselves. In a somewhat loose sense, this is an illustration of the theory of the second best. Adding an additional “distortion” (here, cognitive error) can actually be welfare-enhancing if it corrects a pre-existing market failure of some sort (here, adverse selection).

In light of our findings, it seems that further research in this area is needed. We continue to believe that the central problem that motivated this study - the lack of empirical support for widespread adverse selection - remains as an important empirical anomaly in search of an explanation.
References


Figure 1: PDF for coefficient of risk aversion, $r$
Figure 2: PDF for probability of loss, $p$

Figure 3: Scatter diagram of $p$ and $r$. The green (dash-dotted) line shows the per dollar premium charged by the insurer and the red (solid) line shows the lower bound of $p$ for those who buy insurance.
Figure 4: Joint histogram of $p$ and $r$

Figure 5: Insurer profitability as a function of premium level, $\pi$
Figure 6: Percentage of population buying insurance, as a function of premium level, $\pi$

Figure 7: Average coefficient of risk aversion among insureds, as a function of premium charged, $\pi$
Figure 8: Percentage of population buying insurance as a function of premium charged, by whether insured’s probability of loss exceeds the premium.

Figure 9: Average probability of loss among insureds, as a function of premium charged, $\pi$. 

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Figure 10: Joint histogram of $r$ and $p$ for $\tau = -1$

Figure 11: Insurer’s profit as a function of premium charged, $\pi$ (Perfect negative correlation between $r$ and $p$)
Figure 12: *Percentage of population buying insurance, as a function of premium charged, $\pi$, for perfect negative correlation between $r$ and $p$*

Figure 13: *Average coefficient of risk aversion among those buying insurance as a function of premium charged (perfect negative correlation between $r$ and $p$)*
Figure 14: Average probability of loss among those purchasing insurance, as a function of premium charged, $\pi$ (perfect negative correlation between $p$ and $r$)

Figure 15: Insurer’s break-even premium, as a function of the standard deviation in insureds’ error in estimating $p$, ($\tau_{r,p} = 0$)
Figure 16: Percent of population buying insurance as a function of the standard deviation in insureds’ error in estimating $p$, $(\tau_{r,p} = 0)$

Figure 17: Percent of population buying insurance, as a function of the mean and standard deviation of $p$, $(\tau_{r,p} = 0)$
Figure 18: Percent of population buying insurance, as a function of the mean and standard deviation of $p$, ($\tau_{r,p} = 1$)