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Frege's Conception of Truth as an Object

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Frege's Conception of Truth as an Object

Junyeol Kim, PhD

University of Connecticut, 2020

Abstract: In this dissertation I explore Frege's conception of truth. In particular I defend the thesis that Frege in his mature career takes truth to be an object, i.e., the True *qua* the reference of true sentences. In the literature on truth Frege has been usually taken to be a truth deflationist or a truth primitivist. Indeed Frege leaves a number of comments that sound like typical deflationist claims and his famous indefinability argument is the most discussed argument for primitivism. However, Frege is neither a deflationist nor a primitivist. His deflationist remarks and the indefinability argument are rather his arguments that truth is an object.

For Frege judging—and its verbal counterpart asserting—is always acknowledging the truth of a truth-bearer. Thus, if truth is an object for him, his conception of judgment ought to also be adjusted accordingly. Indeed Frege shows how we can understand judgment *qua* acknowledgment of truth if truth is an object. Specifically, judging that *p* is identifying the True with the reference of '*p*' non-judgmentally, i.e., without judging that *p* is identical with the reference of '*p*'. Frege's conception of judgment impacts his conception of knowledge because the act of judging is that of producing knowledge. In a nutshell our act of judging and the knowledge we produce by it are epistemically grounded in our objectual knowledge of the True. If we take this point into account, we can develop a new reading of Frege's claim that logic is the science of truth.

The suggested reading of Frege's conception of truth and related notions can make a contribution to contemporary philosophical problems. First, Frege's conception of judgment can cope with Peter Hanks's recent criticisms of the Fregean picture of judgment in general. Secondly, his conception of assertion can explain the connection between truth and assertion in a novel way. Lastly, Frege's conception of knowledge can be developed into a new theory of knowledge where objectual knowledge is more fundamental than propositional knowledge.

Keywords: Gottlob Frege, Truth, Judgment, Assertion, Knowledge, Logic

Frege's Conception of Truth as an Object

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2020

ii

APPROVAL PAGE

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Contents

Approval Page	iii
Acknowledgements	iv
1 Frege’s Conception of Truth as an Object	1
1.1 The Truth-Value True as a Reference of Sentences	2
1.2 Previous Readings of Frege on Truth	5
1.3 The Challenges for Taking Truth to be an Object	9
2 Argument 1: Frege is Not a Deflationist	14
2.1 ‘My Basic Logical Insights’	18
2.2 Truth and the Truth-Value True	25
2.3 Does Frege Have a Truth-Predicate?	33
2.3.1 The Metalinguistic Justification for Logical Laws?	33
2.3.2 Greimann’s Argument for the Truth-Predicate in Frege	35
2.3.3 Objection: Sentence and Thought in Frege	38
2.3.4 Assertion Sign ‘ ’: a Prelude to Chapter 5	44
3 Argument 2: Frege is Not a Primitivist	47
3.1 The Circularity Reading	48
3.1.1 The Circularity Reading	49
3.1.2 Criticism	52
3.1.3 Concluding Remarks	58

3.2	Literal Reading of the Indefinability Argument	60
3.2.1	The Analysis of the Passage from ‘Thought’	62
3.2.2	The Motivation for (VIR)	68
3.2.3	The Motivation for (J _{Pro}) and the Lesson from the Argument	71
4	More Objections	75
4.1	The Neo-Kantian Reading on Frege on Truth-Values	76
4.1.1	Argument I. Correspondence between Frege and Russell	77
4.1.2	Argument II. Semantic Uselessness of Truth-Values	81
4.2	The ‘No Entity’ Theory Reading of Frege on Truth	86
4.2.1	The NE-Theory Reading of Frege’s Conception of Truth	88
4.2.2	Frege is not a ‘No Entity’ Theorist	91
4.2.3	Greimann’s Objections against the Object Reading	99
5	Frege’s Conception of Judgment and Assertion	104
5.1	The Judgment-Stroke and the Horizontal	104
5.2	The Tensions	111
5.2.1	Syntactic Tension in Begriffsschrift	111
5.2.2	The Tension with Frege’s Conception of Assertion	112
5.2.3	The Tension with Frege’s Indefinability Argument	113
5.3	Is (A-Id) Frege’s Elucidation of Assertion?	116
5.4	Assertion as Non-Assertive Identification	124
5.5	Appendix: Other Uses of the Horizontal	131
6	Knowledge, Science, and Logic in Frege	134
6.1	Frege’s Conception of Objectual Knowledge	137
6.2	Knowledge of the True and Knowledge-That	141
6.3	Science, Logic, and the True	144
6.4	Appendix: The Aim of ‘On Sense and Reference’	149

7	A Fregean Picture Further Developed	155
7.1	Judgment: Frege vs. Hanks	156
7.1.1	The Truth-Condition of a Judgment	156
7.1.2	Grasping a thought and Judging	160
7.2	Assertion: Truth Norm and Performative Flaw	164
7.2.1	A Review of the Previous Descriptive Explanations	167
7.2.2	A Fregean Performative Explanation	170
7.3	Knowledge: the Frege-Millikanian Theory	172
7.3.1	The Basic Components of the Theory	173
7.3.2	The Frege-Millikanian Theory and the JTB analysis	179
7.3.3	The Ability to Identify _{nj} the True	181
8	Conclusion	185
	Bibliography	190

CHAPTER 1

FREGE'S CONCEPTION OF TRUTH AS AN OBJECT

Gottlob Frege's insight into the nature of truth is almost entirely forgotten in the current literature on truth. Someone might immediately object to this claim by saying that he is well known as a philosopher who inspires truth deflationism—the claim that truth is not a substantive notion—or truth primitivism.^{1,2} Indeed, Frege is well known as such. But that fact is rather the evidence that his insight is forgotten. This is because he is neither a deflationist nor a primitivist. Or so I argue in this dissertation.

¹For instance, Dummett (1978) takes Frege to be the origin of truth deflationism. Also, see Stoljar and Damnjanovic 2014. Asay (2013) describes Frege as a primitivist.

²I am not suggesting that truth deflationism is a single philosophical position about truth. Truth deflationism is rather a category under which a number of different theories—e.g., disquotationalism (Field 1994, 2001; Quine 1986), minimalism (Horwich 1998), the redundancy theory (Ramsey 1927), etc.—fall under. Also, as Bar-On and Simmons (2007) point out, there are different aspects of truth one can be a deflationist about, i.e., the truth-predicate 'true', the concept of TRUTH, and the property of *truth*. Hence, one can be a deflationist about some aspects and anti-deflationist about others. I am just suggesting a *characterization* of the basic spirit that underlies different truth deflationist positions.

I am not denying that Frege inspires truth deflationism or primitivism. In fact he does. In a number of places Frege puts forward claims that could be the main theses of deflationist theories. Furthermore his famous argument for the indefinability of truth (1897, 1918a) is perhaps one of the most discussed arguments for truth primitivism. What I attempt to establish is that all these different aspects of Frege's conception of truth are tied back to his more fundamental claim, i.e., the claim that *truth is an object*. Frege considers truth to be an object and his deflationist- or primitivist-like claims are his *arguments* that truth is an object. He is not a deflationist because the object of truth is a substantive entity. Frege is not a primitivist because logic is the science of truth as such. My aim is to construct an exegesis that captures Frege's conception of truth as an object in his mature career and explains how this conception is connected to his conception of other notions such as judgment, assertion, and knowledge.

In Frege scholarship it is widely agreed that there are some initial reasons to suspect that truth is an object for Frege. Nevertheless, most commentators eventually do not take those reasons seriously. Previous readings mostly reject—or ignore—that truth is an object for Frege. My dissertation is hence a resistance against this interpretative trend. §1.1 introduces what scholars consider to be the initial reasons to suspect that truth is an object for Frege. §1.2 reviews previous readings that reject—or ignore—that truth is an object for him. In addition to engaging with these previous readings, an exegesis that accepts that truth is an object for Frege ought to meet other challenges. §1.3 introduces these challenges and explains how the dissertation is organized.

1.1. THE TRUTH-VALUE TRUE AS A REFERENCE OF SENTENCES

In his mature career Frege makes the famous distinction between the sense and the reference of an expression (1891, 1892c, 1892a, 1893, etc.). The content of every mean-

ingful expression of a language is divided into the level of sense and that of reference. The reference of an expression is an entity referred to by the expression. The sense of an expression contains a 'mode of presentation' (Frege 1892c: 57) of its reference. Frege does not further explain what he means by 'mode of presentation'. Even though the nature of sense is an important subject in Frege scholarship and I deal with it a little bit in due course, our discussion does not heavily depend on it.

What is important to us is the reference of a term. In a language terms are divided into two categories: saturated and unsaturated terms. Unsaturated terms have empty places for argument terms, e.g., 'the author of ...', '... is a prime number', etc. These unsaturated terms refer to functions, i.e., unsaturated entities that have empty places for arguments. If a function is saturated with arguments, it yields an object as its value. Saturated terms such as 'the author of *Grundgesetze*' or 'Gottlob Frege' refer to objects, i.e., saturated entities that do not have empty places for arguments. For Frege things that exist are either objects or functions. Therefore, in Frege's mature framework, the semantic categories go hand in hand with the ontological categories. Saturated terms refer to saturated entities, i.e., objects while unsaturated terms refer to unsaturated entities, i.e., functions.

A sentence can also have a reference. When a sentence has a reference, its reference is a truth-value—the True or the False. The True is the truth-value of true sentences and the False is that of false sentences. The question is whether truth-values are objects or functions. Frege writes:

A sentence contains no empty place, and therefore we must regard what it stands for as an object. But what a sentence stands for is a truth-value. Thus the two truth-values are objects. (1891: 32)³

³For the translation of Frege's published or unpublished articles, I mostly depend on Frege 1970, 1979,

Note that the True is a 'truth-item' and that it is an object. This is the initial reason to think that truth is an object for Frege. The claim that truth is an object is the claim that *truth is the True*.

Indeed Frege is switching back and forth between truth and the True. In 'On Sense and Reference' Frege argues that a sentence must have a reference and its reference is a truth-value. Suppose that we encounter the sentence 'Odysseus was set ashore at Ithaca while sound asleep' (Frege 1892c: 62). Its sense is a thought, i.e., the thought that Odysseus was set ashore at Ithaca while sound asleep. Frege first points out that if the thought is all there is about this sentence, then we do not need to bother to figure out whether its part, e.g., 'Odysseus', has a reference. This is because whether or not the proper name has a reference the thought still remains the same because the name still has a sense whether or not it has a reference. However, we do often attempt to figure out whether a name involved in a sentence has a reference. Why? We ought to notice the compositionality of reference: the reference of a complex expression like a sentence is determined by that of its parts (1892c: 62).⁴ Therefore, if the reference of a part is missing, the whole complex expression does not have a reference. Then the fact that we are concerned about the reference of 'Odysseus' *qua* the part of the sentence shows that we are concerned about whether the sentence has a reference. Hence, we are committed to the existence of a sentential reference. Then, what is the reference of a sentence? Frege writes:

But now why do we want every proper name to have not only a sense, but also a reference? Why is the thought not enough for us? Because, and to the extent

1984, 1997. I specify the year of publication of the work if it is published or the conjectured year of completion if it is originally unpublished. The year of the cited work is followed by the page number of its translation.

⁴A thought does not satisfy this condition. Thus it is not the reference of a sentence but its sense (Frege 1892c: 62).

that, we are concerned with its *truth value*. . . . It is the striving of *truth* that drives us always to advance from the sense to the reference.

We have seen that the reference of a sentence may always be sought, whenever the reference of its components is involved; and that this is when and only when we are inquiring after the *truth value*. (Frege 1892c: 63; italics mine)

Therefore, the reference of a sentence is a truth-value. What is noteworthy is that Frege is switching back and forth between the notion of truth-value and that of truth. This suggests that truth is the True for him.

We can see similar switching back and forth elsewhere. In ‘Logic in Mathematics’ Frege comments:

Showing . . . that *truth* is not a property of sentences or thoughts, as language might lead one to suppose, this consideration confirms that a thought is related to its *truth-value* as the sense of a sign is to its reference. (1914: 234; italics mine)⁵

I explain ‘this consideration’, which is one of Frege’s arguments that truth is an object, in Chapter 2. It appears that Frege is saying that truth is not a property of truth-bearers and *thus* that the relation of a thought to a truth-value is that of sense to reference. This inference seems to work if the True is truth.

1.2. PREVIOUS READINGS OF FREGE ON TRUTH

Hence, we have initial reasons to think that truth is an object for Frege. However, as I have said, most previous readings reject that truth is an object, i.e., the True, for Frege. There are several different reasons why they deny it.

One reason why they deny that truth is an object for Frege is that he seems to use something like a truth predicate. Because sentences are names, they can feature on the sides of the identity sign. For instance, informally, we can say something such as this:

⁵Also see ‘Introduction to Logic’ (Frege 1906a: 194).

'(2+2 is equal to 4) = (3+3 is equal to 4)'. Now consider the predicate '... is (identical with) the True'. It seems to function like the truth predicate '... is true' in the following sense: when and only when we substitute '...' with the names of the True—including all true sentences—the result refers to the True. Frege indeed uses the predicate '... is the True' in his works. For instance, Frege's formal language *Begriffsschrift* has a formal analogue of '... is the True'. In *Begriffsschrift* all well-formed sentences are prefixed by '┆'. The short horizontal line is called 'the horizontal' in the mature version of *Begriffsschrift*. Frege writes:

I regard [the horizontal] a function-name such that

— Δ

is the True when Δ is the True, and is the False when Δ is not the True. (Frege 1893: §5)⁶

So the function referred to by the horizontal is what '... is the True' refers to. Moreover, Frege uses '... is the True' directly. Look at the following passage from *Grundgesetze der Arithmetik*:

According to §12,

$$\left[\begin{array}{l} \Gamma \\ \Delta \\ \Gamma \end{array} \right]$$

would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible; accordingly [the law I]

$$\left[\begin{array}{l} a \\ b \\ a \end{array} \right]$$

⁶For the translation of *Grundgesetze*, I depend on Ebert and Rossberg's (Frege 2013).

(Frege 1893: §18)

Frege is explaining the Law I of *Begriffsschrift*. Notice that Frege's explanation looks like a meta-linguistic justification of an axiom of a logical system. Also, note that the predicate '... is the True' looks like a truth-predicate in such a meta-linguistic justification. We can find similar passages for other laws and inference rules of *Begriffsschrift*, and can also find the same predicate in those passages. If '... is the True' is indeed a truth predicate for Frege, he accepts that truth is something that can be referred to by such a predicate, i.e., a function. Scholars like Greimann (2008), Künne (2008), and Taschek (2008) take this observation seriously and claim that truth is not an object for Frege.⁷

The second argument that truth is a property for Frege is that by making the True the reference of truth sentences, he is committed to the view that truth is a property. For instance, Künne writes:

... The sense of the truth-value name is a mode of presentation of a truth-value. So Frege cannot reasonably deny that all true thoughts can be subsumed under the concept *x is a mode of presentation of the True*. So, by the end of the day, a subsumption view is rehabilitated by his own lights. (Künne 2008: 35)

Here 'concept' can be taken as meaning 'property'; and 'the subsumption view' refers to the view that takes truth to be a property of thoughts. Künne's point is that although Frege certainly resists the subsumption view, he *eventually* comes to accept it. Richard

⁷Greimann has a particular argument for the claim that '... is the True' is Frege's metalinguistic truth-predicate. This argument is based on a reading of Frege's conception of sentence shared by Klement (2001), Landini (1996), and Pedriali (2017). When Greimann argues that truth is not an object for Frege, however, his point is different from the other two scholars' point. This is because he also denies that truth is a property for Frege. For him, truth is neither an object nor a property (Greimann 2004, 2007). I will deal with Greimann's reading later.

Kimberly Heck (2012: 48) provides a similar argument although his point is slightly different.⁸ Mark Textor makes a similar point through a different argument:

To say that logic is not concerned with the concept ξ *is true* is not to deny that there is such a concept. Indeed, Frege has good reason to think that there is such a property. If 'ξ is true' did not refer to a concept, sentences containing it ('(the thought) that seawater is salty is true') would be neither true nor false. However, this is clearly not the case. Therefore, 'ξ is true' must have sense and reference, and there must be a concept ξ *is true*. (Textor 2010: 648)⁹

Another argument that truth is still a property for Frege is that it is solely for some *technical* reasons that he maintains that the True is an object referred to by a sentence. For instance, Gabriel claims that

[Why does Frege argue that the True is an object?] The answer seems to be that it is Frege's theory of identity which forced him to do so. (1984: 375)

Gabriel's point is that it is only because *Begriffsschrift* needs to allow an identity sign to appear between sentences that Frege takes the True *qua* a sentential reference to be an object. To support this claim Gabriel points to the following passage in Frege's letter to Russell:

If the sign of identity is used between sentences, the truth-value must be recognized as the [reference] of [the sentence]. (Frege 1980: 152–3)

According to Gabriel, Frege admits that 'in fact taking truth-values as objects is not an advantage for semantics in general, but only for semantics of a special formal language like [*Begriffsschrift*]' (1984: 376). Gabriel points to Frege's comment in 'On Sense and Reference':

⁸Heck claims that although it is not incorrect to say that truth is an object for Frege, to argue so is 'to miss what is really at issue' (Heck 2012: 48) in Frege. However, throughout this dissertation, I argue that what is at issue in Frege can only be solved by appealing to the point that truth is an object.

⁹Textor is appealing to Frege's short, but seminal, manuscript 'My Basic Logical Insights' (Frege 1915). I will analyze this manuscript in the next chapter. This analysis shows that Textor's argument is unsound.

The designation of truth-values as objects may appear to be an arbitrary fancy or perhaps a mere play upon words, from which no profound consequences could be drawn. (Frege 1892c: 63–64)

The truth-value True, Gabriel claims, is still a property in Frege's *philosophical*—non-technical—framework. Specifically it is a value property like goodness or beauty.

We have reviewed readings that deny that truth is an object for Frege. One might think that in the end not many commentators deny it. However, there is only a short list of commentators who reject that truth is an object for Frege, not because everyone else accepts it, but because everybody else is silent about it. In particular there are only a few works that address whether truth is the True for Frege. Ignorance is the default attitude of the literature on Frege toward the question as to whether truth is an object for him. But this ignorance does not seem to be grounded in the implicit agreement that truth is an object for Frege. It rather seems to be grounded in the consensus that whether truth is the True for Frege is unimportant to understanding his philosophical framework.¹⁰

1.3. THE CHALLENGES FOR TAKING TRUTH TO BE AN OBJECT

Again my aim in this dissertation is to build up an interpretation according to which truth is the True and hence an object for Frege. Such a reading ought to respond to the points made by the previous readings we've just seen. Specifically, first, my reading

¹⁰For instance, see Pedriali 2017 or Schaar 2018. Both commentators talk about both truth and the True; but they are not dealing with what kind of relationship obtains between them. Moreover, Reck (2007) almost never mentions truth-values when he deals with truth in Frege. Why is that so? It is not my intention to track down such a historical cause. My rough conjecture is that the reason consists in the criticism of Michael Dummett—the most influential scholar and one of the philosophical giants in the history of analytic philosophy—about Frege's move to turn sentences into proper names of truth-values. Dummett even considers this move to be a 'disaster' (Dummett 1993: 644). His criticism is not concerned with Frege's identification of truth with the True. However, still Frege's conception of sentences as proper names is the most critical step toward Frege's conception of truth as an object. Therefore, Dummett's criticism can eventually work as a criticism against the latter.

ought to establish that Frege's use of '... is the True' or the horizontal does not imply that truth is a property referred to by such a predicate. Secondly, my reading ought to establish that there is no gap between what Frege says on the surface and what he has deep down in his mind; that the former *is* what he has in his mind. Thirdly, my reading ought to show that accepting that truth is the True for Frege is important to understanding his philosophical framework.

In addition to these, there are further challenges which my reading must meet. One of them concerns Frege's conception of *judgment*. Throughout his entire career, Frege takes it for granted that the act of judging is constituted by that of acknowledging the truth of a truth candidate. In his mature career, truth candidates are thoughts. Thus, judging is acknowledging the truth of a thought for the mature Frege.¹¹ As we will see later, judging as such is actively applying the notion of truth to a thought. But how can Frege take the act of judgment to be that of active application of truth to a thought if truth is an object? Don't we acknowledge the truth of a thought by ascribing the property *truth* to a thought? In short a reading that accepts that truth is an object for Frege must be able to explain in what sense judgment is acknowledgment of truth.

The second challenge concerns Frege's conception of logic and science in general. In 'Thought' he comments:

Just as 'beautiful' points the way for aesthetics and 'good' for ethics, so does 'true' for logic. All sciences have truth as their goal; but logic is also concerned with it in a special way: logic has much the same relation to truth as physics has to weight or heat. To discover truths is the task of all sciences; it falls to logic to discern the laws of truth. (Frege 1918a: 342)

All sciences aim at truth. Among these sciences logic takes truth to be its main subject.

¹¹See Frege 1892c, 1897, 1893, 1906b, 1906a, 1915, 1918a, etc.

In this sense it is the science of truth. If truth is an object, then implausible implications seem to follow from those claims. Surely, we are happy to say that all the sciences aim at truth. However, what would it mean that all those different sciences aim at one object? Different sciences rather seems to aim at explaining different objects and properties. Moreover, logic comes to be the science that deals with a single object, the True. But how can there be a science of an object? Furthermore, if all the other sciences aim at the True anyway, in what sense is logic the only science of the True? A reading that accepts that truth is an object for Frege must be able to answer these questions.

The third challenge concerns the theoretical significance of taking truth to be an object. Say truth is indeed an object for Frege. Then his conception of truth is radically different from that of contemporary philosophy. In contemporary philosophy truth is widely regarded as a (sort of) property. Or, at least, no one works with a framework that considers truth to be a single object. Accordingly, our conception of notions related to truth—e.g., knowledge, assertion, judgment, belief, etc.—is based on a conception of truth radically different from Frege's. Then the following question seems to follow: is there anything *we* can theoretically learn from Frege's conception of truth? If one's interest is just in retrieving Frege's own view, then one does not need to be interested in this question. But my aim in this dissertation is to defend Frege's insights into the nature of truth as theoretically fruitful and valuable. Then I have to provide a positive answer to the above question.

My dissertation is thus an attempt to meet the above challenges. In Chapters 2 and 3 I provide my arguments that truth is an object for Frege in his mature career. Chapter 2 addresses the tension between his deflationist-like claims and his other claims that regard truth as substantive. I argue that we can reconcile Frege's conflicting comments

only when we take it into account that truth is an object for him. Chapter 3 deals with Frege's indefinability argument. I show by a line-by-line analysis that this argument is not an argument that truth is indefinable, but an argument that truth cannot be defined *qua* a property *because* it is not a property.

In these chapters, I provide my responses to most of the previous readings that deny that truth is an object for Frege. Chapter 4 addresses two commentators that are not addressed in Chapters 2 and 3: Greimann—his 'No-Entity' theory reading of truth in Frege—and Gabriel.

Chapter 5 is concerned with Frege's conception of judgment (and its verbal counterpart assertion), i.e., what acknowledging the truth of a thought amounts to if truth is an object. We seek an answer to this question by way of dealing with the tensions related to the sign '⊢'—the sign for making a judgment (or assertion). I will argue that these apparent tensions can be explained away. While explaining these tensions away, we come to see how Frege elucidates judgment as acknowledgment of truth under his conception of truth *qua* an object. Under this elucidation acknowledgment of truth is not truth predication but non-judgmental—Millikanian—identification of the True.

Chapter 6 explains in what sense all sciences aim at the True and logic is the science of the True. The answers are found in Frege's conception of propositional knowledge. Taking truth to be an object, Frege's conception of propositional knowledge, which is tied to truth, differs from traditional conceptions. Specifically Frege takes propositional knowledge to be grounded in our objectual knowledge of the True. If we take this point into account, we can explain why the True is the aim of all sciences and the subject matter of logic.

Chapter 7 deepens and improves Frege's insights revealed in the previous chapters.

In addition to his conception of truth as an object, Frege also provides a new conception of judgment as non-judgmental identification and of propositional knowledge as grounded in objectual knowledge. In this chapter, I first show how Frege's conception of judgment can respond to Peter Hanks (2007, 2011, 2015, 2018)'s recent criticisms against the Fregean conception of judgment in general. Secondly, I show how Frege's conception of assertion can contribute to the issue with the relationship between truth and assertion. Finally, I further develop Frege's conception of knowledge into what I call 'Frege-Millikanian theory of knowledge' by combining Frege's conception with Millikan's theory of objectual knowledge.

CHAPTER 2

ARGUMENT 1: FREGE IS NOT A DEFLATIONIST

There is a tension concerning Frege's comments on the relationship between logic and truth. On the relationship Frege writes:

Decisive for the treatment of this science is how the logical laws are conceived, and this in turn connects with how one understands the word 'true'. (Frege 1893: XIV–XV)¹

Frege is asserting that the notion of logical law is central to the science of logic and is somehow related to how we understand the word 'true'. He holds this idea throughout his career:

... In the field [psychology] investigates it does not study the property 'true' as, in its field, physics focuses on the properties 'heavy', 'warm', etc. This is what logic does. It would not perhaps be beside the mark to say that the laws of logic are

¹The roman numbers are the page numbers of the Foreword of *Grundgesetze*.

nothing but an unfolding of the content of the word 'true'. Anyone who has failed to grasp the meaning of this word —what marks it from others— cannot attain to any clear idea of what the task of logic is. (Frege 1879b: 3)

Frege regards Logic as the science whose *subject matter* is the property *truth* just as the property *heaviness*, for instance, is one of the subject matters of physics. Assuming that 'true' refers to the property *truth*, establishing the laws of logic is understanding *truth*. Hence, one cannot get a grip on the task of logic without understanding the meaning of the word. Frege retains this idea throughout his entire career. In 'Thought' written much later in his career, Frege comments:

Just as 'beautiful' points the way for aesthetics and 'good' for ethics, so does 'true' for logic. ... Logic has much the same relation to truth as physics has to weight or heat. To discover truths is the task of all sciences; it falls to logic to discern the laws of truth. (1918a: 342)

Logic aims to discover the *laws of truth*. Thus there seems to be a substantive sense in which for Frege

(LogT) logic is the *science of truth*.

As we've said, however, Frege in his mature career puts forward claims that could easily be main theses of truth deflationism. These deflationist comments do not really sit well with (LogT). In 'On Sense and Reference' where the distinction between sense and reference is specified, he claims:

... Even the sentence 'The thought that 5 is a prime number is true' contains only a thought, and indeed the same thought as the simple '5 is a prime number.' (1892c: 64)²

Frege is putting forward the following claim:

²We can find similar claims in a number of Frege's mature works such as 1906a, 1914, 1915, 1918a, etc.

(Iden) The thought that p is identical with the thought that the thought that p is true.

(Iden) seems to imply that the predicate ‘ \dots is true’ does not have a substantive sense. Indeed, that appears to be the idea Frege has in mind. In ‘My Basic Logical Insights’, Frege remarks: “‘True’ has a sense that contributes nothing to the sense of the whole sentence in which it occurs as a predicate [i.e., the sentence of the form ‘The thought that p is true’]’ (1915: 251). Let ‘ $\langle p \rangle$ ’ mean ‘the thought that p ’. Frege accepts that

(Defl) The sense of ‘true’ makes no contribution to $\langle \langle p \rangle$ is true \rangle so that $\langle \langle p \rangle$ is true \rangle is identical to $\langle p \rangle$.

He is hence a deflationist about the meaning of the truth-predicate, which is shared by almost all truth deflationists.³ In the same paper, Frege also writes:

[...] ‘True’ only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with is not contained in the word ‘true’ at all \dots [but] in the assertoric force with which a sentence is uttered. (1915: 252)

Frege thus accepts that

(Assert) What logic concerns is contained not in the truth-predicate but in the assertoric force.

Given that predicates such as ‘ x is true’ are unsaturated, and predicates as such refer to properties in Frege’s mature framework, it is puzzling that he accepts (Defl) and (Assert) while he also accepts (LogT). If logic is the science of truth, it seems, it should be the science of the property *truth* referred to by the truth-predicate. If so, what logic concerns appears to be in the truth-predicate in a significant sense. But it appears that Frege denies this by putting forward (Assert). Moreover, if truth is such a substantive

³Commentators such as Künne (2008) read (Iden) in a different way. According to them, Frege’s point is rather that the sense of ‘true’ is already involved in the sense of every sentence. As we will see later, however, this omnipresence reading of (Iden) is not entirely plausible.

property and is referred to by the truth predicate, it is not clear why Frege puts forward (Defl). It even seems paradoxical because, as we will see later, Frege does accept that, if 'true' in ' $\langle p \rangle$ is true' refers to a property, then $\langle \langle p \rangle$ is true \rangle differs from $\langle p \rangle$.⁴ If Frege were a deflationist who accepts that truth *qua* the property referred to by the truth predicate does not exist, we could easily understand why he argues for (Defl). However, (LogT) certainly does not look like a claim such a deflationist can embrace. (LogT), (Defl), and (Assert) do not square well with each other. Frege even makes these conflicting remarks in a single article (1897). This is the tension we encounter in his conception of the relationship between logic and truth.⁵

This tension can be explained away if we take it into consideration that truth is an object for Frege. In particular (Iden), (Defl), and (Assert) constitute Frege's arguments that truth is an object. In §2.1 I develop a thorough analysis of Frege's 'My Basic Logical Insights' where we can find all those three claims together. Analyzing this article helps us to understand the real issue behind this tension. §2.2 suggests my interpretation according to which the tension in question is only apparent. §2.3 addresses a crucial objection to my reading, i.e., the objection that Frege has a substantive truth predicate.

⁴As we will see, Frege makes this point in, for instance, 'On Sense and Reference' (Frege 1892c), 'Logic' (1897), 'Introduction to logic' (1906a), and 'Logic in Mathematics' (1914).

⁵Taschek (2008) and Greimann (2014) deal with this tension. Taschek takes this tension as genuine and attempts to explain why Frege has this tension. Greimann criticizes Taschek's reading and maintains that the tension is only apparent. I do not visit these commentators in this paper. First, Greimann's criticism of Taschek seems quite decisive. Secondly, both commentators do not deal with (Defl). More importantly, neither Greimann nor Taschek takes it into account that the mature Frege identifies truth with the True. Only if we take it into account, however, we can satisfactorily explain away the tension. Pedriali (2017) also deals with this issue.

2.1. ‘MY BASIC LOGICAL INSIGHTS’

Frege takes judging as acknowledging or recognizing a thought to be true.⁶ In ‘My Basic Logical Insights (MBLI)’, he writes:

Making a judgment does not alter the thought that is recognized to be true. . . .
We can always cull out the thought that is recognized as true; the act of judgment forms no part of this. (1915: 251)

This is related to Frege’s elucidation of judgment in terms of the distinction between sense and reference. In ‘On Sense and Reference’, he writes:

But so much should already be clear, that in every judgment, no matter how trivial, the step from the level of thoughts to the level of reference (the objective) has already been taken. (Frege 1892c: 64)

Judgments can be regarded as advances from a thought to [its] truth value. (65)⁷

A sentence has a thought as its sense and a truth-value as its reference. To judge that p is not to add something to $\langle p \rangle$, but to *take a step* from $\langle p \rangle$ to its truth-value.⁸ As Greimann (2014) says, the step toward a truth-value is crucial in Frege’s logic. Criticizing what he dubs ‘the intentionalist logic’, Frege writes:

The intensionalist logicians are only too happy not to go beyond the sense; for what they call the intension . . . is . . . the sense. They forget that logic is not concerned with how thoughts . . . follow from thoughts, that the step from thought to truth-value—more generally, the step from sense to reference—has to be taken. They forget that the laws of logic are first and foremost laws in the realm of [references] and only relate indirectly to sense. (1892a: 122)

⁶Frege characterizes judgment as recognition of truth in his entire career. But Richard Kimberly Heck (2012: §2.3) claims that Frege’s understanding of this characterization has changed. Our discussion in Section 4 shows how the mature Frege understands this characterization.

⁷Frege 1970 uses ‘a’ for the translation instead of ‘its’. However, the original German word is ‘*seinem*’, which means ‘its’.

⁸All these are metaphorical elucidations; but what Frege is trying to establish seems to be clear. We will talk about these metaphorical elucidations more and more as our discussion proceeds.

The so-called intensionalist about logic takes it that logic only concerns how thoughts follow from thoughts. However, logic aims to discern the laws that concern the realm of reference. Therefore, the step from sense to reference should be taken for the sake of logic. This would be why all the well-formed sentences in Frege's logic, *Begriffsschrift*, have the device for making a judgment, i.e., '⊢'. In Frege's conception, logic is only concerned with judgments. Now, assertion 'manifests' (1918a), 'communicates' (1897), or 'gives an expression to' (1906a), a judgment. In other words, assertion is manifested, communicated, and expressed judgment. Therefore the act of assertion does not form a part of the asserted thought, either.

After pointing out the nature of judgment and assertion, Frege all of sudden turns to his points about the truth predicate 'true':

The word 'true' is not an adjective in the ordinary sense. (1915: 251)

For the sake of contrast, Frege first describes how a normal predicate 'salty' works.

If I attach the word 'salty' to the word 'sea-water' as a predicate, I form a sentence that expresses a thought. To make it clearer that we have only the expression of a thought, but that nothing is meant to be asserted, I put the sentence in the dependent form 'that sea-water is salty'. ... I could have it spoken by an actor on the stage as part of his role, for we know that in playing a part an actor only seems to speak with the assertoric force. Knowledge of the sense of 'salty' is required for an understanding of the sentence, since it makes an essential contribution to the thought ... (1915: 251)

Then, he proceeds to the truth-predicate.

If I attach ['true'] to the words 'that sea-water is salty' as a predicate, I likewise form a sentence that expresses a thought. For the same reason as before I put this also in the dependent form 'that it is true that sea-water is salty'. The thought expressed in these words coincides with the sense of the sentence 'that sea-water is salty'. So the sense of the word 'true' is such that it does not make any essential

contribution to the thought. If I assert 'It is true that sea-water is salt', then I assert the same thing as if I assert 'Sea-water is salt'.(1915: 251)

Frege puts forward (Iden) and (Defl). So the truth-predicate differs from other predicates in that it does not make any contribution to the thought expressed by a sentence that includes the predicate. Frege concludes:

This enables us to recognize that the assertion is not to be found in the word 'true', but in the assertoric force with which the sentence is asserted. (1915: 251)

Given the flow of the text, Frege seems to be responding to an objection to his claim that assertion does not constitute a part of the asserted thought. The objection appears to be something like the following: the predicate 'true' in ' $\langle p \rangle$ is true' stands for assertoric force and forms a part of $\langle \langle p \rangle$ is true \rangle . If this objection is true, assertoric force can be a part of the asserted thought and, in that sense, assertion can constitute a part of the asserted thought. Frege's response is that assertoric force cannot constitute a part of the asserted thought by way of 'true' because 'true' cannot make any contribution to a thought due to its semantic inertness.

In fact there is *no* predicate—whether or not a predicate has a substantive sense to contribute to a thought—by way of which assertoric force can constitute a part of the asserted thought. Look at the following passage from 'Logic in Mathematics':

... What we mean by ... 'The thought that $3 > 2$ is true' can be more simply said by ... '3 is greater than 2'. Thus we do not need the word 'true' at all to say this. And we can see that really nothing at all is added to the sense by this predicate. In order to put something forward as true, we do not need a special predicate: we only need the assertoric force with which the sentence is uttered. (Frege 1914: 233)

After making the same point about 'true', Frege quickly puts forward a general claim: there is no predicate such that assertoric force is fixedly attached to it and so we need it whenever we make an assertion. In 'On Schoenflies ...' Frege writes:

[In the sentence 'Two is prime'], we count the copula 'is' as belonging to this part of the sentence. But there is usually something combined with it which here must be disregarded: assertoric force. We can of course express a thought, without stating it to be true. The thought is strictly the same whether we merely express it or whether we also put it forward as true. Thus assertoric force, which is often connected with the copula or else with the grammatical predicate, does not belong to the expressions of the thought (1906c: 177)⁹

So assertoric force does not form a part of the asserted thought. In that sense assertion does not form a part of the asserted thought, either.

In addition to the general fact that no expression can make assertoric force a part of the asserted thought, there is something special about the case of 'true' because 'true' unlike other expressions cannot constitute a part of a thought, i.e., because it does not have a substantive sense to contribute to a thought. Frege speculates about this point in the remainder of MBLI:

The word 'true' has a sense that contributes nothing to the sense of the whole sentence in which it occurs as a predicate.

But it is precisely for this reason that this word seems fitted to indicate the essence of logic. . . . 'True' seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought. And although this attempt miscarries, or rather through the very fact that it miscarries, it indicates what is characteristic of logic. And this, from what we have said, seems something essentially different from what is characteristic of aesthetics and ethics. For there is no doubt that the word 'beautiful' actually does indicate the essence of aesthetics, as does 'good' that of ethics, whereas 'true' only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with is not contained in the word 'true' at all but in the assertoric force with which a sentence is uttered. (1915: 251–2)

⁹Pedriali (2017) does not read this passage literally; he thinks that the assertoric force is still attached to a copula in a sentence for Frege. However, his reading is not fully satisfactory as we will see in §4.

This passage would perhaps be the most puzzling part of MBLI. Commentators navigate this puzzling passage by developing all kinds of interpretations of what Frege means by ‘the essence of logic’ here (cf. Greimann 2014, Pedriali 2017, Taschek 2008). However, I believe, these attempts are misleading. There is nothing mysterious about what Frege means by ‘the essence of logic’. Note that he says that ‘beautiful’ indicates the essence of aesthetics and ‘goodness’ indicates that of ethics. ‘Beautiful’ refers to the property *beauty* with which aesthetics is really concerned; and ‘good’ refers to the property *goodness* with which ethics is really concerned. ‘The essence of logic’ is ‘what logic is really concerned with’. The essence of logic is just truth. Fiddling with what ‘the essence of logic’ means would not help us to understand this passage any further.

So I develop a reading of the passage that does not assign any special meaning to ‘the essence of logic’ except for that of ‘what logic is really concerned with’. First, let us analyze what Frege says at the end of the passage:

The word ‘beautiful’ actually does indicate the essence of aesthetics ... whereas ‘true’ only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with is not contained in the word ‘true’ at all but in the assertoric force with which a sentence is uttered. (1915: 252)

Frege claims that ‘true’ makes an *abortive* attempt to indicate truth. But a word does not make an attempt to indicate anything; it is a speaker who uses an expression that attempts to indicate anything by it. Frege’s point is then that we fail to indicate truth by ‘true’ in the sense that we indicate the property *beauty* by ‘beautiful’. This is because truth is not contained in ‘true’ but in assertoric force. This reversely appears to imply that the property *beauty* is indeed contained in ‘beautiful’ and *thus* that we can indicate the property by the predicate.

Now, when we assert ‘That painting is beautiful’, we indeed successfully talk about

the property *beauty* by virtue of our making 'beautiful' a part of the asserted sentence. This is because 'beautiful' has a substantive sense by virtue of which it refers to the property *beauty*, i.e., because 'beautiful' contains the property *beauty*. Then it appears to be tempting to say the following: Frege's point is that even when we assert ' $\langle p \rangle$ is true', we do not talk about truth by virtue of our making 'true' a part of the asserted sentence. That is because 'true' does not have any substantive sense that would let it refer to truth, i.e., because 'true' does *not* contain truth. Instead we can talk about truth only by virtue of assertoric force or assertion. If this is along the right lines, then we can explain what Frege means by the following:

And this, from what we have said, seems something essentially different from what is characteristic of aesthetics and ethics. For there is no doubt that the word 'beautiful' actually does indicate the essence of aesthetics, as does 'good' that of ethics, whereas 'true' only makes an abortive attempt to indicate the essence of logic (1915: 252)

What is characteristic of aesthetics and ethics is that we can talk about the essence of aesthetics and ethics by virtue of the relevant predicates, i.e., 'beautiful' and 'good'. But what is characteristic of logic is different: we cannot talk about the essence of logic by virtue of the relevant predicate 'true'. We can do that solely by virtue of assertoric force. Assertoric force is essentially different from the predicates like 'beautiful'. The latter has a sense that can constitute a part of a thought. But the former does not form a part of a thought at all because it does not belong to any expression of a thought:

... Assertoric force, which is often connected with the copula or else with the grammatical predicate, does not belong to the expressions of the thought ...
(1906c: 177)

We can also make sense of the beginning of the passage:

... The word 'true' has a sense that contributes nothing to the sense of the whole sentence in which it occurs as a predicate.

But it is precisely for this reason that this word seems fitted to indicate the essence of logic. ... 'True' seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought. And although this attempt miscarries, or rather through the very fact that it miscarries, it indicates what is characteristic of logic. (1915: 251–2)

What is characteristic of logic consists in the point that we can talk about the essence of logic, i.e., truth, only by virtue of assertion. As we've just said, assertion does not form a part of a thought. Now 'true' gives an expression to this distinctive feature of assertion: just like assertion does not form a part of a thought, the sense of 'true' *fails* to form a part of a thought. Frege's point is then not that 'true' can indicate truth or it is a device for assertion. 'True' does not indicate truth; its 'attempt' to do so is only abortive. Nor is it a device for assertion; there cannot be such a device. 'True' merely reminds us, through its semantic inertness, of the fact that we can *only* talk about truth by virtue of assertion that does not constitute a part of a thought.

Recall:

(Assert) What logic concerns is contained not in the truth-predicate but in the assertoric force.

In the suggested reading of MBLI, it means that we can talk about truth not by virtue of the truth-predicate but by virtue of assertoric force. The suggested reading of MBLI also shows that (Iden) and (Defl):

(Iden) The thought that p is identical with the thought that the thought that p is true

(Defl) The sense of 'true' makes no contribution to $\langle\langle p \rangle \text{ is true} \rangle$ so that $\langle\langle p \rangle \text{ is true} \rangle$ is identical to $\langle p \rangle$

put forward the semantic inertness of ‘true’, which is just a reminder of (Assert). (Iden), (Defl), and (Assert) understood as such do not go against (LogT):

(LogT) logic is the *science of truth*.

For those claims do not imply that truth is not a substantive property. Frege’s point is just that we can only talk about truth by virtue of assertion.

However, in what sense can we talk about truth only by virtue of assertion? Without providing a proper answer to the question, the tension under consideration is not yet completely explained away. We already appear to have an hint to this answer. Recall assertion is manifested, communicated, and expressed judgment. Also recall we can take a step from a thought to a truth-value only in our judgment. Then, it is tempting to say the following: we can talk about or indicate truth just in case we take a step from a thought to a truth-value—specifically the True in this case. But then what is the relationship between truth and the True?

2.2. TRUTH AND THE TRUTH-VALUE TRUE

Some terminology. Frege often uses the locution ‘The object O falls under the concept C ’.¹⁰ A concept is a function such that its value is always a truth-value. For instance, the predicate ‘ \dots is prime’ refers to the concept that yields the True given a prime number and the False otherwise. 2 falls under the concept *prime* because the value of the concept is the True when 2 saturates it.

There seems to be an obvious way to connect truth to the truth-value True. Saying that a thought is true is equivalent to saying that the sentence that has the thought as its sense refers to the True. In other words, $\langle p \rangle$ is true just in case ‘ p ’ refers to the True, i.e., p is the True. This leads to the following idea:

¹⁰For instance, see Frege (1892a).

(TP) Truth is a concept under which $\langle p \rangle$ falls when p is the True.

Certainly, if Frege accepts that all true sentences refer to the True, he must also accept that there is a concept (or even concepts) under which all true thoughts fall. We can easily identify such a concept as Künne nicely shows:

The sense of a singular term, Frege famously said, [contains] ‘a mode of presentation ...’ of the object it designates ... So Frege cannot reasonably deny that all true thoughts can [fall under] the concept *x is a mode of presentation of the True*. (Künne 2008: 35)¹¹

Künne concludes:

So, by the end of the day, a subsumption view is rehabilitated by his own lights. (Künne 2008: 35)

However, that there is a concept under which every true thought falls does not imply that the concept constitutes truth. The identification of truth with one of such concepts obtains only if (TP) is true. Frege denies (TP).

In ‘On Sense and Reference’, Frege writes

[1] One might be tempted to regard the relation of the thought to the True not as that of sense to reference, but rather as that of subject to predicate. [2] One can, indeed, say: ‘The thought, that 5 is a prime number, is true.’ But [3] ... nothing more has been said than in the simple sentence ‘5 is a prime number.’ [4] The truth claim arises in each case from the form of the declarative sentence, and [5] when the latter lacks its usual force, e.g., in the mouth of an actor upon the stage, even the sentence ‘The thought that 5 is a prime number is true’ contains only a thought, and indeed the same thought as the simple ‘5 is a prime number.’ It follows that [6] the relationship of the thought to the True may not be compared with that of subject to predicate.

¹¹Künne uses ‘is’ instead of ‘contain’. But Frege does not say that the sense of an object *is* a mode of presentation of the object. The former contains the latter. See Frege (1892c: 57).

For Frege, ‘the relation of subject to predicate’ is ambiguous. It can denote two different relations: the relation of an object to a concept (an object falls under a concept) and that of a concept to another concept (a concept is subordinate to the other concept) (1892a). Because a thought is an object—it is referred to by a saturated expression ‘the thought that p ’—what it denotes in [1] must be the relation of an object to a concept. By [1], Frege is taking into consideration theorists who regard the True as a concept under which a thought can fall. The rest of the passage is an argument against these theorists. The conclusion we have in [6] is that the True is not a concept under which $\langle 5 \text{ is prime} \rangle$ falls. But the argument shows that truth is not a concept, too. Here is how the argument runs:

(P₁) If the True is a concept, then ‘ \dots is true’ denotes the concept.

(P₂) If ‘ \dots is true’ denotes a concept, $\langle p \rangle$ is different from $\langle \langle p \rangle \text{ is true} \rangle$.

(P₃) $\langle p \rangle$ is identical with $\langle \langle p \rangle \text{ is true} \rangle$.

(C₁) \therefore ‘ \dots is true’ does not denote a concept.

(C₂) \therefore The True is not a concept.

(P₃) can be read off from [3]–[5]. [3] is just (P₃). [4] and the first half of [5] are dealing with our misconception of ‘true’ such that, in ‘ $\langle p \rangle$ is true’, ‘true’ adds to $\langle p \rangle$ the assertoric force as a sense added to $\langle p \rangle$. This misconception can hinder people from realizing that $\langle p \rangle$ is identical with $\langle \langle p \rangle \text{ is true} \rangle$. Establishing that the assertoric force is not attached to ‘true’ at all, Frege repeats (P₃) in the second half of [5]. Then, he directly proceeds to the conclusion in [6]. This shows that Frege is committed to (P₂), which is also strongly suggested by ‘but’ between [2] and [3]. Without (P₁), Frege cannot proceed from (C₁) to (C₂). Also, (P₁) seems to explain Frege’s move from [1] to [2].

The above analysis of the argument shows that Textor's argument that ' \dots is true' refers to a concept is misleading:

To say that logic is not concerned with the concept ξ *is true* is not to deny that there is such a concept. Indeed, Frege has good reason to think that there is such a property. If ' ξ is true' did not refer to a concept, sentences containing it ('(the thought) that seawater is salty is true') would be neither true nor false. However, this is clearly not the case. Therefore, ' ξ is true' must have sense and reference, and there must be a concept ξ *is true*. (Textor 2010: 648)¹²

Frege's whole point is that ' \dots is true' must not refer to a concept. If it did, $\langle p \rangle$ would differ from $\langle \langle p \rangle$ is true \rangle . Textor is appealing to the compositionality of the reference of a complex term: if ' $\langle p \rangle$ is true' has a reference, ' \dots is true' should have a reference. On top of this, one can further argue that ' \dots is true' is an unsaturated term and thus it must refer to a concept. However, ' \dots is true' is an exception—presumably the only exception—to these rules. That is the whole point of Frege's above argument.

When we take truth to be a concept or a property, we take it to be the denotation of ' \dots is true'. The above argument says that truth as such is not a concept. Frege writes in 'Logic in Mathematics':

So to say of a sentence, or thought, that it is true is really quite different from saying of seawater, for example, that it is salt. In the former case we add something essential by the predicate, in the former we do not.

Showing \dots that truth is not a property of sentences or thoughts, as language might lead one to suppose, this consideration confirms that a thought is related to its truth-value as the sense of a sign is to its reference. (1914: 234)

Therefore, (TP) is false. Truth is not a concept.

¹²Textor is appealing to Frege's short, but seminal, manuscript 'My Basic Logical Insights' (Frege 1915). I will analyze this manuscript in the next chapter. This analysis shows that Textor's argument is unsound.

Recall that Frege's ontology only recognizes objects and functions. Assuming truth is a function, it must be a concept. Hence, truth must be an object. That is,

(TTV) Truth is the True.¹³

Our language misleads us to consider this object as a concept by labeling it with the unsaturated term '... is true.' Our language is so logically imperfect that it produces a number of confusions of the same kind:

... We can scarcely avoid using such expressions as 'the concept *prime*'. Here there is no trace left of unsaturatedness, of the predicative nature. ... So language brands a concept as an object. (Frege 1906c: 177)

We cannot avoid using expressions like 'the concept *prime*' because of the defect of our language. Likewise,

That we cannot [avoid using '... is true'] is due to the imperfection of language. (Frege 1915: 252)¹⁴

Thus there is strong textual evidence for (TTV). But not many commentators ascribe it to Frege as we have said.¹⁵ Rather commentators tend to deny that (TTV) is correct of Frege's conception of truth. For instance, Greimann (2004, 2007) and Künne (2008) point to these passages.

What, in the first place, distinguishes it from all other predicates is that predicating it is always included in predicating anything whatever. (Frege 1897: 129)

... We cannot recognize a property of a thing without at the same time finding out the thought *this thing has this property* to be true. (Frege 1918a: 328)

¹³Greimann (2007) claims that truth is not even an object for Frege. However, Kim (2019) shows that Greimann's reading is grounded in a misinterpretation of Frege's comments on saturation.

¹⁴Also, see 'Concept and Object' (Frege 1892b). One difference between the case of 'the concept *prime*' and '... is true' is that Frege says that the former indeed refers to an object while he denies that the latter refers to a concept. That again makes '... is true' a sole exception to Frege's rule of reference.

¹⁵A few exceptions would include Burge (1986) and Ricketts (2003).

They regard these remarks as the evidence that for Frege truth is something that can be predicated. Künne even maintains that the above remarks shows that Frege accepts the omnipresence of truth:

(Omni) $\langle\langle p \rangle \text{ is true} \rangle$ is the full articulation of $\langle p \rangle$.

(Omni) is not warranted. Those passages can also be read as supporting the following:

(Red) $\langle\langle p \rangle \text{ is true} \rangle$ is reduced down to $\langle p \rangle$.

Our reading of MBLI in §2.1 seems to show that (Red) is the case. Frege's point there is that the sense of 'true' is not included in any thought—not that its sense is everywhere. Neither show the above passages that Frege recognizes the concept *truth*. If there were the concept *truth* and '... is true' referred to it, $\langle p \rangle$ would differ from $\langle\langle p \rangle \text{ is true} \rangle$. By the above comments, Frege is merely pointing out the peculiarities of the sense and the reference of '... true'. His goal is to suggest the idea that truth cannot be a concept:

... We cannot recognize a property of a thing without at the same time finding out the thought *this thing has this property* to be true. ... The [reference] of the word 'true' seems to be altogether *sui generis*. May we not be dealing here with something which cannot be called a property in the ordinary sense at all? (Frege 1918a: 328)

Frege's comments in which Greimann and Künne's rejection of (TTV) is grounded do not show that he takes truth to be something that can be predicated. Greimann (2004, 2007) has other objections to (TTV). I address those objections in §2.3 of this chapter and Chapter 4. As we will see, those objections can be satisfactorily responded to.

For now, let us accept (TTV) and go back to the tension in question. We can explain why Frege is strongly committed to the following deflationist claims:

(Iden) The thought that p is identical with the thought that the thought that p is true

(Defl) The sense of 'true' makes no contribution to $\langle\langle p \rangle \text{ is true} \rangle$ so that $\langle\langle p \rangle \text{ is true} \rangle$ is identical to $\langle p \rangle$

Those claims are a part of his argument for (TTV)—the central thesis of his conception of truth. Moreover, Frege's commitment to them does not go against (LogT):

(LogT) logic is the *science of truth*.

By (TTV), logic is the science of the True. (Defl) and (Iden) are valuable observations that lead us to identify what logic is the science of. We can also explain why logic needs judgment *qua* advancement to truth-values. The True, or truth, is the *reference* of true sentences. Therefore, we recognize truth only when we take a step from a thought to a truth-value. This is the reason why advancing to truth-values is essentially necessary for the sake of logic, i.e., the science of truth. (Assert)

(Assert) What logic concerns is contained not in the truth-predicate but in the assertoric force.

now means that the advancement to the True, i.e., truth, is only made by the assertoric force, but not by '... is true'.¹⁶ The tension is *completely* explained away. The related exegetical question about the relationship between truth and the True is also answered.

A qualification ought to be added. It is not the case that Frege holds the conception of truth as an object throughout his career. As Heck (2012) points out, the early Frege appears to take truth as a property. Frege's conception of truth as an object only comes along with his sense-reference distinction. This is why the remarks that seemingly go against (LogT) are found only in the mature Frege's works. But those remarks do not

¹⁶One might wonder if (Assert) is compatible with Frege's claim that truth is objective. My suggested reading shows that (Assert) does not go against the objectivity of truth. (Assert) is basically the claim that we can recognize the True via the act of judgment or assertion, not by the sense or reference of the truth-predicate. Whether or not a thought belongs to the True is still independent of whether the True of the thought is recognized by an epistemic subject.

actually go against (LogT) in the mature Frege's framework because, for him, (LogT) only means that logic is the science of the True. The tension is only apparent.

One might ask in what sense logic is the science of the True. This is the question for Chapter 6. However, here I provide a sketch of my answer to it. First, (propositional) knowledge is reducible to the knowledge of the True. In 'On Sense and Reference', Frege writes:

Comprehensive knowledge of the reference [of a proper name] would require us to be able to say immediately whether any given sense belongs to it. (1892c: 58)

The reference of a proper name is an object. Thus Frege is claiming that comprehensive knowledge of an object is achieved when we can immediately say whether any given sense belongs to it. Thus, comprehensive knowledge of the True is achieved when we can immediately say whether any given thought belongs to it. Being able to decide whether a given thought $\langle p \rangle$ belongs to the True seems to be equivalent to knowing that p or that it is not the case that p . To gather propositional knowledge is to pile up our knowledge of the True and attempt to achieve comprehensive knowledge of it.

Now, logic is only concerned with the True. Frege's logic, *Begriffsschrift*, has logical laws and inferential rules. Inferential rules are the rules that lead us from a truth to another truth, which means that, given a thought that belongs to the True, it leads to another thought that also belongs to the True. Logical laws are indubitably true, i.e., they obviously belong to the True. To accept certain thoughts as logical laws is to accept that the truth-value True is the object to which those thoughts obviously belong. We never introduce any other object into logic unless we can deduce its existence from logical laws by inferential rules.¹⁷ Hence, logic purports to expand our knowledge of

¹⁷For instance, the existence of value-ranges follows from the laws of logic.

the True without depending on the knowledge of any other object. It is in this sense that logic only concerns the True and is the science of the True.¹⁸ But again, this is merely a sketch of what will come in Chapter 6.

2.3. DOES FREGE HAVE A TRUTH-PREDICATE?

Closing this chapter I address one more objection to the claim that truth is the True. The objection is that Frege has a substantive Tarskian truth-predicate. This claim has been supported by other scholars like Künne (2008) and Taschek (2008). But Dirk Greimann (2008) provides the most developed formulation of it and a strong argument for it. So I focus on Greimann's discussion here.

2.3.1. THE METALINGUISTIC JUSTIFICATION FOR LOGICAL LAWS?

Does Frege have a meta-linguistic conception of logic? Many commentators say 'No'; they say that Frege's conception of logic differs from the current meta-linguistic conception of logic.¹⁹ Still, commentators like Heck (2012) and Greimann (2008) disagree; they claim that Frege's conception of logic substantively hinges on semantics. Greimann (2008) even insists that we can find a meta-linguistic truth-predicate in Frege's mature works. Recall the following passage of *Grundgesetze*:

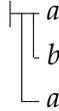
According to §12,

$$\left[\begin{array}{l} \Gamma \\ \Delta \\ \Gamma \end{array} \right]$$

¹⁸The question as to why Frege chooses Laws I~VI among other obviously true thoughts is not answered by it. He might have thought that Laws I~VI are the most obviously true thoughts, or that other obviously true thoughts can be reduced to Laws I~VI (Frege 1893: Forword). In any event, this is a challenge not just for my interpretation of Frege's conception of logical law, but also for other interpretations.

¹⁹For this point, see, e.g., Burge 1986, Ricketts 1996, 2003, Goldfarb 2001, etc.

would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible; accordingly [Law I]



(Frege 1893: §18)

Note that Frege's explanation of Law I looks like a meta-linguistic justification of an axiom of a logical system. Also, note that the predicate ' x is the True' looks like a truth-predicate in such a meta-linguistic justification. We can find similar passages for other laws and inference rules of *Begriffsschrift*, and can also find the same predicate in those passages. Greimann's contention is that this predicate is a meta-linguistic truth-predicate for Frege. I will call Greimann's reading 'the truth-predicate (TP) reading (of ' x is the True')'. The TP reading is important to our discussion because it constitutes a strong objection to my main thesis that truth is an object for Frege. The aim here is to critically examine Greimann's argument for the TP reading.

One might immediately object that ' x is the True' is not a meta-linguistic truth-predicate, but just an identity predicate like ' x is 2' in Frege's formal language. But Greimann argues that the predicate *as such* still can, and does, work as a truth-predicate in the relevant sense. What I want to show here is that Greimann's argument for the TP reading is not satisfactory. §2.3.2 introduces his argument through a certain tension concerning Frege's assimilation of sentences with proper names. His argument is that a plausible way to explain away this tension implies that ' x is the True' is a truth-predicate in a relevant sense. §2.3.3 shows that Greimann's suggested solution cannot be what Frege would be willing to accept. §2.3.4 presents an alternative solution to

the tension strictly grounded in Frege's remarks.²⁰

2.3.2. GREIMANN'S ARGUMENT FOR THE TRUTH-PREDICATE IN FREGE

Greimann (2008: 409) argues that if ' x is the True' is a truth-predicate for Frege in the relevant sense, i.e., if Frege's explanations of logical laws that make use of the predicate are metalinguistic soundness arguments, then we ought to be able to formulate Tarskian T-sentences like the following:

'Grass is green' is true in English iff grass is green.²¹

For Frege, however, sentences can be true or false only in the sense that the thoughts expressed by them are true or false. So the relevant kind of (quasi) T-sentences would be something like this:

The thought that grass is green is true iff grass is green.

Greimann (2008: 410) claims that we can construct a schema for the relevant kind of T-sentences out of ' x is the True':

(FSch) The truth-value of the thought that p is the True iff p .

Therefore, there is no problem with taking ' \dots is the True' as a truth-predicate in the relevant sense. Or so argues Greimann.

An immediate objection, however, would be that (FSch) includes the operator 'the truth-value of'. Frege never uses such an operator in his explanations of logical laws.

²⁰I do not deal with whether Frege's explanations of logical laws and inference rules are meta-linguistic justifications. Greimann's argument that ' x is the True' is a truth-predicate is independent of the point that Frege uses the predicate in his explanation of logical laws that looks like a meta-linguistic justification. As we will see later, examining Greimann's argument leads to an important point with regard to how we should understand *Begriffsschrift*.

²¹J. Weiner (2005) claims that ' x is the True' cannot be a truth-predicate because its extension only involves one and the only object, the True, while what we need for semantics is a predicate under which all the true sentences of an object language falls. Against this, Greimann claims that as far as we can formulate Tarskian T-sentences out of the predicate, it is sufficient for semantics.

It rather seems as if Frege is using it simply as the identity predicate ' $x = \text{the True}$ ', so to speak, as in

(S) $2+2=4$ is the True.

If so, we cannot confirm that Frege is using ' x is the True' like a truth-predicate in the relevant sense (Greimann 2008: 410).

As a response to this objection, Greimann claims that the sense of any sentence ' p ' is identical with that of the phrase 'the truth-value of: (the thought) that p ' in Frege. If so, then even sentences such as (S) are in fact instances of (FSch). This is his argument that ' x is the True' is a Tarskian truth-predicate.²² Let us see why Greimann argues so.

In §2 of *Grundgesetze*, Frege writes:

$'0^2 = 4', '1^2 = 4', '2^2 = 4', '3^2 = 4'$

are expressions of thoughts, some true, some false. I express it like this: the value of the function $\xi^2 = 4$ is the *truth-value* of the true, or that of the false. It is already clear from this that I do not want to assert anything yet when I simply write down an equation, but that I merely *designate* a truth-value; just as I assert nothing when I simply write down ' 2^2 ', but merely *designate* a number.

The point that a sentence refers to a truth-value is relevant to understanding this. A sentence—an equation in this case—always includes a predicate: a word that denotes a concept, i.e., a function whose value is always a truth-value. For instance, in the above equations, we commonly find the predicate ' $\xi^2 = 4$ '. Now, e.g., ' $2^2 = 4$ ' refers to the value of the concept ' $x^2 = 4$ ' saturated with the argument 2. Therefore, it refers to the True. Because the value of a function is an object, truth-values are objects, and

²²I do not dispute Greimann's claim that (FSch) gives us quasi T-sentences. Further, I do not dispute the claim that Frege provides a sort of meta-linguistic justifications of logical laws. The aim of this chapter is to critically examine this particular argument for those claims, which will be explained in the following.

thus the reference of a sentence, or an equation, is an object. Now, equations as such express thoughts. In Frege's *Begriffsschrift*, however, writing an equation down is not asserting that the thought it expresses is true. By writing it down, we only *designate* a truth-value. For asserting, a special sign is necessary:

Above it is already stated that within a mere equation no assertion is yet to be found; with '2+3=5' only a truth-value is designated, without its being said which one of the two it is. Moreover, if I wrote '(2+3=5)=(2=2)' and presupposed that one knows that 2=2 is the True, even then I would not thereby have asserted that the sum of 2 and 3 is 5; rather I would only have designated the truth-value of: that '2+3=5' refers to the same as '2=2'. We are therefore in need of another special sign in order to be able to assert something as true. To this end, I let the sign '⊢' precede the name of the truth-value, in such a way that, e.g., in

$$(B) \quad \text{⊢ } 2^2 = 4'$$

it is asserted that the square of 2 is 4. (Frege 1893: §5)

Expressions like (B) are well-formed *sentences of Begriffsschrift* (*Begriffsschriftsatz*).

If (B) is a sentence, a question can be raised. The reference of a sentence is always a truth-value. Therefore, (B) is also the name of a truth-value. Now, say $\text{⊢ } \xi^2 = 4$ is a concept such that it yields the True as its value if 2 is given as an argument and the False otherwise. Then, it follows that (B) also designates a truth-value as the equations in the earlier passage do. However, Frege clearly denies it:

The assertion sign cannot be used to construct a functional expression; for it does not serve in conjunction with other signs, to designate an object. '⊢ 2 + 3 = 5' does not designate anything; it asserts something. (Frege 1891: 34n)

This raises the question of how (B) can be a name of a truth-value while $\text{⊢ } \xi^2 = 4$ is not a concept.

Greimann maintains that Frege distinguishes sentences from (mere) singular terms for truth-values. Only expressions such as (B) are sentences. Equations such as ' $2^2 = 4$ ' are not sentences but singular terms for truth values. Both sentences and singular terms for truth-values are of course proper names of truth-values; they 'denote (*bedeuten*)' (Greimann 2008: 414) truth-values. However, singular terms for truth-values differ from sentences in that only the former can be used for the speech-act of reference *qua* a proper part of the speech act of assertion; only singular terms for truth-values 'designate (*bezeichnen*)' (Greimann 2008: 414) in this sense.

Then, the sense of an equation such as ' $2^2 = 4$ ', which would be taken as a sentence, must be that of an expression that can be used as a part of a sentence like (B) for the sake of the speech-act of reference *qua* a proper part of assertion. Indeed, Greimann argues, Frege is *paraphrasing* the sentence in the ordinary sense ' $(2+3=5)=(2=2)$ ' into a non-sentential phrase like this: 'the truth-value of: that " $2+3=5$ " refers to the same as " $2=2$ ". Greimann insists that in general the sense of a singular truth-value term '*p*' is identical with that of 'the truth-value of: that *p*'. Therefore, even ' $2+2=4$ is the True' is indeed equivalent to the (quasi) T-sentence 'The truth-value of: that $2+2=4$ is the True'.²³ Greimann concludes: '*x* is the True' is a truth-predicate in the relevant sense.

2.3.3. OBJECTION: SENTENCE AND THOUGHT IN FREGE

In a nutshell, Greimann believes that in Frege ' $2+2=4$ ' can be paraphrased into 'the truth-value of: that $2+2$ is 4'. Consider the following objection to his interpretation.

In a number of places, Frege says that the sense of a sentence in the usual sense is a

²³See Greimann (2007: 136). Here, Greimann argues that because the thought of a sentence is a condition when the sentence refers to the True, ' $2+3=5$ ' indeed has the same sense as 'the truth-value of: that $2+3$ equals 5'. He does not repeat the same argument in his 2008 paper. This argument, however, does not work if our discussion is along the right lines. For the critical discussion of Greimann 2007, see Kim 2019.

thought. Since '2+2=4' is a sentence in the usual sense, its sense must be a thought, i.e., $\langle 2+2 \text{ is } 4 \rangle$. On the contrary, 'the truth-value of: that 2+2 is 4' is not a sentence in the usual sense. So its sense is not a thought. Thoughts differ from senses of non-sentential singular terms. Here is an argument. For an arbitrary thought $\langle p \rangle$, it is always identical with $\langle \langle p \rangle \text{ is true} \rangle$ (Frege 1892c, 1897, 1906a, 1914, 1915, etc.). But the same point is not applicable to a non-sentential singular term. A case in point: 'the truth-value of: that 2+2 is 4'. Its sense differs from $\langle \text{The truth-value of: that } 2+2 \text{ is } 4 \text{ is true} \rangle$.²⁴ If '2+2=4' and 'the truth-value of: that 2+2 is 4' have different senses, then one cannot be the paraphrase of the other.

Greimann has a reply. He does not reject that a thought as the sense of a sentence differs from the sense of a singular term like 'the truth-value of: that 2+2 is 4'. When he says that 'the truth-value of: that 2+2 is 4' is a paraphrase of '2+2=4', he is arguing that the latter as an expression of *Begriffsschrift* is *not* a sentence expressive of a thought. Let me explain. According to Greimann, it is not the case that Frege 'wants to defend the absurd view that in natural languages an expression such as '2² = 4' is used as a singular [truth-value] term [like 'the truth-value of: that 2² equals 4']' (Greimann 2008: 412). In natural language, it expresses $\langle 2^2 \text{ equals } 4 \rangle$, and as such 'can be used to assert something, but not to designate something' (Greimann 2008: 412).²⁵ Greimann argues that Frege turns what we take to be sentences in natural languages into singular

²⁴What only thoughts—but not non-sentential singular terms—have is not assertoric force but positive predication. Positive predication itself does not amount to assertoric force. Thoughts do not carry assertoric force with themselves as we've already seen. Frege's thesis that $\langle p \rangle$ is identical with $\langle \langle p \rangle \text{ is true} \rangle$ is rather his argument that a thought does not contain assertoric force.

²⁵Greimann (2008: 412) also claims that Frege's functional analysis of sentences is only intended to be applied to formal sentences. But this is not really clear. For instance, when Frege (1906c: 177-8) explains the sentence '2 is prime' in terms of the saturation of a concept, does he talk about formal languages or natural languages? Or, is he talking about language in general? Greimann's claim seems to be question-begging without further discussion.

truth-value terms in *Begriffsschrift* because he makes a clear distinction between a mere entertainment of a thought—what he calls ‘predication’—and assertion. With ordinary language sentences such as ‘2+2 is equal to 4’, however, we cannot clearly mark the distinction. For we can use ordinary language sentences to perform both predication—entertainment of a thought—and assertion (Greimann 2008: 412). In order to make the distinction clearly shown, Greimann claims, Frege regards ‘2+2=4’ *qua* term of his formal language to mean ‘the truth-value of: that 2+2 equals 4’, which alone cannot be used for the act of assertion because it is not a sentence. A sentence is constructed and assertion is performed, only when we write ‘⊢’, a special truth-predicate or truth-operator (Greimann 2004, 2007), in front of such a term.

To check the emerging explanation of *Begriffsschrift*, say—following Greimann—‘2+2 equals 4_E’ is an English sentence and ‘2+2=4_F’ a corresponding singular term in *Begriffsschrift*. ‘2+2 equals 4_E’ expresses $\langle 2+2 \text{ equals } 4 \rangle$, and is a sentence in the sense that it can be used to assert but not to designate an object. However, ‘2+2=4_F’ does not express such a thought; it only expresses the sense of a singular term. Therefore, it is not a sentence in the same sense. We come to have a sentence in this sense only if we put ‘⊢’ in front of it. Now, ‘⊢ 2 + 2 = 4’ does express a thought, namely, $\langle \text{The truth-value of: that } 2+2 \text{ equals } 4 \text{ is the True} \rangle$. Therefore, ‘⊢’ is a formal version of ‘*x* is the True’ (Greimann 2004, 2007), and is the only *genuine* predicate—which is also a truth-predicate or -operator—of *Begriffsschrift* in the sense that only with ‘⊢’ we can have a sentence.^{26,27}

²⁶Indeed, the early version of *Begriffsschrift* (Frege 1879a) understands ‘⊢’ in this way. However, in the version in *Grundgesetze*, Frege clearly denies this view because the judgment stroke is not a predicate anymore (the judgment stroke is not identified as a primitive function). Frege has reasons for this change according to Heck (2012: §2.3).

²⁷One problem with taking ‘⊢’ to be a truth-predicate is that ‘⊢ 2’ can also count as a *Begriffsschriftsatz*. Greimann must accept that in such a case ‘⊢’ is not a truth-predicate, but a mere identity predicate.

This explanation presupposes that ‘⊢’ is a predicate whose sense constitutes a part of thoughts. At the same time, the sign ‘⊢’ stands for assertoric force. Therefore, here, ‘⊢’ (i) contributes to the formation of a thought and (ii) indicates assertoric force. In a significant sense, this explanation takes assertoric force to contribute to the formation of a thought. However, this is what Frege emphatically rejects as we have already seen. Recall what he says in ‘On Schoenflies’:

We can of course express a thought, without stating it to be true. The thought is strictly the same whether we merely express it or whether we also put it forward as true. Thus assertoric force, which is often connected with the copula or else with the grammatical predicate, does not belong to the expressions of the thoughts
 ... (Frege 1906c: 177)²⁸

In fact, the main logical defect of natural language concerning assertoric force—and assertion—is that it leads us to believe that assertoric force comes as a part of thought. That is what Frege says in MBLI:

So the word ‘true’ [in the sentences of the form ‘The thought that *p* is true’] seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought. (Frege 1915: 252)

Frege’s point is that the attempt to make assertoric force constitute a part of a thought is defective because assertoric force is *not* a part of a thought. It is unlikely that Frege would let his logic commit the mistake he himself points out—the mistake of making assertoric force a contribution to a thought.

Then, why don’t we say that it is such a mere identity predicate? This leads to a doubt against his reading of ‘... is the True’ because ‘2 is the True’ seems to be a perfectly legitimate. But Greimann does not seem to take this issue seriously. He simply admits that in such a case ‘... is the True’ fails to work as a truth-predicate. Greimann’s basic idea seems to be that as long as the predicate provides all instances of (FSch), the fact that it can play a different role does not prevent us from taking it to be a truth-predicate.

²⁸Also, Frege (1914: 233) says that to assert, ‘we do not need a special predicate’. Greimann’s reading makes ‘⊢’ a special predicate for assertion.

I believe that Greimann is right when he argues that (i) Frege needs a category of ‘expressions that are supposed to express a thought without simultaneously asserting its truth’ (Greimann 2008: 412) and that (ii) ‘ $2+2=4_F$ ’ falls under such a category. The problem is, I believe, that from those true claims, he proceeds to the conclusion that ‘ $2+2=4_F$ ’ must correspond to something like ‘the truth-value of: that $2+2$ is 4 ’ in English. First of all, the latter in English does not express a thought. It involves ‘(the thought) that $2+2$ is 4 ’. However, it is the name of a thought. Strictly speaking, a name refers to its reference and expresses its sense (Frege 1892c: 61; 1893: §2). Thus, ‘(the thought) that $2+2$ is 4 ’ does not express $\langle 2+2 \text{ is } 4 \rangle$ but *refers to* it. What it expresses is the sense of ‘(the thought) that $2+2$ is 4 ’ (Frege 1892c: 66). Given that what a name *expresses* is its sense, a name that expresses a thought is a name that has a thought as its sense, i.e., a sentence in the usual sense. If ‘ $2+2=4_F$ ’ expresses a thought as Greimann says, it must be a sentence in the usual sense, not a non-sentential singular term like ‘the truth-value of: that $2+2$ is 4 ’.

Even if we leave this issue aside, a problem remains: it is not clear at all why Frege ought to turn ‘ $2+2=4_F$ ’ into a non-sentential term in order to deprive assertoric force of it. Why can’t he simply regard ‘ $2+2=4_F$ ’ as corresponding to ‘ $2+2$ is 4 ’ in English to which assertoric force is not added, e.g., the sentence uttered by an actor on a stage (cf. Frege 1892c)? Is that because as soon as we form a sentence and thereby express a thought, we come to make an assertion, i.e., because sentences or thoughts contain assertoric force in themselves? But that is exactly what Frege rejects as we have seen throughout this chapter. It does not appear that in order to detach assertoric force from ‘ $2+2=4_F$ ’ we ought to turn it into a non-sentential term. All we need to do is to realize that assertoric force is not its semantic component. If we realize it, then we can

take $'2+2=4_F'$ to be a sentence that expresses $\langle 2+2 \text{ is } 4 \rangle$ but is not asserted. Assertoric force is added to the thought by $'\vdash'$.

Indeed, $'2+2=4_F'$ or what Greimann calls 'singular truth-value term' in general is a sentence that expresses a thought. After introducing eight primitive functions, Frege writes:

However, not only a reference but also a sense belongs to all names correctly formed from our signs. Every such name of a truth-value *expresses* a sense, a *thought*. For owing to our stipulations [for those primitive functions], it is determined under which conditions it refers to the True. The sense of this name, the *thought*, is: that *these conditions are fulfilled*. Now, a [*Begriffsschriftsatz*] consists of a judgment-stroke with a name, or a Roman marker, of a truth-value (Frege 1893: §32).

First, note that Frege takes a *Begriffsschriftsatz* like

$$(B) \quad \vdash 2^2 = 4$$

to be the combination of the judgment-stroke (the vertical stroke) and a name of a truth-value. Therefore, what Frege calls 'name of a truth-value' is not a *Begriffsschriftsatz*. Rather, something like the following is an instance of such a truth-value name:

$$(N) \quad \text{— } \dot{\varepsilon}(\text{— } \varepsilon)$$

By the stipulations given for the functions involved in (N), we can specify when it refers to the True. Frege's point entails that (N)'s sense is $\langle \text{Those conditions are met} \rangle$, i.e., $\langle \text{The value-range of the concept referred by the horizontal falls under the concept} \rangle$.^{29,30} (N) is a typical case of what Greimann takes as a singular truth-value term: it certainly refers to a truth-value though it is not a *Begriffsschriftsatz*. Frege's remarks here are

²⁹The horizontal sign called 'the horizontal' in *Grundgesetze* refers to a concept such that it yields the True given the True as a value, and the False otherwise.

³⁰The sentence in the latter angled brackets is a natural language analogue of (N).

explicit enough to affirm that such a singular truth-value term ' p ' in general expresses $\langle p \rangle$. A thought is the sense of a sentence. So Frege takes singular truth-value terms to be sentences. A singular truth-value term ' p ' cannot be translated into a non-sentential term like 'the truth-value: that p '. Greimann's argument for the TP reading must be rejected.

2.3.4. ASSERTION SIGN '⊢': A PRELUDE TO CHAPTER 5

The following still has to be explained: only expressions like (B) count as sentences in Frege's *Begriffsschrift* by means of which we can assert but cannot designate a truth-value, while expressions such as ' $2+2=4$ ' only designate a truth-value. It is clear that for Frege, an expression cannot be used alone for asserting even if it expresses a thought. The answer is in one of the passages from *Grundgesetze* we've already seen:

Above it is already stated that within a mere equation no assertion is yet to be found; with ' $2+3=5$ ' only a truth-value is *designated, without its being said which one of the two it is*. (Frege 1893: §5; italics mine)

Designating a truth-value corresponds to writing down a name of a truth-value such as ' $2+3=5$ ' *without* saying whether it refers to the True or the False. Then, to assert that $2+3=5$, i.e., to write down '⊢ $2 + 3 = 5$ ', is to 'say' that $2+3=5$ is the True.

To this end [of asserting] I make use of a vertical stroke at the left end of the horizontal, so that, e.g., by writing

$$\vdash 2 + 3 = 5$$

we assert that $2+3$ equals 5. Thus here we are not just writing down a truth-value, as in

$$2 + 3 = 5,$$

but also at the same time *saying that it is the True*. (Frege 1891: 34; italics mine)

To assert that $2+3$ equals 5 is to identify $2+3=5$ as the True. The act of asserting as such is distinguished from that of designating a truth-value. Thus to write down $\vdash 2+3 = 5$, i.e., to assert that $2+3$ equals 5 , is not to designate a truth-value.

It might be objected that an act of identifying an object O_1 as O_2 is just an act of assertively predicating ' $x = O_2$ ' of ' O_1 '. If so, it might seem, Greimann's reading is partially true—namely the claim that ' \vdash ' is a formal version of ' \dots is the True'. But again that cannot be correct. Return to another passage we have seen:

Moreover, if I wrote ' $(2+3=5)=(2=2)$ ' and presupposed that one knows that $2=2$ is the True, even then I would not thereby have asserted that the sum of 2 and 3 is 5 ; rather I would only have designated the truth-value of: that ' $2+3=5$ ' refers to the same as ' $2=2$ '. (Frege 1891: 34)

What Frege points out is that one cannot 'say' that p is the True even by predicating the identity predicate ' $x = T$ ' of ' p ' where ' T ' is the most obvious name of the True. (It is not clear whether Frege intends to provide a non-sentential paraphrase of ' $(2+3=5)=(2=2)$ '. That point is not relevant to understanding this passage.) This shows that to assert that p , i.e., to 'say' that p is the True, cannot be done by predicating ' x is the True' of ' p '. Thus, ' \vdash ', the device for asserting, cannot be such a predicate.

Then what is it? We will come back to this issue in Chapter 5. Here I provide a brief sketch. I believe that ' \vdash ' is purely a performative for performing a non-predicational act of identification. We are not unfamiliar with the non-predicational and non-assertoric conception of identification. According to Millikan (1998, 2000) or Camp (2002), the act of identifying is neither an act of predicating an identity predicate nor asserting an identity. Identification is an activity of its own kind that is arguably more fundamental than an act of predication or assertion. I believe that Frege is certainly appealing to such a conception of identification when he elucidates that asserting that p is taking a

step from $\langle p \rangle$ to its truth-value in 'On Sense and Reference' (Frege 1892c).³¹ Chapter 5 provides the detail of this reading. What is important is that, given all his remarks we have seen, Frege would rather understand '⊢' as a performative for such a non-predicational and non-assertoric identification. This understanding of the device goes well along with Frege's sharp distinction between predicating and asserting, and with his claim that we do not need any special predicate or copula for the sake of assertion. The expressions like '⊢ $\xi^2 = 4$ ' are not functional expressions, because '⊢' is not a functor at all. What we have by putting '2' in the position of ' ξ ' is not a truth-value, but an act of asserting that 2^2 equals 4, i.e., 'saying' that $2^2 = 4$ is the True. *Begriffsschriftsätze* are not names of truth-values. Thus, they do not designate truth-values.³²

Given the evidence that truth is not a concept for Frege, the predicate '... is the True' is rather to be regarded as an identity predicate of the form ' $x = a$ '.³³ Of course, Frege seems to provide justifications for logical laws that are pretty similar to meta-linguistic ones. Still, because he depends on this identity predicate in those justifications, they are at most precursors or prototypes of meta-linguistic justifications.

³¹If judging is indeed such a fundamental act of identifying, we can make better sense of Frege's claim that 'judgment is something peculiar and incomparable' (Frege 1892c: 65).

³²Note that, in §32 of *Grundgesetze*, Frege does not classify *Begriffsschriftsätze* as names of truth-values. Also, recall that Frege says that a *Begriffsschriftsatz* asserts something. Sentences in the ordinary sense do not assert; we assert with sentences. *Begriffsschriftsätze* are not sentences in the ordinary sense. Greimann's reading, however, compares *Begriffsschriftsätze* to sentences in the ordinary sense. That point is what distinguishes his reading from mine.

³³Our discussion implies that a thought is not a truth-condition. Refer to Maunu (2002)'s discussion regarding this point.

CHAPTER 3

ARGUMENT 2: FREGE IS NOT A PRIMITIVIST

In the last chapter we have dealt with Frege's deflationist claims. Our discussion shows that Frege is not a deflationist. His deflationist comments are rather his arguments that truth is an object, i.e., the True.

Frege is also known as a truth primitivist in the literature. This is mainly because of his famous argument for the indefinability of truth. We can find it in 'Logic' (Frege 1897) and 'Thought' (1918a). The aim of this chapter is to show that this indefinability argument is not an argument that truth is not definable. It is an argument that truth cannot be defined *as a concept* because it is not a concept. Therefore, it is not a primitivist argument, but an argument for Frege's conception of truth as an object.

In Frege scholarship the indefinability argument has been taken to be an argument that if truth is definable then an absurd result is produced. A number of commentators take this absurd result to be a vicious *infinite regress* (Carruthers 1981; Dummett 1993;

Heck 2010, 2012; Heck and May 2018; Kemp 1995; Kremer 2000; Soames 1999; Pagin 2001; Ricketts 1986, 1996). I agree with these commentators. So the reading I develop here also identifies the absurd result as an infinite regress. However, there are other philosophers who believe that Frege is appealing to a sort of circularity in the argument. Baldwin (1995) and Grossmann (1969) adumbrate this idea, and Künne (2003, 2008) fully develops a reading of the argument under which it appeals to a kind of vicious circularity.¹

I start my discussion of Frege's indefinability argument by critically examining the circularity reading of Künne (§3.1). Then I turn to developing my own interpretation of the argument (§3.2).

3.1. THE CIRCULARITY READING

Here is the indefinability argument in 'Thought':

... Could we not maintain that there is truth when there is correspondence in a certain respect? But which respect? For in that case what ought we to do so as to decide whether something is true? We should have to inquire whether it is *true* that an idea and a reality, say, correspond in the specified respect. And then we should be confronted by a question of the same kind, and the game could begin again. So the attempted explanation of truth as correspondence breaks down. And any other attempt to define truth also breaks down. For in a definition certain characteristics would have to be specified. And in application to any particular case the question would always arise whether it were *true* that the characteristics were present. So we should be going round in a circle. (Frege 1918a: 327)

Though Frege starts the argument with (a version of) the definition of truth *qua* correspondence, he generalizes it into other definitions. The argument is clearly a *reductio*;

¹Sluga (2002) provides a historical approach to the indefinability argument—tracking down the origin of the indefinability thesis. This kind of historical approach to the argument seems to be compatible with any of the philosophical readings we will deal with here.

it attempts to establish that, if truth is definable, then an absurd result is produced. The text appears to give a reason to think that the absurd result is an infinite regress. However, Wolfgang Künne writes:

Many commentators misconstrued this objection as a vicious infinite regress argument. [The sentence ‘We should be going round in a circle’] makes it as clear as can be that Frege takes himself to be offering a kind of vicious circle objection. (2008: 13)

In this chapter, we critically examine Künne’s circularity reading. I first establish that the circularity reading depends on the notion of concept that appears to be alien to Frege. I also establish that the circularity reading makes the indefinability argument a non-starter. If my criticism is along the right lines, Frege’s indefinability argument, central to his conception of truth and judgment, ought to be interpreted in a different way.

3.1.1. THE CIRCULARITY READING

The circularity reading concentrates on Frege’s ‘*global*’ (Künne 2008: 12) indefinability argument—the indefinability argument against truth definition in general. The global argument is contained in the second half of the passage from ‘Thought’.

And any other attempt to define truth also breaks down. For [1] in a definition certain characteristics would have to be specified. And [2] in application to any particular case the question would always arise whether it were *true* that the characteristics were present. [3] So we should be going round in a circle. (1918a: 327; numbering is Künne’s.)

According to the circularity reading, ‘definition’ in [1] must mean ‘analytic definition’. When we attempt to define ‘true’, we try not to ‘graft a new meaning upon this word’ (Künne 2008: 13) as we do in physics or mathematics, but to clarify the old meaning of

the word by the conceptual analysis. That is, we are seeking an analytic definition of ‘true’. Now, such an analytic definition of a concept C specifies the characteristics of C , i.e., concepts constitutive of C . Thus, in an analytic definition of the concept *truth*, its characteristics are specified as [1] states.²

Künne writes:

But according to Frege one feature . . . is shared by all adequate analytic definitions: they capture an order of epistemic priority—we come to know that the definiendum applies to an object through coming to know that the definiens applies to it. Consequently, . . . an analytic definition of a predicate P is to be rejected if having the concept expressed by P is a precondition for deciding whether the alleged definiens applies in any given case. If we now assume, as Frege does in [2], that one cannot decide whether the characteristics specified in an alleged definiens of ‘true’ are exemplified in a given case without deciding whether it is true that they are exemplified, the conclusion in [3] follows, for one can hardly decide whether something is thus and so (e.g., true) without having the concept of a thing’s being thus and so (of a thing’s being true). (2008: 14)

Say we have an *analytic* definition of truth like this:

(T) x is true just in case x is Δ .³

²In the earlier version of the circularity reading, Künne takes Frege’s technical use of the term ‘characteristic’ into account. In Frege’s technical use a concept C_1 is a characteristic of a concept C_2 just in case ‘ C_1 ’ shows up in an analytic definition of C_2 and so something that falls under C_1 also falls under C_2 . However, Künne claims that one more condition ought to be added: C_1 is not a sufficient condition for C_2 . If Künne is correct, a concept can have a characteristic in this technical sense only if it has a *conjunctive* definition (‘ x falls under the concept C just in case x falls under $C_1, \dots, \text{ and } C_n$ ’). The earlier version of the circularity reading is based on Künne’s own reading of ‘characteristic’. However, as we will see in the next section, the additional condition is certainly not a part of Frege’s use of the term ‘Characteristic’. Moreover, as Künne correctly points out, this use of ‘characteristic’ strengthened by himself is not relevant here because ‘the very definition he had been concerned with in his local argument, “For all x , x is true iff x corresponds with reality”, lacked this kind of structure’ (Künne 2008: 13). Giving the earlier version of this reading in his *Conceptions of Truth* (Künne 2003), Künne reads ‘characteristic’ in Frege’s technical sense, and admits that the fact that not all definitions of truth have a conjunctive form can work as evidence against this earlier version of the circularity reading or as an objection to Frege’s argument (2003: 132–3). It seems as if in his 2008, Künne thinks that he has a way to bypass this exegetical issue as we will see below.

³Künne distinguishes analytic definitions from constructive or stipulative definitions, and says that

The correspondence definition is an instance of (T) where ‘ Δ ’ is substituted by ‘corresponding to reality’. As Künne points out an analytic definition such as (T) gives the following epistemic priority of the concept Δ .

- (C₁) Having the concept *truth* cannot be a precondition for deciding whether something is Δ .

The circularity reading now ascribes the following claim to Frege:

- (C₂) one cannot decide whether x is Δ without deciding whether it is true that x is Δ

where ‘that x is Δ ’ refers to a truth-candidate whose content is that x is Δ and ‘true’ is a predicate that refers to the property of *truth*. Because a truth bearer is the sense of a sentence, i.e., a thought for Frege in his mature career, (C₂) is equivalent to the following: one cannot decide whether x is Δ without deciding whether $\langle x$ is $\Delta \rangle$ is true. Hereafter, I use the expression ‘The thought that p is true’ instead of the expression ‘It is true that p ’.

Lastly, the circularity reading takes the following as a premise of the argument.

- (C₃) One cannot decide whether something is true without having the concept *truth*.

(C₂) and (C₃) entail that

- (C₄) One cannot decide whether x is Δ without having the concept *truth*.

(C₄) contradicts (C₁). Our assumption ought to be denied: there cannot be an analytic definition of truth of the form (T). This way, Frege’s indefinability argument is appealing to the vicious circularity of an analytic definition of the concept *truth*. Or so argues the circularity reading.

the indefinability argument concerns the former. According to Künne, analytic definitions ‘purport to capture, by means of a compound expression, the sense of an atomic expression already in use’ (Künne 2008: 13).

3.1.2. CRITICISM

3.1.2.1. AN INTERPRETATIVE ISSUE WITH (C₁) AND (C₃)

(C₁) and (C₃) have the phrase ‘having the concept *truth*’. Künne (2008: 13)’s presentation of the circularity reading starts with his explanation of the notion of concept in Frege, and no other conception of concept is explicitly introduced. If so, it appears to be a reasonable assumption that ‘concept’ in (C₁) and (C₃) means ‘Fregean concept’. However, if we read ‘the concept *truth*’ in (C₁) or (C₃) as meaning ‘the Fregean concept *truth*’, it is hard to make sense of them. Certainly, the phrase ‘having the concept *truth*’ concerns us, epistemic subjects, having the concept *truth*. It is perfectly proper to say ‘S has a certain concept’—only if we take ‘concept’ as denoting units of mental representations or mental contents constitutive of our conceptual scheme. Nevertheless, a Fregean concept (F-concept) is not such a component of our conceptual scheme. It is rather an entity in the world; an F-concept is a function whose value is always a truth-value: the True or the False. But if an F-concept is not a unit of our conceptual scheme, what does ‘S has the F-concept *F*’ mean?

One might respond by saying that ‘having’ in (C₁) and (C₃) means ‘knowing’ while ‘concept’ still means ‘F-concept’. Someone might find the sentence (1) ‘S knows the F-concept *truth*’ to sound similar to (2) ‘S knows Gottlob Frege’. It looks as though the verb ‘know’ in sentences like (2) can have two different meanings: it can mean (i) the subject’s acquaintance with an object (Hanks 2007: 145) or it can mean (ii) the subject’s ability to identify an object (Millikan 2000). The verb ‘know’ with the meaning (i) does not seem to produce an intensional context. *S* is acquainted with Frege if and only if *S* is acquainted with the author of *Grundgesetze*. If (T) is true, the F-concept *truth* is

identical with the F-concept Δ because they should be exactly the same *function*. Let 'know_(i)' be 'know' with the meaning (i). If 'having' in 'having the [F-]concept *truth*' means 'knowing_(i)', (C₁) comes down to this:

(C₁*) Knowing_(i) the Fregean concept Δ cannot be a precondition for deciding whether something is Δ .

However, (C₁*) contradicts the circularity reading's own rationale for (C₃): '[...] one can hardly decide whether something is thus and so (e.g., true) without [knowing_(i)] the [F-]concept of a thing's being thus and so (of a thing's being true)' (Künne 2008: 14). Similar considerations apply if we take 'know' to have (ii) as its meaning.

One might still take 'know' in '*S* knows *o*' as producing an intensional context. For instance, one might claim that, even if N_1 is identical with N_2 , the fact that one knows N_1 (N_2) does not entail that one knows N_2 (N_1) if the sense of ' N_1 ' differs from that of ' N_2 '. However, this does not help because we already assume that (T) is a true analytic definition of 'true'. If (T) is really a correct definition in the analytical sense, then the sense of 'true' must be identical with that of ' Δ '—as Künne (2008: 13) acknowledges.⁴

A more reasonable, or perhaps the only, way to make sense of the phrase 'having the concept', I believe, is to hypothesize that 'concept' there means not 'F-concept' but something else (although Künne specifies no other conception of concept). Perhaps, 'concept' in (C₁) and (C₃) might refer to concepts *qua* psychological constructs that constitute our conceptual scheme. Let us call concepts as such 'P-concept'. P-concepts must differ from Fregean senses. To make sense of (C₁) and (C₃), we ought to differentiate the P-concept TRUTH from the P-concept Δ . However, the Fregean sense of 'true' is

⁴In the earlier version of the circularity reading Künne takes 'concept' to mean 'sense' (2003: 132ff). It works in the earlier version because it is based on the assumption that a characteristic of a concept is not a sufficient condition for the concept. See Footnote 2 of this chapter. However, in this later version of the circularity reading where this assumption is dropped, taking 'concept' to mean 'sense' does not help as we've just seen. Indeed Künne (2008) does not say that 'concept means 'sense'.

identical with that of ‘ Δ ’ if (T) is an analytic definition. Therefore, the P-concept TRUTH (the P-concept Δ) must differ from the Fregean sense of ‘true’ (the Fregean sense of ‘ Δ ’). However, it is not evident at all that Frege is appealing to the notion of P-concept—with which we can avoid the paradox of analysis—in the passage from ‘Thought’. Given Frege’s strong anti-psychologism (cf. Frege 1893: Foreword), it is not evident that he ever endorses such a psychological conception of concept. This raises a serious doubt against the exegetical propriety of the circularity reading.

3.1.2.2. PHILOSOPHICAL ISSUES WITH THE CIRCULARITY READING

Even if we concede that Frege adopts the notion of P-concept and that ‘concept’ in (C₁) and (C₃) means ‘P-concept’, the indefinability argument under the circularity reading turns out to be a non-starter. First, it has an implausible premise. Also, it is too difficult to defend all its premises altogether.

The Omnipresence of Truth? Let us examine the way the circularity reading explains how Frege motivates (C₂). The circularity reading claims that (C₂) is an instance of the following schema Frege would be willing to endorse:

- (S) ‘One cannot VERB that / whether p , without thereby VERB-ing that / whether [the thought that p] is true.’ (Künne 2008: 16)

where ‘VERB’ is substituted by a verb for a mental action with a thought. To support this scheme Künne points to (Iden)

- (Iden) $\langle p \rangle$ is identical with $\langle \langle p \rangle$ is true \rangle . (Frege 1892c, 1897, 1906a, 1906c, 1914)

Künne’s point is that given this *identity thesis*, performing an act with $\langle p \rangle$ is just performing the act with $\langle \langle p \rangle$ is true \rangle . Hence, Künne claims, every instance of (S) including (C₂) is true for Frege.

However, (C₃) does not square well with the identity thesis if the latter is read in a certain way. Say that the following is the rationale that underlies the identity thesis:

(Def) The sense of 'true' contributes nothing to $\langle\langle p \rangle \text{ is true} \rangle$, and hence neither $\langle p \rangle$ nor $\langle\langle p \rangle \text{ is true} \rangle$ has what we may take as the sense of 'true' *qua* predicate.⁵

If (Def) is the rationale for the identity thesis, (C₃) is false because to decide whether $\langle p \rangle$ is true is just to decide whether p . Thus, we do not have to have the P-concept TRUTH or know the sense of 'true' in order to decide whether $\langle p \rangle$ is true.

Of course, there is another way to understand the identity thesis:

(OP) The sense of 'true' is hidden even in the sense of a sentence that does not have a truth-predicate, and so $\langle p \rangle$ always comes down to $\langle\langle p \rangle \text{ is true} \rangle$.

If (OP) is the case, deciding whether p always comes down to deciding whether $\langle p \rangle$ is true. To do the former, one should already have the P-concept TRUTH and know the sense of 'true'. If (OP) is the rationale that underlies the identity thesis, the proponent of the circularity reading can defend (C₃) by appealing to the identity thesis.

As our discussion of Frege's 'My Basic Logical Insights' shows, what Frege intends to argue for through (Iden) is rather closer to (Def). His point is that the fact, that 'true' has nothing to contribute to the sense of a sentence where it shows as a predicate, in a sense gives an expression to the fact that the act of assertion does *not* form a part of the asserted thought. However, let us leave this issue aside.

Künne does attribute (OP) to Frege although he does not seem to take it to be a part of the indefinability argument.⁶ What our consideration establishes, however, is that

⁵(Def) seems to be what Frege says in 'My Basic Logical Insights' (Frege 1915: 252).

⁶Künne (2008) claims that Frege attempts to resist the view that truth is a property and that the indefinability argument is one of such attempts. However, Künne also claims, Frege could not but

the indefinability argument under the circularity reading ought to involve (OP) as its part. If the identity thesis were underwritten not by (OP) but by (Def) or something like it, (C₃) would become implausible. However, if the indefinability argument depends on (OP), it appears to be a non-starter. Let '*p*' be an arbitrary sentence. If (OP) is true, then its sense is $\langle\langle p \rangle \text{ is true} \rangle$, i.e., the sense of ' $\langle p \rangle \text{ is true}$ '. Likewise, the sense of ' $\langle p \rangle \text{ is true}$ ' is $\langle\langle\langle p \rangle \text{ is true} \rangle \text{ is true} \rangle$, i.e., the sense of ' $\langle\langle p \rangle \text{ is true} \rangle \text{ is true}$ '. This regress will never stop. The consequence of this regress is that we cannot fix the sense of a sentence, which is by no means plausible. To base the indefinability argument on (OP) is to make it a non-starter.

The Instability of the Argument under the Circularity Reading Even if we concede that (OP) is plausible, the argument is still unstable because it is not possible to justify all its premises at the same time. Let me first note that not every instance of (S) is true. Only given the identity between $\langle p \rangle$ and $\langle\langle p \rangle \text{ is true} \rangle$, the instance of (S) holds only if the mental action denoted by the verb ' ϕ ' that substitutes 'VERB' is *merely thought-sensitive*, i.e., only if the identity between $\langle p \rangle$ and $\langle q \rangle$ is a *sufficient* condition for the identity between ϕ -ing that *p* and ϕ -ing that *q*. If ϕ -ing is not only thought-sensitive but also P-concept-sensitive, i.e., if ϕ -ing that *p* is identical with ϕ -ing that *q* just in case (i) $\langle p \rangle$ is identical with $\langle q \rangle$ and (ii) the P-conceptual content of '*p*' is identical with that of '*q*', the identity between $\langle p \rangle$ and $\langle\langle p \rangle \text{ is true} \rangle$ does not entail the instance of (S) with ' ϕ ': 'We cannot ϕ that *p* without ϕ -ing that $\langle p \rangle \text{ is true}$ '. Therefore, if (C₂) is a true instance of (S), the act of deciding whether *p* is merely thought-sensitive.

If deciding whether *p* is merely thought-sensitive, however, it is difficult to motivate

return to the property view and accept that all we can say about truth is (OP), which shows according to Künne that truth is a peculiar or *sui generis property* of thoughts. This shows that Künne takes (OP) to come after the considerations like the indefinability argument.

that

(C₃[†]) One cannot decide whether something is true without having the P-concept TRUTH,

i.e., (C₃) with the notion of P-concept. Say that I have the P-concept Δ, I know the sense of 'Δ', and recognize the F-concept Δ. Intuitively, it looks as if I can decide whether something is Δ. Then I can also decide whether something is true without having the P-concept TRUTH. Here is an argument. Supposing (T) is an analytic definition of 'true', ⟨x is true⟩ is identical with ⟨x is Δ⟩. Because the act of deciding whether *p* is merely thought-sensitive, deciding whether *x* is Δ is identical with deciding whether *x* is true. Therefore, if I can decide whether *x* is Δ, I can *thereby* decide whether *x* is true. Thus, having the P-concept TRUTH is not necessary for deciding whether ⟨*p*⟩ is true. (C₃[†]) seems to be false if deciding whether *p* is merely thought-sensitive.

Here, appealing to (OP) would not help. If (OP) is the case, of course, deciding whether *x* is Δ comes down to deciding whether ⟨*x* is Δ⟩ is true. Nevertheless, this does not imply that we need the P-concept TRUTH to decide whether *x* is Δ. Given (T), ⟨⟨*x* is Δ⟩ is true⟩ is identical with ⟨⟨*x* is Δ⟩ is Δ⟩. Given that deciding whether *p* is merely thought-sensitive, deciding whether ⟨*x* is Δ⟩ is true is identical with deciding whether ⟨*x* is Δ⟩ is Δ. When I perform the latter, I thereby perform the former. It still seems intuitively true that I can decide whether ⟨*x* is Δ⟩ is Δ as far as I have the P-concept Δ, know the sense of 'Δ', and recognize the F-concept Δ. Thus, the proponent of the circularity reading cannot defend (C₃[†]) by appealing to (OP).

There are two ways of defending (C₃[†]) against the objection. One is to claim that, without the P-concept TRUTH, we cannot decide whether something is Δ even if we have the P-concept Δ, know the sense of 'Δ', and recognize the F-concept Δ. Nevertheless,

it is difficult to see how we can motivate this claim given that (OP) does not help. The other, more plausible, way is to claim that, merely by deciding whether something is Δ , we cannot decide whether something is true. One can motivate this claim by saying that deciding whether p is not merely thought-sensitive but also P-concept-sensitive.

However, if we accept that deciding whether p is also P-concept-sensitive for some reasons, (C₂) comes to be implausible. As we said, only given the identity between $\langle p \rangle$ and $\langle \langle p \rangle \text{ is true} \rangle$, 'VERB' in (S) should be substituted by a verb that refers to a merely thought-sensitive mental act. Of course, one can argue that the P-conceptual content of ' p ', or simply $[p]$, is identical with $[\langle p \rangle \text{ is true}]$. However, this idea seems to be too far gone from what Frege is trying to argue in the passage from 'Thought'. Even if we concede that Frege adopts the identity between $[p]$ and $[\langle p \rangle \text{ is true}]$, it is unclear how we can motivate it. One way we can think of is to claim that because $\langle p \rangle$ is identical with $\langle \langle p \rangle \text{ is true} \rangle$, $[p]$ is identical with $[\langle p \rangle \text{ is true}]$. However, if that is the case, then $[\langle p \rangle \text{ is true}]$ must also be identical with $[\langle p \rangle \text{ is } \Delta]$ given that $\langle \langle p \rangle \text{ is true} \rangle$ is identical with $\langle \langle p \rangle \text{ is } \Delta \rangle$. If so, the P-concept TRUTH is identical with the P-concept Δ . As we have said earlier, however, we cannot make sense of (C₁) (with 'P-concept') and (C₃[†]) if the P-concept TRUTH is identical with the P-concept Δ .

3.1.3. CONCLUDING REMARKS

It is difficult to motivate (C₁), (C₂), and (C₃) altogether (where 'concept' means 'P-concept'). Even if we were able to do so somehow, it ought to depend on an implausible way of understanding the identity thesis, i.e., (OP). The argument under the circularity reading turns out to be a non-starter. Moreover, the circularity reading ascribes to Frege the notion of P-concept that is entirely alien to him. For these reasons, I conclude that the circularity reading should be discarded. Closing my discussion of the circularity

reading, I provide an interpretative point that seems to suggest that the indefinability argument may entirely differ from the argument under the circularity reading.

Recall Frege's argument against the correspondence definition.

... Could we not maintain that there is truth when there is correspondence in a certain respect? But which respect? For *in that case* what ought we to do so as to decide whether something is true? We should have to inquire whether it is *true* that an idea and a reality, say, correspond in the specified respect. (1918a: 327; italics mine)

The point Frege makes is conditional: (CC) *if* truth is defined as correspondence, one cannot decide whether x is true without deciding whether $\langle x$ corresponds to reality \rangle is true. Recall that the circularity reading needs the following:

(C₂C) One cannot decide whether x corresponds to reality without deciding whether $\langle x$ corresponds to reality \rangle is true.

Supposing that truth is defined as correspondence, we can deduce (C₂C) from (CC). However, if Frege is appealing to the identity thesis in this passage, this inference is unnecessary because (C₂C) directly follows from the identity thesis alone. It is unlikely that Frege is putting forward (CC) to acquire (C₂C). If it is (CC) that Frege is putting forward here, it is implausible that he is appealing to the identity thesis here.

A more promising interpretation of the above passage appears to be saying that the premise of the argument against the correspondence definition is not (C₂C) but (CC) and that the identity thesis is irrelevant to the argument. If so, the same goes for the global argument. The premise of the global argument is not (C₂) but this: one cannot decide whether x is true without deciding whether $\langle x$ is Δ \rangle is true *if* truth is defined as a F-concept Δ . The identity thesis is also irrelevant to the global argument. If this exegetical point is along the right lines, then what we ought to ask to understand

the argument is why Frege takes it for granted that defining truth as a F-concept Δ implies that deciding whether $\langle x \text{ is } \Delta \rangle$ is true is necessary for deciding whether x is true. To pursue this exegetical question, I think, will lead to a different reading of the argument.⁷ I turn to developing such a reading in the next section.⁸

3.2. LITERAL READING OF THE INDEFINABILITY ARGUMENT

The indefinability argument is cryptic. It is hard to see what the rationales are for the literal assertions Frege makes in the passages that contain the indefinability argument. Because of this difficulty, secondly, it is difficult to figure out what exactly the premises of this argument are. Given these exegetical difficulties, one who attempts to interpret this argument can choose either of the following two methods. One is to read Frege's points made in his other works into the passages that contain the indefinability argument and reconstruct the argument through this projection (e.g., Künne's circularity reading). The other is to retrieve the premises of the argument by a line-by-line literal analysis of those passages and then examine if what we've retrieved make sense or whether they are well connected to Frege's other points. Both strategies are perfectly legitimate in interpretative works that have their own strengths and weaknesses. In the case of the indefinability argument, nonetheless, there are few works that adopt the second strategy. What I would like to do is to develop an interpretation of the

⁷Heck (2012: §2.3) and Pagan (2001) follow this idea. They claim that the act of deciding whether p is basically the act of judging that p and that Frege's conception of judgment as acknowledgment of truth is related to the argument. Specifically, they argue that given Frege's conception of judgment as such, if truth is a property, acknowledging the truth of $\langle p \rangle$ is assertively predicating the property *truth* of $\langle p \rangle$, which is equivalent to judging that $\langle p \rangle$ is true, i.e., acknowledging the truth of $\langle \langle p \rangle \text{ is true} \rangle$. I believe their interpretation is on the right track. However, as we will see, their interpretation also has a problem. My interpretation will be an attempt to fix the problem with their reading.

⁸§3.1 is a slightly modified version of my paper 'The Circularity Reading of Frege's Indefinability Argument' (Kim 2020) published in *Thought* and has been reproduced here with the permission of John Wiley and Sons.

argument that adopts the second strategy. In other words, I develop a literal analysis of the passages and then connect what are retrieved to Frege's other works.

We can determine the structure of the indefinability argument fairly uniquely by a line-by-line interpretation of the passage from 'Thought'. The literal analysis reveals that the argument presupposes the Fregean conception of judgment—a conception criticized by Hanks (2007, 2011, 2015, 2018).⁹ It also establishes that the argument, assuming the conception is true, concludes that truth is not a Fregean concept, i.e., a property. If my suggested reading is along the right lines, Frege is urging us to retain the Fregean conception of judgment and abandon the common belief that truth is a property of truth-bearers. Frege's argument is not a primitivist argument. It is an argument that truth is not a property but an object.

Under my suggested reading, the conclusion of the argument is that there cannot be a proper truth definition of the following form:

(TD) x is true iff x is Δ .

The argument contained in the passage is the following *reductio*:

Assumption: There is a proper definition of truth of the form (TD).

(T_{Pro}) If there is a proper definition of truth of the form (TD), truth is a property.

(J_{Pro}) If truth is a property, one acknowledges the truth of x iff one acknowledges the truth of $\langle x \text{ is true} \rangle$. ($\langle p \rangle$ means 'the thought that p '.)

⁹Hanks puts forward a non-Fregean conception of proposition based on a non-Fregean understanding of judgment. He claims that the Fregean conception of judgment posits the existence of propositions as abstract entities that have truth-conditions independently of any human activity. According to Hanks, that is a fundamental mistake. I return to Hanks's conception of judgment and his criticism of the Fregean conception of judgment later in §7.1.

(VIR) If one acknowledges the truth of x iff one acknowledges the truth of $\langle x \text{ is true} \rangle$, an absurd result is produced.

\therefore There is no proper definition of truth of the form (TD).

§3.2.1 develops this reading. §3.2.2 and §3.2.3 develop the rationales for (VIR) and (J_{Pro}) respectively. A few notes. ‘An absurd result’ in (VIR) refers to a vicious infinite regress. I explain this point when I develop a rationale for (VIR) in §3.2.2. The Fregean conception of judgment is related to (J_{Pro}). As we will see in §3.2.3, (J_{Pro}) is true if the Fregean conception of judgment is true.

3.2.1. THE ANALYSIS OF THE PASSAGE FROM ‘THOUGHT’

Two preliminaries. First, though the argument is woven in terms of the act of deciding whether something is true on the surface, it is *judging* that it is really concerned with.¹⁰ For Frege, to judge is to acknowledge the truth of a truth-bearer. Recall the question at the beginning of the passage:

For in that case [i.e., if truth is defined as correspondence] what ought we to do so as to decide whether something is true? (1918a: 325)

This presupposes that what it takes to decide whether something is true relies on how truth is defined, i.e., what the nature of truth amounts to. Judging *qua* acknowledging the truth of a truth-bearer relies on the nature of truth exactly in that way: depending on the nature of truth, what it takes to acknowledge the truth of something must be different. Hence, this passage is about judgment. This means that we can paraphrase ‘decide whether x is true’ in the argument into ‘acknowledge the truth of x ’.¹¹

¹⁰For the relationship between the indefinability argument and judgment, see Heck 2007, 2012, Kremer 2000, Pagin 2001, and Ricketts 1996.

¹¹In Frege, deciding that x is not true is also reduced to acknowledging the truth of the thought that x is not true. As we can see in ‘Negation’ (1918b), Frege rejects that there is the act of rejecting that is not reduced to the act of judging. Thus, for him, rejecting that p is judging that the thought that p is not true.

The second point is concerned with truth-bearers in Frege. Truth-bearers for him in his mature career are thoughts, i.e., the senses of sentences, and to acknowledge the truth of the thought that p , or $\langle p \rangle$, is to judge that p . However, Frege initially talks not about thoughts but about ideas in the argument, which means that the argument must not depend on whether truth-bearers are thoughts. As we will see, the argument only requires it to be true that thoughts are *one* kind of truth-bearers.

Now, I turn to retrieving (J_{Pro}) from the passage. Given that Frege is talking about judgment, the question in the passage, ‘In that case what ought we to do so as to decide whether something is true?’, is asking what must go into the place of ‘ ϕ ’ in the following:

(J_{Cor}) If truth consists in correspondence, one acknowledges the truth of a truth-bearer x iff ϕ .

Frege immediately answers:

We should have to inquire whether it is *true* that an idea and a reality, say, correspond in the specified respect. (1918a: 325)

If we translate ‘inquire (i.e., decide) whether \dots is true’ into ‘acknowledge the truth of \dots ’, the that-clause remains. To see what it refers to, we need to take it into account that truth is defined as correspondence to reality, which implies that truth is defined as a property a truth-bearer has. Thus, in ‘It is true that an idea and a reality correspond in the specified respect’, ‘It is true’ must denote the property *truth*. Then, the that-clause must refer to a truth-bearer that has the property: a sort of propositional content Frege calls ‘thought’. So Frege’s above statement can be paraphrased as follows: ‘One should have to acknowledge the truth of $\langle x \text{ corresponds to reality} \rangle$ ’. That is, we have the following:

(J_{Cor}*) If truth consists in correspondence, one acknowledges the truth of a truth-bearer x iff one acknowledges the truth of $\langle x$ corresponds to reality \rangle .

In the generalized indefinability argument, (J_{Cor}*) is generalized into (J_{Pro}). Frege remarks:

For in a definition certain characteristics would have to be specified. (1918a: 325)

First, the term ‘characteristic (mark)’ must be explained. In *Grundgesetze* Frege says:

If one says: “a square is a rectangle in which adjacent sides are equal”, one defines the [property] *square* by stating what properties something must have in order to [have] it. I call these properties characteristic marks of the [property]. (1893: XIV)¹²

The characteristics of a property P are properties constitutive of P that will be specified in the analytic definition of P if there is such a definition.¹³ Therefore, ‘characteristics’ in the passage means ‘characteristics of truth’. This establishes that the indefinability argument targets at truth definitions which specify the characteristics of truth, i.e., those of the form (TD):

(TD) x is true iff x is Δ .

This is also confirmed by the indefinability argument in ‘Logic’ where Frege identifies the target of his argument as follows:

The same [point that an absurd result is produced] would hold of any definition of the form ‘ A is true if and only if it has such-and-such properties’ \dots . (1897: 128)

¹²The original translation uses ‘concept’ instead of ‘property’. I use ‘property’ to simplify our discussion here. Doing so does not distort any of Frege’s points. This is shown by the fact that what Frege calls ‘properties’, i.e., characteristics, also refer to concepts, strictly speaking. We will see Frege’s concept of *concept* in the next section.

¹³A primitive property PP that is not constituted by other properties still has its characteristic, i.e., PP itself. All properties are their own characteristics.

Now, let us return to 'Thought'. Frege writes:

For in a definition certain characteristics would have to be specified. And in application to any particular case the question would always arise whether it were *true* that the characteristics were present. (1918a: 325)

The first statement means that if we have a proper truth definition of the form (TD), truth is a property that has characteristics. The second statement must be the answer to the question as to what we ought to do to decide whether something is true if truth has specified characteristics. The following seems to be what we have in the second statement:

(J_{Gen}) If truth has characteristics, one acknowledges the truth of x iff one acknowledges the truth of \langle those characteristics obtain \rangle .

'Those characteristics obtain' must mean ' x has the properties constitutive of truth, i.e., has the property *truth*' or simply ' x is true'. Moreover, truth having characteristics is equivalent to truth being a property. Hence, (J_{Gen}) comes down to the following:

(J_{Pro}) If truth is a property, then one acknowledges the truth of x iff one acknowledges the truth of $\langle x$ has the property *truth* \rangle , i.e., the truth of $\langle x$ is true \rangle .

(J_{Cor}*) is an instance of (J_{Pro}) because to define truth as correspondence is to define it as a property of truth-bearers. (J_{Pro}) is indeed what the indefinability argument depends on.

Before turning to retrieving other premises, I address an objection to taking (J_{Pro}) to be a premise of the argument. Regarding it as a premise, I've assumed that an entity is a property just in case it has characteristics. Künne (2001) would disagree. He says a characteristic mark M of a property P must satisfy the following condition: ' M is a property, M is \dots a component of an analytic definition [of P], and nothing can [have

P] without [having M]' (2008: 13). If he is right, a primitive property cannot have a characteristic. However, the second condition Künne assigns on characteristics is groundless. All Frege says about characteristics and definitions is that we come to have a definition of a property by specifying its characteristics; that does not imply that only a property that shows up in a definition of P can be a P 's characteristic. Any property 'that an object must have in order to have P ' (Frege 1903a: §150) is a characteristic of P . Künne (2001) takes the second condition to be necessary for the following reasons. Given only what Frege says, a characteristic of P is merely a necessary condition for P . Then, properties like *not both Q and not Q* or *Q or not Q* are characteristics of P . Künne claims that 'the content of any concept would be so rich that nobody could have one concept expressible in English without having all of them' (2001: 276). But this claim presupposes that all characteristics of P must show up in its definition. That is false; all Frege says is that *only* characteristics of P *can* show up in its definition, which does not imply the presupposition at all. A primitive property PP can have a characteristic, i.e., PP itself according to Frege's definition of characteristic.

Given (J_{Pro}), we can easily see that (VIR) is also a part of the indefinability argument:

(VIR) If one acknowledges the truth of x iff one acknowledges the truth of $\langle x \text{ is true} \rangle$, an absurd result is produced.

Recall Frege's argument against the correspondence definition:

We should have to inquire whether it is *true* that an idea and a reality, say, correspond in the specified respect. And then we should be confronted by a question of the same kind, and the game could begin again. So the attempted explanation of truth as correspondence breaks down. (1918a: 325; numbering mine)

The first claim is the consequent of (J_{Cor}^*). It is followed by the claim that an absurd result, which breaks down the correspondence definition, would be produced. So we

have the following:

(VIR*) If one acknowledges the truth of x iff one acknowledges the truth of $\langle x$ corresponds to reality \rangle , an absurd result is produced.

The same reading is applied to the part for the generalized indefinability argument. Hence, we have (VIR). As I said, I explain why the absurd result (VIR) points to is a vicious infinite regress in the next section. Here, what we need to focus is the point that Frege takes the antecedent of (VIR) by itself leads to an absurd result.

Let us take stock of what we have so far:

(J_{Pro}) If truth is a property, one acknowledges the truth of x iff one acknowledges the truth of $\langle x$ is true \rangle .

(VIR) If one acknowledges the truth of x iff one acknowledges the truth of $\langle x$ is true \rangle , an absurd result is produced.

To have the absurd result under the assumption that we have a proper definition of truth of the form (TD), we need the following:

(T_{Pro}) If there is a proper definition of truth of the form (TD), truth is a property.

Ascribing (T_{Pro}) to Frege here seems legitimate because, first, he needs it to have the intended conclusion and, second, the consequent of (T_{Pro}) is obviously entailed by the logical form (TD). Hence, the *reductio* that depends on (T_{Pro}), (J_{Pro}), and (VIR) is the actual argument contained in the passages from 'Thought'.

I turn to motivating (J_{Pro}) and (VIR). In this discussion, we will see that the argument forces us to choose either the Fregean conception of judgment or the intuitive idea that truth is a property of truth-bearers.

3.2.2. THE MOTIVATION FOR (VIR)

Recall (VIR):

(VIR) If one acknowledges the truth of x iff one acknowledges the truth of $\langle x \text{ is true} \rangle$, an absurd result is produced.

The absurd result is a vicious infinite regress. Call the antecedent of (VIR) '(AA)'. One thing to be noted is that the right side of (AA) specifies what we ought to do in order to acknowledge the truth of x . That is how Frege reaches (J_{Pro}). Now, suppose that (AA) is true. Say one attempts to acknowledge the truth of a truth-bearer b . By (AA), one can do so by and only by acknowledging the truth of $\langle b \text{ is true} \rangle$. This is where the argument depends on the assumption that thoughts are at least a kind of truth-bearers. $\langle b \text{ is true} \rangle$ is a truth-bearer because it is a thought, and thus it satisfies (AA). By (AA), one can acknowledge its truth by and only by acknowledging the truth of $\langle \langle b \text{ is true} \rangle \text{ is true} \rangle$. By (AA) again, one can perform the latter by and only by acknowledging the truth of $\langle \langle \langle b \text{ is true} \rangle \text{ is true} \rangle \text{ is true} \rangle$. This regress will continue endlessly. This infinite regress seems to be what Frege is getting at when he says the following:

And then we should be confronted by a question of the same kind, and the game could begin again. (1918a: 325)

It might be objected that the infinite regress is vicious only if acknowledging the truth of $\langle p \rangle$ is not identical with acknowledge the truth of $\langle \langle p \rangle \text{ is true} \rangle$. Indeed, it is tempting to think that the former is identical with the latter for Frege. First of all, judgment is thought-sensitive:

(Th_t) Acknowledging the truth of $\langle p \rangle$ is identical with acknowledging the truth of $\langle q \rangle$ iff $\langle p \rangle$ is identical with $\langle q \rangle$.

In 'On Sense and Reference' (1892c), Frege elucidates judging that p as taking a step from $\langle p \rangle$ to the reference of ' p ', i.e., one of the truth-values, the True and the False. We will see later what this metaphorical elucidation means. However, it already appears clear that it implies (Tht): taking a step from $\langle p \rangle$ is identical with taking a step from $\langle q \rangle$ iff $\langle p \rangle$ is identical with $\langle q \rangle$. Secondly, as we have already seen, Frege does claim that $\langle p \rangle$ is identical with $\langle \langle p \rangle$ is true \rangle . Then, by (Tht), acknowledging the truth of $\langle p \rangle$ is identical with acknowledging the truth of $\langle \langle p \rangle$ is true \rangle . Then, it seems that the infinite regress produced by (AA) is not vicious.

But this objection misses the point that while accepting the *identity thesis* that $\langle p \rangle$ is identical with $\langle \langle p \rangle$ is true \rangle , Frege also accepts the following:

(Diff) If ' \dots is true' denotes a property, $\langle p \rangle$ is different from $\langle \langle p \rangle$ is true \rangle .

As we've already seen, Frege endorses it in his argument in 'On Sense and Reference' that the True is not a property of a thought but its reference. Here is how the argument is reformulated:

(P₁) If the True is a property, then ' \dots is true' denotes the property.

(P₂) If ' \dots is true' denotes a property, $\langle p \rangle$ is different from $\langle \langle p \rangle$ is true \rangle .

(P₃) $\langle p \rangle$ is identical with $\langle \langle p \rangle$ is true \rangle .

(C₁) \therefore ' \dots is true' does not denote a property.

(C₂) \therefore The True is not a property.

If this reformulation of the argument is acceptable, Frege is committed to (Diff), i.e., (P₂). By our assumption that there is a truth definition of the form (TD), truth is a property. By (Diff) and (Tht), acknowledging the truth of $\langle p \rangle$ is different from acknowledging the truth of $\langle \langle p \rangle$ is true \rangle . Hence, the objection to (VIR) does not succeed.

(Diff) is arguably plausible; saying that snow is white certainly seems to be radically different from saying that a thought has a certain property. Presumably, this is why, in the literature, it is hard to find out a theory of truth that takes truth to be a property *and* takes $\langle p \rangle$ to be identical with $\langle \langle p \rangle$ is true \rangle . For instance, Horwich (1998), who takes truth to be a property in a minimal sense —i.e., in the sense that ‘true’ is a legitimate predicate—, still takes $\langle p \rangle$ to be different from $\langle \langle p \rangle$ is true \rangle *because* truth is a property.

A number of readings of the indefinability argument take the identity thesis to be a part of the argument (cf. Künne 2008 or Soames 1999). But Frege’s commitment to (Diff) provides a reason to doubt this claim. In Frege’s own understanding, the identity thesis would not obtain if truth were a property. However, the argument assumes that truth is a property. Under that assumption, Frege cannot depend on the identity thesis without going against his own commitment to (Diff).

Note that the argument from ‘On Sense and Reference’ concludes that truth is not a property of thoughts. The indefinability argument leads us to the same conclusion. Given (J_{Pro}) and (VIR):

(J_{Pro}) If truth is a property, one acknowledges the truth of x iff one acknowledges the truth of $\langle x$ has the property *truth* \rangle , i.e., $\langle x$ is true \rangle ;

(VIR) If one acknowledges the truth of x iff one acknowledges the truth of $\langle x$ is true \rangle , an absurd result is produced;

the assumption that truth is a property will lead to the same infinite regress. Hence, we should deny the assumption that truth is a property of truth-bearers, which implies that truth is not a property of thoughts. Thus, the argument in ‘On Sense and Reference’ and the indefinability argument are different arguments to the conclusion that truth is not a property. The difference is that while the former depends on the identity between $\langle p \rangle$

and $\langle\langle p \rangle \text{ is true}\rangle$, the latter does not. The indefinability argument is Frege's argument that truth is not a property but an object.

3.2.3. THE MOTIVATION FOR (J_{PRO}) AND THE LESSON FROM THE ARGUMENT

Recall (J_{PRO}) :

(J_{PRO}) If truth is a property, one acknowledges the truth of x iff one acknowledges the truth of $\langle x \text{ is true}\rangle$.

Before turning to the motivation for it, I first need to note that Heck (2012) and Pagin (2001) also take (J_{PRO}) to be a premise of the indefinability argument. For instance, Heck writes:

[Say] we do think of truth as a property of thoughts. Then, the idea that judgment is admission of a thought's truth becomes the idea that judging is a form of predication: judging is predicating truth of a thought. . . . Predication in this sense is itself a sort of judgment: To predicate truth of the thought that p is just to judge that the thought that p is true . . . (Heck 2012: 47)

Heck is inferring (J_{PRO}) by the following two propositions:

(Q_1) If truth is a property, one acknowledges the truth of x iff one predicates the property *truth* of x .

(Q_2) One predicates truth of x iff one judges that x is true, i.e., acknowledges the truth of $\langle x \text{ is true}\rangle$.

Pagin (2001: 5) presents the same inference to (J_{PRO}) .

One problem with this inference is that in Frege's conception of predication, (Q_2) is false. For him, predicating is grasping a thought, e.g., predicating a property P of an object o is grasping $\langle o \text{ is } P\rangle$. Grasping a thought does not amount to acknowledging its truth.¹⁴ In this conception of predication (Q_2) is false. Of course, what Heck means by

¹⁴See, e.g., Frege 1892c, Frege 1897, Frege 1906a, etc.

‘predicate’ in (Q₂) is *strong* predication that includes a commitment to the correctness of predication. We can define strong predication within Frege: strongly predicating is acknowledging the truth of the thought grasped by predication. (Q₂) is an instance of this definition. The problem is that (Q₁) is made true only if we read the right side of its consequent as (Q₂) defines. Hence, (Q₁) is simply (J_{Pro}). The inference from (Q₁) to (J_{Pro}) is circular.

At this point let us speculate about what Frege’s own motivation for (J_{Pro}) could be. What is to be noted is that in the passage Frege puts forward (J_{Pro}) as something so obviously true that he does not even provide an explanation. This suggests that (J_{Pro}) is grounded in something to which Frege is so strongly committed that he cannot think of any alternative. One thing Frege is committed to that strongly is his conception of judgment as acknowledgment of truth. This conception has a particular implication about the act of acknowledging a property of an object in general. Let *o* be an object and *P* be a property objects can have. Consider the following descriptions of act-types:

- (A₁) Acknowledging the *P*-ness of *o*,
- (A₂) Judging that *o* is *P*, and
- (A₃) Acknowledging the truth of $\langle o \text{ is } P \rangle$.

The Fregean conception of judgment elucidates (A₂) as (A₃). On top of that, it also claims that it is by and only by performing (A₃) that we perform (A₁). The Fregean conception of judgment entails that if truth is a property, it is by and only by acknowledging the truth of $\langle x \text{ is true} \rangle$ —a case of (A₃)—that one acknowledges the truth of *x*—a case of (A₁)—, which is what (J_{Pro}) asserts. Therefore if one is committed to the Fregean conception of judgment, then one ought to also accept (J_{Pro}). It appears to be a salient possibility that (J_{Pro}) in ‘Thought’ is grounded in the Fregean conception of judgment

to which he is strongly committed.

If the Fregean conception of judgment is indeed Frege's motivation for (J_{Pro}) , however, he is committing a mistake here. For there is a viable alternative to the Fregean conception of judgment. For instance, a non-Fregean conception of judgment has recently been championed by Hanks (2011, 2015, 2018). For Hanks, (A_2) is reduced to (A_1) where (A_1) is predicating the property P of o .¹⁵ One cannot perform (A_3) through performing (A_1) ; the former is an act-type different from the latter. (J_{Pro}) does not obtain for Hanks; (A_1) is not achieved by and only by doing (A_3) .

To recap, Frege puts forward (J_{Pro}) as obviously true because he is firmly committed to the Fregean conception of judgment. But the Fregean conception of judgment is not sufficient for motivating (J_{Pro}) because there is a viable alternative to it. Therefore, Frege's argument cannot really establish that truth is not a property, unless he has a knock-down argument against the alternative.

Nevertheless, Frege's indefinability argument still provides a crucial lesson. The Fregean conception of judgment, under which judging is making a commitment to the truth of a proposition as an abstract entity that represents things in a way, is a popular point of view.¹⁶ Frege's argument shows that the Fregean conception is incompatible with the ordinary view that truth is a property of truth-bearers. For the following is arguably true:

(J_{Pro}^*) If the Fregean conception of judgment is correct, if truth is a property, one acknowledges the truth of x iff one acknowledges the truth of $\langle x \text{ is true} \rangle$.

If we assume that the Fregean conception is true and that truth is a property, we come to

¹⁵For Hanks, predication is inherently committal. See ch.3 of Hanks 2015.

¹⁶See ch.1 of Hanks 2015.

have the vicious infinite regress. We ought to abandon at least one of the assumptions.

Frege's choice is to reject that truth is a property as we have seen. If truth is not a property, the vicious infinite regress is not produced even if the Fregean conception of judgment is correct. This is because acknowledging the truth of a truth bearer is not a case of (A₁) anymore (recall that '*P*' in (A₁) refers to a property). Now, this means that Frege ought to provide an alternative conception of judgment *qua* acknowledgment of truth. He does provide such an alternative in 'On Sense and Reference' and other mature works as we will see in Chapter 5. This reversely shows that Frege is attempting to justify his search for an alternative understanding of truth and judgment when he builds up his arguments against the thought that truth is a property. Therefore, Frege's indefinability argument is not a primitivist argument. It is an argument that truth is an object.

CHAPTER 4

MORE OBJECTIONS

Chapter 2 and Chapter 3 establish that Frege's deflationist and primitivist comments are his arguments that truth is an object. In our discussion, we've already addressed several objections. This chapter addresses two more objections to the claim that truth is an object for Frege. §4.1 deals with what I call 'the neo-Kantian reading'. This reading interprets Frege's philosophy in general by considering the historical connection between Frege and the neo-Kantian philosophers at his time. The advocates of the neo-Kantian reading claim that although Frege is forced by his logic to take a truth-value to be an object, he still takes truth to be a property, specifically a value property, *philosophically*. §4.2 deals with Greimann's reading of Frege on truth according to which truth is neither a property nor an object for Frege. According to Greimann, Frege is rather an advocate of the 'No Entity' theory of truth according to which truth is not an

entity although it still plays a logically significant role.

4.1. THE NEO-KANTIAN READING ON FREGE ON TRUTH-VALUES

Gottfried Gabriel (1984, 2002, 2013a, 2013b) shows the historical connection of Frege to ‘the value-theoretical [Southwestern] school of neo-Kantianism’ (1984: 374). Gabriel argues that, just like neo-Kantians, Frege regards truth as a *value*, and his term ‘truth-value’ should be understood value-theoretically.¹ In fact, it is Wilhelm Windelband who started value-theoretical neo-Kantianism that first introduced the term and used it in a value-theoretical sense. Both Windelband and Frege using ‘truth-value’ is not a coincidence given that they studied under the advisory of Hermann Lotze at Göttingen who talks about the value-difference between truths and untruths.²

The historical connection between Frege and neo-Kantianism appears to be well entrenched. Also, the interpretation of Frege on truth-values grounded in this historical context, what I call ‘the neo-Kantian reading’, provides a different dimension through which we can explore Frege’s philosophical insights into truth-values and truth. I believe that the neo-Kantian reading makes a valuable contribution to Frege scholarship and that the historical connection between Frege and neo-Kantianism deserves more attention. Our discussion is not concerned with this positive aspect of the neo-Kantian reading.

This section concerns the neo-Kantian reading’s take on Frege’s claim that a truth-value *qua* the reference of a sentence is an object. The neo-Kantian reading maintains that Frege is ‘forced’ (Gabriel 1984: 375) by his logic *Begriffsschrift* (Frege 1879a, 1893)

¹According to Gabriel (1984: 374), the fact that Frege is using ‘truth-value’ in a value-theoretical sense is revealed in ‘On Sense and Reference’ (1892c) where Frege argues that the *Bedeutung* of a sentence is a truth-value.

²For this historical connection among Frege, Windelband, and Lotze, see Gabriel 2002, 2013a, and Sluga 1980.

to argue so and that he concedes that ‘taking truth-values as objects is an advantage . . . only for semantics of a special formal language like *Begriffsschrift*’ (Gabriel 1984: 375). In short, the proponent of this reading thinks that Frege takes a truth-value *qua* a sentential reference to be an object solely because of *technical* reasons, so to speak.³ The aim of this chapter is to establish that it is due to *philosophical* reasons that Frege takes a truth-value to be an object.

Specifically I address the neo-Kantian reading’s two arguments that Frege takes truth-values to be objects only for technical reasons. Its first argument (§4.1.1) appeals to the correspondence between Frege and Russell. However, as we will see, the textual evidence does not fully support the neo-Kantian reading. Its second argument (§4.1.2) is that Frege concedes that taking truth-values as objects is an ‘advantage . . . only for semantics of a special formal language like’ (Gabriel 1984: 376) *Begriffsschrift*. But we will see that for Frege, taking a truth-value as an object denoted by a sentence plays an important role in his understanding of judgement that has a philosophical significance in his framework.

4.1.1. ARGUMENT I. CORRESPONDENCE BETWEEN FREGE AND RUSSELL

The neo-Kantian reading argues that as Tugendhat (1970) suggests, Frege’s term ‘*Bedeutung*’, which we have already translated as ‘reference’ consistently, should be understood as meaning ‘significance’—especially when Frege is talking about ‘the *Bedeutung*

³By ‘technical’, I mean ‘logical’ or ‘logic-related’. The neo-Kantian reading appears to distinguish technical contexts in this sense from philosophical contexts in Frege’s works. Although I do not believe that this distinction obtains, I do not tackle it in this paper.

of a sentence'.^{4,5} The neo-Kantian reading's point is that it is not really Frege's intention to assimilate sentences with proper names. The neo-Kantian reading recognizes that by taking truth-values as objects, Frege misleads his readers to think that truth-values are sentential *references*. According to this reading, however, we should not be misled by Frege's taking truth-values as objects.

Replying to Russell's letter, Frege writes:

If the sign of identity is used between sentences, the truth-value must be recognized as the *Bedeutung* of the sentences. (Frege 1980: 152)

The neo-Kantian interpretation (Gabriel 1984: 375-6) takes this passage as where 'the *Bedeutung* of the sentence' means something close to 'the *reference* of the sentence' and Frege is explaining why he is 'forced' (Gabriel 1984: 375) to point to the thought that a truth-values is the reference of a sentence. According to the exegesis, the antecedent of the above statement, which is required to be true by a part of *Begriffsschrift*, i.e., by Frege's theory of identity, implies that truth-values are objects.⁶ The gist of the neo-Kantian reading is that Frege is confessing that he is forced by the technical requirement of *Begriffsschrift* to say that truth-values are objects and thereby misleadingly point to the idea that truth-values are references of sentences.

However, that is not what is going on in this passage. Let us turn to Russell's letter Frege is here responding to. In this letter, Russell writes:

⁴Tugendhat's suggested translation was not well received; it was criticized by many commentators including Dummett (1993: 199-203). Unlike Tugendhat, Gabriel does not deny that '*Bedeutung*' has the meaning of 'reference'. His point is that, on top of that meaning required by *Begriffsschrift*, Frege also means 'significance' or 'value' by the term.

⁵In general it is not an easy question how we should translate '*Bedeutung*' in Frege's terminology. This difficulty has resulted in a number of different translations of it, e.g., 'reference', 'referent', 'denotation', 'meaning', 'nominatum', etc. There are a number of papers on this. See, e.g., Angelelli 1982; Bell 1980; Green, forthcoming, etc. Beaney (2019)'s discussion provides an excellent review of this debate.

⁶It is not clear what Gabriel means by 'theory of identity (in Frege)'. I assume it refers to the parts of *Begriffsschrift* that deal with how the identity sign can be used.

I cannot . . . believe that the True or the False is the [*Bedeutung*] of a [sentence] . . . as a certain person is the [*Bedeutung*] of the name Julius Caesar. (In Frege 1980: 151)

Russell's complaint is certainly that by rendering a truth-value to be the *Bedeutung* of a sentence, we come to assimilate sentences with proper names like 'Julius Caesar'. This shows that Russell is already understanding '*Bedeutung*' as something like 'reference'. If Frege were thinking that it is misleading to take '*Bedeutung*' in 'the *Bedeutung* of a sentence' to mean 'reference', and trying to clarify this misconception by explaining why he could not but say that truth-values are objects, it would have to be the case that Frege at least makes a gesture at this misconception. However, Frege never talks about this alleged misconception. What Frege is doing before the foregoing comment in his letter to Russell (1980: 152) is to recommend Russell to read his 'On Sense and Reference' and recap his arguments in it that a truth-values is the *Bedeutung* of a sentence.⁷ If so, then what Frege is doing when he replies to Russell is not to make an excuse for taking truth-values as objects or as sentential references, but to *argue* that a truth-value is indeed the *Bedeutung* of a sentence when '*Bedeutung*' is understood in the sense Russell attributes to it, i.e., when it means something like 'reference'.

Again, Frege's comment in his letter to Russell immediately follows his recap of the arguments in 'On Sense and Reference' that a truth-value is the reference of a sentence. Then it is only natural to construe his comment as one of those arguments. If it is such an argument, it is certainly an exceptional argument because, in other works, Frege does not motivate the idea that the reference of a sentence is a truth-value through

⁷In particular, Frege is summarizing two arguments. The first one appeals to the point that we are interested in whether the parts of a sentence have references only when we are interested in the truth-value of a sentence. The second one appeals to the compositionality of reference. [A] is Frege's third argument that truth-values are sentential references.

appealing to the point that the identity sign can come between sentences.⁸ However, if we look into the correspondence between Frege and Russell, we can understand why Frege provides Russell with such an argument. Unlike what the neo-Kantian reading says, the antecedent of [A]—that the identity sign can be put between sentences—only implies that sentences are proper names. Now, Russell has definitely read *Grundgesetze*, and so he is highly familiar with Frege's use of the identity sign between sentences. In one letter Russell sent before the above one, he writes:

On p.49 you say that $\Gamma = \Delta$ has a [Bedeutung] if Γ and Δ are proper names for ranges of values or names for truth-values. (In Frege 1980: 134)

Thus, Russell understands that the identity sign can appear between sentences. However, he does not tackle Frege's use of the identity sign between sentences in any of his letters. Then, what Frege is saying by his comment could be this: 'Given that you already agree to the use of the identity sign between sentences, you should be willing to accept that sentential references are objects, in which case it is only natural, given the previous arguments I have given, for you to accommodate that truth-values are those objects referred to by sentences'. If so, what Frege is doing is not to provide a technical reason to take truth-values to be objects, but to point out the incoherence between accepting the use of the identity sign between sentences and denying that sentential references are proper names, and thereby convince Russell that truth-values are sentential references.

Recall that the neo-Kantian reading claims that it is *only* due to technical reasons that Frege takes truth-values as objects. However, Frege's statement in his letter to Russell cannot support such a strong claim because, if our discussion is along the right

⁸For instance, that is not how he motivates the idea even in 'Function and Concept' (Frege 1891) which is written for mathematicians.

lines, it is not even clear that by the statement Frege is trying to give a technical reason to take truth-values as objects.

4.1.2. ARGUMENT II. SEMANTIC USELESSNESS OF TRUTH-VALUES

Gabriel writes:

In fact, taking truth-values as objects is not an advantage for semantics in general, but only for semantics of a special formal language like Frege's [*Begriffsschrift*]. (Gabriel 1984: 376)

If Frege himself also shares this thought, then there is a reason to believe that it is only for technical reasons that Frege takes truth-values as objects. Gabriel looks upon the following as the textual evidence that Frege shares the same thought:

The designation of the truth values as objects may appear to be an arbitrary fancy or perhaps a mere play upon words, from which no profound consequences could be drawn. (Frege 1970: 63–4)

According to the neo-Kantian reading (Gabriel 2013b), truth is a property, or a concept (following Frege's own terminology), for Frege in the philosophical context.⁹ It is a value concept, and this value concept of truth is what Frege would mean by 'truth-value' in the philosophical context. The value of truth is a concept acknowledged in our judgement, which constitutes the main thesis of the 'acknowledgement theory of truth' the neo-Kantian reading ascribes to Frege.

However, it is not justified to argue based on the above statement that Frege thinks that taking truth-values as objects is only beneficial for his *Begriffsschrift*. Note that Frege says that 'it *may appear* to be an arbitrary fancy'. This can be read as meaning

⁹Gabriel (2013b) concedes that Frege denies that truth is a concept, but also argues that the concept of truth is still relevant to understanding Frege's conception of truth. I assume that Gabriel is also appealing to the distinction between the technical context and the philosophical context in Frege, although he does not explicitly do so.

that he in fact thinks that it *is not* an arbitrary fancy. I provide two arguments that this reading is correct.

The first argument is based on the fact that, right after the above statement, Frege presents a philosophical argument that truth-values are objects. Before taking a look at this argument, let me note that the above statement is followed by this:

What I mean by an object can be more exactly discussed only in connexion with concept and relation. I will reserve this for another article (i.e., 'On Concept and Object'). But so much should already be clear, that in every judgement, no matter how trivial, the step from the level of thoughts to the level of reference (the objective) has already been taken. (Frege 1892c: 64)

What Frege is pointing to in the first statement is his ontology according to which all things that exist are classified as objects or functions. A function is unsaturated; it has empty places for arguments. Saturated by arguments, a function yields an object as its value. An object is saturated; it does not have any place for an argument. A concept is a special kind of function whose value for given arguments is always a truth-value. In the last statement of the above passage Frege elucidates judgement in terms of the sense of a sentence, i.e., a thought, and the reference of a sentence, i.e., a truth-value.

We will talk more about this passage soon. What is crucial now is that it is immediately followed by this argument we have already seen:

One might be tempted to regard the relation of the thought to the True not as that of sense to reference, but rather as that of subject to predicate. One can, indeed, say: 'The thought, that 5 is a prime number, is true.' But ... nothing more has been said than in the simple sentence '5 is a prime number.' The truth claim arises in each case from the form of the declarative sentence, and when the latter lacks its usual force, e.g., in the mouth of an actor upon the stage, even the sentence 'The thought that 5 is a prime number is true' contains only a thought, and indeed the same thought as the simple '5 is a prime number.' It follows that the relationship

of the thought to the True may not be compared with that of subject to predicate.
(Frege 1892c: 64)

This is a *philosophical* argument concerning the nature of the True. It does not appeal to any point from *Begriffsschrift*. Also, notice that Frege is using the terms ‘subject’ and ‘predicate’ he believes should be abandoned (Frege 1892a). He does not introduce his official terms ‘concept’ and ‘relation’:

I will reserve [the discussion about concept and relation] for another article [i.e., ‘On Concept and Object’]. (Frege 1892c: 64)

It seems as if he is trying not to touch those *logically proper* notions because he should bring technical points to elucidate them. Hence, he is using more common notions of subject and predicate he in fact is reluctant to use. This strongly suggests that he is trying to avoid appealing to technical or logical points in the following discussion. In the above argument, we can conclude, Frege is trying to establish *philosophically* that the True must not be taken to be a concept under which thoughts fall. By establishing it, he is urging the readers to swallow his suggestion that the True *as an object* is the reference attached to thoughts.

My second argument against the neo-Kantian reading is that truth-values as objects and sentential references play a philosophically crucial role in Frege—specifically in his conception of judgment. Recall the following statement the neo-Kantian reading depends on:

The designation of the truth values as objects may appear to be an arbitrary fancy or perhaps a mere play upon words, from which no profound consequences could be drawn. (1970: 63–4)

Frege admits that taking truth-values as objects can look like an arbitrary stipulation

from which no meaningful consequence can be inferred. As we've seen, this statement is followed by this passage:

What I mean by an object can be more exactly discussed only in connexion with concept and relation. I will reserve this for another article (i.e., 'On Concept and Object'). But so much should already be clear, that in every judgement, no matter how trivial, the step from the level of thoughts to the level of reference (the objective) has already been taken. (Frege 1892c: 64)

In the first two statements Frege appears to be saying that the meaningful consequences of taking truth-values as objects can be fully explained only by bringing his distinction between objects and concepts, which he says he will do in a different paper. Then, the connective 'but' in the last statement suggests that Frege is showing a meaningful consequence of taking truth-values as objects that is *already* clear *before* going to such (partly) technical points. The meaningful consequence concerns judgement; judging is taking a step from a thought to a truth-value. If this reading is along the correct lines, then it is not correct to say that Frege believes that taking truth-values as objects has no profound consequence. Introducing truth-values as objects is rather necessary for understanding the act of judgement.

We need an explanation about why it is an important consequence that judging is taking a step from the level of sense to that of reference. After the argument that the True is the reference of a sentence, Frege writes:

By combining subject and predicate, one reaches only a thought, never passes from sense to reference, never from a thought to its truth-value. One moves at the same level but never advances from one level to the next. (Frege 1892c: 64)

For Frege, the act of predication does not amount to that of judgement. Assume that an actor on a stage says 'Caesar is assassinated'. Predication occurs there; the property

of *being assassinated* is predicated of Caesar. But an assertion, a speech-act counterpart of judgement, does not occur. The question is in what judgement consists if it does not consist in the act of predication.¹⁰ That is the problem Frege faces when he tries to explain the act of judgement. Frege's solution is, as we have seen, to introduce the distinction between sense and reference. With it, Frege can now explain that to judge is to take a step from the level of sense to that of reference. Predicating does not amount to judging because it does not bring us to the level of reference. This appears to be an important philosophical consequence of taking truth-values as objects and sentential references: Frege can meaningfully distinguish predicating from judging.

We still need to explain how the elucidation of judgment as a step toward a truth-value makes judgment distinguished from predication, or more generally, what judgment as such a step amounts to. The next chapter answers these questions. However, it already seems to be clear that Frege has philosophical reasons to take the reference of a sentence to be an object, i.e., the True.

As I've said, it is not my intention to raise a doubt against the historical connection between Frege and value-theoretical neo-Kantianism that I believe the neo-Kantian reading convincingly shows. Neither is it my intention to challenge the neo-Kantian reading's thesis that, for Frege, truth is a value acknowledged in our act of judgement. The only part of the neo-Kantian reading I find hard to accept consists in its claim and arguments that it is only for technical reasons that Frege takes truth-values *qua* sentential references to be objects. The evidence for it is indecisive, and the evidence against it is more abundant and robust. However, the insightful observations of the neo-Kantian reading do not have to come with such a controversial claim. The valuable

¹⁰According to Hanks (2015), it is a hard question the Fregean conception of proposition in general must answer.

contribution of the neo-Kantian reading, I believe, can rather be fully understood only if Frege's claim that truth-values are objects is taken at face value and seriously.¹¹

4.2. THE 'NO ENTITY' THEORY READING OF FREGE ON TRUTH

Frege's arguments we've seen in Chapters 2 and 3 conclude that truth is not a property. We've inferred from this conclusion that truth is an object. However, Greimann (2004, 2007) would object to this inference. He insists that like the *property reading* of Frege on truth, the *object reading* does not square well with textual evidence. His first argument is that 'Thought' (Frege 1918a) in which Frege mainly talks about truth 'is not concerned with determining the logical category of the truth-values; rather, the truth-values are not mentioned at all' (Greimann 2004: 438). This seems to show that Frege's conception of truth is not related to the True. Greimann's second argument (2004: 438; 2007: 141–2) concerns the peculiarity of truth Frege points out. In 'Thought', Frege remarks:

[...] [W]e cannot recognize a property of a thing without at the same time finding the thought *this thing has this property* to be true. So with every property of a thing there is tied up a property of a thought, namely truth. (Frege 1918a: 328)

Even though Greimann does not take this passage to show that truth is a property for Frege, he thinks that Frege is here revealing a crucial part of his conception of truth, which is that predicating truth is always included in predicating anything whatever' (2004: 438). Truth, Greimann claims, is something that can be predicated although it is not a property. But the True—an object—cannot be predicated.

Greimann's third argument against the object reading concerns the indefinability of truth. Frege (1897, 1918a) famously holds that truth is indefinable. The problem is

¹¹It is well known that Dummett (1993) strongly criticizes Frege's assimilation of sentences with proper names. But a number of Frege scholars now do not necessarily agree to those criticisms. See, e.g., Burge 1986, Heck 2012, and Ricketts 2003. Combining their insights with the neo-Kantian reading, I believe, will lead to a deeper understanding of Frege's philosophy.

that the True seems to be definable. Greimann points out that, '[i]n order to overcome the referential indeterminacy of value-[range] terms in his system, Frege identifies in § 10 of *Grundgesetze* with the value-[range] of the function $\text{--- } x'$ (Greimann 2007: 142).¹² Greimann interprets this fact as showing that Frege defines the True as follows: The True = $\hat{\epsilon}(\text{--- } \epsilon)$. If so, the True which is definable cannot be truth.

If this critique is correct and truth cannot even be an object for Frege, we are lost in regards to his conception of truth. If truth is neither a property nor an object, what is it then?

Greimann presents an alternative reading. In this reading whose detail I introduce below, Frege's conception of truth consists in what we may call 'the "No Entity" theory of truth (the NE-theory)', the main claim of which is that truth is neither a property nor an object.¹³ According to the theory, truth is 'no thing'. If we fully understand the NE-theory of truth, Frege's remarks on truth we have seen are not puzzling anymore. Or so argues Greimann.

I call this alternative 'the NE-theory reading'. My first aim is to establish that Frege cannot accept the main claims of the NE-theory. If my argument is sound, then the NE-theory reading should be discarded. Also I show that Greimann's criticism of the object reading can be satisfactorily responded to.

§4.2.1 explains the NE-theory reading. §4.2.2 is my critique of the reading. §4.2.3 responds to Greimann's criticisms of the object reading.

¹²I use 'value-range' instead of 'value-course' following Ebert and Rossberg's translation (Frege 2013) of *Grundgesetze*.

¹³Greimann himself calls this theory 'the assertion theory of truth' because he thinks that for Frege truth is something expressed by assertion. Then Greimann claims that truth as such is neither an object nor a concept. The part where Greimann claims that truth is expressed by assertion essentially depends on his understanding of ' \dots is the True', which we have already criticized in Chapter 2. I do not repeat this part again here. Instead I focus on the part of Greimann's reading where he claims that for Frege truth is a non-entity.

4.2.1. THE NE-THEORY READING OF FREGE'S CONCEPTION OF TRUTH

In §2.3 we have Greimann's argument for the Truth Predicate Reading of '... is true'. He accepts that

(A₁) $\langle p \rangle$ is true if and only if_A the truth-value of $\langle p \rangle$ is the True

where ' p if and only if_A q ' or ' p iff_A q ' entails ' $\langle p \rangle$ is identical to $\langle q \rangle$ and $\langle q \rangle$ constitutes the analysis of $\langle p \rangle$ '. As we have seen ascribing (A₁) to Frege conflicts with his conception of assertion. But let us set aside those problems and focus on the following problematic implication of (A₁): Truth is a property of thoughts. To say $\langle p \rangle$ has the True as its truth-value is to say that $\langle p \rangle$ has the property of having the True as its truth-value. (A₁) looks like the claim that this property of $\langle p \rangle$ constitutes the truth of $\langle p \rangle$. Frege cannot accommodate this because truth is not a property for him as the NE-theory reading itself admits. However, the NE-theory reading claims that the NE-theory of truth has additional components that cancel out this implication. The NE-theory reading finds these components in Frege's conception of *saturation* (of a concept).

A small reminder would have a merit here. What we call 'properties' fall under the category of *concept* in Frege's framework. Concepts are functions the value of which is always a truth-value. Frege (1891, 1892b) looks upon functions as *unsaturated*. To say that a function is unsaturated is to say that a function has empty places that must be *saturated* by arguments in order for the function to provide an object as its value. Concepts are also unsaturated. Saturated, they provide a truth-value as its value. A concept is thus *predicative* (Greimann 2007: 128-9).

On the saturation of (a particular kind of) concepts, Frege writes:

In the sentence 'Two is a prime' we find a certain relation designated: that of subsumption. We may also say the object falls under the concept *prime* [...]. This also

creates the impression that the relation of subsumption is a third element supervenient upon the object and the concept. This isn't the case: the unsaturatedness of the concept brings it about that the object, in effecting the saturation, engages immediately with the concept, without need of any special cement. Object and concept are fundamentally made for each other, and in subsumption we have their fundamental union. (Frege 1906c: 178)

Here, Frege is talking about a particular kind of saturation: *Subsumption*. He seems to be suggesting that subsumption as a kind of saturation is not a genuine relation: Subsumption occurs just because concepts immediately engage with objects by their own nature and hence no special cement or relation is necessary for that. It also seems that Frege identifies this kind of saturation with the relation of *an object o falling under C*. If so, Frege is here arguing that the relation of *o falling under the concept C* is not a genuine relation. That is indeed how the NE-theory reading understands this passage. Greimann, who suggests the NE-theory reading, writes:

The relation that Frege calls "subsumption" is a special case of saturation. According to the system in *Begriffsschrift*, this relation holds between an object and a concept if the result of the concept's saturation by the object *is a fact* [...]. (Greimann 2007: 130; italics are mine.)

The consequence of the concept *prime*'s saturation by the number 2 is 2's being prime. 2's being prime *is a fact*; 2 is prime. Therefore, what Greimann is arguing here is that subsumption holds between 2 and the concept *prime* if 2 is prime. Subsumption as such is naturally identified with the relation of *o falling under C*; '2 falls under the concept *prime*' is equivalent to '2 is prime'. What is crucial is that this kind of relation is found in every sentence. For instance, say John loves Mary. We may say that John stands in the relation *love* to Mary. This looks like an example of another type of saturation: the relation of *an object o₁'s standing in the relation R to an object o₂* (Greimann 2007: 137).

If subsumption or the relation *o falling under C* is not a genuine relation, it seems, this new type of saturation or the relation *o₁'s standing in R to an object o₂* should not be a genuine entity.

On these different relations, the NE-theory reading claims:

Note that because of the very special role that this relation plays in judgment it cannot be substituted by ordinary relations. While the role of ordinary relations is to form the predicative part of judgments, the role of saturation is to connect this part with the non-predicative parts. Therefore, saturation, subsumption and kindred relations like the standing of two objects in a relation [...] form a logical category *sui generis*: they are, anachronistically speaking, not genuine relations, but “non-relational ties” whose sole task is to tie the predicative and the non-predicative parts of judgments together. Let us call these very special relations “logical relations”. (Greimann 2007: 131)

The NE-theory reading continues arguing that logical relations themselves are ‘neither saturated nor unsaturated’ (131). This has an ontological consequence, because Frege’s ontology consists only of the category of saturated entities, i.e. objects, and that of unsaturated, i.e. functions. Still, logical relations fall under none of those categories. They are ‘no-thing’ (131). Accordingly, the NE-theory of truth has the following:

(A₂) Logical relations are neither objects nor concepts.

Lastly, the NE-theory reading (Greimann 2007: 137–8) attributes the following claim to Frege:

(A₃) The truth-value of $\langle p \rangle$ is the True iff_A the logical relation relevant to $\langle p \rangle$ obtains.

The NE-theory reading supports (A₃) by the following passage from *Grundgesetze*.

We say that an object Γ *stands in the relation* $\Psi(\xi, \zeta)$ *to the object* Δ if $\Psi(\Gamma, \Delta)$ is the True. Likewise we say that the object Δ *falls under* the concept $\Phi(\xi)$ if $\Phi(\Delta)$ is the

True. (Frege 1893: 8)¹⁴

The logical relation relevant to $\langle \Phi(\Delta) \rangle$ is subsumption. Frege seems to be saying that the fact that the truth-value of $\langle \Phi(\Delta) \rangle$ is the True consists in the fact that the subsumption obtains between $\Phi(\xi)$ and Δ , and that the fact that the truth-value of $\langle \Psi(\Gamma, \Delta) \rangle$ is the True consists in the obtaining of the logical relation relevant to it. For every thought, there is always a logical relation relevant to it. Based on the above passage, the NE-theory reading maintains that Frege would be willing to say that the truth-value of a thought being the True consists in the obtaining of the relevant logical relation. Hence, (A₃).

By (A₁) and (A₃), $\langle p \rangle$ is true iff_A the logical relation relevant to $\langle p \rangle$ obtains. By (A₂), we have the main thesis of the NE-theory—the claim that truth is neither a concept nor an object (Greimann 2007: 147). Hence, there is no problem with attributing (A₁) to Frege. Given (A₂) and (A₃), (A₁) does not entail that truth is a property.

The NE-theory reading seems to fit the textual evidence we have seen in the last section. It can explain why Frege contends that truth is not a property by saying that truth is the obtaining of a logical relation that is not even a genuine entity. Because truth is not even an entity, it is indefinable as Frege argues. Finally, truth is still predicative because it is a tie among different entities.

However, I argue, Frege is not an NE-theorist of truth.

4.2.2. FREGE IS NOT A 'NO ENTITY' THEORIST

In particular I argue that Frege accepts neither (A₂) nor (A₃):

¹⁴ Greimann claims that 'Frege explains subsumption in terms of truth' (2007: 138). It is not certain that the passage really verifies Greimann's reading. Nor is it certain that Greimann needs this point for his main idea that, for Frege, deciding that a thought is true is just deciding that the logical relation relevant to the thought obtains.

(A₂) Logical relations are neither objects nor concepts.

(A₃) The truth-value of $\langle p \rangle$ is the True iff_A the logical relation relevant to $\langle p \rangle$ obtains.

Let us start with (A₂)

Recall that the NE-theory reading ascribes (A₂) to Frege by appealing to the following passage from ‘On Schoenflies’:

In the sentence ‘Two is a prime’ we find a certain relation designated: that of subsumption. We may also say the object falls under the concept *prime* [...]. This also creates the impression that the relation of subsumption is a third element supervenient upon the object and the concept. This isn’t the case: the unsaturatedness of the concept brings it about that the object, in effecting the saturation, engages immediately with the concept, without need of any special cement. Object and concept are fundamentally made for each other, and in subsumption we have their fundamental union. (Frege 1906c: 178)

The NE-theory reading, I believe, correctly points out that Frege argues that subsumption *qua* a kind of saturation is not a genuine relation. On top of that, the NE-theory reading also assumes that ‘subsumption’ means ‘the relation of *o falling under C*’ and concludes that the relation of *o falling under C* is not a genuine relation. I will contend that the assumption leads to a misinterpretation of this passage. Before that, however, I first flag a problem with the inference of the NE-theory reading.

I remind you of the point that saturating a function is merely providing an argument for the function. In ‘[Comments on Sense and Reference]’, Frege writes:

Accordingly I call the function itself unsaturated, or in need of supplementation, because its name has first to be completed with the sign of an argument if we are to obtain a reference that is completed in itself. I call such a reference an object and, in this case, the value of the function for the argument that effects the supplementing or *saturating*. (Frege 1892a: 119)

Frege's point is evident: A function is saturated if it is provided with an argument for its empty place. A saturated function yields an object as its value.¹⁵

The concept *prime* is saturated by the object 2, i.e., it takes 2 as argument and yields the True as value. Let us call the saturation of a concept by an object 'OC-saturation'. OC-saturation diverges from the relation of *o falling under C*. The concept *prime* is also saturated by 4; it also takes 4 as argument yielding the False as value in this case. However, 4 does not fall under the concept *prime*. Of course, the NE-theory reading is aware of this. That is why it says:

The relation that Frege calls "subsumption" is a special case of saturation. According to the system in *Begriffsschrift*, this relation holds between an object and a concept if the result of the concept's saturation by the object [(i.e., OC-saturation)] *is a fact* [...] (Greimann 2007: 130; Italics are mine.)

The NE-theory reading is taking a case of subsumption as that of OC-saturation where the relation *subsumption also* obtains between an object and a concept, i.e., where the object *also* falls under the concept.

In the second half of the passage from 'On Schoenflies', however, Frege appears to maintain that OC-saturation *in general* is not a genuine relation.

... the unsaturatedness of the concept brings it about that the object, in effecting the saturation, engages immediately with the concept, without need of any special cement. (Frege 1906c: 178)

Say that Frege's claim concerns OC-saturation in general. To directly proceed from it to the conclusion that the relation of subsumption or *o falling under C* is not a genuine relation, the NE-theory reading should make the following inference: 'Since a case of

¹⁵Also see 'Function and Concept' (1891: 24–5) and *Grundgesetze* (1893: 8). Saturation can also be explained in the level of sense. See Frege 1923. Here, saturating or completing is simply fulfilling the empty space of the sense of a concept.

OC-saturation where the relation of *o falling under C* also obtains is always a case of OC-saturation and OC-saturation is a pseudo relation, the relation of *o falling under C* is also a pseudo relation.’ This inference is invalid as the following inference is invalid: ‘Since purified water is always water and water is a natural kind, being purified is a natural kind.’

An immediate response would be that ‘the saturation’ in the previous quotation just means ‘subsumption’—‘OC-saturation where subsumption obtains between the object and the concept or where the object falls under the concept’. This response relies on the fundamental assumption of the NE-theory reading: What Frege refers to by ‘subsumption’ is the relation of *o falling under C*. Probably, the assumption would be based on the first two statements of the passage:

In the sentence ‘Two is a prime’ we find a certain relation designated: that of subsumption. We may also say the object falls under the concept *prime* [...]. (Frege 1906c: 178)

However, these statements must be read differently. A line-by-line examination of the passage would help to see this.

Let me present Frege’s second statement altogether with the omitted part:

We may also say the object falls under the concept *prime*, but if we do so, we must not forget the imprecision of linguistic expression we have just mentioned. (177)

Quoting this passage, I omitted the second half of this statement because the NE-theory reading does not explain what Frege means by ‘the imprecision of linguistic expression we have just mentioned’. However, it is extremely important to see what he means by that. Frege is pointing out a problem with the locution ‘2 falls under the concept *prime*’. The problem is explained in the previous passage.

[W]e can scarcely avoid using such expressions as 'the concept *prime*'. Here there is no trace left of unsaturatedness, of the predicative nature. Rather the expression is constructed in a way which precisely parallels 'the poet Schiller'. So language brands a concept as an object [...]. (177)

The problem is that the phrase 'the concept *prime*' leaves no empty place and hence is a proper name. Strictly speaking, the concept *prime* is an object. This is 'the paradox of the concept *horse*' in 'On Concept and Object' (Frege 1892b): The concept *horse* is not a concept. This must be what Frege refers to by 'the imprecision of linguistic expression we have just mentioned'. Given this problem, we can expect a further problem with the locution '2 falls under the concept *prime*'. It leads to the contention that the relation of subsumption is a relation between *two objects*: 2 and the concept *prime*. This is exactly what Frege is saying:

This also creates the impression that the relation of subsumption is a third element supervenient upon the object and the concept. (178)

However, Frege denies it:

This isn't the case [...]. (178)

Subsumption is not such a relation. But the relation of *o falling under C* is. The number 2 surely falls under the concept *prime*. This is a relation between the two objects: 2 and the concept *prime*. What Frege means by 'the relation of subsumption' cannot be 'the relation *x falling under y*'.

The problem Frege is dealing with, I believe, is a classical one. Say that the relation of subsumption obtains between two objects. Say we express this relation by 'x falls under y'. In the sentence '2 falls under the *concept prime*', we find a relation designated: the relation among 2, the concept *prime* and the relation of subsumption. We may also

say that the relation of subsumption obtains between 2 and the concept *prime*. Note that the relation of subsumption is again an object. This leads to the claim that there is the three-place relation of x obtaining between y and z that obtains between the relation of subsumption, the concept *prime* and 2. Now, in the sentence ‘the relation of subsumption obtains between 2 and the concept *prime*’, we find a relation designated: the relation among 2, the concept *prime*, the relation of subsumption and of three-place obtaining. This regress, an instance of what we often call ‘Bradley’s regress’ (Perovic 2017), is what Frege is getting at. His point is that if we take the relation we designate in ‘2 is prime’, i.e., subsumption, as a relation that obtains between two items, this regress is unstoppable. Subsumption must not be such a relation.

In the second half, Frege suggests an alternative understanding of subsumption:

This isn’t the case: the unsaturatedness of the concept brings it about that the object, in effecting the saturation, engages immediately with the concept, without need of any special cement. Object and concept are fundamentally made for each other, and in subsumption we have their fundamental union. (178)

The alternative is that subsumption is a kind of saturation: OC-saturation. Although Frege is just talking about OC-saturation, I believe that his point is perfectly general. Functions are unsaturated. Due to its nature, a function immediately saturates itself with an argument, yielding an object as the value for the argument. A function does so with each and every possible argument—without any discrimination. No special cement is hence necessary for saturation in general. Perhaps, this may be one reason why Frege maintains that a function should always be totally defined (Frege 1891: 33).¹⁶ OC-saturation is the case of a single object saturating an one-place first level concept, i.e., a concept that receives an object as an argument and yields a truth-value

¹⁶For the detailed discussion of this point, see Blanchette (2012) and Heck (2012).

as the value for the argument. What Frege is arguing is that subsumption is just OC-saturation, which is not a genuine relation. There are different kinds of saturation: The saturation of a two-place first level function by two objects in order, \dots , the saturation of an one-place second level function by a first level function, \dots . By the same token, all of them are not genuine relations.

Now, let us return to the opening of the passage.

In the sentence 'Two is a prime' we find a certain relation designated: that of subsumption. (Frege 1906c: 178)

Substituting 'Two is a prime' with 'Four is a prime' is fine; the same kind of relation is designated by the latter. The relation of subsumption is thus introduced *qua* the relation that is designated by the sentences of the form '*n* is prime'. Then, Frege says:

We may also say the object falls under the concept *prime* [...]. (Frege 1906c: 178)

What Frege is saying seems that the locution '*n* falls (or subsumes) under the concept *prime*' is our attempt to capture the relation that is designated by the sentences of the form '*n* is prime'. Given this locution, it is comprehensible to call such a relation 'subsumption'. Frege's point is that subsumption, *qua* the relation designated by the sentences of the form '*n* is prime', must be sharply distinguished from the relation of *n*'s falling under the concept *prime*. The former is just OC-saturation, while the latter is a two-place relation designated by the misleading locution '*n* falls under the concept *prime*'.

What Frege is doing seems to be clear. He is diagnosing that Bradley's regress is a pseudo problem that results from us failing to see the unsaturatedness of functions. If we understand the unsaturatedness of functions, we can also understand that what obtains between functions and their arguments is not a genuine relation. Therefore,

the regress does not exist. We fail to see this because natural language names functions in the way it names objects as it does in the locution '2 falls under the concept *prime*'. Of course, we cannot avoid that kind of locution in natural language (Frege 1906c: 177). Nonetheless, we need to be careful because the locution misfires. To establish that it misfires is the whole point of the passage.

Now, the ascription of (A₂):

(A₂) Logical relations are neither objects nor concepts

to Frege is unwarranted. That is not what he puts forward in the passage. What he is saying is that *saturation* is not a genuine relation. In the end, Frege has never said that 'logical relations' are not genuine entities.¹⁷

Note that the ascription of (A₃)

(A₃) The truth-value of $\langle p \rangle$ is the True iff_A the logical relation relevant to $\langle p \rangle$ obtains,

to Frege depends on this passage in *Grundgesetze*:

We say that an object Γ *stands in the relation* $\Psi(\xi, \zeta)$ *to* the object Δ if $\Psi(\Gamma, \Delta)$ is the True. Likewise we say that the object Δ *falls under* the concept $\Phi(\xi)$ if $\Phi(\Delta)$ is the True. (Frege 1893: 8)

The NE-theory reading takes it that Frege is providing a serious philosophical analysis—as serious as (A₃). This cannot be the case. It is not acceptable to analyze a precise notion by a notion that is not just imprecise but also fundamentally problematic. Frege would deny these claims if they were to be read as seriously as (A₃).

What Frege is doing here is not to put forward an instance of (A₃), but to provide the *natural language analogues* of the *Begriffsschrift* expressions like ' $\Psi(\xi, \zeta)$ ' or ' $\Phi(\Delta)$ ' in

¹⁷Frege even never coins the term 'logical relation'.

order to elucidate the logical forms of the thoughts expressed by those expressions. He needs these analogues to make his readers understand the meaning of these odd-looking strings of symbols.¹⁸ In that restricted sense, we can say, Frege is explaining the *Begriffsschrift* expressions. However, it cannot be the case that Frege is providing instances of (A₃), which is a philosophical analysis.

Thus, Frege accepts none of the main theses of the NE-theory of truth—because our discussion in §2.3 shows that there is no reason to think that he accepts (A₁). Therefore, it seems implausible to say that Frege is an NE-theorist of truth.

4.2.3. GREIMANN'S OBJECTIONS AGAINST THE OBJECT READING

Frege is not an advocate of the NE-theory of truth. Therefore, the NE-theory reading ought to be given up. Of course, Greimann's criticisms against the object reading still remain. In the remainder of this section, I show that his criticisms can be satisfactorily responded to.

Criticism 1. Frege does not mention truth values at all in 'Thought'.

I argue that this fact does not show that truth is not the True for Frege. He writes:

The *Bedeutung* of the word 'true' seems to be altogether *sui generis*. May we not be dealing here with something which cannot be called a property in the ordinary sense at all? In spite of this doubt I will begin by *expressing myself in accordance with ordinary usage*, as if truth were property, until some appropriate ways of speaking is found. (1918a: 328–9)

Frege is using the term 'property' instead of 'concept'; he is complying with ordinary terminology. This is because 'Thought' along with 'Negation' (1918b) and 'Compound Thought' (1923) is Frege's attempt to advertise his conception of logic to the people who

¹⁸There is a further reason why he is using this kind of locution, which is related to the previous subject-predicate analysis of content. For this reason, see Frege 1892a.

are not familiar with his formal system. Therefore, he is trying not to use any technical term like ‘concept’ here. Given this context, Frege’s explicit declaration that he would follow the ordinary usage of ‘true’ rather appears to show that he is uncomfortable with the ordinary usage according to which truth looks like a property. Also, the fact that he does not mention truth-values here seems to only mean that he does not have to depend on truth being the True in order to make his points in this paper.

In ‘Thought’, the points directly relevant to the nature of truth are centered around the first few pages (1918a: 325–9) of the paper. All the points in these passages seem to boil down to Frege’s thesis that truth cannot be a property. The rhetorical question, ‘May we not be dealing here with something which cannot be called a property in the ordinary sense at all?’, is found at the end of these passages. Furthermore, Frege here repeats the indefinability argument he gives in ‘Logic’ (1897). As we have seen, the main point of this argument is that truth is not a property. Now, if it is Frege’s aim to convince people of the point that truth is not a property, then he must not depend on truth being the True. If he were to maintain that truth is not a property because it is the True, nobody would buy into it; for it is a sloppy argument that truth is not a property because it is an object. Rather, Frege ought to establish that, whatever we may mean by ‘true’ or ‘truth’, it cannot be a property. If that is what Frege is doing in ‘Thought’, the fact that he does not mention truth values in ‘Thought’ does not show that he denies that truth is the True.

Criticism 2. Truth must be something that can be predicated.

This criticism is grounded in the following passage from ‘Thought’:

[...] [W]e cannot recognize a property of a thing without at the same time finding the thought *this thing has this property* to be true. So with every property of a thing

there is tied up a property of a thought, namely truth. (Frege 1918a: 328)

However, we should note that the fact that the truth-predication is ubiquitous is one of the reasons why Frege rhetorically asks: 'May we not be dealing with something which cannot be called a property in the ordinary sense at all?' (Frege 1918a: 328) If so, we cannot comprehend Frege's comment 'with every property of a thing there is tied up a property of a thought [i.e. the property of *truth*]' at face-value. Moreover, what Frege is pointing out by the ubiquity of truth-predication seems to be the identity between $\langle p \rangle$ and $\langle \langle p \rangle \text{ is true} \rangle$. We should recall that, from that point, Frege infers that truth cannot be a property. It is hence not warranted to infer from the previous passage that truth should be able to be predicated.

Criticism 3: Truth is indefinable while the True is definable.

Certainly, Frege stipulates that the True is the value range of the horizontal, i.e., $\hat{\epsilon}(-\epsilon)$. However, it is not evident that Frege is really giving a definition of the True there. In general, it is very difficult to understand what is going on in §10 of *Grundgesetze* (Heck 2012: 51). As Greimann points out, Frege is dealing with the referential indeterminacy of value-range terms, which is left by Frege's initial stipulation about value-range terms that ' $\hat{\epsilon}\Phi(\epsilon) = \hat{\alpha}\Psi(\alpha)$ ' is coreferential with ' $\hat{\alpha}\Phi(\alpha) = \Psi(\alpha)$ '. This stipulation gives us Basic Law V: $(\hat{\epsilon}f(\epsilon) = \hat{\alpha}g(\alpha)) = (\hat{\alpha}f(\alpha) = g(\alpha))$. The problem is that this stipulation for Law V is not sufficient to decide whether an object, which is not given by a value-range term, is identical with a value-range.¹⁹ But the True and the False are not given by value-range terms. Hence,

[...] the question whether one of the truth-values is a value-range cannot possibly

¹⁹This question sounds similar to the Caesar problem from *Grundlagen* (Frege 1884). Indeed, Frege's discussion around §10 is deeply related to the Caesar problem. See Heck (2012: Ch.3).

decided on the basis of ' $\hat{\epsilon}\Phi(\epsilon) = \hat{\alpha}\Psi(\alpha)$ ' having the same reference as ' $\neg\Phi(a) = \Psi(a)$ '. (Frege 1893: 17)

Based on this point, Frege establishes that 'ironically enough' (Heck 2012: 91),

without contradicting our equating ' $\hat{\epsilon}\Phi(\epsilon) = \hat{\alpha}\Psi(\alpha)$ ' with ' $\neg\Phi(a) = \Psi(a)$ ', it is always possible to determine that an arbitrary value-range be the True and another arbitrary value-range be the False. (Frege 1893: 17)

Frege's 'definition' of the True comes right after this:

Let us therefore stipulate that $\hat{\epsilon}(\text{--- } \epsilon)$ be the True and that $\hat{\epsilon}(\epsilon = (\neg a = a))$ be the False. (Frege 1893: 17)

Frege's 'definition' of the True hence comes as a *completely arbitrary* stipulation. It is by no means evident that this stipulation, coming out of nowhere, is really intended to be a genuine definition of the True.²⁰ It is possible that Frege makes this stipulation only for the sake of his argument that all the expressions of *Begriffsschrift* are referential. At the very least, it is controversial whether Frege takes this stipulation as a genuine definition of the True constituting a part of *Begriffsschrift*.

It rather seems that Frege distinguishes truth values from value-ranges. He quickly stops talking about value-ranges after he realizes that his attempted solution to Russell's paradox is not successful. However, Frege still talks about truth values after that. For instance, in '[Notes for Ludwig Darmstaedter]', he writes:

All sentences that express a true thought have the same [reference] and all sentences that express a false thought have the same [reference] (the True and the False) (Frege 1919: 255)

²⁰ In Frege's logicism, numbers are defined as value-ranges. To justify these definitions, Frege provides a long discussion about traditional attempts to define numbers and a detailed analysis of number statements. This can be one reason why it is hard to believe that this one-line stipulation is intended to be a genuine, serious, definition of the True. According to Blanchette (2012), Frege did have quite a sophisticated conception of conceptual analysis and this stipulation is far from Frege's conception of conceptual analysis.

At this point, Frege stops talking about value-ranges. However, he is still talking about truth-values. If so, it seems, Frege distinguishes truth-values from value-ranges.

Even if the stipulation in *Grundgesetze* is intended by Frege to be a definition of the True, however, Greimann's objection fails to threaten the object reading. Given the exegesis of Frege's indefinability argument, what he really argues when he says that truth is indefinable is that truth cannot be a property and hence cannot be defined *qua* a property. Saying that truth is indefinable *qua* a property is compatible with saying that truth is definable *qua* an object. Therefore, Frege's indefinability of truth does not go against the object reading in any event.

One might object that, presenting the indefinability argument in 'Thought', Frege does write:

Any other attempt to define truth also breaks down. (1918a: 327)

If this means that truth is not definable *qua* any kind of entity, the definability of the True shows that, for Frege, truth is not the True. But the above statement is immediately followed by this:

For in a definition certain characteristics would have to be specified. (1918a: 327)

Therefore, Frege is talking about definitions that ascribe characteristics to truth. As we've already seen, it is only a property that can have characteristics.²¹ The characteristic of a property *P* is the property constitutive of *P* that is specified in the definition of *P*. Thus, Frege is arguing here that a definition that ascribes characteristics to truth, i.e., a definition that takes truth as a property, must fail. In other words, he is arguing that truth is indefinable *qua* a property.²²

²¹Instead of 'characteristic', Künne uses 'mark'.

²²§4.2 is a modified version of my paper 'Frege's Conception of Truth: Two Readings' (Kim 2019) published in *Ergo*, which is an open-access journal.

CHAPTER 5

FREGE'S CONCEPTION OF JUDGMENT AND ASSERTION

In the previous chapters we have reviewed Frege's arguments that truth is an object and responded to the previous readings that deny that truth is an object for him. Now I turn to addressing various challenges my suggested reading ought to meet. The first challenge is to explain how we can understand Frege's conception of judgment as acknowledgment of truth if truth is the True. I answer this question via dealing with the puzzles that concern *Begriffsschrift's* device for judgment/assertion, i.e., '⊢'.

5.1. THE JUDGMENT-STROKE AND THE HORIZONTAL

All the well-formed sentences of Frege's *Begriffsschrift* (1879a, 1893) have the following form:¹

⊢ ξ

¹To avoid too much use of italicization, I do not italicize 'Begriffsschrift' anymore in this chapter.

As I've said, there are tensions related to the symbol '⊢', i.e., Begriffsschrift's device for asserting.² Let us start with in what sense it is a device for asserting.

In 'Function and Concept' Frege remarks:

[In my logic] '5 > 4' and '1 + 3 = 5' just give us expressions for truth-values, without making any assertion. This separation of the act from the subject-matter of judgment seems to be indispensable; for otherwise we could not express a mere supposition—the putting of a case without a simultaneous judgment as to its arising or not. We thus need a special sign in order to be able to assert something. To this end I make use of ['⊢'], so that, e.g., by writing

$$\vdash 2 + 3 = 5$$

we assert that 2+3 equals 5. (1891: 34)

If we write down '2+3=5', we do not yet perform the act of asserting in Begriffsschrift. We only express the 'subject-matter' of the act 'separated' from the act. We assert that 2+3 is 5 only when we put '⊢' in front of '2+3=5'. In this sense '⊢' is a *performative* for asserting.³

'⊢' is constituted by two signs—the vertical and the horizontal stroke. The horizontal stroke is called 'the content-stroke' in the early Begriffsschrift (Frege 1879a) and 'the horizontal' in the mature Begriffsschrift, i.e., the version in 'Function and Concept' (Frege 1891) and *Grundgesetze* (1893). The vertical stroke is called 'the judgment-stroke' in both versions. The judgment-stroke and the horizontal line play different roles. The sign '⊢' is thus a *compositional* performative.

²The shape of the symbol in the early version of Begriffsschrift (Frege 1879a) is slightly different from that in the mature version in *Grundgesetze* (1893). I follow the typesetting of the latter.

³Smith (2000, 2009) deepens the conception of performative that underlies '⊢'. But I do not introduce his view here because all I need is a performative in the sense Frege specifies in the above passage from 'Function and Concept'. Smith completely dismisses the point that the assertion sign '⊢' embodies an act of identification, namely, the act of identifying the True with the reference of a sentence. However, as we will see, that is the most important point about the assertion sign.

But the role of ' \vdash ' *qua* a performative for asserting entirely depends on the judgment-stroke. ' $\text{— } 2 + 3 = 5$ ' is a legitimate Begriffsschrift expression.⁴ However, we cannot make any assertion by writing it down. The assertion that 2+3 equals 5 is made only when we add the judgment-stroke to the left side of ' — '. It is because the judgment-stroke is a performative that ' \vdash ' is a performative:

... Only the judgment-stroke contains the assertion. (Frege 1893: §5)

However, the judgment-stroke cannot come alone. ' $\vdash 2 + 3 = 5$ ' is not even a legitimate expression. It ought to accompany the horizontal line to function as a performative for the act of assertion. If ' \vdash ' is a performative for asserting because the judgment-stroke is such a performative, why should the judgment-stroke come with the other sign? Why can't we make the assertion that 2+3 equals 5 by writing down ' $\vdash 2 + 3 = 5$ '?⁵

Here is how the early Begriffsschrift addresses this question. On the content-stroke, Frege writes:

Not every content can become a judgment by placing — before its symbol; for example, the idea 'house' cannot. We therefore distinguish between judgeable and unjudgeable contents.

The [content-stroke] [...] binds the symbols that follows it into a whole, and assertion, which is expressed by the means of [the judgment-stroke] at the left end of the [content-stroke], relates to this whole. [...] The content-stroke serves generally to relate any symbol to the whole formed by the symbols that follow the stroke. What follows the content stroke must always have a judgeable content. (Frege 1879a: §2)

⁴In the mature Begriffsschrift, the horizontal should be longer when it comes alone (Frege 1893: 9).

⁵Heck and Lycan 1979 is a pioneering work that examines this question. They seek an answer by focusing more on the logical/mathematical aspects of Frege's philosophy. Here, I am seeking an answer by focusing more on Frege's philosophical insights. I believe that Heck and Lycan's answer is compatible with my answer in this paper.

So the content-stroke plays two different roles. First, the content-stroke combines the ideas, expressed by the symbols that follow it, into a judgeable whole. For example, the content-stroke in ' $— 2 + 3 = 5$ ' combines the ideas expressed by '2', '3', '5', '+' and '=' into a judgeable whole—the proposition that 2+3 equals 5. Frege writes:

In [the case of ' $— A$ '], we paraphrase using the words 'the circumstance that' or 'the proposition that'. (Frege 1879a: §2)

Secondly, the content-stroke provides the judgeable whole as the content of a judgment when we attach it to the judgment-stroke. If we write down the judgment-stroke at the left end of ' $— 2 + 3 = 5$ ', we thereby perform the act of judgment with the proposition—constructed by the content stroke—that 2+3 is equal to 5. The judgment-stroke ought to be followed by the content-stroke because, were it not for the content-stroke, there would be no judgeable *whole* with which we can make a judgment by writing down the judgment-stroke.

However, this answer cannot be applied to the horizontal in the mature Begriffsschrift because the content-stroke and the horizontal are entirely different from each other.⁶ After *Grundlagen* (Frege 1884) is published, Frege makes the distinction between the sense and the reference of a term. As we've already seen, the horizontal in the mature Begriffsschrift is related to this distinction that is not introduced at the time of the early Begriffsschrift. Specifically, the horizontal refers to a concept. The concept referred to by it, or simply the 'H-concept', yields the True when the argument is the

⁶In *Grundgesetze* Frege writes: 'Earlier I called [the horizontal] the *content-stroke*, when I combined under the expression "judgeable content" that which I now have learnt to distinguish as truth-value and thought.' (1893: §5 fn.2). Thus, the change from the content-stroke to the horizontal is related to the development of Frege's understanding of content. Frege regards the change from the content-stroke to the horizontal as one of the 'consequences of a deep-reaching development of [his] logical views' (1893: X).

True and the False otherwise. So the following refers to the True:

$$— 2 + 2 = 4$$

The following refers to the False:

$$— 2 + 3 = 4$$

The following also refers to the False:

$$— 2$$

Thus, the H-concept is the concept *being (identical with) the True*. Given the difference between the content-stroke and the horizontal, the answer to the above question —‘If ‘┆’ is a performative for asserting because the judgment-stroke is such a performative, why should the judgment-stroke come with the other sign?’—in the mature Begriffsschrift must be different from the answer concerning the content-stroke. The tension I mentioned earlier concerns Frege’s answer to the question why the horizontal must follow the judgment-stroke.

Now, let us turn to Frege’s explanation of why the horizontal ought to follow the judgment-stroke. In *Grundgesetze* he writes:

Above it is already stated that within a mere equation no assertion is yet to be found; with ‘2+3=5’ only a truth-value is designated, without its being said which one of the two it is. (Frege 1893: §5)

Therefore, to write down ‘┆ 2 + 3 = 5’, i.e., to assert ‘2+3=5’, is to say which one of the

two truth-values $\|2+3=5\|$ is:⁷

To this end [of asserting] I make use of a vertical stroke at the left end of the horizontal, so that, e.g., by writing

$$\vdash 2 + 3 = 5$$

we assert that 2+3 equals 5. Thus here we are not just writing down a truth-value, as in

$$2 + 3 = 5,$$

but also at the same time *saying that it is the True*. (Frege 1891: 34; italics mine)

To assert '2+3=5' is to say that $\|2 + 3 = 5\|$ is the True. Here Frege appears to provide a general elucidation of assertion: asserting that p is constituted by *identifying* $\|p\|$ with the True. This elucidation suggests the following explanation of the judgment-stroke and the horizontal:

- (AI₁) The horizontal is the identity predicate necessary for the act of (verbally) identifying $\|p\|$ with the True, i.e., asserting that $\|p\|$ is the True;
- (AI₂) The judgment-stroke is the marker of assertoric force so that we assert that $\|p\|$ is the True by attaching it to the horizontal.

I call this explanation 'the AI (Assertive Identification) explanation'. Though the AI explanation seems to be plausible, it conflicts with a number of other aspects of Frege's mature framework. First, if the AI explanation is along the right lines, Begriffsschrift encounters a syntactic problem. Secondly, it goes against Frege's central thesis about

⁷Since sentences are names of truth-values for Frege, they can take name position in other sentences. 'Grass is green is the True' or 'Snow is purple is identical to there is a largest prime number' is difficult to parse, however, and arguably ungrammatical in ordinary English. I'm using the double bars to aid parsing. They mark where names of truth-values start and end within a sentence, but have no semantic significance. They are grouping devices, similar to parenthesis in arithmetic or formal logic. In particular, they do not denote a function.

assertion repeated in his mature works. Lastly, the AI explanation violates the lesson from Frege's indefinability argument—the lesson about how the vicious infinite regress is produced. §5.2 explains these tensions.

These tensions lead us to serious doubts against the AI explanation. But can there be an alternative if Frege takes an assertion to be such an identification? The aim of this chapter is to show that there is indeed an alternative. This alternative is based on a non-assertive conception of identification according to which an identification is not an identity assertion but a *non-assertive identification*. If an assertion is a non-assertive identification and '⊢' is a device for a non-assertive identification, then all the tensions we will see in §5.2 are completely explained away. The chapter also aims to provide the evidence that Frege endorses such a non-assertive conception of identification when he elucidates an assertion as an identification. In our discussion we come to see how Frege's conception of judging as acknowledging the truth of a thought is modified under his objectual conception of truth. In particular, judging that p as such is equivalent to identifying the True with $\|p\|$ *non-judgmentally*.

After explaining the tensions with '⊢' in §5.2, I respond in §5.3 to the objection—grounded in a popular understanding of '⊢'—that to take an assertion to be constituted by an identification is only Frege's provisional measure for a certain technical issue in *Begriffsschrift*. I respond to this objection by establishing that Frege's elucidation of asserting as identifying is related to his conception of judging that is also elucidated as identifying in 'On Sense and Reference'. §5.4 presents my interpretation of '⊢' as a performative for non-assertive identification.

5.2. THE TENSIONS

5.2.1. SYNTACTIC TENSION IN BEGRIFFSSCHRIFT

Typically, in logical systems, sentential well-formedness is preserved under the substitution of an expression included in a sentence with an expression of the same type. Assume ‘ Px ’ and ‘ Qx ’ are predicates and ‘ a ’ a singular term in a logical system. Say its syntactic rules imply that ‘ Pa ’ is a well-formed sentence. Then, the logical system dictates that the result of substituting ‘ Px ’ with ‘ Qx ’ in ‘ Pa ’, i.e., ‘ Qa ’ is also a well-formed sentence because ‘ Qx ’ is also a predicate. Let us call this feature of a logical system ‘syntactic substitutivity’.

According to Taschek (2008: 394), there is a reason why Begriffsschrift ought to have syntactic substitutivity. For Frege, logic ought to display correct logical structures of thoughts. This partly means that logic ought to display the correct *inferential roles* of different terms clearly. Frege’s classification of expressions into names and different types of functors is for exhibiting their inferential roles. Therefore, ‘[t]o assume that two distinct expressions are of the same logical type is to assume that there is no inferentially relevant difference between them [...]’ (Taschek 2008: 394), whatever their specific references may be. Therefore, it is a reasonable expectation that Begriffsschrift would have syntactic substitutivity. It satisfies this expectation in most cases.

However, the horizontal attached to the judgment-stroke is a plain exception to syntactic substitutivity *if* the AI explanation of ‘ \vdash ’ is correct. According to the explanation the horizontal in the sign is still a predicate, specifically a first-level one-place concept term.⁸ Suppose that ‘ $\Phi(\xi)$ ’ is a first-level functor and ‘ Δ ’ is a name of an object

⁸The horizontal is a first-level functor because the H-concept maps an object to an object. A second-level functor maps a first-level functor to an object and a third-level functor maps a second-level functor

that satisfies $\Phi(\xi)$. Then,

$$(1) \vdash \Phi(\Delta)$$

is a well-formed sentence. If the horizontal in (1) is indeed a predicate and syntactic substitutivity is a strict rule of Begriffsschrift, the substitution of the horizontal in (1) with ' $\Phi(\xi)$ ' is also supposed to yield a Begriffsschrift sentence. However,

$$\vdash \Phi(\Phi(\Delta))$$

is not well-formed. The horizontal in ' \vdash ' is thus an exception to syntactic substitutivity *if* the AI explanation is correct.

5.2.2. THE TENSION WITH FREGE'S CONCEPTION OF ASSERTION

One of the points Frege repeatedly makes about assertion is that we ought to dissociate the act of assertion and assertoric force from predicates.⁹

Assertoric force which is often connected with the copula or else with the grammatical predicate, does not belong to the expressions of the thought ... (1906c: 177).

In order to put something forward as true, we do not need a special predicate: we only need the assertoric force with which the sentence is uttered. (1914: 233)

We can find similar claims in a number of Frege's mature works—as we have seen in Chapter 2.

According to the AI explanation, the judgment-stroke marks assertoric force and the horizontal is the predicate to which assertoric force is attached. It is based on the thought that identifying $\|p\|$ with the True is attaching assertoric force to ' \dots is the

to an object and so on.

⁹See p.184 of Frege 1979.

True' in ' $\|p\|$ is the True'. Say Frege indeed believes that

(A-Id) Asserting that p is *constituted* by identifying $\|p\|$ with the True.

If Frege also endorses the AI explanation, 'identifying $\|p\|$ with the True' in (A-Id) must refer to the act of adding assertoric force to ' \dots is the True' in ' $\|p\|$ is the True'. If asserting that p is constituted by such an act, we rather need the identity predicate in order to assert. But that goes against Frege's own claim that we do not need a special predicate for asserting. Furthermore, if asserting is always adding assertoric force to ' \dots is the True', assertoric force is monopolized by the identity predicate. Begriffsschrift makes this implication very explicit if the judgment-stroke marks assertoric force as the AI explanation states. Recall that ' $\mid \Phi(\Phi(\Delta))$ ' is not a Begriffsschrift sentence. Writing it down is not asserting even though we have a predicate and the marker of assertoric force. There rather seems to be a significant sense in which assertoric force belongs to the horizontal. But again, that conflicts with Frege's own claim about assertion.

5.2.3. THE TENSION WITH FREGE'S INDEFINABILITY ARGUMENT

Arguably, the AI explanation of ' \mid ' conflicts with what Frege attempts to establish by his indefinability argument. In my suggested interpretation the gist of the indefinability argument is that if truth is a property of thoughts and Frege's conception of judgment is correct, a vicious infinite regress is produced. To recall how this infinite regress is produced, consider the following three types of actions again:

- (A₁) Acknowledging the P -ness of o ,
- (A₂) Judging that o is P , and
- (A₃) Acknowledging the truth of $\langle o$ is $P \rangle$.

In the Fregean conception of judgment (A₁) is equivalent to (A₂) and we perform (A₂) by (A₃). Now, if truth is a property of $\langle p \rangle$, judging that p , i.e., acknowledging the truth

of $\langle p \rangle$ is a case of (A₁). Thus, one can judge that p only by acknowledging the truth of $\langle \langle p \rangle \text{ is true} \rangle$, i.e., judging that $\langle p \rangle$ is true. Now to judge that $\langle p \rangle$ is true, I need to judge that $\langle \langle p \rangle \text{ is true} \rangle$ is true. This infinite regress continues forever.

We can re-describe this lesson of the indefinability argument in terms of predication. Although Frege only has the weak conception of predication under which to predicate is to form or grasp a thought, we can define predication in the strong sense under his framework: to strongly predicate a property P of an object o is to be committed to the truth of $\langle o \text{ is } P \rangle$, i.e., to acknowledge the truth of $\langle o \text{ is } P \rangle$. Let us assume that we judge that p by strongly predicating a property P of $\langle p \rangle$. Then, the latter is equivalent to acknowledging the truth of $\langle \langle p \rangle \text{ is } P \rangle$, i.e., judging that $\langle p \rangle$ is P . Thus the infinite regress is produced; for judging that $\langle p \rangle$ is P is judging that $\langle \langle p \rangle \text{ is } P \rangle$ is P . In this reconstruction the lesson from the indefinability argument can be broken down into two parts. First, if an act of judging is a committal act of predicating, an infinite regress is produced. Secondly, if truth is a property of thoughts and Frege's conception of judgment is correct, an act of judging comes to be a committal act of predicating.

The regress reading seems to be at least an arguable exegetical position.¹⁰ The issue is that the AI explanation has a result that can easily be expected given the first part of the lesson from the indefinability argument. Recall: Frege seems to endorse that

(A-Id) Asserting that p is *constituted* by identifying $\|p\|$ with the True.

The AI reading basically considers the act of identifying $\|p\|$ with the True to be the committal act of predicating the property *being identical with the True* of $\|p\|$ —where the horizontal stands for the relevant property and the judgment-stroke stands for a

¹⁰Whether or not the above infinite regress is the absurdity to which the indefinability argument appeals, Frege's conception of judgment produces this regress if truth is a property. Thus, in any event, this infinite regress is a reason why Frege must avoid the AI explanation of '⊢' if he takes assertion to be constituted by identification.

commitment by marking assertoric force. That is why to write down ‘ $\vdash p$ ’ is to *assert* that $\|p\|$ is the True under the AI explanation. If identifying $\|p\|$ with the True in (A-Id) is predicating *being the True* of $\|p\|$, Frege is committed to the following: asserting that p is constituted by asserting that $\|p\|$ is the True. However, asserting that $\|p\|$ is the True is constituted by asserting that $\|\|p\|$ is the True $\|$ is the True. The regress will continue endlessly. In the indefinability argument, Frege warns us of the infinite regress that we come to have by making the act of judgment constituted by that of (mental) committal predication. Nevertheless, he is committed to the same kind of regress by endorsing the AI explanation, i.e., by making the act of assertion constituted by that of (verbal) committal predication.

Let me address two objections quickly. One might object that the regress from the AI explanation is not vicious because ‘ \dots is the True’ or the horizontal is a logical analogue of ‘true’ in English. The point of this objection is that because $\langle p \rangle$ is identical with $\langle \langle p \rangle$ is true \rangle for Frege, $\langle \|p\|$ is the True \rangle is also identical with $\langle p \rangle$. As we have seen, however, when Frege is claiming that ‘true’ is semantically inert, what he claims is that ‘true’ does not refer to any property. In other words, if ‘true’ were to refer to a property, it would not be semantically inert. But the horizontal or ‘ \dots is the True’ refers to a legitimate property, i.e., the property *being the True*. There is further evidence that these identity predicates are not semantically inert: identity predicates in general cannot be semantically inert for Frege. His explanation of the difference in cognitive value between ‘ $a = a$ ’ and ‘ $a = b$ ’ given in ‘On Sense and Reference’ (1892c) can be taken to be grounded in the idea that $\langle x = a \rangle$ and $\langle x = b \rangle$ are different. This idea presupposes that identity predicates such as the horizontal or ‘ x is the True’ have substantive senses that can be differentiated. So identity predicates are not semantically inert like ‘true’

in English. Hence, $\langle p \rangle$ cannot be identical with $\langle ||p|| \text{ is the True} \rangle$.

It might be also objected that the infinite regress in this case is not vicious because predicating the property *being the True* of $||p||$ is identical with predicating it of $|||p|| \text{ is the True}||$ because $||p||$ is just $|||p|| \text{ is the True}||$, i.e., the True. But avoiding the viciousness of this infinite regress in this way is not plausible. This is because by the logic of this objection we must accept that predicating the identity of $||2=2||$ is predicating it of $||2+3=5||$, i.e., that asserting that 2 is 2 is identical with asserting that 2+3 is 5. We would rather consider ' $||p||$ ' in 'predicating the property *being the True* of $||p||$ ' to be like 'Bruce' in 'I believe Bruce is the Batman'. But then, the infinite regress is vicious.

The infinite regress we have from the AI explanation seems vicious. Or, at least, Frege ought to take it to be vicious. If our interpretation of the indefinability argument is correct and if Frege adopts both (A-Id) and the AI explanation, he is committing himself to the very infinite regress which he warns us of in the indefinability argument.

5.3. IS (A-ID) FREGE'S ELUCIDATION OF ASSERTION?

Let us take stock here. The AI explanation of ' \vdash ' is problematic in various ways. First, it makes the horizontal attached to the judgment-stroke a plain counterexample to syntactic substitutivity of Begriffsschrift. Secondly, if (A-Id)

(A-Id) Asserting that p is *constituted* by identifying $||p||$ with the True.

is Frege's elucidation of assertion, the AI explanation conflicts with his repeated comments on the nature of assertion, and with the insight the indefinability argument is arguably attempting to establish. As I have said, I consider these tensions to be the evidence against the AI explanation.

However, there is an objection to the claim that we should give up the AI explanation due to these tensions. According to this objection, (A-Id) is not Frege's *philosophical*

elucidation of assertion. It insists that the AI explanation is only a provisional measure for a technical issue in Begriffsschrift and that (A-Id) is at most a 'background story' for this provisional measure. The technical issue is related to syntactic substitutivity. See this well-formed sentence:

$$\vdash 2 + 2 = 4$$

'2+2=4' is a singular term. If Begriffsschrift has syntactic substitutivity, the result of substituting '2+2=4' with any singular term should also be a sentence. The smooth-breathing ' $\dot{\epsilon}$ ' refers to a (second-level) function that maps a (first-level) function to its *value-range*. The value-range of a function is an object. For instance, ' $\dot{\epsilon}(\text{--- } \epsilon)$ ' refers to the value-range of the H-concept.¹¹ If we substitute '2+2=4' with ' $\dot{\epsilon}(\text{--- } \epsilon)$ ' in the above sentence of Begriffsschrift, we have:

$$\vdash \dot{\epsilon}(\text{--- } \epsilon)$$

This is indeed a well-formed sentence in Begriffsschrift.

Say the following is a Begriffsschrift sentence:

$$\mid 2 + 2 = 4$$

It seems fine to say that, by writing this down, we assert that 2+2 is equal to 4. Now, recall that sentences are singular terms for Frege. Given the syntactic substitutivity of

¹¹In §11, Frege stipulates that $\dot{\epsilon}(\text{--- } \epsilon)$ is the True after he establishes that it is possible to stipulate that a truth-value is an arbitrary value-range. See Heck (2012: Ch.3) which provides an excellent explanation about why Frege makes this stipulation. I here use this particular non-sentential name ' $\dot{\epsilon}(\text{--- } \epsilon)$ ' to bypass the alleged factivity of judgment in Frege. The factivity of judgment does not matter for our discussion.

Begriffsschrift, the following would also have to be a well-formed sentence:

$$| \dot{\varepsilon}(\text{--- } \varepsilon)$$

It is not clear what kind of assertion we make here. It does not seem to make any sense to say 'I assert that the value-range of the H-concept'. We can make an assertion only with the sense of a *sentence*, i.e., a thought, but not with the sense of a non-sentential name like ' $\dot{\varepsilon}(\text{--- } \varepsilon)$ '.

The horizontal attached to the judgment-stroke can prevent this difficulty. Writing down the Begriffsschrift sentence

$$\vdash \dot{\varepsilon}(\text{--- } \varepsilon)$$

can be regarded as asserting that $\dot{\varepsilon}(\text{--- } \varepsilon)$ is the True where the predicate 'is the Truth' given by the first horizontal. Because ' $\text{--- } \dot{\varepsilon}(\text{--- } \varepsilon)$ ' is a sentence in an intuitive sense, there seems to be no problem with saying that writing the above sentence of Begriffsschrift is asserting. According to scholars like Taschek (2008), the horizontal attached to the judgment-stroke works as a *sentence generator* in that ' $\text{--- } \xi$ ' is always a sentence of the form ' ξ is the True' whatever may come into the place of ' ξ '. Even if we substitute a non-sentential name like ' $\dot{\varepsilon}(\text{--- } \varepsilon)$ ' with ' ξ ' in ' $\vdash \xi$ ', the horizontal ensures that what follows the judgment-stroke is a sentence. The horizontal attached to the judgment-stroke solves the potential difficulty with taking sentences as singular terms.¹²

Recall (A-Id):

(A-Id) Asserting that p is *constituted* by identifying $\|p\|$ with the True.

Taking 'identifying $\|p\|$ with the True' to mean 'asserting that $\|p\|$ is the True' as the AI

¹²Dummett (1993: 315) also writes that the horizontal 'in effect turns any singular term into a sentence.'

explanation implies, the objection under consideration argues that Frege endorses (A-Id) merely to resolve the technical issue we have seen, i.e., to introduce the horizontal as a sentence-generator. Then, (A-Id) ought to be regarded as a suggestion like this: '*In Begriffsschrift* let us pretend that (A-Id) is true so that we can avoid the technical issue'. The gist of this objection is that such a *provisional* measure does not have to be in tension with Frege's *philosophical* claims. Surely, the tension between the horizontal in '⊥' and syntactic substitutivity remains unresolved. Still, allowing a few counterexamples to syntactic substitutivity appears to be a small price to pay if, by doing so, Frege can retain his conception of sentences as names and the undeniable fact that asserting is always asserting a sentence.

I concede that the suggested reading is an open option. Nonetheless, I am reluctant to accept it. One reason is that Frege never says that (A-Id) and the AI explanation are provisional measures for logic although it is reasonable in this case to expect that he would do so. For him the nature of assertion is important to understanding logic. That is why Frege makes a number of comments on it. Now, the endorsement of (A-Id) is a big decision on the nature of assertion; it makes assertion fall under the kind of activity we can perform with a non-sentential name, i.e., identification. Given the importance of assertion for Frege, if (A-Id) and the AI explanation were just provisional and did not reflect his thoughts about its nature, he would flag it explicitly. But Frege never does so.

More crucially, there is strong evidence that (A-Id) is his philosophical elucidation of assertion. This evidence is found in the relation of asserting to judging in Frege. He writes:

- (2) The acknowledgement of the truth of a thought - the *act* of judgment

(3) The manifestation of *this* judgment - assertion (1918a: 329; italics mine)

Frege also says that an assertion 'communicates' (1897: 139) or 'gives an expression to' (1906a: 185) an act of judgment.¹³ I attempt to establish that this relation holds for Frege because for him the act of judging that p is that of (mentally) identifying $\|p\|$ with the True. If so, then there is a reason to believe that (A-Id) is not a mere provisional measure for Begriffsschrift but Frege's philosophical claim based on his understanding of the relationship between asserting and judging.

In 'On Sense and Reference', Frege writes:

But so much should already be clear, that in every judgment, no matter how trivial, the step from the level of thoughts to the level of reference (the objective) has already been taken. (Frege 1892c: 64)

Judgments can be regarded as advances from a thought to [its] truth value. (65)¹⁴

According to this elucidation, to judge that p is to take a step or advance from $\langle p \rangle$ to its reference. This elucidation of judgment is metaphorical. One observation is that he does not try to explain this metaphor further—as if he thinks that sufficient hints on it are already provided. Indeed, I believe, he leaves hints on this metaphor in the earlier parts of the paper.

The first hint is that we can take the same kind of step even with a non-sentential name:

... it is doubtful whether the name 'Odysseus' ... has reference Whoever does not admit the name has reference can neither apply nor withhold the predi-

¹³I acknowledge that in Frege the act of judging ought to be distinguished from the epistemic status we assign to a thought by judging. The distinction is important to understanding Frege's anti-psychologism of logic. As 'Thought' clearly shows, what is manifested by an assertion is not a judgment as a special epistemic status, but a judgment as an act. In the following discussion, 'judgment' uniquely means 'the act of judging'.

¹⁴Frege 1970 uses 'a' for the translation instead of 'its'. The original word is '*seinem*' that means 'its'.

cate. But in that case it would be superfluous to *advance to the reference of the name*. (1892c: 62; italics mine)

It is the striving for truth that drives us always to *advance from the sense to the reference* [of a name such as 'Odysseus']. (63; italics mine)

So Frege takes the act of judging to fall under the category of the act we can perform with not only sentences but also non-sentential names. This observation already points to the possibility that Frege considers the act of judgment to be that of identification which we can *also* perform with *non-sentential names*.

Let us go further. Frege states that there is no point in taking a step from the sense of a name to its reference if we believe that the name does not have a reference. This statement is related to the second hint on the metaphor 'taking a step', which concerns the epistemic gap between the sense of a name and its reference. After providing his explanation of the difference in cognitive value between ' $a = a$ ' and ' $a = b$ ' based on his sense-reference distinction, Frege writes:

Comprehensive knowledge of the reference would require us to be able to say immediately whether any given sense belongs to it. To such knowledge we never attain. (Frege 1892c: 58)

Frege is arguing that it is necessary for the comprehensive knowledge of an object to have the *ability* to decide whether any sense given to us belongs to the object. He appears to conclude that we never have such knowledge because this condition has never been met. Then, this implies that the knowledge of the sense of a name does not immediately lead to the knowledge of its reference. For, if the former directly led to the latter, we could say if any sense *given* to us belongs to a certain object. The sense of a name does not even inform whether the name has a reference:

In grasping a sense, one is not certainly assured of a reference. (58)

We can comprehend $\langle \text{Odysseus} \rangle$ in 'Odysseus fought in the Trojan war'; but we do not know if the name has a reference. In these senses, there is an epistemic gap between a name's sense and its reference. Taking a step from a name's sense to its reference is related to this epistemic gap

Let us check what the gap between a name's sense and its reference involves. Let 'N' be an arbitrary name whose sense is grasped by us. First, $\langle \text{N} \rangle$ alone does not tell us whether

(Gap₁) 'N' has a reference.

Let 'N_a' be any name such that we already grasp its sense $\langle \text{N}_a \rangle$ and recognize the existence of its reference. Say $\langle \text{N} \rangle$ differs from $\langle \text{N}_a \rangle$. $\langle \text{N} \rangle$ alone does not say whether

(Gap₂) $\langle \text{N} \rangle$ belongs to N_a.

Now, what does the act of 'saying' or deciding whether $\langle \text{N} \rangle$ belongs to N_a amount to? To answer, we ought to take it into account that Frege is putting forward the epistemic gap between a name's sense and its reference right after he puts forward the distinction between sense and reference to explain why the cognitive value of ' $a = b$ ' is different from that of ' $a = a$ '. The epistemic gap between sense and reference constitutes his explanation of how the difference between $\langle a \rangle$ and $\langle b \rangle$ yields the cognitive difference between ' $a = a$ ' and ' $a = b$ '. In the case of ' $a = a$ ', as soon as we grasp $\langle a \rangle$, we come to know that ' $a = a$ ' is true if ' a ' has a reference. We cannot do so in the case of ' $a = b$ '. Say we grasp $\langle b \rangle$ and recognize the existence of b . Even if we grasp $\langle a \rangle$, we cannot know whether ' $a = b$ ' is true if ' a ' has a reference, because $\langle a \rangle$ does not tell us whether it belongs to b . Only by additional investigations one can decide whether $\langle a \rangle$ belongs to b and eventually whether a is b . Given this reading, we can say, deciding whether $\langle a \rangle$ belongs to b must be an act of producing the knowledge that ' $a = b$ ' is true (or false).

In general, deciding whether (Gap₂) is true amounts to deciding whether

(Gap₂*) N is identical with N_a.

Note that trying to decide if (Gap₂*) is the case has no point if we deny (Gap₁)—just as taking a step from ⟨Odysseus⟩ to its reference is ‘superfluous’ if ‘we do not admit the name “Odysseus” has reference’ (Frege 1892c: 62). Then, taking a step from the sense of ‘N’ to its reference must be deciding whether (Gap₂*) is the case for a certain ‘N_a’.

Judging that *p* is constituted by taking a step from ⟨*p*⟩ to its truth-value. The latter is deciding whether ⟨*p*⟩ belongs to a truth-value, specifically, the True. In particular, it is deciding *that* ⟨*p*⟩ belongs to the True. To decide that ⟨*p*⟩ belongs to the True is to decide that $\|p\|$ is the True, i.e., identifying $\|p\|$ with the True. Judging that *p* is thus constituted by identifying $\|p\|$ with the True. Given Frege’s endorsement of (A-Id)—the claim that asserting that *p* is *constituted* by identifying $\|p\|$ with the True—we can now explain why Frege claims that an act of assertion to ‘communicate’, ‘give an expression to’, or ‘manifest’ an act of judgment. An assertion *qua* a *verbal* identification is a manifested, expressed, and communicated form of the identification that we *mentally* perform in our act of judgment. Now, ‘⊢’ *qua* an assertion device makes this relationship *explicit* by *representing* the act of judging that *p* *qua* that of identifying $\|p\|$ with the True.¹⁵ If so, (A-Id) does not look like a mere provisional measure for Begriffsschrift. It is rather tightly connected to Frege’s conception of judging that is also tightly connected to his distinction between sense and reference and his understanding of the cognitive value of identity sentences.

¹⁵Frege takes ‘⊢’ to be a ‘Begriffsschrift representation of a judgment’ (1893: §5). One might wonder if ‘judgment’ here means a judgment *qua* an epistemic status. See Footnote 19. First, it is possible for Frege to mean both the act of judging and the epistemic status. Hence, choosing one reading is not necessarily rejecting the other reading. Secondly, Frege calls ‘⊢’ a representation of a judgment right after he says a judgment is the acknowledgment of the truth of a thought that is taken by him to be ‘the act of judgment’ (1918a: 329).

The suggested reading of the relationship between assertion and judgment in Frege has an additional exegetical merit. Specifically it can explain away an apparent tension between assertion, judgment, and lying.

In 'A lied in saying he had seen B,' the subordinate clause designates a thought which is said (1) to have been asserted by A (2) while A was convinced of its falsity. (Frege 1892c: 66fn)

If a liar assertively says '57+68 is 122' to deceive a child who has not learned to do addition with numbers larger than 5, what the liar is doing is asserting that 57+68 is 122. This seems to conflict with Frege's claim that assertion manifests judgment. The liar does not manifest the judgment that 57+68 is 122. It is precisely because the liar never judges that 57+68 is 122 that the liar is a liar. But this tension can be explained away now. Assertion is manifested identification one can perform in one's judgment. In that sense assertion manifests judgment. If it is in that sense that assertion manifests judgment, the fact that one asserts that p does not entail that one judges that p .

If my reading of the relationship between judgment and assertion in Frege is along the right lines, the tensions in §5.2 cannot be explained away by saying that (A-Id) is not Frege's genuine elucidation of assertion. The possibility we need to consider now is that the AI explanation is not how Frege implements (A-Id) in Begriffsschrift.

5.4. ASSERTION AS NON-ASSERTIVE IDENTIFICATION

The AI explanation leads to the tensions we have seen because it takes the act of identifying $\|p\|$ with the True to be that of *asserting* that $\|p\|$ is the True, which is equivalent to attaching assertoric force to '... is the True' (the tension with Frege's remarks on assertion) or predicating *being the True* of $\|p\|$ (the tension with the indefinability argument). What Frege needs in order to avoid these tensions is an alternative to *assertive*

identification, i.e., *non-assertive* identification.

Consider the speaker-reference in the following circumstance. Say I am talking to my friend Tom in front of my department building. Tom and I see a person walking toward us. She is my other friend Maggie. But I misidentify her as Mary who is also my friend and say to Tom ‘Mary is coming here’. I am speaker-referring to a person by ‘Mary’. But whom am I speaker-referring to? The person I am intending to talk about by uttering this sentence is Maggie who is actually coming toward the department building. If so, then it appears to be plausible that I am speaker-referring to Maggie by ‘Mary’. Nevertheless, saying only this fails to capture a crucial aspect of the situation: Because I am using the sentence literally, my intention is, at least partially, to talk about Mary. Therefore, I am intending to talk about Maggie *qua* Mary. The best way to capture the situation seems to be saying this: (SI) I am speaker-referring to Maggie by ‘Mary’ and *thereby* (mistakenly) identifying Maggie with Mary. Then, I am intending my assertion to concern both Mary and Maggie. If we buy into (SI), then we regard my speaker-reference to Maggie by ‘Mary’ as a verbal identification of Mary with Maggie. Nevertheless, it appears implausible that I am asserting that Maggie is Mary by making this speaker-reference.¹⁶ By assertively uttering ‘Mary is coming here’, I make only one assertion. My speaker-reference is only a proper part of it. My speaker-reference seems to be a non-assertive identification if it is indeed an identification.

But do not get me wrong: I am not advocating a conception of speaker-reference, but only exploring the possibility of non-assertive identification. This speculation does have a predecessor: Ruth Millikan (1998, 2000, 2017). She suggests a non-*judgmental*

¹⁶Scholars like Heck (2012) and Textor (2010) even claim that, for Frege, to acknowledge the truth of $\langle p \rangle$ is to ‘refer to the True via $\langle p \rangle$ ’. Although they do not specify what kind of reference they mean by ‘refer’ there, it must be speaker-reference.

conception of identification. According to Millikan, to identify an object is to use our *substance* concept of the object, which is an ability or know-how to re-identify the object (2000: 9). Specifically, the substance concept of an object is an ability to re-identify the object by various media like perceptions or language. Detailing the notion of substance concept goes beyond the scope of this chapter. The point is that re-identifying an object by the substance concept of it is not making an identity judgment. Millikan writes:

... You don't make claims when you stand up to walk just because it's possible you could trip and fall. Similarly, you don't make claims when you develop substance concepts or when you mark identities in thought. Erroneous identification is not failure on the level of know-that but failure on the level of know-how. ... There is no such thing as an identity judgment. ... Grasping an identity is not remotely like harboring an intentional attitude [i.e., making a judgment]. Similarly, mistaking an identity is not harboring a false belief [i.e., making a false judgment]. (2000: §12.7)

In Millikan (2000: §1.7)'s framework, my speaker-reference to Maggie by 'Mary' can be considered to be an attempt to re-identify Mary verbally, i.e., a *verbal* application of my substance concept of Mary. My identification goes wrong because my substance concepts of Mary and of Maggie are not strong enough to enable me to identify Mary in the given situation. Therefore, my speaker-reference is a non-judgmental identification performed verbally, which seems to amount to a non-assertive identification. Thus, in this theory, we come to have the notion of a non-assertive identification *qua* a verbal use of a substance concept of an object. It is not my intention to defend Millikan's theory of identification. However, her discussion shows that it is arguably possible to develop the notions of non-judgmental identification and of non-assertive identification.

I take it to be defensible that we can develop the notion of non-assertive identification. What is important is that if we apply the notion of non-assertive identification to

'⊢', we can develop an entirely different interpretation of it while retaining the thought that it is a performative for identification. Consider the following:

$$(2) \text{ — } 2 + 3 = 5$$

The sense of (2) is an identity thought: $\langle ||2+3=5|| \text{ is the True} \rangle$ where the sense of the horizontal constitutes a part of this identity thought. Now, the AI explanation takes the judgment-stroke in

$$(3) \text{ ⊢ } 2 + 3 = 5$$

to attach assertoric force to the horizontal *qua* a predicate of the identity sentence (2). This is how the AI explanation makes '⊢' a device for *predicative* identification that is equivalent to *assertive* identification. Instead, let us say the judgment-stroke in (3)

(i) cancels the effect of the horizontal *qua* a predicate, i.e., constructing the identity sentence (2)

and thereby (ii) turns the horizontal into a performative for identifying $||2+3=5||$ with the True *non-predicatively* and thus *non-assertively*.

Call such a performative 'T-identifier'. If '⊢' is the T-identifier, to write down (3) is to identify $||2+3=5||$ with the True without predicating the property *being the True* of $||p||$ and thus without asserting that $||2+3=5||$ is the True. The horizontal alone is just an identity predicate, and hence it is not a performative. It is (a part of) the performative only when it is attached to the judgment stroke. So the judgment-stroke constructs a performative out of an identity predicate, of the form ' $x = O$ ', connected to it. The *act* of assertion is contained only in the judgment-stroke because the judgment-stroke is a *performative-forming* operator. The performative we acquire through combining

the judgment-stroke and ' $x = O$ ' is an O -identifier. Logic requires the T-identifier; for assertion is the logically fundamental activity and it is constituted by non-assertive identification of *the True*.¹⁷ Hence, the judgment-stroke ought to be followed by the identity predicate ' x is the True', i.e., the horizontal.

If this interpretation of ' \vdash ' is correct, then the potential tensions which we have seen in §2 *entirely* disappear. First, the horizontal attached to the judgment-stroke is not a counterexample to syntactic substitutivity because it is not a functor at all. Through the judgment-stroke attached to it, it is turned into the T-identifier, which is purely a performative. Geach (Anscombe and Geach 1961) is almost correct in looking upon ' \vdash ' as a primitive sign; for it is primitive *qua* a performative in that, without either of the judgment-stroke and the horizontal, it cannot be a performative. What Geach is missing is a rationale for its compositionality, which is provided by our reading. Secondly, if the suggested reading is along the right lines, there is no tension between ' \vdash ' and Frege's conception of assertion. For the horizontal attached to the judgment-stroke is not a part of identity sentences of the form ' x is the True'. It is a part of the performative ' \vdash '. Therefore, although the judgment-stroke produces assertoric force by turning the horizontal into the T-identifier, that does not imply that assertoric force belongs to the horizontal *qua* an 'expression of thoughts'. Because the horizontal attached to the judgment-stroke is not a predicate, it does not make Frege committed to the thought that we need a special predicate for assertion. Of course, we need the horizontal to formulate the T-identifier. However, that does not entail that we need the predicate for asserting. What we need for asserting is just the *act* we perform with

¹⁷This point only applies to Frege's mature career. His early conception of judgment and assertion is different. In the early *Begriffsschrift* Frege takes judgment and assertion to be predication of truth or facthood (1879a: §2). The indefinability argument Frege develops in his mature career shows that his conception of judgment/assertion has changed.

the T-identifier, not the ingredients of the T-identifier. Lastly, (A-Id) does not produce the kind of the vicious infinite regress Frege points to in his indefinability argument. Accepting (A-Id) leads to the regress only when we are also committed to the idea that identifying $\|p\|$ with the True is predicating the property *being the True* to $\|p\|$ that is equivalent to asserting that $\|p\|$ is the True. Reading \vdash as the T-identifier, we are not committed to it anymore.

Is the suggested explanation of ' \vdash ' what Frege has in mind? There is some evidence that it is. Recall the passage from 'Function and Concept':

... By writing

$$\vdash 2 + 3 = 5$$

we assert that 2+3 equals 5. Thus here we are ... *saying that it is the True*. (1891: 34; italics mine)

Describing the act of identification of $\|2 + 3 = 5\|$ with the True, Frege uses 'say' ('*sagen*') instead of 'assert' ('*behaupten*'). This seems to show that Frege is reluctant to consider this identification to be an identity assertion. One might claim that Frege is merely hiding the circularity of his elucidation of assertion here. Maybe. But if Frege is trying to hide the circularity, that only indicates that he is already aware of it and realizes that the relevant act of identification must not be that of assertive identification.

Notice that for Frege, judging that p —like asserting that p —must be (mentally) identifying $\|p\|$ with the True *non-judgmentally*. If it were the act of judging that $\|p\|$ is the True—i.e., predicating the property *being the True* of $\|p\|$ —the vicious infinite regress we see in the indefinability argument would be produced. Keeping this point in mind, let us return to Frege's elucidation in 'On Sense and Reference' that the act of producing the knowledge that a is b is that of deciding that $\langle a \rangle$ belongs to b . If what Frege has in mind were judgmental identification, he would be able to say simply that we produce

the knowledge that a is b by judging that a is b . However, Frege instead elucidates our cognitive activity for achieving the identity knowledge as the activity of relating one name's sense to the reference of the other name—even endorsing the cryptic metaphor 'taking a step'. Frege seems to be trying hard to formulate the idea of non-judgmental identification.¹⁸ He applies this metaphorical elucidation to the act of judgment, which implies that he takes the act of judgment as that of non-judgmental identification, i.e., an identification that is not an identity predication. The sign '⊢' represents a judgment. If a judgment is such a non-judgmental/non-predicative identification for Frege, that provides a good reason to take the sign to be the T-identifier—the performative for an identification that is not constituted by an identity predication.

Furthermore, '⊢' as it is given in Begriffsschrift can be considered as the T-identifier. It is the official explanation of '⊢' that by writing down '⊢ $2 + 3 = 5$ ' we identify $\|2+3=5\|$ with the True. Hence, the sign is a performative for identification with the True. The horizontal attached to the judgment-stroke cannot be replaced by a functor of the same type, or of any other type, which implies that it is not a predicate anymore. Hence, the sign '⊢' as it is precisely fits our description of it as the T-identifier.

Our discussion shows how Frege understands judging as acknowledging the truth of $\langle p \rangle$. In a nutshell, we acknowledge the truth of $\langle p \rangle$ by non-judgmentally identifying the True with $\|p\|$, saying that $\langle p \rangle$ belongs to the True, or taking a step from $\langle p \rangle$ to the True. Understanding judging in that way, Frege can happily combine his idea that truth is an object and that judging is acknowledging the truth of $\langle p \rangle$. Furthermore, he can avoid the vicious infinite regress he is pointing to in the indefinability argument.

¹⁸There is a similarity between Frege and Millikan relevant to this point. Millikan claims that when we accept ' o_1 is o_2 ', what we do is not to make an identity judgment but to realize the 'overlap' (2000: 144) of two contents on a single object. Realizing the overlap of contents, which Millikan calls 'co-identifying', seems to be quite similar to Frege's deciding whether a sense belongs to the reference of a known name.

5.5. APPENDIX: OTHER USES OF THE HORIZONTAL

Closing this chapter I would like to deal with an important objection to my suggested reading of the horizontal attached to the judgment-stroke. The objection is that my reading must be rejected because it can't explain other uses of the horizontal, e.g., why the negation-stroke in '¬' (the vertical line) must accompany horizontals (the lines on the sides of the negation-stroke).

This objection assumes that there must be a single unified explanation that can be applied to all the different uses of the horizontal in Begriffsschrift. However, I do not share this assumption. The horizontal is just an identity predicate that can serve different purposes. Then, it is at least equally plausible to assume that different uses of the horizontal are underwritten by different rationales. I justify this assumption by addressing the case of the negation-stroke and the horizontals attached to it.

First, we ought to note that the negation-stroke alone does not refer to a function. Say it refers to a function. It must be one that can have an argument of the form '— Δ', i.e., a first-level function that maps an object to an object. For Frege a function must always be *totally* defined:

It is essential that ... 'a + b' should always have a reference whatever signs for definite objects may be inserted in place of 'a' and 'b'. This involves the requirement as regards concepts that, for any argument, they shall have a truth-value as their value ... The requirement of the sharp delimitation of concepts thus carries along with it this requirement for functions in general that they must have a value for every argument. (Frege 1891: 33)¹⁹

If the negation-stroke itself refers to a function then the function must also be defined for, say, 2. If the function denoted by the negation-stroke is defined for 2, then '¬ 2'

¹⁹There is a debate about whether the domain of Begriffsschrift is restricted. See Blanchette 2012 and Heck 2012 for this debate.

must be a legitimate expression of Begriffsschrift. But it is not a legitimate expression at all. Thus, the negation-stroke alone does not refer to a function. It is a special sign that constitutes a function-name only with horizontals attached.

Then what is it that the negation-stroke as such a special sign embodies? I think it embodies what Frege (1918b: 357) calls 'a negating word'. A negating word has special features: it 'may occur anywhere in a sentence' (1918b: 353) and depending on which part of a sentence it is attached to, a resulting thought varies. Therefore, a negating word alone cannot denote a function. It can refer to a function only when it is combined with a part of a sentence (e.g., 'All cats are not dogs' and 'Not all cats are dogs'). The negation-stroke *qua* a negative word is always attached to the horizontal '... is the True' on its right side and thereby constitutes '... is not the True'. Thus '¬' refers to 'a concept under which all objects fall with the sole exception of the True' (1893: §6). The fact that the negation-stroke can only have the horizontal on its right side means just that '... is not the True' is the only negation necessary for logic. Now, the horizontal on the left side of the negation-stroke is related to the point that logic does not need negative judgment (1918b: 355–7). Now if we write down

$$\neg 2 + 3 = 4$$

we judge that it is not the case that $2+3$ is 4. In that case, can't we consider '¬' to be a sign for negative judgment—*denying* that $2+3$ is 4? The answer is 'No'. '¬' is not a performative for negative judgment due to the left horizontal: '¬' comes from '¬¬' by the *fusion of horizontals* that combines consecutive horizontals into one. The one and only performative in Begriffsschrift is '⊢' that is for (positive) judgment.

Thus Frege's own elucidation of negation leads to the rationale for the horizontals attached to the negation-stroke, which is different from the rationale that underlies the horizontal attached to the judgment-stroke. This result supports my assumption that different uses of the horizontal have different rationales. Of course, there are other uses of the horizontal to be explained, i.e., the horizontals in the conditional-stroke or in the universal quantifier. Here, the vertical stroke is the conditional-stroke and all the short horizontal lines are horizontals. Although I do not pursue this explanation here, I indicate the fact that the answer to the question why the horizontals must be attached to the conditional-stroke can be retrieved in Frege's conception of conditional thought (cf. Frege 1923).

CHAPTER 6

KNOWLEDGE, SCIENCE, AND LOGIC IN FREGE

It is not an easy task to decode Frege's conception of logic. The first question concerning it is whether Frege has the Tarskian semantic conception of logic. As we have seen, Greimann (2008) insists that Frege has the Tarskian semantic conception of logic—identifying '... is the True' as Frege's truth-predicate. Nevertheless, as we have seen, Greimann's argument that '... is the True' is Frege's truth-predicate is not satisfactory. Commentators such as Goldfarb (2001), Ricketts (1996), and J. Weiner (2005) argue that Frege does not have a truth-predicate. For instance, Ricketts writes:

[Frege's] logical laws do not describe valid forms of argument; they are about neither sentences nor thoughts sentences express; they do not use a truth-predicate.
(Ricketts 1996: 124)

The gist of Ricketts's argument is that Frege does not have the Tarskian conception of logic that requires a meta-linguistic truth-predicate:

[Frege] has then no semantic conception of logical consequence, in the post-Tarskian sense of 'semantic'. (Ricketts 1996: 124)

I agree with these commentators. Frege cannot have a truth-predicate because truth is not an entity that can be referred to by a predicate because it is an object. Hence, it seems implausible to think that Frege has the Tarskian conception of logic.

Then what kind of conception of logic does Frege have? Here is one view.

More than this, Frege lacks any general conception of logical consequence, any overarching conception of logic. Frege has only a retail conception of logic, not a wholesale one. He tells us what logic is by identifying specific laws and inferences as logical. ... [Frege's conception of logic] does not state a defining criterion of the logical. (Ricketts 1996: 124)

Frege's conception of logic is retail, not wholesale. He simply presents various laws of logic and logical inference rules, and then demonstrates other logical laws on the basis of these. He frames no overarching characteristic that demarcates the logical laws from others. (Goldfarb 2001: 30)

According to Goldfarb and Ricketts Frege has the 'universalist conception of logic' that provides only a single necessary condition for being a logical law: a purely logical law is a 'maximally general truth' (Ricketts 1996: 124). Logical laws are topic-universal; they are 'the laws of truth for every object, concept, and relation' (Ricketts 1996: 127). Of course, not all maximally general truths are logical truths. Therefore, 'maximally general truth' is not an exhaustive description of 'logical truth'. Frege does not provide a definition of logical truth because he does not have any 'overarching' conception of logic. Or so argue the proponents of the universalist reading.

In the universalist exegesis of Frege's conception of logic, truth does not seem to play a substantive role. This exegesis does not appear to square well with the fact that Frege takes logical laws to be the laws *of truth*. However, the universalist reading does

not take this claim to establish that truth plays a substantive role in Frege's conception of logic. For instance, Goldfarb writes:

Now Frege does say, 'Logic is the science of the most general laws of truth.' But he does not intend this as the demarcation of logic, only as a 'rough indication of the goal of logic'. (2001: 31)

Also, Ricketts writes:

The laws of geometry set forth generalizations concerning spatial concepts and relations—they are, so to speak, the laws of truth concerning these concepts and relations. Similarly, the fundamental laws of any special science set forth true generalizations, laws of truth, for some range of concepts proprietary to that science. By their topic-universality, the laws of logic are the laws of truth for every object, concept, and relation—they are the laws of truth simpliciter. (1996: 127)

Thus, in the universalist reading, Frege's claim that logical laws are the laws of truth is explained in terms of the generality of those laws, but not in terms of truth.

I can accept that Frege does not have an overarching conception of logic. However, I hesitate to accept that it is generality that plays all the substantive roles in his conception of logic. Recall the passage from 'Thought':

All sciences have truth as their goal, but logic is also concerned with it in a quite different way: logic has much the same relation to truth as physics has to weight or heat. To discover truths is the task of all sciences; it falls to logic to discern the laws of truth. (1918a: 325)

The point of his claim is not that there is no particular object or property logic takes as its main subject. Frege's point is that truth is the main subject of logic. Furthermore, recall how Frege explains the laws of *Begriffsschrift*. His explanations—which appear to be pretty similar to meta-linguistic justifications—heavily depend on the True (and the False). Given that truth is the True for Frege, it rather seems reasonable to think that truth plays a crucial role in his understanding of logic.

The challenge we face here is to construct an interpretation of Frege's conception of logic in which he assigns to truth a substantive role in demarcating logic while we can remain neutral about whether he has a semantic conception of truth like the Tarskian.¹ We can construct such a reading by taking it into account that truth is an object for Frege—specifically by answering the questions that pop up from the above passage of 'Thought' when we take Frege's truth to be the True. We've already raised these questions in Chapter 1. Frege claims that all sciences aim at truth. Therefore, there is a single object which all the sciences aim at. How should we make sense of this claim? Furthermore, if truth is the aim of all sciences, in what sense is logic the sole science that has truth as its main subject? The answers to these questions eventually lead to the new reading of Frege's conception of logic. To answer those questions we need to look into Frege's mature conception of objectual knowledge, propositional knowledge, and the relationship between them. I start with his conception of objectual knowledge.

6.1. FREGE'S CONCEPTION OF OBJECTUAL KNOWLEDGE

Recall the following passage from 'On Sense and Reference'.

The sense of a proper name is grasped by everybody who is sufficiently familiar with the language or totality of designations to which it belongs; but this serves to illuminate only a single aspect of the reference, supposing it to have one. Comprehensive knowledge of the reference would require us to be able to say immediately whether any given sense belongs to it. To such knowledge we never attain. (1970: 57–8)

'The reference' in 'comprehensive knowledge of the reference' means 'the reference of a proper name', i.e., 'object'. Hence, Frege is talking about comprehensive knowledge

¹In my thought Richard Kimberly Heck (2007, 2010, 2012) attempts to understand Frege's conception of logic in this way. That is, Heck seems to think that although Frege does not have the Tarskian conception, he still assigns a crucial role to the semantic notions like truth in his logic. What I am trying to in this section is to develop his idea further.

of an object in this passage. If we have comprehensive knowledge of the reference of a proper name 'O', i.e., the object O, we have the ability to say immediately whether any given sense belongs to O. Let us analyze this claim.

First, let us consider the nature of the act of saying that the sense of a proper name belongs to an object. A special case of this act is judging that p : saying that $\langle p \rangle$ belongs to the True. Frege metaphorically elucidates judging as such to be taking a step from a thought to its truth-value in order to stress the point that it is a *non-judgmental* act of identifying. Hence, saying that $\langle p \rangle$ belongs to the True is taking a step from $\langle p \rangle$ to its truth-value, i.e., identifying the True with $\|p\|$ non-judgmentally. Recall that Frege applies the metaphorical elucidation 'taking a step' to proper names in general, e.g., 'Odysseus'. Accordingly, to say that the name of a proper name 'N' belongs to O is to take a step from $\langle N \rangle$ to its reference, i.e., to identify N with O *non-judgmentally*.

Given that saying that $\langle N \rangle$ belongs to O is identifying N with O, it is clearly the act by which we come to *know the identity* between N and O. The second thing to be considered is the nature of knowledge of identity produced by the act of saying $\langle N \rangle$ belongs to O. One might think that the knowledge produced is just the *propositional* knowledge that N is O. However, that is false. To see this point we first need to check the relationship between judging that p and knowing that p . In 'Sources of Knowledge of Mathematics and the mathematical natural Sciences', Frege states that he counts only 'judgment proper' (1924: 267) as knowledge. Given that knowledge is rather an epistemic state while a judgment is an epistemic act, what Frege means seems to be the following: the state of knowledge is produced by the act of judgment.² Specifically, we can say, one produces the knowledge that p by judging that p .

²For the relationship between knowledge and judgment in Frege, see Ricketts 1996 and Textor 2010.

If so, it follows that the propositional knowledge that N is O is produced by judging that N is O. The act of producing the knowledge of identity that N is O is that of saying that $\langle N \rangle$ belongs to O. If the act of saying that $\langle N \rangle$ belongs to O is that of producing the propositional knowledge that N is O, then the former must be the act of judging that N is O, i.e., an act of *judgmental* identification. However, the act of saying that $\langle N \rangle$ belongs to O is not such a judgmental identification. Therefore, the former cannot be the act of producing the propositional knowledge that N is O. Again, the act of saying that $\langle N \rangle$ belongs to O produces our knowledge of the identity between N and O. But the knowledge produced is *non-propositional* knowledge of identity. Now, I will notate the non-propositional knowledge of the identity between O_1 and O_2 as 'the knowledge_{np} that O_1 is O_2 ' where 'np' means 'non-propositional'. Our discussion can be summed up as the following proposition:

(NPKI) The act of saying that $\langle N \rangle$ belongs to O is that of producing the knowledge_{np} that N is O.

Two clarificatory points. First, it is possible to produce the propositional knowledge that O_1 is O_2 . We can produce it by judging that O_1 is O_2 . The point we've made is that judging that O_1 is O_2 is not saying that $\langle O_1 \rangle$ belongs to O_2 (or saying that $\langle O_2 \rangle$ belongs to O_1) and that we can perform the latter without performing the former. Second, we ought to clarify the nature of the knowledge produced by the act of saying that $\langle N \rangle$ does *not* belong to O. Given that saying that $\langle N \rangle$ belongs to O is identifying N with O non-judgmentally, it is tempting to claim that saying that $\langle N \rangle$ does not belong to O is *distinguishing* N from O *non-judgmentally*. If so, then the knowledge produced by saying that $\langle N \rangle$ does not belong to O is the knowledge_{np} that N is not O. As we have non-propositional knowledge of identity, we have non-propositional knowledge

of non-identity.³

One more point to be taken into account is that we cannot say whether $\langle N \rangle$ belongs to O only by grasping $\langle N \rangle$. This follows from the epistemic gap between the sense of a name and its reference. The epistemic gap includes the fact that $\langle N \rangle$ alone does not say anything about whether N is identical with the references of names whose senses are known to us. Therefore, one can say whether $\langle N \rangle$ belongs to O only when in addition to grasping $\langle N \rangle$ one does something else that lets one fill the epistemic gap between the sense and its reference.

Now return to Frege's point under consideration: if one has comprehensive knowledge of O , one has the ability to decide immediately whether any sense given to one belongs to O . As we have said, one cannot decide whether $\langle N \rangle$ belongs to O only by grasping the sense. Then it appears plausible to regard comprehensive knowledge of O as what provides us with the ability to decide whether any sense belongs to O .

Frege says:

To such knowledge we never attain. (1970: 58)

Thus we only have *non-comprehensive* knowledge of objects. It seems as if to say that one has non-comprehensive knowledge of O is to say that there are names such that one cannot say whether the senses of those names belong to O . Here is how I unify Frege's points. In general, knowledge of an object is what provides us with the ability to say

³The act of distinguishing the True from, say, $\|2+3=4\|$ non-judgmentally is not the act represented by $\ulcorner 2 + 3 = 4 \urcorner$. The latter represents the act of identifying the True with $\| \|2+3=4\|$ is not the True $\|$ non-judgmentally. One might object that recognizing the act of (a) distinguishing the True from non-True's non-judgmentally goes against Frege's denial that there is an act of (b) negative judgment. But (a) is different from (b); the former is an act with the True while the latter seems to be an act with the False. Logic does not need to have a performative for (b) because it is not clear whether (b) exists and even if it exists, it is unnecessary for the sake of logic to take (b) into account (Frege 1918b). (a) does not seem to be necessary in logic because logic only cares about judgment and in the level of judging what we do when we distinguish the True from $\|2+3=4\|$ is to identify the True with $\| \|2+3=4\|$ is not the True $\|$.

whether a given sense belongs to the object. If we have comprehensive knowledge of *O*, we have the *strongest* ability to do so, i.e., the ability to say whether any sense given to us belongs to *O*. Now we can introduce the *degree* or *level* of one's knowledge of an object and also about *extending* one's objectual knowledge. Specifically, the more extended one's knowledge of *O* is, the more names one can correctly identify as the names (or non-names) of *O*. We never have comprehensive knowledge of an object; we never have the strongest ability to say whether a given sense belongs to an object. Thus, for us, there are always names such that we cannot tell whether the senses of those names belong to an object.

That much is, I believe, what we can retrieve about Frege's conception of objectual knowledge from the passage of 'On Sense and Reference'. I understand that this way of talking about objectual knowledge can sound implausible and even crazy. However, Frege is not the only one who holds this seemingly peculiar conception; Millikan (1998, 2000, 2017) does as we've briefly mentioned. We will have a chance to talk about this in §7.3 where we develop Frege's conception of knowledge by combining Millikan's theory of identity. For now, all I am saying is just that Frege has such a conception of objectual knowledge.

In the next section I turn to Frege's conception of propositional knowledge. As we will see propositional knowledge turns out to be grounded in our objectual knowledge as such, specifically, our knowledge of the True.

6.2. KNOWLEDGE OF THE TRUE AND KNOWLEDGE-THAT

Since the truth-value True is an object, we can apply the previous points to the True. In other words, we can talk about 'knowledge of the True'. Given the epistemic gap between the sense of a sentence '*p*', i.e., $\langle p \rangle$ and its truth-value, one cannot say whether

$\langle p \rangle$ belongs to the True only through grasping $\langle p \rangle$. Our knowledge of the True is what provides us with the ability to say whether a given thought belongs to the True. When we use this ability and say that $\langle p \rangle$ belongs to the True, we judge that p . By judging that p , we come to have the knowledge of the identity between $\|p\|$ and the True. Therefore, the act of judging that p is grounded in the ability to say that $\langle p \rangle$ belongs to the True. Because this ability is provided by our knowledge of the True, the activity of judgment is epistemically grounded in knowledge of the True.

Thus judging that p produces the knowledge of the identity between $\|p\|$ and the True. But this knowledge of identity is not propositional knowledge of identity. Recall we produce the propositional knowledge that p by judging that p . Assume that judging that p produces the propositional knowledge that $\|p\|$ is the True. Then, judging that p is judging that $\|p\|$ is the True, i.e., identifying the True with $\|p\|$ *judgmentally*. But judging that p is identifying $\|p\|$ with the True *non-judgmentally*. Contradiction. Hence, the knowledge of identity we produce by judging that p is the knowledge_{np} that $\|p\|$ is the True.

We produce the propositional knowledge that p by judging that p . But the cognitive result we produce by judging that p is the knowledge_{np} that $\|p\|$ is the True. Therefore, the propositional knowledge that p is just the knowledge_{np} that $\|p\|$ is the True. That is why Frege elucidates propositional knowledge in terms of the relationship among a thought, a truth-value, and judging.

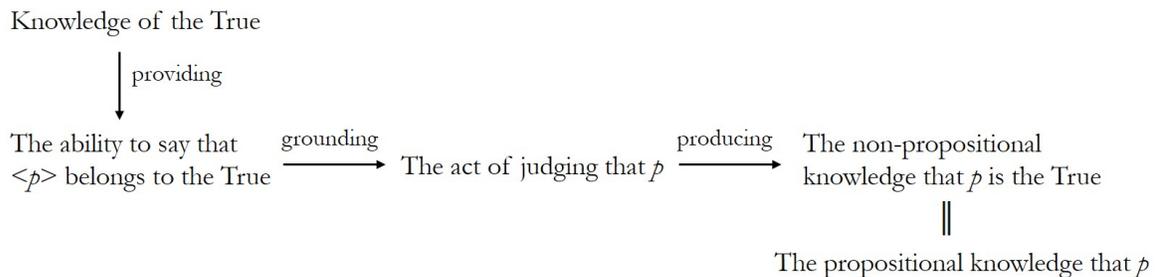
We can never be concerned only with the reference of a sentence; but again the mere thought alone yields no knowledge, but only the thought together with its reference, i.e. its truth value. Judgments can be regarded as advances from a thought to a truth value. (Frege 1892c: 65)

One might wonder if we should posit a process that turns the knowledge_{np} that $\|p\|$ is

the True into the propositional knowledge that p . However, Frege says nothing about such a process. Here, the point that Frege directly regards judgment as knowledge seems to have significance. Perhaps what he means by that is that there is nothing more other than judgment we should perform in order to acquire propositional knowledge. What we have by judging is just propositional knowledge. Hence, we can ascribe the following to Frege:

(PKID) The propositional knowledge that p is the knowledge_{np} that $\|p\|$ is the True.⁴

The knowledge_{np} that $\|p\|$ is the True is produced by the act of judging that p . Judging is epistemically grounded in the ability to say whether $\langle p \rangle$ belongs to the True, which is in turn provided by one's knowledge of the True. By (PKID) propositional knowledge is grounded in knowledge of the True, i.e., objectual knowledge. Hence, we have the following chart.



⁴As I've said, we can still have propositional knowledge that O_1 is O_2 . We can have it by judging that p . The propositional knowledge that O_1 is O_2 is thus the knowledge_{np} that $\|O_1$ is $O_2\|$ is the True. Propositional knowledge of identity is still non-propositional knowledge of identity—of a different object so to speak. Then, what is the relationship between the knowledge_{np} that O_1 is O_2 and the knowledge_{np} that $\|O_1$ is $O_2\|$ is the True? The answer, I think, is that if one has the latter, then one has the former. By that, I am not pointing out the fact that $\langle \|O_1$ is $O_2\|$ is the True \rangle entails $\langle O_1$ is $O_2 \rangle$. The entailment I put forward is a much stronger relationship we may perhaps call 'epistemic entailment'. But it still seems plausible to think that this epistemic entailment obtains between the two pieces of identity knowledge_{np}.

In conclusion, propositional knowledge is grounded in our knowledge of the True. Based on this point we can explain in what sense all the sciences aim at the True and in what sense logic is the only science whose main subject is the True.

6.3. SCIENCE, LOGIC, AND THE TRUE

If we have comprehensive knowledge of the True, then we have the strongest ability to say whether a thought belongs to the True—the ability to say whether *any* given thought belongs to the True. Were infinite time provided, we would be able to have all pieces of propositional knowledge—achieving omniscience. But we never have such an ability; we only have non-comprehensive knowledge of the True. Hence, there are always thoughts such that we cannot say whether they belong to the True. Someone's knowledge of the True is more extended than other people's knowledge of the True. As one's knowledge of the True becomes more extended, one comes to be able to say about more and more thoughts whether they belong to the True.

In the sciences, we come to produce new pieces of propositional knowledge, e.g., that the Earth goes around the Sun, that nothing can be faster than light, that Fermat's last theorem is true, etc. Therefore, the pursuit of the sciences strengthens our ability to say whether a given thought belongs to the True. To put it differently, by way of the sciences, we are extending our knowledge of the True. The sciences, including logic, are eventually our intellectual activities to extend our knowledge of the True. Thus, in Frege's framework, there is a significant sense in which the sciences aim at the True. They aim at extending our knowledge of the True.

So logic in the same sense aims at the True. However, that can't be the reason why logic is the science *of* the True. In what sense is logic the only science that undertakes the True as its main subject? A short answer is: logic is the science of the True because

it examines how far we can extend our knowledge of the True when we only have knowledge of truth-values, i.e., the True and its counterpart the False. Here is a bit longer answer. In logic we do not presuppose knowledge of objects other than truth-values. We recognize objects other than truth-values if our knowledge of truth-values is sufficient for allowing us to recognize them. In logic we extend our knowledge of the True within the range which our knowledge of truth-values and of objects recognizable by our knowledge of truth-values can cover. Let me explain further.

First, let us review the overall structure of *Begriffsschrift*. The laws of logic are the thoughts whose truth cannot be rationally doubted. They obviously belong to the True, i.e., the sentences that express logical laws (law-sentence) are the most obvious names of the True.⁵ By writing down '⊢' in front of those law-sentences, i.e., by asserting that those logical laws are true, we identify the references of law-sentences as the True non-judgmentally. Inferential rules necessarily bring us from these obvious names of the True to other names of the True. Hence, in logic, we basically extend our knowledge of the True only via inferential rules and definitions.

The crux is that we do not need to recognize objects other than the True and the False in order to recognize the truth of the logical laws. One might raise an objection based on Law V:

$$(\dot{\epsilon}f(\epsilon) = \dot{\alpha}g(\alpha)) = \text{⊃} f(\alpha) = g(\alpha)$$

(Its natural language analogue would be: 'The value-range of $f(x)$ is identical with that of $g(x)$ iff for all x $f(x)$ is identical with $g(x)$ '). The objection is that, to recognize the truth of Law V which introduces the condition for the identity of value-ranges, one ought to

⁵Say Law I is given by the following: '⊢ L_1 '. What I mean by 'Law I' is the thought expressed by ' L_1 '. ' L_1 ' is a law-sentence that expresses Law I.

antecedently recognize value-ranges and then have ‘intuitions’ or ‘observations’ about them. However, see what Frege says about Law V:

... If one function (of first-level with one argument) and a second function are so constituted that both always have the same value for the same argument, then one may say instead: the value-range of the first function is the same as the value-range of the second. We then recognise something in common both functions and this we call the value-range both of the first function and of the second function. That we have the right so to acknowledge what is common, and that, accordingly, we can convert the generality of an equality into an equality (identity), must be regarded as a basic law of logic. (1903a: §146)⁶

Frege is saying that if we recognize functions that always have the same value for a given argument, we can recognize the value-ranges of those functions and their identity conditions. Thus, what we need to antecedently recognize in order to recognize value-ranges are only functions. Note that as soon as we recognize truth-values, we come to be able to recognize functions. For instance, if we recognize truth-values, we are already in the position where we can recognize the horizontal function — x that yields the True if the True is given as an argument and the False otherwise. If we recognize functions, we are already in the position where we can recognize the truth of Law V. Thus, our recognition of value-ranges can be grounded solely in our recognition of truth-values.⁷ So even in the case of Law V, the only objects we need to antecedently recognize for acknowledging the truth of Law V are truth-values.⁸ Then, by way of Law V and individual functions, we come to acknowledge the existence of particular value-

⁶The same point is made in Frege 1891 and 1893.

⁷This can be a reason why truth-values are the ‘most fundamental’ logical objects (Shramko and Wansing 2018: §1.3).

⁸In §147 of *Grundgesetze II* (1903a), Frege is complaining about ‘lawless creation of numbers by many mathematicians’, contrasting it against his way of recognizing—not creating—value-ranges through Law V. I believe that this point is closely related to our recognition of truth-values. For examining this issue in Frege, see Ebert and Rossberg (2019).

ranges. Logic hence concerns knowledge of value-ranges because our knowledge of truth-values is sufficient to produce our knowledge of value-ranges. That is the sole legitimate reason why objects other than truth-values can be introduced in our logical investigation.

Let us synthesize what we have. In logic, we start with acknowledging that law-sentences are obvious names of the True. They are the most obvious names of the True; for we can recognize the identity between the True and the reference of law-sentences solely through our recognition of the True and the False. From these obvious names of the True, we proceed to other names of the True. When we do so, we never recognize the existence of objects other than truth-values unless our recognition of those objects can be solely grounded in our recognition of truth-values (e.g., value-ranges). Hence, logic aims to extend our knowledge of the True as far as our recognition of truth-values allows us to go.

Other sciences are different from logic. The laws introduced by other sciences surely belong to the True if they are indeed true. However, we cannot recognize their truth solely by our recognition of truth-values. To recognize the truth of other scientific laws, we need to depend on our cognitive abilities such as sense perception or the Kantian intuition (Frege 1924) where we can observe or intuit objects other than truth-values and their properties. Sciences other than logic hence extend our knowledge of the True beyond the range our recognition of truth-values can bring us to.

In conclusion, logic can be considered to be the science of the True because its aim is to extend our knowledge of the True solely through our recognition of the True and the False. Other sciences aim to extend our knowledge of the True beyond what our recognition of truth-values tells us. Therefore, other sciences cannot be taken to be the

science of the True in the sense that logic is the science of the True—although they still aim at the True in a substantive sense.

Recall the universalist interpretation of Frege on logic according to which the notion of truth does not play a substantive role in distinguishing logic from other sciences. The reading says that the only distinctive feature of logical laws, provided by Frege himself, is its utmost generality. That is not true. Logical laws are also the laws such that the recognition of the truth of those laws does not require anything other than our knowledge of truth-values. Thus, the notion of truth plays a significant role in setting the boundary of logic. At the same time, however, that does not mean that Frege has a meta-linguistic conception of logic like Tarski's. Frege's objectual conception of truth does not require such a semantic conception of logic antecedently.

Our discussion shows that Frege is an advocate of what we may call 'the scientism of truth'. He does not attempt to provide any explicit definition of truth not because he is a primitivist but because truth is what we ultimately investigate into in the sciences for him. Our knowledge of truth or the True is constantly expanded in the sciences and we come to know more and more about it. Logic plays an important role in our effort to expand our knowledge of the True. It solely focuses on truth and thereby lets us fix what we refers to by 'truth': truth is the object to which logical laws obviously belongs to. One might ask why we can't just take it as the definition of truth. We can do so. But doing so does not do any work in our intellectual activities. What matters is to expand our knowledge of truth constantly by way of the sciences. In this sense, Frege is suggesting the scientism of truth.

Probably one of the crucial results of our discussion here is Frege's epistemology. In our discussion he turns out to have pretty a substantive conception of knowledge. This

goes against the general consensus in Frege scholarship according to which Frege is not interested in epistemology and epistemology does not play a significant role in his philosophy.⁹ Moreover, his conception of knowledge is not only substantive but also radically different from any contemporary theory of knowledge. In Frege's conception of knowledge propositional knowledge is reduced to non-propositional knowledge of the identity of an abstract object—the True—and is also ultimately grounded in the objectual knowledge of this abstract object. This idea deserves thorough examination. In the next chapter (§7.3) I provide a sketch of a theory of knowledge based on Frege's conception of knowledge.

6.4. APPENDIX: THE AIM OF 'ON SENSE AND REFERENCE'

A number of important points of my suggested reading—Frege's conception of truth, judgment, and knowledge—are retrieved from the paper 'On Sense and Reference'. In the suggested reading an aim of this seminal article, which has not been acknowledged by commentators before, seems to be emerging. Before turning to the contributions of Frege's philosophy to contemporary problems, I would like to make this aim explicit.

The paper starts with the problem 'Frege's Puzzle', i.e., the question as to how we can explain the difference in cognitive value between ' $a = a$ ' and ' $a = b$ '. Frege writes:

... $a = a$ and $a = b$ are obviously statements of differing cognitive value; $a = a$ holds *a priori* and, according to Kant, is to be labelled analytic, while statements of the form $a = b$ often contain very valuable extensions of our knowledge and cannot always be established *a priori*. (1892c: 57)

⁹This consensus seems to be caused partly by Dummettian reading of Frege, which has been dominant in Frege scholarship. Dummett (1993: 669) compares Frege to Descartes—considering both of them as initiating a philosophical revolution. While Descartes revolutionizes philosophy by putting epistemology at its center, Frege does so by putting *language* at its center. Dummett holds that, because Frege is mainly interested in language, he addresses epistemological issues 'only in a limited fashion'. The consensus that Frege does not have epistemology appears to be rooted partly in Dummett's interpretation.

This cannot be his elucidation of the relevant cognitive difference. ‘ $2 = 1 + 1$ ’ can be established *a priori*. Frege also knows about this. He is saying that the truth of ‘ $a = b$ ’ cannot *always* be established *a priori*, which means that *sometimes* the truth of such a sentence can be established *a priori*. Frege here just appears to be pointing to a way to *initially realize* the cognitive difference between ‘ $a = a$ ’ and ‘ $a = b$ ’.

Of course, Frege’s explanation of the cognitive difference between ‘ $a = a$ ’ and ‘ $a = b$ ’ depends on the sense-reference distinction. But how does this distinction explain the cognitive difference? A tempting answer appears to be this: we can know that ‘ $a = a$ ’ is true *analytically*, i.e., purely thanks to its sense, while we cannot do so in the case of ‘ $a = b$ ’. However, this cannot be Frege’s explanation, either.

$$\left(\begin{array}{c} \Gamma \\ \vdots \\ \Delta \\ \vdots \\ \Gamma \end{array} \right) = [(\dot{\epsilon}f(\epsilon) = \dot{\alpha}g(\alpha)) = \neg f(a) = g(a)]$$

The left side expresses Law I and the right side expresses Law V. They express different thoughts because Law I is different from Law V. Hence, it is a case of ‘ $a = b$ ’. However, it can still be established analytically. Hence, just as the apriority explanation above does not work, the analyticity explanation based on sense does not work either.

However, obviously the distinction between the sense and the reference of a term is an important part of Frege’s explanation of the cognitive difference. We’ve already sketched how the distinction contributes to the explanation of the cognitive difference in Chapter 5. But let us move slowly and look at the text. Right before suggesting the sense-reference distinction Frege writes:

What is intended to be said by $a = b$ seems to be that the signs or names ‘ a ’ and

'*b*' designate the same thing, so that those signs themselves would be under discussion; a relation between them would be asserted. But this relation would hold between the names or signs only in so far as they named or designate something. It would be mediated by the connexion of each of these signs with the same designated thing. But this is arbitrary. Nobody can be forbidden to use any arbitrary producible event or object as a sign for something. In that case that sentence $a = b$ does not refer to the subject matter, but only to its mode of designation; we would express no proper knowledge by its means. (Frege 1892c: 56–7)

Frege is criticizing the claim—he once accepted—that by asserting ' $a = b$ ' we express the relationship between '*a*' and '*b*' as signs with different shapes. His worry is that this kind of relationship can be arbitrarily proliferated because we can arbitrarily introduce a new sign so as to designate an object. Why is this worrisome? Notice that Frege is claiming that when we assert that ' $a = b$ ' we express some *knowledge*. If by asserting ' $a = b$ ' we express the relationship between different signs as different shapes, the knowledge we express is not proper knowledge because that kind of 'knowledge' is something we can achieve by arbitrary manipulation. The knowledge we attempt to express by ' $a = b$ ' should be genuine, non-arbitrary, knowledge that requires genuine epistemic effort of us. This establishes that what Frege is concerned about here is not just the cognitive difference between ' $a = a$ ' and ' $a = b$ ' but also the nature of the knowledge of identity *in general* and the nature of the act of identification, i.e., the act by which we achieve identity knowledge.

The notion of sense comes into the picture when Frege attempts to clarify the nature of the act of identifying *a* with *b*. In a nutshell, to identify *a* with *b* is to say that $\langle a \rangle$ ($\langle b \rangle$) belongs to *b* (*a*). The issue of arbitrariness is solved here because the senses of names cannot be arbitrarily manipulated by us—because the relationship between the sense of a name and its reference is completely objective. The cognitive difference between

' $a = a$ ' and ' $a = b$ ' is finally explained. Consider ' $a = a$ ' first. Suppose that we already grasp the sense of ' a ' and recognize the existence of its reference. We thereby say that $\langle a \rangle$ belongs to a . That is exactly all it takes for us to see the truth of ' $a = a$ '. However, in the case of ' $a = b$ ', we must additionally grasp $\langle b \rangle$, recognize its reference, and try to say whether $\langle b \rangle$ belongs to a .¹⁰ Therefore, we have the cognitive difference.

At this point, we ought to note that Frege never makes the following claim: because $\langle a = a \rangle$ and $\langle a = b \rangle$ are different thoughts, the judgment that a is a is different from the judgment that a is b . This claim would lead us to a much simpler way to explain the cognitive difference between ' $a = a$ ' and ' $a = b$ '. However, Frege does not go that way. He never describes the act of identifying a with b as the act of judging that a is b . Instead Frege is describing it as the act of saying that $\langle a \rangle$ ($\langle b \rangle$) belongs to b (a) and later as taking a step from the sense of a name to its reference. By that, as we have seen, Frege is pointing to the idea that the act of identifying a with b , which he is trying to capture, is not an act of judgmental identification. The knowledge of identity we produce by such a non-judgmental identification is non-propositional knowledge of identity. The question is why Frege appeals to non-judgmental identification and non-propositional knowledge of identity—instead of appealing to the difference between $\langle a = a \rangle$ and $\langle a = b \rangle$ and the thought-sensitivity of judgment.

The answer concerns Frege's considerations we've seen in Chapters 2–5. He realizes

¹⁰In my reading of 'On Sense and reference', it might be said, the identity relationship turns out to be that of co-referentiality. But that is not the case. The identity relationship is completely objectual; it obtains of the same object. May (2012) defends the same interpretative claim. However, I think, his arguments are flawed. May's argument heavily depends on the premise that Frege's stipulation of the True as the value-range of the horizontal concept is a genuine definition of the True. What our discussion shows is that it cannot be; the True is the object whose nature must be revealed by our logical investigation, not a stipulated object. Also, May takes it that relating identity to co-referentiality always depends on meta-linguistic assumptions. Not necessarily. Frege relates identity to co-referentiality of thoughts via the analysis of objectual knowledge, not via a meta-linguistic assumption.

that truth cannot be a property and that the act of judging—the logically fundamental activity—cannot be the act of predicating the property *truth* of a thought. Thus, Frege needs an alternative conception of judgment where judging is not reduced down to predicating the property *truth* of a thought. He finds this alternative in the notion of non-judgmental identification: judging that p is saying that $\langle p \rangle$ belongs to the True, i.e., taking a step from $\langle p \rangle$ to its truth-value. Judging that p produces the propositional knowledge that p , which is the knowledge_{np} that $\|p\|$ is the True. The vicious infinite regress—pointed out in the indefinability argument—is blocked and truth is not a property but an object. All these are exactly what Frege is pointing out in the middle of 'On Sense and Reference' after he explains that the sense of a sentence is a thought and its reference is a truth-value.

If our discussion is along the right lines, 'On Sense and Reference' ultimately aims to clarify the nature of truth and that of judging *via* the nature of identity knowledge and that of the act of identifying. That is the ultimate aim for which the sense-reference distinction is designed. The article is not a side-job that is merely remotely related to Frege's logicist project. It is at the center of the logicist project insofar as judging is at the center of his conception of logic.

One might object that if my suggested reading is along the right lines, we cannot explain why Frege takes judgment to be 'something quite peculiar and incomparable' (Frege 1892c: 65). Of course, he elucidates judgment as non-judgmental identification. But nothing more is said about judgment as such. So what we have in the paper comes down to the two points: judgment as such is not predication and truth is not a property. Frege does not have a language to *explain* the nature of identification and of judgment any further. That is why he is using only metaphorical expressions like 'taking a step

from a sense to its reference' or 'a sense belongs to an object'. That is understandable. All Frege cares about is judgment and truth *insofar as* logic requires us to clarify their nature. He appears to think that he've said enough to clarify their nature *for the sake of logic*—because what logic requires is to show that judgment is not predication and that truth is not a property—and thus he does not go any deeper. That might be the reason why Frege leaves the further nature of judgment 'peculiar and incomparable'.

One might also object that if my reading of 'On Sense and Reference' is correct, then we cannot explain why Frege is testing his sense-reference distinction with different kinds of sentences. I disagree. The fact that he is doing rather supports my reading. Frege writes:

The supposition that the truth-value of a sentence is its reference shall now be put to further test. (Frege 1892c: 65)

Frege ought to test this particular hypothesis in order to confirm that his elucidation of judging that p , i.e., taking a step from $\langle p \rangle$ to its reference or saying that $\langle p \rangle$ belongs to the True, indeed works for all kinds of $\langle p \rangle$. Rather, this reading appears to provide the paper with a more unified and consistent structure.

CHAPTER 7

A FREGEAN PICTURE FURTHER DEVELOPED

We have developed an exegesis of Frege's conception of truth as an object and looked into how the notions related to truth—judgment, assertion, knowledge, logic, etc.—are elucidated under this conception. In this last substantial chapter I turn to examining how Frege's philosophy we've retrieved can contribute to contemporary philosophy. Of course, to provide full answers to that question will take multiple books. However, I would like to provide a showcase for the potential of Frege's philosophy here.

Specifically I deal with how Frege's conceptions of judgment, assertion, and knowledge can be further developed. In regards to judgment I address Peter Hanks's recent criticisms of the Fregean conception of judgment in general. I show that Frege's own version—unlike other versions of the Fregean conception—can cope with some crucial

criticisms of Hanks. In regards to assertion I deal with the problem of the relationship between assertion and truth. I argue that if we accept Frege's conception of assertion, we can explain why we have a *pro tanto* reason to try to assert only truths. Finally, I develop the 'Frege-Millikanian theory of knowledge' by combining Frege's conception of knowledge with Millikan's theory of objectual knowledge.

7.1. JUDGMENT: FREGE VS. HANKS

Hanks (2007, 2011, 2015, 2018) recently suggests a theory of propositions that arguably goes against the Fregean conception of proposition in general. According to the latter, propositions are abstract entities that represent the reality in certain ways and thereby bear truth-conditions. Hanks's theory of propositions regards (assertoric) propositions as act-types—types of judgments. Hanks partly supports his theory by criticizing the Fregean conception of judgment. Our discussion concentrates on Hanks's criticisms. I argue that Frege's own conception provides satisfactory replies to Hanks's criticisms.

7.1.1. THE TRUTH-CONDITION OF A JUDGMENT

The Fregean conception in general considers propositions, i.e., thoughts in Frege, as primary bearers of truth-conditions.¹ Hanks insists that, under the Fregean conception, judgments come to have truth-conditions *only* because they are related to thoughts in a certain way. Therefore, Hanks (2015: 42) concludes, the Fregean conception ought to answer the question of propositional unity, i.e., the question as to how an abstract

¹Fregean propositions are force-free contents. According to the Fregean conception, content is sharply distinguished from force. Hanks denies the content-force distinction. Hanks's propositions include force as their parts. See Hanks 2007, 2011, and 2015 (esp. Chapter 4).

entity like a thought can have a truth-condition independently of any human activity.^{2,3} Hanks's point is that the question of propositional unity is perhaps too difficult (2007: §2; 2015: ch.2; 2018: §3); it has never been answered satisfactorily. According to him, it is a fundamental mistake to explain the truth-condition of a judgment by appealing to that of a thought; it is to put the cart before the horse. Hanks maintains that we ought to explain the truth-condition of a thought by that of a judgment. The act of judging is representing the world in a particular way, and so it has a correctness condition. Its correctness is its truth. Therefore, judgments are primary and independent bearers of truth-conditions (Hanks 2015: 66). Thoughts inherit their truth-conditions from the truth-conditions of judgments.⁴

It is controversial whether Hanks's criticisms against the attempted explanations of propositional unity are fair. But the issue I would like to address is something different: why can't Fregeans accept that a judgment has its truth-condition by its own nature, i.e., independently of thoughts? For Fregeans, to judge that p is to acknowledge the truth of $\langle p \rangle$. Say that to acknowledge the truth of $\langle p \rangle$ is to take $\langle p \rangle$ to be true. Then, judging that p is correct iff $\langle p \rangle$ is true. It seems as though Fregeans can insist that, whether or not $\langle p \rangle$ is actually a truth-bearer, judging that p has a correctness condition equivalent to the truth-condition of $\langle p \rangle$ solely because it is acknowledging the truth of $\langle p \rangle$. Even if

²Frege's answer to the question of propositional unity appeals to the metaphorical notion of saturation. See, e.g., Frege 1891, 1892b, 1892a, 1906c. Hanks (2015: §2.1) criticizes it by saying that Frege's answer leaves propositional unity primitive and impenetrable and it does not help us to understand how we represent the world in our mental acts. There are a number of recent papers on this issue. See, e.g., King 2013, King, Soames, and Speaks 2014, Pickel 2019, etc.

³According to Rojczak and Smith (2003: 173), Frege attempts to objectify knowledge by detaching thoughts from the mind. According to them Frege's move belongs to 'part of a larger historical process' (2003: 173) in which a number of different philosophers and logicians at the time of Frege play crucial roles.

⁴According to Hanks, (assertoric) propositions are not contents but act-types, namely, types of judgments as mental acts. See Hanks 2011 and ch.3 of Hanks 2015.

thoughts turn out not to be bearers of truth-conditions, that only means that $\langle p \rangle$ cannot be true because it cannot be a truth-candidate, and so judging that p is incorrect. This shows that even if Fregeans fail to explain propositional unity, they can still explain why judgments have correctness conditions equivalent to truth-conditions.

According to Hanks, however, Fregeans cannot make this move. He writes:

Judgment, on the Fregean view, is an act of endorsing [or acknowledging] a proposition. . . . We cannot say that to endorse a proposition is to accept it as true. To accept a proposition p as true is either to judge that p is true or it is a neutral, non-committal act of attributing truth to p . If accepting p as true is to judge that p is true then we've analyzed one judgment, judging that p , in terms of another, judging that p is true. This leads to a regress. . . . So, the act of endorsing a proposition cannot be analyzed as accepting a proposition as true. . . . It looks as though the Fregean is going to have to view judgment as a primitive attitude one can bear to a proposition. (Hanks 2015: 45)

The second horn of the dilemma obviously cannot function as an analysis of judgment. Therefore, the major reason why Hanks (2015: 16) believes that the Fregeans ought to leave judgment *qua* acknowledgment of truth 'peculiar and incomparable'—i.e., primitive—consists in the *vicious infinite regress* Frege is appealing to in the indefinability argument.

But Hanks is not entirely correct in saying that Frege leaves judgment primitive. Frege does provide an elucidation of judgment according to which it is non-judgmental identification. Frege is saying that judgment is peculiar and incomparable not because he has nothing to say about it, but because he considers non-judgmental identification as a fundamental, unanalyzable, type of act. One might object that in the end what it takes to stop the vicious infinite regress is to make the relationship between truth and judgment non-analyzable and primitive. However, that is not the case. What it

takes to block the vicious infinite regress is to provide the elucidation of judgment under which it is not truth predication. That is because the vicious infinite regress is produced only if judging that p is predicating the property *truth* of $\langle p \rangle$. Frege is well aware of this point. What our discussion shows is that he attempts to clarify the nature of identification *to the extent* that it becomes clear that what he has in mind is non-judgmental identification. Taking judgment to be such non-judgmental identification Frege could avoid turning judgment into truth predication. Of course, Frege does take thoughts to be legitimate bearers of truth and truth-conditions—not because they have the property *truth* but because sentences refer to the True by virtue of their thoughts. Therefore, he still needs to answer the question of propositional unity in one way or another. However, in any event, Frege does not need to be committed to the claim that the truth-condition of judgment is derived *only* from that of thoughts.

Now, Frege can elucidate why judgments have truth-conditions as follows. Recall: to judge that p is to identify $\|p\|$ with the True non-judgmentally. It is correct to identify $\|p\|$ with the True non-judgmentally just in case p is the True. Therefore, judging that p is correct just in case p is the True. Now, p is the True iff p . Accordingly, judging that p is correct iff p . This inference shows that judging that p has its correctness condition where correctness is just truth not because it derives its correctness condition from the truth-condition of a thought but because it is an act of *identification* that has its own correctness condition. In Frege's conception, the independence of judgments' truth-conditions from thoughts' truth-conditions is even clearer. The correctness condition of identification obtains whether or not thoughts actually have truth-conditions. One might object that p is the True iff $\langle p \rangle$ is true and hence that the correctness condition of identification is merely a disguised truth-condition of $\langle p \rangle$. That objection presupposes

that ‘... is the True’ is a metalinguistic truth-predicate and that ‘true’ is also such a predicate in Frege. However, both assumptions are not warranted.

7.1.2. GRASPING A THOUGHT AND JUDGING

There is Hanks’s other important complaint against the Fregean conception that can be answered by Frege. The Fregean conception requires there to be an act of grasping—or entertaining—the thought the truth of which we are about to acknowledge. To grasp a thought is to identify and single out a thought among many others. As such, grasping a thought is not making a commitment to the truth of the thought. It is rather a neutral, non-committal, mental act that is commonly found not just in the act of judging but also in the act of asking, hypothesizing, etc. On the act of grasping a thought, Hanks writes:

... Prior to judging that p , we do things like ask whether p , or hypothesize that p , ... These are the kinds of actions that Frege points to when he wants to illustrate entertainment. The error comes in making these actions preconditions for judgments. In some cases acts of judgment are preceded by entertainment-like preliminaries, but they do not have to be. Entertainment has some intuitive appeal, but only because sometimes we do things that look like Fregean entertainment. (2015: 35–6)⁵

His point is that although sometimes an entertainment-like stage precedes a judgment, it is a mistake to posit that every judgment is necessarily preceded by such a stage.

We need to be a little cautious when we understand Hanks’s criticism. His criticism is not that the Fregean conception entails that an entertainment *temporally* precedes a judgment in each and every actual case, which appears to be implausible. Hanks admits that the Fregean’s two-stage understanding of judgment, grasping a thought

⁵Indeed, in many previous readings, grasping a thought is described as Hanks here describes. For instance, see Bouveresse (1982: §1)’s or Currie (1980)’s discussion.

and acknowledging its truth, is based on ‘abstractions from the overall act of forming a judgment . . . which we can distinguish as theorists’ (Hanks 2018: §2). Hanks’s point is that in this theoretical framework, entertainment *logically* precedes judgment: we cannot acknowledge the truth of a thought without antecedently grasping the thought. He seems to take this implication to be an ‘error’.

I am not really sure that it is erroneous to say that entertainment of a thought is a precondition for judgment. But let us concede for the sake of argument that it is indeed an error. Also, it ought to be pointed out that Frege does seem to say that entertaining $\langle p \rangle$ is a precondition for judging that p :

Whenever anyone recognizes something to be true, he makes a judgment. What he recognizes to be true is a thought. It is impossible to recognize a thought as true before it has been grasped. (Frege 1915: 251)

What I want to argue is that given Frege’s conception of judgment, he does not have to be committed to the claim that entertaining $\langle p \rangle$ is a precondition for judging that p . Of course, the fact that one judges that p entails the fact that one grasps $\langle p \rangle$ while the latter does not entail the former. But that does not necessarily entail that judging that p is to be logically preceded by grasping $\langle p \rangle$. Under Frege’s conception of judgment, judging that p can sometimes come to be a precondition for grasping $\langle p \rangle$.

Say I’ve read Olga Tokarczuk’s *The Last Stories*. I see my friend who studies Polish literature having a conversation with her colleague. My friend utters ‘Olga Tokarczuk’s *Ostatnie historie* explores death’. I gently intervene and ask ‘Just like *The Last Stories*?’ My friend asserts ‘*Ostatnie historie* is *The Last Stories*’. Because I trust her knowledge in Polish literature, I accept her assertion as true. What do I accept here? By making the assertion my friend is saying that $\langle \textit{Ostatnie historie} \textit{ is } \textit{The Last Stories} \rangle$ belongs to the True. By *accepting* her assertion I am also saying that $\langle \textit{Ostatnie historie} \textit{ is } \textit{The Last Stories} \rangle$

belongs to the True. Thus my acceptance of my friend's assertion is my judgment that *Ostatnie historie* is *The Last Stories*. In my judgment my grasping the relevant thought occurs. However, is grasping it a precondition for my judgment? I do not think so. It rather seems that before my judgment I could not grasp the thought. Before I hear my friend's assertion I do not even know whether 'the sense of "Ostatnie Historie"' refers to anything. Hence at this stage I cannot grasp $\langle \textit{Ostatnie Historie is The Last Stories} \rangle$. Now I hear the assertion. But my hearing it does not yet let me grasp me the thought. If I were to take what my friend utters to be gibberish I would not be able to recognize the existence of the sense of '*Ostatnie is The Last Stories*'. My recognition of the sense is made when I accept my friend's assertion. By saying that $\langle \textit{Ostatnie historie is The Last Stories} \rangle$ belongs to the True I recognize the existence of the sense of '*Ostatnie historie is The Last Stories*', i.e., $\langle \textit{Ostatnie historie is The Last Stories} \rangle$, and thereby recognize that of '*Ostatnie historie*'. Hence in this case my judgment is arguably a precondition for my grasping the relevant thought.⁶ Of course, even in this case, grasping $\langle p \rangle$ must occur with judging that p because the latter is saying that $\langle p \rangle$ belongs to the True. However, that does not mean that grasping $\langle p \rangle$ is a precondition for judging that p .

One might object that this is just an exceptional case and in most of cases grasping $\langle p \rangle$ is a precondition for judging that p . I do not think that this is just an exceptional case. What the previous case shows is that we can recognize other speakers' acts of

⁶This point is related to a main thesis of neo-logicism in philosophy of mathematics. Neo-logicism that inherits its main ideas from Frege holds the 'syntactic priority' thesis: the claim that 'if we speak truly, the structure of reality inevitably mirrors the contours of our speech' (MacBride 2003: 108). For this point, also see Wright 1983. If we accept this principle, the fact that we take a certain syntactic structure with a singular term ' s ' to be true provides a sufficient reason to think that there is an item in the world that can be identified as s . What I am arguing here is that Frege can say the same thing about sense: the fact that we take ' $a = b$ ' (where b is not a name known to use antecedently) to be true provides a sufficient reason to think that ' b ' has a sense. If so, we can backtrack the sense of ' b ' after we take the identity sentence to be true.

non-judgmental identification and through these recognitions we can grasp the senses of terms and sentences we've never heard of before. I believe that the use of this ability occurs much more often than this objection assumes and plays an important role in our language learning. A baby must learn who is 'Mama' by recognizing that people refer to her mother by that sound. Also, we appear to learn new abstract or theoretical terms by recognizing other speakers' assertions of sentences that have those terms. In such cases we grasp thoughts partly constituted by the senses of those terms through recognizing the judgments expressed by those assertions—as we do in the above case.

One might object that the distinction between grasping a thought and judging is still an artificial product of Frege's conception of judgment that posits the existence of thoughts. I disagree. The gap between a name's sense and its reference has epistemic reality. We know *something* about the name 'Odysseus'. First of all we know that it would be a name of a person if there were Odysseus. We can also use the name as such in a number of sentences. What is the most important thing is that we understand that the use of the term is not arbitrary—just as its sense is supposed not to be arbitrary. Nevertheless, whatever we know about it does not let us decide whether the name has a reference or whether Odysseus is identical with the reference of a name whose sense and reference are known to us. If we call whatever it is that we understand about 'Odysseus' 'the sense of "Odysseus"', we are in a position where we grasp ⟨Odysseus⟩ but fail to seal the sense-reference gap. In this way, Frege's notion of grasping a name's sense is grounded in the epistemic reality concerning our knowledge of names. Now, exactly the same elucidation of grasping can be applied to the act of grasping a thought *if* we accept that sentences are names. The act of asking, hypothesizing, etc. can be taken to show that there can be a gap between a thought and its truth-value just like

the gap between \langle Odysseus \rangle and Odysseus.⁷

A due question is then what are the reasons to take sentences to be names. As we've seen Frege provides a number of arguments that sentences are proper names of truth. On top of that, based on this new conception of truth and sentence, Frege develops quite new conceptions of related notions like judgment, assertion, knowledge, logic, etc. These new conceptions, which appear to deserve thorough examination, provide more reasons to take the possibility seriously that sentences are names of truth.

7.2. ASSERTION: TRUTH NORM AND PERFORMATIVE FLAW

Say that, for any false assertion, we have a *pro tanto* reason to take it to be flawed.⁸ A popular way to explain this is to depend on the notion of *constitutive norm*.⁹ A number of theorists (M. Weiner 2005; Whiting 2013, 2015; Williamson 1996, 2000) maintain that assertion is normatively constituted activity and that the constitutive norm of assertion is—or implies—that one must assert that p only if $\langle p \rangle$ is true.¹⁰ If we accommodate this normative conception of assertion, any false assertion violates the norm constitutive

⁷If so, those acts are not presented as preliminary stages for judgments.

⁸Williamson (2000) uses the terms like 'entitled to feel resentment' instead of 'taking it as flawed'. However, using this kind of term seems to be misleading. Whether we feel resentment or not is not necessarily connected to whether an assertion is flawed or not. Some of the criticisms against the constitutive norm approach seem to be caused by this misleading term. For instance, see Johnson (2015a: §2.2).

⁹The intuition that a false assertion is flawed is taken as supporting the thought that it is a constitutive norm of assertion that one must assert that p only if the proposition that p or $\langle p \rangle$ is true. See M. Weiner (2005) or Whiting (2013, 2015). It is more often argued that assertion is flawed when the asserter does not know that the asserted proposition is true; and this is taken as supporting that it is a constitutive norm of assertion that one must assert that p only if one knows that p (Littlejohn 2014; Williamson 1996, 2000). There is a debate about the truth norm and the knowledge norm of assertion. However, this debate does not influence the discussion in this paper. Even if we take the knowledge norm as the constitutive norm of assertion, a false assertion will still be taken as flawed.

¹⁰M. Weiner (2005) and Whiting (2013, 2015) argue that the constitutive norm of assertion is truth, while Williamson (1996, 2000) and Littlejohn (2014), who argue that knowledge is the constitutive norm of assertion, would say that the constitutive norm of assertion *implies* that truth is the constitutive norm of assertion. For the former, truth is the '*basic*' (Montminy 2013: 40) constitutive norm of assertion while it is not for the latter.

of assertion and thus there is a *pro tanto* reason to consider it to be flawed.

However, this would certainly not be the only possible way of explaining that we have a *pro tanto* reason to take a false assertion as flawed. Another way would be to endorse the following type of explanation:

(Performative Explanation: PE) Assertion is essentially related to truth in a particular descriptive manner. Given this way assertion relates itself to truth, asserting a false proposition is always a *flawed performance* of assertion.

Let us assume that for any false assertion we have a *pro tanto* reason to consider it to be flawed.¹¹ The task I undertake here is to construct a descriptive elucidation of the relationship between assertion and truth on which we can build up a plausible PE for the *pro tanto* flaw of a false assertion. ‘Descriptive elucidation’ means an elucidation that relates truth to assertion, but does not entail that truth is the constitutive norm governing assertion. For instance, Wright (1992) explain that

(1) To assert that *p* is to present $\langle p \rangle$ as true.¹²

(1) does not entail that truth is the constitutive norm of assertion. (1) falls under what I call ‘descriptive elucidation’.

One might wonder why we should bother to develop a PE based on a descriptive elucidation of the relationship between assertion and truth. Think of a person who shares the intuition that we have a *pro tanto* reason to consider a false assertion to be flawed, but does not share the thought that an assertion is an essentially norm-laden action. For such a theorist, to develop a version of PE would be a viable option for

¹¹A similar idea has been suggested and defended with respect to the state of belief. Theorists like Shah (2003) and Velleman (2000) argue that a belief in a false proposition is a flawed state. Lynch (2009a, 2009b) claims that it is correct to believe $\langle p \rangle$ just in case $\langle p \rangle$ is true.

¹²Ascribing (1) to Frege, Bar-On and Simmons (2007) provides an extensive and helpful discussion about the conception of asserting as presenting-as-true.

explaining the *pro tanto* flaw of a false assertion. A PE, of course, can have a normative implication, which is that *if one desires* to assert in a flawless manner, one must assert a truth. This implication seems relatively less controversial than the claim that assertion is essentially norm-laden.

Even for theorists who happily embrace that an assertion is essentially norm-laden, there are some reasons to be interested in developing a PE. Timothy Williamson writes:

Given a speech act *A*, one can ask ‘What are the rules [i.e. the constitutive norms] of *A*?’. Given an answer, one can ask ‘what are non-circular necessary and sufficient conditions for a population to perform a speech act with those rules?’. (2000: p239-40)

Circular conditions for performing a speech act *A* with the constitutive norm ϕ would look like this: One performs *A* if and only if one performs a speech act the constitutive norm of which is ϕ . Williamson says that finding non-circular conditions for performing a given speech act is a ‘more ambitious’ (Williamson 2000: 239) and philosophical project. I understand the idea underlying Williamson’s comments as follows. Suppose that one asks why a false assertion is flawed. It appears better to answer this question by saying ‘Assertion is related to truth in such and such a way and because of this relationship truth is the norm governing an assertion’ than by saying ‘A false assertion is flawed’ or ‘We should assert truth’—as it seems better to answer why stealing is immoral by saying ‘Stealing can be described as such and such an act and our general moral principles imply that such an act is impermissible’ than by simply saying ‘It is just immoral’ or ‘It is a given moral norm not to steal’.¹³ To search for such a better

¹³It seems that in the literature on moral issues, it is a common project to develop the former kind of answer to why a certain type of action is impermissible. For instance, Dougherty (2013) tries to answer why rape is impermissible in the above way. He explains rape as having sex without the other’s consent and that is what makes rape impermissible. This kind of project is often found in the literature on moral

looking answer certainly seems like a more ambitious and philosophical project.¹⁴ If Williamson's idea thusly understood is plausible, then one who thinks of assertion as norm-laden still has a reason to be interested in developing a PE because a plausible PE can be a source for the more satisfying way to answer why a false assertion is flawed or why truth is a constitutive norm of assertion.¹⁵

Therefore, as far as we hold that there is a *pro tanto* reason to take a false assertion as flawed, there are reasons for attempting to develop a PE. My aim in this section is to establish that Frege's conception of assertion as non-assertive identification of the True provides us with a plausible PE. I first review the previous descriptive explanations of assertion and show why those explanations fail to provide a PE. This process will provide us with a better idea about what a successful PE would look like. Then I turn to constructing a PE from Frege's conception of assertion.

7.2.1. A REVIEW OF THE PREVIOUS DESCRIPTIVE EXPLANATIONS

Let us start with Wright's elucidation:

- (1) To assert that *p* is to present $\langle p \rangle$ as true. (Wright 1992)

issues. To the question as to why killing a person is impermissible, Marquis (1989) responds by saying that killing a person is destroying a future-like-ours, which is impermissible.

¹⁴MacFarlane (2011) is motivating this kind of project in a similar way. MacFarlane points out that the constitutive norm of assertion, if any, does not tell us anything about what it is to make an assertion or how to make an assertion MacFarlane (2011: 86). However, we can independently pursue the goal of answering these unanswered questions. Given the list of the conditions when assertion is incorrectly performed, we can duly expect these independently developed descriptions of assertion to explain why assertion is incorrectly performed under those conditions MacFarlane (2011: 93–4).

¹⁵Recently, theorists argue that assertion does not have any constitutive norm. For instance, see Cappelen (2011) or Casey Johnson (2015a: Chs.1 and 2; 2015b). These criticisms against the constitutive norm approach to assertion attack the data for this approach. Those data mostly come from individual intuitions about assertions made in certain contexts, e.g. assertions about lottery tickets. A plausible PE of the *pro tanto* flaw of a false proposition can provide a brand-new support for the constitutive norm approach. Such a PE is based on the essential relationship between assertion and truth and hence can be used to show that any false assertion is flawed. This seems to constitute a reason to think that truth is a constitutive norm of assertion. This support from a PE for the constitutive norm approach is not influenced by the criticisms of the above-mentioned theorists.

This platitudinous elucidation seems to fail to show that a false assertion is a flawed performance of assertion.¹⁶ I can present my colleague Tom as running now by drawing a perfect picture of him running although he is not running right now. However, the fact that Tom is not running right now does not make my presentation of him as running a flawed performance of presentation. Then, it is not evident why we should think that the fact that the proposition presented as true is not true makes it a flawed performance of presentation to present it as true. Hence, (1) fails to provide a plausible PE (performative explanation) for the *pro tanto* flaw of a false proposition. Hence, it is not the kind of elucidation we are looking for.

One might object that presenting-as-true, unlike presenting-as-running, produces the assertoric force and hence amounts to assertion. If so, we may be able to say that presenting ⟨Obama is the current president of the United States⟩ as true is flawed as asserting that Obama is the current president of the United States is flawed. However, this does not help because the question we are dealing with is why a false assertion is flawed. Furthermore, it is unclear that presenting-as-true necessarily produces the assertoric force. The sentence ‘The thought that Obama is the current president of the United States is true’ seems to present ⟨Obama is the current president of the United States⟩ as true, ‘even if the sentence is not uttered assertorically’ (Pagin 2016: §5.1).

A more promising elucidation seems to be this:

(2) To assert that *p* is to make a commitment to the truth of ⟨*p*⟩ (Wright 1992).¹⁷

One might contend that (2) is what we are looking for, saying that to make a commitment to the truth of a false proposition is indeed flawed. I can agree that a subject’s

¹⁶(1) is taken by Wright (1992) as a platitude about truth. These platitudes work as ingredients for a network analysis of the concept TRUTH in truth pluralism.

¹⁷MacFarlane (2011: §.4) provides a helpful discussion of this idea.

making a commitment to the truth of a false proposition supports that the subject is flawed epistemically and/or often morally. Nonetheless, it is not evident that making a commitment to the truth of a false proposition is a flawed performance of making a commitment. Even if a subject makes a false assertion for certain reasons, e.g. due to misleading evidence she has, the subject makes a commitment to the truth of the proposition issued in a flawless manner. That is why her assertion is taken seriously. It is in that the *commitment* is flawed that we say that it is flawed to make a commitment to the truth of a false proposition. The commitment usually comes along with a variety of responsibilities, and perhaps the falsity of the asserted proposition makes some of those responsibilities unfulfilled. The unfulfilled responsibilities provide reasons to think that the commitment, and hence the subject, are flawed. However, this does not necessarily mean that the subject *makes* a commitment to the truth of a false proposition *in a flawed manner*. If so, (2) cannot be what we are looking for. What we need is a description of assertion that explains why asserting a false proposition is a *flawed performance* of assertion.

Note (2) describes assertion in terms of the effects of assertion; a commitment to the truth of $\langle p \rangle$ is an effect of asserting that p (MacFarlane 2011: 91). How do we produce such an effect then? Truth is usually regarded as a property of propositions, and thus a tempting answer seems to be that we make such a commitment by predicating the property *truth* of $\langle p \rangle$.¹⁸ This provides another way to describe assertion in terms of truth:

(3) To assert that p is to predicate *truth* of $\langle p \rangle$.

There are two different conceptions of predication.¹⁹ In the first, to predicate a property

¹⁸Below, '*truth*' means 'the property *truth*'.

¹⁹These two different conceptions of predication are now familiar to us thanks to the debates between

P of an object o is to compose the proposition that o is P . If we conceive predication in this way, however, we cannot explain why asserting a false proposition is flawed performance of assertion. Even though Dostoevsky is not an American writer, predicating *American writer* of him in this sense can be flawlessly performed. In the second, to predicate P of o is to assert that o is P . To predicate *truth* of a false proposition in this sense might be regarded as a flawed predication—if we already accept that a false assertion is flawed. But what we need is the explanation about why asserting a false proposition is flawed.

Thus none of (1), (2), and (3) is the elucidation of assertion we are looking for; they do not provide a PE for the *pro tanto* flaw of a false assertion. We can retrieve such a PE from Frege's conception of assertion.

7.2.2. A FREGEAN PERFORMATIVE EXPLANATION

In Frege's conception, to assert that p is to identify the True with $\|p\|$ non-assertively—without asserting that $\|p\|$ is the True. An assertion as such is expressive of a judgment as a non-judgmental identification of the True. Recall that the act of judging is grounded in our ability to identify the True. Thus judging is using the ability to identify the True. Specifically it is a *mental* use of the ability. Then, we can regard the act of asserting as a *verbal* use of the ability to identify the True insofar as asserting is an attempt to express a judgment. If we describe assertion as such, then we describe it in terms of its process. Then, what is the result of this process? Again in the case of judging *qua* a mental use of the ability to identify the True, what we achieve by judging that p if it is correctly performed, i.e., if $\|p\|$ is indeed the True, is the knowledge_{np} that $\|p\|$ is the

Scott Soames and Peter Hanks (APA Session on *New Thinking about Propositions*, St. Louis, Missouri, February 21, 2015).

True. In the case of asserting *qua* a verbal use of the ability to identify the True, what we achieve by asserting that p if $\|p\|$ is the True is the *expression* of the knowledge_{np} that $\|p\|$ is the True.

Thus in Frege's conception assertion is regarded as a process of identifying the True and assertion as such has a correctness condition in terms of the result of the process constitutive of assertion. If the process P , which has C as the condition for the process's being correctly performed, fails to meet C in a certain case, that constitutes a reason to take P to be performed incorrectly—in a flawed manner—in the case. Then we have the following result: if one asserts that p and $\|p\|$ is not the True, one's asserting that p is performed in a flawed manner. Let us see whether this claim obtains case-by-case.

Say that S asserts that p and $\|p\|$ is not the True. Now, we can think of two cases. In one case S would not assert that p if S knew that it is not the case that p . In the other case S would assert even if S knew that it is not true that p . In the first case S asserts that p because S does not know that it is not true that p . In other words, S 's ability to identify the True is not sufficiently strong to say that $\langle p \rangle$ does not belong to the True. In other words S 's ability to identify the True is flawed. A flawed ability to carry out a process results in a flawed execution of the process. In this case S 's assertion that p is grounded in a verbal use of S 's flawed ability that is relevant to the assertion. Thus, S 's assertion is a flawed performance of assertion *qua* identification of the True when S does not know that the relevant thought is false.

In the other case S might already know that it is not true that p or does not know whether p . In this case, S is using her ability to identify the True in an improper way. In particular S uses her ability in order to identify the True with $\|p\|$ although S knows that her proper use of the ability would yield a different result or leaves the possibility

open that her proper use of the ability might turn out to lead to a different result. A *crooked use* of an ability to carry a process yields a *crooked performance* of the process. *S*'s assertion is grounded in *S*'s crooked use of the relevant ability. Thus it is a flawed performance of assertion.

One might object that in the second case *S*'s assertion does not use the ability to identify the True. If this objection is correct, then we cannot explain why *S*'s assertion is a flawed performance of assertion by appealing to *S*'s use of the ability. But I disagree. The ability to identify the True is the ability to single out the True and label it with a certain expression. Even in the second case *S*'s utterance of '*p*' singles out the True and labels it with '*p*'. That is why *S*'s utterance is *S*'s assertion. Hence, *S*'s assertion even in this case is grounded in the use—though crooked—of *S*'s ability to identify the True. Such a crooked use of the ability is still a use of it.

Hence we come to have a PE for the *pro tanto* flaw of a false assertion by accepting Frege's conception of assertion. Under the conception to assert is to identify the True. Assertion as such is grounded in our verbal use of the ability to identify the True. We make a false assertion because our ability to identify the True is flawed or we use the ability in a crooked way. In any case our act of identifying the True is performed in a flawed manner. Therefore a false assertion is always performatively flawed.

7.3. KNOWLEDGE: THE FREGE-MILLIKANIAN THEORY

Lastly, I sketch what I call 'the Frege-Millikanian theory of knowledge'. Even though the theory is grounded in Frege's conception of knowledge, it is not solely *his* theory. Frege never attempts to build up a theory of knowledge, and hence his conception of knowledge has explanatory gaps. I believe that we can seal off these gaps by consulting Millikan's theory of objectual knowledge and identity knowledge that is pretty similar

to Frege's conception of them. Thus, the theory of knowledge we come to have in the end can rightly be called 'the Frege-Millikanian theory'.

7.3.1. THE BASIC COMPONENTS OF THE THEORY

Let us start with the objectual-knowledge (o-knowledge) of the True. We've said that the o-knowledge of the True gives us the ability to identify the True non-judgmentally. What is this o-knowledge of the True? Frege does not provide a specific answer to this question. At this point it is beneficial to point out the parallel between Millikan's theory of o-knowledge and Frege's conception. In the latter, we've just said, the o-knowledge of an object *O* *provides* the ability to identify *O*. Millikan's theory also connects the o-knowledge of *O* to the ability to identify *O*. But she puts a more immediate relationship between them; for her the o-knowledge of *O* is *just* the ability to identify *O*. As we've said, Millikan calls this ability 'the substance concept of *O*' (Millikan 2000). The substance concept of *O* is the ability to identify *O* via a variety of media including perceptions and *language*. Here 'identification' always means non-judgmental identification.

So Millikan leads us to a simple way to fix the nature of o-knowledge. The following is one of the theses of the Frege-Millikanian theory of knowledge:

(FMK₁) The o-knowledge of *O* is the ability to non-judgmentally identify *O* via a variety of media.

Let 'identification_{nj}' and 'identify_{nj}' refer to the act of non-judgmental identification. 'The ability to say that $\langle t \rangle$ belongs to *O*' in Frege's terminology is 'the ability to identify_{nj} *O* via the medium of '*t*' (or its sense $\langle t \rangle$) in this theory. By (FMK₁) the o-knowledge of the True is the ability to identify_{nj} the True via various media.

As we've said (FMK₁) might sound extremely implausible. Isn't the o-knowledge

of O just a collection of the 'propositional knowledge (p-knowledge) that O has such-and-such a property'? It cannot be denied that there are pieces of p-knowledge *about* an object. Millikan, nonetheless, points out that if we cannot identify the object O and reliably re-identify it in various circumstances, we cannot pile up such p-knowledge about O. Our ability to identify O is a precondition for our acquiring such p-knowledge about O. What Millikan is doing is to define this ability as the o-knowledge of O. This ability is *distinctively objectual* knowledge in the sense that it cannot be reduced down to p-knowledge about O. Frege, I believe, is also looking for such distinctively objectual knowledge via a different route. He appears to believe that the identity knowledge we attempt to express by the claims of the form ' $a = b$ ' has a distinctively non-propositional character because it is not produced by the act of judging that a is b . Frege takes this identity knowledge to spring from our ability to identify an object and labels what provides this ability with 'o-knowledge'.

The parallel between Millikan and Frege goes through and through. Both of them attempt to provide a case for the existence of non-judgmental identification. Moreover, they both attempt to put our knowledge of identity at the center of our cognitive life. It is an ironic fact that Millikan identifies her main antagonist as the 'Fregean' for whom identification is always judgmental identification. Certainly in many readings Frege is a Fregean in this sense. However, Frege is not a Fregean under my suggested reading. He is rather a Millikanian.

Although Frege is a Millikanian in many aspects, there are also differences between these two philosophers. In a certain sense Frege's conception of knowledge of identity is less radical than Millikan's theory. Her point is not just that our act of identifying is non-judgmental. Her point is that there is nothing like an identity thought of the form

$\langle a = b \rangle$ or that there is no identity relationship that holds of the same object. All there is concerning what we have called 'identity' is the relationship between an object and the representational content that represents the object. Hence, there is nothing like the p-knowledge that a is b . Frege would not accept it. For him there are identity thoughts $\langle a = b \rangle$ and there is the relationship of identity that holds of the same object. Thus we can make a judgmental identification, i.e., a judgment that a is identical with b and can have the p-knowledge that a is b .

Still, Frege is more radical than Millikan in a different aspect. This is because unlike Millikan, Frege makes our p-knowledge grounded in our o-knowledge—specifically our o-knowledge of the True. First, following Frege, the Frege-Millikanian theory of knowledge puts forward the following:

(FMK₂) If the identification_{nj} of O via a proper name ' O_p ' is successful, it produces the knowledge_{np} that O_p is O .

Judging that p is identifying_{nj} the True via ' p '. Thus, we have this:

(FMK_{2J}) If the act of judging that p is successful, it produces the knowledge_{np} that $\|p\|$ is the True.

The knowledge we produce by judging that p is the p-knowledge that p . Thus we have the following:

(FMK₃) The p-knowledge that p is the knowledge_{np} that $\|p\|$ is the True.

The Frege-Millikanian theory permits the p-knowledge that a is b . However, that does not entail that there is p-knowledge of identity distinguished from non-propositional knowledge of identity. The p-knowledge that a is b is merely the knowledge_{np} that $\|a$ is $b\|$ is the True. The p-knowledge that a is b is the non-propositional identity knowledge of the True while the knowledge_{np} that a is b is that of the object denoted by ' a ' and ' b '. Finally,

(FMK₄) To identify_{nj} O via a proper name 'O_p' is to use the o-knowledge of O, i.e., the ability to identify_{nj} O.

By (FMK₄), to judge is to use our o-knowledge of the True. Hence, our p-knowledge is epistemically grounded in our o-knowledge of the True.

Two points must be clarified: the ability to identify_{nj} an object and the knowledge_{np} that O₁ is O₂. First, in what does the ability to identify_{nj} an object consist? We recognize and identify objects by our cognitive abilities such as perceptual abilities. Accordingly, it seems natural to think that the ability to identify_{nj} an object consists in these cognitive abilities. Thus the Frege-Millikanian theory puts forward the following:

(FMK₅) The ability to identify_{nj} O via a medium M is epistemically grounded in the cognitive ability by which we can identify_{nj} O via M.

Say that I identify_{nj} my wife, who is approaching me, through my visual perception. In this case, I am using the ability to identify_{nj} my wife via a visual perception and this ability is epistemically grounded in my perceptual ability that is one of my cognitive abilities. I say: I am using the *perceptual* ability to identify_{nj} my wife. The perceptual ability to identify_{nj} my wife constitutes a *sub*-ability of the *general* ability to identify_{nj} my wife. There can be different sub-abilities of this general ability that are grounded in various cognitive abilities. For instance, my ability to identify_{nj} my wife via linguistic expressions like 'Mary' or 'the wife of Junyeol' is my *linguistic*—or *conceptual*—ability to identify_{nj} her that depends on my linguistic ability. My linguistic ability to identify_{nj} my wife is also a sub-ability of the general ability to do so.

It goes along with Frege's own view to regard the ability to identify_{nj} an object as constituted by such sub-abilities grounded in cognitive abilities. Frege himself thinks that objects are in the end given to us through our cognitive abilities. He writes:

Stars are objects of sense perception. (1925: 279)

He considers sense perception to be one of the sources of knowledge (Frege 1924)—the faculties (Anlage) for knowledge.²⁰ What Frege is asserting is that stars as astronomical objects can be identified by our perceptual ability. He also states that we can identify objects—like what he calls ‘the base plane’ (1925: 279)—by our geometrical ability that we may take to be similar to the Kantian intuition. Hence, it goes along with Frege’s intention to think of the ability to identify_{nj} an object as grounded in our cognitive abilities to do so.

Let us turn to the knowledge_{np} that O_1 is O_2 . This knowledge_{np} is produced by the act of identifying_{nj} O_1 with O_2 . We perform this act when the object O , which is the reference of both ‘ O_1 ’ and ‘ O_2 ’, is given to us as the reference of ‘ O_1 ’ (‘ O_2 ’) and then we realize that O is also the reference of ‘ O_2 ’ (‘ O_1 ’). This act of identification_{nj} is grounded in our ability to identify_{nj} O . That is, we identify_{nj} O_1 with O_2 by exerting the ability. If we successfully perform the act and achieve the knowledge_{np} that O_1 is O_2 , then our ability to identify_{nj} O is already sufficiently *strong* to let us produce that knowledge_{np}.

What I mean by ‘strong’ here is a property of an ability—how developed an ability is. Say my wife can drive a car better in an area with high mountains than I can; then my wife’s ability to drive a car is stronger than my ability to drive a car. For a task T the completion of which requires a certain ability A , whether a subject can complete T of course primarily depends on whether the subject has A . However, it is also dependent on whether the subject’s A has a strength sufficient to carry out the task. For instance, I might not be able to drive all the way up to the peak of Mount Washington not because I lack the ability to drive, but because my ability to drive is not sufficiently strong to

²⁰Here, ‘faculty’ is a translation of ‘Anlage’. In the translation of ‘Sources of Knowledge’ (Frege 1979: 267–274), ‘Anlage’ is translated into ‘disposition’. But ‘Anlage’ means an innate property of an entity that can be developed into its ability. Thus ‘faculty’ also works as its translation.

let me do so.

If our ability to identify_{nj} O is already sufficiently strong to enable us to identify_{nj} O_1 with O_2 , shouldn't we rather say that we already have the knowledge_{np} that O_1 is O_2 ? Is there any theoretical space for making sense of this knowledge_{np} supposed to be added by our identification_{nj}? I think that there is—if we consider the difference between merely possessing an ability and having actual experience of using the ability successfully. Think of a pitcher who tries to learn to throw a curveball. She learns the grip and the throw from her coach. One day the pitcher finally comes to throw curveballs successfully. At one point before that, she would already form and improve the ability to throw a curveball. However at the point when she throws successful curveballs that fall down beautifully, she experiences the successful use of her ability and that experience in turn would strengthen her ability further. Actually experiencing the successful use of an ability is even often a huge progress in one's effort to improve the ability. Taking this point into account, I say:

(FMK₆) The knowledge_{np} that O_1 is O_2 is the experience of the successful use of the ability to identify_{nj} the object by means of ' O_1 ' (or O_2).

Therefore, the knowledge_{np} that O_1 is O_2 contributes to strengthening one's ability to identify_{nj} the relevant object. By (FMK₆) we can explain what our p-knowledge adds to our cognitive repertoire. The p-knowledge that p is the knowledge_{np} that $\|p\|$ is the True, which is the experience of the successful use of our ability to identify_{nj} the True by way of ' p '. Hence, it helps to strengthen our ability to do so and thereby strengthen our o-knowledge of the True.

7.3.2. THE FREGE-MILLIKANIAN THEORY AND THE JTB ANALYSIS

Now I would like to examine how the Frege-Millikanian theory of p-knowledge can accommodate the insights of a traditional analysis of it, i.e., the JTB analysis according to which the p-knowledge that p is the justified true belief that p . This step would help us to familiarize ourselves with this new framework of knowledge.

As we've seen the Frege-Millikanian theory entails:

(FMK₆P) The p-knowledge that p is the experience of the successful use of one's ability to identify_{nj} the True by way of ' p '.

Given (FMK₆P), how can we accommodate the insight that underlies the belief condition of the JTB analysis? The idea that underlies the belief condition appears to be that p-knowledge is an attitude toward the relevant thought that can be stored in our cognitive repertoire and to which the property *truth* and the property *being justified* can be attributed. In (FMK₆P) what plays the role of belief is the experience of the use of the ability to identify_{nj} the True. Though this experience itself is not an attitude toward a thought, it is what produces something like an attitude toward a thought. Specifically, the experience of the use of the ability to identify_{nj} the True by way of ' p ' produces and strengthens one's disposition to re-identify the True via ' p ' and thereby via '< p >' over and over again. Moreover, this experience can be restored in our cognitive repertoire. Finally, as I will explain shortly, the truth condition and the judgment condition are captured by the point that p-knowledge is the experience of the *successful use* of our ability to identify_{nj} the True. Hence, the experience of the use of the ability, which constitutes p-knowledge, is tightly connected to the other condition of p-knowledge—the *successful use condition*.

Now, when is the use of one's ability to identify_{nj} the True by way of ' p ' successful?

One obvious condition is the correctness condition of the act of identification, i.e., the following:

(FMK₇) One's use of the ability to identify_{nj} *O* by way of '*T*' is successful only if *T* is *O*.

Thus, it follows that

(FMK₇P) One has the p-knowledge that *p* only if $\|p\|$ is the True.

(FMK₇P) corresponds to the truth condition of the JTB analysis of p-knowledge. However, this cannot be called 'truth condition' if we follow Frege's objectual conception of truth according to which truth is the True. Thus, we would rather call it 'the identity condition'. In any event, the identity condition preserves the insight that underlies the truth condition of the JTB analysis: the factivity of p-knowledge.

The identity condition, however, is not sufficient. A successful use of the ability to identify_{nj} an object is possible only when the ability is strong enough. Specifically,

(FMK₈) One's use of the ability to identify_{nj} *O* by way of '*O_p*' is successful only if the ability is strong enough to let us identify *O_p* with *O*.

Thus, we have the following:

(FMK₈P) One has the p-knowledge that *p* only if one's ability to identify_{nj} the True is sufficiently strong to enable us to identify_{nj} the True with $\|p\|$.

Let us call (FMK₈P) 'the strength condition'. The strength condition seizes the insight that underlies the justification of the JTB analysis: p-knowledge must be supported by truth-conducive elements. The strength of our ability to identify_{nj} the True is certainly a truth-conducive element under Frege's objectual conception of truth. Moreover, the strength condition also seems to capture the lesson from the Gettier Problem (Gettier 1963). Since the Problem, it has been widely agreed that justification is insufficient—or

even unnecessary—for p-knowledge. P-knowledge also requires a subject to have the ability to *track truth*.²¹ Given (FMK_{8P}), p-knowledge in the Frege-Millikanian theory requires a subject to have an ability to track the True. In quite a direct sense the ability condition also preserves the insights from the Gettier Problem.

To recap, the Frege-Millikanian theory of knowledge accommodates the insights of the JTB analysis of p-knowledge. I turn to clarifying a little bit more about our ability to identify_{nj} the True.

7.3.3. THE ABILITY TO IDENTIFY_{NJ} THE TRUE

That we have the ability to identify_{nj} the True implies that we have an access to the True. But how do we acquire the initial access to this abstract object?²² Frege's answer to this question is related to what he calls 'logical object' (Frege 1903b: §147). Logical objects are the objects we can access solely through our '*logical faculty*' (1903b: §147). Truth-values are supposed to be logical objects (Shramko and Wansing 2018: §1.3). If so, they are what we can access only through our logical faculty. However, Frege does not say much about 'logical faculty' although he definitely takes our inferential ability to constitute a part of our logical faculty (Frege 1925: 279).

To say we *can* access the True solely by our logical faculty is, however, not to say we *do* or *must* initially access the True by our logical faculty. Hence, Frege-Millikانيين can accept the possibility that a subject accesses the True via other routes. One possible route is via people's use of language. As we've said an assertion is a verbal counterpart of a judgment. An assertion as such can be taken to be a verbal identification_{nj} of the

²¹The idea of reliabilism has been suggested by a number of theorists in a number of different forms. Some classical works include Armstrong 1973, Dretske 1971, Goldman 1976, 1986, Nozick 1981, etc.

²²Frege never says that the True is an abstract object. However, given that it is not an object we can access by our perceptual abilities, it is mostly likely an abstract object.

True. Frege-Millikanians can say that a subject can acquire the initial access to the True via a competent speaker's (correct) assertion, just like we can acquire the initial access to Caesar by a competent speaker's (correct) use of the proper name 'Caesar'. This is basically Millikan's idea: She (2000: §1.10) believes that language enables its speakers to access things in the world. If we accept this idea, we can say that one route through which we acquire the initial access to an object is our ability to recognize the reference of a term via other people's use of the term. That is our *linguistic* ability to identify_{nj} an object. So we can access the True by our ability to recognize the reference of a sentence via other people's use of the sentence, i.e., our linguistic ability to identify_{nj} the True. As we've said in §7.1 on judgment, it is thanks to this ability that Frege's conception of judgment does not take grasping $\langle p \rangle$ to be a precondition for judging that p .

The ability to identify_{nj} the True thus includes the linguistic ability to identify_{nj} the True. This ability can also be labeled 'the *conceptual* ability to identify_{nj} the True' because it is the ability to identify_{nj} the True when it is given by a thought that can be considered as *conceptual contents*. Now, the conceptual ability to identify_{nj} the True is essentially related to our inferential faculty. Frege writes:

To make a judgment because we are cognizant of other truths as providing a justification for it is known as inferring. (Frege 1879a: 3)²³

Thus our inferential faculty enables us to proceed from previous judgments to a new judgment. So we can say: we have the inferential ability to identify_{nj} the True, i.e., the ability to identify_{nj} $\|q\|$ with the True by virtue of identifying_{nj} $\|p_1\|, \|p_2\|, \|p_3\|, \dots, \|p_n\|$ with the True.

We've identified two sub-abilities to identify_{nj} the True: the linguistic/conceptual

²³This is Frege's early manuscript 'Logic' written in 1879. The mature Frege retains the same conception of inference as 'transition to a new judgment' (Frege 1893: 1) from a previous one.

ability and the inferential ability. Can there be other ways to identify_{nj} the True? For instance, can we identify_{nj} the True by perceptual experience? The True is an abstract object, and hence the answer appears to be 'No'. The ability to identify_{nj} the True is essentially to identify_{nj} the True linguistically/conceptually. But our perceptual ability is a cognitive ability, which is closely related to p-knowledge. Hence, it is reasonable to require a theory of p-knowledge to make sense of 'perceptual knowledge'. Frege appears to be aware of this requirement because he considers sense perception as one of the sources of knowledge. He writes:

A sense impression is not in itself a judgment, but becomes important in that it is the occasion for our making a judgment. (Frege 1924: 267)

But perceptual ability does more than merely providing an occasion for our making a judgment to the extent that the defects of perceptual ability lead to errors.

[T]here are at our disposal a diversity of means for correcting the judgment gained from the [illusive vision]. (Frege 1924: 268)

We need to note that Frege is saying that a judgment is *gained from* a visual perception. From his remarks, we can read off at least three different theses regarding the relationship between our perceptual ability and our ability to identify_{nj} the True conceptually.

- (PC₁) The perceptual ability is not directly related to the ability to identify_{nj} the True conceptually. It merely enables us to perceive objects in the world and their properties, and thereby provides chances for making judgments.
- (PC₂) The perceptual ability can directly guide us to use our ability to identify_{nj} the True conceptually for making judgments concerning perceptible things.

(PC₃) One of the sub-abilities of the perceptual ability constitutes a sub-ability of the ability to identify_{nj} the True conceptually. Our judgments about perceptible things are results of using this per/conceptual ability.

None of these theses, which can be developed in a number of different ways, appears to be incompatible with the Frege-Millikanian theory. If so, Frege-Milliknians have a variety of options for the relationship between the perceptual ability and the ability to identify_{nj} the True conceptually.

Here I've attempted to give a sketch of the Frege-Millikanian theory of knowledge. Although we should clarify much more—e.g., the notion of ability—to develop it into a full theory, it goes beyond the scope of this dissertation to do so. But its potential already seems to be emerging. For one it seems to be able to explain our cognitive life in general by putting objectual knowledge at its center. Also, while doing so, it seems to recognize the insights of the traditional analysis of knowledge and to be compatible with lots of current theories of knowledge.

CHAPTER 8

CONCLUSION

When I first started this research project, my aim was to construct a neo-Fregean theory of truth based on Frege's comments on truth. However, in the middle of the project, I had to change my aim. For I realized that Frege's insights into the nature of truth were left unexplained or even unrevealed. That is why my dissertation ended up focusing mostly on the exegesis of Frege's conception of truth and related notions.

Above all, Frege is an *objectualist* of truth who considers truth to be an object—the True *qua* the reference of true sentences. Even though he is widely known as a truth deflationist and he definitely makes deflationist claims, he is not a deflationist. As we have seen Frege's deflationist comments are his arguments that truth is not a property but an object. Likewise, even though Frege is widely known as a primitivist of truth and he does argue that truth is indefinable, he is not a truth primitivist. In the

literal reading we've developed, the gist of Frege's indefinability argument turns out to be that truth cannot be defined as a property because it is not a property. Hence, Frege's primitivist argument is also his argument for objectualism. Arguing that Frege is an objectualist of truth, I explained the problems with the previous readings that do not recognize Frege as an objectualist. These readings, despite their insightful observations, have fundamental limits because they do not fully appreciate what is at the center of Frege's conception of truth—his objectualism.

One might retort that my reading reads too much into Frege's works. Frege is only concerned about his logicist project—at least until he realizes that Russell's paradox cannot be solved by the way he suggests—and about logic insofar as it is related to his logicist project. The point of this objection is that Frege has no reason to go deep into the nature of truth to the extent that I believe he goes. The discussion in Chapter 5 can be considered to be a response to this objection. For Frege judging and its verbal counterpart asserting are the basic units of logical inferences. What he realizes in his mature career is that judging and asserting must not be predicating the property *truth*. Frege finds an alternative in the view that takes truth to be an object. By taking truth to be an object he can say that judging and asserting that p are identifying_{ni} the True with $\|p\|$. Moreover the discussion in Chapter 6 provides a crucial logical role of truth-values as objects. Accepting that truth-values are objects, Frege can make sense of value-ranges *qua* logical objects. Specifically, by recognizing truth-values, we can also recognize functions and eventually value-ranges as objects.¹ In Chapter 7, I show that Frege's conception of judgment and assertion also has philosophical benefits.

Because judging is producing propositional knowledge for Frege, his unique con-

¹Although we don't discuss them here, there are further logical benefits of taking truth-values to be objects. See Burge 1986 and Ricketts 2003.

ception of judging naturally leads to a unique conception of propositional knowledge. Chapter 6 has explored Frege's conception of propositional knowledge. As we've seen he puts objectual knowledge at the center of his conception of knowledge. Objectual knowledge of an object O provides us with the ability to identify_{nj} O , which we use when we identify O with the reference of a given term. The identity knowledge we produce by the use of this ability is non-propositional. Judging is using our ability to identify_{nj} the True. The non-propositional identity knowledge of the True we produce by judging is propositional knowledge in Frege. This interpretation of his conception of knowledge leads to the answers to the questions as to in what sense the sciences aim at the object of truth and as to in what sense logic is the unique science for which truth is the main subject-matter. In Chapter 7 we develop Frege's conception of knowledge into the Frege-Millikanian theory of knowledge.

Closing the dissertation I need to point out that we've not answered the question as to what the True is. Of course, we now have a number of different ways to describe it. The True is the reference of true sentences. Also, it is a logical, abstract, object. But what is more important is that the True is ||If 2 is an even number then it is an even number||, ||2 + 3 is equal to 5||, ||Nothing is faster than light||, ||Kim is a PhD candidate at UConn||, etc. It is an object our knowledge of which is getting stronger and more expanded continuously. If the True is such an object, it may not be possible to define it and provide an exhaustive theory about its nature. The True is the ultimate aim our cognitive activities strive for, not what we already fully grasp.

What is also important is that the debates in the current literature on truth are still meaningful and must go on even under the objectualist conception of truth. What we are looking for in our theory of truth is whether there is a substantive property owned

by truth-bearers we take to be true. Deflationists say “No”. Substantive theorists like correspondence theorists or coherence theorists claim that there is a single substantive property owned by true truth-bearers. Pluralists state that there are multiple different properties owned by different true truth-bearers.² We can ask a similar question here: Is there a substantive property shared by thoughts which we say that belong to the True or from which we take a step to the True? We ought to notice that this question does not concern a substantive property constitutive of the property *truth*; for there is nothing like the property *truth*. Then what is the question about? I believe that there can be a number of different answers. Here I introduce two answers. One is as follows:

Philosophico-Psychological Understanding

Through the above question we are backtracking the tendencies we have when we attempt to say that thoughts belong to the True—whether we tend to identify the True with $\|p\|$ when $\langle p \rangle$ has a particular property. This backtracking can show how we actually tend to think about the True.

The other is as follows:

Epistemological Understanding

Through the above question we are trying to figure out how our cognitive abilities, which let us identify_{nj} the True, function. In particular we are asking whether there are properties of thoughts our cognitive abilities distinctively track down in their pursuit of the True. This inquiry is an *epistemological* inquiry into the nature of the True *itself* because what we are trying to find out is which properties of thoughts count as the *epistemic marks* of the True. That would in the end partly show what kind of object the True is—just as the fact that visual perceptions can count as the evidence for Venus shows what kind of object Venus is.

²I intend this statement to be neutral between Wright (1992, 2013)-Lynch 2000, 2009b, 2013 style moderate pluralism and strong pluralism (Kim and Pedersen 2018). Both of them accept that there are multiple properties. But moderate pluralism says that there is a single property shared by all different truth-bearers—what we may call ‘minimal truth’. However, strong pluralism denies the existence of such a single property.

I think that there can be other answers to why we would attempt to figure out what we used to call 'truth properties' in Frege's objectualist conception of truth. But it already seems clear that our debates about the so-called truth properties are still meaningful in a new guise under Frege's objectualism and objectualism itself does not provide us with an answer. Hence, those debates must still go on under Frege's objectualism of truth.

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