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Study of Supercontinuum and High Repetition Rate Short Pulse Generation

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The rapidly increasing data traffic nowadays has benefited a lot from the dramatic progress of optical telecommunications such as dense wavelength division multiplexing (DWDM) and time-division multiplexing (TDM), which employ broadband light source and high-repetition ultrashort optical pulse respectively to carry information in optical fibers. High speed all optical data processing and switching components will also be important in future fiber-optic communication systems since conventional electro-optical parts have reached their bottleneck both speed-wise and efficiency-wise. In this PhD thesis work, I propose a dispersion varying scheme realized by non-uniformly tapering fibers or waveguides to enhance supercontinuum generation. The physical mechanism to generate broadband continuum such as dispersion, nonlinearity, and soliton dynamics have been explained. Numerical demonstration has been done in both lead silicate microstructured optical fibers and chalcogenide planar waveguides. A fiber ring laser system with rational harmonic mode-locking and nonlinear polarization rotation of a highly nonlinear photonic crystal fiber has been designed and experimentally demonstrated to generate stable ultrashort optical pulse train at a repetition rate of 30 GHz with low noise level. All-optical encryption operating at 250 Gb/s using optical Boolean logic gates based on the two-photon absorption (TPA) in bulk semiconductor optical amplifiers (SOAs) has been demonstrated. The effects of TPA on the performance of optical logic gates based on quantum-dot SOAs have also been explored. TPA can improve the operating speed up to 320 Gb/s.
Study of Supercontinuum and High Repetition Rate Short Pulse Generation

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APPROVAL PAGE

Doctor of Philosophy Dissertation

Study of Supercontinuum and High Repetition Rate Short Pulse Generation

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iii
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# TABLE OF CONTENTS

## Chapter 1 Introduction

1.1 A Brief Introduction of Fiber-Optic Communications ................................................................. 1

1.2 Supercontinuum Source and Ultra-Short Optical Pulsed Train ............................................. 6

1.3 Overview of This Thesis .............................................................................................................. 7

## Chapter 2 Supercontinuum Generation in Dispersion-Varying Microstructured Optical Fibers

2.1 Introduction .............................................................................................................................. 10

2.2 Modeling of Supercontinuum Generation .............................................................................. 12

2.2.1 Physical Mechanisms ........................................................................................................ 12

2.2.2 Generalized Nonlinear Schrödinger Equation .................................................................... 14

2.3 Dispersion Engineering of Microstructured Fibers ................................................................. 15

2.3.1 Dispersive Wave Generation and Dispersion-Varying Scheme ..................................... 15

2.3.2 Dispersion Engineering ..................................................................................................... 19

2.4 Simulation Results .................................................................................................................. 20

2.5 Conclusion ............................................................................................................................ 29

## Chapter 3 Mid-Infrared Supercontinuum Generation in Tapered Planar Waveguides

3.1 Introduction ............................................................................................................................ 30

3.2 Waveguide Design .................................................................................................................. 31

3.3 Simulation Results and Discussions ...................................................................................... 38
Chapter 4 High-Repetition Rate Ultrashort Pulse Generation Using Hybrid Mode-Locking ......................................................................................................................... 46

4.1 Introduction ........................................................................................................................................................................ 46
4.2 Experiment Set-up .............................................................................................................................................................. 48
4.3 Hybrid Mode-Locking Principle ............................................................................................................................................ 51
  4.3.1 Harmonic and Rational Harmonic Mode-Locking ................................................................................................................. 51
  4.3.2 Nonlinear Polarization Rotation ........................................................................................................................................ 52
4.4 Numerical Simulation ............................................................................................................................................................ 56
4.5 Experimental Results ............................................................................................................................................................ 60
4.6 Conclusion .............................................................................................................................................................................. 65

Chapter 5 High Speed All-Optical Encryption Based on Two-Photon Absorption in Semiconductor Optical Amplifiers ...................................................................................................................... 66

5.1 Introduction ........................................................................................................................................................................... 66
5.2 PRBS Model ............................................................................................................................................................................ 68
5.3 Key-Stream Generators ....................................................................................................................................................... 70
5.4 SOA Gain and Phase Dynamics .......................................................................................................................................... 75
5.6 Conclusion .............................................................................................................................................................................. 88

Chapter 6 Effects of Two-Photon Absorption on All Optical Logic Operation Based on QD-SOA .................................................................................................................................................... 89

6.1 Introduction ........................................................................................................................................................................... 89
Chapter 1 Introduction

This introductory chapter is intended to provide an overview of fiber-optic communications. Section 1.1, a brief introduction of fiber-optic communications will be provided. Section 1.2 discusses the supercontinuum source and high-speed pulsed laser source which would benefit the fiber-optic communication systems. In section 1.3, a brief overview of the entire thesis work will be introduced.

1.1 A Brief Introduction of Fiber-Optic Communications

Light has been used as a carrier of information in communication systems since the invention of laser, in 1960. The early work on unguided optic communication systems (non-fiber) has many disadvantages. However, it provided much of the fundamental theory and many of the actual components required for fiber-optic systems. It was not until 1970 that the first truly low-loss fiber was developed and fiber-optic communications became practical [1]. In 1966, Charles Kao and Charles Hockham with the Standard Telecommunication Laboratory in England proposed the first dielectric fiber and showed losses in existing glass was due to contaminants which could potentially be removed. In 1970, the fiber loss was reduced to ~20 dB/km in the wavelength region near 1 μm [2] and the GaAs semiconductor lasers which are able to operate continuously at room temperature were developed at about the same time. Since then, commercial fiber-optic communication systems were deployed and revolutionized the telecommunication industry afterwards.
A basic fiber-optic communication system consists of a transmitter, a receiver and a fiber-optic link. Generally, a laser diode (LD) or a light-emitting diode (LED) is used as the transmitter which is modulated to convert the electrical message into the optical signal. This optical signal is then guided through the optical fiber. Fiber amplifiers are used to compensate for the loss after certain transmission distance to maintain sufficient signal power level. At the receiver end, photodetectors are able to convert the optical signal back to the electrical signal and the message is extracted.

Compared to systems based on electrical cables, fiber-optic communication system has advantages, the most import of which include high data-transmission capacity, low transmission loss, low cost per transported bit, light weight and immune to electromagnetic interference. Since its first implementation, fiber-optic communication has gone through several generations and the transmission capacity has grown significantly during the last three decades due to the emerge of new technologies such as erbium-doped fiber amplifier (EDFA), wavelength division multiplexing (WDM), digital coherent transmission and space-division multiplexing (SDM). This trend is illustrated in Figure 1.1. With the state of art digital coherent transmission technology, a single channel data rate of 100 Gb/s has been commercially available. The fiber-optic communication systems developed over the years operate in four “telecom windows”: 850 nm, 1310 nm, 1550 nm and 1600 nm. These “telecom windows” are associated with the optical loss properties in conventional silica fiber as illustrated in Figure 1.2. Wavelength range from 1530 nm to 1565 nm is called conventional band (C-band) which is most widely utilized in long-haul terrestrial and submarine communications due to the low loss (0.2 dB/km). Nowadays, research has been extended to L-band (1565 nm to 1625 nm) to furtherly increase the transmission capacity with WDM.
Figure 1.1 The increase of fiber-optic transmission capacity over the last three decades through key technology breakthrough.
Figure 1.2 The transmission windows of silica optical fibers.
Figure 1.3 The growth of fiber-optic transmission capacity.
1.2 Supercontinuum Source and Ultra-Short Optical Pulsed Train

Wavelength division multiplexing (WDM) is a technology which multiplexes multiple optical carrier channels on a single optical fiber to increase the data rate dramatically [3]. WDM has significantly increased the transmission capacity of optical fiber and revolutionized the cost per bit of transport. As shown in Figure 1.3, the fiber transmission capacity has grown significantly due to the development of WDM. Dense wavelength division multiplexing (DWDM) refers to optical signals multiplexed within the C-band normally with a 50 GHz channel spacing. Current state of art DWDM is able to reach a transmission capacity of 10 Tb/s for a single fiber. Supercontinuum (SC) is an ideal source to obtain well-managed optical carriers for DWDM fiber-optic communication systems, in which the input optical spectrum is dramatically broadened after propagation through nonlinear media [4, 5]. It is obtained through a combination of dispersion and nonlinearity of the propagation media and it provides an effective way to obtain a large number of coherent wavelength channels. Besides, SC with a spectrum extending further into the mid-infrared (MIR) region can be utilized in many other applications such as molecular spectroscopy, fluorescence microscopy and laser radar [6].

Ultrashort optical pulses in picosecond or femtosecond ranges are comprised of a broad bandwidth of mutually phase-locked wavelengths, which are beneficial to applying DMDW technique. The generation of stable ultrashort optical pulses is also crucial for optical time division multiplexing (OTDM) in high-speed optical communications. OTDM is a technique where several optical signals are combined, transmitted together, and separated again based on different arrival times [7]. The pulse duration must be very narrow to avoid overlap between adjacent pulses when the
transmission speed is high to ensure error-free information transmission. Ultrashort optical pulse train at high repetition rate can be generated from a hybrid mode-locked fiber laser. Ultrashort optical pulses also have application in all-optical Boolean logic gates such as XOR, AND, OR gates. All-optical logic gates are expecting to replace electrical circuits in future Tera-bit telecommunication systems including signal generation and data encryption [8, 9]. Semiconductor optical amplifier (SOA) based Mach-Zehnder interferometer (MZI) is able to modulate ultrashort optical pulses to realize all-optical logic operation. Quantum dot (QD) SOA and SOA based on two-photon absorption (TPA) have both been demonstrated to enhance the operation speed to 250 Gb/s.

1.3 Overview of This Thesis

In this doctoral dissertation, I present my research work on all-optical simulation including supercontinuum generation and logic gates and demonstration of ultrashort pulsed fiber laser. All these devices have been numerically simulated or experimentally demonstrated. In chapter 2, I describe the general procedure for supercontinuum generation simulation by solving the generalized nonlinear Schrödinger equation which governs the propagation of a short pulse in optical waveguides and fibers considering linear and nonlinear optical effects. The detailed mechanisms involved in supercontinuum generation such as dispersion, self-phase modulation, cross-phase modulation, dispersive wave radiation, and stimulated Raman scattering have been discussed. Specifically, I numerically demonstrate enhanced supercontinuum generation in a non-uniform microstructured optical fiber with increasing diameters of air holes in the cladding. This non-uniform structure is able to enhance the output supercontinuum spectrum to over ~6 μm.
In chapter 3, supercontinuum based on planar waveguides with highly nonlinear material is presented. A 2-cm planar rib waveguide made of As$_2$S$_3$ with gradually increased etch depth is simulated for broadband supercontinuum. Combining the high nonlinearity of the chalcogenide material and optimized dispersion profile engineered by the tapering of etch depth, this planar waveguide is able to broaden the spectrum of the injected pulse centered at 1.55 $\mu$m to 1-7 $\mu$m. Supercontinuum is significantly enhanced into the mid-infrared.

In chapter 4, I design a high-repetition-rate ultrashort pulsed fiber laser source. This hybrid mode locked laser combines rational harmonic mode-locking from the RF driven LiNiO$_3$ Mach-Zehnder modulator and passive mode-locking from the nonlinear polarization ration effect of a highly nonlinear photonic crystal fiber. Rational harmonic mode-locking is an efficient way to generate high-repetition-rate optical pulse trains while passive mode-locking has been demonstrated to generate ultra-short pulses with pulse width at sub-picoseconds. By combining these two methods, a 30 GHz pulse train is generated and the pulse width is shortened from 5.8ps to 1.9 ps. Numerical simulation of the pulse evolution has also been conducted.

In chapter 5, I show my design and simulation of a scheme to realize all-optical encryption and decryption using key-stream generators and optical XOR gates. All-optical logic operations are based on ultrafast fast response from the two-photon absorption in semiconductor optical amplifiers. The key used for encryption is acquired from the all-optical pseudo random bit sequence (PRBS) which is constructed by a linear feedback shift register (LFSR) and a XOR gate.
Three other sophisticated key-stream generators including alternative step generator and shrinking generator are designed and simulated. Results show this scheme is able to realize the function of encryption and decryption at 250 Gb/s.

In chapter 6, the effects of two-photon absorption (TPA) of quantum-dot semiconductor optical amplifier (QD-SOA) on all-optical logic operation are studied. QD-SOA based Mach-Zehnder interferometer (MZI) has been demonstrated to realize all-optical Boolean logic operation such as XOR, AND, OR etc. at high speed (250 Gb/s). During the propagation of sub-picosecond pulses, TPA will generate carriers in the QD-SOA. This additional carrier pumping can improve gain recovery time and thus reduce pattern effects of the output. Rate equation model is used to simulate carrier transition in the QD-SOA including TPA. Simulation results show this scheme is suitable for all-optical logic operation at 320 Gb/s considering TPA.
Chapter 2 Supercontinuum Generation in Dispersion-Varying

Microstructured Optical Fibers

2.1 Introduction

Supercontinuum generation (SCG) is a process of spectral broadening of sufficiently intense incident pulses propagating through nonlinear media. Recently, broadband supercontinuum (SC) sources are useful for many applications in the field of optical metrology, optical coherence tomography, optical imaging, dense wavelength division multiplexing (DWDM), and many others [1-4]. The primary mechanism of this spectral broadening is demonstrated to be a combination of action including self-phase modulation (SPM), cross phase modulation (XPM), stimulated Raman scattering (SRS), four-wave mixing (FWM), soliton fission and dispersive properties of the fiber [5]. The output SC spectrum is very sensitive to the dispersive and nonlinear properties of optical fiber. To achieve broadband SC, the control of the fiber dispersion is more important than achieving the maximum possible nonlinearity. Therefore, there has been extensive research focusing on dispersion tailoring to maximize the continuum bandwidth [6-8].

Microstructured optical fiber (MOF), also known as holey fiber, has led to significant progress in ultra-broadband SCG. It has a solid core surrounded by an array of air holes rendering the cladding region with an average refractive index lower than that in the core. Various types of glass such as lead-silicate [9], fluoride [10] and chalcogenide glass [11] have been introduced as core materials in MOFs. The guidance properties of MOFs make it possible to engineer the dispersion characteristic, which is critical to achieving broadband SC. Simply by changing the dimension of
the solid core or air holes, the zero-dispersion wavelength (ZDW) is modified, and multiple ZDWs can be formed. Price et al. studied SCG in non-silica glass MOFs and demonstrated that small core MOFs with two ZDWs can achieve broadest continuum spectra when pumped at wavelength approximately midway between two ZDWs [5]. Solitons undergo Raman-induced frequency shift (RIFS) towards the longer ZDW soon after the fission and amplify dispersive wave (DW) generated across the longer ZDW by the phase matching condition. In [12], Klimczak et al experimentally demonstrated SC spreading from 0.8 μm to 2.6 μm by pumping in the anomalous dispersion region in a dispersion engineered lead-bismuth-galate oxide glass microstructured fiber. Recently, non-uniform waveguides with continuously varying dispersion profiles have been designed to generate SC with wider bandwidth. Chen et al. [10] achieved coherent broadband continuum spanning 1.5-3 μm in a ZBLAN fluoride fiber with exponentially decreasing core size to shift the second ZDW. In [13], Hu et al described a method for mid-infrared SCG in a non-uniform planar waveguide with increasing etch depth to vary the dispersion profile, and a both broadband and flat near-octave spanning spectrum was obtained.

In this chapter, I present a microstructured fiber with increasing air hole diameters along the propagation distance. This allows the second ZDW to shift continuously towards longer wavelengths. Thus, the phase matching condition is also continuously modified accordingly, and, the generated dispersive wavelength can be guided towards a longer wavelength eventually to generate a broad spectrum. By launching hyperbolic secant pulses at 1.55 μm, output spectrum extends from ~1 μm to over 5 μm. Considering the multiphonon absorption edge of the material, output spectrum ends at ~4.7 μm.
2.2 Modeling of Supercontinuum Generation

Supercontinuum generation in optical fibers involves the interplay between nonlinear and linear effects that occur during the propagation of an optical field in the fibers [5]. Generally, according to the dispersion regime of pumping wavelength, SCG can be divided into two categories: pumping in the normal dispersion and pumping in the anomalous dispersion region. When pumping in the normal dispersion region, the spectral broadening mechanism is mainly due to modulation instability such as four-wave mixing; while in the anomalous dispersion region, it is soliton dynamics which dominates the spectral broadening process. Modeling supercontinuum has been widely studied by numerically solving the generalized nonlinear Schrödinger equation, which takes into account various kinds of physical mechanisms such as loss, dispersion, self and cross-phase modulation, soliton effects, Raman scattering and etc. Here I provide a general description of important optical phenomenon contributing to the spectral broadening process.

2.2.1 Physical Mechanisms

Dispersion is a linear effect but one that plays a crucial role in influencing the character of nonlinear interactions in a fiber. It arises from the fact that the phase velocity and group velocity of light propagating in a fiber depend on the optical frequency. The propagation constant $\beta$ can be expanded into Taylor series around the center frequency $\omega_0$ of the input pulse:

$$\beta(\omega) = n_{eff}(\omega) \frac{\omega}{c} = \sum_{k \geq 0} \frac{1}{k!} \beta_k (\omega - \omega_0)^k$$

(2.2.1)

$$\beta_k = \frac{d^k \beta}{d \omega^k} \bigg|_{\omega=\omega_0}$$

(2.2.2)
\(\beta_2\) is the group velocity dispersion (GVD) and higher order dispersion terms are the coefficients associated with the above Taylor series expansion. The wavelength range where \(\beta_2 > 0\) is referred to as the normal dispersion regime, whereas the wavelength range where \(\beta_2 < 0\) is referred to as the anomalous dispersion regime. The wavelength where \(\beta_2 = 0\) is referred to as zero dispersion wavelength (ZDW). When modeling SCG, the dispersion is a combination of both material dispersion and waveguide dispersion. Therefore, to do dispersion engineering, one can modify the waveguide material as well as the waveguide dimensions. More discussion about ZDW and waveguide dispersion are provided in the later section.

Self-phase modulation (SPM) and cross-phase modulation (XPM) can be observed if one only considers the Kerr nonlinearity. The modulation to the refractive index caused by time dependent intensity can cause a time dependent phase delay to the same pulse (SPM) or a co-propagating pulse (XPM). SPM behaves differently in normal and anomalous dispersion regimes. The interaction of XPM in supercontinuum generation can be rather complex as the two interacting pulses can be propagating in different regimes of dispersion. In the anomalous dispersion regime, the nonlinear chirp induced by SPM and linear chirp from GVD can combine to form an either stable or periodic evolution of optical solutions. The order of soliton is determined by:

\[
N = \sqrt{\frac{L_D}{L_{NL}}} = \sqrt{\frac{\gamma P \tau^2}{|\beta_2|}}
\]  

(2.2.3)

High-order solitons are intrinsically unstable and can decay into a series of lower-order solitons in the presence of perturbation. As solitons Raman shift to a longer wavelength, when close to the ZDW, some soliton power will be transferred across the ZDW to the normal dispersion regime to
generate dispersive waves (DWs). Soliton dynamics including soliton fission and non-solitonic radiation (dispersive wave generation) is an important part in supercontinuum generation pumped in anomalous dispersion regime. The SCG discussed below is focused on pumping in the anomalous dispersion regime.

2.2.2 Generalized Nonlinear Schrödinger Equation

The SCG can be numerically described by the generalized nonlinear Schrödinger equation (GNLSE) under the slowly varying envelop approximation [14], which is the fundamental formula that governs optical pulse propagation in nonlinear media.

\[
\frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \sum_{k} \frac{i^{k-1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma (1 + \frac{i}{\omega_0 \partial T}) (A \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT')
\]

(2.2.4)

This equation is derived from Maxwell’s equations and takes into account of linear and nonlinear optical effects such as loss, dispersion, self and cross-phase modulation, Raman scattering and etc. In GNLSE, \(A(z,T)\) is the complex slowly varying amplitude of the pulse envelope in the reference frame moving with the group velocity \(v_g\) of the input pulse, and \(\alpha\) is the fiber loss, \(\gamma\) is the nonlinear parameter defined as \(\gamma = \frac{2\pi n_2}{\lambda A_{\text{eff}}}\), where \(n_2\) is the nonlinear refractive index, \(\lambda\) is the central wavelength of the pulse, and \(A_{\text{eff}} = \frac{\iint |E|^2 dx dy}{\iint |E|^4 dx dy}\) is the effective mode area of the fiber. \(\beta_k\) are the Taylor series expansion coefficients of the propagation constant \(\beta(\omega)\) around the center frequency \(\omega_0\), where \(\beta(\omega) = n(\omega)\omega/c\), \(n(\omega)\) is the frequency dependent refractive index and \(c\) is the speed of light. The nonlinear response function \(R(T) = (1-f_R)\delta(T) + f_R h_R(T)\) takes into account both the instantaneous electronic response indicated by the delta function, and the delayed Raman contribution represented by \(h_R(T)\). The Raman response function is generally acquired by fitting
experimental data. For SF57 glass, $f_R = 0.1$ and the Raman response function can be expressed as [15]

$$h_R(t) = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2} \exp\left(-\frac{t}{\tau_2}\right) \sin\left(\frac{t}{\tau_1}\right)$$

(2.2.5)

where $\tau_1 = 5.5$ fs and $\tau_2 = 32$ fs. The left side of GNLSE models the linear propagation effects, while the right-side models the nonlinear effects. The GNLSE has been well studied and a couple of methods have been proposed to solve it. The most famous one is the split-step Fourier method [14].

2.3 Dispersion Engineering of Microstructured Fibers

Microstructured optical fibers have many nice properties that make them ideal for dispersion engineering and thus for supercontinuum applications.

2.3.1 Dispersive Wave Generation and Dispersion-Varying Scheme

Soliton dynamics is an important mechanism for supercontinuum pumped in anomalous dispersion regime. When the solitons are close to the ZDW, some of the soliton power will be transferred across the ZDW to the normal dispersion regime to generate dispersive waves (DWs) satisfying a phase-matching condition (PMC):

$$\frac{\gamma P_{sol}}{2} = \sum_{k \geq 2} \frac{\beta_k(\omega_{sol})}{k!} (\omega_{DW} - \omega_{sol})^k$$

(2.3.1)

where $\omega_{sol}$ and $P_{sol}$ are the frequency and peak power of the solitons respectively. The energy of the soliton leaks by phase-matching across the ZDW to the normal dispersion region and the dispersive waves can only be generated at phase-matched positions. If one can vary the phase
matching conditions along the propagation of solitons, then it will be possible to guide the dispersive waves to be generated in longer wavelengths. This suggests a possible way to enhance the supercontinuum by continuously modifying the PMC.

Figure 2.1 illustrates the general idea of enhancing the supercontinuum bandwidth by varying the second ZDW along the propagation distance. As the second ZDW shifts to a longer wavelength progressively, the PMC will also be continuously modified. The solitons keep red shifting and the generated DW wavelengths can be guided towards longer wavelengths. Therefore, the output spectrum can be significantly broadened.
Figure 2.1 Conceptual illustration of the continuum generation in the dispersion varying structure.

Figure 2.2 Cross section of the microstructured optical fiber. d is the air hole diameter; Λ is the hole to hole distance (pitch).
Figure 2.3 (a) Dispersion profile and (b) effective mode areas of SF57 microstructured optical fibers with different air hole diameters. The pitch $\Lambda$ is 1 $\mu$m.
2.3.2 Dispersion Engineering

Microstructure fabrication technique has been widely applied to non-silica glasses. Here I choose SF57 as the material for the MOF. SF57 is a commercially available lead-silicate glass widely used to fabricate MOFs for SCG or other nonlinear applications. The nonlinear refractive index of SF57 measured at 1.06 μm is $4.1 \times 10^{-19}$ m$^2$/W, which is much higher than pure silica glass ($2.7 \times 10^{-20}$ m$^2$/W at 1.06 μm) [16]. Although it is still lower than that of chalcogenide glass, SF57 shows higher thermal and crystallization stability while exhibiting low softening temperatures. The material loss of SF57 glass is 1.6 dB/m at 1.55 μm [17]. SF57 has a multiphonon absorption edge of ~5 μm [18]. Since the OH impurities in oxide glasses can lead to high absorption in the mid-infrared region, dehydration techniques should be applied during the fabrication process to reduce the loss (<50dB/m) [19]. Due to the short propagation distance of our designed microstructured fiber ~3 cm, the waveguide related losses can be considered to be negligible [19]. I predict the refractive index of SF57 glass by the generalized Sellmeier equation based on the measured data

$$n^2(\lambda) = 1 + \frac{A_1 A_2^2}{\lambda^2 - B_1} + \frac{A_2 A_3}{\lambda^2 - B_2} + \frac{A_3 A_2}{\lambda^2 - B_3}$$

(2.3.2)

where $\lambda$ is the wavelength of the light propagating in the material, and $A_1$=1.817, $A_2$=0.429, $A_3$=1.072, $B_1$=0.014, $B_2$=0.059, and $B_3$=121.420.

The total dispersion in the fiber depends on both material and waveguide contributions; hence it can be tailored by varying the parameters of the waveguide. Microstructured optical fibers have a solid core and the cladding are filled with air holes as shown in Figure 2.2. The quantity $d$ is defined as the air hole diameter and $\Lambda$ is the hole to hole distance (pitch). The effective refractive index of the cladding region is lowered due to the presence of air holes. Therefore, the light guiding
mechanism is similar to that in step-index fiber. It is known that MOF can be endlessly single mode if the air filling fraction f=d/Λ <0.4 [20]. Even for MOFs with a large air filling fraction ~0.9, when the light launching condition of the MOF is optimized for maximum input coupling, 95% of the light will propagate in the fundamental mode [21]. Therefore, it is a good approximation to consider the continuum output only in the fundamental mode. By varying the diameter of air hole while fixing the pitch, the dispersion profile can be altered. Figure 2.3 shows the different dispersion profiles and effective mode areas for MOFs having different air hole diameters with constant pitch Λ=1 μm. As d increases from 0.35 μm to 0.7 μm, the second ZDW shifts to a longer wavelength into the mid-infrared region. SF57 material dispersion is also plotted in Figure 2.2 for comparison, which only has one ZDW at around 2 μm.

In my designed fiber, the diameters of the air holes increase linearly along the propagation length z with the relationship d = d₀(1+z/z₀), where initial air hole diameter d₀ = 0.36 μm and z₀ = 3.48 cm. The pitch is fixed at 1 μm. This microstructured fiber has a non-uniform structure. The air hole diameters at input (z=0) and output (z=3cm) sides are 0.36 μm and 0.67 μm respectively. The initial injected pulse we used in all the simulations has a hyperbolic secant field profile A(0,T)=√P₀sech(T/T₀), where T₀=56.7 fs corresponding to the full-width-at-half-maximum (FWHM) 100 fs and the peak power is P₀=6 kW. In the simulation, we use 2¹⁴ points and a temporal resolution of 1.6 fs giving a time window of 26 ps.

2.4 Simulation Results
Figure 2.4 Spectral evolution with propagation distance (a) and the output spectrum (b) for SF57 microstructured fiber with $d=0.40$ $\mu$m, $\Lambda=1.0$ $\mu$m.
Figure 2.5 Spectral evolution with propagation distance (a) and the output spectrum (b) for SF57 microstructured fiber with $d=0.50 \, \mu m$, $\Lambda=1.0 \, \mu m$. 
Figure 2.6 Spectral evolution with propagation distance (a) and the output spectrum (b) for SF57 microstructured fiber with non-uniform air hole diameter increasing from 0.36 μm to 0.67 μm along its length to shift the second ZDW.
I first study spectral evolution in the microstructured fiber with uniform air holes. Figure 2.4 shows the results after 3-cm propagation in fiber with constant air hole diameter 0.40 μm. This fiber has the second ZDW at 2.12 μm as shown in Figure 2.3. The seed wavelength 1.55 μm is midway between the two ZDWs and a little closer to the first ZDW. This pumping method has been demonstrated to be the optimum condition to generate supercontinuum efficiently by enabling soliton dynamics. We use a logarithmic density plot to represent the generation and evolution of spectral components. As shown in Figure 2.4, the output spectrum extends to ~3 μm. After temporal compression, the initial soliton decays into fundamental solitons due to perturbations of higher order dispersion and Raman effects. These fundamental solitons then undergo frequency red shift due to intra-pulse Raman scattering and transfer energy to dispersive waves by phase-matching across the second ZDW to the normal dispersion region. However, the transfer of energy from solitons to dispersive waves results in a “spectral recoil” effect that can stabilize the soliton frequency in the vicinity of second ZDW. The solitons stop red shifting and the dispersive waves can only be generated at phase-matched positions. As we can see in Figure 2.4(b), there is a dip in power between the spectral components generated by solitons (peak at ~2 μm) and that by dispersive waves (peak at ~3 μm).

Next, I investigate the microstructured fiber with fixed air hole diameter 0.50 μm which has two ZDWs far away from each other. As seen from Figure 2.3, the second ZDW is far from the seed wavelength 1.55 μm. The SC only spans the 1-2.5 μm range in Figure 2.5. In the long wavelength region, no dispersive waves are generated in this case. Power transfer requires that the soliton overlap spectrally with the dispersive wave; whereas in this case soliton formed in the anomalous
dispersion region may not redshift as far as the second ZDW spectrally; thus, no power is transferred beyond the second ZDW. The spectral broadening on the long wavelength is mainly due to soliton-self-frequency shift (SSFS) and stimulated Raman scattering (SRS).
Figure 2.7 (a) Spectral evolution at selected propagation distance as shown for the non-uniform microstructured fiber. (b) Effect of loss on the output continuum.
Figure 2.8 Output spectra for different shifting rates of air hole diameter: $d = d_0[1 + (z/z_0)^k]$, where
(a) $k=1/4$; (b) $k=1$; (c) $k=2$; (d) $k=4$. 
The simulated SCG in the SF57 non-uniform microstructured fiber that we stated earlier in section 2.3 is shown in Figure 2.6. The SC spectrum at the output of $z=3$ cm extends to over $\sim 5 \ \mu m$. The input pulse undergoes soliton fission and dispersive wave emission across the upper ZDW as stated before. However, the shifting of the second ZDW along the fiber length makes it possible that the DWs are generated towards longer wavelengths with continuously modified phase-matching condition. The solitons also red shift to longer wavelengths since the recoil effect is suppressed when the second ZDW progressively shifts to longer wavelengths. Solitons can transfer energy to DWs over a large regime in the mid-infrared. Thus, the SC spectrum at output is significantly broadened. Figure 2.7 (a) shows output spectrum at selected distance in this 3cm non-uniform fiber. Note that the output spectrum looks less dense at wavelengths larger than $\sim 3 \ \mu m$ and there is no clear dip around 2-3 $\mu m$ region. This is because the energy of the solitons transfers to the DWs and spreads out over a larger spectral region. Since the continuum extends over the material multiphonon edge, the multiphonon absorption should be considered, and result is shown in Figure 2.7 (b). It can be seen that the output spectrum reaches $\sim 4.7 \ \mu m$ limited by the transmission window. The continuum bandwidth is much wider than that of microstructured fibers with uniform air holes. Figure 2.8 compares the output spectra for various shifting rates of air hole diameters with $d=d_0[1+(z/z_0)^k]$ in which $d_0=0.36 \ \mu m$ and $k$ is set to be $\frac{1}{4}, 1, 2, 4$, respectively. The parameter $z_0$ is adjusted so that $d$ remains $0.67 \ \mu m$ at the end of the fiber. It can be seen that the linear shifting gives the broadest SC spectrum. The generated continuum spectrum is sensitive to the dispersive properties along the fiber, which is determined by the geometry of the air holes. This suggests that the second ZDW should shift at an optimal rate (not too fast or too slow). If the ZDW varies too rapidly, there may not be spectral overlap between solitons and DWs; hence the solitons cannot transfer energy to the DWs across the longer ZDW, and the spectrum cannot spread out; on the
other hand, if the ZDW shifts too slowly, the solitons may lose most of the energy before the DWs with longer wavelengths are emitted. In [11], SC spanning from 2.0 to 6.1 μm was generated in a 9 cm chalcogenide microstructured fiber. However, their initial seed pulse was at 3.5 μm which is not at the telecommunications wavelength; also, the length of the fiber is longer than we used here. In [22], SC was produced in dispersion tailored all-solid soft glass photonic crystal fibers pumped at 1550 nm. However, the spectrum broadening was mainly caused by self-phase modulation and four-wave mixing rather than soliton dynamics. This non-uniform structure offers a new way for dispersion management and may be important for particular applications requiring broadband supercontinuum source.

2.5 Conclusion

In conclusion, I have numerically studied supercontinuum generation in a 3 cm SF57 glass microstructured optical fiber with non-uniform structure. This fiber has air holes with linearly increasing diameters in the cladding along its length and the second ZDW is continuously shifting towards a longer wavelength. The results show the continuously modified phase-matching condition can efficiently broaden the output continuum spectra. With input pulse at 1.55 μm, the generated continuum spans from ~1μm to ~4.7 μm limited by the material transmission window. If the transmission window of the material is wider, the spectra can even extend to over 5 μm.
Chapter 3 Mid-Infrared Supercontinuum Generation in Tapered Planar Waveguides

3.1 Introduction

Supercontinuum generation (SCG) in the mid-infrared (MIR) is of growing fundamental interest due to its wide applications in optical metrology, optical imaging, and broadband sources [1]. As stated in the previous chapter, dispersion and nonlinearity are key factors in broadband supercontinuum generation. So far, theoretical and experimental supercontinuum (SC) investigations have mainly focused on fiber based geometries, particular photonic crystal fiber, in which the dispersion can be easily engineered towards a particular purpose [2]. Compared to fibers, planar waveguides have their advantages in low cost fabrication and the potential for integrated optics. For this reason, SCG in planar geometries with high nonlinearity and tailored dispersion has gained much attention recently [3-5]. Zhang et al [3] reported an on-chip octave-spanning supercontinuum generation in a silicon on silica waveguide. The waveguide was designed to exhibit four zero dispersion wavelengths. However, silicon suffers from two-photon absorption (TPA) which limits the achievable output bandwidth. In [6], supercontinuum with ultrawide spectral width up to two octaves was demonstrated experimentally in a chalcogenide rib waveguide by femtosecond pump pulses at 4 μm. In [7], Hu et al designed a dispersion-varying SF57 planar waveguide and reported both broadband and flat near octave SC spanning 1.3-2.5 μm.

In the MIR range, the onset of losses in silica motivates the use of nonsilica compound glasses with high transparency for the generation of broadband continuum. Chalcogenide based glasses
become promising candidates due to their large nonlinearity and wide transmission window in mid-infrared. Ultra-high nonlinearity ($\sim \gamma 93.4/W/m$) was reported in a tapered $\text{As}_2\text{Se}_3$ fiber [8]. However, these glasses exhibit strong normal dispersion at the telecom wavelengths, implying the need for longer wavelength pump lasers. Through proper waveguide design, the zero-dispersion wavelength (ZDW) could be shifted to wavelengths below 1.5 $\mu$m and multiple ZDWs could be formed. As stated in the previous chapter, a dispersion-varying scheme can enhance supercontinuum generation by guiding the generated dispersive waves to longer wavelength. In this chapter, I apply this idea in the dispersion engineering of the planar waveguide and design an axially non-uniform $\text{Air-As}_2\text{S}_3\text{-MgF}_2$ planar rib waveguide. This waveguide has a varying etch depth to shift the second ZDW gradually. By launching hyperbolic secant pulses at 1.55 $\mu$m into this waveguide, supercontinuum spanning from $\sim 1$ $\mu$m to $\sim 7$ $\mu$m is obtained.

3.2 Waveguide Design
Figure 3.1 Cross section of the proposed Air-As$_2$S$_3$-MgF$_2$ planar rib waveguide. $W$, $d$ and $h$ are the rib width, outer slab thickness and etch depth, respectively.
As stated before, I have designed an axially non-uniformly tapered Air-As$_2$S$_3$-MgF$_2$ planar rib waveguide to allow the continuous shift of the second ZDW. The cross section of this rib waveguide is shown in Figure 3.1. A layer of As$_2$S$_3$ glass is deposited onto a MgF$_2$ substrate with the outer slab thickness $d$, etch depth $h$ and rib width $W$. Due to the high absorption of the silica glass beyond 4 μm, MgF$_2$, sapphire and chalcogenide glass are normally used as the substrate material in the MIR applications. MgF$_2$ has a transparency window up to 7.7 μm [9] and sapphire over 5 μm [10]. Here we choose MgF$_2$ to utilize its wide transmission window and low refractive index. Since MgF$_2$ is extremely fragile and easy to break, it may pose a limit to the waveguide length. The linear refractive index of As$_2$S$_3$ glass at 1.55 μm is 2.4, which results in an index contrast of 75% to the MgF$_2$ substrate, allowing for tight modal confinement. The nonlinear refractive index of As$_2$S$_3$ is $n_2=3\times10^{-18}\text{m}^2/\text{W}$, which is much higher than that of pure silica glass ($2.7\times10^{-20}\text{m}^2/\text{W}$) [11]. This high intrinsic nonlinearity not only reduces the pump power requirement for SCG but also enhances the soliton self-frequency shift rate [12]. As$_2$S$_3$ has been demonstrated to have a wide transmission window in mid-infrared with multiphonon edge over 8 μm [13]. At 1.55 μm, the material loss is less than 1 dB/m [14]. With careful environment control during waveguide fabrication, the SH absorption can be maintained at a low level. Low loss, low TPA and high transparency make As$_2$S$_3$ glass a promising material for mid-infrared SCG. Therefore both fibers and planar waveguides fabricated of As$_2$S$_3$ glass have been used in SCG and many other applications [4, 15].
Figure 3.2 Dispersion profiles of various As$_2$S$_3$ planar waveguides with different etch depths $h$ for the fundamental TM mode. The rib width $W$ and outer slab thickness $d$ are 2 $\mu$m and 0.5 $\mu$m, respectively.

Figure 3.3 The nonlinear parameter ($\gamma$) as a function of waveguide etch depth for the waveguide with the same parameters used in Figure 2.
I predict the refractive index of As$_2$S$_3$ glass by the generalized Sellmeier equation \( n^2 = 1 + \sum_{i=1}^{5}[A_i \lambda^2 / (\lambda^2 - L_i^2)] \), where \( \lambda \) is the wavelength of the light propagating in the material, and the fitting coefficients \( A_1 = 1.898, A_2 = 1.922, A_3 = 0.877, A_4 = 0.119, A_5 = 0.957, L_1 = 0.023, L_2 = 0.063, L_3 = 0.123, L_4 = 0.203, L_5 = 750 \) [16]. This equation is assumed to be valid in the entire MIR range of our simulation. As mentioned before, the dispersion of bulk As$_2$S$_3$ glass is strongly normal at near infrared wavelengths and the ZDW is beyond 3 μm. However, the total dispersion is a combination of both material properties and waveguide geometries. Reducing the transverse dimensions will increase the wavelength dependence of the mode effective index and can result in anomalous waveguide dispersion. The waveguide slab thickness and etch depth are the preferred variables used for dispersion engineering. In this chapter, I use etch depth to control the waveguide dispersion. Figure 3.2 shows dispersion profiles of TM mode (vertically polarized) for the rib waveguide with fixed width 2 μm, slab thickness 0.5 μm and various etch depths. The dispersion is calculated by using a full-vector finite difference mode solver for the fundamental TM mode. In practice, a polarization controller can be used to select either the TE or TM mode. As we can see from the Figure 3.2, the anomalous waveguide dispersion can offset the normal material dispersion to form two zero dispersion wavelengths below 3 μm. As the etch depth \( h \) increases, the second ZDW is shifted to a longer wavelength. Varying the waveguide etch depth also changes the mode effective area, which then alters the nonlinear parameter. It is shown in Figure 3.3 that \( \gamma \) decreases as the etch depth keeps increasing for constant width (W) and slab thickness (d).

The SCG can be numerically described by the generalized nonlinear Schrödinger equation (GNLSE) under the slowly varying envelope approximation,
\[ \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \sum_{k \geq 2} \frac{i^{k-1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma (1 + \frac{i}{\omega_0} \frac{\partial}{\partial T})(A \int_{-\infty}^{+\infty} R(T') \left| A(z, T - T') \right|^2 dT'), \quad (3.2.1) \]

For As$_2$S$_3$ glass, $f_R = 0.1$ and the Raman response function acquired from experimental data fitting can be expressed as [17]

\[ h_R(t) = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2} \exp\left(-\frac{t}{\tau_2}\right) \sin\left(\frac{t}{\tau_1}\right), \quad (3.2.2) \]

where $\tau_1 = 15.5$ fs and $\tau_2 = 230.5$ fs.

Based on the characteristics of the total dispersion, I have designed an axially non-uniformly tapered structure for SCG as illustrated in Figure 3.4. This 2 cm planar waveguide has rib width 2 \( \mu \)m, slab thickness 0.5 \( \mu \)m and increasing etch depth from 0.3 \( \mu \)m to 0.5 \( \mu \)m along the propagation distance following the relationship $h = h_0[1 + \left(\frac{z}{z_0}\right)^{1/4}]$. $h_0$ is 0.3 \( \mu \)m, corresponding to the etch depth at the input. Note that low loss As$_2$S$_3$ planar waveguides of similar dimension have been fabricated for SCG and other applications [4]. The tapered waveguide can be fabricated using a multi-step etching process. It involves covering the rib with a mask and etching the rib height to a certain depth, followed by re-masking and re-etching to a larger depth. The process can be repeated a few times to fabricate a rib waveguide with varying etch depth similar to my desired one. The tapering rate will depend on the etching time/etching rate. Because of the short length of my designed waveguide (only 2 cm), the propagation loss is considered to be negligible. The initial injected pulse I used in all the simulations has a hyperbolic secant field profile $A(0, T) = \sqrt{P_0 \text{sech}(T/T_0)}$, where $T_0 = 28.4$ fs corresponding to the full-width-at-half-maximum (FWHM) 50 fs and the peak power is $P_0 = 2$ kW.
Figure 3.4 Schematic of the tapered Air-As$_2$S$_3$-MgF$_2$ planar rib waveguide with longitudinal varying etch depth.

Figure 3.5 Output spectrum for As$_2$S$_3$ planar waveguide with fixed etch depth $h=0.35$ μm (i.e. $k=0$, no taper). The first and second ZDWs of this waveguide are 1.4 and 2.0 μm, respectively. The rib width is 2 μm and the outer slab thickness is 0.5 μm.
3.3 Simulation Results and Discussions

I first study SCG in an As$_2$S$_3$ planar waveguide with constant etch depth at 0.35 μm with the same width and slab thickness. As we can see from Figure 3.2, this waveguide has two zero dispersion wavelengths at 1.4 μm and 2.0 μm. The central wavelength of input pulse is at 1.55 μm, which is located in the middle of the two ZDWs and a little closer to the first ZDW. This is believed to be the optimum pumping condition for SCG. Figure 3.5 shows the output spectrum in which the SC spans from 1 μm out to 3.4 μm. Here in the long wavelength region, the SCG originates from a two-stage mechanism. The input pulse undergoes soliton fission, and the fundamental solitons red shift to the second ZDW due to Raman shift effect. In the second stage, DWs are emitted across the second ZDW toward the normal dispersion regime where the phase-matching condition is satisfied. The solitons transfer energy to the DWs and their frequencies are stabilized due to a spectral recoil effect [18]. As we can see in Figure 3.5, there is a dip in power between the spectral components generated by solitons (around ~1.9 μm) and that by dispersive waves (peak around ~3 μm). The broadened output spectrum only extends to ~3.4 μm, limited by the fixed second ZDW.
Figure 3.6 (a) Spectra at selected propagation distance for the proposed tapered planar waveguide described in Figure 5 with $k=1/4$ and $h_0=0.30 \ \text{m}$. (b) The etch depth as a function of propagation distance is shown.
Figure 3.6 shows the spectral evolution of the incident pulses travelling in the tapered planar waveguide described by Figure 3.4. The effective mode area at the input is equal to $1.31 \, \mu m^2$, yielding a nonlinear parameter $\gamma$ of $9.3 \, /W/m$. It can be seen that the generated spectrum has a greater bandwidth, reaching from $\sim 1 \, \mu m$ to $\sim 7 \, \mu m$. The dimension of the waveguide at the input is designed so that a second ZDW exists. Therefore, the input pulses undergo soliton fission and DW emission across the upper ZDW as stated before. However, for the tapered waveguide with an increasing etch depth, its second ZDW is shifted along the propagation distance, leading to a continuous modification of the phase-matched wavelength for DW emission. Solitons can transfer energy to DWs over a larger regime and DWs can be produced towards longer wavelengths. The spectral unevenness caused by the gap between solitons and DWs is successfully overcome and the energy transfer efficiency is enhanced. Thus, the SC at the output is significantly broadened.

As we can see from Figure 3.6, there is no clear dip in power around 2-3 $\mu m$ region in the generated output spectrum. Note that this result is comparable with that presented in [19], where SC extending to $\sim 4.8 \, \mu m$ was obtained in a 4 cm SF57 fiber taper seeded by femtosecond pulses with peak power 6 kW. I have predicted broader output spectrum from a planar waveguide with a substantially lower pumping power and a shorter propagation distance. In [3], flat SC extending from 1.2 to 2.5 $\mu m$ was obtained from a dispersion tailored silicon chip. However, it turned out SCG in silicon could seriously suffer from two-photon absorption in some cases. In [4], SC bandwidth spanning 1.4 octaves was produced in a chalcogenide planar waveguide with similar dimension as the one used here. However, the spectrum broadening was mainly caused by four-wave mixing rather than soliton dynamics. Figure 3.6 (b) shows that for $k = 1/4$, the etch depth $h$ increases rapidly initially ($\sim 0.05 \, mm$) until $h$ reaches $\sim 0.35 \, \mu m$. When fabricating this device, one can also begin with an etch depth at $0.35 \, \mu m$ and then start tapering following the rest of the
curve (using the equation for $h(z)$) in Figure 3.6(b). The simulation shows that there is nearly no difference in the generated supercontinuum between the two cases.
Figure 3.7 Output spectra for different up tapering rates: $h = h_0 \left[ 1 + \left( \frac{z}{z_0} \right)^k \right]$, where (a) $k=1/6$; (b) $k=1/4$; (c) $k=1/2$; (d) $k=1$ (linear). The waveguide etch depths at the input and the output are 0.3 $\mu$m and 0.5 $\mu$m.
Since the geometry of the planar waveguide determines the dispersion characteristic, it plays a critical role in the SC process. Figure 3.7 compares the output spectra for various planar geometries with different up tapering rates, following \( h = h_0[1 + (z/z_0)^k] \) in which etch depth at the input \( h_0 = 0.3 \) μm and \( k \) is set to be 1, 1/2, 1/4, 1/6, respectively. The parameter \( z_0 \) is adjusted so that \( h \) remains 0.5 μm at the exit. The waveguide rib width is 2 μm and the slab thickness is 0.5 μm for all calculations. It can be seen that the form of tapered waveguide described by Figure 3.6 gives the broadest SC spectrum. Since the rate at which the second ZDW increases is determined by the rate of up tapering of the waveguide, the tapering rate is directly linked to the rate of redshift of the solitons. As solitons move to spectral regions of lower nonlinearity and transfer energy to DWs, the redshift rate gets slower and the tapering rate should also slow down with propagation distance to allow solitons to catch up [12]. Therefore, the generated spectra for \( k = 1/2, 1/4, 1/6 \) are much wider than that of linear type (\( k = 1 \)). This is shown in Figure 3.8.
Figure 3.8 The calculated 40 dB supercontinuum spectral bandwidth for various k values.
Figure 3.8 shows the calculated 40 dB bandwidth of the generated spectrum for various k values. The waveguide etch depth h follows the same relationship with propagation distance z as described in Figure 3.7. The figure shows that k=1/4 is the optimum value for achieving the broadest supercontinuum spectrum. Figure 3.7 and 3.8 suggest that the second ZDW should shift at an optimum rate (not too fast or too slow). If the ZDW shifts too rapidly, there may not be spectral overlap of solitons and DWs; hence the solitons cannot transfer power to the DWs across the ZDW, and the spectrum cannot spread out; on the other hand, if the ZDW shifts too slowly, the solitons may lose most of the energy before the DWs with longer wavelengths are emitted [20]. Therefore, through a careful design of the tapered waveguide structure, we can obtain SC with a large bandwidth.

### 3.4 Conclusion

I present and numerically demonstrate supercontinuum generation in a non-uniformly tapered planar waveguide structure. A tapered Air-As$_2$S$_3$-MgF$_2$ planar rib waveguide with increasing etch depth is numerically modelled by solving a generalized nonlinear Schrödinger equation where the dispersion, nonlinear coefficient and Raman contribution are included. The waveguide has zero dispersion at two wavelengths and the second ZDW is gradually shifted to a longer wavelength to modify the phase-matching condition for emitted dispersive waves. With input pulse at 1.55 μm, the generated spectrum spans from ~1 μm to ~7 μm. This on-chip broadband SC source may be important for certain applications in integrated optical system.
Chapter 4 High-Repetition Rate Ultrashort Pulse Generation Using Hybrid Mode-Locking

4.1 Introduction

The generation of stable high-speed pulse train with ultrashort pulse width is very important for high-bit-rate fiber optic telecommunication system. During the past few decades, the generation of high speed short pulses has been studied extensively [1-5]. Among various platforms, active harmonic mode-locking has proven to be an effective way to generate high-repetition-rate pulses by incorporating an electro-optical intensity modulator, such as LiNiO$_3$ modulators or electric absorption modulators (EAM) inside the laser cavity. In particular, the implementation of rational harmonic mode-locking technique is able to overcome the modulator bandwidth limitation and further increase the repetition rate [1, 6]. For pth order rational harmonic mode-locking, the cavity-loss modulation frequency is intentionally detuned $f_c/p$ away from the exact harmonics of the cavity’s fundamental frequency $f_c$, resulting in a pulse train with a repetition rate $p$ times larger than that for harmonic mode-locking. However, with rational harmonic mode-locking, the temporal pulse duration is usually limited to several picoseconds. Furthermore, the amplitudes of pulse train suffer from severe fluctuations in the time domain and are not generally equal for $p>2$. This amplitude unevenness is not good for fiber optic communication systems. Several researchers have carried out experiments to either compress the optical temporal width or suppress the supermodes noise to improve the laser stability. Lin et al proposed a scheme to equalize the pulse amplitudes of a 40 GHz rational harmonic mode-locked pulse train by reshaping the semiconductor optical amplifier gain within one modulation period [7]. Ma et al demonstrated a pulse width
compression scheme by guiding the pulse train generated from a rational harmonic mode-locked fiber ring laser to pass through a nonlinear amplifying loop mirror twice [5]. Li et al demonstrated theoretically and experimentally pulse-amplitude-equalization in 4th rational harmonic pulses based on nonlinear polarization rotation [8].

Compared to actively mode-locked lasers, passively mode-locked fiber lasers have the advantage of generating pulse train at ultrashort pulse width. Recently, compact passively mode-locked fiber lasers have been successfully constructed to generate subpicosecond pulses by using a saturable absorber (SA) such as semiconductor saturable absorber mirror (SSAM) [9], graphene [10], nonlinear polarization rotation (NPR) technique [11, 12] and nonlinear fiber loop mirror [13]. The NPR technique combined with a polarizer can induce an intensity depended loss in the cavity and has been used to achieve ultrashort pulses in fiber lasers. In [11], Luo et al constructed a L-band passively mode-locked fiber laser utilizing the NPR technique and generated pulses with full width at half maximum (FWHM) 458.7 fs. However, the pulse repetition rate is only at 8.6 MHz. Liu et al reported the generation of a stable passive 23rd harmonic mode-locked pulse train at 230 MHz with a pulse width of 0.44 ps [12]. Despite the fact that those NPR based passively mode locked fiber lasers can produce ultrashort femtosecond pulses, they suffer from the drawback of low repetition rate (only at MHz level) with respect to the total cavity length, which limits their applications in high speed fiber optic communications. A possible solution would be to build a hybrid mode-locked scheme to combine these two mode-locking methods [3, 14]. Li et al both numerically and experimentally demonstrated that by incorporating a charcoal nano-particle saturable absorber into the rational harmonic mode-locked laser cavity, they were able to generate a pulse train at high repetition rate (20 GHz) and short pulse width (~3.2 ps) [3].
In this chapter, I report a hybrid mode-locked erbium-doped fiber ring laser by combining the active rational harmonic mode-locking and the NPR based passive mode-locking. A high speed 30 GHz pulse train with improved stability and narrower pulse width is generated. In this hybrid scheme, by carefully adjusting the polarization controllers the full width at half maximum (FWHM) of a 30 GHz pulse train is shortened to ~1.9 ps compared to ~5.8 ps with only rational harmonic mode locking and the pulse amplitude is equalized simultaneously.

4.2 Experiment Set-up

Figure 4.1 shows the experimental setup of the proposed hybrid mode-locked fiber ring laser system. The gain of the fiber laser is provided by an EDFA, which consists of a 23-m long Erbium-doped fiber (EDF) and a 980-nm pump laser diode. A LiNbO$_3$ Mach-Zehnder modulator (MZM) driven by ~10 GHz radio frequency (RF) signal is utilized for rational harmonic mode-locking. Because the loss of the LiNbO$_3$ modulator is polarization sensitive, a polarization controller PC3 is inserted at the input port of the modulator. An optical isolator in the cavity is to ensure unidirectional propagation of the laser mode. A highly nonlinear photonic crystal fiber (PCF), an inline polarizer and two PCs (PC1 and PC2) are used to generate nonlinear polarization rotation effect. If we remove polarization controllers PC1, PC2 and the polarizer, then there is no NPR effect in the cavity. The polarization controllers used here utilizes stress-induced birefringence of the fiber to create independent wave plates to alter the polarization of the transmitted light. The fractional wave plate is created by winding a short length of single mode fiber around a spool. Typically, the insertion loss is very low (~0.2 dB). Compared to polarization controllers fabricated
using bulk components, which use the $\lambda/4$ or $\lambda/2$ wave plates based on the birefringence of a crystal, fiber polarization controllers (stress induced) do not require fiber-to-free-space coupling; thus they are very easy to incorporate in a fiber cavity with low loss [15]. The laser output is coupled out using a 90:10 coupler. All the components are connected by standard single-mode fibers (SSMFs).
Figure 4.1 Experiment setup of the hybrid mode-locked fiber ring laser based on the combination of rational harmonic mode locking and the passive nonlinear polarization rotation technique. EDFA: Er-doped fiber amplifier, MZM: Mach-Zehnder modulator, PC: polarization controller, OC: optical coupler.
4.3 Hybrid Mode-Locking Principle

The idea of using hybrid mode locking is to combine the high repetition achieved by active mode-locking and ultrashort pulse width achieved by passive mode-locking to generate short pulse train at high repetition rate. Here I will briefly talk about the working principle of harmonic and rational harmonic mode locking, which are ways of active mode locking and passive mode locking based on the nonlinear polarization rotation of a photonic crystal fiber.

4.3.1 Harmonic and Rational Harmonic Mode-Locking

Harmonic mode locking is standard way to generate pulse train with high repetition rate. Assume a fiber ring laser with cavity length L, then the fundamental cavity frequency is \( f_c = \frac{c}{(Ln_{\text{eff}})} \), where \( c \) is the velocity of light in vacuum and \( n_{\text{eff}} \) is the effective refractive index of the fiber. This fiber ring laser typical has a modulator driven by external RF signal of frequency \( f_m \) to modulate the loss of the cavity. For harmonic mode locking, \( f_m = nf_c \), where \( n \) is an integer and the repetition rate of generated pulse train is equal to \( f_m \). Considering the loss and gain in the fiber ring laser and using the Fourier analysis method, one can get the expression for the envelope of mode locked pulse [16]:

\[
a(\omega) = A\exp\left(-\frac{\omega^2\tau^2}{2}\right) \tag{4.3.1}
\]

\[
a(t) = \frac{\sqrt{2\pi}}{\tau} A\exp\left(-\frac{t^2}{2\tau^2}\right) \tag{4.3.2}
\]

\[
\tau = \left(\frac{2g}{M}\right)^{1/4} \left(\frac{1}{\omega_m\Omega_g}\right)^{1/2} \tag{4.3.3}
\]

where \( g \) is the gain at the center frequency, \( \omega_m = 2\pi f_m \), \( M \) is the modulation depth and \( \Omega_g \) is the...
gain bandwidth. The solution is a Gaussian pulse for harmonic mode locking pulse and the pulse width is proportional to $f_m^{-1/2}$.

When the modulation frequency $f_m$ is detuned $f_c/p$ away from the exact harmonics of $f_c$, where $p$ is an integer, then it is called rational harmonic mode-locking. In this case $f_m = (n + 1/p)f_c$, the pulses have to circulate in the cavity $p$ more times before it can emit as output. Therefore, the repetition rate of the generated pulse train is increased by $p$ times. Following a similar analysis of harmonic mode locking, we can conclude the solution for rational harmonic mode-locking is also a Gaussian pulse given by Equation (4.3.2) and the pulse width $\tau$ is expressed as:

$$\tau = \left(\frac{2g}{M}\right)^{1/4} \left(\frac{1}{(n+1/p)\omega_M f_g}\right)^{1/2}$$  \hspace{1cm} (4.3.4)$$

From the equation above, we can see the pulse width parameter $\tau$ is now proportional to $(n + 1/p)^{-1/2}$. Since $n$ is an integer much larger than 1, one can conclude the pulse width for rational harmonic mode-locking is almost the same as that for harmonic mode-locking.

### 4.3.2 Nonlinear Polarization Rotation

Here the passive mode-locking is realized by the nonlinear polarization rotation. The PC-PCF-PC-polarizer structure can introduce the intensity-dependent loss and the transmission principle is shown in Figure 4.2. The PCF used here has dispersion parameter $\beta_2 = -1.66 \text{ ps}^2/\text{km}$, $\beta_3 = -0.03 \text{ ps}^3/\text{km}$ and nonlinearity $\gamma = 11/\text{W/km}$. The birefringence $\Delta n$ is $3.5 \times 10^{-5}$. In Figure 4.2, $\alpha_1$ is the angle between the fast axis of the PCF and the polarization direction of the input signal before entering the PCF. $E$ is the electric vector of input signal. $\alpha_2$ is the angle between the fast axis of
PCF and the polarization direction of the in-line polarizer. The Kerr nonlinearity of the PCF can generate a rotation of polarization state, which depends on the pulse intensity. The transmission introduced by NPR can be expressed as[8]:

\[ T = \cos^2 \alpha_1 \cos^2 \alpha_2 + \sin^2 \alpha_1 \sin^2 \alpha_2 + \frac{1}{2} \sin 2 \alpha_1 \sin 2 \alpha_2 \cos (\Delta \varphi_L + \Delta \varphi_{NL}) \]  (4.3.5)

\[ \Delta \varphi_L = (n_x - n_y) / \beta L \]  (4.3.6)

\[ \Delta \varphi_{NL} = - \left( \frac{1}{3} \right) \gamma P L \cos \alpha_2 \]  (4.3.7)

where \( \beta = 2\pi / L \) is the propagation constant; \( \Delta \varphi_L \), \( \Delta \varphi_{NL} \) are the linear and nonlinear phase changes; and \( L \), \( n_x \), \( n_y \), \( \gamma \) are the length, linear birefringence coefficient of fast axis and slow axis and nonlinear coefficient of the PCF. \( P \) is the instantaneous power of input signal. The quantities \( \alpha_1 \) and \( \alpha_2 \) which determine the transmission through the NPR structure can be adjusted by changing the two polarization controllers PC1 and PC2.
Figure 4.2 Operation principle of NPR. E: electric field, x: fast axis of PCF, y: slow axis of PCF, PC: polarization controller.

Figure 4.3 Conceptual illustration of effects of different transmission curves induced by NPR technique on pulse shaping.
As shown in Equation (4.3.5), we can plot a cosine curve relation between the transmittivity and the instantaneous power. The value of $\alpha_1$ and $\alpha_2$ will not only decide the offset and the amplitude of the cosine curve, but also affect the period of the cosine curve. Tuning PC1 and PC2 properly to change the values of $\alpha_1$ and $\alpha_2$, then the transmission curve can be varied as shown in Equation (4.3.5). When the rational harmonic mode-locked 30 GHz pulses go through the NPR mechanism, they experience pulse shaping. Therefore, different transmission curves will result in different pulse shaping mechanisms as suggested by Figure 4.3. Fang et al utilized this pulse shaping mechanism induced by the NPR transmission properties and demonstrated flat-top pulse generation in a fiber ring laser [17]. The three transmission curves in Figure 4.3 correspond to three different $\alpha_1$ and $\alpha_2$ sets. If the NPR induced transmission curve is the dotted line as shown in Figure 4.3, the low intensity part of the pulse (pulse wings) will have high transmission while the high intensity part (pulse center) will experience low transmission. In this case, the NPR somehow acts as a pulse equalizer which can reduce the intensity fluctuations of the pulse train; however due to the high transmission of pulse wings, it will also broaden the pulse width meanwhile. The dashed line in Figure 4.3 corresponds to a state that the high intensity part (pulse center) of the input pulse experiences little loss, which the low intensity parts (pulse wings) undergo high loss. The NPR in this state has the same functionality as a saturable absorber, thus only leading to pulse compression. Compared to a saturable absorber, the NPR technique has more flexibility in acquiring various transmission curves. When the transmission curve is adjusted to the solid line shape, where the pulse wings still undergo great loss, the pulse center will experience a bit higher loss than its adjacent part. Therefore, at this state, the pulse width is narrowed due to the high absorption of the leading and trailing edges and if the pulse peak intensity suddenly rises or falls due to the environmental disturbance or mode competition, the transmission loss of the NPR will also rise or
fall correspondingly. As a result, the instantaneous amplitude fluctuations of the pulses are suppressed. Thus, compressing the pulse width and equalizing the pulse train amplitudes can be realized simultaneously.

### 4.4 Numerical Simulation

I have conducted numerical simulation of the hybrid mode-locked fiber laser. For optical pulses with short temporal width, the generalized nonlinear Schrödinger equation (GNLSE) governs their propagation along the fibers[18]:

$$
\frac{\partial A(z, \tau)}{\partial z} + \frac{\alpha}{2} A(z, \tau) + \sum_{k \geq 2} \frac{i^{k-1}}{k!} \beta_k \frac{\partial^k A(z, \tau)}{\partial \tau^k} =
$$

$$
\frac{g}{2} A(z, \tau) + \frac{g}{2 \Omega_g} \frac{\partial^2 A(z, \tau)}{\partial \tau^2} + i \gamma A(z, \tau) |A(z, \tau)|^2
$$

where $A(z, \tau)$ is the slow varying pulse envelope; $z$ is the propagation distance; $\tau$ is the time delay parameter; $\alpha$ is the fiber loss and $\gamma$ is the nonlinear parameter. $\beta_k$ represent the dispersion coefficients and I include up to the third dispersion parameter in the simulation. $\Omega_g$ denotes the gain bandwidth of the EDF and the saturation effect of the EDF is taken into account by expressing the gain factor as $g = g_0/(1+E/E_{sat})$, where $g_0$ is the small signal gain, $E$ is the pulse energy and $E_{sat}$ is the EDF gain saturation energy. The GNLSE is solved by applying the split-step Fourier method [19]. The values for $\beta_2$, $\beta_3$, $\gamma$, $E_{sat}$ and $g_0$ for EDF used in this numerical simulation are: $-0.13 \times 10^{-3} \text{ ps}^2/\text{m}$, $0.135 \times 10^{-3} \text{ ps}^3/\text{m}$, $3.69 \text{ W}^{-1} \text{ km}^{-1}$, 0.1 pJ and 1.09 dB/m. For pulse evolution in the PCF and the SSMF which composes the ring laser cavity, we just simply set $g=0$. Equation (4.3.5) models the effect of NPR. The quantities $\alpha_1$ and $\alpha_2$ which determine the transmission through the NPR
structure can be adjusted by changing the polarization controllers PC1 and PC2. During each round trip after the polarizer, the output pulse envelope \( A(z, \tau)_{out} = \sqrt{T(\alpha_1, \alpha_2)} A(z, \tau)_{in} \), in which \( T(\alpha_1, \alpha_2) \) as a function of \( \alpha_1 \) and \( \alpha_2 \), is evaluated by Equation (4.3.5) – (4.3.7). In the simulation, we set \( \alpha_1=45^\circ \) and explore the effects of various \( \alpha_2 \) values. The output coupler is modeled by \( A(z, \tau) = RA(z, \tau) \). Since we use a 90/10 coupler in our fiber ring laser system as in Figure 4.1, R is \( \sqrt{0.9} \).

The Lithium Niobate Mach-Zehnder modulator driven by an RF signal can be described by the following single-pass transmission function [20]:

\[
T = \cos^2 \left( \pi \frac{V_b + V_m \sin(2\pi f_m \tau)}{V_\pi} \right)
\]

(4.4.2)

in which \( V_\pi \) is the voltage required for a phase shift of \( \pi \) between the two arms and it’s 6 V; \( V_b \) is the DC voltage bias; \( V_m \) and \( f_m \) are the amplitude and frequency of the RF signal, respectively. Simulation is initiated by launching into the system a seed pulse with small amplitude. The pulse evolution within the ring cavity is then iteratively modeled until a steady state is reached after many roundtrips. The asymptotic state is independent on the seed pulse.
Figure 4.4 Simulated pulse generation from the fiber ring laser implementing (a) only the rational active harmonic mode-locking and (b) hybrid mode-locking with NPR in the cavity, $\alpha_1 = 45^\circ$ and $\alpha_2 = 30^\circ$. 
Figure 4.5 Numerical simulation results of the evolution of the pulse width in the fiber ring laser.
(a) Only rational active harmonic mode-locking. (b) Hybrid mode-locking with NPR, $\alpha_2=30^\circ$. (c) $\alpha_2=120^\circ$. (d) $\alpha_2=85^\circ$. 
Figure 4.4 depicts the simulated pulse development of the output pulses with only rational harmonic mode-locking and hybrid mode-locking, respectively. Figure 4.5 shows the evolution of pulse width (as it makes the round trips in the cavity) with different polarization configurations (corresponding to different $\alpha_2$ values). Figure 4.5(a) is the active mode-locking only case and the generated pulse width is ~5.63 ps. In Figure 4.5(b) the NPR works strongly as a pulse compressor and the generated pulse width is compressed to ~1.88 ps. In Figure 4.5(b) we can see, after some cycles, the pulse width is broadened to ~6.56 ps. The NPR in this state only acts as an amplitude equalizer and as a result the pulse width is broadened. In Figure 4.5(d), the NPR does not affect the pulse width significantly and the output pulse width is ~5.25 ps.

### 4.5 Experimental Results

After carefully tuning the PCs and the frequency of the RF signal, a 30 GHz pulse train with ultrashort pulse width can be generated. When the fundamental frequency of the cavity $f_c$ and the modulation frequency $f_m$ satisfies the condition $f_m = (n+1/p)f_c$, where $n$ and $p$ are both integers, the laser resonator operates in the rational harmonic mode-locking (RHML) regime and the pulse train with a repetition rate of $pf_m$ can be produced. Here in our case, $f_m$ is set at ~10 GHz to realize 3rd order rational harmonics. If we remove polarization controllers PC1, PC2 and the polarizer, we can assume that the NPR has no effect on pulse shaping or there is no NPR in the cavity. Figure 4.6 shows the auto-correlation trace of the output pulse train with only RHML (a) and of hybrid mode-locking with NPR (b). Without NPR, the generated pulse train has a calibrated pulse width ~5.8 ps which is very close to our numerical calculated value 5.63 ps. However, with NPR in cavity and after the careful tuning of the PCs, the pulse width of the generated 30 GHz pulse train is
shortened to ~1.9 ps. The compressing ratio is as high as 67%. The NPR inside the cavity can greatly improve the pulse shortening mechanism of the fiber ring laser due to the high loss it induced to the pulse wings. The central part of the pulse experiences relatively low loss compared to the pulse wings. This agrees well with the numerical simulation as it shows that with $\alpha_2=30^\circ$, the pulse width could be compressed to 1.88 ps. In [21], a saturable absorber (single-walled carbon nanotube) was combined with nonlinear polarization evolution to generate ultra-short pulses at 93 fs at 38.1 MHz repetition rate using passive mode-locking. I have generated pulses with 1.9 ps pulse width at 30 GHz repetition rate using active mode-locking. Carbon nanotubes can be fabricated using relatively simple techniques. The nanotube composite can be sandwiched between two fiber connectors, which can be conveniently incorporated in a fiber cavity [22]. However, as saturable absorbers, their characteristics depend on the particular sample fabrication process, which makes the corresponding mode-locked laser performance variable. The saturable absorption recovery time of nanotubes is slow compared to the nonlinear response of a Kerr-medium (NPR mechanism). The pulse shortening mechanism in this paper is rational harmonic active mode locking and the nonlinear polarization rotation and that of Ref [21] the primary mechanism is passive mode-locking using saturable absorption followed by nonlinear polarization evolution. In Figure 4.6, the temporal spacing between the peaks is ~33.2 ps which also confirms the repetition rate at 30 GHz.
Figure 4.6 Auto-correlation trace of the pulse train output from the fiber ring laser. (a) With only rational harmonic mode-locking, the 30 GHz pulse width is ~5.8 ps. (b) Hybrid mode-locking with NPR in the cavity, the 30 GHz pulse width is compressed to ~1.9 ps.
Figure 4.7 RF spectra of the generated pulse trains at 30 GHz. (a) Only rational harmonic mode-locking. (b) Hybrid mode-locking with NPR in the cavity.

Figure 4.8 The measured oscilloscope trace of the generated 30 GHz pulse train. (a) Only rational harmonic mode-locking. (b) Hybrid mode-locking with NPR in the cavity.
Figure 4.7 compares the RF spectra of the generated 30 GHz pulse train with only RHML (a) and of hybrid mode-locking with NPR (b). The highest peak in both figures correspond to the operating frequency of the fiber ring laser which is ~30 GHz. As we can see in Fig 7(a), without NPR, there are sidebands in the RF spectrum. These are the supermodes which cannot be removed by tuning the modulation frequency $f_m$ and the polarization controller PC3 in the cavity. However, no supermodes are observed in Fig 7(b) with the presence of NPR in the cavity. The signal-to-noise ratio is over 25 dB after the NPR incorporated. Because the supermodes competition directly leads to pulse fluctuation between adjacent peaks, the reduction of the supermodes in the RF spectrum can improve the stability of optical pulse generation in high order rational harmonic mode locking [1]. The reduced amplitudes of the supermodes in the fiber ring laser is the direct result of the specific transmission property induced by the NPR effect. This is also confirmed in Figure 4.8. As shown in Figure 4.8(a), without the PC1, PC2 and the polarizer in cavity, the generate 3rd order rational harmonic mode locking pulse train suffers severe pulse amplitudes unevenness. The pulse amplitude carries a periodical envelope mainly due to the mismatch between the driving RF signal frequency and the fundamental cavity frequency [23]. With the introduction of NPR into the cavity, pulse amplitude is equalized. As we can see in Figure 4.8(b), there is nearly no amplitude fluctuations between adjacent pulse peaks and the amplitude of the pulse train is almost even. Therefore, by careful tuning of the PC1 and PC2, the equalization of the pulse amplitude and the compression of the pulse width are realized simultaneously, as a result of the desired transmission characteristic of the NPR technique.
4.6 Conclusion

In conclusion, I have designed and experimentally demonstrated a hybrid mode-locked fiber ring laser, which combines rational harmonic mode locking technique and a nonlinear polarization rotation scheme, to generate an optical pulse train with improved stability and shortened pulse width. The laser is operated at 30 GHz using an intra-cavity LiNbO$_3$ Mach-Zehnder modulator at the frequency of $\sim$10 GHz to achieve 3$^{rd}$ rational harmonics. The generated pulse width is compressed from $\sim$5.8 ps to $\sim$1.9 ps and the pulse amplitude fluctuations are significantly reduced for this hybrid scheme. Pulse evolution in the laser ring cavity has also been numerically simulated by solving the generalized nonlinear Schrödinger equation, which shows good agreements with the experimental results.
Chapter 5 High Speed All-Optical Encryption Based on Two-Photon Absorption in Semiconductor Optical Amplifiers

5.1 Introduction

Encryption and decryption have long been used to facilitate secure communication. Encryption is the conversion of data into a cipher text using an encryption algorithm that cannot be easily understood by unauthorized user. Decryption is to convert encrypted data back into its original form, so it can be understood. This whole process usually needs a key-stream. The pseudorandom bit sequence (PRBS) can be used as the key-stream for encryption and decryption.

In future high-speed communication systems, all-optical data processing will be important [1]. The pseudorandom bit sequence (PRBS) which was first introduced in electronics, is characterized by its simplicity of generation, good repeatability and statistical properties [2]. It thus received wide application, including in simulation of noise in signal transmission, data encryption/decryption, and in bit error rate testers (BERTs). A pseudorandom bit sequence (PRBS) can be generated using a linear feedback shift register (LFSR) [3]. To generate a stable optical PRBS sequence using LFSR, an optical XOR logic gate is needed. In recent years, demonstrations of high speed all-optical XOR logic gates using different schemes were reported, including using semiconductor optical amplifier loop mirror (SLALOM) [4], ultrafast nonlinear interferometer (UNI) [5], and the SOA based Mach-Zehnder interferometer (SOA-MZI) [6, 7]. However, all these schemes have a limitation of signal bit rates of up to 40 or 80 Gb/s. A scheme for higher data rate logic using quantum dot (QD) based semiconductor optical amplifier has been studied [8, 9]. It can operate at
data rates of 250 Gb/s. Another process that can result in a fast phase change of a probe signal is two-photon absorption (TPA) of a pump beam [10, 11]. Pump-probe experiments have shown that phase changes take place in a duration ~1 ps or less when the pump and probe signals are injected into a semiconductor optical amplifier [10, 11].

In this chapter, I present a model to simulate the encryption and decryption process using all-optical gates based on TPA of pump beams. This work shows a faster scheme for encryption using two photon absorption (TPA) based optical logic than has been demonstrated previously. The scheme is suitable to data rates of 250 Gb/s. The encryption algorithm and key we used in this model are XOR operation and PRBS generators respectively. Both the high speed all-optical XOR and PRBS generators are realized by using SOA-MZI based on two-photon absorption (TPA) of pump beams. The concept of XOR and PRBS using SOA based MZI has been described and demonstrated earlier [8-13]. The principle new element of adding two-photon absorption (TPA) is that the phase change ($\delta \phi$) due to the pump beam has a fast component, the magnitude of which is given by [12]:

$$\delta \phi = -\left(\frac{1}{2}\right) \beta \alpha_2 S(t)L$$  \hspace{1cm} (5.1.1)

where $\beta$ is TPA coefficient, $\alpha_2$ is linewidth enhancement factor associated with the TPA process, $S(t)$ is the optical pump pulse intensity, and $L$ is the effective length of SOA active region, which is the length of gain medium in SOA. The negative sign represents the observation that the TPA induced phase change is in opposite direction from that for gain change induced phase change. The input average power of the SOA used in this paper to obtain significant two-photon absorption is 20 mW in 0.5 ps (FWHM) which corresponds to a peak power of 160 mW at 250 Gb/s. The
induced phase changes are large enough for temporal interference of probe pulse travelling through the two arms of the MZI. In this chapter, I also design and investigate three additional secure key-stream generators: three LFSRs cascaded design, alternating step generator and shrinking generator. Results show that encryption and decryption using key-stream generators can be realized at high data rates at 250 Gb/s.

5.2 PRBS Model

Due to the compact and stable structure of the SOA-MZI based XOR gate [13, 14], it can be used to build the PRBS generator. The PRBS generator used here is based on a linear feedback shift register (LFSR) and optical XOR logic gates, shown in Figure 5.1 (a). The principle of logic XOR operation utilizing cross-phase modulation (XPM) process in SOAs has been previously discussed and analyzed [13]. The primary difference here is that the input data pulses are of sufficiently short width, and has higher peak power, so as to provide enough power to produce two-photon absorption and the resulting in fast phase change. The average power of the data pulses is about a factor of 20 higher than that used in the previous work and the pulse duration is shorter (<1ps). The XOR logic process using this ultrafast scheme has previously been studied and analyzed [15]. This work is based on the result of [15] and use more LFSRs and XOR gates to realize practical high speed all-optical encryption and decryption.
Figure 5.1 Schematic diagram of PRBS generator. (a) Block diagram of a LFSR; (b) functional unit, two SOA-MZIs operating as XOR and AND gates. BPF: bandpass filter
As shown in Figure 5.1 (a), the LFSR has m data storing units (delay lines in optics), each unit is capable of storing one binary data bit for one clock period [16]. The whole system is synchronized with a clock. At each period, the nth and mth bits go through a XOR process, the XOR result gets reshaped and wavelength converted back to operation wavelength at an AND gate and is fed back to the delay line. The output PRBS signal can be tapped from the end of the LFSR. Figure 5.1 (b) shows the design of the functional unit. The first MZI serves as an all-optical logic XOR gate for the two bits (m,n), while the other MZI serves as logic AND gate. The AND gate uses the same MZI scheme except that two input data streams are injected into port 5 and 7 respectively. A low power CW light is used into port 6 to cancel out the background noise to make results better in quality. This background noise is the “average background” phase shift caused by the XOR output stream. Since phase change on the clock signal in port 7, caused by the XOR signal, has a long recovery time, it persists even when XOR=0. This CW light can balance the “average background” phase shift in SOA 3, thus making the phase change induced on the clock occur over a short time duration. Similarly, an INVERT operation can be realized by XOR with “1”.

After a band-pass filter centered at λ2, the XOR result of data m and n enters the input port “5” of the logic AND gate. The other arm (port 6) has a low power CW signal as input. The clock signal (all “1” bits) at wavelength λ1 is injected into the center port. The AND operation is the “AND” between the center port input and XOR output which reproduces the XOR output at λ1 to circulate in the LFSR.

5.3 Key-Stream Generators
Figure 5.2 (a) Schematic diagram of encryption process. (b) Schematic diagram of decryption process. The key streams are the same.

Figure 5.3 Block diagram of cascaded designed key-stream generator. PRBS stands for figure 1(a).
The XOR algorithm is widely used in commercial software security programs. In this model, I use the TPA based SOA-MZI logic XOR operation to realize the encryption and decryption of a text message. By applying the bitwise XOR operation between every character in the text and a given key, we can get the ciphertext. Since applying XOR operation twice restores the original, decryption is to reapply the XOR operation to the ciphertext with the same key. The process is shown in Figure 5.2.

In this encryption/decryption scheme, the key-stream plays a critical role in the security of the whole process. Pseudorandom sequences generators based on LFSRs in this way have long been used in stream ciphers because they are well-suited for hardware implementation, produces sequences having large periods and good statistical properties, and are readily analyzed using algebraic techniques. However, since a LFSR is a linear system, the output sequence (the key) is easily predictable; hence PRBS based on a single LFSR is simple for crypt analysis. For example, given a stretch of known plaintext and corresponding ciphertext, an eavesdropper can intercept and recover a stretch of LFSR output stream used in the system, which can be fed the intercepted stretch of output stream to recover the remaining plaintext. Therefore, combining several LFSRs or using the output of one (or more) LFSRs to control the clock of one (or more) other LFSRs give better key-stream generators which effectively reduces this problem. Large period, large linear complexity and good statistical properties are three necessary conditions for a key-stream generator to be considered cryptographically secure [17].
Figure 5.4 Design of the alternating step generator (ASG). (a) Schematic diagram of ASG; (b) schematic of the functional unit in (a).
For a PRBS based on one LFSR, its repetition bit period is \( T = 2^m - 1 \). Basically, the PRBS sequences are different from truly random bit sequences in that the latter has a continuous spectrum while the former has a discrete spectrum with harmonics. The frequency space between two neighboring lines is given by [13]

\[
\Delta f = \frac{f_n}{2^{m-1}}
\] (5.3.1)

As \( m \) increases, the frequency space becomes smaller which indicates the generated PRBS spectrum becomes more continuous and the output can better represent a truly random signal. However, when \( m \) increases, the number of different inputs for the LFSR becomes larger. For example, if \( m = 7 \), there are \( 2^7 - 1 \) different kinds of input for the LFSR. Thus, it becomes difficult to choose the input for a single LFSR in order to generate different PRBSs when \( m \) becomes larger. However, the cascaded design above (Figure 5.3) uses the output of a previous stage as the input of the next stage. It solves the difficulty of choosing and changing the input for one single LFSR. Furthermore, this cascaded design uses three LFSRs, hence capable of generating longer period (large \( m \)) keys.

To get a high linear complexity, the common practice is combining the output of several output sequences in some nonlinear manner. The danger here is that one or more of the internal output sequences -often just outputs of individual LFSRs- can be correlated with the combined key-stream and attacked using linear algebra. This is called a correlation attack. Normally, there is a trade-off between correlation immunity and linear complexity [18]. The alternating step generator (ASG) investigated in this paper has a long period and large linear complexity and can generate more secure keys consequently. The ASG uses three LFSRs, which we call LFSR A, LFSRB and LFSR
C for convenience. PRBS A, PRBS B, PRBS C are their output bit sequences correspondingly. The design is shown in Figure 5.4. The output of LFSR B decides which of the other two is to be used in the ASG output. If LFSR B outputs a “1”, the output of LFSR A goes to the final ASG output; if LFSR B outputs a “0”, the output of LFSR C is in the final ASG output. This functional unit is shown in Figure 5.4(b). It combines PRBS A and PRBS C according to the bit output of LFSR B. Therefore, the output sequence of ASG is much harder to break. We also investigate a relatively new generator: shrinking generator. The shrinking generator uses a different form of clock control. It uses two LFSRs, LFSR A and LFSR B. When LFSR B bit is “1”, the eventual shrinking generator output is the same as the bit of LFSR A; when LFSR B bit is “0”, the A bit is discarded, and there is no output. Therefore, the final output is a “shrunken” version of the A bits. This scheme is reasonably efficient and makes the key much harder to break [19]. Overall, the three key-stream generators investigated here can serve as more secure key-stream generators than a single LFSR.

5.4 SOA Gain and Phase Dynamics
Figure 5.5 Calculated gain and phase change in a SOA due to a series of pulses shown in (a). (b) Gain modulation as a function of time, (c) phase modulation due to gain modulation as a function of time. The gain plotted is the value of $G$. (d) Total phase modulation including two-photon absorption as a function of time. Note the periodic total phase change is primarily due to two-photon absorption in (d).
Figure 5.6 Calculated h factors in a SOA due to a series of pulses shown in Figure4 (a). h is the sum of $h_l$, $h_{CH}$ and $h_{SHB}$. 
As noted before, we use SOA-MZI considering two-photon absorption in pump beams to realize high speed all-optical XOR operation and PRBS. The output of MZI in this scheme is given by:

\[
P_{\text{out}}(t) = \frac{P_{\text{in}}}{4} \left\{ G_1(t) + G_2(t) - 2\sqrt{G_1(t)G_2(t)} \cos[\varphi_1(t) - \varphi_2(t)] \right\} \tag{5.4.1}
\]

where \( P_{\text{in}} \) is the optical power of the input clock signal, \( G_1 \) and \( G_2 \) are the gains in the two arms of SOA-MZI, \( G(t) = \exp(h_l + h_{CH} + h_{\text{SHB}}) \) for each SOA. \( \varphi_1(t) - \varphi_2(t) \) is the phase difference of the clock signal in two arms.

The temporal gain and phase change in the SOA has been analyzed by a numerical solution of the SOA rate equations taking into account two-photon absorption. The time dependent gain of the SOA satisfies the temporal gain rate equations [20, 27]:

\[
\frac{dh_1(t)}{dt} = \frac{h_0 - h_1(t)}{\tau_c} - \frac{P(t,0)}{E_{\text{sat}}} \left[ e^{h_{\text{total}}} - 1 \right] \quad \tag{5.4.2}
\]

\[
\frac{dh_{CH}(t)}{dt} = -\frac{h_{CH}(t)}{\tau_{CH}} - \frac{\varepsilon_{CH}}{\tau_{CH}} \left[ e^{h_{\text{total}}} - 1 \right] S(t,0) \quad \tag{5.4.3}
\]

\[
\frac{dh_{\text{SHB}}(t)}{dt} = -\frac{h_{\text{SHB}}(t)}{\tau_{\text{SHB}}} - \frac{\varepsilon_{\text{SHB}}}{\tau_{\text{SHB}}} \left[ e^{h_{\text{total}}} - 1 \right] S(t,0) - \frac{dh_l(t)}{dt} - \frac{dh_{CH}(t)}{dt} \quad \tag{5.4.4}
\]

where \( h(t) \) is an integral of optical gain over the length of SOA and \( h_{\text{total}} \) equals the sum of \( h_l \), \( h_{CH} \) and \( h_{\text{SHB}} \). \( \tau_c \) is the carrier lifetime, \( \exp[h_0] = G_0 \) is the unsaturated power gain and \( E_{\text{sat}} \) is the saturation energy of the SOA. \( E_{\text{sat}} = P_{\text{sat}} \tau_c \), \( P_{\text{sat}} \) is the SOA’s saturation power. \( P(t) = k S(t) \) is the input pump power inside the SOA active region, \( k \) is the SOA active region cross section, here we use 0.5 \( \mu \text{m}^2 \). \( S(t,0) \) is the instantaneous input optical intensity inside the SOA and \( h_l \), \( h_{CH} \), \( h_{\text{SHB}} \) are the \( h \)-factor values for carrier recombination, carrier heating (CH) and spectral hole burning (SHB) respectively. Equations (5.4.3) and (5.4.4) account for the intra-band carrier dynamics (i.e. carrier
heating and spectral hole burning effects). For the $\beta$ value used here (20 cm/GW), the change in intensity due to two-photon absorption is low, hence that term can be neglected in this model. The carrier density induced phase change is given by:

$$\phi(t) = -\frac{1}{2} [\alpha h(t) + \alpha_{CH} h_{CH}(t)] - \frac{1}{2} \beta \alpha_2 S(t)L$$ \hspace{1cm} (5.4.5)

where $\alpha$ is the traditional linewidth enhancement factor, $\alpha_{CH}$ is the carrier heating linewidth enhancement factor. The SOA and TPA related parameters are shown in table 1. Note that a low gain of 10 dB reduces the amount of gain induced phase change which is slower than the two-photon absorption related phase change. The total phase dynamics used here include both the fast (TPA induced) phase change and the relatively slow gain induced phase change. Due to the large input power (average power > 20 mW), the TPA induced fast phase modulation becomes the major contribution of the total phase shift. The calculated change in gain and phase as a function of time when a pulse train at 250 Gb/s is injected into the SOA is shown in Figure 5.5. It shows that the dominant phase change is due to two-photon absorption. Figure 5.6 shows the contributions of $h_t$, $h_{CH}$ and $h_{SHB}$ to the total $h$ factor. The gain contribution of spectral hole burning and carrier heating are relatively small compared to that for carrier recombination.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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</thead>
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<tr>
<td>Psat</td>
<td>30 mW [20,27]</td>
<td>Saturation power of the SOA</td>
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<tr>
<td>G0</td>
<td>10 dB</td>
<td>Unsaturated power gain</td>
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<tr>
<td>τc</td>
<td>300 ps [20,27]</td>
<td>Carrier lifetime</td>
</tr>
<tr>
<td>τSHB</td>
<td>100 fs [20,27]</td>
<td>Spectral hole burning time</td>
</tr>
<tr>
<td>εCH</td>
<td>0.08 μm²/W [20,27]</td>
<td>CH gain suppression factor</td>
</tr>
<tr>
<td>α</td>
<td>5</td>
<td>Traditional linewidth enhancement factor</td>
</tr>
<tr>
<td>αCH</td>
<td>1 [20]</td>
<td>Linewidth enhancement factor</td>
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<td>α2</td>
<td>-4 [14-16]</td>
<td>TPA linewidth enhancement factor</td>
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<tr>
<td>β</td>
<td>20 cm/GW [15,18]</td>
<td>TPA coefficient</td>
</tr>
<tr>
<td>L</td>
<td>5 mm</td>
<td>SOA effective length</td>
</tr>
</tbody>
</table>

Table 5.1 Values of parameters used in simulation.
5.5 Simulation Results

In the simulation, we assume the input data stream pulses to be Gaussian pulses, i.e.,

\[ P(t) = \sum_{n=-\infty}^{+\infty} a_n \frac{2\sqrt{\ln 2} P_0}{\sqrt{\pi \tau_{FWHM}}} \exp\left(-\frac{4 \ln 2 (t-nT)^2}{\tau_{FWHM}^2}\right) \]  

(5.5.1)

where \( P_0 \) is the energy of a single pulse, \( a_n \) represents nth data in the input data stream, \( a_n = 1 \) or \( 0 \).

In the simulation, we choose \( P_0 = 0.01 E_{sat} \), pulse energy is normalized to the SOA saturation energy. The power axes of all the simulation in this paper are in arbitrary units, the actual power in Watt is proportional to the axes power by the value of \( E_{sat} \). Also, we assume all the input signals are of RZ types.

The simulated results of encryption and decryption for the TPA based XOR operation are shown in Figure 5.7. Input data representing the original message (chosen as “100001101” pattern) is on the top left (a) and a key (chosen as “1011001110” pattern) is shown on top right (b). The encrypted data which is the XOR operation between the input data and the key is shown on bottom right (d). The decrypted data which is supposed to be the same as the original data is shown on bottom left (c). Apparently, the top left and the bottom left traces are identical as it should be. Thus, all-optical encryption and decryption based on TPA in semiconductor optical amplifiers operation at 250 Gb/s is feasible. We also calculated the quality factor (Q factor) of the decrypted data in Figure 5.7. The Q factor is 7.2, which means a good quality. The Q-factor is given by \( Q = (S_1-S_0)/(\sigma_1+\sigma_0) \), where \( S_1, S_0 \) are the average intensities of the expected “1”s and “0”s and \( \sigma_1, \sigma_0 \) are the standard deviations of those intensities. The primary reason for noise in this calculation (i.e. presence of \( \sigma_1, \sigma_0 \), which lowers the Q-factor defined above) is pattern effects resulting from the long recovery times of gain.
and gain induced phase change. The calculation in this paper does not take into account the noise effects of amplified spontaneous emission (ASE) from SOA. The ASE causes spontaneous-spontaneous beat noise and signal-spontaneous beat noise. In addition, if one were to measure the error rate of the output, the dark current of the photodiode, short noise, and the thermal noise need to be considered. An analysis of all these noise terms is given in Chapter 6 of Ref. [13].

The key used for encryption and decryption -all-optical PRBS- generated by a 7-bit optical LFSR is simulated by modeling the logic XOR and AND operations in the presence of fast two-photon absorption induced phase change. The result is shown in Figure 5.8. The initial input is seven “1”s. We have included in our PRBS scheme an amplifier (e.g. SOA) right after the AND gate output (Figure 5.1(a)). The gain of this amplifier could be adjusted to set the average power of the feedback signal to the original input value. We also calculate the Q factor of this PRBS sequence, which is 6.8, indicating a good quality of the data.
Figure 5.7 Simulation results for encryption and decryption. (a) Input data message; (b) key; (c) decrypted data; (d) encrypted data. The results are for a 250 Gb/s data rates.

Figure 5.8 Simulation result of PRBS sequences generated by 7-bit LFSR, operating at 250 Gb/s. The input of the LFSR is seven “1”s.
Figure 5.9 Simulation of the cascaded design key-stream generator. (a) Simulation result of PRBS A; input is all “1”s. (b) Simulation result of PRBS B; input is the “15th-21st” of the output of PRBS A; (c) Simulation result of PRBS C; input is the “15th-21st” of the output of PRBS B.
Figure 5.10 Simulation result of the alternating step generator (ASG). (a) Simulation result of PRBS A sequences; input is “1111111”. (b) Simulation result of PRBS B sequences (control sequence); input is chosen as “1010001”. (c) Simulation result of PRBS C sequences, input is chosen as “0010110”. (d) Simulation result of the output sequence of the ASG.
Figure 5.11 Simulation result of the shrinking generator. (a) Simulation result of PRBS A sequences; input is “1111111”. (b) Simulation results of PRBS B sequences (control sequences); input is “1010001”. (c) Simulation results of the output of shrinking generator.
The simulation result of cascaded design key-stream generator is shown in Figure 5.9. The input of the latter LFSR is the output of the previous LFSR. Figure 5.10 shows the simulation result of the alternating step generator. PRBS B sequence serve as the control sequence and decides which bit of sequence A or sequence B is taken in the ASG output. The input of LFSR A is “1111111”; the input of LFSR B and C are chosen to be different from A, here we use “1010001” and “0010110” respectively. The output sequence is shown in Figure 5.10 (d). The simulation result of the shrinking generator is shown in Figure 5.11. PRBS B sequence also serves as the control sequence. The inputs of LFSR A and LFSR B are the same as those in the alternating step generator. Since in shrinking generator bits are output at a rate that depends on the appearance of 1’s in PRBS B sequence, as we can see in the Figure 5.11, the bits length period of the output is much shorter than that of sequence A. From probabilistic perspective, the output bit rate is on average 1 bit for each 2 bits of PRBS B governing the PRBS A. Therefore, the repetition bit period reduces to about half of the original value, $2^{7-1}$. In order to get the same period key-stream as $2^7$, one way we could do is to use bigger bit LFSRs (m larger than 7).

The total phase dynamics used in the model here includes both the fast (TPA induced) phase change and the relatively slow cross phase modulation and cross gain modulation. The latter processes are slow and hence they cause “pattern effect” at high speeds i.e. the phase change caused by an individual bit (e.g. bit 1) depends on the prior sequence of bits. Due to high injected power (average power more than 20 mW), the TPA induced fast phase modulation becomes a major contribution of the total phase shift (Figure 5.5), and the pattern effect resulting from long recovery time becomes relatively small, resulting in fast phase recovery. The basic mechanism for phase change
is carrier induced change in refractive index. The TPA process generates hot carriers in the active region of the SOA. These carriers have energies of \(~0.8\text{ eV}\) (which approximately equal the band gap of the active region) which are considerably larger than the energy barrier (\(~0.3\text{ eV}\)) at the cladding layer/active layer interface. Assuming ballistic transport, this allows the hot carriers (\(v \sim 2.5 \times 10^6 \text{ m/s}\)) to escape the active region with a time scale of the \(~0.15\text{ps}\) for a cladding thickness of \(~0.3\ \mu\text{m}\). Since the TPA induced carriers are present only for a short duration (both generation time-determined by input pulse length, and escaper time are short), the phase change also occurs for a short duration.

5.6 Conclusion

In this work, I have designed and modeled high speed all-optical encryption and decryption systems using key-stream generators and an XOR gate using semiconductor optical amplifier (SOA) based Mach-Zehnder interferometer. The data pulses are short enough and have high average power so that the peak power is large to cause significant two-photon absorption and consequent fast phase change. Simulation results show that this all-optical model can realize the basic function of encryption and decryption at high bits rates up to 250 Gb/s. I also investigate and design three more complicated key-stream generators, i.e. three LFSRs cascaded design generator, alternating step generator and shrinking generator. Results show that these key-stream generators operating at 250 Gb/s are feasible and are capable of generating secure keys.
Chapter 6 Effects of Two-Photon Absorption on All Optical Logic Operation

Based on QD-SOA

6.1 Introduction

All-optical signal processing is expected to be important in future tera bit rate telecommunication networks [1]. All optical logic gates operating at high speeds will play important roles in future all optical networks, including signal regeneration, all optical packet routing and data encryption[2,3]. Recently, there have been extensive research on various schemes of optical logical gates including traditional Boolean logic functions such as XOR, OR, AND, XNOR, Set-Reset, and other logical circuits such as parity generator and checker [4-18]. These schemes include using light beam interference in silicon photonic crystal[6], photonic crystal ring resonator[4], four-wave mixing in semiconductor optical amplifier (SOA) [5], binary phase shift keyed signal [18], single quantum-dot semiconductor optical amplifier based Mach-Zehnder interferometer [8, 17] and dual semiconductor optical amplifier Mach-Zehnder interferometer (SOA-MI)[7, 11, 13]. Among these schemes, the SOA based MZI has the advantage of being stable, compact, and simple, which has been demonstrated to be capable of all optical switching such as XOR, Set-Reset and flip-flop[7,11,13]. Especially with the introduction of SOA based on quantum-dot (QD) active region, the all optical switching speed reached ~250 Gb/s[7, 13]. The QD excited state carriers can act as a reservoir which results in an ultrafast gain recovery. Berg et al [19] claimed that QD-SOA does not support tera bit all optical logic operation due to the slow recovery of carriers in wetting layers. However, in their model, they assumed that the direct injection of carriers into wetting layer of a QD-SOA was only from the injected current. This is not the case during the propagation of sub-
picosecond optical pulses because two-photon absorption (TPA) will generate carriers in the bulk region of a QD-SOA, which contributes to the carrier dynamics. TPA in normal SOAs has already been utilized to realize all-optical logic operation in previous studies [15, 20]. By taking TPA into account, Ju \textit{et al} demonstrated that the pattern effects are reduced when a train of optical pulses is injected in a QD-SOA; thus it is possible to operate at a higher speed [21]. Ju’s simulation also has its limitations. It neglects gain saturation and nonlinear effects and is only based on inter-band carrier transitions. These effects dominate the device’s gain dynamics above certain input power level.

In this chapter, I study the effects of TPA on optical logic operation based on QD-SOA-MZI. Rate equation approach has been widely used to simulate the carrier dynamics on QD-SOA systems assuming ultrafast dynamics. I present a model to consider wetting layer carriers refill through TPA as well as nonlinear effects affecting the gain and phase dynamics of the QD device. Our results show that with the consideration of TPA, optical logic gates based on QD-SOA-MZI have an improved output qualify and are capable of operating at 320 Gb/s.

6.2 Schematic of All-Optical Logic Gates
Figure 6.1 Schematic diagram of the QD-SOA-MZI based XOR gate. BPF: bandpass filter centered at $\lambda_2$. Two phase shifters are used to induce a $\pi$ phase difference in two arms.

Figure 6.2 A schematic of carrier dynamics with TPA in an InAs/GaAs QD-SOA. Quantum dots are embedded in the wetting layer.
Due to the compact and stable structure, SOA-MZI has been widely used in optical logic gates\[22, 23\]. Figure 6.1 presents a schematic diagram for QD-SOA-MZI based optical XOR gate. The principle of logic XOR operation utilizing the cross gain modulation (XGM) and cross phase modulation (XPM) processes in SOAs has been discussed and analyzed \[11, 22\]. The function of A XOR B is realized as follows. Data streams A and B are carried by two optical pulse streams at wavelength $\lambda_1$. They are injected into port 1 and 2, respectively. There is a clock stream centered at $\lambda_2$ injected into port 3 and it is evenly split into the two arms of the MZI. Initially, the MZI is unbalanced with a phase difference $\pi$ between these two branches. As the two clock streams travel in the QD-SOAs, their phases and amplitudes are modulated. When they recombine at port 4, their interference will produce different results under different initial conditions. For example, if the input A is ‘0’ and B is ‘0’, then these two clock streams experience the same gain and phase shift in QD-SOAs and when they recombine at port 4, considering a phase difference $\pi$, they will undergo destructive interference and the output result is ‘0’. If A is ‘1’ and B is ‘0’, the gain and phase modulation of the two clock streams are different and their interference will produce ‘1’. This realizes the functionality of XOR.

We can also use this same scheme to realize the logic AND operation. By inputting data stream A and B into port 1 and 3, we will get a pattern of A AND B out from port 4. A low power CW light is normally injected into port 2 to cancel the background noise of data stream A out.

### 6.3 QD Device and Rate Equation

The device we choose here to construct the all-optical logic gate is the commonly discussed
InAs/GaAs QD-SOA, in which InAs quantum dots are embedded in GaAs layers[24-26]. The gain of this type of device around 1.55 μm is typically ~15 dB and the noise figure is low at ~7 dB [24]. Also the gain of this device is nearly polarization independent[26]. Here I use the three-level rate equation model to simulate the carrier transitions in the device. Figure 6.2 illustrates the optical gain, the TPA process and carrier transitions between the wetting layer (WL), the QD excited state (ES), and the QD ground state (GS). The device gain is determined by the carrier density of the QD ground state. The TPA generates carriers in the bulk region. These carriers then relax to the WL, and eventually are captured into QDs on ultrafast timescale [21]. Generally, carriers in the barriers are free to move in 3D and are captured very rapidly by the 2D wetting layer at a relaxation timescale of ~ 70 fs [21, 27]. Ju et al have proven that the carrier dynamics in the bulk region due to TPA can also refill the WL and QDs on ultrafast timescales and thus significantly reduce pattern effects for optical signal-processing operating at Tbit/s [21, 28]. They introduced a three coupled rate equations model including the barrier region, as well as the WL and QDs. In our model, we ignore the barrier dynamics and assume that carriers are injected directly from the contacts into the WL [19]. Since the only recipient of the pump current is the WL, and the QD excited state serves as a carrier reservoir for the ground state with ultra-fast carrier relaxation to the latter, the device gain dynamics is affected by their carrier densities and transition rates.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{wr}$</td>
<td>Recombination lifetime of WL</td>
<td>0.2 ns[29]</td>
</tr>
<tr>
<td>$\tau_{esr}$</td>
<td>Recombination lifetime of ES</td>
<td>0.2 ns[29]</td>
</tr>
<tr>
<td>$\tau_{gsr}$</td>
<td>Recombination lifetime of GS</td>
<td>0.1 ns[29]</td>
</tr>
<tr>
<td>$\Gamma_d$</td>
<td>Active QD region confinement factor</td>
<td>0.1</td>
</tr>
<tr>
<td>$a$</td>
<td>Differential gain</td>
<td>$8.6 \times 10^{-15} \text{cm}^2$[19]</td>
</tr>
<tr>
<td>$\tau_{w-e}$</td>
<td>Relaxation lifetime from WL to ES</td>
<td>3 ps[3]</td>
</tr>
<tr>
<td>$\tau_{g-e}$</td>
<td>Relaxation lifetime from GS to ES</td>
<td>10 ps[3]</td>
</tr>
<tr>
<td>$n_{es}$</td>
<td>Density of carriers in ES</td>
<td>$7.2 \times 10^{-18} \text{cm}^3$[29]</td>
</tr>
<tr>
<td>$n_{gs}$</td>
<td>Density of carriers in GS</td>
<td>$3.6 \times 10^{-18} \text{cm}^3$[29]</td>
</tr>
<tr>
<td>$A_d$</td>
<td>Cross-sectional area of the active layer</td>
<td>$1.54 \times 10^{-13} \text{cm}^2$[29]</td>
</tr>
<tr>
<td>$A$</td>
<td>Modal area</td>
<td>$1.4 \times 10^{8} \text{cm}^2$[21]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>TPA coefficient</td>
<td>70 cm/GW[21]</td>
</tr>
<tr>
<td>$L$</td>
<td>Active region length</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Linear linewidth enhancement factor</td>
<td>4</td>
</tr>
<tr>
<td>$\alpha_{CH}$</td>
<td>CH linewidth enhancement factor</td>
<td>1.2[30]</td>
</tr>
<tr>
<td>$\varepsilon_{CH}$</td>
<td>Gain suppression factor of CH</td>
<td>$0.3 \times 10^{-17} \text{cm}^3$[31]</td>
</tr>
<tr>
<td>$\varepsilon_{SHB}$</td>
<td>Gain suppression factor of SHB</td>
<td>$7.5 \times 10^{-17} \text{cm}^3$[31]</td>
</tr>
</tbody>
</table>

Table 6.1 Parameters used in the simulation.
The change in carrier densities of the three energy levels including the TPA process are described by the following coupled rate equations[19, 21, 29]:

\[
\frac{dw}{dt} = \frac{I}{eVN_{wm}} - \frac{w}{\tau_{wr}} - \frac{w}{\tau_{w-e}} (1 - h) + \frac{N_{esm}}{N_{wm}} \frac{h}{\tau_{e-w}} (1 - w) + \frac{\kappa}{2\hbar\omega N_{wm}} \left[ \frac{S(t)}{A} \right]^2,
\]  

(6.3.1)

\[
\frac{dh}{dt} = -\frac{h}{\tau_{esr}} - \frac{N_{wm}}{N_{esm}} \frac{w}{\tau_{w-e}} (1 - h) - \frac{h}{\tau_{e-w}} (1 - w) + \frac{N_{gsm}}{N_{esm}} \frac{f}{\tau_{g-e}} (1 - h) - \frac{h}{\tau_{e-g}} (1 - f),
\]  

(6.3.2)

\[
\frac{df}{dt} = -\frac{f}{\tau_{gsr}} - \frac{f}{\tau_{g-e}} (1 - h) + \frac{N_{esm}}{N_{gsm}} \frac{h}{\tau_{e-g}} (1 - f) - \frac{\Gamma_d}{A_d} a (2f - 1) \frac{1}{N_{gsm}} \frac{S(t)}{h\omega},
\]  

(6.3.3)

where \( w, h \) and \( f \) represent the occupation probability of the wetting layer, the QD excited state, and ground state, respectively; \( N_{wm}, N_{esm}, \) and \( N_{gsm} \) are the maximum densities of carriers in each state; the spontaneous radiation lifetime of each state is denoted by \( \tau_{ar} \) (“a” being “w”, “es” or “gs”); \( \tau_{a-b} \) denotes the relaxation time between any state “a” and state “b”; \( \Gamma_d \) is the active layer confinement factor; \( I \) is the injected current; \( a \) is the differential gain; \( V \) is the volume of the active layer; \( A_d \) is the effective cross-sectional area of the active layer; \( \kappa \) is the TPA coefficient; \( \hbar \) is the reduced Planck constant; \( A \) is the modal area and \( S(t) \) is the total input light power. The TPA generated carriers are taken into account by the last term in Equation (6.3.1).

The gain of QD-SOA including nonlinear process such as carrier heating (CH) and spectral hole burning (SHB) effects is expressed as[32, 33]:

\[
g(t) = \frac{a(N - N_t)}{1 + (\varepsilon_{CH} + \varepsilon_{SHB}) S(t)},
\]  

(6.3.4)

where \( N \) and \( N_t \) are the GS carrier density, the transparent GS carrier density respectively; \( \varepsilon \) denotes the gain suppression factor. The refractive index of the active region is affected by the injected light and the change of temperature due to carrier heating. As a result, it will cause a phase change to any probe wave injected into the QD-SOA [18]:

95
\[ \varphi(t) = -\frac{1}{2} \left( \alpha G_L(t) + \alpha_{CH} \Delta G_{CH}(t) \right), \tag{6.3.5} \]

where \( G_L(t) \) is the linear gain factor of the device given by \( e^{\gamma(t) L} \), \( L \) being the effective length of the active layer; \( \alpha \) is the linewidth enhancement factor of the device corresponding to band-to-band transition and \( \alpha_{CH} \) is the linewidth enhancement factor of the device related to carrier heating process [31, 34]. The linewidth enhancement factor due to spectral hole burning is \(-0 \) [13].

As noted previously, I use a QD-SOA-MZI to realize high speed all optical XOR and AND operation. The output of this scheme from the combination of two data streams can be expressed as:

\[ P_{out} = \frac{P_{cb}(t)}{4} \left[ G_1(t) + G_2(t) + 2\sqrt{G_1(t)G_2(t)} \cos(\varphi_1(t) - \varphi_2(t) + \varphi_0) \right], \tag{6.3.6} \]

where \( P_{cb} \) is the light power of the input clock signal; \( G_1(t) \) and \( G_2(t) \) are the calculated total linear gain factors. The primary parameters used in this simulation are presented in Table 6.1.

### 6.4 Simulation Results

To solve the rate equations in the QD-SOAs numerically, we use the explicit Runge-Kutta method. I use a step size of 0.02 in the time domain and the number of maximum steps is \( 10^6 \). In this simulation, a PRBS signal is used as input data and we assume the data stream pulses to be Gaussian pulses. The single pulse energy is 0.5 pJ and full width at half maximum (FWHM) is 1ps. The injected current to the QD-SOA is 250 mA. Figure 6.3 shows the calculated gain and phase dynamics in a QD-SOA due to a series of input pulses using the model mentioned in Section 3. The injected pulses cause a periodic change of occupation probability of the ground state and excited state in the QD as indicated in Figure 6.3(b). The ultrafast recovery of the occupation
probability is due to the ultrafast carrier capture into the QDs. The corresponding gain and phase recovery are depicted in Figure 6.3(c) and (d). These ultrafast gain and phase recoveries ensure the ultrafast all-optical logic operation. Figure 6.4 and Figure 6.5 show the results of XOR performance without and with the consideration of TPA, respectively. Here we use the pseudo-eye-diagrams to show the output quality. The calculated “1”s and “0”s are superimposed on each other to plot the eye diagram. The output quality can also be quantitatively characterized by the Q factor, which is defined as $Q = (P_1 - P_0)/(\sigma_1 + \sigma_0)$ [22]. Here $P_1$ is the average peak power of output signal “1”s and $\sigma_1$ is the standard deviation of all “1”s. $P_0$ and $\sigma_0$ are defined analogously for output “0”s. The output bit error rate (BER) is related with the Q factor by: $BER \approx (2\pi)^{1/2}\exp(-Q^2/2)/Q$ [7]. The primary reason for noise in this calculation is pattern effects resulting from the long recovery time of gain and gain-induced phase change. As we can see from Figure 6.4 and Figure 6.5, with TPA, the XOR output has a clearer eye diagram and thus a higher Q factor compared to the case without TPA. This is because that the TPA induced carrier pumping leads to a change in carrier recovery dynamics in QDs, which reduces the gain recovery time. Thus, pattern effects are reduced and a higher Q factor is achieved.
Figure 6.3 calculated gain and phase change in a separate QD-SOA due to a series of pulses. (a) Input pulses at the speed of 320 Gb/s. (b) Occupation probability of the excited state and ground state as a function of time. (c) Change of gain factor as a function of time. (d) Phase change as a function of time.
Figure 6.4 Simulation results of XOR gates operating at 320 Gb/s without the consideration of TPA. The input single pulse energy is 0.5 pJ and pulse width is 1 ps.

Figure 6.5 Simulation results of XOR gates operating at 320 Gb/s with the effects of TPA. The input single pulse energy is 0.5 pJ and pulse width is 1 ps.
Figure 6.6 Simulation results of AND gates operating at 320 Gb/s without the consideration of TPA. The input single pulse energy is 0.5 pJ and pulse width is 1 ps.

Figure 6.7 Simulation results of AND gates operating at 320 Gb/s with the effects of TPA. The input single pulse energy is 0.5 pJ and pulse width is 1 ps.
Figure 6.8 (a) The dependence of Q factor on single pulse energy. The injected current is fixed at 250 mA and pulse width is 1 ps. (b) The dependence of Q factor on injected current. The input single pulse energy is fixed at 0.5 pJ and pulse width is 1 ps. (c) The dependence of Q factor on pulse width/bit period ratio at 320 Gb/s. The injected current is fixed at 250 mA and single pulse energy is set at 0.5 pJ. (d) The dependence of Q factor on data rate. The single pulse energy is 0.5 pJ and injected current is 250 mA. The ratio of the pulse width to bit period is fixed.
All-optical AND gates are realized with the same scheme of XOR gates as described in section 6.2. The performance of AND gates is showed in Figure 6.6 and Figure 6.7. It also indicates the TPA in quantum dots can improve the output quality of the AND gates.

To furtherly investigate the effects of TPA on optical logic operations, I compare the dependence of Q factor of XOR gates on various parameters for these two cases. The calculated Q factor shows significant dependence on the injected current, initial pulse width, pulse energy and data rate as suggested in Figure 6.8. As we can see in Figure 6.8(a), by fixing the inject current at 250 mA and FWHM 1ps, the output quality degrades when one increases the single pulse energy of the input data for both with and without TPA cases. Without the effects of TPA, the Q factor drops very fast when pulse energy is increased from 0.1 pJ to 0.5 pJ; while with TPA taken into account, the decrease of Q factor becomes slower. This is easy to understand. As we increase the single pulse energy, the peak power of the pulses is increased. The TPA process is enhanced when the injected pulse train has a higher peak power, which will mitigate the depletion of carrier density of the active region of the device. As a result, the dropping rate of output Q factor will become lower. In Figure 6.8(b), Q factor increases with the increase of the injected current until the current reaches a certain level (~275 mA with TPA and 350 mA without TPA) and then it saturates. With TPA, it saturates at a smaller current level compared with that without TPA. The injected current produces carriers to the wetting layer; thus, in each energy level in the quantum dot the carrier density is able to recover to its initial state. The more carriers produced, the faster the recovery time will become. This can reduce the pattern effects due to long recovery time of the logic operation. The TPA process also contributes to the carrier recovery thus the Q factor can saturate at a lower current...
level than that without TPA. The Q factor decreases with the increase of pulse width and data rate for both cases. Typically, digital transmission systems require the BER to be lower than $10^{-9}$, which means the Q factor should remain above the critical limit of six. This can be easily realized by reducing the input pulse width. We calculate that with TPA included, when the input pulse width is reduced to 0.9 ps with the single pulse energy unchanged, the output Q factor reaches 6.4. As indicated in Figure 6.8(c), as long as the injected pulse width is below 0.9 ps, this scheme is capable of generating results with low error rate ($Q > 6$) at the speed of 320 Gb/s. The overall Q factor of the TPA case is always higher than that without TPA. This demonstrates that TPA plays an important role in QD gain bleaching and reducing pattern effects for ultrafast optical logic operation.

6.5 Conclusion

I have studied the two-photon absorption (TPA) effects on all-optical logic operation based on QD-SOA MZI using the rate equation model. TPA contributes to the ultrafast carrier dynamics by generating carriers captured into QDs on ultrafast timescale. This additional carrier pumping can improve gain recovery time in the QDs and thus reduce pattern effects in all-optical logic operation. Our simulation shows the output has a higher Q factor when TPA effects are included. Thus QD-SOA based MZI scheme is suitable for all-optical logic operation at 320 Gb/s.
Appendix A Publications and Presentations

A.1 Journal Publications and Book Chapters


A.2 Conference Presentations and Proceedings


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Chapter 1


Chapter 2


Chapter 3


Chapter 4


Chapter 5


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