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Investigation of the Bond Behavior of Steel Reinforcement Bars Embedded in Ultra High Performance Concrete under Static Loads using Finite Element Modeling

Manish Roy

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The bond properties of reinforcement bars (rebar) embedded in concrete depends on the tensile behavior of the surrounding material and its capability of resisting micro cracks. Since the tensile behavior of Ultra High Performance Concrete (UHPC) is dominated by the amount and the orientation of fibers as well as the type of fibers, an effort has been made in this study to investigate the influence of the fiber volume fraction \( V_f \) and the fiber orientation on the bond behavior of steel rebar embedded in UHPC under a static loading condition using finite element simulation. Owing to the inclusion of discrete fibers, the characteristic of UHPC is highly anisotropic even at a macro level and it is important that the material model of UHPC captures that anisotropy properly. While modeling fibers discretely is time consuming and involves a lot of computational power, the present study proposes a computationally efficient way of modeling UHPC. In this approach, UHPC is considered as a composite material with the matrix modeled as a homogeneous material and the fibers modeled as smeared reinforcement. The directional vector of the smeared reinforcement represents the orientation of the fibers inside the matrix. ATENA, a finite element program, is used for this purpose. The material properties of the fibers are calibrated using the stress-stain data obtained from uniaxial direct tensile tests of UHPC. The calibrated fiber properties are then used to model pullout tests of a rebar embedded in UHPC. The bond stress versus slip properties of the rebar are validated with the experimental pullout test results. The calibrated rebar properties along with the fiber properties are then used to model uniaxial direct tensile tests of UHPC with embedded rebar. Once the tensile test model of the
composite is validated using the experimental data, parametric studies are conducted to determine the effect of $V_f$ and fiber orientation on the uniaxial tensile behavior of rebar-reinforced UHPC. Based on the parametric studies, the dependence of the structural ductility on $V_f$ is discussed and a recommendation for the minimum strain to attain ductility is made.
Investigation of the Bond Behavior of Steel Reinforcement Bars Embedded in Ultra High Performance Concrete under Static Loads using Finite Element Modeling

Manish Roy

B.E., Jadavpur University, 2000
M.S., West Virginia University, 2011

A Dissertation
Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy at the University of Connecticut

2019
To my beloved mother
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1. INTRODUCTION

Background and motivation

Ultra High Performance Concrete (UHPC) is an advanced cementitious material characterized by its high compressive strength and tensile strength, enhanced post-cracking ductility, and improved durability properties as compared to the Normal Strength Concrete (NSC) and High Performance Concrete (HPC) [1,2]. Hence, UHPC has the potential to be used as a construction material replacing traditional concrete in reinforced concrete applications, especially in critical high shear regions and beam-column junctions. Moreover, by virtue of its excellent bond property with NSC [3] and steel reinforcement bars [4], it can also be used as a repairing material.

UHPC has attracted a significant amount of attention from the research community over the last couple of decades. Encouraging results from different experimental programs conducted around the world have made several state and federal agencies in the US show interest in UHPC and its application in promoting a more resilient and sustainable infrastructure [5].

Although UHPC has the capability to be designed for stand-alone structural applications without conventional steel reinforcement bars (rebar), UHPC with embedded rebar can reduce the material cost of a project by partially replacing the expensive steel fibers. However, in order to design rebar-reinforced UHPC for structural applications, it is important to understand the bond property between UHPC and rebar as it dictates the requirement of development length and splices in structural members.

The bond properties of rebar embedded in concrete depends on the tensile behavior of the surrounding material and its capability of resisting micro cracks. Since the tensile behavior of UHPC is dominated by the amount and the orientation of fibers (as shown in Figure 1 and Figure
2, respectively) as well as the type of fibers, an effort has been made in this study to investigate the influence of the fiber volume fraction \(V_f\) and the fiber orientation of UHPC on the bond behavior of steel rebar embedded in UHPC under a static loading condition using finite element simulation.

Figure 1 Effect of fiber volume fraction on the tensile strength of UHPC
(parallel fiber orientation) (adapted from [6])

Due to the inclusion of discrete fibers, the characteristic of UHPC is highly anisotropic even at a macro level and it is important that the material model of UHPC captures that anisotropy properly. While modeling fibers discretely is time consuming and involves a lot of computational power, the present study proposes a computationally efficient way of modeling UHPC. In this approach, UHPC is considered as a composite material with the matrix modeled
as a homogeneous material and the fibers as smeared reinforcement. The directional vector of the smeared reinforcement represents the orientation of the fibers inside the matrix. ATENA (v. 5.6.1) [7], a finite element program, is used for this purpose.

![Graph showing the effect of fiber orientation on the tensile strength of UHPC](image)

**Figure 2 Effect of fiber orientation on the tensile strength of UHPC (2.5% steel fibers) (adapted from [8])**

The fiber properties and the rebar properties including the bond stress versus slip relationship are calibrated using the stress-stain data obtained from uniaxial direct tensile tests of UHPC [4] and the force versus slip data obtained from pullout tests of rebar [4], respectively. The calibrated rebar properties as well as the fiber properties are then used to model uniaxial direct tensile tests of rebar-reinforced UHPC. Once the tensile test model of the composite is validated using the experimental data [9], parametric studies are conducted to determine the
effect of $V_f$ and the fiber orientation on the uniaxial tensile behavior of rebar-reinforced UHPC. Based on the parametric studies, the dependence of the structural ductility on $V_f$ is discussed and a recommendation for the minimum strain to attain structural ductility is made.

**Goal of the research**

The goal of the present study is to investigate the influence of fiber volume fraction and fiber orientation on the bond behavior of reinforcement bar embedded in ultra high performance concrete.

**Tasks**

The following tasks are performed in order to achieve the above goal:

a. Calibration of the stress versus strain data of fibers by simulating uniaxial direct tensile tests of UHPC.

b. Calibration of the bond stress versus slip data of rebar embedded in UHPC by simulating pullout tests.

c. Investigation of the influence of fiber volume fraction and fiber orientation on the pullout behavior of rebar embedded in UHPC.

d. Validation of stress versus strain curves of uniaxial tensile tests of rebar-reinforced UHPC vis-à-vis experimental results.

e. Investigation of the influence of fiber volume fraction and fiber orientation on the tensile behavior of reinforced-UHPC.

f. Recommendation for optimum fiber volume fraction to achieve structural ductility.
**Dissertation Organization**

This dissertation is organized into five chapters as follows:

Chapter 1: The background and the motivation, the goal, the tasks, and the organization of this dissertation are presented in this introductory chapter.

Chapter 2: This chapter describes the process for calibration of the stress versus strain data of fibers by simulating uniaxial direct tensile tests of UHPC.

Chapter 3: This chapter deals with the calibration of bond stress versus slip data of rebar embedded in UHPC by simulating pullout tests. It also investigates the influence of fiber volume fraction and fiber orientation on the pullout behavior of rebar embedded in UHPC.

Chapter 4: This chapter talks about the simulation of uniaxial tensile tests of UHPC with embedded rebar and the effect of the fiber volume fraction and the fiber orientation on the said uniaxial behavior of the composite.

Chapter 5: This chapter summarizes the key findings of the research.
2. CALIBRATION OF UHPC – UNIAXIAL TENSILE TEST

As mentioned in chapter 1, UHPC is an anisotropic material, since the orientation of the fibers influences the tensile strength of UHPC significantly. Figure 2 shows the variation in tensile stress versus strain curves of a typical UHPC tensile specimen (225 MPa compressive strength and 2.5% steel fibers) for different types of fiber orientation. Hence, it is important that the material model of UHPC captures the anisotropic property properly. Since modeling fibers discretely (Figure 3a) is time consuming and involves a lot of computational power, the present study proposes a computationally efficient way of modeling UHPC. In this approach, UHPC is considered as a composite material with the matrix modeled as a homogeneous material and the fibers as smeared reinforcement (Figure 3b). The directional vector of the smeared reinforcement represents the orientation of the fibers inside the matrix. ATENA (v. 5.6.1) [7], a finite element program, is used for this purpose. The calibration process along with the validation is described below.

![Figure 3 Schematic representation of modeling approach for fibers. a. Discrete fiber modeling, b.](image)

Figure 3 Schematic representation of modeling approach for fibers. a. Discrete fiber modeling, b.
Smeared fiber modeling

Material model for UHPC

‘Reinforced Concrete Model’ in ATENA is used as the material model for UHPC. The concrete matrix is modeled with volume elements and the fibers are modeled as smeared reinforcement with 1D elements. However, the smeared reinforcement is not added at the constitutive level; rather it is modeled as a separate element with nodes connected to those of the concrete elements. Perfect bond is assumed between the smeared reinforcement and the UHPC. The total material stiffness of UHPC is the sum of the material stiffness of the matrix \((D_c)\) and that of the fibers as smeared reinforcement (Equation 1).

\[
D = D_c + \sum_{i=1}^{n} D_{si} \tag{Equation 1}
\]

The material stiffness matrix of the \(i\)th smeared reinforcement \((D_{si})\) is calculated by Equation 2.

\[
D_{si} = p_i E_{si} \begin{bmatrix}
\cos(\beta_i)^4 & \cos(\beta_i)^2 \sin(\beta_i)^2 & \cos(\beta_i)^3 \sin(\beta_i)^1 \\
\cos(\beta_i)^3 \sin(\beta_i)^1 & \cos(\beta_i)^1 \sin(\beta_i)^3 & \cos(\beta_i)^2 \sin(\beta_i)^2 \\
\end{bmatrix}
\]

where, \(\beta\) is the angle between the global axis x and the \(i\)th reinforcement direction, \(E_{si}\) is the elastic modulus of the fibers, and \(p_i\) is the fiber ratio \((p_i = A_i / A_c)\). The stress versus strain curve of the fibers is calibrated to correctly simulate the actual effect of the discrete fibers.

The material model for concrete is a fracture-plastic model that combines constitutive models for fracturing (tensile) and plastic (compressive) behavior. The fracture model is based
on the classical orthotropic smeared crack formulation and the crack band theory employing
Rankine failure criterion and exponential softening. Both the rotated and the fixed crack model
can be used in the simulation. In this study, the fixed crack model is used. The plasticity model is
based on the Menétrey-Willam failure surface using the return mapping algorithm for the
integration of the constitutive equations. The combined algorithm allows for both the plastic as
well as the fracture model to be developed and formulated separately. The model can simulate
cracking, crushing, and crack closure due to crushing in other material directions.

The properties of the UHPC used for the calibration are summarized in Table 1.

<table>
<thead>
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<th>Table 1 UHPC properties (adapted from [4])</th>
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<tr>
<td>Matrix</td>
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<tr>
<td>14-day compressive strength</td>
</tr>
<tr>
<td>14-day direct tensile strength</td>
</tr>
</tbody>
</table>

Additional material properties used in the simulation are summarized in Table 2.

<table>
<thead>
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<th>Table 2 Additional material properties used in the simulation</th>
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<tr>
<td>Parameter</td>
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<td>Elastic modulus of fibers ( (E_{sl}) )</td>
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<tr>
<td>Elastic modulus of UHPC ( (D) )</td>
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<tr>
<td>Poisson’s ratio of UHPC</td>
</tr>
<tr>
<td>Fracture energy of UHPC matrix</td>
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One element investigation

In order to test the UHPC material model, one element investigation of a direct tensile test with the boundary conditions as shown in Figure 4 is carried out first. GiD (v. 11.0.8) [11] is used as the pre-processor in developing the model for the uniaxial tensile tests of UHPC. Since the calibrated UHPC material properties are used in the pullout as well as the composite tension tests (with #3 and #4 rebar), the element size needs be kept unchanged as far as possible for all the three types of tests. According to the ATENA Troubleshooting Manual, unrealistic crack might result if a whole concrete element falls between two reinforcement nodes. In other words, the edge length of a concrete element should not be less than the diameter of the rebar. Since the highest diameter of the rebar used in this study is 12.5 mm (#4 rebar), the concrete element size for the calibration purpose is chosen as 12.5 mm and is kept close to that size for the other two types of tests. Symmetry boundary conditions are used along all three planes and the load is applied as a prescribed displacement along Z axis. The displacement is monitored at one of the corner points on the loading surface and the load is determined by monitoring the summation of the reaction of all the nodes of the loading surface. The stress is calculated by dividing the total reaction by the cross-sectional area of the loading surface and the strain is calculated by dividing the displacement by the length of the element along Z axis. The resulting stress versus strain graph is compared with that of the experiment in Figure 5. The Poisson’s ratios along x and y axes are plotted in Figure 6. From Figure 5 and Figure 6, it is evident that the material model works as expected.
Figure 4 Loading and boundary conditions for one element investigation

Figure 5 Comparison of stress versus strain response for UHPC
(2% parallel fibers)
Multi-element investigation

The same UHPC model is then used for the multi-element investigation of a direct tensile test and the corresponding stress versus strain curves are plotted in Figure 7. It is evident from that figure that the strain in the softening region decreases with the increase in the specimen size although the same element size is kept in each case as that in the one element investigation. This is because the crack width of the major crack opened during the softening of the specimen is smeared along the length of the specimen in order to determine the strain and hence, the strain decreases as the specimen size increases. Therefore, in order to get the correct stress versus strain curve, the post-peak strain needs to be adjusted. The strain is adjusted using the smeared crack approach as explained below.
UHPC under direct tension – smeared crack approach

UHPC typically exhibits strain-hardening behavior under direct tension. Once the first cracking strength is reached (stage-I in Figure 8), the stress increases with the strain and multiple cracking occurs (stage-II in Figure 8) until the stress reaches the maximum value (peak stress). After that, a major crack, defined as the critical failure crack, opens up and it leads to the softening of the material (stage-III in Figure 8). The softening branch of the stress-strain curve is
represented by a stress versus crack opening displacement relationship. In order to calibrate the material model correctly with the experimental stress-strain curve under direct tension, the stress versus strain data is directly used in the material model until the peak stress. After the peak stress, i.e., for the softening branch, the crack opening displacement of the specimen is calculated first from the experimental stress versus strain data using Equation 3, Equation 4, and Equation 5. Then the strain is calculated based on the size of concrete elements in the simulation following the smeared crack approach based on fracture energy (Equation 6).

![Figure 8 UHPC under direct tension](image-url)
\[ \epsilon_{crpc} = \epsilon_t - \frac{\sigma_t}{E_{cc}} \]  
(at peak stress)  

Equation 3

\[ \epsilon_{cr} = \epsilon_t - \frac{\sigma_t}{E_{pc}} - \epsilon_{crpc} \]  
(beyond peak stress)  

Equation 4

\[ w_{cr} = \epsilon_{cr} \times h \]  

Equation 5

\[ \epsilon_{tpp} = \frac{w_{cr}}{h_e} \]  
(beyond peak stress)  

Equation 6

where, \( \epsilon_t \) = nominal strain (from experimental data), \( \sigma_t \) = nominal stress (from experimental data), \( \epsilon_{crpc} \) = cracking strain at peak stress, \( E_{cc} \) = initial Young’s modulus, \( \epsilon_{cr} \) = cracking strain beyond peak stress, \( E_{pc} = 7.5 \text{ GPa} \) (reduced stiffness), \( w_{cr} \) = crack width, \( h \) = specimen length, \( h_e \) = element length, and \( \epsilon_{tpp} \) = post-peak strain (converted from the crack opening displacement).

The adjusted stress versus strain data is then used to simulate the uniaxial direct tensile test of the actual test specimen used in the experiment as described below.

**Simulation of Direct Tensile Test on the whole specimen**

The calibrated fiber stress versus strain values are used to simulate a direct tensile test on the whole specimen (25 mm x 50 mm x 400 mm long) used in the experiment by Roy et al. [4]. A picture of the test set-up is shown in Figure 9.
To save on the computational time, only 1/8\textsuperscript{th} of the specimen between the gauges (175 mm) is modeled with symmetry boundary conditions along the planes of symmetry (Figure 10). The resulting stress versus strain curve is compared with that of the experiment in Figure 11. It is evident from that figure that the said curve follows the experimental curve closely.

Figure 9 Tensile test set-up [4]
Figure 10 FE model of the tensile specimen with the symmetry boundary conditions

Figure 11 Stress versus strain response with calibrated fiber properties
Crack propagation under tension

Figure 12 through Figure 16 show the propagation of cracks in the specimen at different load steps. In each figure, the left part shows the plot of load versus displacement and the right part shows the crack pattern. Figure 12 shows the specimen in an un-cracked state, where the load is just short of the cracking strength of the matrix. As soon as the matrix reaches its cracking strength (Figure 13), all the elements are cracked (multiple cracking) and the crack width increases with the increase in load until the composite reaches its strength (Figure 14). Right after the composite reaches its strength, one major crack opens in the top row of elements and the crack width for all other elements starts reducing (Figure 15). Figure 16 shows the state of the specimen at a later stage where the width of the major crack increased significantly and that of the other cracks became almost zero.

![Graph and 3D model](image-url)

**Figure 12** The crack propagation in UHPC under tension (load step 10)
Figure 13 The crack propagation in UHPC under tension (load step 11)

Figure 14 The crack propagation in UHPC under tension (load step 100)
Figure 15 The crack propagation in UHPC under tension (load step 101)

Figure 16 The crack propagation in UHPC under tension (load step 200)
**Effect of fiber volume fraction on the tensile behavior of UHPC**

Once the fibers are calibrated vis-à-vis the experimental stress versus strain data for 2% fibers, the stress versus strain curves for other fiber volume fractions are compared in Figure 17 (1% fibers) and Figure 18 (3% fibers). While the peak stress for 1% fibers is 25% below that of the experimental curve, the peak stress in case of 3% fibers is 4% above that of the experimental curve. Since $V_f$ is the only parameter that is changed from the case with 2% fibers to these two cases and the stress versus strain curves for 2% and 3% fibers match closely with that of the experimental data, it can be argued that the experimental stress versus strain data for 1% fibers might not reflect the correct behavior of the material. The fiber stress-strain values, thus calibrated, are used in the simulation of pullout tests in the next chapter.

![Figure 17 Effect of fiber volume fraction on the tensile behavior of UHPC (1% fiber)](image-url)
Figure 18 Effect of fiber volume fraction on the tensile behavior of UHPC (3% fiber)
3. CALIBRATION OF BOND-SLIP RELATIONSHIP – PULLOUT OF REBAR

Background

Rebar allows transfer of tensile stresses across cracks through a combination of a) chemical adhesion, b) frictional resistance, and c) bearing of the ribs on the concrete. Chemical adhesion between the concrete and the steel is the first resistance to be overcome when a small tensile load is applied to the rebar and it ranges from 0.5 to 1.0 MPa in conventional concrete (CC) [12]. Frictional resistance arises due to the micro-irregularities on the surface of the steel, the wedge action of granular materials between the rebar and the concrete, and the component of the bearing force acting parallel to the bar rib (Figure 19a) [12] and it ranges from 0.4 to 10.0 MPa in CC [13,14].

However, the bearing of the ribs plays a much larger role in developing the bond strength as compared to friction and adhesion. Once the adhesion is overcome upon the application of a tensile load, the bar slips slightly and the ribs of the rebar bear against the concrete at an angle creating two force components: a) parallel to the length of the rebar and b) perpendicular outward from the length of the rebar (Figure 19b). The perpendicular component of the bearing force causes a tensile ring of radial stresses to develop along the perimeter of the bar leading to radial cracks, also known as, longitudinal cracks or splitting cracks (Figure 19c). The aforesaid bond mechanisms are explained in detail elsewhere [12,15-17]. Owing to the fibers in Fiber Reinforced Concrete (FRC), the tensile ring is redistributed around the whole matrix after initial cracking [18]. With the increase in the load, the bar slips further and as soon as the fibers get pulled out, longitudinal cracks develop along the bar axis, which corresponds to the maximum bond strength. At this stage, fibers play a very important role. If the longitudinal cracks are bridged by the fibers without being opened excessively (Figure 19d), a relatively ductile pullout
failure will occur. Otherwise, a sudden splitting failure will happen with further opening of the longitudinal cracks [18].

Figure 19 Bond mechanisms (idealized). a Friction ($V_f$), b Bearing of the rib ($V_b$), c Radial longitudinal cracks in CC, d Crack bridging in Fiber Reinforced Concrete (FRC)

(adapted from [12,15,18,19])

UHPC has been found to have a much higher bond stress than CC so far as the rebar has enough cover to prevent a splitting failure and the bond stress increases with the increase in the compressive strength of concrete ($f'_c$) with RILEM [20] recommended (modified) $4.5db$ concrete cover, where $db$ is the diameter of the bar, and that the slip at the peak bond stress in UHPC
(without fiber) is generally lower than that of CC due to the higher modulus of elasticity in UHPC [19,21,22]. Although many researchers have carried out bond tests with concrete containing fibers, the orientation of fibers did not get much attention in their work [19,21,23-26].

However, at 4.5\textit{db} cover, the concrete cover itself can prevent the longitudinal splitting failure by resisting the radial tensile stresses. Since fibers in FRC do not get activated at 4.5\textit{db} cover, they have virtually no effect on the bond behavior [21]. Fibers prove to be effective when covers are small enough to induce a splitting type of failure before the rebar can develop full bond strength [24,27-30]. Aarup et al. [27] observed that an embedment length of 6.25\textit{db} with 1.8\textit{db} cover achieved the maximum bond stress of 23.6 MPa with a pullout failure (not rebar rupture). In their study, they used Compact Reinforced Composite (CRC) with \( f_c' = 165 \text{ MPa} \) and varied the fiber contents between 3\% and 6\%. Cheung and Leung [28] used no fiber and 2\% fibers with 5\textit{db} and 8\textit{db} embedment length and a constant cover of 3.25\textit{db} in high strength fiber reinforced cementitious composites (\( f_c' = 150 \text{ MPa} \)). They observed splitting failure for all the cases. The average pullout strength increased by 144\% for 5\textit{db} and 154\% for 8\textit{db} embedment lengths when the fiber volume fraction was increased from 0 to 2\%. Leutbecher [24] used a constant embedment length of 1.5\textit{db} in UHPC (\( f_c' = 150 \text{ MPa} \)) and noticed that at 2.5\textit{db} cover, the maximum pullout strength increased by 70\% when the fiber content was increased from 0 to 1\%. Saleem et al. [30] tested pull-out specimens of #10 and #22 rebar with 8\textit{db}, 10\textit{db}, 12\textit{db}, and 18\textit{db} (only for #22) embedment lengths and cover of 0.4\textit{db} and 0.2\textit{db}, respectively. They used ultra high strength concrete having \( f_c' = 174 \text{ MPa} \) for specimens with #22 rebar and 18\textit{db} embedment length and \( f_c' = 166 \text{ MPa} \) for all other specimens. Their results showed that the development lengths of #10 and #22 rebar were 12\textit{db} and 18\textit{db}, respectively. Fehling et al. [29] performed pullout tests on 12 mm diameter ribbed bar embedded in UHPC having \( V_f = 1.5\% \) and
$f'_c = 170$ MPa. Embedment length and concrete cover (varying from $1db$ to $2.5db$) were the parameters in their investigation. The major concrete failure modes observed during the tests were cone failure, splitting, and V-type splitting failure. They observed that there was almost no residual stress at a slip of 7 mm for a brittle concrete cone failure. They concluded that the splitting failure and the V splitting failure were preferred as the fibers were activated and acted as confinement to facilitate a more ductile failure. Yuan and Graybeal [31] conducted direct tension pullout tests of deformed reinforcing bar (lap spliced) embedded in UHPC at 1 day or 7 days after casting ($f'_c = 135$ MPa at 7 days after casting). The primary parameters in their investigation included the embedment length of reinforcing steel, concrete side cover, bar spacing, compressive strength of UHPC, and type and size of deformed bar. They observed that the bond strength increased with the increase in the embedment length of the bar, the concrete side cover, and the compressive strength of UHPC, respectively, while it decreased with the increase in bar diameter and spacing. They also found out that the bond strength was higher in case of high strength bars that did not yield before bond failure. Lagier et al. [32] investigated the influence of fiber content ($V_f$) on the bond strength of tension lap splices. They noticed that an increase in fiber content delayed the onset and propagation of first macro-cracks in lap splice leading to increased bond strength. They further reported that for a given splice length of $10db$, the ultimate bond stress increased by 29% and 53% due to an increase in $V_f$ from 1% to 2% and 4%, respectively. Holschemacher et al. [22] assessed the bond behavior of conventional as well as ‘deep-ribbed’ rebar in ultra high strength concrete (UHSC) using pull-out specimens. The parameters used in their experiments included rebar diameter, reinforcement type, surface geometry of the rebar, the concrete cover size, and the loading rate. They reported that UHSC having crushed aggregates with a maximum grain size of 5 mm showed no negative effect with
respect to splitting or bond stress. UHSC with ‘deep-ribbed’ rebar showed better ductility compared to UHSC with conventional rebar. They also observed that the faster the loading rate, the higher the bond stress values and the larger the displacement at maximum bond stress.

Many other researchers have investigated the influence of different parameters such as embedment length of rebar, diameter and type of rebar, concrete strength, and concrete cover on the bond behavior between rebar and UHPC [25,33]. However, research on the effect of fiber orientation and fiber content of UHPC on the bond stress has been very limited [26,34]. Since number of fibers and their orientation influence the crack bridging effort in UHPC [8,35], it is expected that those two parameters would also have an impact on the bond behavior between rebar and UHPC, especially at a low concrete cover. The goal of this chapter is to calibrate the bond stress versus slip data obtained in the pullout experiment and further investigate the effect of fiber volume fraction ($V_f$) and fiber orientation on the pullout behavior of rebar embedded in UHPC having a low cover.

Geometry and conditions of the FE model

The geometry of the pullout test, investigated in the present study, is based on the specimen used in the experiment by Roy et al. [4]. The geometry of the specimen is shown in Figure 20. The experimental set-up is shown in Figure 21. Due to the symmetry, only one-half of the specimen is modeled to save on computational time. The half-scale model is shown in Figure 22.
Figure 20 Geometry of the pullout specimen

Figure 21 Pullout test set-up [4]
According to the ATENA Troubleshooting manual [36], applying a load or constraints directly to a reinforcement point outside the concrete is problematic. Hence, a small elastic volume is added around the end of both the pullout bar and the support bar. The load as well as the constraints is applied directly to the elastic volumes as applicable. The Young’s modulus of the elastic volume is considered to be very high such that the deformation of the elastic volume is negligible compared to the deformation of other parts of the model.

The concrete and the elastic volumes are modeled with 8-node linear hexahedra elements. The reasons to select hexahedral elements over tetrahedral elements are a) first-order tetrahedral elements would require extremely fine mesh for sufficiently accurate results, which in turn would increase the analysis time, b) quadratic tetrahedral elements would use full integration technique and hence, the analysis time would be longer, c) hexahedral elements would have a better convergence rate than that of tetrahedral elements, and d) a good mesh of hexahedral elements would usually provide a solution of equivalent accuracy at less computational cost.
Since it is difficult to guess the orientation of the cracks in an element, an aspect ratio close to 1 is kept. The minimum edge length of a concrete element is 11.25 mm and the maximum edge length of a concrete element is 12.5 mm. The element size in the elastic volumes is kept as 12.5 mm. The meshed concrete as well as the elastic volumes is shown in Figure 23. The reinforcement bars are modeled with 1-D element with axial stiffness only.

![Figure 23 Meshed assembly](image)

Symmetry boundary conditions are applied to the surfaces at the plane of symmetry of the specimen. To prevent any unrealistic response, the lateral movements of the elastic volumes are restrained perpendicular to the pullout bar as well as the support bar. The end of the elastic volume around the support bar is constrained in all the three directions to simulate the testing condition of the pullout specimen. The boundary conditions are shown in Figure 24 and Figure 25. The load is applied as a prescribed displacement on the end of the elastic cube around the pullout bar as shown in Figure 26. The displacements on the pullout bar is measured at the bar end inside the elastic cube and at the junction of the bar in air and the concrete. Then the displacement at the pullout screw position is interpolated from these two values. The
displacement on the side bar is measured at the bar end. The slip is calculated by subtracting the displacement of the side bar from that of the anchorage bar. The load is calculated by summing up the reactions of all the nodes on the loading surface.

Figure 24 Boundary conditions for concrete

Figure 25 Boundary conditions for the elastic volume at the pullout bar end
Materials

The SOLID Elastic material model in ATENA is used for the elastic volumes. The Young’s modulus of the elastic volumes is considered to be 20 times that of steel material such that the deformation of the elastic volume is negligible compared to the deformation of other parts of the model. The material properties of the elastic volumes are shown in Table 3.

Table 3 Material properties of the elastic volumes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>4,000,000 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio of UHPC</td>
<td>0.3</td>
</tr>
</tbody>
</table>
1D Reinforcement material model is used for the pullout bar, the side bar, and the support bar. The material properties are taken from [4] and are summarized in Table 4. The diameter of the support bar is chosen such that the pullout bar is pulled out of the concrete first. Hence, no slip is considered for the support bar as well as the side bar. The stress versus strain curve from the experiment [4] is used as the input to the program. The said curve is shown in Figure 27. The bond force versus slip data from the experiment (Figure 28; adapted from [4]) is converted to bond stress versus slip data and input to the program. For the calibration purpose, the data for pullout specimen with 12db embedment length, 2% fiber amount, and perpendicular fiber orientation are considered. The conversion is done using Equation 7.

<table>
<thead>
<tr>
<th>Table 4 Material properties of the reinforcement bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>Yield stress</td>
</tr>
<tr>
<td>Ultimate stress</td>
</tr>
<tr>
<td>Diameter of the pullout bar</td>
</tr>
<tr>
<td>Diameter of the side bar and the support bar</td>
</tr>
</tbody>
</table>
Figure 27 Rebar stress versus strain data

Figure 28 Bond force versus slip data (adapted from [4])
\[ \tau_i = \frac{P_i}{\pi \times db \times l_e} \]  

where, \( \tau_i \) and \( P_i \) are the bond stress and the pullout load for a particular slip, respectively. 

\( l_e \) is the initial embedment length of the rebar. The end of the pullout bar inside the elastic volume is restricted from having any slip.

' remarksConcrete Model’ in ATENA is used as the material model for UHPC. The concrete matrix is modeled with volume elements and the fibers are modeled as smeared reinforcement with 1D elements. However, the smeared reinforcement is not added at the constitutive level; rather it is modeled as a separate element with nodes connected to those of the concrete elements. Perfect bond is assumed between the smeared reinforcement and the UHPC.

The total material stiffness of UHPC is the sum of the material stiffness of the matrix (\( D_c \)) and that of the fibers as smeared reinforcement (Equation 8).

\[
D = D_c + \sum_{i=1}^{n} D_{si}
\]  

Equation 8

The material stiffness matrix of the \( i^{th} \) smeared reinforcement (\( D_{si} \)) is calculated by Equation 9.

\[
D_{si} = p_i E_{si} [ \begin{array}{ccc}
\cos(\beta_i)^4 & \cos(\beta_i)^2 \sin(\beta_i)^2 & \cos(\beta_i)^3 \sin(\beta_i)^1 \\
\cos(\beta_i)^3 \sin(\beta_i)^1 & \cos(\beta_i)^1 \sin(\beta_i)^3 & \cos(\beta_i)^2 \sin(\beta_i)^2
\end{array} ]
\]  

Equation 9

where, \( \beta \) is the angle between the global axis x and the \( i^{th} \) reinforcement direction, \( E_{si} \) is the elastic modulus of the fibers, and \( p_i \) is the fiber ratio (\( p_i = A_{si}/A_{c} \)). The calibrated stress versus
strain data obtained earlier (chapter 2) is used here. The properties of the UHPC used are summarized in Table 5.

<table>
<thead>
<tr>
<th>Table 5 UHPC properties (adapted from [4])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix</td>
</tr>
<tr>
<td>14-day compressive strength</td>
</tr>
<tr>
<td>14-day direct tensile strength</td>
</tr>
<tr>
<td>Fiber volume fraction (for the calibration)</td>
</tr>
<tr>
<td>Fiber orientation (for the calibration)</td>
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</tbody>
</table>

Additional material properties used in the simulation are summarized in Table 6.

<table>
<thead>
<tr>
<th>Table 6 Additional material properties used in the simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Elastic modulus of fibers ($E_s$)</td>
</tr>
<tr>
<td>Elastic modulus of UHPC ($D$)</td>
</tr>
<tr>
<td>Poisson’s ratio of UHPC</td>
</tr>
<tr>
<td>Fracture energy of UHPC matrix</td>
</tr>
</tbody>
</table>

Comparison with experimental data

A number of analyses were run using the above material parameters and varying the fiber volume percentage for each of the directional vector along x and y axes (perpendicular to the
load direction) and a decent match was obtained vis-à-vis the experimental bond force versus slip curve with the following combination: fiber volume fraction along $x = 1.5\%$, along $y = 0.5\%$, and $z = 0\%$. This conforms to the casting procedure used in the experiment. The comparison is shown in Figure 29.

![Figure 29 Comparison of bond force versus slip (2% perpendicular fibers)](image)

The model is then calibrated for the other two types of orientation, namely, parallel fiber orientation and random fiber orientation, vis-à-vis the experimental force versus slip data. A good match was obtained in each case for the following combinations: a) parallel fiber orientation – fiber amount along $x = 0.5\%$, along $y = 0\%$, and along $z = 1.5\%$; b) random fiber orientation – fiber amount along $x = 0.5\%$, along $y = 0.1\%$, along $z = 0.3\%$, and along each of
the 4 directions pointing toward the middle of the octants = 0.275%. The comparison is shown in Figure 30 and Figure 31, respectively.

Figure 30 Comparison of bond force versus slip (2% parallel fibers)
Figure 31 Comparison of bond force versus slip (2% random fibers)

**Parametric study – effect of fiber orientation**

The peak forces from the above graphs are plotted against the respective fiber orientation type in Figure 32 to investigate the effect of fiber orientation on the peak force. From that figure it is evident that the pullout force for the random orientation and the perpendicular orientation is 16% and 23% higher than that for the parallel orientation, respectively. Due to the low concrete cover, the specimens fail due to radial splitting cracks (Figure 19c and d). Hence, perpendicularly oriented fibers could bridge the cracks more effectively as compared to the other two types of fiber orientation.
Figure 32 Effect of fiber orientation on the peak pullout force for 2% fibers

The load versus slip graphs are combined together and compared vis-à-vis the combined experimental curves in Figure 33a and Figure 33b. It is evident that the slip at the peak load is maximum for perpendicular orientation of fibers and minimum for parallel orientation of fibers. The value for the random fiber orientation lies in between. These results confirm that the crack bridging effort is the most effective when fibers are oriented perpendicular to the applied tensile load and the least effective when they are parallel to the load direction.
Parametric study – effect of fiber volume fraction

Once the model is validated for 2% fibers, it is run for other fiber percentages (with random fibers). The peak forces are plotted against the fiber volume fractions in Figure 34. It is evident from that figure that the increase in fiber volume fraction from 0% to 1%, 2%, and 3% increases the pullout load by 92%, 166%, and 181%, respectively. As the fiber volume fraction increases for a particular fiber orientation, the probability of number of fibers crossing the cracks increases, which prevents excessive opening of the cracks. As a result, the composite can resist more stress and strain and hence, the bond stress and the pullout load increase.
Figure 34 Effect of fiber volume fraction on pullout force for random fibers

Figure 35 shows the load versus slip response for different fiber volume fractions for random orientation. It is evident from both the figure that the maximum load and the associated slip increase with the increase in fiber content.
Failure patterns

Figure 36, Figure 37, and Figure 38 show the failure patterns vis-à-vis the fiber orientation for 2% fiber volume fraction. It is evident from these figures that the number of splitting cracks is the highest in case of perpendicular orientation due to effective crack bridging. In case of parallel orientation, the fibers were aligned with the load direction and hence, were not very effective in crack bridging. Hence, the width of the splitting cracks is largest in this case.
Figure 36 Crack pattern for perpendicular fibers (2% $V_f$)

Figure 37 Crack pattern for random fibers (2% $V_f$)
The bond stress versus slip values, thus calibrated, are used in the simulation of tensile tests of reinforced UHPC in the next chapter.
4. UNIAXIAL TENSIILE BEHAVIOR OF REINFORCED UHPC

Background

This chapter focuses on the uniaxial tensile behavior of rebar-reinforced ultra high performance concrete (reinforced UHPC). In an un-cracked state, reinforced UHPC under tensile loading behave elastically with a perfect bond (Figure 39a). This is similar to conventional reinforced concrete (RC) under uniaxial tension. As the tensile load is increased, it leads to the development of micro cracks as soon as the matrix reaches its cracking strength locally (Figure 39b). For the purpose of this study, micro cracks are defined as any crack with an upper limit of 10 μm width; beyond this width limit, cracks are considered to be macro cracks [37]. In comparison to RC where the tensile load across the crack is only transferred by the rebar, reinforced UHPC allows the transfer of the tensile load across the crack by the combined effort of fiber-reinforcement and rebar. This crack-bridging effect of the fibers increases the composite stiffness beyond the tension stiffening effect of rebar reinforced concrete [38]. With a further increase in the tensile load, strain-hardening UHPC exhibits multiple cracking (Figure 39c). It is worth noting here that this phenomenon of multiple cracking differentiates strain-hardening cementitious composites (e.g., UHPC containing at least 1.5 vol.% steel fibers of aspect ratio 65 [39]) in its composite tensile behavior from conventional fiber reinforced concrete (FRC). The multiple cracking of the matrix continues with the increased tensile load until the tensile strength of UHPC is reached. Then the fiber-reinforced matrix starts softening, which leads to the development of a macro crack (Figure 39d). At this stage or with further increase in the tensile load, the yielding rebar starts strain-hardening, resulting in the formation of multiple macro cracks (Figure 39e) followed by rebar softening and ultimately, failure (Figure 39f).
Figure 39 Mechanics of strain-hardening reinforced UHPC under tension (adapted from [40]): (a) Uncracked; (b) Fiber bridging; (c) Multiple matrix cracking due to strain hardening; (d) Macro cracking due to matrix softening; (e) Multiple macro cracking due to rebar hardening; (f) Rebar failure/softening.

Figure 39 demonstrates how the fiber reinforcement influences the development of micro and macro cracks under increased tensile loading. The effect of fiber reinforcement is controlled by the type, the amount, and the orientation of fibers [41] present in the composite. The fiber reinforcement not only enhances the tensile load transfer along the load direction as illustrated in Figure 39, but it also improves the bond properties between the rebar and the fiber composite [3]. Under tensile loading, the bond between the matrix and the fibers, as well as the splitting cracks, develops along the load direction. Hence, fibers, oriented perpendicular to the load direction, improve the bond properties more effectively than that aligned with the load direction [4]. This phenomenon motivated the authors to investigate the effect of fiber orientation on the overall performance of reinforced UHPC under tensile loading.
Several researchers investigated the behavior of reinforced UHPC under tensile loading. Redaelli [42] performed direct tension tests on real-scale (160 mm × 160 mm cross-section with 1 m measurement length) UHPC dog bone-shaped specimens reinforced with ordinary steel bars (16 mm diameter). He found that the cracks opened at the serviceability-limit state were thin and closely spaced (spacing ~20 to 100 mm). He also observed that the tension-stiffening effect in reinforced UHPC was more pronounced than that in RC, resulting in a higher stiffness of the composite. Moreover, reinforced UHPC might have a positive financial impact due to the possible reduction in the amount of expensive steel fibers added to the matrix. Leutbecher and Fehling [43] showed that rebar reinforced UHPC with as low as 0.9% fiber volume fraction ($V_f$) could demonstrate strain-hardening behavior with very small crack spacing and crack widths, whereas a typical UHPC may require sufficiently large amount of fibers ($V_f > 1.5\%$) [42] on its own to achieve strain-hardening and favorable crack width. This is of significant importance because the amount of expensive steel fibers dominates the cost of UHPC. A significant reduction in $V_f$ can lead to a significant decrease in material cost. Kunieda et al. [44] conducted uniaxial tensile tests on reinforced ultra-high performance strain hardening cementitious composite (UHP-SHCC) specimens having compressive strength ($f_{c}'$) of 95 MPa and 1.5% $V_f$. They observed that all the UHP-SHCC specimens showed strain-hardening behavior with multiple cracking. Similar experiments on rebar-embedded FRC, carried out by other researchers [45-47], showed favorable results with respect to crack spacing and crack width. A review of the aforesaid literature suggests that the interaction between rebar and concrete in RC or the interaction among rebar, concrete, and fibers in conventional FRC is well understood. However, the interaction between rebar and strain-hardening UHPC needs to be investigated further in
order to understand the effect of strain-hardening characteristic on the composite tensile behavior.

**Brittle Failure of Reinforced Strain-Hardening UHPC?**

Strain-hardening UHPC is characterized by multiple cracking and significantly enhanced energy absorption capacity until failure [39]. Prima facie, reinforced strain-hardening UHPC is expected to behave as a highly ductile material due to the ductile behavior of both the fiber-reinforced matrix and the hardening rebar. However, the following conditions could lead to a rather brittle failure and thus, motivated this research to investigate further.

The softening behavior of strain-hardening UHPC is characterized by the formation and subsequent opening of a macro crack similar to that of FRC. If the softening behavior of the fiber-reinforced UHPC matrix (i.e., the slope of region A-B in Figure 40) and thus the decrease in force \( \Delta F_m \) due to the decrease in stress resistance \( \Delta \sigma_m \) is more pronounced than the hardening behavior of the rebar (i.e., the slope of region C-D in Figure 40) and thus the increase in force \( \Delta F_r \) due to the increase in stress resistance \( \Delta \sigma_r \), then opening of only one macro crack might lead to a local rebar failure (region E-F in Figure 40). In other words, if during softening, \( \Delta F_m \) (decrease in force in the fiber-reinforced UHPC matrix) > \( \Delta F_r \) (increase in force in the rebar) (Figure 41), then the load carrying capacity of the reinforced composite will be reached as soon as the first macro crack forms and hence, the formation of only one macro crack will lead to a sudden failure of the composite. This yield-point localization without forming other rebar yield points leads to a loss of ductility of the composite [48], which might pose a threat to the structure at the ultimate limit state [42].

In summary, one of the following two conditions occurs when the UHPC matrix reaches the peak tensile strength under uniaxial tensile loading:
If $|\Delta F_m| < |\Delta F_r|$ \rightarrow$ formation of multiple macro cracks \rightarrow$ increase in ductility \hspace{1cm} \text{Equation 10}

If $|\Delta F_m| > |\Delta F_r|$ \rightarrow$ formation of one macro crack \rightarrow$ loss of ductility \hspace{1cm} \text{Equation 11}

where

$$\Delta F_m = \Delta \sigma_m \times A_m$$ \hspace{1cm} \text{Equation 12}

$$\Delta F_r = \Delta \sigma_r \times A_r$$ \hspace{1cm} \text{Equation 13}

and $A_m$ and $A_r$ are the area of the matrix and the rebar, respectively.

Figure 40 Comparison of stress versus strain curves of UHPC, reinforced UHPC, and reinforcement steel.
The multiple cracking of the matrix followed by the formation of one macro crack provides sufficient ductility to the composite for controlling cracks at the serviceability limit state. However, it is important that the composite has sufficient ductility to attain high strain levels at the ultimate limit state [43]. Improved ductility at high strain levels through tailored fiber-reinforcement, accompanied by the formation of multiple macro cracks, would ensure the safety of the structure at the ultimate limit state. Leutbecher [24] recommended lowering the fiber content and the use of rebar with pronounced hardening to facilitate multiple macro cracking, similar to conventional RC. Redaelli [42] suggested the use of rebar with enhanced and continuous strain-hardening property in order to improve ductility. Sturwald and Fehling [49] proposed an increased amount of rebar reinforcement at higher fiber dosages in order to make bar hardening more pronounced than the softening behavior of the fibers. Thus, for ultimate limit-state design, the influence of the fibers, as well as the rebar reinforcement, has to be considered to attain a ductile composite behavior at failure [50]. Although other researchers [21,42,51] have encountered a similar problem of strain localization at the ultimate limit-state, research on the effect of orientation and amount of fibers in UHPC as well as the type of rebar
reinforcement has been very limited [52]. Hence, an effort has been made in this research to characterize the behavior of rebar-embedded strain-hardening UHPC under uniaxial tension with a major focus on the effect of the amount and orientation of fibers.

**Geometry and conditions of the FE model**

The geometry of the uniaxial tensile test, investigated in the present study, is based on the specimen used in the experiment by Roy et al. [9]. The geometry of the specimen is shown in Figure 42. The experimental set-up is shown in Figure 43. Due to the symmetry, only one-eighth of the specimen is modeled to save on computational time. The 1/8-scale model is shown in Figure 44.

![Figure 42 Geometry of the tensile specimen ([9])](image)
Since applying a load or constraints directly to a reinforcement point outside the concrete is problematic, a small elastic volume is added around the end of the reinforcement bars. The load as well as the constraints is applied directly to the elastic volume. The Young’s modulus of the
The elastic volume is considered to be very high such that the deformation of the elastic volume is negligible compared to the deformation of other parts of the model.

The concrete and the elastic volume are modeled with 8-node linear hexahedra elements. The reasons to select hexahedral elements over tetrahedral elements are a) first-order tetrahedral elements would require extremely fine mesh for sufficiently accurate results, which in turn would increase the analysis time, b) quadratic tetrahedral elements would use full integration technique and hence, the analysis time would be longer, c) hexahedral elements would have a better convergence rate than that of tetrahedral elements, and d) a good mesh of hexahedral elements would usually provide a solution of equivalent accuracy at less computational cost. Since it is difficult to guess the orientation of the cracks in an element, an aspect ratio of 1 is kept. The edge length of a concrete element is 12.5 mm. The element size in the elastic volume is also kept as 12.5 mm. The meshed concrete as well as the elastic volume is shown in Figure 45. The reinforcement bars are modeled with 1-D element with axial stiffness only.

Figure 45 Meshed assembly
Symmetry boundary conditions are applied to the surfaces at the planes of symmetry of the specimen. To prevent any unrealistic response, the lateral movements of the elastic volume are restrained perpendicular to the reinforcement bars. The boundary conditions are shown in Figure 46 and Figure 47. The load is applied as a prescribed displacement on the end of the elastic volume as shown in Figure 48. The displacement on the concrete is measured at a distance of 200 mm from the edge (elastic volume side) and the strain is calculated by dividing this displacement by half the gauge length (300 mm). The load is calculated by summing up the reactions of all the nodes on the loading surface and the strain is determined by dividing this load by the cross-sectional area of the rebar.

Figure 46 Boundary conditions for concrete
Figure 47 Boundary conditions for the elastic volume

Figure 48 Load as a prescribed displacement
Materials

The SOLID Elastic material model in ATENA is used for the elastic volume. The Young’s modulus of the elastic volume is considered to be 20 times that of steel material such that the deformation of the elastic volume is negligible compared to the deformation of other parts of the model. The material properties of the elastic volumes are shown in Table 7.

Table 7 Material properties of the elastic volume

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</tbody>
</table>

1D Reinforcement material model is used for the reinforcement bars. The material properties are taken from [4] and are summarized in Table 8. The side bar is continued up to the gauge start such that the specimen fails within the gauge length of 300 mm. Hence, no slip is considered for the side bar. The stress versus strain curve from the experiment [4] is used as the input to the program. The said curve is shown in Figure 49. The bond stress versus slip data calibrated in chapter 3 is input to the program. For the calibration purpose, the data for pullout specimen with $12\,db$ embedment length, 2% fiber amount, and parallel fiber orientation are considered. No slip of the main bar is considered inside the elastic volume.

Table 8 Material properties of the reinforcement bars

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>221,858 MPa</td>
</tr>
<tr>
<td>Yield stress</td>
<td>700 MPa</td>
</tr>
</tbody>
</table>
Ultimate stress | 1102 MPa
---|---
Diameter of the main bar | 9.375 mm (#3)
Diameter of the side bar | 12.5 mm (#4)

‘Reinforced Concrete Model’ in ATENA is used as the material model for UHPC. The concrete matrix is modeled with volume elements and the fibers are modeled as smeared reinforcement with 1D elements. However, the smeared reinforcement is not added at the constitutive level; rather it is modeled as a separate element with nodes connected to those of the concrete elements. Perfect bond is assumed between the smeared reinforcement and the UHPC. The total material stiffness of UHPC is the sum of the material stiffness of the matrix ($D_c$) and that of the fibers as smeared reinforcement (Equation 14).
\[ D = D_e + \sum_{i=1}^{n} D_{sl} \] \hspace{1cm} \text{Equation 14}

The material stiffness matrix of the \( i \)th smeared reinforcement \( (D_{sl}) \) is calculated by Equation 15.

\[
D_{sl} = p_i E_{si} \begin{bmatrix}
\cos(\beta_i)^4 & \cos(\beta_i)^2 \sin(\beta_i)^2 & \cos(\beta_i)^3 \sin(\beta_i) \\
\cos(\beta_i)^2 \sin(\beta_i)^2 & \sin(\beta_i)^4 & \cos(\beta_i)^1 \sin(\beta_i)^3 \\
\cos(\beta_i)^3 \sin(\beta_i) & \cos(\beta_i)^1 \sin(\beta_i)^3 & \cos(\beta_i)^2 \sin(\beta_i)^2
\end{bmatrix}
\]

\hspace{1cm} \text{Equation 15}

where, \( \beta \) is the angle between the global axis x and the \( i \)th reinforcement direction, \( E_{si} \) is the elastic modulus of the fibers, and \( p_i \) is the fiber ratio \( (p_i = A_s/A_c) \). The calibrated stress versus strain data obtained earlier (chapter 2) is used here. The properties of the UHPC used are summarized in Table 9.

<table>
<thead>
<tr>
<th>Table 9 UHPC properties (adapted from [4])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix</td>
</tr>
<tr>
<td>14-day compressive strength</td>
</tr>
<tr>
<td>14-day direct tensile strength</td>
</tr>
<tr>
<td>Fiber volume fraction (for the calibration)</td>
</tr>
<tr>
<td>Fiber orientation (for the calibration)</td>
</tr>
</tbody>
</table>
Additional material properties used in the simulation are summarized in Table 10.

**Table 10 Additional material properties used in the simulation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus of fibers ($E_{s1}$)</td>
<td>210,000 MPa</td>
</tr>
<tr>
<td>Elastic modulus of UHPC ($D$)</td>
<td>44,650 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio of UHPC</td>
<td>0.2</td>
</tr>
<tr>
<td>Fracture energy of UHPC matrix</td>
<td>5.4e-5 MN/m (adapted from [10])</td>
</tr>
</tbody>
</table>

**Comparison with experimental data**

Figure 50 and Figure 51 show the comparison of stress versus strain curves vis-à-vis the experimental results for parallel fibers and random fibers (2% fiber volume fraction), respectively. It is evident from those figures that the FEA results match closely with the experimental results, although the softening curve for 2%-para is a little off. Also, both the figures show that the composite stiffness is much higher than that of the bare bar. Since no experimental data is available for 2%-per specimen, the composite curve is plotted vis-à-vis the bare rebar curve only (Figure 52). Since perpendicular fibers are not at all effective for crack bridging, the stress versus strain curve basically follows that of the rebar right after the UHPC starts softening signifying that the load is taken by the rebar alone.
Figure 50 Stress versus strain curve (2%-parallel)

Figure 51 Stress versus strain curve (2%-random)
Parametric study – effect of fiber orientation

The peak stress values are plotted against the respective fiber orientation type for 2% fibers in Figure 53 to investigate the effect of fiber orientation on the peak stress. From that figure it is evident that the peak stress for the random orientation and the parallel orientation is 27% and 51% higher than that for the perpendicular orientation, respectively. Since the models are subjected to uniaxial tension, the composite with fibers arranged parallel to the applied load had the highest probability of fibers crossing the cracks compared to the composites with random and perpendicular fiber orientation, thereby registering the maximum peak stress.
The effect of orientation of the fibers on the stress versus strain response is shown in Figure 54 for 2% \( V_f \). It is evident from the figure that the strain-hardening modulus registers the highest value when the fibers are aligned with the load direction and the lowest value when the fibers are arranged perpendicular to the load direction. These results confirm that the crack bridging effect is most effective when fibers are oriented parallel to the applied tensile load and the least effective when they are perpendicular to the load direction. However, the bond between UHPC and the rebar is better when the fibers are perpendicular to the load direction rather than parallel \([4]\). This explains the improvement in ductility (defined as the strain at the peak stress) for the composite when the fiber orientation is changed from parallel to perpendicular with respect to the load direction.
Parametric study – effect of fiber volume fraction

The peak stress values are plotted against the fiber volume fractions in Figure 55 though Figure 57. It can be seen from Figure 55 that the peak stress of reinforced-UHPC with parallel fibers increases by 2%, 9%, and 56% when the $V_f$ is increased from 0.5% to 0.75%, 1%, and 2%, respectively. In case of reinforced-UHPC with randomly oriented fibers (Figure 56), the peak stress is increased by 12%, 14%, and 36% when the $V_f$ is increased from 0.5% to 0.75%, 1%, and 2%, respectively. The peak stress in Figure 57 (with perpendicular fibers) remains almost the same (~0.5% difference) when the $V_f$ is increased from 1% to 2%. Since fibers transfer the tensile forces across cracks, higher fiber volume fraction increases the probability of the number of fibers crossing the cracks, thereby increasing the load carrying capacity of the UHPC as well as
the composite. For perpendicular fiber orientation, however, the stress increase is almost negligible because the fibers have little to no effect on the composite strength as soon as the UHPC matrix cracks.

Figure 55 Effect of volume fraction on the peak stress of reinforced UHPC (parallel orientation)
Figure 56 Effect of volume fraction on the peak stress of reinforced UHPC (random orientation)

Figure 57 Effect of volume fraction on the peak stress of reinforced UHPC (perpendicular orientation)
Figure 58, Figure 59, and Figure 60 show the effect of fiber volume fraction on the stress versus strain response of the composite with parallel fiber orientation, random fiber orientation, and perpendicular fiber orientation, respectively. It is evident from the aforesaid figures that the strain-hardening modulus increases with the increase in fiber volume fraction. This is due to the improvement in the crack-bridging effect with the increase in $V_f$, thereby registering higher stresses at lower strains. However, the composite loses its ductility when $V_f$ is increased further beyond a particular value (e.g., 0.75% for parallel fibers (Figure 58) and 1% for random fibers (Figure 59). This is because the softening of UHPC becomes more pronounced with the increase in $V_f$ as compared to the hardening of A1035 rebar. For example, in Table 11, $\Delta F_m$ for 1%-par-A1035 specimen increases from 12.5 kN to 22.9 kN with the increase in crack width from 0.15 mm to 0.4 mm (see exposure class [53]) based on the stress versus crack-width opening relationship (Figure 61) of the UHPC. However, the value of $\Delta F_m$ is still lower than $\Delta F_r$ even at a higher crack width and thus leading to a ductile behavior. But in case of 2%-par-A1035 specimen, $\Delta F_m$ (42.9 kN) exceeds the value of $\Delta F_r = 28.5$ kN when the crack width is 0.4 mm. Hence, the specimen starts losing ductility as the crack width increases and becomes unstable as soon as $\Delta F_m$ surpasses $\Delta F_r$ corresponding to a crack width of 0.22 mm. It is worthwhile to note the difference between the material ductility and the structural ductility here. Even though UHPC with fibers and steel rebar materials are separately considered to be ductile under tensile loads, a combination of these two materials may not always impart ductility to the resulting structure as evidenced here and hence, shows the importance of this study. For the composite with perpendicular fibers (Figure 60), ductility does not depend on $V_f$ as the perpendicular fibers have no effect on the uniaxial tensile behavior of the composite.
Figure 58 Effect of volume fraction on the stress versus strain response (parallel orientation)

Figure 59 Effect of volume fraction on the stress versus strain response (random orientation)
Figure 60: Effect of volume fraction on the stress versus strain response (perpendicular orientation).

Table 11: Force mechanism for the stress versus strain response

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$\Delta F_r$ (kN)</th>
<th>$\Delta F_m$ (kN)</th>
<th>Crack Width (mm)</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%-par-A1035</td>
<td>28.5</td>
<td>12.5</td>
<td>0.15 $^c$</td>
<td>$</td>
</tr>
<tr>
<td>1%-par-A1035</td>
<td>28.5</td>
<td>22.9</td>
<td>0.4 $^d$</td>
<td>$</td>
</tr>
<tr>
<td>2%-par-A1035</td>
<td>28.5</td>
<td>21.9</td>
<td>0.15 $^c$</td>
<td>$</td>
</tr>
<tr>
<td>2%-par-A1035</td>
<td>28.5</td>
<td>42.9</td>
<td>0.4 $^d$</td>
<td>$</td>
</tr>
</tbody>
</table>

$^a \Delta F_r = (f_t - f_y)A_r$. $^b \Delta F_m = (f_t' - \sigma_w)A_m$; $\sigma_w$ is the stress in UHPC at a specific crack width ($w$) (Figure 61). $^c$ seawater; wetting and drying (Exposure data from [53]). $^d$ Dry air or protective membrane (Exposure data from [53]).
Recommendation for $V_f$

In reinforced concrete design, the steel reinforcement bars are assumed to attain a minimum strain of 0.5% in order to have a tension-controlled design such that the compression load is carried by the concrete and the tensile load is carried by the rebar alone. In order for the fibers in the UHPC to carry a part of the tensile load in case of rebar-reinforced UHPC structural members (utilizing the high tensile strength of UHPC), it is suggested that the composite attains a minimum strain of 1% at the peak stress enabling the members to have sufficient ductility. In Figure 62, the strain at peak stress is plotted against $V_f$ for the composite specimens. If 1% strain at the peak stress is considered as the threshold value for ductility, it can be recommended that UHPC with a low fiber volume faction (~0.5%) should be used in conjunction with strain-hardening rebars (such as A1035 bars) in order to achieve ductility for 0.9% reinforcement ratio.
Failure Pattern

Figure 63 and Figure 64 show the macro crack patterns in the composite for 0.5% parallel fibers and 2% parallel fibers, respectively. It can be seen from these two figures that the number of through macro cracks for 0.5% fibers (4) is much higher than that for 2% fibers (1). This is because the specimen with 0.5% fibers has much better ductility due to rebar hardening as compared to the specimen with 2% fibers. This result corroborates the recommendation made above.
Figure 63 Macro cracks for reinforced-UHPC with 0.5% parallel fibers

Figure 64 Macro cracks for reinforced-UHPC with 2% parallel fibers
Comparison between 1/8 symmetry and 1/4 symmetry models

In this study, only 1/8th of the specimen is modeled due to the three symmetry planes along the x-, y-, and z-axes. Since a major crack opens up at a position where the side bar discontinues (Figure 64), one analysis is run by using the 1/4 symmetry model (using the full length of the specimen along the load direction) to see if the crack patterns will change significantly. Figure 65 shows the macro crack patterns for the 1/4 model (for 2% parallel fibers). By comparing Figure 64 and Figure 65, it can be said that the macro crack patterns do not differ significantly. Hence, applying a symmetry plane along the length of the rebar is not only computationally efficient but also gives accurate results.

Figure 65 Macro cracks for reinforced-UHPC with 2% parallel fibers
(1/4 symmetry model)
5. CONCLUSIONS

Amid the growing interest in the application of rebar-reinforced ultra-high performance concrete (UHPC) in the US, the present study investigated the influence of the amount and the orientation of fibers on the bond behavior of rebar-reinforced strain-hardening UHPC using finite element simulation.

The conclusions from the study are summarized below:

- A computationally efficient method for modeling UHPC is proposed. In this method, the fiber is modeled as smeared reinforcement and the UHPC matrix is modeled as a bulk material.
- The fiber stress versus strain data is calibrated by simulating the uniaxial tensile test of UHPC.
- The bond stress versus slip data is calibrated using the pullout test of rebar embedded in UHPC.
- It is observed that the pullout load increases with the increase in fiber volume fraction.
- For a given fiber volume fraction, UHPC with fibers oriented perpendicular to the load direction develops the highest pullout load and UHPC with fibers oriented parallel to the load direction registers the lowest pullout load. The pullout load values with random fibers lie in between.
- The calibrated fiber and rebar properties are then used to simulate the uniaxial tensile test of rebar-reinforced UHPC (composite tensile test).
- The composite tensile strength increases with the increase in fiber volume fraction for a given fiber orientation.
For a given fiber volume fraction, the UHPC with fibers oriented parallel to the load direction shows the highest peak tensile stress and the UHPC with fibers oriented perpendicular to the load direction records the lowest peak stress. The peak stress values with random fibers lie in between.

Stress versus strain curves of the composite show that the modulus of the composite in the strain-hardening region increases with the increase in fiber content. However, ductility of the composite decreases with the increase in fiber volume fraction beyond a certain value. In order to achieve enhanced ductility, it is recommended that the UHPC composite attains a minimum strain of 1% at the peak stress. Using the reinforcement ratio (0.9%) in the present study, it is recommended to use UHPC with 0.5% fibers along with A1035 bars.

For a particular fiber volume fraction, the strain-hardening modulus records the maximum value for the composite with parallel fibers and the minimum value for the composite with perpendicular fibers. The value for random fiber orientation is in between.
REFERENCES


[53] ACI Committe 224. Control of Cracking in Concrete Structures. 2002;ACI 224R-01.