Advancing the Learning of Algebra for ALL: Case Studies of Teachers’ Efforts Toward Equitable Math Teaching

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Increasing opportunities for all students to learn, think and communicate mathematically is a vital educational priority. The learning of algebra, in particular, empowers students with foundational knowledge that is essential for upper level mathematics courses that give students choice in their educational paths. Yet, myriad indicators suggest that preparation and access to this mathematics coursework is not the same for all. Teachers, central in facilitating equitable math learning experiences (NCTM, 2014), require preparation to support this critical educational need. We must determine what teachers need to know to expand equitable teaching efforts. While principles from research on equitable teaching provide a beginning knowledge base, this knowledge base is incomplete. Teachers must also know how to adapt this knowledge base into their practice, requiring descriptive, “well-documented events” (Shulman, 1986) that can be used to prepare them in equitable teaching.

The purpose of this qualitative study was to provide an in-depth, close-up examination of practices of four algebra teachers identified as having stances aligned with equitable teaching practices. The goal was to uncover how teachers used what they know about their students (inside and outside the classroom) to meet their students’ particular learning needs – considering both cognitive and sociocultural perspectives to frame what teachers “knew” about their students. To accomplish this, I employed ethnographic and case study methods within a situated practice framework. All teachers worked in classrooms with high representation of underserved students. Within-case and cross-case syntheses uncovered central learning phenomena and produced analytical explanatory models for teachers’ efforts to advance their students’ learning.
The findings demonstrated that each teacher used particular sets of forms of knowledge of the student (mathematical and nonmathematical) in situated ways. Further, teachers’ uses of their knowledge of the student were described, along with the teachers’ perceptions of these interventions. The analytical models for each teacher and across cases incorporated the forms of knowledge of the student used, the teachers’ perceptions of this use, their teaching goals and their perspectives on learning. Overall, teachers highly prized students’ mathematical thinking and worked to develop long-term skills that empowered students to succeed in school.
Advancing the Learning of Algebra for ALL:
Case Studies of Teachers’ Efforts Toward Equitable Math Teaching

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B.S., Yale University, 1996
M.S., Southern Connecticut State University, 2003

A Dissertation
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Doctor of Philosophy Dissertation

Advancing the Learning of Algebra for ALL: Case Studies of Teachers’ Efforts Toward Equitable Math Teaching

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### TABLE OF CONTENTS

#### CHAPTER 1: OVERVIEW OF THE STUDY

- Introduction .............................................................................................................. 1
- Background .............................................................................................................. 4
  - Situated Equitable Math Teaching Characteristics .............................................. 5
  - Differences in Scholarship: Cognitivists vs. Socioculturalist ............................ 8
- Problem Statement .................................................................................................. 10
- Research Questions ................................................................................................. 11
- Study Methods ........................................................................................................ 12
- Key Terms and Disciplinary Focus ........................................................................ 13
  - Equity .................................................................................................................. 13
  - Knowledge of the Student ................................................................................... 14
  - Why Study Algebra .............................................................................................. 15
  - Dissertation Overview .......................................................................................... 15

#### CHAPTER 2: LITERATURE REVIEW

- Summary .................................................................................................................. 17
- Definition of Equity ................................................................................................ 18
- Review Criteria ....................................................................................................... 20
- Core Characteristics of the Equitable Math Teacher ............................................. 20
  - Reflective and proactive about equity ................................................................ 21
  - Maintains High Expectations for All Students .................................................... 27
  - Build on Students’ Lived Experiences ................................................................ 30
- Situated Practices and Knowledge for Equitable Math Instruction ...................... 39
- Forms of Knowledge of the Student ......................................................................... 41
- The Student: A need for integrative understanding .............................................. 43
- Theoretical Framework ............................................................................................ 47
  - Situated Practice .................................................................................................. 48
  - Critical Race Theory .............................................................................................. 49
  - Integrated Framework ......................................................................................... 51

#### CHAPTER 3: METHODS

- Methods .................................................................................................................... 53
  - Participants and Schools ...................................................................................... 53
  - Access to Sites and Implications on Methods ....................................................... 65
  - Instruments and Data ............................................................................................ 66
  - Data Analysis ......................................................................................................... 74
  - Limitations in Data Collection ............................................................................. 83
  - Validity and Reliability .......................................................................................... 85
- Positionality ............................................................................................................. 86
  - My Perspective on Learning ................................................................................ 86
  - Personal and Professional Experiences ............................................................... 87
CHAPTER 4: CASE FINDINGS ........................................................................................................ 91
Within Case Analysis: Participants ........................................................................................... 91
Within Case Analysis: Findings Format .................................................................................... 91
Case 1: Beth – ‘Learning to the Nth Power’ .............................................................................. 95
   Background information: Algebra 2 (Level 2) course and their students’ learning paths ......... 95
   Background information: Specifics on Beth’s course ............................................................ 98
   Background information: Beth’s students and classrooms .................................................. 100
   Findings: Beth’s teaching goals ............................................................................................ 102
   Findings: Forms of knowledge of the student used by Beth .................................................. 103
   Findings: Beth’s teaching interventions .............................................................................. 112
   Findings: Central phenomena in Beth’s practice ................................................................. 125
Case 2: Shannon – “You Go Ahead and Explain It” .............................................................. 134
   Background information: Sheltered Language Instruction Program (SLI) and its math course options .......................................................... 134
   Background information: Specifics on Shannon’s course ................................................. 137
   Background information: Shannon’s students and classrooms ....................................... 141
   Findings: Shannon’s teaching goals ................................................................................... 142
   Findings: Forms of knowledge of the student use by Shannon ........................................... 144
   Findings: Shannon’s teaching interventions .................................................................... 147
   Findings: Central phenomena in Shannon’s practice ........................................................ 162
Case 3: Eddy – “I Will Not Stop” ............................................................................................. 170
   Background information: Algebra 1 Academics course and their students’ learning paths .......... 170
   Background information: Specifics on Eddy’s course ....................................................... 172
   Background information: Eddy’s students and classrooms ............................................. 176
   Findings: Eddy’s teaching goals ....................................................................................... 177
   Findings: Forms of knowledge of the student used by Eddy .............................................. 179
   Findings: Eddy’s teaching interventions ......................................................................... 184
   Findings: Central phenomena in Eddy’s practice ............................................................ 199
Case 4: Dena – “Makes Sense? If not, come see me” ............................................................ 206
   Background information: Quantitative Analysis course and learning paths at the college ........ 206
   Background information: Specifics on Dena’s course ...................................................... 207
   Background information: Dena’s students and classroom ............................................. 209
   Findings: Dena’s teaching goals ....................................................................................... 212
   Findings: Forms of knowledge of the student used by Dena ............................................ 212
   Findings: Dena’s teaching interventions ....................................................................... 215
   Findings: Central phenomena in Dena’s practice ........................................................... 229

CHAPTER 5: CROSS-CASE ANALYSIS – ‘Your Thinking Can Empower You’ .... 237
Cross-Case Analysis ................................................................................................................. 237
| Knowledge of the Student                     | 237 |
| Comparisons on Use of Knowledge of the Student | 245 |
| Factors Supporting or Challenging Teachers  | 248 |
| Comparisons of teachers’ learning goals     | 255 |
| Cross Case Analysis Model                    | 259 |
| Summary                                      | 267 |
| CHAPTER 6: DISCUSSION AND IMPLICATIONS       | 269 |
| Discussion of Cross-Case Findings            | 271 |
| Forms of Knowledge of the Student            | 271 |
| Forms of knowledge used: Cognitivist or socioculturalist? | 274 |
| Implications from the Cross-Case Model       | 275 |
| Discussion of Case Findings                  | 275 |
| Additional Implications                      | 284 |
| Recommendations and Final Remarks            | 286 |
| REFERENCES                                   | 290 |
| APPENDICES                                   | 306 |
| APPENDIX A. Pre-Observation Interview Questions with Rationale | 306 |
| APPENDIX B. Data Collection Instruments      | 308 |
| APPENDIX C. Sample Questions Used for Post-Observation Interviews and their Purpose | 310 |
| APPENDIX D. Descriptions of Initial Theoretical Codes | 312 |
| APPENDIX E. Classroom Snapshot: The Bet      | 315 |
| APPENDIX F. Sample from Text Trail Between Steve and Beth Through the App | 317 |
| APPENDIX G. The Guardian Incident            | 319 |
| APPENDIX H. Sample Classroom Conversations & Forms of Knowledge of the Student | 322 |
| APPENDIX I. Conversation on Lesson Progress [Eddy and Brenda] | 327 |
| APPENDIX J. Asante’s Case: The Falling Skydive | 328 |
| APPENDIX K. Sample Activity from Eddy’s Video Lessons | 332 |
| APPENDIX L. Daniel’s Case                    | 334 |
| APPENDIX M. Classroom Snapshot: Working with a deck of cards | 338 |
| APPENDIX N. Common Forms of Knowledge        | 342 |
# LIST OF TABLES

Table 3.1  Recruitment Interview Statements .......................................................... 55
Table 3.2  Sample Recruitment Interview Responses ............................................. 56
Table 3.3  Number of Student Permission Forms by Participant’s Classroom .......... 66
Table 3.4  Initial List of Codes from Theory .............................................................. 76
Table 4.1  Teacher Cases ............................................................................................ 91
Table 4.2  Description of Case Background Subsections ........................................ 92
Table 4.3  Topics Covered During Beth’s Observational Period ............................. 93
Table 4.4  Beth’s Referenced Students and Key Characteristics ......................... 103
Table 4.5  Forms of Knowledge of the Student Used by Beth ................................ 104
Table 4.6  Topics Covered During Shannon’s Observational Period ................. 139
Table 4.7  Graded Components in Shannon’s Algebra 1 Unit on Systems of Linear Equations .......................................................... 140
Table 4.8  Shannon’s Referenced Students and Key Characteristics .................. 143
Table 4.9  Forms of Knowledge of the Student Used by Shannon .................... 144
Table 4.10 Description of Lesson Activities in Eddy’s Self-Paced Program .......... 173
Table 4.11 Lesson Topics Assigned to Eddy’s Academic Students .................... 175
Table 4.12 Eddy’s Referenced Students and Key Characteristics ...................... 178
Table 4.13 Forms of Knowledge of the Student Used by Eddy ............................ 179
Table 4.14 Topics Covered During Dena’s Observational Period ....................... 209
Table 4.15 Dena’s Referenced Students and Key Characteristics ..................... 210
Table 4.16 Forms of Knowledge of the Student Used by Dena .......................... 212
Table 5.1  Common Forms of Knowledge of Student Used Across Cases .......... 239
Table 5.2  Teachers’ Value of Common Forms of Knowledge of the Student ........ 246
Table 5.3  Teachers’ Learning Goals Indicating Empowerment ........................... 257
LIST OF FIGURES

Figure 3.1  Schematic Demonstrating Pairing of Codes Through Simultaneous Coding ......................................................... 77
Figure 3.2  Three Main Domain Areas Used Through CCM ................................................................. 78
Figure 4.1  Students’ Math Course Learning Paths for Algebra 2- Level 2 Students in Sundryville High School ........................................................................... 97
Figure 4.2  Forms of Knowledge of the Student Informing Beth’s Interventions ........ 114
Figure 4.3  Model of Learning Phenomena in Beth’s Case .............................................................. 129
Figure 4.4  Students’ Math Course Learning Paths in SLI Program in Sundryville High School ........................................................................... 137
Figure 4.5  Forms of Knowledge of the Student Informing Shannon’s Interventions .... 149
Figure 4.6  Model of Learning Phenomena in Shannon’s Case .................................................. 165
Figure 4.7  Students’ Math Course Learning Paths in Mixville High School .................. 171
Figure 4.8  Forms of Knowledge of the Student Informing Eddy’s Interventions ........ 186
Figure 4.9  Model of Learning Phenomena in Eddy’s Case ....................................................... 201
Figure 4.10 Students’ Math Course Learning Paths at Beacon Community College ... 208
Figure 4.11 Forms of Knowledge of the Student Informing Dena’s Intervention .......... 217
Figure 4.12 Model of Learning Phenomena in Dena’s Case ...................................................... 231
Figure 5.1  Analytical Model Depicting the Central Learning Phenomena Within the Larger Context ........................................................................... 260
CHAPTER 1: OVERVIEW OF THE STUDY

Introduction

Increasing opportunities for all students to learn, think, and communicate mathematically is a vital educational priority. We know that strong mathematical skills are needed to support society and its economy. We also know that participating successfully in this economy requires quantitative skills (Schmidt, 2012) and heightens the need for higher education. This economic participation improves individuals’ social mobility, as well as their ability to fully participate in society and make greater social contributions. Further, evidence suggests that bachelor’s degree completion rates, regardless of field of study, may improve with enrollment into math courses at the Algebra II level and above (Adelman 1999, 2006). Putting all of this together, we can assert that access to and preparation for those high school level mathematics courses, and in particular algebra, may have an impact on individual’s future career prospects and their ability to fully participate in our society more generally. But when there are gaps in this access and preparation, which have continued to favor some students over others, we must also recognize that there are issues of social justice and fairness that cannot be ignored.

The mathematics community recognizes the importance of access to developing strong mathematical foundations for all students (e.g., Mathematical Association of America, 2018; National Council for Teachers of Mathematics [NCTM], 2014), but myriad indicators suggest that this access is not the same for all U.S. students. Results from assessments, such as the National Assessment of Education Progress (NAEP), demonstrate wide and persistent gaps between White students and Black and Hispanic students. For example, the 2015 eighth grade NAEP data indicated that at least 43% of White students reached a proficiency level in mathematics. In stark contrast, only 13% of Black students, 19% of Hispanic students and 6% of
English language learners (ELLs) shared this same proficiency level (National Center for Education Statistics [NCES], 2015). U.S. students’ White-Black and White-Hispanic score gaps have also been pervasive in international tests (Berliner, 2006; Schmidt, 2012). While assessment average data cannot provide a substantive understanding of students’ individual experiences, these trends point to patterns of educational inequalities that can inhibit social mobility and civic participation.

These disparities must prompt us to seek a more nuanced understanding of our students’ mathematical learning experiences so that we recognize not only gaps in performance, but also gaps in opportunities to learn. Flores (2007), for example, examined these mathematical learning opportunities to demonstrate students’ reduced access to qualified teachers, high expectations, and per student funding. These deficit areas have been addressed by scholarship, and have brought attention to the educational systems’ and the educational policies’ inability to address the particular needs of students from underserved populations (e.g., Darling-Hammond, 2004, 2007; Ladson-Billings, 1997; Oakes, 1990, 1995; Schoenfeld, 2002). These opportunity gaps can have a strong effect on students’ math learning experiences.

This study focused on classroom instruction as a primary vehicle to expand our efforts in equitable mathematics education. Teachers are essential (Boaler, 2002; Flores, 2007; NCTM, 2014): they can modify the instruction to be culturally responsive (Bonner, 2014), develop classroom norms that foster high cognitive demand (Boaler, 2002), and provide specific support (Celedón-Pattichis, & Ramirez, 2012; Turner, Dominguez, Maldonado, Empson, 2013). Given the centrality of teachers in students’ learning experiences, we must ask ourselves, what would it

---

1 In this document, the term ‘students from underserved populations’ refers to students from typically marginalized ethnic minorities (e.g. Black, Hispanic, Native American, etc.) and/or students from low socio economic level (SES) that are experiencing lower access to mathematical learning.
take to create the conditions for more equitable mathematics instruction nationwide? What must our teachers know and draw upon for their preparation for this critical instructional need?

But teaching and the teaching practices that have been found to advance student learning, are situated practices (R. Gutiérrez, 2002). That is, what a teacher does to advance learning can be found to be extremely effective in a particular classroom, but possibly found to be ineffective in another classroom. Because of the situated nature of teaching, R. Gutiérrez (2002) proposed an equity agenda that included a descriptive understanding of practice that takes into consideration “the everyday dilemmas that teachers face, the power that they wield, the influence of local contexts, and the relationships between humans” (p. 175). Moreover, because students’ backgrounds and life experiences may be quite different within classrooms and across classrooms, there is a great deal to learn about how teachers capitalize on this diversity, making use of what they have come to know about their students in practice.

This practice-based understanding is critically needed because it provides teachers with essential forms of knowledge of “how” and “when” to apply what they know about their students through instruction. Shulman (1986) described this type of knowledge as case knowledge, contending that teachers need to draw upon on practice-based understanding through “specific, well-documented and richly described events” to “illuminate both, the practical and the theoretical” (p. 11). Much like doctors or lawyers, this contention relies on the notion that teachers are professionals that not only use theoretically based knowledge from research with implications for practice, but they also draw on knowledge from the wisdom of practice itself, to derive theoretical implications (Shulman, 1986; 1987).

Applying Shulman’s (1986) knowledge for teaching framework here, I argue that the field of equitable math teaching needs to draw upon case knowledge to gain a situational
understanding of equitable teaching practices that we know from extant research-based efforts (e.g. *propositional* knowledge in Shulman’s framework). When teachers are able to draw upon both types of knowledge (case and propositional knowledge), they can gain the *strategic* (Shulman, 1986) knowledge that actually allows them to handle the complexities of practice; where teachers confront “particular situations or problems, whether theoretical, practical, or moral, where principles collide and no simple solution is possible” (p. 13).

This dissertation study sought to increase our case knowledge base of equitable math instruction through a descriptive understanding of how practicing teachers in grades 9 – 14, used their knowledge of their students to advance their learning in courses in the algebra sequence. Algebra is a fundamental course that provides the content and reasoning tools (e.g. numeric, algebraic and graphically) that prepare students for higher level math courses in high school and college. A descriptive understanding of how teachers advance the learning of algebra, particularly with students from underserved populations, empowers the mathematics community as a whole. This understanding best positions the mathematics community to proactively help restore learning access and provide greater educational opportunities for all.

**Background**

Many scholars have explored issues of equitable teaching. A review of the extant literature surfaced three contextually dependent, core features of the equitable math teachers’ practice. The review also surfaced fundamental differences in theoretical frames that drove these inquiries, which fell into two broad categories of cognitivist and sociocultural perspectives of learning. I summarize the highlights of this literature below.
Situated Equitable Math Teaching Characteristics

As noted above, three core characteristics of the equitable math teacher were identified in a review of relevant research: (1) reflective and proactive about equity, (2) maintains high expectations for all, and (3) builds on students’ lived experiences. I describe these characteristics below.

Reflective and proactive about equity. The first characteristic is reflective and proactive about equity. Equitable teachers are reflective about their beliefs and about the ways that their beliefs impact their decisions and behaviors in the classroom (Rousseau & Tate, 2003). When teachers engage in reflection, they are able to critically question behaviors that are guided by “sameness as fairness” ideologies (K. Gutiérrez, 2008) that not only ignore the realities of students from underserved populations (Rousseau & Tate, 2003), but also have been found to perpetuate inequalities in the classroom (e.g. Hand, 2010; Planas and Civil, 2002; Straehler-Pohl & Pais, 2014). Through the process of reflection, teachers can gain a new perspective about the field of math education that questions notions of neutrality (Brelias, 2015; Ladson-Billings, 1995b; Martin, 2013; Moses & Cobb, 2001; Rousseau & Tate, 2003) and that helps them reconstruct and pursue educational experiences that bestow students social and economic mobility (e.g. Ball, Goffney & Bass, 2005; Frankenstein, 1990, 1995, 2014; Gutstein 2003; Moses & Cobb, 2001). Although the literature has yielded little empirically based understanding as to how reflection incites proactive behaviors toward equity, it does provide the theoretical understanding that reflection empowers teachers with a new lens (Larrivee, 2000) and enables teachers with the critical process of developing awareness and performing checks on the effect of their teaching practices (Rousseau & Tate, 2003; Howard, 2003).
Maintains high expectations for all. The second essential characteristic is maintains high expectations for all students. Regardless of students’ differences, equitable teachers maintain high expectations for all. This may seem an obvious goal, but there is abundant empirical evidence demonstrating that students from underserved populations are subjected to lower expectations and reduced curricula (e.g. Straehler-Pohl, Fernández, Gellert & Figueiras, 2014; Hand, 2010; Oakes, 1990; 1995; Schiller, Schmidt, Muller & Houang, 2010). Although scholarship in equitable mathematics teaching has shown that maintaining high expectations can be challenging (e.g. Stein, Grover & Henningsen, 1996), it does yield equitable learning outcomes (e.g. Boaler & Staples, 2008; Gutstein, 2003; Moses & Cobb, 2011). Equitable math teachers believe that all students can succeed and they modify their teaching in different ways to maintain high expectations depending on their students’ needs and what they have learned from their teaching environments (e.g. Ball et al., 2005; Bonner & Thomasenia, 2012; Civil, 2014; Moses & Cobb, 2011).

Builds on students’ lived experiences. The third and final essential characteristic of equitable teachers is that they build on students’ lived experiences. Students are individuals with very different life and learning experiences that have helped characterize who they are. When these same students are in the mathematics classroom, they typically learn in certain ways and through certain norms that might not be in alignment with their own personal experiences outside the classroom (Jorgensen, Gates & Roper, 2014). Scholarship has exhibited two different approaches to “bridging” these differences. Some work has argued for making the culture of mathematics more accessible to students (e.g. Ball et al., 2005; Boaler, 2002; Silver, Smith & Nelson, 1995). Other work has argued for the need to completely re-define how the mathematics is taught (e.g. Bonner, 2014; R. Gutiérrez, 2012; Martin, 2013). Both approaches have addressed
gaps in experiences between student and classroom, suggesting that by using this more holistic approach to understand the learner, students’ identities as mathematical learners are further developed and reinforced.

These core characteristics of the equitable math teacher, however, provide us with empirical evidence that support our understanding of equitable math teaching as situated practice (Lave, 1991). Bonner and Thomasenia’s (2012) work exemplifies this point. They studied three different classrooms with three different teachers that implemented cultural responsive teaching in different ways. While one teacher capitalized on cultural and community traditions using classroom practices like singing, another teacher capitalized on students’ interests in working with cars. Although this is an oversimplification of the more nuanced argument the authors made, my intent is to highlight a fundamental characteristic in equitable teaching: if teaching is situated practice, then equitable approaches are also situated and can neither be generalized nor replicated without adaptation.

Through a situated practice lens, this need for adaptation underscores the complexity in conceptualizing what a teacher should know, especially in light of our high diversity of contexts. With this understanding, R. Gutiérrez (2002) argued that we could advance the field of equity in math education by recognizing its situated nature, thus, focusing on “what it takes to enact particular practices, especially ones that relate to certain kinds of students” (p. 171), but also recognizing that not all conditions in a teachers’ practice may be known or even predicted.

Therefore, the core characteristics of equitable math teaching and their associated work can be understood as basic principles or patterns that teachers can draw upon for their own learning and growth in their efforts to advance their students’ learning. This knowledge base, however, can be best described as propositional knowledge (Shulman, 1986) in that it is
conceptual and decontextualized. In order for teachers to be able to know “what it takes to enact particular practices”, their knowledge base must include case knowledge – through a descriptive understanding of the context, and of why and how these practices can be applied in day-to-day math instruction. This includes knowing what has worked and has not worked in practice. I thus argue that we want to be knowledgeable of core equitable teaching practices that have been found to work from research, but that in order to expand efforts in as many classrooms as possible, we need a growing inventory of case knowledge so as to also build a practice-based knowledge base for a more complete, understanding of equitable math instruction.

This study also addressed an additional gap in scholarship associated with theoretical alignments in research of equitable math teaching. I describe this next.

**Differences in Scholarship: Cognitivists vs. Socioculturalists**

The field of research in equitable math education has been guided primarily by cognitivist and sociocultural perspectives of learning. Work guided by cognitivist perspectives has advanced the field by growing our knowledge base on essential content areas that are needed in knowing how to teach math. Ball, Thames and Phelps’ (2008) work on the mathematical knowledge for teaching (MKT) framework, for example, brought to the forefront a more complete understanding of these content knowledge areas. While their work was based on elementary and middle school math, their findings are considered applicable to secondary math, and other scholars have extended the work into secondary mathematics education. The MKT framework has also informed additional work on quality education (e.g. Charalambous & Hill, 2012; Goffney, 2010; Hill, 2007, 2010) that has been used to measure the extent of a teachers’ use of specialized content knowledge to teach math. Issues equity have been advanced by
cognitivist perspectives by bringing attention to the equitable access to quality instruction (e.g. Hill & Lubienski, 2007).

Work guided by sociocultural perspectives of learning, has also brought to our attention issues of access and opportunity, as well as how mathematics is understood by students in formal and informal settings, and how social settings support learning. Ladson-Billings’ work (1995b) on culturally relevant pedagogy, for example, elucidated scholarship on issues of hegemony that threatened students’ access to learning through neutral conceptions of learning.

From an epistemological standpoint, there are fundamental differences in these two bodies of work. Cognitivist perspectives are primarily associated with “the individual mind in isolation, context free problem solving and mental representations and reasoning” (Tenenberg & Knobelsdorff, 2014, p. 2). Sociocultural perspectives, on the other hand, view learning as a result of the individual’s social interaction within a context (Tenenberg & Knobelsdorff, 2014). R. Gutiérrez (2013) – as a socioculturalist -- asserts:

The very practices that are taken up in the classroom and the meaning of doing mathematics are inextricably tied to the constellation of other identities that students bring to the classroom. Such an acknowledgement opens the doors for us to see that holding an equity stance means recognizing that as a mathematics teacher, one teaches mathematics and so much more than mathematics that influences students’ development. (p. 18)

In the end, my argument is that we need both traditions in learning about equitable teaching and learning of math. Cognitivist work has advanced the field through its stronger focus on mathematical conceptual development. Socioculturalist work has advanced the field through a stronger focus on issues of access by challenging traditional notions of neutrality. As reflected in R. Gutiérrez’s (2013) assertion above, socioculturalists have argued for the need to tend to additional aspects of the learner that are not necessarily just mathematical. This tendency to focus on distinct aspects of the same learner pointed to a gap in scholarship that this study
sought to address and that is at the heart of this study’s problem statement. Our understanding of students, who are central stakeholders in the learning process and who help co-construct the situated classroom learning experiences has been disjoint. Thus, a second significance of this study is that it also sought to address equitable teaching through a more holistic understanding of the student.

**Problem Statement**

We argue that students need to learn mathematics in light of who they are and the diverse gifts that they bring to their experiences every day. (Aguirre, Mayfield-Ingram & Martin, 2013).

Aguirre et al. (2013) propose an inspiring goal in equitable mathematics teaching. However, research and experience tell us that teachers face daily demands and responsibilities that challenge their ability to get to know their students in meaningful ways and to apply that knowledge of their students to support students’ mathematical learning. If this is the case, how do we feasibly operationalize this goal in practice?

Although research efforts have not gone in vain, these efforts have mostly furthered a disjointed understanding of how to use students’ “diverse gifts”, based on their epistemological alignment. Research efforts aligned with sociocultural perspectives have brought attention to students’ lived experiences and issues of access. Research efforts aligned with cognitivist perspectives have brought attention to students’ individual mathematical background knowledge and learning. Due to fundamental differences in how these theoretical perspectives conceptualize the nature of learning, there has been little work, if any, providing a descriptive understanding of how to advance students’ mathematical learning while holistically capitalizing on all aspects of the learner.
More recent cognitive work has focused on delineating the forms of knowledge that math teachers need through a math quality and equity (MQE) framework (Goffney, 2010). MQE explicitly highlights the importance of knowledge of the student for teaching:

I argue that in order to design and enact high quality equitable instruction, teachers must build bridges between what the students know and what they need to learn which requires knowledge of students, culture, and content. Thus, to provide equitable instruction, teachers must rely both on a solid knowledge of the subject matter as well as knowledge of their students’ cultural lived experiences, and bring sensibilities and awareness of issues related to equity. (Goffney, 2010, p. 14)

Goffney’s assertion brings us closer to a critical intersection on forms of knowledge about the student that are paramount to equitable learning. Teachers must leverage mathematical and non-mathematical forms of knowledge of their students in order to make learning more accessible. This is an equitable goal that is yet to be captured through research.

The problem in the field of research in equitable math education is that we do not have an integrative, cogent understanding of how these forms of knowledge of the student are sought and capitalized on through instruction. This problem compels us to examine practice in equitable mathematics teaching to seek a more in-depth, contextual, practice-based understanding of what teachers have found that works in advancing students’ learning. With this purpose in mind, the field of research can proactively (R. Gutiérrez, 2002) position itself to increase connections between research and practice, and can make equitable teaching more accessible to the mathematics teaching field as a whole.

**Research Questions**

This study sought to describe practicing teachers’ perspectives about students. The research questions in this study positioned students as an essential knowledge resource that informs the practice of teachers as they strive to advance students’ mathematical learning. Four research questions focused the inquiry:
• What forms of knowledge of the student (e.g., mathematically foundational, identity, community, etc.) do practicing algebra teachers use to leverage the learning of algebra for students from underserved populations?

• How are these multiple and diverse forms of knowledge of the student applied in practice to support the teachers’ algebra learning goals for their students?

• In what ways do these teachers perceive their use of these forms of knowledge as helpful in supporting their students’ learning of algebra?

• What models can be developed to understand these teachers’ practice as they attempt to advance their students’ learning within their situated context of instruction?

**Study Methods**

In this qualitative study, I worked with four teachers who opened their classroom doors to help illuminate equitable mathematics teaching. Two teachers taught Algebra 1. One teacher taught Algebra 2. One teacher taught a Quantitative Analysis course based on problem solving at a community college. Each demonstrated an equitable teaching stance, and an awareness of issues in equitable learning access. To answer the research questions, I combined both ethnographic and case study methods. The ethnographic methods helped ground my findings in the teacher’s environment. For example, I observed each participant for the length of a unit of study, using those data to confirm the teachers’ perspectives on their instruction. Case study methods yielded descriptive and situated depictions of the practice of each teacher. In a cross-case analysis, I was also able to triangulate data and compare and contrast patterns across cases for an understanding of the central phenomena that captured what the teachers knew about the students and on how they used it to advance their learning. I provide a more detailed description of the methods in Chapter 3.
Key Terms and Disciplinary Focus

In this section I define two key terms: equity and forms of knowledge of the student. I also provide my rationale for my choice in disciplinary focus: algebra.

Equity

Prior to starting this dissertation work, my definition of equity in math education would have been simple – to advance the learning of math for all students. Given the growing diversity in classrooms, the task on its own is demanding. But the political climate in which I have conducted this work made it ever so clear to me that this already challenging task is even more daunting. There are growing divides in our society that threaten our basic understanding and appreciation for diversity. Life experiences deviating from those of the mainstream are being trivialized much too often, creating a new social norm that is not just unfortunate, but simply wrong. What happens in classrooms does not operate in a vacuum, impenetrable from the larger sociopolitical issues at play. Throughout the study, I was deeply aware of the broader world in which the teachers and students found themselves, one in which #BlackLivesMatter, the devastation of Puerto Rico, threats to DACA funding, and hate crimes were part of the regular news. All of this impacts students’ mathematical learning, their aspirations and prospects for future educational choices, and their teachers’ work.

With these concerns in mind, I adopt Secada’s (1989) definition for equity which states: “Equity in mathematics education, therefore, should be construed as a check on whether or not the actions taken in teaching mathematics to students and the social arrangements resulting from those actions are just” (p. 24). This definition captures, in my opinion, the essence for our work towards equitable math learning – namely, a self-reflective assessment on whether our practices are fair. The process of teaching is replete with constant decision-making with both short-term
and long-term outcomes that often times are unintended. Teachers must carefully consider their actions within their practice to best meet our students’ learning needs so that outcomes are just. But what does justice look like in a specific classroom? Here Goffney’s (2010) remarks help. In essence, equitable teachers “must build bridges between what the students know and what they need to know” (p. 14) and this includes not just content knowledge, but also knowledge about the students and about their culture. Goffney’s (2010) assertion underscores the importance on students’ learning as a holistic approach where teachers need to capitalize on what they can get to know about their students tending to both mathematical and non-mathematical aspects. In doing so, teachers will themselves get an opportunity to learn and have a more informed understanding of their learners and their lived experiences. This, in turn, will provide teachers with a firmer and more grounded basis for checking and assessing if their teaching decisions have indeed led to just outcomes.

Knowledge of the Student

Teachers know many things about their students, including their mathematical strengths, family, culture, and funds of knowledge. Some of this knowledge is abstract, for example the types of misconceptions students typically have about fractions. Some of it is situated, for instance, how a particular student prefers to be called on in class. I created and used a preliminary list of potential categories of knowledge of the student that could be found through this study based on forms that have appeared in scholarship (these are enumerated in Chapter 3). I was open to, and also expected this list to grow because of the situated nature of the teachers’ practice. I acknowledge that this term “knowledge” has been frequently associated with cognitivist work, but I clarify that I used it for lack of a more descriptive term. I opened myself
to the possibility of finding new constructs to describe what a teacher has come to learn about his or her students.

**Why Study Algebra**

There are many reasons why we need to focus on algebra. To start with, a large extent of the work on mathematical learning has focused on elementary school or middle school levels (e.g., MKT, Cognitive Guided Instruction). Yet, algebra is a core high school graduation requirement. It is part of the Common Core Standards of Math (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and is assessed in the SAT examination and other subject specific tests used for high school completion and college admissions. Equitable efforts in mathematical learning must be vested at all levels.

Although these are all great reasons to support the need to increase attention to the algebra classroom, I choose algebra because it is a foundational course for all higher level math courses in high school and college (Conley, Drummond, deGonzalez, Rooseboom, & Stout, 2011; Usinskin, 1995). From a standpoint of access, I believe that all students must receive a strong preparation up to at least algebra. This preparation is what actually empowers all students with choice in their future educational endeavors, including college.

**Dissertation Overview**

In Chapter 2, I provide a more in-depth review of the literature that informed the research agenda, as well as the project’s theoretical framework – an integrated framework based on situated social practice framework (Lave, 1991) and critical race theory in education (CRT) (Ladson-Billings & Tate, 1995). In Chapter 3, I focus on my research methods. In Chapter 4, I report my findings from each of the four cases. For each teacher case, I present an analytical model that was used to describe the central learning phenomena. In Chapter 5, I report findings from the cross-case analysis. I also include an analytical model that I used to describe the overall
central learning phenomena across all cases. In Chapter 6, I conclude with my discussion of the findings in relation to the research questions. I also provide implications and recommendations for the fields of research and practice in equitable math teaching, based on the study findings.
CHAPTER 2
Literature Review

“Effective teaching is the nonnegotiable core that ensures that all students learn mathematics at high levels” (NCTM, 2014, p. 1).

In 2014 the NCTM issued their Principles to Action outlining the necessary measures to make mathematical learning accessible to all students. In doing so, the NCTM placed teachers at center stage to equitable mathematics education. While the field of research in math education would agree with this charge, the field may also find itself with multiple interpretations on what it means to be an effective teacher, particularly for students from underserved populations. This chapter reports findings from an inquiry process that was originally guided by one question – recognizing the multiplicity of contexts, what must our teachers know and draw upon to be prepared in equitable mathematics teaching? This basic question led to additional findings that together provide a background for this study and a research agenda that is driven to expand equitable math teaching instruction while also increasing much needed connections between the research and the practice field.

I start this chapter with a summary of the literature review. I then provide my definition and personal understanding of equity. While I have included this definition before in Chapter 1, I revisit it again here because in many ways, it has informed my review and critical interpretation of the literature. Next, I describe my review criteria. I then expound my findings from the literature review in the order listed in the summary. I end this chapter with a theoretical framework to study equitable mathematics teaching. The framework is based on Lave’s (1991) situated social practice and it incorporates contextual issues of power that are best captured through a critical race theory lens (Tate, 1995, 1997; Ladson - Billings & Tate, 1995).
Summary

My review of scholarship on practicing math teachers rendered a set of core characteristics for equitable math instruction. These are: (1) reflective and proactive about equity, (2) maintains high expectations for all, and (3) builds on students’ lived experiences. Through a situated practice lens we can understand how certain teaching practices and/or approaches have met their intended outcomes for a given context of instruction. Teachers, however, need to also know how to adapt and implement these practices in their own contexts of instruction to be able to meet their students’ needs. Thus, our knowledge base from theoretical principles is incomplete without complementing it with practice-based knowledge. I join Shulman’s (1986) call for building a case knowledge base for teaching to argue that in order to prepare teachers for equitable teaching we need descriptive and contextually based accounts from practice from which to further develop teachers’ knowledge of how to adapt and implement equitable teaching practice. I also argue that teachers’ knowledge must draw upon students, not only because they are central stakeholders in the teaching and learning process, but also because in discussions of equity, we want to build on their lived experiences inside and outside the classroom to meet their diverse needs. I revisit literature to describe forms of knowledge of the student. I then point to a gap in scholarship associated with a tendency in research to ascribe to disjoint perspectives of learning, which further justifies our need for a more integrative understanding of how to advance students’ learning.

Definition of Equity

There are growing divides in our society that threaten our basic understanding and appreciation for diversity. Life experiences deviating from those of the mainstream are being trivialized much too often, creating a new social norm that is not just unfortunate, it is simply
wrong. And so, what happens in the classroom that does not operate in a vacuum from the larger sociopolitical issues at play, requires also a grounded understanding from the teacher as to how his or her decisions have an impact on students’ mathematical learning and their prospects for future educational choices. With these concerns in mind, I adopt Secada’s (1989) definition for equity which states: “Equity in mathematics education, therefore, should be construed as a check on whether or not the actions taken in teaching mathematics to students and the social arrangements resulting from those actions are just” (p. 24).

The process of teaching is replete with constant decision-making with both short-term and long-term outcomes that often times are unintended. Secada’s (1986) definition captures the essence for our work towards equitable math teaching – namely, a pause and self-assessment of our teaching so that outcomes in our students’ learning are just.

This definition describes a broader goal for equity in math education, but it also requires a level of specificity as to how it can be operationalized into classroom instruction. Although my primary definition of equity is Secada’s (1989), I incorporate Goffney’s (2010) remarks into my understanding of equity for day-to-day classroom teaching. Goffney (2010) asserted:

I argue that in order to design and enact high quality equitable instruction, teachers must build bridges between what the students know and what they need to learn which requires knowledge of students, culture, and content. Thus, to provide equitable instruction, teachers must rely both on a solid knowledge of the subject matter as well as knowledge of their students’ cultural lived experiences, and bring sensibilities and awareness of issues related to equity (p. 14).

Goffney’s (2010) assertion captures, the notion that teachers need to capitalize on what they can get to know about their students holistically and that they need to tend to both mathematical and non-mathematical aspects of their students. In doing so, teachers will themselves get an opportunity to learn and have a more informed understanding of their learners and their lived
experiences. This in turn, will provide teachers with a firmer and more grounded basis for checking and assessing if their teaching decisions have indeed led to just outcomes.

**Review criteria**

This review of the literature focused on empirical work. Various bodies of work were reviewed (e.g. adult learning, middle and high school math education) focusing first on the practicing equitable math teacher. Since not all relevant work explicitly stated “equity or equitable”, I searched for work that also included the practice of teachers with students described as: “diverse”, “in urban settings”, “from underserved or underrepresented subpopulations”, “low SES”, “typical ethnic minorities”, “Latin@” and “Black or African American”. This broad approach led to a more nuanced review of related work, found through a snowball process that through a triangulation process rendered an understanding of typical challenges in equitable teaching as well as practices and teaching approaches that have been found to work in the field. I also note here that work on professional development in equitable teaching for the practicing teacher is scarce (Battey & Franke, 2015). I used work on pre-service teacher preparation to support my understanding of theoretical implications. I did not, however, use results from studies on pre-service teaching in the triangulation process to identify indicators of successful teaching practices in the field.

**Core Characteristics of the Equitable Math Teacher**

Scholarship in math education recognizes the pressing need for math learning that is equitable (Strutchens et al., 2012). Concerns for math achievement disparities between white students and students from typical ethnic minorities and/or low socioeconomic standing (SES) in the United States have been widely and consistently documented (e.g. Schmidt, 2012). Researchers have also noted the educational systems’ and educational policies’ inability to
address the particular needs of these students (e.g. Darling-Hammond, 2004; Ladson-Billings, 1997; Oakes, 1995; Schoenfeld, 2002). Despite empirical evidence of equitable math teaching practices (e.g. Boaler & Staples, 2008; Bonner, 2014; Bonner & Thomasenia, 2012; R. Gutiérrez, 1999; Ladson-Billings, 1995a), we are still in need of an in-depth understanding of how these teachers implement these practices in their “moment-by-moment” (Lampert, 2001) teaching. We must ask ourselves, how do they do it? How do equitable math teachers manage their day-to-day ongoing decision making to advance their students’ learning? These questions guided my inquiry process as I looked for core characteristics of the equitable math teacher. This process helped validate three key teaching features that theoretically support teacher’s ongoing equitable behaviors. I describe each of these teaching features in the sections that follow using a situated practice lens to also demonstrate potential challenges in our efforts to expand equitable teaching instruction from these features’ inherent situated nature.

**Reflective and proactive about equity.** John Dewey stated that reflective thought constitutes the “active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends” (Dewey, 1910, p. 6). Applying this definition to the practice of teaching can translate reflection into the active, persistent and careful consideration of teachers’ personal beliefs and forms of teaching knowledge in light of how they were originated (e.g. why do teachers come to hold a belief and/or how did a teacher acquire this form of knowledge?); and of how these beliefs and forms of knowledge have translated into what teachers experience in the classroom.

Teaching, however, is not a linear process. It is highly interactional (Vygotsky 1978, Voigt 1994, Yackel & Cobb 1996). As such, teachers must make almost instantaneous behavioral decisions in response to what they observe and subsequently perceive at the moment,
in order to support students’ learning experiences. Teachers need reflective thought as a check point where they step back from their fast paced day-to-day decision making to consider their overall behaviors that are many times enacted unconsciously. “As teachers think more deliberately, articulating the rationale that underlies their teaching decisions, they begin to name and confront the dilemmas and contradictions they face on a daily basis” (Larrivee, 2000, p. 297).

**Critical reflection.** What teachers “see” or “evidence” in their classroom, can be relative to their earlier conceptions and particularly held beliefs. And so what we, including teachers, have come to believe and know depends on what we have experienced (Vygotsky, 1978), which often times is different from our students’ lived experiences. Critical reflection has been deemed an important tool that helps teachers confront pre-conceived notions about students’ access to learning opportunities as a result of race and social status, question their own notions of the role of education and particularly, their own roles as agents in defining education (Felton & Koestler, 2015; Larrivee, 2000; Howard, 2003). Howard (2003) contends, for example, that critical reflection is essential in developing critically relevant pedagogy. Through reflection, teachers can engage in a critical inquiry process that allows them to question issues of hegemony and preference for practices that respond to the mainstream, rather than the minority. Teachers can develop and implement culturally relevant pedagogies when they view cultural differences as assets that must be capitalized on in the classroom (Howard, 2003).

Rousseau and Tate (2003) proposed the practice of reflection as an essential tool in helping teachers increase opportunities to learn for their students. Their work provides us with an example of how math teachers’ pre-conceived notions and unexplored ideologies can affect their behavioral decisions and impact students’ learning experiences. They studied a group of
high school math teachers in a school setting that was predominantly white with 27% minority student representation. The lower track math courses, however, were overrepresented by minority students. The researchers were interested in understanding teachers’ attitudes in relation to their practices. The teachers exhibited a neutral perspective towards their students. Similar to the “sameness as fairness” (K. Gutiérrez, 2008) term, color-blind mentalities ignore the realities of students of color and from underserved populations (Rousseau & Tate, 2003). These neutral mentalities, driven by the belief that everyone should be treated equally, actually produce additional educational inequalities (Rousseau & Tate, 2003). Their research findings demonstrated that the teachers in their study did acknowledge patterns of low performance but placed the blame on students’ socioeconomic standing and their families’ lack of involvement. The teachers, however, did not make connections between this status and their students’ racial backgrounds. The authors showcased a classroom snapshot where the researchers observed a class where two African American students were the “only two students in the class in need of help, went for an entire 50-minute class period without any instructional interaction with the teacher” (Rousseau & Tate, 2003, p. 214). The teacher did not allow the students to help each other and also explained that she expected them to take initiative to ask for help. According to the researchers, the teachers in this study did not notice how their practices marginalized their students, and placed blame instead on students’ lack of ownership and responsibility over their education. They called this “pattern of action and inaction” as “allowing students to fail” (p. 241). Rousseau and Tate proposed that math teachers must practice critical reflection in order to avoid the replication of the same social patterns of inequality that their students were already subjected to outside their classroom.
There are many other studies that have revealed the marginalization of students as a result of teaching behaviors triggered by normalized ideologies that bestow preferential status to the mainstream (e.g. Hand, 2010; Planas and Civil, 2002; Straehler-Pohl & Pais, 2014). This work is referenced here because it demonstrates how deeply rooted teacher ideologies can be (Rousseau and Tate, 2003). Critical reflection, as proposed by Tate and Rousseau (2003) would have helped teachers explore their practices and their effect on their students.

Rousseau and Tate (2003) argued that teachers can be better positioned to respond to all forms of student marginalization by engaging in a form of reflection that is “social reconstructionist” (p. 212). This type of reflection prompts teachers to “look both inward at their individual practice and outward at the institutional, cultural, and political contexts in which their practice is situated” (p. 211). Engagement in this practice allows teachers to critically question issues such as: the effect of color-blind ideologies in the classroom, the role of math in society and culture and how students’ particular differences (e.g. ethnic, linguistics and socioeconomic backgrounds) affect their learning of mathematics. The practice of reflection might lead to a new perspective, one that questions the presumption that math and math education are neutral fields of study (Brelias, 2015; Ladson-Billings, 1995a; Martin, 2013; Moses & Cobb, 2001; Rousseau & Tate, 2003). Although this reflective perspective is not the sole factor in the path towards becoming an equitable teacher, it is essential in helping teachers pursue instructional experiences that proactively seek emancipation and bestow students with social and economic mobility (e.g. Frankenstein, 1990, 1995, 2014; Gutstein 2003; Moses & Cobb, 2001).

**Reflection in practice.** Research on practicing equitable math teachers has provided evidence of the use of reflection as an essential component of their practice (e.g. Bonner, 2012; Gutstein 2003, 2006). Bonner and Thomasenia (2012) used a grounded case study approach to
conceptualize culturally responsive teaching. They followed the practice of a math teacher, Mrs. Finley. She was selected as a participant for multiple reasons that included community recommendations and her track record for turning at-risk students into grade-level students. Her alumni described her teaching as “transformational” (p. 28) and she was highly sought after as the teacher that “turned at-risk students into grade level math students”. As a culturally responsive teacher, according to the researchers, Mrs. Finley adopted the communication style that was characteristic of her student population. She frequented popular locations in the community and re-defined a mathematics teaching and learning experience guided by the particular social capital and cultural practices of her students and their community. Bonner and Thomasenia’s (2012) conception of culturally responsive teaching fundamentally rested on four core areas: knowledge, communication, relationship/trust, and constant reflection and revision (p. 29). These core areas were not mutually exclusive; they were considered to work in concert with each other and were almost impossible to observe in isolation (Bonner & Thomasenia, 2012). Mrs. Finley was described as constantly reflecting and making adjustments to the other three essential components within her ongoing practice. She debriefed students as well as school personnel on what worked during the day. She looked for areas where she had not been as successful and this triggered her response to understanding her students better and looking for alternative ways to help them learn. Bonner and Thomasenia’s (2012) description of Mrs. Finley, demonstrate a master reflective practitioner that through long –term reflection developed an organic and instantaneous deliberate approach (Larrivee, 2000) in her teaching.

Research on teacher education in equitable math teaching practices has revealed ample use of reflection (e.g. Battey & Frankey, 2015; Bianchini & Brenner, 2010; Felton & Koestler, 2015; Felton – Koestler, 2015; De Freitas, 2008; Foote, McDuffie, Turner, Aguirre, Bartell &
Drake, 2013; Leonard, Brooks, Barnes-Johnson, & Berry, 2010; Planas and Civil, 2009; Wager and Foote, 2013). In most of the work referenced here, if not all, reflection has been accompanied by transformation and action; where action was represented by a particular approach to equitable teaching. The participants in Planas’ and Civil’s (2009) study, for example, engaged in a professional development program that was inquiry based. They reflected as a group on the practices that were or were not particularly effective for their immigrant students. They also reflected on their ownership over the success of these students. For example, in one of their sessions, the teachers themselves recognized that examining the language used in the problems they selected for their students was not enough to help their students succeed. Through reflection, they recognized that they needed to take “into account everything that will allow the students to do the task” (p. 400). Planas and Civil (2009) noted that their participating teachers experienced a transformational effect in the form of empowerment. “By gaining empowerment, conflicts are not necessarily overcome and old practices are not totally eliminated. Instead, the teachers become more reflective on these conflicts and practices so that they are nearer to actions of change by assuming new roles” (Planas & Civil, 2009, p. 407). Based on the researchers’ assertion, the process of reflection did not seem to induce the elimination of all practices, but it increased teachers’ awareness of conflicts, preparing them to address changes in practice.

**Reflection as a situated practice.** Practicing equitable math teachers have devised their own situated approach to attend to equity. The participants in Planas and Civil’s (2009) study were interested in addressing the needs of their immigrant students and as a result, they sought out help through professional development. Bonner and Thomasenia’s (2012) participating teacher had already achieved a deep understanding of her students’ needs when she was selected
for study. Yet, she practiced ongoing reflection to look for ways to better align her practices to particular student needs. Their need and use of reflection was situated. Teachers’ and students’ experiences and identities are diverse. When teachers reflect on students’ identities and experiences, they do not reflect on textbook generalized descriptions of people. Otherwise their experiences are trivialized (Battey & Franke, 2015; Leonard et al., 2010). This brings back to light the understanding that there are subcultures within cultures and that even within families, members’ experiences and formed identities will differ based on their lived personal experiences. Because of this, teachers’ reflective thoughts are situated on the particular experiences lived within their context of instruction.

**Maintains High Expectations for All Students.** In concept, most math teachers, if not all, would agree with the statement that all students should receive a strong foundation that prepares them for higher level math courses and higher education. What teachers believe in theory, however, and what they enact in practice can be very different (e.g. Stigler & Hiebert, 1997, 2000). For example, using TIMSS data, Stigler and Hiebert (1997) found that math teachers in the U.S. believed that their instruction developed conceptual connections. Observational data from their practice, on the contrary, revealed highly procedural teaching approaches, devoid of meaning making experiences. There are also ample studies, exposing teachers’ use of a lower curriculum in lower-tracked classrooms (e.g. Cogan, Schmidt & Wiley, 2001; Oakes, 1995; Schiller, Schmidt, Muller & Houang, 2010). Unfortunately, students from minority subpopulations and lower socioeconomic standing have been found to be overrepresented in these lower tracks (e.g. Oakes, 1990, 1995; Rousseau & Tate, 2003).

This particular section of the review focuses on equitable teachers’ approaches to maintaining high expectations for all students. They encounter particular challenges within the
context of their practice. Yet, they manage to facilitate meaningful mathematics learning experiences.

**High expectations for all in practice.** Scholarly work on equitable math teaching indicates the need to maintain high expectations for all students (e.g. Boaler & Staples, 2008; Celedón-Pattichis & Ramirez, 2012; Gutstein, 2003). Equitable math teachers who are working to maintain high expectations for all students respond to students’ differences and needs. This does not necessarily mean that students will necessitate a different treatment all the time, nor does this mean that a standard way to treat and teach some students will always result in equitable outcomes. In many cases, equitable math teachers have had to re-define the learning expectations for all students.

Boaler and Staples (2008), for example, studied a high school math department that re-defined the teaching and learning experience by using open-ended learning problems that allowed for full student engagement across ability-levels. Students worked in groups with defined responsibilities that helped them develop ownership over their own learning. While the researchers reported a discrepancy between students’ achievement in high stakes assessment data and students’ in-class alternative forms of assessment, the students ultimately succeeded by progressing to higher mathematics levels.

Another example of the use of alternative indicators for student learning was found in in the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project. The researchers based their understanding of student learning on a “student mediation model” (Stein, et al, 1996, p. 457). That is, their project design was not fundamentally guided by assessment data. The design of the QUASAR project was based on what students actually do in the classroom. In the researchers’ perspective, teaching influenced students’ cognitive processes
and these cognitive processes resulted in student learning. It was believed that through consistent engagement in opportunities to reason and make sense of mathematics, students would develop a deeper understanding of math and evidence strong problem solving skills (Stein, et al., 1996). Student learning was operationalized through a curriculum based on mathematical tasks (Boston & Smith, 2009; Stein at al., 1996); which were defined as classroom activities that focused students’ attention on a particular mathematical idea (Stein et al., 1996). These mathematical tasks differed in levels of cognitive demand. Consequently, when students sustained engagement in high cognitive demand tasks, their level of mathematical understanding was also understood to remain at the highest cognitive levels.

More recent work on equitable teaching has been primarily guided by socio-cultural theories of learning. Ladson-Billings (1995a, 1997), for example, studied a math teacher that used culturally relevant teaching strategies. This teacher “demonstrated the possibility of using the students’ prior knowledge as a bridge to new learning” (p. 704). She was efficient in her use of class time and “treated all students as if they were intellectually exceptional” (p. 703). She orchestrated a classroom learning experience that fostered meaning-making and full learning engagement. Ladson-Billing’s (1995a) study presented an additional outcome from teaching approaches that focused on meaning making. The students were also able to recognize when they were taught in meaningful ways. Unfortunately, this gave way to unintended negative consequences. When students moved on to the next school level, they found themselves dissatisfied with the low expectations and procedural teaching in their new school (Ladson-Billings, 1995a) and tried to get back to the teacher’s classroom. This finding points to students’ interest to learn, to recognize what is a successful learning experience, and to want more.
**High expectations for all as a situated practice.** Whether inspired by cognitivist perspectives of learning (e.g. Henningsen & Stein, 1997; Hill & Lubienski, 2007) or by sociocultural theories of learning (e.g. Civil, 2014; R. Gutiérrez, 2002, Gutstein, 2003; Ladson-Billings, 1995b, 2014; Strutchens et al., 2012), equitable math teachers believe that all students can learn and can be successful doers of mathematics. This practice, however, can be understood as situated based on the multiple teaching approaches, teachers’ understanding of their students’ need and their particular contexts of practice. This has included: capitalizing on what students do in the classroom by re-thinking the types of problems and curriculum that students grapple with (e.g. Boaler 2002; Boaler & Staples, 2008) and appreciating students as intelligent and capable individuals (e.g. Ladson-Billings, 1995a).

**Builds on students’ lived experiences.** The third core characteristic of the equitable math teacher is closely associated to teachers’ central goal to see all students as rich individuals whose identities and lived experiences are not just validated, but capitalized on and built upon (Aguirre, et al., 2013) to increase opportunities to learn.

Socio-cultural learning theories have heavily influenced the work supporting this characteristic. One central tenet of sociocultural theory is that learning arises through interactions with the environment (Vygotsky, 1978). Keeping in mind the increased diversity in students’ experiences when classrooms’ student compositions are also diverse (e.g. culture, language, SES), equitable math teachers understand that the classrooms’ social norms (Yackel & Cobb, 1996), as well as the culture of school mathematics (Martin, 2013), may not be familiar to all students. As a result, students may be marginalized from learning experiences that (intentionally or unintentionally) privilege others’ experiences.
Based on the literature review, two approaches in math education were found that responded to differences in student’ lived experiences. One line of work has been guided by the need to make the culture of school mathematics more accessible to students (e.g. Boaler, 2002; Jorgensen et al., 2014; Silver & Smith, 1995; Yackel & Cobb, 1996). Another line of work has been guided by the need to re-define the mathematics taught and how it is taught in the classroom, so as to provide an inclusive learning environment (e.g. Boaler, 2000; Boaler & Staples, 2008; Bonner, 2014; R. Gutiérrez, 2012). Both approaches highlight the importance of sociological aspects in mathematics learning.

The latter approach, however, places a stronger emphasis on moving away from traditional notions of what learning mathematics is by capitalizing on students’ diverse characteristics and interests to co-construct the learning experience (Aguirre et al., 2013; Bonner, 2014, Bonner & Thomasenia, 2012). The co-construction of learning experiences is particularly important in discussions of equity because in building on students’ lived experiences the student and the learning of mathematics are both transformed (Bonner, 2014; Bonner & Thomasenia, 2012; Tate 1995). Students are not assimilated into a standard notion of a mathematical learner because the learning experience does not follow a standard notion of learning mathematics.

This review highlights two central approaches found in the research field that fundamentally seek to transform the learning of mathematics and to build on students’ lived experiences. These two broad categories are: culturally relevant and/or culturally responsive teaching, and critical mathematics. While other approaches are recognized, the categories shown here are believed to comprehensively demonstrate situated ways that practicing teachers have used to build on students’ lived experiences.
**Culturally relevant teaching.** Gloria Ladson-Billings is commonly known for developing and coining the term “culturally relevant teaching”. Her main concerns at the time (Ladson-Billing, 1995b) she proposed this pedagogy revolved around constraints associated with approaches that made education more culturally congruent to students from ethnic minorities. Ladson-Billings (1995b) argued that educational approaches of this kind were driven to train minorities to succeed in “main stream culture” (p. 467). Instead of accommodating students’ cultures to the school culture, which was a practice that implicitly ratified notions of cultural deficits, Ladson-Billings (1995b) proposed culturally relevant pedagogy as a way to address student achievement while also helping students affirm their cultural identity and develop a critical perspective.

The process by which Ladson-Billings (1995b) developed culturally relevant pedagogy is less commonly known, but relevant to this literature review on equitable teaching practices. She used a grounded theory approach and ethnographic methods to study the practice of eight teachers that met various selection criteria. The study methods involved four phases of data collection that started with a base interview on teachers’ beliefs about teaching, classroom management and styles. This phase was followed by teacher observations, video recordings and video tapings of their practice. The researcher also interviewed participants before and after recordings. The final phase involved a collective approach where the participating teachers would analyze video tapings of each other and would interpret each other’s practices. The resulting construct of culturally relevant pedagogy was fundamentally formed from the study of the practice of equitable teachers whose teaching years ranged from 12 to 40 years of experience in classrooms of African American students. In summary their collective work yielded a pedagogy with essential characteristics that included: the believe that all students can succeed

32
academically, where teachers are community members that give back to their community, a pedagogy that is “always in the process of becoming” (p. 478), a pedagogy that developed a community of learners that encouraged each other to learn and “maintained fluid student-teacher relationships” (p. 480) and the belief that teachers facilitate learning where knowledge is “not static, it is shared, recycled, and constructed” (p. 481).

Since then, much research guided by culturally relevant pedagogies has furthered the field of research in math education in ways to support students’ identities and lived experiences while helping them succeed academically (e.g. Gutstein, Lipman, Hernandez & de los Reyes, 1997; Hubert, 2014; Leonard et al., 2010; Lipman, 1995; Tate, 1995). It is important to note that these efforts to build on students’ lived experiences have always included as a central tenet the fact that mathematical learning and academic success are an expectation. Work showcasing the equitable teaching practice of building on students’ lived experiences has also embraced the second core characteristic of maintaining high expectations.

**Culturally responsive teaching.** Culturally responsive teaching is described as a view of learning that incorporates “intellectual, academic, personal, social, ethical, and political dimensions all of which are developed in concert with one another” (Gay, 2000, p. 44). This suggests an integrated understanding of students as individuals. As such, connections between students’ homes and school environment are essential (Gay, 2000). In essence there are similarities between culturally responsive and culturally relevant pedagogies of teaching (Strutchens et al., 2012), and either pedagogy has been deemed feasible to address issues of math education equity for students from underserved populations (Strutchens et al., 2012). Culturally responsive teaching has furthered the field for research in math education in ways to support
students’ identities while helping them succeed academically (e.g. Aguirre & Zavala, 2013; Bonner, 2014; Bonner & Thomasenia, 2012; Hernández, Morales & Shroyer, 2013).

**Culturally relevant/Culturally responsive teaching in practice.** Bonner & Thomasenia’s (2012) work on culturally responsive teaching was described earlier in this document to highlight the use of reflective practices within a culturally responsive framework. In a later publication, Bonner (2014) provided more detailed descriptions of culturally responsive teaching, Bonner’s (2014) publication showcased three different classrooms where the teachers modified their practices in different ways based on their particular context. In one classroom, a teacher (recall – Mrs. Finley) taught using African American traditions in choir singing to engage students and facilitate their learning. She re-defined the classroom norms by adopting a communication style that was embedded in her students’ culture and lived experiences. Another teacher brought model cars to explore mathematical concepts. She did this because her students spent a large amount of their time outside of school working with cars. In the third case, the teacher encouraged partner talk to work through problems in an all-girls classroom, shifting power in the learning process to her students. There were many more strategies described that together, supported cultural responsiveness. These culturally responsive approaches were highly situated, and depended on the collective make-up of each classroom.

**Critical mathematics.** Critical mathematics represents a broad category of pedagogical approaches in math education that is fundamentally driven by a “vision of social and educational justice and equality” (Tutak, Bondi & Adams, 2011, p. 2). In essence, critical math educators reject the notion that math is neutral and opt to instill values of critical inquiry in their classroom that give way to student empowerment and liberation. Many researchers of critical math pedagogy credit Paulo Freire for influencing their work through critical pedagogy (e.g.}
Frankenstein, 1990, 2014; Gutstein, 2006), though it is recognized that other researchers around the world have made great contributions to its inception (Tutak, Bondi & Adams, 2011). Consistent with Freire’s work, critical math pedagogies seek to not only impart knowledge in the student, but to develop a critical consciousness that transforms students into agents of change, or as Gutstein (2006) described it –“reading and writing the world” (p. 334). Said transformation has an inherent impact on students’ identities, through an empowerment that seeks to question oppressive structures. Critical math pedagogies build on students’ lived experiences because they (1) recognize sociopolitical issues that have oppressed students from underserved populations and (2) build on this understanding to develop students’ critical consciousness and agency in the learning process. I also note that I construe any work in the field denominated as ‘teaching for social justice’ to belong to domains of critical math pedagogy because of its alignment to the critical inquiry of social issues.

*Critical mathematics in practice.* Although much research in critical math documents success of its enactment in teachers’ practice (Brelias, 2015; Frankenstein, 1990, 2014; Gutstein, 2003; Rubel, Limb, Hall-Wieckert, & Sullivan, 2016), other work provides realistic portrays of the challenging nature of such implementation (Brantlinger, 2013, 2014; Gregson, 2012; Gutstein, 2003; Pais, Fernandez, Matos & Alves, 2012). For example, Gutstein (2003) described the implementation of a social justice program along with a standards based curriculum in seventh and eighth grade mathematics. He taught at an urban public school with a student population that was 99% Latino and also 98% low-income. He used a National Science Foundation (NSF) funded curriculum, called Mathematics in Context (MiC). This curriculum encouraged the use of informal approaches in order to prioritize student’s mathematical understanding over the application of procedures. Gutstein (2003) noted that this curriculum was
well in alignment with social justice projects because he wanted students to learn “mathematics meaningfully” (p. 47). He used social justice projects in an attempt to help students connect the curriculum to their lives and “make sense of social phenomena” (p. 47). Gutstein’s (2003) choice for social justice projects supported a math educational environment that transcended classroom learning as it helped students use math as a tool to understand the world and equity issues. At the same time, more complex issues for both teacher and students were self-reported. The teacher was challenged and had to learn how to help students cope with feeling powerless as a result of their increased awareness of unfair conditions (Gutstein, 2003). He also acknowledged that his projects were less mathematically challenging than the curriculum in use. Upon reflection, he recognized that he told students how to do some calculations. He also understood, however, that the projects served a different role in helping students use math as a tool to understand greater issues in society and that trade-offs were needed. Gutstein’s (2003) accounted experience provided a candid assertion that reflected the ongoing challenges of attempting to make the learning experience more accessible, while at the same time, preparing students with a strong conceptual base.

**Attention to language.** This review on practices that build on students’ lived experienced would be incomplete without attention to students’ language, which often times is not the same as the one used for instruction in mainstream classrooms (English). Equitable math teachers tend to students’ linguistic differences,

In linguistically diverse classrooms, consideration must be made for language use, especially because math as a discipline has its own characteristic register with specific mathematical meaning dependent on linguistic construction (Adler, 1999; Phakeng & Moschkovich, 2013; Schleppegrell, 2007). Moreover, in today’s educational context,
communication is deemed a central practice in mathematics classrooms in order to construct viable arguments and critique the work of others (see Common Core State Standards of Mathematics (CCSSM), The NGA Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010). Whether a classroom has or has not adopted these standards, language use in the math classroom is central to teaching and learning math. Phakeng and Moschkovich’s (2013) description of linguistically diverse students’ needs portrays this point well:

As Gee [1999] would put it, students are essentially learning how to act, interact, think, value, talk, write, and read in mathematically appropriate ways with appropriate props in the appropriate places. If they are learning mathematics in a language that is not their home language, then their task is even more demanding, because they have to learn to do all of the above in a new language that they are still learning. (p. 126)

Equitable math teachers understand that English Language Learners (ELLs) need particular support because, as with all students, they need access to engage in discourse and in the overall practice of the community of math learners that will consequently further develop their identity as mathematics learners (Turner et al., 2013).

An example in practice. Let’s use a classroom example from Moschkovich’s (2002) work to demonstrate this. Two bilingual Latina students were exploring the “steepness” in the graphs of linear functions. The equations for these functions were $y = x$ and $y = -0.6x$. One student was not sure how to make this determination, while her working partner associated the $x$-axis to the ground. She used her gestures to demonstrate that the line $y = -0.6x$ was closer to the ground and she used the term “empinada” from Spanish to describe the steepness of the lines. As Moschkovich (2002) explained, the term “empinada” is a technical term in mathematics in Spanish. This demonstrated that the helping student not only had a strong mathematical understanding, but she also had already developed the mathematics register (Schleppegrell,
2007) in Spanish. In this case, the student seemed to have already developed an identity consistent with a doer of mathematics, and was comfortable communicating mathematically in Spanish. The teacher would have most likely needed to work with this student in making future mathematics learning more accessible by providing experiences for her to further develop cognitive academic proficiency, CALP, (Cummins, 1980) in English. Not all students come to the mathematics classroom with personal experiences that are aligned with the technical knowledge of school. In these cases, the teacher would most likely need to re-define the mathematical learning experience. And so, the decision made by the teacher would highly depend on his or her understanding of the learning scenario. Moschkovich (2002) argued that a situated socio-cultural perspective can “broaden the analytical lens” (p. 206) for teachers of ELL’s. This perspective allows them to ascertain both their students’ difficulties as well as their resources.

*Builds on students’ lived experiences as a situated practice.* Moschkovich’s (2002) perspective is in alignment with all other points made above regarding the situated nature of the practice of the equitable math teacher. The challenges and successes of implementing a critical math pedagogy that I have described demonstrate the situated nature of this equitable math teaching practice. Similarly, implementing culturally relevant or culturally responsive practices is a highly situated endeavor.

In this literature review I have only used culturally relevant, culturally responsive, critical math pedagogies and attention to language to portray the situated nature of equitable teachers’ practices. Other pedagogies, like the use of funds of knowledge (see also Aguirre et al., 2012; Civil & Bernier, 2006; González, Andrade, Civil & Moll, 2001) would have presented a different facet of equitable teaching with different, yet, situated characteristics.
Situated Practices and Knowledge for Equitable Math Instruction

The three characteristics described up to this point (reflective and proactive about equity, maintains high expectations for all students, and builds on students’ lived experiences), serve as broad common descriptors of the equitable teacher as rendered from scholarship. These characteristics, however, do not provide an “easy to follow recipe” as to how to use them for every mathematics classroom. This is an impossibility, given the large set of student and teacher differences across all classrooms. Using a situated practice lens, I argue that teachers, in their own situated practice, have read their teaching environment and their students’ difficulties and resources (mathematical foundations, cultural, community based, linguistic, etc.). According to what these teachers have read, they have adjusted their approaches to best induce mathematical learning. The desired end result from any approach chosen, is a student that is a confident doer of mathematics that negotiates mathematical meaning and that is fully prepared to engage in the classroom “Discursive” (Gee, 2008) practices of higher level math courses.

I take on R. Gutiérrez’s (2002) perspective on the teacher’s practice and its centrality towards an equity agenda. R. Gutiérrez (2002) contended that the teacher’s practice is an ongoing dynamic process that is grounded in its context. The teacher’s practice, according to R. Gutiérrez (2002), is a combination of what the teacher brings to the classroom and the teachers’ “membership in local communities” (p. 171). Using this conception of the teacher, she argued that math education’s equity agenda needs to focus more on “what it takes to enact particular practices, especially ones that relate to certain kinds of students” (p. 171), while keeping in mind that these practices are many times framed by a variety of conditions, many of which cannot be predicted. This conception of the teacher’s practice brings us closer to the realities of
day-to-day activity that in some way have foregrounded the equitable teaching showcased through scholarship.

Taking on this situated perspective brings on additional, yet, necessary complexities that we need to address in determining “what it takes” to expand efforts in equitable math teaching to our diverse classroom contexts. Teachers need to be prepared and develop the necessary knowledge base to enact equitable teaching practices. The knowledge base from scholarship provides teachers with principles on what has worked to advance students’ learning, but it does not provide them with knowledge on *how* to enact them.

Using Shulman’s (1986) knowledge growth in teaching framework, we can also understand teachers’ work as a practice that requires different forms of knowledge. Theoretical knowledge from scholarship is essential in informing what works and does not work, but this knowledge is not limited to principles, which were described by Shulman (1986) as propositional knowledge. There is also theory needed in how to enact these principles, derived from a more historical, contextually grounded knowledge base (i.e. case knowledge). The teachers’ practice is thus, analogous to the practice of a lawyer or a doctor, whose knowledge base requires a combination of three forms: propositional knowledge, case knowledge and strategic knowledge. I best describe these forms in the following way – Propositional knowledge supports teachers with theories with implications for practice. Case knowledge supports teachers with theories derived from practice. These two, complement each other, so that teachers can make determinations in all situations (strategic knowledge), even those that seem novel simply because they do not match other instances or situations we have studied before.

I argue that in order to expand our efforts in equitable math teaching, we need to empower teachers with a comprehensive knowledge base that includes case knowledge – the
knowledge that will support their application and adaptation of our scholarly knowledge so they can responsively address their students’ diverse needs. Because the three core characteristics of the equitable math teacher respond to students’ need, I also argue that we need case knowledge on how teachers enact their practices and advance their students’ learning of math, based on what they know about their students.

**Forms of Knowledge of the Student**

I propose studying the practice of the teacher as naturally occurring in context (R. Gutiérrez, 2002), using the teachers’ own perspective on how given practices help advance students’ learning. I also propose focusing on the teachers’ practice on the student, as an essential knowledge source to their practice. This entails adopting how the teacher, whom is viewed here as the implementer of his or her practice, has come to understand that certain practices meet their intended goal of learning based on what they have come to know about their students. With this approach, students, take a center stage through the teacher’s practice. R. Gutiérrez once stated:

> However, I echo Ball and Bass (2000) that beliefs, knowledge, and curricular materials alone do not dictate teacher practice. Rather, because teachers’ beliefs and knowledge emerge and are grounded in their participation in workplace settings, we must attend to these contexts (p. 171).

Thus, what teachers have come to know about their students is viewed here as a source of capital, as part of the teacher’s workplace setting that helps inform his or her practice. I now briefly illustrate different ways that teachers’ knowledge of their students has informed their practice.

Bonner and Thomasenia’s (2012) study of a teacher’s practice is an example from scholarship that provided evidence of how a teacher reflected on both collective and individual forms of knowledge of the student. For example, the teacher re-designed her overall teaching
style to make learning more accessible to an African American community of students. In doing so, the teacher reflected in a *collective* form of knowledge of the student that rendered a culturally responsive approach. Her new practices responded to her students’ community and cultural heritage. This same teacher was also found to reflect regularly to find ways to meet her students’ *individual* needs. She paid attention to how her students learned and this prompted her to target specific needs based on her day-to-day interactions with her individual students. Thus, her students’ day-to-day mathematics learning experiences were re-constructed through the teachers’ practice, addressing, both collective and individual forms of knowledge of the student.

When teachers build on students’ lived experiences (this was found to be an essential characteristic of the equitable teacher), they must incorporate a rich set of diverse forms of knowledge of their students. Some are mathematical, where a teacher targets students’ areas of strengths and areas that need improvement (Aguirre, et al., 2013). Others are mathematical but sourced outside the classroom, like funds of knowledge from cultural practices (González, et al.; 2001) or funds from family members (Civil & Bernier, 2006). But funds of knowledge is only a subset of this rich set of diverse forms of knowledge. Other forms of knowledge are non-mathematical, like students’ positioning in society as members of marginalized subpopulations (Gutstein, 2003; Planas & Civil, 2002) or associated with the community that the school services (Moses & Cobb, 2001).

When considering equitable mathematics teaching, the forms of knowledge used from the student can be extensive and highly dependent on the context of the teacher’s practice. Without a purposeful alignment to a particular perspective of learning, the notion of “knowledge of the student” is construed here as a holistic set of information from the student that can be mathematical or non-mathematical, collective or individual. The overall objective would be to
produce a descriptive understanding of how knowledge of the student informs the day to day realities of practice of the equitable teacher in his or her goals to advance students’ mathematical learning.

The Student: A need for an integrative understanding

I have proposed to focus on students as a central form of knowledge for the teachers’ practice. But teachers also need to know what to use about their students through instruction. As I reviewed the literature, which is a critical source of knowledge on what teachers have used about their students, I noticed a gap. Scholarship in the study of the practicing math teacher, particularly in the United States, has been primarily guided by cognitivist and sociocultural perspectives of learning. These two perspectives are disjoint. In this section, I provide examples of work to illustrate how both perspectives have both advanced and also limited the field of research.

The mathematics knowledge for teaching (MKT) framework (Ball et al., 2008) is an example of work guided by cognitivist perspectives of learning. It was generated from fifteen years of studying the practice of elementary and middle school mathematics teachers. This framework delineates different forms of content knowledge that teachers use in order to teach math. Ball et al.’s (2008) work built on Shulman’s (1987) earlier work by providing a more complete understating of the forms of knowledge that are needed in teaching. Whereas Shulman (1987) brought attention to pedagogical content knowledge at a time when only content was the primary focus in teaching, Ball et al. (2008) demonstrated that particular forms of knowledge in content and pedagogy are needed for teaching math. MKT’s findings have also been extremely important because they demonstrated that the practice of math teachers involves specialized forms of knowledge that go above and beyond what is commonly understood as math content.
In defining these categories of knowledge for teaching, MKT enabled the field of math education to identify and eventually measure teachers’ use of these forms of knowledge through their instruction. These measures led to a new research area in math education known as quality education (e.g. Charalambous & Hill, 2012; Goffney, 2010; Hill, 2007; 2010; Hill & Lubienski, 2007). In theory, quality education centers on the idea that the extent of MKT used by a teacher can be used to determine how qualified a teacher is to teach math. Quality education is relevant to equitable teaching because it brings attention to the notion that teachers need to know the subject they teach and how to teach it to advance the learning of all students.

Culturally relevant pedagogy (Ladson-Billings, 1995b) is yet another significant framework that rather than cognitive perspectives, has been guided by sociocultural perspectives of learning. As I described earlier in this review, this framework was originated from Ladson-Billings’ (1995b) work with eight teachers. The teachers collaborated to reach a shared understanding of the teaching practices they attributed to their students’ success. The teachers found that their teaching practices were driven by the beliefs that all students can succeed, that their pedagogy was “always in the process of becoming” (p. 478), and that they pulled knowledge out from their students to advance their learning. These findings have influenced a great extent of research in equitable teaching because they uncovered a different dimension on what is particularly effective in the learning of African American students. One of the main implications of culturally relevant pedagogy in the field of equitable teaching has been that our conceptions of effective teaching need to go above and beyond “one size fits all” mentalities. Because of this, its central tenets have been applied to advancing the learning of students from other typically underserved populations.
As noted, MKT and culturally relevant pedagogy are two frameworks that exemplify two distinct perspectives on learning within the field of math education and educational research, more broadly. According to Tenenberg and Knobelsdorf (2014), cognitivism is primarily associated with “the individual mind in isolation, context free problem solving and mental representations and reasoning” (p. 2). Fundamentally speaking, if we can identify forms of knowledge to advance the learning of all students as proposed in MKT, we operate under the assumption that everyone, regardless of experiential differences should be able to learn equitably. Issues of equity would thus be reduced to issues of access to teachers that possess strong MKT. Sociocultural theory, on the other hand, views the mind (and learning) as cultural products, where evolvement takes place through tools and social interactions (Tenenberg & Knobelsdorf, 2014; Vygotsky, 1978). Much aligned with sociocultural theory, culturally relevant pedagogy does not view knowledge as fixed. It conceptualizes knowledge as something “shared, recycled and construed” (Ladson-Billings, 1995b, p. 481). Issues of equity revolve around challenging traditional notions of math teaching that have favored the learning of mainstream students. Because of this, culturally relevant pedagogy addresses issues of hegemony that could be overlooked through more neutral conceptions of knowledge.

Neither perspective ignores the central tenet of its counterpart. Cognitivist math education researchers have highlighted the importance of social interactions (e.g. Cobb, 1994; Lampert, 2001; Silver et al., 1995). Ball, et al. (2008), for example, noted that their MKT framework needed more work to understand its cultural specificity. Additionally, researchers from another well-known cognitivist research project, the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR), affirmed that students’ cultural heritages and diverse ways of thinking must be acknowledged and used to support equitable
learning efforts (Silver, et al., 1995). Likewise, sociocultural math education researchers have consistently highlighted the importance of providing a strong mathematical foundation in equitable teaching (e.g. Aguirre & Zavala, 2013; Bonner, 2014; Bonner & Thomasenia, 2012; Gutstein, 2003; Hernandez et al., 2013; Ladson-Billings, 1995a; Moses & Cobb, 2001).

Yet, research produced from both perspectives has yielded little understanding of exactly how to incorporate in practice the central tenet of their counterpart. Research aligned with cognitive perspectives has mostly highlighted math conceptual development. The quality education framework, for example, constraints its definition of a “qualified mathematics teacher” to the teacher’s funds of MKT. In fact, a critical synthesis of the literature on equitable teaching revealed little empirical evidence on how teachers’ mathematical knowledge can support equitable teaching. We do know, however, that all teachers need to be competent in the subject they teach and how to teach it (Ball et al., 2008; Ma, 2010). But cognitive research has not provided descriptive depictions of exactly how to use student specific differences such as race and culture in the day-to-day mathematical conceptual development that takes place through instruction. Research aligned with sociocultural perspectives, on the other hand, is motivated by the belief that the key to combat current inequalities is to recognize and highlight students’ differences in learning (e.g. Aguirre, Mayfield-Ingram, & Martin, 2013; Bonner & Thomasenia, 2012). These differences are the result of students’ diverse experiences and social interactions inside and outside the classroom (Civil, 2014; K. Gutiérrez, 2008; R. Gutiérrez, 2013). Although sociocultural research has stressed the criticality of mathematical learning (e.g. Ladson-Billings, 1995b), it has not provided descriptive depictions of how to advance students’ mathematical conceptual development.
Thus, via epistemological differences, scholarship in equitable mathematics teaching has advanced through seemingly parallel paths. This study did not propose adopting either perspective of learning, rather, it proposed adopting the teachers’ perspective about their students’ learning, through their practice on what they have paid attention to. Some of these aspects about the student, based on the teachers’ perspective, might reflect alignment with either perspective of learning. In Chapter 3, I enumerate potential aspects about the student (i.e., forms of knowledge) that I started the research process with, recognizing that the process itself would generate new aspects about the student or possibly confirm some from scholarship. Due to the theoretical nature of these potential aspects about the student that I started with, they reflect alignment with sociocultural or cognitive perspectives of learning.

In summary, I have highlighted in this chapter core characteristics of the equitable math teacher. Using a situated practice lens, I have describe the relationship between teachers’ core equitable teaching practices and their context of instruction. I have also noted how scholarship’s use of distinct perspectives on learning have advanced, but also limited our understanding of effective teaching. In an effort to advance the field through a more holistic understanding of the student, I proposed that we study equitable teaching as a situated practice, using the teacher’s perspective on what he or she has come to know about his or her students to advance students’ learning. I now end this chapter with a theoretical framework that is informed by my review of the literature.

**Theoretical Framework**

Conceptualizing that teaching is a situated practice is not new. R. Gutiérrez (2002) stated that “just as teaching is situational, so, too, is teacher ability” (p. 172). She posited that a teacher that reflected great ability in helping students in a given context would not necessarily be as
effective if asked to teach in a different context. She used this position to argue that in
discussions of equity, when we ask ourselves what are good teaching practices, we must also ask
ourselves “for whom the mathematics is effective and under what conditions” (p. 172). Thus, the
practice of the teacher, as described by R. Gutiérrez (2002) in the quote above, is not a list of
characteristics in isolation from the context where the practice is enacted. Rather, it is a co-
constructed phenomenon from the interactions between the mathematics, the teacher, the
students and the context. R. Gutiérrez’s (2002) position on the practice of the equitable teacher
as the key for an equity agenda, is well aligned with the goals of this study. Although she did not
explicitly propose specific theoretical frameworks to support her equity agenda, the focus on
teachers’ behavioral decisions and daily activity in context are best captured through a situated
practice theoretical framework.

In the sections that follow, I further expound upon the notion of situated practice. It is
the main theoretical framework that underpins the problem statement, research questions and
design of this study. I also present critical race theory as a secondary, but necessary framework
for this study.

**Situated practice.** Lave’s anthropological approaches to study learning have advanced
the field of education in many ways. She has used settings outside the institution of schooling to
study how and what can be learned through the process of activity. This approach has brought
attention to the learning that takes place in context, dissociated from other more formal and
traditional forms of learning that are found in schools. Although Lave’s contributions are many,
this proposed study is particularly guided by Lave’s (1991) situated social practice framework.
Lave (1991) asserted that “this theoretical view emphasizes the relational interdependency of
agent and world, activity, meaning, cognition, learning, and knowing” (p. 67). More
specifically, what separates situated social practice from other points of view, is that it “claims that learning, thinking, and knowing are relations among people engaged in activity in, with, and arising from the socially and culturally structured world” (Lave, 1991, p. 67). This perspective helps us understand the practice of equitable teachers in context, highlighting the relational interdependency between all factors that help define students’ mathematical learning experiences (e.g. teachers, students, curriculum, beliefs, physical environment, etc.). If we position the teacher as the implementer of this practice, we must then study the teacher in activity, with and arising from the socially and culturally structured world of the teacher.

Lave (2012) recently called for a critical stance on research. She argued that the field must develop “new research that asks what the processes are by which persons are produced and produce themselves in historical and political terms” (p. 169). This proposal responds to this call by paying particular attention to sociopolitical issues such as hegemony and opportunities to learn through the equitable teacher’s practice. Because of this, I consider critical race theory a necessary lens that helps us address said sociopolitical issues that are impossible to ignore in discussions of equity for students from underserved populations (R. Gutiérrez, 2013). Lave’s (2012) call reflects an alignment with critical race theory, which is a secondary, but necessary theoretical framework employed in this study.

**Critical Race Theory.** Darling-Hammond (2007) asserted that “students will not learn to higher levels unless they experience good teaching, a strong curriculum and adequate resources” (p. 258). In doing so, she brought to light ever present educational disparities from the No Child Left Behind (NCLB) Act of 2001. The NCLB act was purposely designed with the goal to increase educational outcomes for all children by mandating higher achievement outcomes from schools. While in concept, this law would have helped reduce inequalities in
educational systems, there were unintended consequences such as the lowering of educational standards, and penalties on schools that had actually achieved improvements for students from underserved populations (Darling-Hammond, 2007). There was a particular reduction in the quality of education available in schools for low SES and students of color (Darling-Hammond, 2007). Although there was evidence suggesting that under NCLB the White-Latino achievement gap had been slightly reduced, there was also evidence that minority concentrations in schools increased, affecting negatively the Black-White achievement gap (Hanushek & Raymond, 2005). This particular episode in the history of the U.S. educational system, is reflective of the principles that underpin critical race theory (CRT). CRT posits that racism is endemic. As a framework, it uncovers issues of hegemony and seeks the elimination of racial oppression as well as all other forms of oppression (Dixson & Rousseau, 2005).

Oppression, however, can take place in covert ways in the educational system (Dixson & Rousseau, 2005). When we assume a neutral stance that ignores the realities of marginalized students, we ignore our responsibility to address the already existing inequalities in the nation’s school systems. Using a CRT lens, we can assert that additional oppression took place through the NCLB act by instituting the same accountability guidelines for everyone. The same guidelines were instituted in spite of the fact that higher spending schools were better positioned to meet the requirements of the law with spending ratios that were at least two or three times greater than lower spending schools (Darling-Hammond, 2007).

Though not much work has made use of CRT in mathematics education research (R. Gutiérrez, 2013), the extant work does help explain covert and overt disparities in mathematics education. Tate (1995) argued, for example, that while African American students’ experiences in the mathematics classroom reflect inequities in the availability of technology resources and
teacher qualifications, there are more nuanced inequities by virtue of teachers’ choices in traditional pedagogical practices that are not in alignment with African American students’ experiences. Tate brought to light issues of racism in the mathematics classroom. Martin (2013), on the other hand, used a CRT lens to illustrate a more endemic racism phenomenon that encompassed the overall math education field (e.g. practicing educators, national organizations and scholarship). Martin (2013) urged the field to consider “what kind of project is math education?” (p.328) and “whose interests are served by the project?” (p. 328). These are pivotal questions that prompt us to reflect on whose realities have been tended through mathematics education. As we advocate for the advancement of all, we must pay attention to students' race and lived experiences. Otherwise, we exert a covert form of expression through exclusion.

Particularly for researchers, Martin (2013) asked, “how do we continue to make sense of the highly racialized nature of mathematics education, both as a knowledge-producing domain and as an activity experienced by teachers and students on a daily basis?” (p. 328). This is a central question that points to a prevalent gap in the field of research in math education and that is brought to light through a CRT lens. The study of the practice of the equitable teacher must tend to the activity that is experienced by teachers and students on a daily basis and that also helps co-construct the teacher’s practice within its context of instruction.

**Integrated framework.** CRT and situated practice - with a critical stance (Lave, 2012) - are compatible. Both frameworks fundamentally call for careful attention to context and collective processes. CRT views conceptions of neutrality as detrimental to learning experiences. Through a situated practice framework, these conceptions of neutrality can ignore the sociopolitical context of the teacher’s practice in classrooms with students from underserved populations. Due to their compatibility, together, situated practice and CRT are able to provide a consistent lens
for studying the practice of the equitable math teacher. These two frameworks are also able to particularly buttress the problem in the field of math education that this study proposes to address.
CHAPTER 3

Methods

This study sought to understand how practicing algebra teachers use their knowledge of their students from underserved populations to advance students’ mathematical learning by documenting teachers’ perceptions. The nature of the research questions called for interpretive inquiry. In particular, this study employed complementary case study and ethnographic field methods within teachers’ contexts of practice (Yin, 2014). Ethnographic field methods were used to document teachers’ practice, which is contextualized and situated. Case study research methods – which often draw on ethnographic methods – were used to explore teachers’ perceptions and to interpret their practice and perceptions (Yin, 2014). The sections that follow delineate the specific methods that were used.

Participants and Schools

I recruited and selected participants using a purposive sampling (Merriam, 1998) process. My goal was to recruit between two to four participants. Potential participants were selected from an initial broad pool constituted from all teachers who taught in middle schools, high school or community colleges serving sufficiently culturally and/or SES diverse populations (at least 25% representation from non-white and/or low SES students) within the same state. Selection from this larger pool required that teachers meet three primary criteria: (1) recommendations from relevant sources, (2) teachers’ self-report of their value of knowledge of their students in their teaching practice and evidence suggesting an equity stance that supported equitable teaching values and that demonstrated an awareness of issues in equitable learning

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2 School administrators, community and/or educational organizations that support underserved students, parents and/or students and teacher education programs and professional development organizations
access., and (3) teaching in a math course in the algebra sequence and/or in a unit requiring algebraic applications.

I sought recommendations to help narrow the larger pool of candidates. I asked recommenders to identify math teachers who they considered to be: (1) dedicated to their profession (e.g., R. Gutiérrez, 1999; Planas & Civil, 2009), (2) committed to their students’ learning of math (e.g., Boaler & Staples, 2008; Ladson-Billings, 1995a), and/or (3) known to be highly resourceful in finding ways to help their students learn (e.g., Bonner, 2014). I chose these criteria because they increased the likelihood of finding teachers exerting behaviors associated with the research questions. I deemed a teacher who is dedicated and committed to students’ learning to be more likely to align his or her behaviors to student learning, than a teacher who is not considered dedicated to his or her profession or to students’ learning. I also deemed a resourceful teacher to be more likely to look for different ways to support students’ learning than a teacher who was not considered to be resourceful. I asked recommenders to nominate teachers who had at least two years of teaching experience. Where possible, I sought multiple recommendations for teachers.

I conducted recruitment interviews to confirm the teacher’s perspectives on equitable teaching and their use of knowledge of their students for indicators of equitable teaching perspectives that were aligned with the literature. The interviews followed a protocol that asked teachers to first share a little bit about their teaching background. I then gave teachers four statements associated with the literature on equitable math teaching to respond to (see table 3.1).

The recruitment interviews were semi-formal interviews (Merriam, 2009). Because of this, additional conversation took place after each prompt in order to obtain a more thorough understanding of the teacher’s equity stance and how this stance informed his or her teaching
practices. During these interviews, I asked teachers to choose their own pseudonym, which was used throughout the study.

Table 3.1. Recruitment Interview Statements

<table>
<thead>
<tr>
<th>Question</th>
<th>Expected Outcome</th>
<th>Supporting Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prompt: “I have four statements that I would like to read to you. I will read one at a time. I would like to hear your thoughts on each of them. I may follow up with a few questions to clarify my understanding.”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. The educational system favors some students more than others.</td>
<td>Teacher demonstrates awareness of equity issues in the system</td>
<td>R. Gutiérrez (2013); Martin (2013); Tate (1995); Zevenbergen (2005)</td>
</tr>
<tr>
<td>2. I believe that the way that math has been traditionally taught in schools favors some students over others.</td>
<td>Teacher demonstrates awareness of equity issues that affects mathematics education</td>
<td>Frankenstein (1990; 2014); Ladson-Billings (2014); Martin (2013); Straehler-Pohl &amp; Pais (2014); Tate (1995)</td>
</tr>
<tr>
<td>3. I do not think it is important to change my teaching practices as long as they are effective to most of my students.</td>
<td>Teacher demonstrates equitable values on student potential by describing ways that he/she accommodates ALL students, instead of just most</td>
<td>Boaler (2000; 2002); Bonner (2014); Bonner &amp; Thomasenia (2012)</td>
</tr>
<tr>
<td>4. I believe that when I use what I know about my students’ experiences (inside and/or outside the classroom, I am more effective in helping them learn.</td>
<td>Teacher demonstrates use of knowledge of their students</td>
<td>Aguirre et al. (2013); Aguirre &amp; Zavala (2014); Civil (2014)</td>
</tr>
</tbody>
</table>

I present below two sample responses to the first prompt from the recruitment interview for the purpose of demonstrating how the expected outcomes from the recruitment interview questions were applied (see table 3.2). The first response shown was considered to have a stronger alignment to theory. The second response shown, given by a participant who was not invited to participate, was considered to have a weaker alignment to the literature.

Both responses suggest that teachers’ perceive that schools are striving to meet students’ needs. Dena’s response was judged to have a stronger alignment to theory because it reflected an
awareness of challenges outside the classroom that still have an effect on students’ access to learning in the classroom, particularly lower SES students. Jenn’s response was considered to have a lower alignment; although Jenn’s comments suggest an awareness that different students have different needs, her response lacked an understanding of how students from underserved populations could have inequitable access to learning systemically.

Table 3.2. Sample Recruitment Interview Responses

<table>
<thead>
<tr>
<th>Prompt #1: “The educational system favors some students more than others.”</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Response with Stronger Alignment</strong></td>
<td><strong>Response with Weaker Alignment:</strong></td>
</tr>
<tr>
<td>Participant: DENA (one of the four selected participants)</td>
<td>Participant: Jenn (the fifth participant that was not invited to participate)</td>
</tr>
<tr>
<td>“Absolutely. The way the educational system is designed and the way that the resources are put into it, really favors students who have a lot of support at home or come from a family environment or culture in which they have access to resources and energy to support their learning as opposed to students who maybe come from a family background where education was not valued, or the resources were not there to support them just because of conflicts in regards to what needs to happen for survival. You may have single parent households, or households where the parents have multiple jobs or they may have finished or not finished high school so they do not have the resources to support their children.” (recruitment interview).</td>
<td>“I disagree with that statement. I think that with the type of inclusion that we have in our school systems, every student requires a different type of instruction and a different type of teaching and care. We spend a lot of time across the board challenging those that need it and try to build those that are aren’t…There are different ways to meet everyone’s needs”</td>
</tr>
<tr>
<td><strong>Follow up for clarification:</strong> “Does this have an effect in opportunities to learn in the classroom?”</td>
<td><strong>Follow up for clarification:</strong> “Do you think that everyone’s needs are met efficiently?”</td>
</tr>
<tr>
<td>Dena spoke about the fact that often times, there is help outside of school or other supports offered that students simply cannot partake on because of competing priorities outside the classroom.</td>
<td>“I think that we do the best we can to reach each student. I am sure there are students, well, there are still some that we are trying to figure out what they need, but for the most part, just in school in general we do everything we can to reach each students’ needs” (recruitment interview).</td>
</tr>
</tbody>
</table>

The third and final criterion – that participants were teaching algebra -- was used to maintain a common content background. It also helped provide consistency with the research questions and the perspective that algebra is a gateway to mathematical learning and educational
choice. Together, the three criteria were used to increase the potential incidence of observable teaching behaviors that would help describe what teachers get to know about their students and how they use it to advance their learning of math. Consistent with a situated practice theoretical framework that recognizes the multiplicity of equitable teaching approaches, a single “perfect” candidate was not believed to exist.

Teacher recommendations preceded arrangements for recruitment interviews, but the processes of obtaining informed consent for the in-depth study and obtaining access to sites, dictated the order in which participants were actually confirmed for study participation. Some teachers met all three criteria, but access at their places of work was not granted. In other cases, access to conduct the study at the institution was granted and the teachers met all criteria, but the teachers chose not to participate in the in-depth study. From the original broader pool of potential teachers in institutions meeting the demographic requirements, a total of 28 teachers were recommended. Eleven of these teachers participated in recruitment interviews. Confirmation to participate in the larger in-depth study was given once both, the teacher’s agreement to participate and permission to conduct the study at the site were received.

In the end, four teachers participated in the in-depth study. All four demonstrated a strong alignment with an equity stance. The sampling process yielded a very select group of teachers from which rich and thorough descriptive depictions of their practice were obtained. Description of each participant follows.\(^3\)

**Beth.** Beth is a female teacher, between 25 and 35 years old. She described herself as being half Puerto Rican and half white; she reported a strong association with her Puerto Rican heritage. Beth was nominated by the secondary math supervisor of Sundryville District who

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\(^3\) Names of teachers and students have been replaced by pseudonyms. Fictitious names have also been used for all educational institutions.
emphasized that she was an exemplary teacher. Beth has taught for six years, all at Sundryville High School.

Beth went to high school and college in the same state where she now teaches. She has two Bachelor’s degrees, one in Math and one in Spanish. While in college, she served as a substitute teacher at the high school where she graduated from. Upon graduation from college, she started a doctoral program in math education out of state. She did not enjoy the research aspects of the program and felt that her passion was in teaching. Beth decided not to finish the doctoral program, and used her doctoral credits towards a master’s program in mathematics education. She was accepted into an alternative state certification program for high school math teaching that ran through a summer. She interned at Sundryville High School while she completed her certification requirements and was hired by the school upon completion of the certification program. For three consecutive years, Beth taught at Sundryville High School and moved for the summers back to the university where she started graduate school so she could complete the remaining credits for her master’s degree. She is currently pursuing a second master’s degree in technology for the purpose of advancing professionally. During the study, Beth also taught math at night in an adult education program held at Sundryville High School.

Beth’s use of knowledge of her students, as suggested initially by recruitment interview data, focused on efforts in curriculum design. She explained that her students needed a customized type of curriculum to meet Common Core expectations in Algebra 1 and to develop college and career readiness skills in Algebra 2. In order for her students to successfully learn, Beth believed that she needed to “stick to a standard, but still reach out” and ask herself, “Can I differentiate?” (Beth, recruitment interview). In working with her particular population of students, she had to “set a reachable bar, reflect on best practices” and ask herself, “How do we
help out?” (Beth recruitment interview). Through working with her students, she learned to adjust her frame of mind and not make assumptions about what a student has been taught. Beth noted, “my room became a place to grow and be a better person” (Beth, recruitment interview).

**Shannon.** Shannon is a white female teacher, also between 25 and 35 years old, who teaches at Sundryville High School. As with Beth, she was nominated by the district’s secondary math supervisor. Shannon was the first person that the supervisor recommended because of “all the great things she does to support her English Language Learner students” (Secondary math supervisor, phone conversation). Shannon has taught for five years at Sundryville High School within the school’s Sheltered Language Instruction program for their English Language Learner students.

Shannon went to high school in the same state and college where she now teaches. She interned at a local high school to complete her certification requirements and her bachelor’s degree for secondary math teaching. Upon graduation, she was hired as a long term substitute teacher at the school where she interned to complete her teaching certification requirements. Within that same year, Sundryville High School offered Shannon a full time position in their mainstream classrooms. In her second year of teaching, Shannon shared teaching responsibilities between mainstream and Sheltered Language Instruction (SLI) classrooms. When an opening became available to be a dedicated math teacher in the SLI program, Shannon asked to be re-assigned to it. Since then, the program has continued to grow. After three years, Shannon teaches five math courses which include Math Intervention and Pre-Algebra. The SLI and special education program are the only programs that offer high school students Pre-Algebra in Sundryville High School. All other students, regardless of their performance in middle school, are placed into a ninth grade Algebra 1 course. Shannon reported enjoying working with her
students and wanting to develop herself professionally to better support their needs. At the time of the study, Shannon was starting a master’s degree program in Teaching English to Students of Other Languages (TESOL).

Based on her recruitment interview responses, Shannon reflected a holistic understanding of her students’ educational needs. Shannon shared that the program has to be prepared to receive students regardless of their educational background and that often times, some students have not been in a classroom for long periods of time because of their travel conditions. Shannon said that she believes in one of the school mottos which is “teach to every child, every day” (Shannon, recruitment interview) and that it does not matter what that learning is. It could be social interactions, how to deal with emotions, how to take notes, how to be prepared, just “teach them something” (Shannon, recruitment interview). She has learned from her students’ growth that “if you put enough time and effort, they will achieve their goals” (Shannon, recruitment interview). She has also learned that she has to be always prepared, ready to provide easy modifications and find different ways to explain math to better support her students’ learning. Aside from tests and assessments, she feels that she knows when her students have learned when she sends them off team and the mainstream classroom teachers comment on their great work ethic and respect.

**Beth’s and Shannon’s School.** Beth and Shannon worked in Sundryville High School. Sundryville High School is the only public high school in its district without admission requirements (e.g., lottery). Of its 1,650⁴ students, 41% were Hispanic or Latino, 38% were Black or African American, 16% were White and 5% were from other ethnic backgrounds. Of

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⁴ All figures on demographics have been rounded up to the next significant digit based on how it was reported in publicly available state reports. This was done to protect the confidentiality and the identification of participating schools.
the total enrollment, eight percent of its students were identified as English Language Learners (ELLs). It is also noteworthy that although state demographics reflected lower proportions of ELL students at the high school level, the growth trends as of 2015 reflected its highest rates for ELL students at the high schools (state\textsuperscript{5} department of education profile report, 2015). The school started a SLI program five years ago, which houses instruction in all subjects along with language instruction support. The program is not a bilingual program, and all instruction still takes place in English; that said, it is designed to provide a learning community where teachers of the same students meet almost every day to discuss students’ progress and support their needs.

Fifty six percent of students are eligible for free or reduced meals.

**Eddy.** Eddy is a white male teacher between the ages of 45 and 55. Originally, Eddy’s department chair, Dominic, was in the recruitment pool. He felt he was not in the position to participate, but, he highly recommended Eddy for his commitment to students’ learning. Based on my questions, Dominic thought that Eddy would be an excellent candidate (Dominic, recruitment interview). Eddy has taught at Mixville High School for three years as an Algebra 1 teacher.

Eddy is a graduate of Mixville High School. Teaching was a second career. He was originally a mechanical engineer for twelve years, working in aerospace applications. Upon receiving his certification through an alternative route to certification program offered by the state, Eddy worked as a middle school teacher for two years at a science magnet school, where he taught sixth grade math. Eddy then changed to his current position at Mixville High School. It was more conveniently located, “just miles from my house” (Eddy, recruitment interview). At Mixville High, Eddy has gotten involved with curriculum design, which he described as a

\textsuperscript{5} The specific state was removed as required by the approved IRB protocol.
“variation of the Common Core” (Eddy, recruitment interview). He has also been involved in assessment initiatives to move towards a mastery-based program for the district’s math program. At the time of the study, Eddy was also completing requirements for certification in educational leadership.

During the recruitment interview, Eddy shared his concern for helping his students develop the tools to be successful in school. The system, according to Eddy, can favor some students over others because it is “set up for things to be done in a certain way and some students come in knowing how to play that game already, they know how to do school and are successful” (Eddy, recruitment interview). Eddy considers success in math to be particularly important because it represents a core subject. Since all students start high school with Algebra I in their district, passing his course increases their likelihood of graduating. Eddy stated that if students obtain at least five high school credits in their freshman year, they are 80% more likely to graduate from high school (Eddy, recruitment interview). Based on his experiences, he has learned that in order for students to get to Calculus AP, they need to develop other non-academic skills that are more associated with budgeting their time, learning how to cope with adversity and not being afraid of getting something wrong and then trying again. When I asked Eddy to tell me more about “being afraid to get something wrong,” he went on to explain how he had re-designed his classroom to be a self-paced learning environment where students were required to persist at math problems until they obtained 80% proficiency. He finds himself believing in his students’ abilities more than what they believe in themselves. He does not understand why or when, but that at some point someone had told his students that they were not good in math. Because of this, Eddy uses “Dweck’s mindset of - you do not know it yet, but if you keep working at it, you can do it” (Eddy, recruitment interview).
**Eddy’s School.** Eddy teaches five classes of Algebra 1 at Mixville High School, which is one of two public high schools in his district. There are other free magnet and technical high school options, located in the same district, but they have admission requirements and are managed by other regional and/or state offices. Of Mixville’s approximately 1,000 students, fifty one percent are Hispanic or Latino, thirteen percent Black or African American, thirty one percent White, and five percent were from other ethnic backgrounds. Of the total enrollment, nine percent of its students were identified as English Language Learners (ELLs). Seventy percent of students are eligible for free or reduced meals.

**Dena.** Dena is a white female between the ages of 45 and 55 who teaches full-time at Beacon Community College. I had worked alongside Dena for many years as we were board members of a teaching professional organization. My department chair at the college where I teach recommended Dena. Not only did she meet all three criteria, but my department chair noted that Dena had been influential in helping her reconsider her own perspective on equitable teaching. They had worked together in a state level committee that was charged with designing instructional supports for college students who did not meet college readiness measures. Through this joint work, Dena helped her rethink her idea of what is fair by considering issues of opportunities and access.

Dena has more than 15 years of teaching experience; fourteen have been full-time. She obtained a B.S. in finance with a minor in math in the state that the study took place. Dena started her teaching career overseas in Kenya through the U.S. Peace Corps program as a high school math teacher. She then taught for a year at a private college that specialized in supporting students with dyslexia. Dena left this position to complete a master’s degree program in curriculum and instruction in math education. She also obtained her certification for teaching
high school math. Upon graduation, she returned to the same private college and taught for a second year. After that, Dena moved to the state where this study took place and worked at higher education institutions teaching math part-time. In the meantime, she also completed a doctoral program in Special Education with a focus on adults with learning disabilities and math. She worked as an educational consultant for a few years after obtaining her Ph.D. and she also continued to teach math in community colleges. Dena has been teaching full time math at Beacon Community College for the past nine years.

Recall Dena’s response presented earlier in this chapter in which she says that not all students have the same access to education as a result of low resources and competing priorities to meet basic survival needs. Dena’s responses, overall, reflected a substantial understanding of students, including equity issues associated with educational access for different sub-populations of underserved students, students’ competing responsibilities, mathematical maturity, different learning styles, and their ability to handle personal challenges such as math anxiety. In her responses, she described how she uses this knowledge to modify her lectures, incorporating pedagogical strategies that support universal design (Burgstahler, 2001). These strategies, in her opinion, “provide equal access regardless of learning style” (Dena, recruitment interview). She believes it is important to change and modify instruction to help all students and that in order to do so, “you must reflect as you go through the process of what works” because “what you do in the classroom may have different outcomes on students” (Dena, recruitment interview). I observed Dena in a quantitative analysis course that combined a broad set of mathematical topics.

**Dena’s college.** Beacon Community College is a two-year public college. In the semester that the study was conducted, it serviced approximately 1,700 students of which about
one third maintained full-time enrollment. Based on Beacon’s 2016 report form the Integrated Postsecondary Education System (IPEDS) data (https://nces.ed.gov/ipeds), the college serviced 28% nonwhite students of which 9% were Hispanic or Latino and 9% were Black or African American. Beacon’s high representation in students from low SES was reflected in the 39 percent of students who received Pell Grant (2016 IPEDS data, National Center for Education Statistics).

Access to Sites and Implications on Methods

All procedures required by the Institutional Review Board (IRB) and by each individual educational institution were followed. Although the study participants were all teachers, because I conducted observations, the IRB required additional procedures to obtain student consent (or parental permission in the case of minors) for documenting students’ individual discussions and/or using copies of their written school work. Permission for the use of data varied by classroom and student, limiting the available data, an issue I revisit in the limitations section of this chapter.

Eddy’s classroom posed the most substantial limitations because his instruction was designed to provide one-on-one attention to each student as they all worked with a self-paced computer-based platform. Eddy offered that I work with him in two of his classes to increase my access to data from his students and practice. I warned him that while I appreciated his offer, this also meant that we could be possibly doubling the time spent together outside the classroom to discuss what I had observed. He smiled and said, “Let’s make it a good one” (Eddy, pre-observation interview). Eddy also agreed to use a personal recording device. We followed a working protocol for recordings, where he would announce the student’s name and the lesson number they were discussing as he approached each student. This protocol facilitated my ability
to not only better understand the context of their interactions when I reviewed recordings, but it also helped me parse out any unauthorized data.

The number of permissions received for each participant’s classroom is shown in table 3.3, along with their corresponding classroom size.

Table 3.3. Number of Student Permission Forms by Participant’s Classroom

<table>
<thead>
<tr>
<th>Participant’s Classroom</th>
<th>Number of Classroom Students</th>
<th>Number of Permission Forms Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beth</td>
<td>25</td>
<td>17</td>
</tr>
<tr>
<td>Shannon</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Eddy – Period 6</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>Eddy – Period 7</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>Dena</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

**Instruments and Data**

The unit of analysis was the teacher, who was understood to be situated in nested contexts of classroom, school, and community – that is, “because teachers’ beliefs and knowledge emerge and are grounded in their participation in workplace settings, we must attend to these contexts” (R. Gutiérrez, 2002, p. 171). The primary data sources were: teacher interviews, classroom observations, check-ins with the teachers to confirm understanding of observations and teacher decision-making, ethnographic field notes, audio recordings of lessons, and video recordings of the board and/or of the visuals used for instruction. A wide range of artifacts (e.g., samples of student work and assessments, copies of virtual discussions from mobile applications, handouts, school flyers and informational pamphlets, etc.) were collected and reviewed to support my analysis of teachers’ practice and contexts. Artifacts varied depending on each teachers’ practice and are described in more detail in this chapter by teacher.
Data Collection Goals and Assumptions. I used the data sources delineated in this section, and their eventual analysis, to obtain a holistic and descriptive understanding of teachers’ practice in order to get to capture their decisions, including — from their own perspective and “teaching eyes” — how they used their knowledge and beliefs about students. This goal was made explicit early on in teachers’ involvement in the study. The study description that was disseminated to seek for recommendations and to invite participants stated that “teachers would be joining me in an inquiry process.” I did not construe teachers as passive subjects, but rather, collaborators who would help me assemble accounts of their experiences (Gubrium & Holstein, 2012). This understanding influenced how the data were collected. For example, on many occasions, the teachers themselves chose the examples of student work for me. I also regularly asked them to help me understand their day to day work through examples that they selected. In essence, I saw myself as a participant (Holstein & Gubrium, 1995) in a joint inquiry. As such, I paid close attention not just to what they shared, but also to how and in what context it was being shared with me (Gubrium & Holstein, 2012).

Additionally, while the research questions called for the interpretation of how teachers’ knowledge of students was used to advance students’ learning, I was sensitive to the fact that what students learned was highly dependent on the teacher’s goals. That is, we could say that each teacher wanted their students to learn math, but that learning could be best described as an ultimate goal that was supported by a myriad of behaviors and skills that these teachers wanted to see their students develop. Some behaviors and skills could be strictly mathematical, others were not. These behaviors and skills, were in turn dependent on the teachers’ perceptions and interpretations of what is important to learn, why it is important to learn it and how it is best learned.
These assumptions had implications for data collection, which involved an ongoing in-process analysis to determine if the interpretation of patterns and observations were supported by plausible explanations. Throughout the data collection and analysis process, then, I documented the teachers, students, their interactions and what they interacted around and about. I asked the teachers questions to inform my efforts to interpret what I observed through their teaching perspective. As the process unfolded, my questions became more selective and pointed as I came to develop insight into their perspective. This process of data collection and analysis was like constructing a map by finding, and setting in place, landmarks that I eventually verified in a second stage of data analysis. Thus, while I describe data analysis below, it is best understood as being interdependent and intertwined with data collection. I now briefly describe each data source.

**Teacher Interview: Pre-Observation.** A pre-observation semi-structured interview (Merriam, 2009) was conducted to obtain information about the teachers’ teaching background, teaching style, and use of their knowledge of students. These interviews were designed to last less than one hour (see Appendix A). The questions in the protocol were typically followed by additional prompts to seek clarification based on each teacher’s response. Data from these interviews provided preliminary categories on the types of knowledge of the student that each teacher used to advance students’ learning. I used a start list of codes based on the literature (see Data Analysis) and added new codes to capture additional forms of knowledge that emerged from the interview. The data from this interview also provided preliminary categories on teaching behaviors that were considered to be valued by the teachers. In some cases these desired behaviors were found to be in direct connection to forms of knowledge of the student and sometimes they were not.
The recruitment interviews, in conjunction with the pre-observation interviews provided a preliminary form of focus for future data collection. Because of this, I reviewed the recruitment interviews prior to conducting the pre-observation interviews to look for possible areas that needed clarification as early as possible in the data collection process. While this preliminary focus did not prevent the collection of data that could have seemed unrelated at first, the focus was needed for the purpose of maintaining alignment with the research questions. I best describe this focus as a “pair of glasses” that helped clarify a path that was unfolding as I walked through it.

For example, when Beth reported that it was important for her students to be “college and career ready” (Beth, recruitment interview and pre-observation interview), I sought clarification on what that meant in terms of observable behaviors and skills, why she thought it was important for her students to have them, and what she did to support the development of these skills. From these early interviews, I came to understand that “being college and career ready” – to Beth -- was associated with a combination of math proficiency and non-mathematical academic skills. It then became my goal in the subsequent data collection process to look for patterns and behaviors associated with what that proficiency was, what those skills were, and how they were both associated to Beth’s conception of what it meant to be college and career ready. I also was open to the possibility that other behaviors and/or goals could be eventually found in association to Beth’s conception of college and career readiness. In alignment with the research questions, I also looked for incidence of different forms of knowledge of the student and how she used them to support this goal.

**Classroom observations.** I observed each teacher for the duration of at least one full unit of study, for a minimum of three weeks (see Appendix B for a summary of all data sources
and total observations/interviews). In cases where the teacher was absent, I obtained samples of planned work that would be facilitated by the substitute teacher and I also took pictures of any instructions and/or reminders that the teachers arranged for students to copy.

I positioned myself as an observer-participant (Emerson, Fretz & Shaw, 2011), with limited interaction with students and teachers. In three of the four cases, the teachers had already pre-determined where I would sit. I wanted to reduce the likelihood that my presence would affect the natural day-to-day classroom interactions. In Eddy’s case, I had a choice on location. The room was set up with tables to have students work individually on their portable computers. There was an outer ring of empty chairs positioned along the classroom walls. I chose a chair closer to Eddy’s desk, because it allowed me to maintain oversight over the camera that was positioned to capture the board.

To support documentation of observations, I audio recorded all lessons. I also used a video camera that was set on a small tripod and positioned to capture the board and any visuals the teachers used. In Eddy’s classroom, the use of video did not add new data after the first 15 minutes of class because his lessons were designed to provide a short all-class activity that required the use of the smartboard, but after that, the rest of the period was spent on individual work online while Eddy visited with each student for individual instructional support. Eddy agreed to use a personal voice recorder that he carried on him so we could record his individual discussions and interactions with students. The recorder was the size of a computer flash drive that he hung around his neck.

In Beth’s classroom, she already typically recorded herself as well as her work on the smartboard. This was part of her regular teaching practice because she wanted to make her lessons available through an online drive to students who were absent or in in-school suspension
(ISS), or other in-school support programs. Although I still captured my own audio and visual recordings in her class, her regular practice to record herself provided me with classroom data on the few days that I was not able to be physically present.

I made acclimation visits to three of the four classrooms. The acclimation visits served the purpose of helping me self-assess what I am noticing as a data collection instrument myself, get a sense of the classroom dynamics, sketch seating charts and device a tentative plan to best code students. I used the visit to ensure that I had as much visual access, and to ensure that the video was capturing the board, but still positioned in a way that would not be disruptive. After each acclimation visit, I revisited my jottings and field notes the same day. I re-read the research questions and created a sketch of the classroom along with a preliminary plan on how to code the students. I was not able to perform acclimation visits in Dena’s community college classroom because the institutional permission to conduct the study was received and approved by IRB at around the same time that the unit of study was scheduled to start. Despite this challenge, I was able to collect data on the full unit in Dena’s classroom. Additionally, Dena’s classroom had only eight students and planning needs for observation became minimal. I met Dena before her first observation and we discussed in detail the lesson objectives she had for that first observation, as well as the format she followed for instruction as this was a once-a-week, three hour class. We also walked together to the classroom and spent time testing the recording equipment.

During regular classroom observations, I jotted notes, paying particular attention to “fragments of action and talk to serve as focal points” that would be later on used to write accounts of observed events (Emerson, Fretz & Shaw, 2011, p. 31). I also marked the times from the audio recorder. I focused on teachers’ behaviors associated with using their knowledge of
students. At the end of the day, I would then look over my notes, review any recordings if necessary and write additional comments from observations. On multiple occasions, I generated audio recorded memos to make sense of the data and to plan possible questions for check-ins with the teachers. I also wrote brief memos to make sense of observations, document my thinking and reflect.

**Teacher check-Ins.** One-on-one check-ins were performed with each teacher during the course of the observational period. Although I did offer to meet by phone and/or do check-ins by email, all teachers preferred to meet while at school. Check-ins varied in length because they were done at the teachers’ convenience. Due to their typical daily demands and responsibilities, which were sometimes unpredictable, we often opted to schedule the check-ins in advance. This did not guarantee that we would be able to meet at the accorded time, but it allowed me to plan what to prioritize in terms of clarifications and questions. On many occasions, unforeseen changes in the teachers’ schedules allowed me to join them in their daily work outside the classroom such as offering student support for in-school suspension students (Beth), participating in department meetings (Beth), participating in library duty (Beth), tutoring during prep periods (Shannon and Beth), offering help sessions after school (Eddy), making up tests during office hours (Dena), and tutoring in the college’s help center (Dena). This helped me gain a more grounded perspective of teachers’ involvement with their students outside the classroom, how this involvement informed what they got to know about their students, and how that particular knowledge reflected association to what they do inside and outside the classroom to support students’ learning.

During check-ins, we also discussed their lesson plans, especially if they had made decisions to make changes to accommodate different concerns. For example, Eddy decided to
change his 80% rule on online work proficiency in the last three weeks of class. This had been his guideline all year long. He also changed the schedule on required lessons. We discussed during check-ins why he made these changes, and in what ways the changes were associated to either what he knew about his students or to his overall goals in learning. Beth, on the other hand, made changes to how she graded and provided feedback on one quiz. We discussed why she made this change in association with what she knew about her students and how that change tied to her overall goals for her students’ learning.

While revisiting classroom incidents, observations or any other data, my questions were driven to look for association to what the teachers knew about their students, their overall learning goals and unit specific goals, and where possible, to the behaviors they were expecting of students.

**Teacher interviews: post-observation.** A final semi-structured interview (Merriam, 2009), after all observations and preliminary analysis, was conducted to confirm or challenge observed patterns and to triangulate data. These were prolonged interviews (Yin, 2014), ranging from 2.8 hours to 3.8 hours (see Appendix B for interview duration by teacher). Three of the four interviews were conducted over the course of multiple days. The post-observation interview was used to conduct any member checks that were not completed through check-ins. The main objective of this interview, however, was to test and confirm patterns that surfaced outside the teachers’ self-report of their use of their knowledge of the student (see Appendix C for illustrative questions and associated prompts). These questions inherently varied by teacher, situated in their contexts of practice and on their particular course for data collection and preliminary analysis.
Artifacts. Other relevant sources of data were collected for the purpose of understanding the classroom, school, and community contexts. These artifacts also varied by teacher (see Appendix B). For example, in Eddy’s case it was important for me to review the lessons that students were completing online. Since I did not have administrative access to the students’ online platform, I created a data bank with all videos using the publicly available online program where Eddy created links from. I then asked Eddy to review my data bank to ensure that I had accurately collected all videos. I also took pictures of the students’ lesson completion sheets which they used to list the lessons they had completed by date. This gave me an understanding of students’ individual pace in relation to the expected overall lesson coverage for the semester.

Samples of student work were collected for all teachers. While the particular samples shared by teachers varied by classroom, the purpose was the same. By reviewing and discussing with teachers their students’ work, I was able to capture their perspective on student progress, including their interpretation of what and how students were learning, as well as what they were able to know about their students through classwork and performance assessments.

Data Analysis

Data analysis employed case study research methods (Yin, 2014) to provide empirical evidence supporting the teachers’ use of knowledge of their students to advance their mathematical learning. As noted in the previous section, preliminary data analysis took place and also informed data collection. Later analyses used within-case and cross-case study methods to evidence and support the underlying “deep structure” of the phenomenon for an explanatory framework (Miles & Huberman, 1994). The subsections that follow describe the data analysis methods in the order in which they were introduced, with the understanding that the overall process was iterative in nature.
**Memos.** Memos were used throughout the research process. For example, I wrote brief memos to aid my thinking and document my decision-making in the recruitment process. I wrote memos to reflect on patterns that I observed within a particular teacher’s practice or across teachers’ practices. I revisited particular classroom incidents and/or my understanding of how different forms of knowledge were found through memos. I also wrote memos to confirm my coding scheme and overall analysis methods (Miles & Huberman, 1994).

**Data coding through data collection.** Data coding and its corresponding analytic processes were applied throughout data collection. Although a combination of coding schemes were used for data analysis, I started coding using forms of knowledge of the student. This was a form of selective coding (i.e., selecting one core variable to delimit the data (Glaser, 1978)). I chose to start with this approach to ensure early alignment with the research questions and their focus on the teachers’ use of knowledge of the student.

I used a *start list* (Miles & Huberman, 1994) of theoretical codes for forms of knowledge of the student (see table 3.4). Descriptions and examples for these codes are also included in Appendix D. Coding and analysis prior to observations (i.e., recruitment and pre-observation interviews) helped provide a preliminary focus on the forms of knowledge of the student that could potentially evidence use through teachers’ practice. I created new categories as I found new forms of knowledge that did not match any from the start list. In cases where the teacher used a unique or characteristic way of describing a student trait, I used in vivo coding (Saldaña, 2009) to identify instances where the teacher had used the same actual words to describe incidents. For example, Beth used the term “college and career ready” on multiple instances. Sometimes she used it in association with mathematical forms of knowledge and sometimes she used it in association with non-mathematical forms of knowledge. By using in vivo coding, I
was not only able to pair the different dimensions of what this trait meant to Beth, but I was also able to align our discussions and common understanding through her frame of reference (Briggs, 1986).

Table 3.4. Initial List of Codes from Theory

<table>
<thead>
<tr>
<th>Categories</th>
<th>Codes</th>
<th>Supporting Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student Mathematical Knowledge</strong></td>
<td>MK</td>
<td></td>
</tr>
<tr>
<td>MK: Typical Errors (e.g., MKT)</td>
<td>MK- ERR</td>
<td>Ball et al. (2007)</td>
</tr>
<tr>
<td>MK: Foundational Gaps</td>
<td>MK – GAP</td>
<td>Aguirre et al. (2013)</td>
</tr>
<tr>
<td>MK: Strength Areas</td>
<td>MK- STR</td>
<td>Aguirre et al. (2013)</td>
</tr>
<tr>
<td>MK: Alternative Approaches/Thinking</td>
<td>MK – Alt</td>
<td>Ball et al. (2007)</td>
</tr>
<tr>
<td>MK: Interests</td>
<td>MK – INT</td>
<td>Aguirre et al. (2013)</td>
</tr>
<tr>
<td>MK: Language of Math</td>
<td>MK- LANG</td>
<td>Aguirre &amp; Zavala (2014); CCSSM (2010)</td>
</tr>
<tr>
<td>MK: Mathematical forms and structure</td>
<td>MK – FRM</td>
<td>CCSSM (2010)</td>
</tr>
<tr>
<td>MK: Unknown</td>
<td>MK – UKN</td>
<td></td>
</tr>
<tr>
<td><strong>Student Non-Mathematical Knowledge</strong></td>
<td>NON</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gonzales, Andrade, Civil &amp; Moll (2001)</td>
</tr>
<tr>
<td>NON: Language only</td>
<td>NON – LANG</td>
<td>Aguirre &amp; Zavala (2014); Celedón-Pattichis &amp; Ramirez (2012); Moschkovich (2002)</td>
</tr>
<tr>
<td>NON: Learning Attitude</td>
<td>NON- LAT</td>
<td>R. Gutiérrez (1999); Hand (2010)</td>
</tr>
<tr>
<td>NON: Interests outside classroom</td>
<td>NON – INT</td>
<td>Aguirre et al. 2013</td>
</tr>
<tr>
<td>NON: Community based</td>
<td>NON- COMM</td>
<td>Moses &amp; Cobb (2001)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Aguirre et al. 2012</td>
</tr>
<tr>
<td>NON: SES (from living conditions)</td>
<td>NON – SES</td>
<td>R. Gutiérrez (1999)</td>
</tr>
<tr>
<td>NON: Unknown</td>
<td>NON- SES</td>
<td></td>
</tr>
<tr>
<td><strong>ALL CODES ABOVE MAY APPLY TO EITHER COLLECTIVE OR INDIVIDUAL FORMS OF KNOWLEDGE</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An initial review of the data after coding the recruitment and the pre-observation interviews reflected that the forms of knowledge of the student were often times appearing in
association to a teacher behavior. This prompted me to adopt a second set of codes for the teachers’ behaviors. This approach also maintained alignment with the research questions because their focus was not just on finding evidence of the teachers’ knowledge of the student, but also on how they used this knowledge to advance students’ learning. I used these two coding schemes across the data collection process through simultaneous coding (Saldaña, 2009). Figure 3.1 shows a simple schematic of an example demonstrating how simultaneous coding led to the pairing of teacher behaviors to what the teachers knew about their students.

![Figure 3.1. Schematic Demonstrating Pairing of Codes Through Simultaneous Coding](image)

The simultaneous coding approach on knowledge of the student and teachers’ behaviors helped guide my questions in check-ins to understand how both behaviors and knowledge were situated within each teacher’s learning goals. These learning goals were sometimes holistic (e.g., Beth’s goal of developing overall college and career readiness) and sometimes unit specific (e.g., Beth’s decision to teach simplification of radical expressions by first re-writing them with fractional exponents). Keeping in mind that teaching and learning is an interactional process (Lampert, 2001), discussions of teachers’ learning goals included discussions of patterns of observed student behaviors. This was important because it helped me look for evidence of student behaviors that either supported or that did not support the teachers’ self-reported learning
goals. During both check-ins and post-observation interview, we discussed these behaviors as a means to further understand the teachers’ perspective of students’ learning experiences.

Open coding (i.e., a first cycle open-ended approach to make sense of the data) (Saldaña, 2009) was applied to the overall data set to look for additional themes that surfaced throughout data collection. Two main categories were found and used to further understand the teachers’ practice: factors that helped support teachers’ practice and factors that challenged teachers’ practice.

**Constant Comparative Methods (CCM).** The use of knowledge of the student to advance students’ learning was confirmed through constant comparative methods (Strauss & Corbin, 1998) and triangulation of data (Merriam 1998) around three main domain areas: the teachers’ knowledge of their students, behaviors, and goals for learning, which included expected student behaviors reflecting learning. Figure 2 represents the three domain areas with arrows to reflect the ongoing cyclical approach in making sense of the data. The use of CCM helped ensure that the knowledge of the student identified empirically-found patterns of association to learning goals and to their use.

Figure 3.2. Three Main Domain Areas Used Through CCM
Forms of knowledge that did not show association to learning goals or use through the teachers’ practice were then discarded from further analysis. This process of elimination shed light on the most prevalent forms of knowledge in use for each teacher’s practice and further refined understanding of the interplay between the three domains for each teacher.

Consider an example. Beth took time on multiple occasions to have with the whole class what she called “genuine conversations” (Beth, observational period) about their level of preparation for class. Using this instance alone without CCM would have yielded a superficial understanding that these conversations (the teacher’s behavior) were aligned with a learning goal to be prepared for class, and a knowledge of students’ preparation level. I refer to this understanding as superficial, because it is not contextualized on Beth’s particular goals, what she has come to understand about her students and why she chooses to have these conversations. By comparing other instances where Beth addressed preparedness, along with her description of why she chooses to have these conversations, we get to understand that Beth was trying to address a dissonance in culture between the students and the school (coded as NON-DISS) and that her students have other competing priorities outside the classroom (coded as NON-PRIOR). She wants students to come prepared because, otherwise, the time she gets to work with them is reduced, if not gone, because anyone who did not watch the video at home in her flipped classroom would have to sit through the video in class while the rest that prepared “gets to work with her” (Beth, check-ins and post-observation interview). CCM along with triangulation helped provide a grounded and more descriptive understanding that was based on multiple instances and discussions with Beth about her goals, what she does, and why. This understanding, however, did not provide yet, full insight to the deeper structure underpinning Beth’s practice and use of knowledge of her students without the application of within-case
analysis methods (Miles & Huberman, 2014). Plausible explanations, however, were tested through the post-observation interviews.

**Within-case analyses.** After the observational period, I performed within-case analyses on each teacher’s practice using an explanation-building case study analysis approach (Yin, 2014). At this stage, I revisited the coding scheme I used while collecting data. I revisited classroom incidents and interview segments. I used pattern codes (Miles & Huberman, 1994) to help explain the teachers’ strongest and most characteristic behavioral patterns that exhibited association with forms of knowledge of the student and learning goals. Pattern codes were used to “re-package and aggregate the data” (Miles & Huberman, 1994, p. 92). Following the example on conversations above, I generated a preliminary list of explanations. I show two sample pattern codes that were used to deepen understanding at this stage in Beth’s within case analysis:

- **CONVNON:** Conversations used to address non-mathematical learning goals for students that still affected mathematical learning.

- **CONVMK:** Conversations used to address mathematical learning goals for students

Since these explanations had been found to be in direct association to learning goal(s), I referred to these as “teacher interventions” directly associated with teacher knowledge of the student. Additional constant comparative methods were applied at this stage with a particular goal of finding patterns and underlying trends within these interventions. In the case of Beth, for example, this analysis revealed dual roles for interventions that merged mathematical and non-mathematical goals for her students. A preliminary explanation in Beth’s case was that both dimensions (mathematical and non-mathematical) were equally needed to advance students’ learning of math, and that by addressing both of them through her interventions, Beth was
facilitating the mutual strengthening of each. This explanation was found to be only partially correct at the second stage of the within case analysis. Findings from Beth’s case, are described in more detail in Chapter 4.

The second stage of analysis involved generating an illustrative model for each teacher’s case. The models were generated to capture the deeper structure and phenomenon behind each teacher’s use of knowledge of their students to advance their learning. These models were developed iteratively, revisiting each model after generating a new model. I started with a model for Beth’s practice. This provided a preliminary model that would undergo changes after subsequently working with my second case. I chose Eddy as my second case because he seemed to be the most different case in relation to Beth’s. This was a purposeful choice to challenge my thinking and test my plausible explanations for each case.

As I revisited an earlier model, I looked for gaps by “walking each student” through their corresponding teacher’s model. That is, I tested classroom incidents and teachers’ accounts of their work with their students to see if they could be fully explained by the individual models. Any disconnects found between accounts and their model was considered an indication that the model was incomplete. The ultimate goal was to achieve full saturation of data.

After confirming full saturation for each illustrative model, analytic memos were written to describe and explain the use of knowledge of the student by each teacher. These memos represented a summative stage in the analytic process that was grounded in the underlying deeper structure of the phenomenon that was empirically found for each teacher case.

**Cross-case analysis.** A cross-case synthesis (Yin, 2014) was performed to look for similar and/or contrasting profiles among cases. I focused on the following profile areas:
• Comparisons of forms of knowledge of the student. This included forms that matched the same code descriptions as well as forms that were completely different across cases. For example, I looked across cases for how the form of knowledge on non-mathematical learning attitudes (i.e., NON-LAT code) appeared in each of the teachers’ practice. I looked for similarities and differences in the teachers’ descriptions of what they had observed in their students, their self-reports on behaviors that indicated the use of that particular form of knowledge and what I had myself observed in their classrooms associated with it.

• Comparisons between factors that supported or challenged the teachers’ practice and their ability to use their knowledge of their students. These factors had been found through open coding in the data collection process.

• Comparisons of pattern codes from the within case analyses for each teacher. While some of these comparisons were done through comparisons of how teachers used forms of knowledge of the student, not all pattern codes were necessarily associated to the same form of knowledge of the student across cases.

• Comparisons of the teachers’ learning goals holistically (i.e., what they consider is important to learn in math) and for the particular unit of study through the observational period. Particular attention was given to the math specific content areas that teachers prioritized through either self-reports, or as reflected in their teaching practices in the observational period. Although not intentionally sought after through the recruitment process, the courses taught by the participating teachers provided representation in the sequential progression of the teaching of algebra. Two teachers taught Algebra 1 in two different districts. One teacher taught Algebra 2 and one teacher taught a college level
math course that had algebra as a pre-requisite for enrollment. This provided and additional dimension for comparisons on content across cases. For example, based on what teachers have come to learn about their students, what particular content areas and/or learning goals do these teachers consider to be essential for their students’ long-term success in future math courses and why?

A cross-case illustrative model was constructed to analyze the deeper structure explaining the underlying phenomenon in the use of knowledge of the student for all cases. This model was iteratively generated as the within-case teachers’ models were also being iteratively generated. Work on the individual models prompted consideration of gaps in the cross-case model and vice versa. All models functioned as analytical tools used to synthesize understanding from all incidents within the data set and to test explanations within cases and across cases. At the same time, the models captured and illustrated findings stemming from the use of case study methods. A description of each model is presented in Chapter 4.

As a final stage in the cross-case analysis, analytic memos were written to describe the overall phenomenon supported by the model. These memos focused on the overall explanatory phenomenon that was empirically supported across cases.

Limitations in Data Collection

There are acknowledged limitations in this study. Some are associated with obtaining full permission for the use of data for all students in the teachers’ classrooms. The target number of teacher cases was reached for this study, but the need to obtain signed parental permission and/or student consent to transcribe and share data associated with individual students limited my access to all collectable data for each individual student. This limitation only affected the use of individual data (e.g., transcriptions from classroom discourse, samples of student work)
for the students that did not provide signed permissions in two of the four participating classrooms (Eddy’s and Beth’s classroom) (see table 3 for information on number of students and number of responses received).

Eddy foresaw the challenge of receiving signed permissions back from all students when we discussed the study procedures. Because of this, he offered that we work on two of his classrooms instead of just one. With this accommodation in place, the total number of signed permissions from Eddy’s practice surpassed the number of students in any of his individual classrooms. In Beth’s case, about one third of the permissions were not received. After reviewing all data, I did not need to discard any of the individual data because signed permissions were received for all students who we spoke about and/or that required transcriptions. Some of these missing permissions belonged to students absent from class due to different types of programs (e.g., disciplinary suspensions, illnesses, and/or alternative settings within school for students with needs). The data set without contributions from these students was considered to be comprehensive enough to capture a descriptive depiction of each teacher’s practice.

From a methodological standpoint, it can be argued that the use of case study research methods may pose concerns for the generalizability of findings. These particular methods were chosen for the purpose of obtaining in-depth descriptions and a more holistic understanding of the use of knowledge of the student by practicing math teachers in grade levels 9 to 14 with a stance on equity. A potential limitation lies within the same advantage of using these methods. While participants shared very particular characteristics that have allowed for a more grounded in depth understanding of their situated work, the number of participants could be construed as small in relation to the larger population of teachers with similar traits (e.g., equity stance) and
contextual reference (e.g., schools with higher representation of students from underserved populations). It can also be argued however, that the use of multiple cases strengthened the validity and reliability of the study because it “captures the holistic essence of the subject studied” (Noor, 2008, p. 1604). Findings from each case were based on a replication approach (Yin, 2014), where a plausible explanation was tested and confirmed across cases. This effort to seek consistency across cases rendered more robust findings (Yin, 2014). This was so much so, that by the time I tested the data in the final fourth case, the cross-case model was saturated. I was able to describe all data for the final fourth case with the model created iteratively with the first three cases.

Finally, my role as a researcher and as a data collection instrument myself could be considered a limitation as a source of bias. No research study can ever be free of bias, but that it is important to reflect on one’s positionality and to disclose the perspectives and assumptions that we carry with us into the research process. I have described the assumptions that have informed my methods earlier in this chapter. I also include my positionality statement at the chapter’s conclusion.

**Validity and Reliability**

Construct validity was addressed in this study by employing multiple sources of evidence (Yin, 2014) and triangulating convergent data. I worked with a peer debriefer (Creswell & Miller, 2000), who was a doctoral student in curriculum and instruction in math education, as well as an experienced teacher and teacher educator, to review alignment to the research questions throughout the data collection process. I also conducted a focus group with students from a doctoral level course in research in math education to evaluate my start list of codes. Most of these doctoral students also had multiple years of experience in teaching mathematics.
Pattern matching and explanation building techniques helped safeguard the internal validity of the study. I also embedded member checks with teachers throughout the data collection process. I explained to each teacher at the study’s onset that often times I would ask questions that may seem as if I was repeating information they had communicated before, but that the purpose was to check for my understanding and to ensure that I had captured accurately their communications. Member checks were typically preempted in our discussions by the phrase – “and as a check…”.

Finally, reliability was addressed by creating a case study database (Yin, 2014). The case study database served as a central repository for all collected data. It preserved the data while also maintaining it in retrievable form (Yin, 2014).

Positionality

Research is never neutral (D’Ambrosio, et al., 2013). In light of this assertion, I write this statement to disclose different perspectives that I have acquired through multiple personal and professional experiences and how I have taken them into consideration in my methods.

My Perspective on Learning

I am a socioculturalist. Unfortunately, the field of math education has scarce work on cognition aligned with sociocultural epistemologies. Cobb (1994) noted that “mathematical learning should be viewed as both a process of active individual construction and a process of enculturation into the mathematical practices of wider society” (p. 13). I agree. I disagree, however, with the notion that individual construction can only be addressed through cognitivist perspectives. Vygotsky’s (1978) work supports this individual construction. His main difference with a Piagetian perspective is that Vygotsky ascribed purpose to individual activity (such as internal speech) on developing individual learning. This individual activity hardly ever
takes place in isolation from the environment; it is often times incited by some form of external environmental stimulus (Vygotsky, 1978).

I use cognitive frameworks such as MKT, understanding its delineation of knowledge as a best attempt to categorize a broad set of cognitive tools that are tied to a given context of instruction. I think that work accomplished through cognitive perspectives is very important, but I also think that issues of equity in math education necessitate attention to students’ different experiences. Through these different experiences, students have come to develop their individual thinking and their understanding of math. Despite my siding on sociocultural perspectives, I made a purposeful effort to incorporate a wide array of forms of knowledge of the student in the start list based on research in equitable math education that was influenced by both perspectives of learning, cognitivist and sociocultural. I wanted to start with as much representation as possible from different theoretical perspectives, recognizing also that new forms could emerge through the research process.

**Personal and Professional Experiences**

I was born and raised in Puerto Rico. My parents, as first generation college students, valued education highly. My mom, in particular, was a teacher. I originally came to the United States only to pursue my college education in chemical engineering, but I chose to stay in the states to work in industry. I considered teaching at that time, a retirement dream. I chose not wait on that dream when the president of my company asked me to join a “fast-track” program for young executives. His offer prompted me to reflect on what and why I worked. I realized that my personal gains were not making the difference I actually yearned to make for the greater good. Within a week, a private high school hired me without certification three days before the year started. Education is important to me because it gave me the power of choice. It allowed
me to explore my interests because I had a preparation that did not limit my options. I taught high school math for six years and I am now in my twelfth year of teaching at a community college.

The community college where I teach is located in an urban city that is also a Hispanic Serving Institution (HSI). I have taught all levels of math in the algebra and calculus progressions, including developmental\(^6\) math courses. I have consistently experienced the over-representation of ethnic minorities (e.g., Hispanic, Black), low SES and first-generation college students in my developmental courses (Mesa, Wladis & Watkins, 2014). I echo Moses and Cobb (2001) as well as Schoenfeld (2002), by affirming that having students without the preparation and/or access to algebra is a civil rights issue. My perspective is slightly different from Moses and Cobb’s (2001) in that I do not necessarily advocate that all students must learn algebra strictly because it will give them greater economic mobility in a technological society. I think this type of access is extremely important, but I also think that even more important is having a strong preparation in mathematics so students can have choice in their educational pursuits.

As a developmental math instructor, I can attest to students’ challenges with low performance and low graduation rates (Bailey, Jeong, & Cho 2010). We make empty promises when we tell students that higher education will grant them social mobility and will enable them to make greater civic contributions, while we also know that only those with strong mathematics foundations will reap these benefits. Proper preparation early on empowers students with choice.

Because of my professional experiences, I occupy what Corbin and Buckle (2009) have described as the *insider-outsider* space. In terms of positionality, it is a space on its own (Corbin

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\(^6\) Non-credit remedial math courses required for college students whose entrance exam placement scores demonstrate low foundations to engage in collegiate level math courses
& Buckle, 2009), because I sometimes held an insider perspective (i.e., a math teacher working with highly diverse populations with a stance on equity). At the same time, I sometimes also held a researcher perspective (i.e., strongly influenced by notions of situated practice), feeling compelled to capture my participants’ viewpoints. I capitalized on my insider role when I described the study to potential participants. I invited them to join me in an inquiry process to help depict the realities of their practice, while I also explained who I was as a graduate student and as teacher. My choice was purposeful. I wanted them to know that the invitation was coming from a colleague with an interest in expanding efforts on equity for the benefit of other teachers and their students.

But insiders do not necessarily hold the same perspectives, opinions, or in this case, the same teaching goals for their students. I admit sensing an initial discomfort when my participants’ goals were very different from mine. Eddy’s goals, for example, had a strong alignment with students’ need to pass his course for the purpose of meeting graduation requirements. This goal was very different from mine. In this awareness, I made a conscious effort to make sure I captured my participants’ voice as best as possible. Member checking was an essential tool for me, where I would read to my participants what I understood about their practice to make sure I had captured it correctly. The researcher in me would jump in to seek understanding of the situated nature of my participants’ decision-making. In a sense, my discomfort became an internal cue to search for additional understanding. It is possible that because of my positionality, both Eddy’s and Dena’s post-observation interviews were the longest. In the case of Eddy, it was because of our differing teaching goals. In the case of Dena, it was because of the familiarity in our teaching contexts as community college instructors. I admit that with Dena, I may have overcompensated through my questioning approach - looking
for a more explicit self-report from her. I wanted to avoid making assumptions based on my personal experiences.

I hold the deepest sense of gratitude to my participants because they willingly and selflessly opened their classrooms doors to me and to this research. They joined an inquiry process for the improvement of all. I kept their altruism in mind as I worked to best capture and describe their daily practice. I owed them that.
CHAPTER 4

CASE FINDINGS

This Chapter reports the findings for each of the four cases studied. Chapter 5 is dedicated to the cross-case analysis that was conducted on all four cases. The section that follows describes the participants and the format used to report the case findings. I have dedicated a separate section for each case within this chapter. I begin the cases with what I consider the most complex and detailed case, Beth.

**Within Case Analysis: Participants**

There were four cases corresponding to each teacher participant. I list the four participant teachers in table 4.1 below, along with the name of their institution and mathematics course where their observation period took place. I include additional descriptions about the institutions and the teachers in the Methods Chapter (Chapter 3).

Table 4.1. Teacher Cases

<table>
<thead>
<tr>
<th>Teacher Case</th>
<th>Institution and Course from the Observation Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beth</td>
<td>Sundryville High School: Algebra 2</td>
</tr>
<tr>
<td>Shannon</td>
<td>Sundryville High School: Algebra 1 in a Sheltered Language Instruction Program</td>
</tr>
<tr>
<td>Eddy</td>
<td>Mixville High School: Algebra 1</td>
</tr>
<tr>
<td>Dena</td>
<td>Beacon Community College: Quantitative Analysis Course</td>
</tr>
</tbody>
</table>

**Within Case Analysis: Findings Format**

The findings for each case were divided into two main sections. The first section provides necessary background information to situate the findings in their particular institution, the mathematics course under study and the students that composed each class. Table 4.2 provides a description of the type of content that was reported in each subsection.

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7 I used pseudonyms for participants and for their institutions.
Table 4.2. Description of Case Background Subsections

<table>
<thead>
<tr>
<th>Background Subsection</th>
<th>Content Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Course and their students’ learning paths</td>
<td>This subsection provides information about the course in relation to the curricular or programmatic needs that it meets at its hosting institution. It also describes the math learning paths that the students in these courses would have followed and/or have access to.</td>
</tr>
<tr>
<td>2 Specifics on the Teacher’s course</td>
<td>This subsection provides particular information about the design of the course as implemented by the teacher. It also includes details about the particular unit that was studied during the observation period.</td>
</tr>
<tr>
<td>3 Students and their Classroom</td>
<td>This subsection provides additional information about the class make-up and a description of the classroom environment.</td>
</tr>
</tbody>
</table>

The second section reports the findings for each case in response to the research questions. I revisit the research questions below:

- What forms of knowledge of the student (e.g., mathematically foundational, identity, community, etc.) do practicing algebra teachers use to leverage the learning of algebra for students from underserved populations?
- How are these multiple and diverse forms of knowledge of the student applied in practice to support the teachers’ algebra learning goals for their students?
- In what ways do these teachers perceive their use of these forms of knowledge as helpful in supporting their students’ learning of algebra?
- What models can be developed to understand these teachers’ practice as they attempt to advance their students’ learning within their situated context of instruction?

The format for this findings section used four primary categories (i.e., teacher’s learning goals, forms of knowledge of the student, teaching interventions, and central phenomena). I also describe each of these categories (i.e. subsections) of the findings section in table 4.3
Table 4.3. Findings Subsections and their Content in Response to the Research Questions

<table>
<thead>
<tr>
<th>Findings Subsection</th>
<th>Content Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Teacher’s Teaching goals</td>
<td>I triangulated the teaching goals from the full case study data for each teacher. I used their teaching goals to further understand the ways that they considered important to advance their students’ learning. I found that their use of the forms of knowledge of the student depended on these goals.</td>
</tr>
<tr>
<td>RQ1</td>
<td></td>
</tr>
<tr>
<td>2 Forms of knowledge of the student found in a Teacher’s case</td>
<td>The forms of knowledge of the student were reported in table form. Although their use was not quantified, they were listed in an order that reflects relevance in use. I also denoted for each form of knowledge if it was found to have a direct association to any from the original categories in the start list (denoted with an O), or if the form of knowledge was not from the start list (denoted as N).</td>
</tr>
<tr>
<td>RQ1</td>
<td></td>
</tr>
<tr>
<td>3 Teaching interventions used by the Teacher based on forms of knowledge of the student</td>
<td>Each teacher exhibited central behaviors in how they used forms of knowledge of their students. I have called these central behaviors: interventions.</td>
</tr>
<tr>
<td>RQ2</td>
<td>This subsection includes: a) A definition for each intervention situated in the context of each case and generated using the full case study data. b) A description of how the forms of knowledge of the students were used to inform each intervention.</td>
</tr>
<tr>
<td>4 Central phenomena in the Teacher’s practice</td>
<td>Findings from the first three subsections above were used to understand the central learning phenomena taking place in each case. In this subsection I describe each teacher’s perspective on their students’ learning, derived from the full case data and in association with their interventions. I also describe the interplay between the teacher’s interventions, situated in the teacher’s perceptions on how their students learn. I end the findings section with an analytical model that I used to study the central phenomena describing how the teacher’s used forms of knowledge of the student and the resulting learning experience for their students.</td>
</tr>
<tr>
<td>RQ3 &amp; RQ4</td>
<td></td>
</tr>
</tbody>
</table>

The teachers’ learning goals for their students elucidated on each teacher’s conception of what it meant to advance their students’ learning within their practice. The teachers’ learning goals in combination with the forms of knowledge of the student that were found to be used by the
teachers, together, respond to research question number one (RQ.1). The teaching interventions, which were informed by forms of knowledge of the student and used to advance their students’ learning respond to research question two (RQ2). The central phenomena grew out teachers’ perspectives on their students and their learning, thus responding to research question three (RQ3). Analytic models were constructed for each teacher based on all of the case data, describing the central learning phenomena originating from teachers’ use of knowledge of their students, responding to research question four (RQ4). Research questions three and four are further addressed within the cross-case analysis (Chapter 5) and the implications from the overall findings (Chapter 6).
Case 1: Beth – “Learning to the Nth Power”

The observational period was conducted over a period of four weeks in an Algebra 2 classroom in May and June of 2017. Beth was on her sixth year of teaching at Sundryville High School.

**Background information: Algebra 2 (Level 2) course and their students’ learning paths.** Sundryville High School had recently eliminated all Pre-Algebra courses. This meant that all ninth graders\(^8\) started their high school math coursework in Algebra 1, regardless of their performance in middle school math. The school offered three learning tracks. Students identified as the top performers were enrolled into an Honors track. The students identified as next highest performing were enrolled into a level 1 track. The students identified as lowest performing were enrolled into a level 2 track. Regardless of track, the math sequence of coursework progressed from Algebra 1 to Algebra 2 and then Geometry. This meant that students would be taking Algebra 2 as tenth graders and Geometry in their eleventh grade of high school. The Algebra 2 classroom under observation was in the level 2 track.

According to Beth, the math department had worked to design the overall math program to prepare students for enrollment into Pre-Calculus in their senior year (post-observation interview). Students from the level 1 and the level 2 tracks could move up to a Calculus track through recommendations of teachers. These students were offered a Geometry course during their summer session after taking Algebra 2. They would then take Pre-Calculus in their eleventh grade and Calculus in their senior year. Most students in the level 2 track, however, enrolled into a Topics in Math course during their senior year after completing Geometry. This Topics course was a self-paced online course with adaptive features that students took in a

\(^8\) Except for students in special programs, like the Sheltered Language Instruction Program (see Shannon’s case).
computer room with support from a teacher. Their teacher monitored their progress and answered questions while students worked on their lessons. The course was designed to match the same math topics as those in the Intermediate Algebra course that were offered at a state community college nearby. According to Beth, their minimum curricular goal for all their students was to have them place into a college level math course (post-observation interview).

The high school had made articulations with the local community college so that their students in the Topics course could take the college’s placement exam at the end of the year. If their placement scores matched the placement requirements at the college, students could get college credit for the Intermediate Algebra course from the community college. Based on my interview data with Dena (participant four), I was able to confirm that the community college under articulations with Sundryville High School also used the same online adaptive program for their own students.

A depiction of the learning paths for Algebra 2 – Level 2 students at Sundryville High School is shown in Figure 4.1. The learning paths I depict here focus on Beth’s descriptions of learning opportunities for her Level 2 students. Figure 4.1 does not include other possible opportunities for track movement for Level 1 and Honors students, but the math options at the senior year level were the same for all tracks. Beth stated that “it’s Topics or Pre-Calc or AP Calc, so there is no other senior level elective class” (post-observation).

Figure 4.1 highlights two features of the learning paths that Beth described as meeting her students’ learning needs. These were: flexibility and high expectations. Beth explained that she supported a tracking system because it allowed for modifications in the curriculum that met students’ needs where they were. According to Beth, however, the teaching needed to maintain rigor and the expectation that students were being prepared for the highest placement possible.
Beth supported tracking systems as long as they incorporated the flexibility to allow for student movement across tracks.

Figure 4.1. Students’ Math Course Learning Paths for Algebra 2- Level 2 Students in Sundryville High School
*Beth’s Algebra 2 – Level 2 course (observation period)

I asked Beth to help me understand how students could achieve the preparation to engage in upper level courses despite track and performance differences. Beth shared with me her earlier experiences when she was in charge of piloting the state’s common core curriculum in her Algebra 1 courses. She noticed discrepancies between her students’ ability to keep up with the lesson pace and those of students from more affluent school systems (post-observation interview). She said she used this experience to inform her design of their Algebra 1 curriculum. Beth maintained the rigor, but in order to make the learning more accessible, she reduced the number of families of functions they learned. This added classroom time to develop the same
skillset that was needed for upper level math courses. Her indicators for student preparation in these upper level courses are described in more detail in the findings subsections within this case.

**Background information: Specifics on Beth’s course.** Beth noticed that when the school moved to start all students in Algebra 1 in ninth grade, that the performance of the honors and level 1 track students in their Algebra 2 courses was not as affected as that of the level 2 track students (post-observation interview). Beth was concerned to see that their Algebra 2 level 2 students were “failing that course on average” (post-observation interview). She took it upon herself to work with a local community college to understand their expectations on curriculum content so as to re-design the curriculum for the school’s Algebra 2 courses. Although the curriculum re-design also took into consideration the topics that were expected to be taught through the Common Core, Beth worked to align the types of problems that students would engage in in the high school course with those in the course at the local community college. For each unit of study, Beth outlined how many lessons would be spent by topic. She also created a pamphlet that included common guided notes that could be used in all level 2 courses (regardless of who taught the course). Students were given a copy of this pamphlet at the start of each unit. In addition to guided notes for classroom use in each lesson, the pamphlet also included the following items: practice worksheets, homework sheets for each day, classroom games (e.g. scavenger hunt, tic–tac–toe), sample SAT questions, and unit review sheets. Beth described her goals for her students to be, “college and career ready” through the new Algebra 2 level 2 curriculum. At the time of the study, the school had only been using the new curriculum materials for two years.

The observational period was conducted during their eighth unit on Radical functions. Table 4.3 summarizes the topics that were taught in this unit. The unit was designed to require
19 days in total. Two of these days were dedicated to review, and one day was dedicated to their unit test.

Table 4.3. Topics Covered During Beth’s Observational Period

<table>
<thead>
<tr>
<th>Topic</th>
<th>Topic Taught</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic 1</td>
<td>Review on Properties of Exponents (integer exponents only)</td>
</tr>
<tr>
<td>Topic 2</td>
<td>Radical Expressions and Rational Exponents:</td>
</tr>
<tr>
<td></td>
<td>Rewriting expressions in either form and applying properties of exponents to simplify expressions</td>
</tr>
<tr>
<td>Topic 3</td>
<td>Radical Functions and Equations:</td>
</tr>
<tr>
<td></td>
<td>Evaluating function values and solving equations</td>
</tr>
</tbody>
</table>

The course grades were based on quarters, dividing the academic calendar year into four parts. Unit eight was one of the two units taught in the last quarter of the year (quarter four). The other unit was an earlier unit on rational functions. The grades during the unit under observation were calculated using the following items: three quizzes, homework checks, classwork (based on completeness of the unit pamphlet) and a unit test. Halfway through the unit Beth changed how she reported the homework and the classwork in the gradebook. This incident is revisited later within the findings sections for its association to what Beth did in response to what she knew about her students.

Lastly, I also note that although Beth was a primary contributor to the curriculum redesign, there was ample evidence supporting her ongoing work with colleagues on what was done to support the learning experiences in the Algebra 2 level 2 course. For example, when Beth told her students that she was thinking of ways to change how she reported their grades, she also told them that she was in the process of discussing alternatives with a colleague and that she would get back to them. Beth also invited me to attend her Algebra 2 level 2 data team meeting. In these meetings, the teachers discussed their individual lesson pace, particular challenges they...
were encountering and things they thought had worked well. They also collaborated to review their final exam questions and to make modifications together.

**Background information: Beth’s students and classrooms.** Beth’s Algebra 2 class had approximately 25 students. This figure is not exact because Beth explained that there were some students in special programs that were in her roster, but that they were not necessarily doing coursework in her classroom. My data from classroom observations reflects a student attendance that fluctuated between 18 and 21 students. Of the 21 students, nine were females and twelve were males. Based on appearance, classroom interactions and comments, most of the students were of color (9 Black, 8 Hispanic, 2 White, 2 Asian).

During the classroom observation period the seating configuration changed three times. At the beginning, the room was divided into two sets of rows that faced each other. A middle wide aisle divided the room into two sections. Beth positioned herself within the aisle at the back of the room. She stood behind a podium in this middle aisle to facilitate classroom discussions. Later into the observational period, the orientation of the rows in the left side of the room was changed to face the front board. The classroom was split into two primary stations. The side that faced the front board (on the left) was dedicated for students that needed to watch her video lessons. The lessons were often times assigned as homework because Beth used a flipped lesson model (i.e., a model where students watch their lessons online and use class time instead for practice or other learning activities). The students that had watched the video as expected of them, sat on the right side of the room in small groups to practice and get help from Beth. In the third configuration that I observed, the classroom continued to be divided into two main areas, but the seats were clustered depending on how students decided to work with each other. I never heard Beth assign groups, nor tell students where to seat. Seating seemed to be
mostly based on preference. There were a few instances where Beth allowed students to sit near her as she stood at the podium, instead of working in groups. In some cases it involved a student that was frequently distracted, and in other cases it involved students that wanted more of her help. In cases where students sat near her because they wanted more of her help, I noticed that Beth would alternate between helping them and those that approached her from their group work. Working with Beth seemed to take on a form of privilege that helped reframe the times when Beth asked students to move next to her due to distractibility. In one class, for example, Beth asked Carlos to move next to her. Carlos exhibited throughout the observational period tendencies to get distracted. After he told her he did not want to move, Beth said with a smile, “it’s the best seat on the house” (observational period).

This year Beth was piloting a flipped model of instruction. She created video lessons that she uploaded into the classroom’s online drive. The lessons were about 35 minutes to 40 minutes in length. She explained that she attempted this model in an effort to maximize her time in practice with students in the classroom. According to Beth, her students often came to class without their homework done and they would tell her that it was because they could not do it on their own. Beth said, “I figured, everyone can take notes” (post-observation interview). Beth explained that in theory, this flipped model would have taken away any of her students’ challenges associated with problem difficulty when they worked independently at home. It would have also allowed Beth and her students to increase their time working together. On multiple occasions, Beth expressed to me and to her students the importance of maximizing engagement during class time (check-ins, observations, pre- and post-observation interviews). In one class in particular, Beth projected on the smartboard the following message as students worked in groups: “Time is nonrefundable”. 

101
Beth’s self-reported objectives in flipping classroom instruction were evident during the observational period. I found during observations, however, that she frequently accompanied her work in flipping the classroom by reminders and talks to students to encourage and increase the incidence of lesson completion before coming to class. During the pre-observation interview, for example, Beth described having to “start all over again developing their habits” because they did not have flipped instruction during their last unit when it was taught by a student teacher. Specifics about Beth’s teaching challenges are revisited as part of the cross-case analysis at the end of this chapter.

The findings sections that follow make reference to some of Beth’s students to provide select examples of their learning experiences. In an effort to assist the reading of the findings section, I provide in key characteristics about these students based on the full case data in table 4.4.

**Findings: Beth’s teaching goals.** Beth’s goals were first triangulated from interview and observational data. I then confirmed these goals through member checks in the post-observation interview. I list Beth’s goals below, but I revisit them again when I describe the teaching interventions that Beth implemented to support these goals.

1. To maintain rigor in students’ math learning experiences

2. To help students learn to “make good choices” that will sustain their engagement in learning (i.e. prioritize their learning inside and outside the classroom)

3. To foster “genuine moments of learning” for her students

4. To develop college and career readiness which included skills such as: responsibility, strong math foundations and ownership over their learning.
Table 4.4. Beth’s Referenced Students and Key Characteristics

<table>
<thead>
<tr>
<th>Students</th>
<th>Particular Characteristics Referenced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darnell</td>
<td>Darnell typically sat in the front of the room, but he also moved around the room frequently. He asked for a pencil almost every day of the observational period. Hannah brought a box of pencils one day and gave it to Beth at the end of class so that she could use them for Darnell. Darnell is also a student that Beth identified as demonstrating improvement in the work he wrote to support his answers. He had also improved his grades in the class. Darnell is referenced here to exemplify Beth’s use of her students’ “witty” demeanors to re-direct them to do their work. A classroom incident on a bet between Beth and Darnell is presented in Appendix E.</td>
</tr>
<tr>
<td>Selena</td>
<td>Selena sat near Beth. While most students worked in groups, Selena worked alone with Beth’s help. She is referenced here as an example of a student with a particular personal condition that Beth worked with to support her learning and that demonstrated improved confidence and performance. Her case is described within the model description at the end of this findings section.</td>
</tr>
<tr>
<td>Carlos</td>
<td>Carlos sat in the front of the room and often worked with Darnell. Carlos struggled with distractions and sometimes motivation. He is referenced here as an example of a student that Beth gave particular individual attention.</td>
</tr>
<tr>
<td>Steve</td>
<td>Steve sat on the right side of the room. During the observational period he did not make contributions to whole class discussions. Steve is referenced here as a student that Beth worked with through the phone App. Beth also explained that as he continued to ask questions, his work continued to improve.</td>
</tr>
<tr>
<td>Hannah</td>
<td>Hannah sat on the left side of the room. During group work, she worked with two other students closer to the back of the room near me. Hannah was an English Language Learner (ELL). She is referenced here as an example of a student that Beth worked with by supporting her communication of her thinking.</td>
</tr>
</tbody>
</table>

**Findings: Forms of knowledge of the student used by Beth.** In table 4.5, I summarize the forms of knowledge of the student that Beth used to advance her students’ learning. I also include their corresponding definitions formulated from and contextualized in Beth’s case data.
Table 4.5. Forms of Knowledge of the Student Used by Beth
Mathematical – MK and Non-Mathematical – NON

<table>
<thead>
<tr>
<th>Form of Knowledge</th>
<th>MK (math) vs. NON (non-math) or BOTH</th>
<th>Definition and Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Culture(^{O}) and Cultural Dissonance</td>
<td>NON</td>
<td>Culture refers to students’ practices, norms and lived experiences. Beth used different dimensions of student’s culture. Some were associated with aspects that Beth positively appreciated about her students and others where aspects that she worked to change because they were in dissonance with her educational goals. When I asked Beth, “why would anybody want to teach at your school?, this was her response: “But in terms of way of being, there is not an apathy for…how do I say it? For life. They come with energy. They have a great energy about them. Whether that’s for good or evil, sort of speak, on any given day that’s different. But they got…like there’s a culture. I love the culture that they come with – minus the rap music. Mmh…” (Beth, post-observation interview) Beth appreciated their sense of humor, their “love of sarcasm” and added that “maybe it is something I can relate to”. (Beth, post-observation interview). She shared the fact that she could talk about the music she loves (e.g. bachata) and that her students would understand it and even comment on having gone to concerts featuring this music. Beth used some of these cultural aspects in the classroom. During the observation period I heard her reply back and respond to their sarcasm by recognizing it, and by turning it around to re-direct students to their work. I share in Appendix E an incident that I have titled “The Bet”. Beth lost a bet to Darnell because she needed him to work on a worksheet that he claimed he had already completed and had received a good grade on. She raised the bet afterwards for a donut and a small coffee, explaining that the coffee had to be small “because they were on a teacher’s budget” (Beth, observation period). There were also aspects about their culture that Beth expressed having challenges with. She shared an incident where she asked a student to not use foul language, but the student explained that it was a song he was singing. She asked him to not to sing it in their classroom because of the language it used. During the post-</td>
</tr>
</tbody>
</table>

\(^{O}\) O = from start list
\(N\) = form of knowledge not on original start list
observation interview, she explained that she did not tolerate disrespect and that this included respect for women which in her opinion, was lacking in the lyrics of some of the songs that her students listened to. She added that a few days later she saw the father park his car to pick up his son with the same music streaming loudly. She shared this to explain that she could only focus on what happens in the classroom. (Beth, post-observation interview).

Culture was found in dissonance with the culture of school when Beth needed her students to do work and study outside their classroom. For example, she wanted students to study over the weekend for their final exams, but they laughed and told her that it was going to be 90 degrees and that they would be going to the beach. Beth said that she has learned that their experiences are very different from hers. She said that she had one parent that was college educated and that because of that, studying was made a priority for her. This did not mean that she would stop reminding them about prioritizing studying. She considered it a situation that she has accepted, but that it would not change the fact that she would hold them accountable for their work (Beth, post-observation).

Beth described “changing her frame of reference” in cases where students’ experiences where different from hers and that she has chosen to look for ways to help them learn how they can behave differently (Beth, recruitment interview).

<table>
<thead>
<tr>
<th>Learning Attitudes</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perseverance</td>
<td></td>
</tr>
</tbody>
</table>

Mathematical – This form of knowledge refers to students’ ability to sustain engagement in problem solving. During class, she approached students directly if they were not working. I heard her say in one class, “if the problem is blank it means you need to ask questions”. She emphasized the need to attempt all problems (observation period and check-in). In one class, she returned a quiz and opted not to grade it because she explained that too many problems were left blank and that she wanted them to continue trying and learning.

Nonmathematical – Beth expected her students “to work hard” (post-observation) as part of her conception of being “college and career ready”. She said, “and again, that comes full circle to college and career ready. You are a student that goes to any community college and works for ten minutes and then puts his feet up, that student is not staying” (post-observation interview).

Beth had dual expectations at the mathematical and nonmathematical level, reflecting convergence between both types
of perseverance efforts in her students. If a student was having difficulty persevering through a problem (mathematical), Beth would then give them explicit instructions as to what students needed to do to persevere. These instructions were not necessarily mathematical. She would say for example, “if the problem is not correct and you do not know why, you need to ask me or ask a classmate”, “if a problem is left blank, it means you are not done”. Sometimes her explicit instructions were mathematical. For example, as she worked through a problem that students were leaving blank in class, she said “notice that I looked at a piece at a time…piece by piece folks”. My interpretation is that Beth responded to what she noticed from her students that they needed to learn to persevere. Perseverance was a skill that Beth worked to develop in her students. Beth stated, “So if I don’t support the training just for you to work for more than 5 minutes, now you decide you are done, that’s not even a math thing. That is can I be a student 101” (post-observation interview).

<table>
<thead>
<tr>
<th>Foundational Gaps</th>
<th>MK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundational gaps were found in students’ abilities to: perform operations with fractions, multiplying signed numbers, apply properties of exponents and interpret the mathematical structure of some expressions. These particular gaps were evident in their connections to the particular mathematical goals of the unit which relied on the relation between radical expressions and their associated rational (fractional) exponential form.</td>
<td></td>
</tr>
</tbody>
</table>

During the observation period, Beth answered questions to students making connections to concepts. Sometimes the connections were quick prompts, like “negative times negative is positive” (observation period). Other connections required her to pause and revisit procedures. For example, a student asked for help on a problem that required that he simplified the following: $y^{25} / y^{3/10}$ Beth reviewed how to find common denominators step-by-step, so that they could subtract the exponents.

During her teaching, Beth attempted to make connections to students’ pre-requisite knowledge by re-writing the meaning of expressions. For example, she would write that $2^4$ was equal to $2*2*2*2$, but in some cases it was not clear if there was a connection made when the student did not evidence having the pre-requisite knowledge to begin with.

In a check-in, Beth explained that she has found herself, “fighting against a lot of tricks” that were given to students at some point in their earlier experiences, but that they turned into an issue now that she needs her students to recognize operations (e.g. “keep-change-change”). Given what she has come to learn about her students,
Beth said that has changed her thinking – “if she has no expectations, then she has no disappointments”. Instead, Beth said that she chooses to create a safe learning place where students can communicate what they don’t understand so that she can say to them, “let’s go over it again, no problem” (Beth, check-in).

In a different class, a student asked Beth, “miss, does factoring ever go away?” Beth answered with a smile, “factoring never goes away, does the alphabet ever go away?”

<table>
<thead>
<tr>
<th>Language and Structure of Math&lt;sup&gt;0&lt;/sup&gt;</th>
<th>MK</th>
</tr>
</thead>
<tbody>
<tr>
<td>This form of knowledge refers to both verbal and written forms of the language of math. In a class Beth said, “Math is a language”. She frequently asked students to read out loud the expressions they were working with. She also asked students to write their work and called their written work “a conversation with her” (observation period).</td>
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In having students read their expressions out loud, Beth said that students would better recognize the operations that were being applied (check-in). For example, in one class, a student read out loud $b^{12}$ as “b 12”. Beth jumped in and said, “remember this is not battleship…b to the…”. In that same class, she asked another student to read the expression $2/b^4$. The student read out loud “2 over be to the $4^{th}$”. Beth said to the student, “over is not an operation, use your language” (observation period)

During check-ins, Beth described to me the aspects in mathematical structure that she stressed based on what she knew about her students. For example, in class students had to rewrite with positive exponents the following: $x^{2/10}y^{1/6}$

She told students, “remember, everything is technically divided by 1”. Beth explained in the check-in that students tended to forget that integers are rational numbers. They might see the number 5, but not necessarily consider it as $5/1$.

Other examples included: recognizing what is the base in an expression and the effect of the parenthesis (e.g., $-3^2$ vs. $(\text{-}3)^2$), identifying roots from rational exponents and rewriting exponential expressions as their corresponding factors.

Beth paid attention to students’ mathematical language and used it to make connections to the operations that were particularly represented within this unit on radical expressions and rational exponents. This from of knowledge reflected association to foundational gaps in that students were not evidencing making connections to earlier mathematical knowledge they were expected to have already acquired. They did evidence applying Beth’s prompts as if it was new knowledge to them.
Personal Interests and/or Convictions

Beth made reference to two aspects about her students that I have classified together under personal beliefs and/or convictions.

Beth learned that her students tend to have a strong sense of social justice (check-in). She explained that they are observant of her practices in the classroom. Because of this, Beth attempted to hold consistency in her teaching and also in the consequences she announced that would take place when an expectation was not met. She said that she “never promises something that she cannot follow through” (check-in). She also explained that she tried to make sure that no one felt excluded from the learning experience. This is the reason she gave me for asking two students that were speaking Patois (creole language) to speak instead in English. She was concerned that some of their classmates would have felt excluded or wondering if they were talking about them (check-in). At the same time, she did support the use of other languages when students were working on math problems.

Beth also appealed to what she described as students’ interest and sense of being a good person (post-observation interview). I had asked Beth about her use of psychology that I had observed in the classroom. In her response, Beth brought up the fact that the mind is very powerful and that she uses it to maximize class learning time instead of using time to handle discipline. She asserted: “I mean, really, just obedience doesn’t exist from within. So if it’s me and 25 for 45 minutes…if I can engage their mind, if I can engage their morals, and that intrinsic, ok, being a good person is connected to being a good student right now…” (post-observation interview). Beth’s use of psychology was associated with her understanding that students inherently wanted to be good and do well, but that at the same time, they did not evidence the behaviors she needed them to engage in if they were confronted through a disciplinary stance.

‘Being a good person’ seemed associated with students’ sense of social justice. Beth’s comments to her students were framed for the good of all. For example, in one class Carlos was playing with a fidget spinner. After addressing distractions, Beth said, “Carlos, the whole class is waiting on you”. Carlos chose to not cooperate. Beth sent him out of the classroom. During the check-in, Beth explained that their time in class was too valuable and that it was not fair to other classmates if that time was spent on only one student just to get him to cooperate.
### Competing Priorities and/or SES challenges

This form of knowledge refers to other responsibilities outside the classroom that Beth has noticed her students undertake.

According to Beth, for example, some students may have younger siblings that they are in charge of in the afternoons until a parent gets home from work (post-observation interview). This means that her students would have to pick up siblings off the bus, feed them and watch them until night time. Beth was aware of situations of this kind and described them as competing priorities in that her students would have a much more limited time to do their work or to stay after school for help.

### Mathematical Strengths

This knowledge refers to students’ demonstrated mathematical strengths in the classroom. In some cases the strength was indicated by the fact that they finished their work early and they would get confirmation from Beth that their work was correct. These students were often times asked by Beth to be a “teacher helper” (observation period). Other times Beth asked them to help classmates around the room. During a check-in she explained that it having them help others, helped them learn it better.

In other cases, a student demonstrated a strength by completing a particular problem that other students had experienced difficulty with. Beth would share with the class that the student had just completed that problem and that if anyone needed to know more about it, to go see that student in the room (observation period).

### Mathematical Confidence and/or Self-efficacy

This form of knowledge refers to students’ self-perception of their abilities in math impacting their actual performance in math.

In Beth’s practice, mathematical self-efficacy and/or confidence was best described as being able to problem solve independently without help from the teacher and/or their notes. In one class, for example, Beth told her students that they could use their notes for an assignment. At the same time, she said that she was challenging them to do it without their notes. Beth stated, “if you do need them…I care about you realizing, OK – I can do these questions. But I need you to have this in your heads. I am not trying to be mean, I am being real with you. For every question that you needed your notes to solve it – that would be one you got wrong without notes” (observation period). My interpretation based on class observation is that Beth wanted students to recognize that they were capable to do the problems, but she also wanted them to do what she called, “quality work”, where they were able to work independently making sense out of their problems.

Beth identified one particular student, Mikkel, as having low self-confidence. I had noticed that when he worked in groups, he would...
go back to the board to change answers he had placed based on other classmate’s comments. One day, Beth read a list of names of students that she wanted to meet to talk about the condensed Geometry summer program. Mikkel’s name was not in the list. He asked Beth if he could participate. Beth told him that he could, but that he needed to understand that the program covered one chapter per week. She said he would need to be comfortable learning that much new material at that pace without questioning himself. Beth told me during the check-in that he had the foundations and motivation to do well in the program, but that she was concerned for his level of confidence.

<table>
<thead>
<tr>
<th>Mathematical Accuracy&lt;sup&gt;N&lt;/sup&gt;</th>
<th>MK</th>
<th>This form or knowledge is associated to the mathematical practice within the common core. Beth explained that her department had picked this particular practice as the one to focus on in their students’ learning. When Beth described how she graded student’s work, she explained that it was important to give credit for partial credit because it allowed students to see how close they were to the final answer. In these same discussions, however, she would note that if the problem was not correct, that “the accuracy was not there” (check-in). In Beth’s case, students’ accuracy was indicated by students’ correctness in final answers.</th>
</tr>
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<tbody>
<tr>
<td>College and Career Readiness&lt;sup&gt;N&lt;/sup&gt;</td>
<td>Both</td>
<td>This form of knowledge was more closely associated to Beth’s teaching goals for students. It is a term that she used frequently throughout check-ins and interviews. Based on the full data set, this form of knowledge was a combination of indicators that included the following: perseverance, mathematical foundations (sometimes described by Beth as a developed math ability through rigor), responsibility and ownership over their learning. I include this form of knowledge here, as a set of attributes that Beth looked to develop in her students. Thus, Beth used this form of knowledge whenever she worked with her students to develop any of these attributes.</td>
</tr>
<tr>
<td>Personal Characteristics or Particular Conditions&lt;sup&gt;N&lt;/sup&gt;</td>
<td>Both</td>
<td>This form of knowledge refers to particular characteristics about students and/or conditions that Beth worked with to help advance her students’ learning. One student, Selena, struggled with anger management. Beth explained how she worked with her in collaboration with the Vice-principal’s office to support her learning. Beth worked with her throughout the year. The results from their work were evident in her unit test performance and in her confidence level (Beth, post-observation).</td>
</tr>
</tbody>
</table>
Other characteristics included: challenges with distractions and maintaining engagement (Carlos) and discomfort in communicating understanding (Steven).
Other conditions included: students with in-school suspensions and students in special behavioral programs.

| Mathematical Progress<sup>N</sup> | This form of knowledge refers to Beth’s interpretations of how well students were learning. In Beth’s practice, I noticed a lot of emphasis on “meaningful learning”. Beth held in one class a 20 minute discussion on the importance of learning meaningfully from doing their homework. She had been grading a quiz and shared with the class that she had noticed a difference between their homework grades and their quiz grades. She reminded them that they got partial grades on their homework based on efforts, but that their efforts in the homework only reflected how complete their work was. She told them that, “homework that looks done does not mean that it is actually learned” (observation period).

Beth also paid attention to the overall class progress through their discussions and engagement in class. At the end of one class Beth told her students when the bell rang, “You guys did fantastic work today, I have seen improvement in your level of confidence, and you are working not just for speed, but for accuracy”.
On the test day, Beth told the class as they worked on their tests, “As I look around, I have been seeing nice work. Now that you have been working for about half hour, move back to other sections. Remember you can move around the test”.

| Relational<sup>N</sup> | NON | This knowledge refers to what Beth has learned that worked in how she relates to her students.

During the pre-observation interview, Beth stated that she learned that her students “genuinely like” that she demonstrates that she cares about them. Beth explained that she wants them to know that they are not just a name in the roster. She asserted, “So, reaching out and showing empathy as much as I can in certain situations has gone a long way, because otherwise it just lacks all emotion and teaching is about emotion” (pre-observation interview). She has also changed her approach to reading her students behaviors as a result of this. For example, Beth said that if she saw a student with her head down, instead of asking her why she was not working, she first asked her if everything was ok.

| Mathematical Errors<sup>M</sup> | MK | This form of knowledge refers to typical errors from students. Beth would follow up examples, by changing the same problems a little and anticipate typical errors. For example, in one class she had explained how to simplify the cubic root of \(-216(x-5)^3\). After completing that problem, she asked the class who they thought the problem would have been different if the radicand was instead -
216(x-5)^6. During the check-in, Beth explained that a typical student error was for students to then rewrite (x-5)^2 as x^2 - 25.

I also noticed that Beth used students’ typical errors to choose how she would instruct students to simplify an expression. For example, in one check-in, I asked her why she taught students to always rewrite expressions in radical form into their equivalent rational exponent form. Beth explained that she has noticed that students make more mistakes in radical form because they ignore the index. Students, in her experience, interpret the radical as a square root. She preferred to have them re-write the expressions in rational form so that the first thing they did was read the index (Beth, check-in).

Given the large incidence of foundational gaps in students’ classwork, I also note I found that a great extent of the typical errors were associated to foundational gaps and to understanding the structure of the mathematical expressions.

| Family Support | This form of knowledge refers to the different ways that Beth knew her students were getting support from their family. It is much more specific form within the form of knowledge of the family in the start list. Beth sent home forms that students filled out in class with their test grade and their own reflection of how well the prepared for their tests. Beth also spoke in class to students about changing the way she reported their homework and classwork grades so that their parents with have a more accurate understanding of their learning progress. |
| Language | This form of knowledge refers to Beth’s knowledge of students’ added demand in learning math in a language (English) that is not their first language. Beth shared her work with a student (Hannah) that had demonstrated improvement in her test grades. Beth said that her tests used to average 50 and now her averages are in the 70’s (post-observation interview). When I asked Beth what she thought helped Hannah, she said that it was their practice together to help her communicate her thinking. Beth described asking her one item at a time, “ok, what did you do here”. Beth explained that the more she communicated her thinking, the more she developed comfort in the practice and the more Beth, herself, was able to understand her mathematical needs. With time, Hannah was also able to demonstrate her thinking in written form and evidence an improved performance in her tests. |
Findings: Beth’s teaching interventions. Beth exhibited six central teaching behaviors in using the forms of knowledge of the student from table 4.5. I describe these teaching behaviors as interventions going forward in this document because they represent a set of particular measures that Beth implemented to advance her students’ learning based on what she knew about her students. These interventions were: (1) expands the boundaries of the learning space, (2) holds meaningful mathematical conversations, (3) uses psychology and reflective techniques (4) demonstrates genuine care, (5) holds high expectations and (5) maximizes engagement and work time.

In the subsections that follow, I provide an operational definition for each intervention. These definitions are based on Beth’s context of instruction and on her descriptions of how she used what she knew about her students and why. For each intervention, I also report the forms of knowledge of the student that informed each interventions’ intended purpose in Beth’s practice. These are summarized in Figure 4.2.

1. Expands the boundaries of the learning space: Definition. Beth expanded the boundaries of the learning space by providing different types of opportunities to learn outside the classroom. She also expanded the boundaries of the learning space by creating opportunities to continue learning even after they had moved on to a new unit of study.

Outside the classroom, Beth used multiple ways to expand the boundaries of the learning space. She stayed after school for help. She also offered students the opportunity to come during their shared lunch blocks to work together. For example, she told a student that wanted to meet for help, “I have a meeting today during lunch block. Can we have lunch on Thursday or Friday?” (observation period). Beth also used technology to expand the boundaries of the learning space. She piloted a flipped model of instruction that year so that students could have
Figure 4.2. Forms of Knowledge of the Student Informing Beth’s Interventions

- Conflicting Priorities, Mathematical Progress, Personal Characteristics or Conditions, Perseverance, Foundational Gaps, ELL needs, Cultural Dissonance, Confidence
  - Expands the Boundaries of the Learning Space

- Mathematical Language and Structure, Foundational Gaps, Mathematical Confidence, Perseverance, Personal Characteristics and ELL needs
  - Holds ‘Meaningful’ Mathematical Conversations

- Mathematical Progress
  - Uses Psychology and Reflective Techniques

- College and Career Readiness
  - Demonstrates Genuine Care

- Relational
  - Holds High Expectations

- Personal Interests
  - Maximizes Engagement Time

- Mathematical Progress
  - Mathematical Gaps, Mathematical Perseverance, Mathematical Strengths, Mathematical Errors, Personal Conditions, Cultural Dissonance, Conflicting Priorities
what she described as an achievable task at home and in preparation for each class. The videos were accessible anywhere as long as there was internet access. Beth video recorded many of her lessons in class so that they were accessible to her students that were absent. She also used a free App so that students could send her questions as texts. In one class she told students, “If I am awake, I will answer it” (observation period). Students sent her pictures of their work, for her to point out where they needed to re-consider their approach or to point any errors. She used this App to also send students reminders about homework and to announce when the videos had been uploaded to their common online drive. Beth allowed me to enroll into her class in this App. I was getting messages just as all students were receiving her messages. While responses were sent directly to Beth, I did get copies of their emojis (e.g. thumbs up’s, sad faces, smiling faces) in response to her announcements.

In the classroom, Beth also expanded the boundaries of the learning space by providing additional opportunities to learn after assessments. The observation period took place while students learned unit eight, but during the first two weeks of the observation period, Beth was still discussing learning alternatives for unit seven, the previous unit. She had students complete a form that they were expected to share with their families indicating their grade and their level of preparation. Students needed to complete this form and also attend either a review session after school or watch her review session online in order to have a second chance to take a new test from unit seven. While the second test session would not have fully replaced their first test, it would have given them an opportunity to continue learning after the unit had ended and reflect new learning. During the observation period Beth withheld from returning her second quiz in graded form to her students. She said that if she had written all comments to them on their errors, they would have only taken in the negative feedback instead of reviewing carefully her
quiz comments. Instead, Beth had a conversation in class with students about their progress. She had them work in groups to prepare for a quiz re-take. She also had them work on the quiz she withheld and had them make quiz corrections in preparation for their re-take. While Beth’s approach to have students learn the topics in the second quiz was not in her original plan, it reflects her expectation on learning which was not limited to an assessment session.

*Forms of knowledge informing ‘expands the boundaries of the learning space’*. Figure 4.2 above, depicts the forms of knowledge that reflected association with this intervention. The forms of knowledge informing this intervention are many. Beth originally implemented the flipped classroom because she wanted to increase students’ learning time outside the classroom. She said that her students were not doing homework at home and that when she asked them why, they would tell her that it was too hard or that they did not know how to do it (math progress and foundational gaps). Beth also noticed that some students could not necessarily stay after school to get help from her because they had family responsibilities to take care of (conflicting priorities). She also explained during the post-observation interview that her students were used to different practices educationally from the ones she was raised with (cultural dissonance). Because of this, her students were less likely to follow the school model she had herself experienced where students took notes in class and then they went home and did their practice questions. With a flipped lesson model, Beth hoped to shift the lesson and note-taking to take place outside the classroom. This was a task she said her students would be capable of doing. When we revisited her practice on flipped lessons at the post-observation interview, Beth added that she noticed an additional outcome from her intervention that she had not originally planned for. She said that her students “ran out of excuses” to say they could not do their homework. Beth explained that “at the end of the day, they are still children” (post-observation interview).
She also noted that she continued to have students come unprepared to class. In response to this challenge, she said that she would be instituting a set of questions the following year to have students reflect on why they were coming to class without watching the videos.

Beth noted additional benefits from flipping the classroom that helped her students in different ways. When we discussed the success from Hannah a student that was ELL (language), Beth said that one of the advantages from flipping the classroom was that she was able to have more one-on-one time with her students and to have conversations to support them (post-observation interview). The added time spent with Beth allowed students to address learning needs and have opportunities to work with Beth and with each other. This helped develop their perseverance and confidence to continue their own learning outside the classroom (see Selena’s case in the model description at the end of this section). Students with extended absences, such as school suspensions or those being sent by Beth to their behavioral counselors could also access Beth’s lessons online (personal conditions). Beth ended her videos with messages such as this one, “and for those of you that were not here with us today, have a great day” (observation period).

Through the texting App, Beth made her support also more accessible outside the classroom. Beth identified one student in particular that in her opinion had improved in class and placed himself in a stronger learning position because of the App. Beth explained that Steve was not comfortable asking questions in class or coming for help. I asked her why. She said that in her opinion, it was his way of being (personal characteristic). Beth said that she had “multiple iterations” of conversations after class and in the hallway with him about the need to ask her questions. She told him, “if we are not going to use time in class to ask questions, I respect that, free will. I can’t force you to ask questions, but questions need to be asked” (post-
observation interview). After asking questions through the App, Beth was able to help him. The change did not take place overnight. Beth said that “it was about a month and a half of improvement on effort. Once he actually started to put things on paper, then I could actually help him with what he was doing wrong and then, the accuracy followed” (post-observation interview). Beth also told me that Steve shared his first-ever question in class on the last day of school when they reviewed for the final exam. I include sample App communications between Steve and Beth in Appendix G.

2. **Holds meaningful mathematical conversations: Definition.** This intervention refers to the mathematical discussions that took place in the classroom and through the app among everyone in Beth’s classroom. Beth held individual conversations with students. She also created a classroom norm where everyone was expected to work with everyone by discussing their work. Beth referred to their discussions both orally and in written form as conversations. She also used two similar terms to describe the conversations she expected. These terms were: ‘meaningful’ and ‘genuine’. Beth expected her student to discuss how they attempted their problems. On various occasions, she tasked them to go to the board and write their answers, and also to discuss discrepancies in their answers and those on the board. Beth also told her students that she wanted them to “have an honest conversation with themselves” to make sure that they were “being in the moment” and paying attention to what they were writing. Beth also described her own conversations with her students as a way to understand their thinking and as a necessary starting place for learning improvement (e.g. Steve’s case in intervention 1 and Hannah in table 4.5 - Language).

*Forms of knowledge informing ‘holds meaningful conversations’*. Figure 4.2 above, depicts the forms of knowledge that reflected association with this intervention. Beth stressed
with her students the importance of understanding the mathematical language and meaning that was embedded in the structure of the expressions and equations that they worked with. During class, she asked students to read out loud the expressions, paying attention to their interpretation of what the symbols were representing. I asked Beth during a check-in, why she asked students to do this. She said, that in verbalizing and putting in words that describe what they saw, students were also making sense of what they were working with and of what they were thinking themselves. In communicating their thinking, whether it was verbally or in written form, Beth addressed students’ foundational gaps and their confidence. She also addressed personal characteristics (see Steve’s conversations through the App) and additional learning demands associated with English language acquisition (see Hannah’s case in table 4.5).

By expecting students to write work to support their thinking (“written conversations”), Beth also taught students how to persevere in productive ways. She asked them to write their work as a way to guide their thinking through problem solving. In other cases, students were not making progress when they were focusing on understanding old work. Beth told her students, “if your old work is misguiding you, erase it and start over” (observation period).

3. Uses psychology and reflective techniques: Definition. This intervention describes a behavioral pattern in the ways that Beth communicated with her students and in the types of tasks she asked them to engage in. I used the terms “psychology” and “reflective” because the patterns pointed to experiences that prompted students to reconsider and be more aware of their behaviors. These behaviors were holistic in that they pertained to issues on how to behave in the classroom, but also on how to best position themselves to learn successfully. I now present two examples, to best portray this intervention.
The first example was taken from a class incident where Beth put on the overhead projector the billboard for the movie ‘The Guardian’. She had started grading the quizzes and noticed a discrepancy in grades between the homework and the quiz grades. After expressing surprise to find out that her students had not seen the movie, she told students the general plot to make her point. She wanted them to know that getting high homework grades was not important if they were themselves not prioritizing on learning and demonstrating their knowledge in the quiz. I include a transcription of this incident in Appendix G. I also note that the dialogue reflects Beth’s style in communicating with her students. Her questioning style evoked student thinking, and engagement in her message. While she was explicit about caring about them, she also wanted them to be the ones communicating what she cared about and what she considered important.

As a second example, I describe Beth’s reflective questions that she gave students to answer after completing their unit eight test. Beth asked students to describe: if they felt prepared going into the test, how they had studied for their test, if they had completed their review for the test, and to write the grade they were expecting on the test. She used these questions to help students develop self-awareness and make connections between their level of preparation and their performance on their tests (post-observation interview). Beth and I reviewed some of these responses during the post-observation interview. Selena, for example, wrote that she had not studied. She wrote that she had done all the homework assignments and that she had also completed the review problems for the test. I asked Beth why she thought that Selena wrote that she did not study if she had done her review problems. Beth explained that she would have expected her to also check all her answers. Beth pointed that this was indicative of
her confidence level. She predicted getting an 80 on the test and that was actually the grade she obtained.

*Forms of knowledge of the student informing ‘uses psychology and reflective techniques’.*

Figure 4.2 above, depicts the forms of knowledge that reflected association with this intervention. This intervention used students’ mathematical progress, relational knowledge about Beth’s students, and their college and career readiness. As seen in the examples provided above, Beth reflected an interest to not only help her students do well or improve in their learning (mathematical progress), but to also position them to become aware of their role in learning. This involved engaging their sense of responsibility, and their sense of ownership over their learning which Beth had described as indicators of students’ college and career readiness.

4. **Demonstrates genuine care: Definition.** This intervention refers to Beth’s demonstration of care on multiple dimensions about her students. Beth demonstrated care about her students’ learning, and the behaviors she considered important to support their learning. Beth also demonstrated care for her students as people. I have worded this intervention to include the term “genuine” because it is the descriptor that Beth used when she communicated with me and with her students. Beth delegated this care, meaning that, in the classroom she instituted norms that reflected care for each other and care for themselves.

*Forms of knowledge of the student informing ‘demonstrates genuine care’.* Figure 4.2 above, depicts the forms of knowledge that reflected association with this intervention. When I asked Beth what she had learned about her students that had helped her advance their learning, Beth said that she had learned that her students liked that she cared about them (pre-observation interview). This reflects her use of her students’ personal interests. Beth applied this
intervention in almost all of her interactions with her students (relational). When Beth greeted her student viewer through her recording, Beth was demonstrating genuine care for her students under personal conditions that prevented them from attending class. When Beth discussed ways to help them overcome their low performance in the quiz, Beth demonstrated care for their mathematical learning progress and their overall ongoing learning. Through Beth’s stress on the need to maximize their work time and to have meaningful conversations, Beth was showing her students that what happened in the classroom required genuine care. In one class Beth asked her students to have “their parents, their aunts, their grandparents or anybody that cared and loved them at home” to look over their answers to their test reflective questions. In this incident Beth was demonstrating care for family support. My personal interpretation is that Beth’s explicit ways to show that she cared translated into a classroom environment that valued learning and collective support.

5. Holds high expectations: Definition. Based on Beth’s descriptions during the post-observation interview, this intervention refers to an unwavering expectation for students to learn the content and to develop the foundations to demonstrate preparation in their subsequent math course. I had built this definition through patterns in my observational data. I then tested my definition when we discussed Beth’s challenges. One of these challenges was that her students were not reflecting the needed preparation level for her course at the start of the year and that she suspected that there had been changes to the curriculum she had designed for their Algebra 1 courses. I decided to test my understanding of how Beth holds high expectations by asking her to explain the type of improvements needed in Algebra 1. Beth’s main points were that the time in the classroom needed to prioritize working with students to have them practice and engage in math learning. She also pointed that the topics taught had to be aligned with what students
would be needing in subsequent courses. Her descriptions in essence captured how Beth maintained high expectations with her students. I added myself the term ‘unwavering’ to this description because I had also recognized a sense of perseverance in the way that Beth worked with her students. For example, Beth expanded the boundaries of learning after tests or quizzes because learning those topics was an expectation that had not been met and that Beth wanted to be met.

*Forms of knowledge of the student informing ‘holds high expectations’*. Figure 4.2 above, depicts the forms of knowledge that reflected association with this intervention. This intervention was informed by students’ **mathematical progress**, **mathematical accuracy**, **mathematical perseverance**, and their **college and career readiness skills**. These forms of knowledge are associated to content and to academic skills that are typically understood to support the learning of this content. Although the data set evidences Beth’s high expectations in almost all components of her practice, my interpretation is that Beth held high expectations for herself. With regards to what Beth knew about her students, the data set reflected a particular focus on needs in learning content. Beth recognized that her students carried mathematical “deficiencies” (post-observation interview), and she made it a priority to prepare her students to engage at what she described “the highest level possible” (post-observation interview).

**6. Maximizes engagement time: Definition.** Beth looked for different ways to maximize her student’s engagement time. She established classroom norms that expected work in the classroom. Beth also maximized engagement time by having students work with each other and by asking students to take on roles as helpers and to “make their way around the room” (Beth, observation period). This intervention reflected support from the administration. When I asked Beth for factors that support her practice, she said that having behavioral counselors and a
dedicated vice-principal allowed her to make plans for students that were not prepared to learn on a given day. I had seen this arrangement in practice as early as the first day when I conducted an acclimation visit. Beth was managing classroom discussions by having students pass a red ball. If a student received the ball, the student was expected to give input and/or to “phone a friend” if the student needed help. A student refused to engage in class with a confrontational tone. Beth said to her, “this is your choice, if you are not ready to be with us and learn, it is best for you to go”. The student picked up her belongings. As she walked out the door, Beth asked her to write the homework that she needed to bring back the next day. I was puzzled by the interaction because I was not used to see a student’s oppositional behavior matched with a calm exit and a reminder to do homework. During the check-in, Beth explained that she had made arrangements for the student to do work at the counselor’s office so that her student would not just miss class, but have an alternative place to continue learning. This was also the case with students placed on in-school suspension. Beth and her colleagues had a dedicated service period for in-school suspension. When they headed out to the room, they would tell each other what their students needed to work on and would then follow up with their students in the suspension room. I was able to do in school suspension duty with Beth one day. She expected maximum engagement of all students in the suspension room too.

Forms of knowledge informing ‘maximizes engagement time’. Figure 4.2 above, depicts the forms of knowledge that reflected association with this intervention. This intervention was informed by mathematical forms of knowledge such as: foundational gaps, perseverance, strengths and errors. Through maximized learning engagement, Beth helped students overcome gaps, errors and persevere. She also had students that exhibited particular strengths help classmates and facilitate discussions with them. I asked Beth in a check-in why she had
students work together. My original understanding was that Beth valued collective components in teaching. Her answer confirmed a different intent. She said, “because there is only one of me and 25 of them” (check-in). Having students help each other was a way for Beth to maximize everyone’s learning engagement.

This intervention was also informed by other nonmathematical forms of knowledge. As seen in the examples above, Beth maximized learning engagement consistently, paying attention to students under particular conditions such as those in suspensions or needing behavioral counseling (personal conditions). Additionally there was an association to forms of knowledge on dissonance with school culture and on conflicting priorities. As described on intervention 1 (expands the boundaries of the learning space), Beth experienced challenges with having students consistently do their homework. She also stated on multiple occasions that “their 46 minutes” were the only times within her control (check-ins). Maximized learning engagement was found to respond to challenges on completion of homework and ongoing preparation that were expected to take place outside the classroom.

**Findings: Central phenomena in Beth’s practice.** I now revisit Beth’s interventions to take a closer look at central phenomena in her practice. These phenomena include her perspective on learning, the interplay between her interventions and her teaching goals and her understanding of her students’ mathematical learning experiences.

**Beth’s Interventions and her perspective on learning.** Beth made a strong use of psychology, as well as reflective tasks and prompts for her students. Her use of psychology was evident for both, mathematical learning needs and non-mathematical learning needs. Mathematically, Beth prompted and expected her students to pay attention to why and how they interpreted and applied the mathematical expressions that they worked with. In non-
mathematical aspects, Beth would also communicate to students through questions or by posing statements that prompted students to consider how their behaviors affected others (or themselves) and their consequences. For example, on various occasions I heard Beth tell her students the phrase, “perception is reality”. She used this phrase to help students re-think situations and to re-think how these situations had an impact on others (check-in and post-observation interview). At the post-observation interview, I shared with Beth that I had noticed this pattern in her behaviors. Beth’s responses best reflected her perspective on learning.

I asked Beth if there was anything in her background or experiences that led her to this choice. Although my original question did not point to either mathematical or non-mathematical aspects of her students, Beth’s response incorporated both. She explained that when she was in her senior year in high school she took Pre-Calculus, but she wasn’t “that AP Calc AP student” (post-observation interview). She said she did like numbers, but that she had “her struggles”. Beth stated:

So I think that helped me, because I wasn’t the genius kid, I had to focus on how am I thinking about this. And then in college, I worked in the math lab a lot, so even working with classmates, I had to figure out “why are they getting so stuck?” (post-observation interview)

Mathematically, Beth recognized that learning needed a sense of self-awareness and focus on how thinking was taking place. This was consistent with the ways that Beth asked her students to approach their work in the classroom. Beth created a learning environment that facilitated a focus on self-awareness in how thinking was taking place. Beth’s psychology-type of strategies transcended into non-mathematical learning areas as she helped students find ways to develop behaviors that would support their learning. During the interview Beth continued to explain that where she taught had an impact on her need to look for ways to help her students develop behaviors to support their learning. She noted there was a difference between where she taught
and where she had learned as a student. She described her own learning experience as “good morning, you will now work on problems one through fifty even, and keep it quiet”. She said that would work for half of her students, “and the other have would go insane” (post-observation interview).

**Interplay between Beth’s interventions.** Beth expanded the boundaries of the learning space to make students’ learning more accessible. She also demonstrated maintaining high expectations in learning that supported her need to look for ways to have her students continue learning even after assessments had been taken and graded. Beth maximized their learning engagement in the classroom, but in also expanding the boundaries of their learning space, she looked for ways to develop the understanding that learning needed to continue to take place outside the classroom. Beth learned from her students that they responded to her when she demonstrated genuine care for them, and she used this to re-frame their roles as students, as something that was good and as something that they needed to care about as much as she did. The interventions I have described up to this point made learning more accessible, but in a way that positioned students to reciprocate to Beth’s efforts to support their learning. “Meaningful learning”, using Beth’s own words, was further supported in two different ways. Beth used psychology and reflective techniques to help students focus on their thinking and this focus transcended to both mathematical and nonmathematical aspects of their learning. She facilitated meaningful mathematical conversations to foster classroom norms where students focused on how they were thinking, making particular use of the structure and language of math.

During the pre-observation interview, Beth described herself as consistent and flexible. While there was evidence in the data to support both descriptors, my interpretation is that Beth held unwavering high standards on her own practice and this, in turn, resulted in her consistency.
Beth demonstrated flexibility in that she found different ways to help students meet her learning expectations. My interpretation, however, was that Beth demonstrated being highly resourceful and creative. She looked for alternative ways to make learning more accessible. She made use of as many resources as possible (e.g. family, guidance counselors and students themselves) to advance her students’ learning.

**Model of the learning phenomena in Beth’s case.** Up to this point I have presented and described the forms of knowledge of the student used by Beth as well as how these have informed her interventions in the classroom. I now present a model for students’ learning experiences in Beth’s classroom. I used the model in Figure 4.3 as an analytical tool to further understand and explain the learning phenomena in Beth’s practice. I describe the components in this model. I end this section with examples from students’ learning experiences depicted through the model.

**Interventions.** Beth used different forms of knowledge of her students to inform the teaching interventions she implemented in her practice. These forms of knowledge and their association to each intervention were depicted in Figure 4.2. Beth’s interventions revealed a form of dual preparation. While students learned math, they also learned how to support their own learning through different forms (e.g. asking questions, coming for help, writing their thinking).

**Buy-In.** In the process of implementing these interventions, I noticed a high incidence of attempts from Beth to help students learn how to support their own learning. Through our discussions of her particular students’ experiences at the post-observation interview, I confirmed a pattern I had observed from classroom observations and check-ins. Beth’s students were at different stages in a gradual development of personal skills to support their own learning. The
The first step was buy-in or trust in the learning process, which Beth had particularly supported through her demonstration of genuine care. She had also supported it through her high expectations. Students did not necessarily develop full buy-in or trust at the beginning. This was more of an initial step into a cyclical process of additional work between Beth and her students.

**Figure 4.3. Model of Learning Phenomena in Beth’s Case**

*Skills to persevere mathematically and non-mathematically.* Beth’s continuous work through her interventions tended to students’ needs mathematically and non-mathematically. Students were expected to support this learning through their preparation, their questions and through their communication of their thinking inside and outside the classroom. In Beth’s case these perseverance behaviors were closely related to her conception of college and career.
readiness skills. Beth valued them highly because they did not just serve as supports for learning, they often times preceded learning (see Steve’s case below). Thus, while there was mathematical learning in the classroom, meaningful learning needed attention to the other non-mathematical components. But, the data shows that these skills were not developed overnight. Beth described her work as one that needed consistency and time. This meant that the process required for students to also continue persevering in learning both mathematically and non-mathematically.

**Experienced success.** Students’ work with Beth led to an initial improvement that helped develop additional buy-in and trust.

**Ownership over their learning.** The model reflects a cyclical phenomenon where the consistent and continuous engagement in: buy-in – perseverance – success led to an outcome on students’ ownership over their learning. This ownership however, entailed different types of behaviors depending on the student. In some cases, ownership required that a student learned to manage a personal challenge (Selena) in order to continue supporting her own learning. In other cases, ownership required developing behaviors that a student may not have been comfortable with before (Steve). I used double arrows for this outcome in the model because Beth described the development of these behaviors as taking place over time.

I end this section by revisiting some of Beth’s students to describe their learning experiences through this model.

**Selena.** Selena used to work on her own. A few times I saw Hannah ask Selena a question where she sat, but for the most part, Selena was one of the few students that did not join a group. I had noticed during the observational period that she asked Beth questions and that Beth would sometimes provide a quick answer. Other times Beth would take her answer to the
board and work out the problem with her. While Selena waited to get help from Beth, she would move her arms and body as if she was swaying to a song in her head. She often moved her arms as if she played the drums. Up until the post-observation interview, my field notes on Selena recorded the questions from Selena to Beth and their interactional dialogue to work through a problem. During the post-observation interview, I asked Beth to describe her thoughts on how her students did on the test and to pick any particular student examples that in her opinion reflected learning improvement. Beth picked Selena.

I did not recognize her by the name, because Selena did not look on appearance Hispanic, but her name and her parent’s name (from the informed consent form) reflected otherwise. Beth proceeded to explain to me how she had made arrangements with the Vice-Principal’s office because of Selena’s behaviors. Beth explained that she would get angry as she worked on problems and would start swearing. Although expanding the boundaries of the learning space was a form of support for Selena, based on the observational data, Beth worked with her through meaningful mathematical conversations. Beth also said that in her case, she learned to “let her ride it out”. Beth had her work on consequences with the office and then start fresh the next day. Beth also noted that seeing Selena persevere through the test without outbursts was an accomplishment. Selena reflected trust in Beth in the way that she looked to work with her. According to my data, Selena had also reflected perseverance in the observation period. There were no outbursts that disrupted class. In our discussions about Selena, Beth pointed out that she not only had a good grade (80), she had also reflected confidence in her skillset through her responses to Beth’s reflective questions at the end of the test. Beth had done all her homework and had completed the review problems. She had demonstrated ownership over her learning.
Steve. Steve was another student that worked independently in the classroom. He did not seat near Beth though. I had noted that he often times put on head phones during classwork. He did not wear the headphones during class discussions, but he also did not contribute answers to these discussions. Beth identified him as another student that had reflected improvement. He used to not ask questions in class. Beth said that she continued to encourage him to come for help and to ask questions, until he started to use the texting App. A short excerpt from Steve’s texting trail is included in Appendix F. The full trail evidences Steve’s questions throughout the year. His first question was: “ion [sic] understand. all these notes, still don’t [sic] know how to find range smh. i think yuh [sic] shou [sic] post more of specific notes that actually tells me how to do the steps than class videos” (Steve, app text trail). Steve’s first question in the app reflected a lack of buy-in into Beth’s flipped lessons. Steve’s eventual continued use of the App evidences how the intervention of “expands the boundaries of the learning space” helped Steve. Beth’ responses also demonstrated genuine care. This was the beginning of Beth’s response to Steve’s first post:

Steve 2:22am is admirable!! Thank you for trying. Class videos and what I gave for the notes is exactly what's needed to do this. What's missing is you asking questions in class! Ask!!! Waaaaaay back when we started this topic, as soon as you weren't seeing where something came from, I need you to ask. (Beth, app text trail).

This response also demonstrates Beth’s efforts to help Steve develop the necessary behaviors to support his learning. In explaining to Steve when he needed to ask questions, Beth was trying to help Steve develop a set of skills that would help support his learning. This was a form of skill that empowered Steve to persevere through learning. Beth explained that with time, Steve used the app more regularly. Beth explained his learning as a two step process. She said, “well, it started with an improvement on effort, accuracy came later” (post-observation interview). In this case, Steve’s learning cycle demonstrated the need to develop nonmathematical skills that
were associated to perseverance and that resulted in his ultimate learning. In developing a regular cycle of communication, doing his work outside of class and checking his understanding through the app and pictures of his work, Steve demonstrated ownership over his learning.

Hannah. Specifics about Hannah’s case were presented in table 4.5 (Language). I revisit her case here to showcase a different outcome from the cycle of consistent work with Beth. In Hannah’s case, according to Beth, she benefited from a flipped classroom because it allowed Beth to give students such as Hannah more one-on-one attention and time to engage in individual conversations. In Hannah’s case, she needed to develop the skillset to communicate her thinking. This was a skill that she needed in order to persevere in her learning. Beth also added that as Hannah spoke more about her work, she was able to also write more to support her understanding. Beth said that Hannah was reflecting improved performance in her tests. In class, there were multiple instances where Hannah approached classmates for help and also worked with classmates in groups. My interpretation is that Hannah made improvements through her learning cycle. She was making progress in the process of developing the skillset necessary to continue persevering and evidencing full ownership over her learning.
Case 2: Shannon – “You Go Ahead and Explain It”

The observational period was conducted over a period of four weeks in an Algebra 1 classroom during the months of April and May of 2017. Shannon was on her third year of teaching in the Sheltered Language Instruction program at Sundryville High School.

Background information: Sheltered Language Instruction Program (SLI) and its math course options. The SLI program was implemented as a way to increase educational support for students that are English Language Learners (ELL). All instruction was in English, but students took their classes separate from the mainstream classrooms. The SLI teaching team had seven teachers and two bilingual tutors. Shannon was the only math teacher in this team. Students placed into this program primarily based on scores from their language arts assessment (e.g., LAS Links). Some students were automatically placed into the program if their first language was known to be different from English and if their arrival to the country was recent (demonstrating need for language support). Once students placed into the SLI program, they had English language acquisition goals embedded into their typical student learning objectives into all disciplines. This meant that Shannon taught math, but she also provided language acquisition support. Placement out of the program was determined on a case-by-case basis, but the most common approach, according to Shannon (check-in), was to have students go off team for science and/or math courses while they remained on the SLI team for other courses like language arts and social studies. They considered this practice in their program to allow students to better transition into mainstream classrooms. Although final decisions to have a student go off team were done at the team level, Shannon’s input for staying on or off team for math was heavily taken into consideration (post-observation interview).
Sundryville High School’s SLI program was growing. Some students came directly from the middle school. Other students came into the high school as recent arrivals into the country (or from other school districts) during the summer or throughout the academic year. Shannon described a high level of transiency for her students prior to their arrival to Sundryville High School. She also said that in her experience, students gravitated towards this school because of the support systems in place. Transiency seemed to be lowered once students arrived to her school (Shannon, check-in). This also meant that the size of her classes continued to grow.

During the year of the observational period, Shannon had received 32 new students that were not accounted for through middle school enrollment numbers. These students had arrived later in the year (Shannon, pre-observation interview). Shannon was in charge of placing students into their math courses once students were determined to be part of the SLI team.

At the time of the study, there were three different course options in the program. These were: Math Intervention, Pre-Algebra and Algebra 1. The Math Intervention course was the lowest level of placement. Shannon designed this course herself as well as the test to determine placement into this course. While the primary objective of this course was to develop foundations of arithmetic, the course also aimed to develop familiarity with particular English language terms that are frequently used in math. Shannon showed me some of her instructional materials during a check-in, and explained that students needed to become familiar with these terms, particularly for the purpose of comparing quantities (e.g. “greater vs. less”, “more than”, “same”, etc.). She used pictures along with the words to help students understand their meaning.

In cases where the terms were complete opposites or associated along a continuum, her instructional materials also positioned the terms so as to develop a sense of sequence and/or degree of association. Based on Shannon’s experience, students that placed into this course
tended to need more instructional support in understanding classroom norms like, coming prepared to the classroom with their school supplies. Shannon explained that in some cases, her students had never been in a classroom or had been out of school for an extended period of time as a result of their long journeys into the country. The Pre-Algebra course focused on algebraic skills such as: simplifying expressions, evaluating expressions, solving linear equations and basics of linear functions. I reviewed the eighth grade middle school exit test that was created by the high school’s Algebra 1 data team. Based on this review, my interpretation is that the Pre-Algebra course focused on Algebra 1 topics (e.g., solving linear equations, graphing and writing equations of lines, finding slopes and intercepts).

Being the single math teacher in the SLI program carried some limitations and some benefits for Shannon and her students. For example, by contract, Shannon was only allowed to teach three preps. Many of Shannon’s Algebra 1 students were in ninth, tenth and eleventh grade, but they were not sufficiently proficient in English to place out of the SLI team upon completion of that course. Shannon asked for permission from her union so that she could be allowed to teach Algebra 2 the year after the study was being conducted. Since the other three courses were still needed, the additional Algebra 2 course represented a fourth prep that was not within her contractual stipulations. Shannon explained that she wanted to provide this choice for her students within the SLI program. I asked Shannon if the program had ever offered a Geometry course for her students. She explained that the school’s Algebra 1 and Algebra 2 courses were sequenced back to back, meaning that all students that completed Algebra 1 were typically scheduled to take Algebra 2 the following year. She also added that the new SAT exam had reduced its focus on geometry. She was more concerned for ensuring that her students followed the same sequence as the mainstream students because she could not necessarily predict
when a student would be ready for movement off team. Shannon wanted her students’
preparation to match as best as possible that of the mainstream students, which was a big priority
to Shannon (Shannon, check-ins, pre-observation and post-observation interview). As the single
math teacher, this also meant that students in the program always had Shannon as their math
teacher. Shannon did not consider this a limitation. She actually considered it an advantage
because it allowed her to follow up on any particular areas that needed strengthening and it
allowed them to get to know each other very well.

Placement into a course did not depend as much on students’ grade level. According to
Shannon, students that brought their transcripts with them were the easiest to place, but this was
not the case for many of them. The students in the Algebra 1 course from the observational
period were for the most part ninth and tenth graders. Some started in Pre-Algebra the year
before, and others started the program that same year in Algebra 1 (e.g. they arrived after school
started). Elia was the oldest student in Shannon’s Algebra 1 class as an eleventh grader. Figure
4.4 depicts a general progression in math courses within the SLI program, recognizing that
students’ starting course could have been anywhere within this progression.

![Math Intervention → Pre-Algebra → Algebra 1* → Algebra 2 (NEW)](image)

Figure 4.4. Students’ Math Course Learning Paths in SLI Program in Sundryville High School
* Shannon’s Algebra 1 course (observation period)

**Background information: Specifics on Shannon’s course.** Shannon’s course
attempted to match the same topics as those taught in the mainstream Algebra 1 courses. The
topics in these courses were aligned with the topics from the common core. Shannon constantly
compared her pace to that of the mainstream courses, but she also recognized that she would not be able to cover as many topics as the mainstream courses would. Her lessons required more time. Shannon explained that she purposely allocated additional lesson time per topic because her students needed it to process information. All instruction was in English, and this was a language that her students were in the process of acquiring and/or becoming proficient with (recruitment interview, check-ins). I conducted the last of the post-observation interview sessions after final exams had been graded. At that point I was able to compare the lesson coverage between the mainstream classrooms and those in the SLI program. Out of the targeted ten units, the mainstream classrooms where able to get to their ninth unit (quadratic functions). The SLI program students were able to get to their eighth unit (properties of exponents). The observational period was conducted during their seventh unit on systems of linear equations. All Algebra 1 courses started with unit zero at the beginning of the semester for a review of integer operations and other eighth grade topics.

I include a list of the topics covered during the observation period in table 4.6. I note that the unit did not include real-life applications of systems of linear equations. This was consistent with mainstream classrooms. I asked Shannon why this was the case. She said that it was a decision at the Algebra 1 team level. She also added that she had heard that there were considerations to eliminate the unit altogether. Because of this, she was happy that at least they were continuing to keep the unit in the list of topics for the course (Shannon, check-in). Shannon was not able to make Algebra 1 team meetings at the school level because her block for meetings was typically used for SLI team meetings. The SLI team met three days per week.
Table 4.6. Topics Covered During Shannon’s Observational Period

<table>
<thead>
<tr>
<th>Topic Taught</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solving Systems of Linear Equations by the Graphing Method</td>
</tr>
<tr>
<td>Solving Systems of Linear Equations by the Substitution Method</td>
</tr>
<tr>
<td>Solving Systems of Linear Equations by the Elimination Methods</td>
</tr>
</tbody>
</table>

Classroom activities varied by day, but they always included time for group work. On days of new instruction, Shannon made use of the smartboard to guide students through her lesson. She would ask questions as the lesson progressed to check for understanding. She would also walk students through examples and then have them work on examples themselves in pairs and/or in groups. When she discussed answers to questions, she would watch students’ behaviors. In one class she said, “I see a few of you erasing your work. That must mean that there are questions” (observational period). Shannon also walked around as students worked on their problems. In cases where she noticed that students had similar questions, she would regroup everyone and share her explanations for these questions. Some days were fully dedicated to completing what they called, student interactive notebooks. These were guided sheets that students filled in with procedural instructions for each topic. The sheets were then cut and glued onto a notebook so that students kept all procedures organized by unit. Maintaining these interactive notebooks was part of the course grade. I describe the graded components that Shannon used during the observation period in table 4.7.
Table 4.7. Graded Components in Shannon’s Algebra 1 Unit on Systems of Linear Equations

<table>
<thead>
<tr>
<th>Graded Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interactive Notebooks</td>
<td>There was a grading chart for interactive notebooks. The notebooks needed to have an index by date. They needed to be complete with all numbered items and they needed to be organized using color schemes. These color schemes helped make a distinction between procedures and examples.</td>
</tr>
<tr>
<td>Homework</td>
<td>Homework assignments were not typically long, often times requiring that students finish worksheets they had started in class. Based on my observations, students typically had four or five questions to complete at home. A few of the students finished their work during class time. These worksheets were sometimes the same used by mainstream classrooms. Shannon described making modifications when she felt that the problem selection did not provide comprehensive or more rigorous examples (e.g. examples that required students to recognize if the systems were consistent or not, through algebraic methods). Students with a homework average lower than a 70% were given a required log sheet that counted as a quiz grade. Students needed to evidence with these sheets that they had come in for help to either the tutors in the lab (housed in the library), to her, or to other teachers.</td>
</tr>
<tr>
<td>Quizzes</td>
<td>There were three quizzes (one for each main objective included in table 4.6). Shannon typically answered questions before a quiz and then provided an additional practice activity afterwards. One of the quizzes was given in her absence and administered by a substitute teacher.</td>
</tr>
<tr>
<td>Unit Tests</td>
<td>There was one unit test. Unit tests were often times the same as those used by the mainstream classrooms, but Shannon described making modifications when the problems were not in alignment with the type of practice provided. For example, if a problem involved the use of fractions and none of the practice from the lessons had required it, she would take them out from the test. In this particular unit test, Shannon used the same test as that of the mainstream classroom and found herself making a problem optional (instead of required) because the solution was an ordered pair that had fractions. The tests included multiple choice questions to help students practice for the SATs. Although Shannon explained in class that they needed to show work for all problems in multiple choice format, she gave credit if students had selected the correct letter option without work to support their answers.</td>
</tr>
</tbody>
</table>
Background information: Shannon’s students and classrooms. Shannon’s algebra class had ten students. There were two male students and eight female students. Except for Lexa, whose first language was Russian, all other students spoke Spanish as their first language. Based on Shannon’s descriptions, four of her students had just arrived within that academic year. The class had a combination of ninth and tenth graders. Elia was the only eleventh grader in the classroom.

The room entrance was on the left side of the room, set along the wall with the front board. The classroom had five rows of seats facing the front board. Students sat on the left three rows, closer to the room entrance. The other two rows were left empty. These two empty rows were closer to Shannon’s computer desk. I sat at a small table off from Shannon’s desk. The back wall was mostly large glass windows with a ledge where Shannon placed crates for students to store their worksheets and graded work. The wall that was closest to Shannon’s desk had charts that traced average class scores on their language assessment tests and their discipline performance tests (the Marker⁹). The charts also included the target scores to demonstrate progress.

All students exhibited full engagement in their lessons during the observation period. They helped each other. Sometimes students would go over to another group when a classmate had asked for their help. I also did not notice any classroom disciplinary issues during the observation period. The only times I saw a student off task was when he or she got permission from Shannon to do work for another class after completing all the work assigned for that day. Having noticed such level of commitment in class, I asked Shannon if this was a typical pattern. She explained that her students valued school and learning (check-in). Their challenges were

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⁹ The Marker is a fictitious name I have given to their district math assessment. Students with scores on the language assessment tests that were below a certain threshold, were not given the Marker test.
more closely associated with outside of school issues, like difficult living conditions. The three students in Shannon’s class that were at risk of not passing the course, were according to Shannon, performing low because they did not come in for help or did not make up work from their frequent school absences.

The findings sections that follow make reference to some of Shannon’s students to provide select examples of their learning experiences. In an effort to assist the reading of the findings section, I provide in table 4.8 key characteristics about these students based on the full case data.

**Findings: Shannon’s teaching goals.** Shannon’s goals were first triangulated from interview and observational data. I then confirmed these goals through member checks in the post-observation interview. I list Shannon’s goals below, but I revisit them again when I describe the teaching interventions that Shannon implemented to support these goals.

1. To have students know the Algebra 1 content and develop the necessary mathematical skills to be prepared for their subsequent Algebra 2 course (within or outside the SLI team)
2. To have students demonstrate advocacy for their learning as indicated by their ability to: ask questions, come prepared to class, and seek help outside the classroom when needed
3. To develop students’ organizational skills to support their ongoing learning at the high school and in college or in their future careers
Table 4.8. Shannon’s Referenced Students and Key Characteristics

<table>
<thead>
<tr>
<th>Students</th>
<th>Particular Characteristics Referenced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lexa</td>
<td>Lexa’s first language was Russian. Shannon described her as a student with a strong math ability and strong organizational skills (i.e. she came prepared to class). Lexa had just arrived that quarter to their class. I referenced Lexa here as the only student that did not speak Spanish.</td>
</tr>
<tr>
<td>Elia</td>
<td>Elia was one of the oldest students in the classroom. She often times finished her work early. She helped her classmates and reflected eagerness to explain her understanding. Elia’s brother was in the same classroom (Hector). Elia watched over her brother in class. In one class, for example, he was falling asleep. Elia turned to him and said, “eso te pasa por estar jugando videos hasta las cuatro de la mañana” [that’s what happens when you stay up until four in the morning playing videos]. Elia and Hector had just arrived to the country within months of the observation period and were acquiring English language skills very quickly. Shannon noted that their parents were very supportive about their education. I reference Elia here as a student that shared her understanding in class.</td>
</tr>
<tr>
<td>Antonio</td>
<td>Antonio demonstrated a strong ability to recognize patterns and to understand conceptually the procedures they learned in class. He was often times helping his classmates, looking over their work and giving them suggestions on how to go about solving a problem. For example, in one class, one student continued to transpose her x’s and y’s to graph a point. Antonio told her, “pon el punto en el cuatro primero para mejor ubicarte” [place the dot on the four first (that was the x-coordinate) so that you can get your location better]. I reference Antonio as a student that was eager to help classmates and share his mathematical thinking.</td>
</tr>
<tr>
<td>Ana</td>
<td>Ana was another student described by Shannon as older (in age) than her peers. Ana had gone for an extended period of time without being in school before her arrival to Sundryville High School. She had taken Pre-Algebra with Shannon within the SLI program the year before. Ana was frequently absent to class. Shannon also shared that Ana was used to carrying typical adult responsibilities with siblings and extended family. I reference Ana here as an example where Shannon used a relational form of knowledge as part of her criteria in determining a students’ math placement.</td>
</tr>
<tr>
<td>Grisselle</td>
<td>Grisselle was Ana’s best friend. They sat near each other and always worked together in class. Grisselle was also absent often from class. She helped take care of her older sister’s child. I reference Grisselle because she was part of Shannon’s decision making process for Ana’s placement.</td>
</tr>
<tr>
<td>Yadira</td>
<td>Yadira lived with her aunt. The rest of her family was still in her country of origin. Yadira took care of her aunt’s child (her cousin) and was also in charge of most of their house chores while her aunt worked. I reference her here as a student that made sculptures for Shannon. Yadira was one of the two students that Shannon recommended for exiting the SLI program in math at the end of the year. My discussions with Shannon on what makes a student successful in the mainstream classroom were grounded on Yadira’s (and her classmate’s) characteristics.</td>
</tr>
</tbody>
</table>
**Findings: Forms of knowledge of the student used by Shannon.** I summarize the forms of knowledge of the student that Shannon used to advance her students’ learning in table 4.9. I also include their corresponding definitions formulated from and contextualized in Shannon’s case data.

Table 4.9. Forms of Knowledge of the Student Used by Shannon
Mathematical – MK and Non-Mathematical – NON

<table>
<thead>
<tr>
<th>Form of Knowledge</th>
<th>MK (math) vs. NON (non-math)</th>
<th>Definition and Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language^O</td>
<td>Both</td>
<td>Mathematical – This form of knowledge refers to students’ use of mathematical language throughout the unit of study. Students constantly used mathematical language in both Spanish and English. Some of these included: coefficient, slope, parallel, perpendicular, opposites, intercepts, standard form, etc. Non-mathematical – As a sheltered language instruction classroom, this form of knowledge was used every day. Students contributed to classroom discussion in English because Shannon wanted to make sure that Lexa (only non-Spanish speaker) was not left out (check-in).</td>
</tr>
<tr>
<td>Mathematical Thinking^N</td>
<td>MK</td>
<td>This form of knowledge is associated to students’ thinking process while they engaged in doing math. During class activities (outside whole-group discussions), students were encouraged to discuss with each other their thinking in their native language. Shannon explained that allowing students to describe their mathematical thinking in their first language as more effective for them (post-observation interview).</td>
</tr>
<tr>
<td>Personal Conditions and/or Challenges^N</td>
<td>NON</td>
<td>Shannon described different types of living situations that her students handled on a daily basis as a result of the migratory status. Some students lived with extended family while their close family relatives lived in their country of origin. This status added responsibilities to contribute to the upkeep of their temporary homes. Some students took care of siblings or of</td>
</tr>
<tr>
<td>(Migratory Status)$^N$</td>
<td>their siblings’ children. Other students worked late hours at restaurants and/or service type of businesses. These circumstances threatened students’ ability to balance their school work with their personal conditions.</td>
<td></td>
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</tr>
<tr>
<td>Emotional Wellbeing$^N$</td>
<td>Shannon explained that students sometimes had good days and sometimes they had bad days in terms of how they felt emotionally. This form of knowledge was directly associated to students’ personal challenges in their migratory status.</td>
<td></td>
</tr>
<tr>
<td>Relational$^N$</td>
<td>This form of knowledge refers to the ways that Shannon has learned to relate to her students and to get to know them. She used humor, which seemed to be well received based on my observations of their interactions. Her humor often times used what she had noticed about them in the classroom. For example, one student had shared with her that she thought that parallel lines were an unfulfilled love story of two people that never touched. During review day, Shannon placed a heart next to a system with parallel lines and said that she was doing it because it reminded her of her students’ comment. At the end of the course in the final exam, that same student placed a broken heart and a sad face on a problem with parallel lines to remind Shannon of their shared joke. This form of relational knowledge is associated to “emotional wellbeing” in that Shannon needed to ‘read’ her students to understand if they had a good or bad day. Shannon gave her students space when they were having a bad day (check-in).</td>
<td></td>
</tr>
<tr>
<td>Mathematical Structure and Symbolic Representation$^O$</td>
<td>Shannon paid attention to students’ interpretation of mathematical structure. In one class, for example, Shannon was going over a system of linear equations that did not have a solution. A student noticed that the lines had the same slope. Shannon used the students’ input to bring connections to what they knew about parallel lines. Shannon also looked for their interpretation of the structure of the equations they worked with to help them make additional connections. For example, if the equation of a linear function indicated that the slope was negative, but the student graphed the line with a positive slope, Shannon would ask the student to show her their algebraic work. She would seek their interpretation from their equations and/or written work. She also emphasized the need for students to write their work, which involved algebraic manipulations.</td>
<td></td>
</tr>
<tr>
<td>Knowledge Category</td>
<td>Knowledge Type</td>
<td>Description</td>
</tr>
<tr>
<td>--------------------</td>
<td>----------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Interpersonal Relationships&lt;sup&gt;N&lt;/sup&gt;</td>
<td>NON</td>
<td>This form of knowledge refers to particular relationships among classmates. For example, Hector and Elia are siblings in her classroom. Shannon was very aware of Elia’s protective nature to watch out and take care of her brother. When Hector seemed inattentive in class, Shannon would check with Elia or let Elia herself press on Hector to make sure that he was attentive. Shannon was also aware of friendships between students and looked for ways to ensure that they influenced each other positively (post-observation interview). A more detailed example is provided in Ana’s case.</td>
</tr>
<tr>
<td>School Readiness&lt;sup&gt;N&lt;/sup&gt; (Cultural Dissonance)</td>
<td>NON</td>
<td>This form of knowledge refers to a difference in students’ lived experiences and Shannon’s or those at the school. Shannon had noticed that many of her students have not been in school for an extended period of time, especially when they have been traveling or transient as a result of migratory status (check-ins, pre-observation interview). Shannon described student indicators that help her determine if/when a student is ready to go off team. One of these indicators was organizational skills (check-ins, post-observation interview). I include these organizational skills as part of a broader category of “school readiness”. Shannon described other associated skills that I did not observe with the particular students in class. I include them here as a form of collective knowledge with the understanding that it is not necessarily applicable to all students: coming to school prepared with supplies, knowing where to store personal belongings, etc.</td>
</tr>
<tr>
<td>Ownership Over their Learning&lt;sup&gt;N&lt;/sup&gt;</td>
<td>NON</td>
<td>This form of knowledge is a combination of indicators that Shannon described as: willingness to come for help, to ask question and/or to advocate for their learning. Shannon placed as much importance to these indicators as to content knowledge when deciding if a student should go off team for Math (post-observation interview).</td>
</tr>
<tr>
<td>Alternative Thinking Approaches&lt;sup&gt;O&lt;/sup&gt;</td>
<td>MK</td>
<td>This form of knowledge refers to students’ particular ways to approach a problem mathematically that may be different from what Shannon had suggested in class. When highlighting the value of conversations in students’ first language, Shannon shared that she appreciates when she describes a problem to a student and then she hears that same student describe it in a different way that is still correct to another classmate in Spanish.</td>
</tr>
<tr>
<td>Learning Attitudes&lt;sup&gt;O&lt;/sup&gt;</td>
<td>MK</td>
<td>This form of knowledge refers to students’ eagerness and willingness to learn. Shannon described herself as fortunate in being able to work with students that exhibited interest to learn.</td>
</tr>
<tr>
<td>-------------------------------</td>
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<td>----------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematical Strength Areas&lt;sup&gt;O&lt;/sup&gt;</th>
<th>MK</th>
<th>This form of knowledge refers to the particular areas where a student demonstrated strengths mathematically. For example, Antonio exhibited a strong ability in understanding conceptually the unit topics. At the same time, he was challenged by careless mistakes and by not paying attention to minor errors in his work. Shannon described his strengths and weaknesses and used it to explain to him what he was good at and what he needed to improve on. Antonio was also particularly helpful in explaining his thinking to classmates and in helping them with their questions during group work.</th>
</tr>
</thead>
</table>

| Foundational Gaps and/or Mathematical Weaknesses<sup>O</sup> | MK | This form of knowledge refers to particular areas where a student reflected mathematical weaknesses. During the observational period, some of these weaknesses were: operations with signed integers, operations with fractions and procedural knowledge (e.g. solving linear equations, etc.). Operations with fractions were minimal because most of the problems focused on integers. In the few occasions that fractions were used, students found difficulty with them regardless of ability level.  

The students that Shannon described as needing most of her help were also the ones evidencing a higher frequency in foundational errors. |
|-------------------------------------------------------------|----|----------------------------------------------------------------------------------|

<table>
<thead>
<tr>
<th>Personal Interests&lt;sup&gt;N&lt;/sup&gt;</th>
<th>NON</th>
<th>This form of knowledge describes students’ personal interests outside the classroom. For example, Shannon shared with me that many of her students are very creative. For example, one of her students, Amara, sculpts. She showed me some of the presents that Amara made for her. One of them was a figure of a mother embracing her child.</th>
</tr>
</thead>
</table>

**Findings: Shannon’s teaching interventions.** Shannon demonstrated five central teaching behaviors in using the forms of knowledge of the student from Table 4.9. I describe these teaching behaviors as interventions going forward in this document because they represent a set of particular measures that Shannon implemented to advance her students’ learning based on what she knew about her students. These interventions were: (1) uses a flexible lesson pace,
(2) builds on relationships, (3) modifies instructional materials from the mainstream classroom (4) establishes classroom norms that encourage group support and conversations in students’ first language, and (5) provides a comprehensive support program.

In the subsections that follow, I provide an operational definition for each intervention. These definitions are based on Shannon’s context of instruction and on her descriptions of how she used what she knew about her students and why. For each intervention, I also report the forms of knowledge of the student that informed each interventions’ intended purpose in Shannon’s practice. These are summarized in Figure 4.5.

1. *Uses a flexible lesson pace: Definition.* Shannon allocated more lesson time to each unit objective than in the mainstream classrooms because her students needed more time to process information in English (Shannon, check-in and post-observation interview). For example, she pointed that sometimes it took her twice the time to explain instructions and to get students to understand what she was asking them to do. Shannon expressed relief in that she had been given the freedom to determine her own lesson pace. This was different from the pace expectations in the mainstream classrooms where all classes needed to allocate the same number of lessons to unit objectives. Still, during the check-ins and during the post-observation interview, Shannon’s responses reflected a need to balance competing goals: allowing students’ needs to dictate pace, versus following pace to ensure that topic coverage was comprehensive. Shannon noted –“sometimes you just have to move on” (post-observation interview).
Figure 4.5. Forms of Knowledge of the Student Informing Shannon’s Interventions
For the particular class under observation, Shannon had also pointed that students’ English skills “were not as low” (Shannon, check-in). I wanted to understand her perspective on how the flexible pace actually helped these students learn, given that they did not seem to need as much time to understand instructions as what she had described for other classrooms with lower English skills. Her response shed light on the need for a flexible pace to develop students’ comfort level to engage in learning, regardless of stage or extent of English language acquisition.

This is what Shannon said:

So I would have to give instructions a couple of times. But you still have some students whose language skills are still a little bit lower. And as you saw, they help each other out. We obviously get off topic, you saw that. But I feel that in order to be a good teacher and have them respect you and get curriculum and the material across to them, you have to have a relationship, they have to be comfortable in your classroom. I feel that I have created that classroom environment where they don’t have a problem asking me. They don’t have a problem stopping us, and asking me or asking a question in front of each other, or asking a question to each other (Shannon, post-observation interview). Shannon’s assertion was consistent with my observations from her classroom.

Students would take in her explanations, then turn around to a classmate and either translate what Shannon said, or explain it in their own words. This response, along with classroom observation data support an understanding that Shannon was trying to do much more than add time to process language, Shannon was adding time to build relationships with students so that they would be more willing to process and share their mathematical thinking. Flexible lesson pace as an intervention is thus strongly associated with two other central interventions in Shannon’s practice that are described here in the findings section (i.e. interventions two and four).

*Forms of knowledge informing ‘uses a flexible lesson pace’. Figure 4.5 above, depicts the forms of knowledge that reflected association with this intervention. This intervention was primarily informed by students’ language needs as they continued to acquire and develop proficiency in English. As Shannon noted in her assertion above, she also used additional time*
to build relationships (relational) with her students and to develop a sense of comfort. Through this feeling of comfort, students were able to share their mathematical thinking and also advocate for their learning. I interpreted students’ efforts to advocate for their learning as a form of ownership over their learning.

Based on my observations and on my discussions with Shannon about individual students, I also found other forms of knowledge that Shannon used to inform this flexible pace. Shannon’s students handled challenging living conditions (personal challenges). The flexible lesson pace gave students additional time to get caught up and/or to get through emotional upsets (emotional state) that challenged their ability to engage in school learning. As described above, Shannon had to balance the flexibility in lesson pace with her competing goal to maintain a pace closer to that of the mainstream classrooms. Shannon explained that sometimes she had to move on to new topics. In these cases, she expected those students that needed additional time to come for help outside the classroom which relied on their ability to own their learning. Shannon stated, “and at that point, that’s on them” (post-observation interview).

2. Builds on relationships: Definition. This intervention refers to Shannon’s efforts to build relationships with her students so that they felt comfortable in the classroom and in the learning process. These relationships were more closely aligned to working relationships than personal relationships. In one of our check-ins, Shannon clarified that the personal issues that her students handled required in many cases professional support that she was not qualified to provide. Despite her focus on a working relationship, some students reflected having feelings of appreciation for her as a person. Yadira, for example, made art pieces for Shannon. I had seen them in Shannon’s classroom throughout the observation period, but never considered asking
where they came from. Shannon showed them to me when she described students’ characteristics she appreciated about them. She was referring to Yadira’s high level of creativity.

During the post-observation interview Shannon shared that she was in the process of writing a paper for a graduate course on what makes a good teacher. She said that what makes a better teacher is to have a trusting relationship with her students. I asked her to tell me more about it. She said:

It’s all about the experience and having the relationships necessary to be a good teacher. Being cold and not approachable, if your students are scared to ask you a question, if they are not comfortable coming to you personally or for academics, then them coming and asking a question, if they are not comfortable and say, ‘oh, I don’t understand this but I am not going to go ask her because I am not comfortable’, then you and the student have both lost an opportunity to have that student learn. (Shannon, post-observation interview)

I also asked her to clarify what she meant by a lost opportunity to learn. Shannon explained that as a teacher, she would miss the opportunity to understand what they need and to give them an experience to gain knowledge. She also pointed that being their single math teacher allows her to understand her students more. By having her students multiple years, Shannon was able to read “when they have a good day or a bad day” (post-observation interview).

*Forms of knowledge informing ‘builds on relationships’.* Figure 4.5 above, depicts the forms of knowledge that reflected association with this intervention. Based on Shannon’s self-reports, this form of knowledge is primarily informed by personal aspects of the students like: how to relate to them, interests outside the classroom, and their emotional state (i.e., relational, personal interests, emotional state). During the observational period I heard Antonio tell Shannon that he had just started a new job. They talked about where and when he would be working. On a different day I heard Elia ask Shannon – “How do you know that what you are going to study is going to be your real career if you are not doing great at it?” Elia had been given permission to start a year early a nursing course at the high school under their technical
education program (Shannon, check-in). Shannon explained to Elia that she was always good in math, but even though math came easy to her, she sometimes needed to put more effort when she was doing her upper level math courses. She told Elia to not let her experience discourage her and that she should see it through.

Although not self-reported by Shannon, I noticed from my classroom observations that she used their mathematical thinking as a way to relate to her students. Shannon shared with the class particular areas she noticed they needed improvement on (after walking around). She also showed them what she had noticed from their class contributions and remembered their individual mathematical approaches (alternative thinking approaches). In doing so, Shannon built relationships where students felt comfortable coming to her, sharing their thoughts and asking her questions. I best interpret this phenomenon as a combination between a form of appreciation for their contributions to the learning process, but also a reaffirmation that they worked together because it was important to improve and continue learning. The example I provided in table 4.9 under the “relational” form of knowledge reflects Shannon’s use of their mathematical understanding.

Not all relationships between Shannon and her students demonstrated positive results. Shannon and I discussed the case of Ana after I asked Shannon to give me her insight on behavioral patterns I had noticed in her interactions to help Ana and Grisselle (interpersonal relationships). Shannon would talk only to Grisselle. She would look over both of their work, but spoke directly only to Grisselle. Shannon explained that Ana did not like to get help from her. She suspected that because Ana was used to being in charge of many family members, that she was not comfortable being in the position of needing help. Shannon gave input to Ana
through Grisselle. When Shannon moved on to a different group, Grisselle explained what Shannon had communicated to Ana.

3. **Modifies instructional materials from the mainstream classroom:** *Definition.*

This intervention describes the modifications that Shannon made to instructional materials used by the mainstream classroom based on what she knew about her students. This form of knowledge is associated with the first intervention on ‘uses a flexible lesson pace’ because Shannon extended the length of their worksheets and also added special projects that had an impact on the pace of the lesson coverage. Shannon made modifications in the following ways: she instituted a group project assignment for the end of each unit so that students would present to the class what they had learned, she added more problems to the worksheets from the mainstream classroom, she ensured that her problem additions provided a more comprehensive set of problems, and she instituted a math lab grade so that students that had a grade below a 70% in their homework were required to come for help outside the classroom.

Shannon did not have students make projects for the unit in the observational period. She explained that they were running out of time and she really wanted her students to complete one more unit by the end of the semester. I did see some of the posters from earlier units and we discussed the reasons why she added that type of activity to their lessons. Shannon displayed many of these posters on the wall closer to the entrance of the room.

*Forms of knowledge of the student informing ‘modifies instructional materials from the mainstream classroom’.* Figure 4.5 above, depicts the forms of knowledge that reflected association with this intervention. Shannon made different types of modifications to instructional materials based on what she considered a priority in her students’ needs. The group projects helped students develop their oral language skills in English. In this regard, the
presentations were informed by students’ language (non-mathematical). Since I was not able to see any of the presentations, I cannot provide evidence for the use of mathematical language. Based on the project posters I saw, and on Shannon’s classroom norms, I suspect that students would have applied their mathematical language because Shannon’s teaching stressed highly and expected students’ use of mathematical language. Shannon added more problems to worksheets so that students would develop a habit of doing additional practice at home (school readiness). Shannon also wanted to provide opportunities for students to meet her and/or seek help outside the classroom. She wanted her students to develop these skills because, in her opinion, they would be needed in mainstream classrooms and for lifelong learning (post-observation) (ownership over their learning). She did not want the homework to be too extensive because she also recognized that not all students’ homes shared the same level of stability (check-in) (personal challenges).

I note here that there were modifications that Shannon wished she would have made, but did not make them. For example, she said she wished there were more examples with fractions because in her opinion, students need them (post-observation interview). The mainstream classrooms did not review fractions in unit zero. Shannon included them in this review at the beginning of the year, but she took away problems with fractions later on in the semester. She said she felt bad, but that at the same time, she chose not to include fractions because they do not include them in the mainstream classroom. Shannon said, “I know I am contradicting myself…but if they are not going to be exposed to them in Algebra 1 and they are not going to be held accountable in Algebra 2, then why should I?” (post-observation interview). I had also asked Shannon if there was a rationale at the department level for not including real-life applications for systems of linear equations. During this check-in, Shannon shared the Algebra 1
team’s discussions to consider eliminating the unit altogether. She said she heard concerns for low performance in this unit, but that it was related to her by a colleague because she was not present at the meeting. I then asked Shannon if she felt that students were missing out by not having word problems. She said that she thought so. Shannon explained that she included applications in their introduction of linear functions in pre-algebra and showed me sample worksheets. She also added that when they worked on real life applications students sometimes came up with “neat examples” (Shannon, check-in). She shared one incident where a student worked at a shop and he started to model how much soap he used to wash his hands based on how long he spent working.

4. Establishes classroom norms that encourage group support and conversations in students’ first language: Definition. Shannon supported a classroom norm where students were expected to help each other. Shannon walked around to answer questions as students worked in groups. Other times she would seat at her desk to have students come individually for help. She would inspect their work and point to them where they had made a mistake or gave them more direction on the procedures they applied. Whole class discussions where in English, because Shannon wanted all students to feel included and understand each other (Shannon, check-in). While students predominantly spoke Spanish as their first language, one student, Lexa, spoke Russian. But even during whole class discussion, a student would be heard saying, “Que’ dijo?” [What did she say?], and a classmate would either repeat or paraphrase what Shannon said in Spanish. This was a central intervention in Shannon’s classroom. In sharing her perspective on their classroom conversations, Shannon asserted:

And if Elia can answer the question, sometimes I don’t even have to answer the question. One of them will say – ‘oh, she said this’ and they answer it in Spanish. And I can say the same thing five times in English, and I always feel that no matter what, they always end up getting it better if it is explained to them in their native tongue… she will even say
like, ‘miss hold on’ or ‘sorry, can I explain that? And I will be like, ‘Yeah, Yeah. You go ahead and explain it’. (post-observation interview)

Forms of knowledge of the student informing ‘establishes classroom norms that encourage group support and conversations in students’ first language’. Figure 4.5 above, depicts the forms of knowledge that were associated with this intervention. This intervention revealed and amalgam of forms of knowledge associated with students’ communication of their mathematical thinking. Students used both nonmathematical and mathematical language during their conversations. They explained what they did in a problem to help a classmate. They also explained their work in the process of getting help from a classmate or form the teacher. The data set for this intervention is extensive. In summary, these conversations used the following forms of knowledge: students’ mathematical thinking, alternative approaches to problem solving, foundational gaps, mathematical strengths and mathematical structure and symbols. Based on my interpretation of behavioral patterns, conversations among students tended to be much more detailed than those conversations with Shannon when she sat at her desk for individual help. In these cases, students were found to approach Shannon for a quick indication of where they may had gone wrong in a problem. Shannon would place her finger on their work to point where they needed to revisit their approach. This was followed by students’ quick responses like, “ah, yes” or “I got it”. Students would then help each other discussing how to fix the problem afterwards. I was wondering about the impact of this intervention on Lexa since her first language was not Spanish. Based on my observations, Lexa worked most often with Antonio. According to Shannon, Antonio did not have the strongest English skills, but he had stronger math skills (post-observation interview).

Shannon spoke some Spanish, but she stated that she was not fluent enough to follow a full conversation in Spanish (check-in). I make this distinction here because the data set that is
rich with examples on **alternative thinking approaches** and **mathematical strengths and weaknesses** corresponded to students’ conversations in class. While Shannon may not have gotten a full appreciation of the nuances and depth of her students’ mathematical discussions, she valued them and she facilitated a learning experience that she had found to help her students learn more effectively. At the post-observation interview Shannon explained that having students explain their thinking was important. Shannon asserted:

> And sometimes I don’t even know what they said. But if it is math content, like they’ll be explaining it and I will be like, yup. Sometimes it is the same explanation and sometimes I find it interesting that I will explain to one, and then that person goes on to explain it to someone else a little bit different, and I go, yeah, I could have explained it that way too. (post-observation).

Shannon described these conversations as a “good check” for her because the students’ explanation is an indication to her that “the person really understands it” (post-observation). I include in Appendix H sample classroom instances that showcase classroom conversations evidencing different forms of knowledge of the student (including **learning attitudes**)

**5. Provides a comprehensive support program: Definition.** This intervention refers to Shannon’s work as an active contributor to her SLI program team. Shannon shared what she observed about her students with her colleagues during their weekly meetings. The teachers worked together to look for ways to support their students. These types of supports were academic and also non-academic. The team discussed their students’ academic progress regularly. They also looked for resources like outside speakers and specialists to meet their students at the school. For example, one day during the observational period, one of the teachers stopped by Shannon’s classroom to ask her if her room was free after her class had been dismissed. They needed a room where to hold a talk for their students with a visitor that specialized on the Dream act. Little by little the teachers brought in their students into
Shannon’s room during her free block, until her room was full. On a different day outside the observation period, they brought a mobile dental clinic to provide free checks on their students because they had noticed that students had been complaining of pain. For one of Shannon’s students in particular, this represented a life-saving experience because they found a condition that had gone untreated too long.

The SLI team also discussed when a student was ready to exit the program. Shannon gave input on her students. During the post-observation interview we discussed cases of students that she had recommended for exiting the program in math. Our discussions on her decision making process shed light on her understanding of what makes an ELL student successful in the mainstream classroom. I present these findings under the forms of knowledge of the student that inform this intervention.

*Forms of knowledge of the student informing ‘provides a comprehensive support program’.* Figure 4.5 above, depicts the forms of knowledge that reflected association with this intervention. Given the program’s holistic nature, this intervention was informed by both nonmathematical and mathematical forms of knowledge of the student. The nonmathematical forms of knowledge were mostly informed by *personal challenges* that got in the way of students’ ability to prioritize their school learning. The teachers discussed the students’ learning progress in their different disciplines as well as their behaviors. I found myself having to pause my audio recorder during check-ins. Other teachers would to stop by Shannon’s classroom unannounced to talk about incidents in their classrooms or things they had noticed about their students. Based on my observations, many of their discussions revolved around their students’ progress in *English language* acquisition. During one check-in, a teacher stopped by to tell Shannon that one of their students had understood her joke in her class. After she left, Shannon
turned to me and said, “this is how we know if they are starting to pick up English, they start to get sarcasm and humor. When they laugh back or react to our jokes, then we know they are getting it” (check-in).

During the post-observation interview I was able to discuss with Shannon her students’ end-of-the year progress, as well as her basis for recommending a student to exit or stay in the SLI program. If a student’s language skills were not high enough (as measured by language assessment scores), students would be automatically recommended to stay in the SLI program. Sometimes, however, the scores were neither strong enough nor weak enough to serve as a definitive indicator of placement based on language. In these cases, Shannon used a combination of indicators that were both mathematical and nonmathematical. These indicators reflected Shannon’s understanding of what makes an ELL student successful in the mainstream classroom. Shannon considered students to be successful when they demonstrated school readiness and organizational skills that supported their overall learning process. For example, these students took notes in class that captured important points from lecture and discussions that made them helpful for review afterwards\(^\text{10}\). Students that were successful also needed to evidence comfort with their language skills to be able to communicate their mathematical thinking to their teachers. In order to communicate their thinking, they also needed to have ownership over their learning so that they could advocate for themselves and look for ways to get help outside the classroom. Because of students’ need to advocate for their own learning, Shannon tried to avoid student placement into mainstream classrooms where teachers did not come in before or after school to help their students. She had what she called her “go to teachers”. These were

\(^{10}\) As opposed to notes that strictly captured what was on the board without consideration of meaning. Shannon explained that this was a common practice for students with very low English acquisition levels (check-in).
teachers that not only provided support outside the classroom, the also sought to get educated about their students and about ways to best support them.

Shannon used of a nonacademic form of knowledge when deciding whether to have Ana repeat Algebra 1 or have her move on to Algebra 2 with the rest of her classmates. Shannon used her understanding of the effectiveness of the relationship between Ana and her friend Grisselle (interpersonal relationship). Ana’s course grade was low, but Shannon’s main indicators to have Ana repeat the course were foundational gaps that she was not overcoming and her relationship with Grisselle which seemed to have a negative effect on Grisselle. Shannon said that Ana had a lot of absences and that although other classmates had many absences, Ana was not coming for help outside the classroom to make-up what she had missed. Grisselle was evidencing the same challenges as Ana, but somehow Grisselle was not reflecting as many gaps as Ana. Shannon expressed concern for the relationship between Grisselle and Ana. She said that the year before, they considered holding them both back, but that they decided to have them move forward together to algebra 1 because they helped each other in the classroom. This year, they worked together, but the relationship seemed one-directional. Shannon’s description of their work relationship matched my observations. Grisselle helped Ana, but Ana remained as a receiver of help without improvement. Shannon considered their interrelationship to no longer be effective and to pose risk on Grisselle’s progress. She also thought that in having Ana repeat the course, she could also place Ana in the position of helping others, which is a position she was used to in her household for family needs.

Shannon also capitalized on positive relationships between students (interpersonal relationships) to determine placement. She recommended exiting two of her students together for math because by knowing they had a good relationship, she also expected that they would
support each other. This type of support was important to Shannon, because, even with
ownership over their learning, their students still struggled when they went off team (Shannon,
post-observation interview). Shannon asserted: “When they go off team, even our strongest
students struggle because they are surrounded by English speakers, they have new teachers,
there’s a new atmosphere” (post-observation interview). She then added that it was easier for
them and for their students when they were part of the program because they got to know so
much about them over multiple years.

**Findings: Central phenomena in Shannon’s practice.** I now revisit Shannon’s
interventions to take a closer look at central phenomena in Shannon’s practice. These
phenomena include her perspective on learning, the interplay between her interventions and her
teaching goals and her understanding of her students’ mathematical learning experiences.

**Shannon’s Interventions and her perspective on learning.** Shannon’s perspective on
learning revealed a dual role between English language acquisition and learning math. She did
not necessarily think that language skills were needed as strongly as in history courses because
those courses “rely more on words” (post-observation interview). According to Shannon,
students could rely more on procedures to describe their thinking in math courses. Shannon’s
descriptions, however, of what makes an ELL student successful in the mainstream classroom
brings us closer to her understanding of what are successful learning experiences. Language
acquisition was not so important for the purpose of communicating within a set structure (as it
would be in an English or History course). Language acquisition was important for the purpose
of maintaining a comfort level to communicate thinking. Shannon asserted:

> The other thing is, if you have the language you are able to explain it. Like explaining
any content, like English content, whatever. You will be more into the lesson that is being
taught. You are more engaged because you are able to converse back and forth, or you

162
can explain concepts in that language to another student. You are reaffirming your understanding of the content.

My interpretation of Shannon’s assertion is that students need a sense of comfort in order to engage in the learning process. They engage in the learning process by communicating their thinking. Their engagement is thus mediated (or possibly moderated) by their level of comfort. Language is important because having a certain level of language proficiency builds this needed sense of comfort for learning engagement.

**Interplay between Shannon’s interventions.** I use Shannon’s perspective of learning as a lens to further understand the interplay between her interventions. Shannon used what she knew about her students to create a learning experience that was built on support. A flexible pace gave support to accommodate the increased demand on ELL students as a result of learning math content in English. Through the flexible pace, Shannon prioritized learning for understanding over trying to maintain the same lesson pace of the mainstream classrooms. She built relationships with her students to establish trust, which was another way to support students’ comfort in learning. These relationships were developed in mathematical and non-mathematical ways. Her modifications on learning materials addressed proficiency needs in language acquisition as well as needs in preparation for Algebra 2. They also helped students develop a level of personal advocacy that they would need in order to succeed in the mainstream classroom. Thus, through self-advocacy, students were being empowered to develop ways to support themselves. Conversations reflected a form of learning engagement that depended on students’ comfort level. The more comfortable a student, the more the student would engage in the same conversations that would also help reaffirm their learning. Finally, Shannon’s membership of the SLI team helped maintain a comprehensive support network for her students’ academic and nonacademic needs.
Model of the learning phenomena in Shannon’s case. Up to this point I have presented and described the forms of knowledge of the student used by Shannon as well as how these have informed her interventions in the classroom. I now present a model for students’ learning experiences in Shannon’s classroom. I used the model in Figure 4.6 as an analytical tool to further understand and explain the learning phenomena in Shannon’s practice. I describe the components in this model, as well as possible paths within the model. This model, in particular, depicts two different paths with different outcomes. I end this section with examples from students’ learning experiences depicted through the model, depending on the path their experiences represent.

Interventions. Shannon used different forms of knowledge of her students to inform the teaching interventions she implemented in her practice. These forms of knowledge and their association to each intervention were depicted in Figure 4.5. One of her interventions, ‘provides a comprehensive support plan’ was implemented through her work in the SLI team. Because of this, the model uses arrows into and out of the SLI team to reflect the interactive nature of Shannon’s work with her students as a contributing member of the team. Shannon’s interventions had a dual effect in developing both personal trust with her students and support.

Personal Trust and/or Support. Shannon’s students demonstrated two types of behaviors as a result of the trust and support that they received through her interventions. In the model these outcomes were shown as (1) practice on readiness behaviors (e.g. coming to class with their worksheets, taking notes, etc.) and (2) their prioritizing and advocating for their learning needs (e.g. coming for help after absences, completing their homework, and getting help when experiencing difficulty understanding and/or low performance). These two types of behavioral outcomes were important indicators that Shannon looked for when determining if a student
would place out of the SLI program. They were necessary forms of self-support behaviors that, according to Shannon, were needed in combination to having a good content foundation and comfort in communicating in English. I note here that Shannon also identified organization as a necessary skill, but the data set did not provide evidence to support the presence or absence of students’ organizational skills.

Figure 4.6. Model of Learning Phenomena in Shannon’s Case

*Practice on readiness Behaviors.* The ultimate development of these readiness behaviors, according to Shannon, took time. Shannon’s practice reflected consistency in supporting students’ practice of these behaviors. There is a closed loop of arrows in the model.
that represents a cyclical type of phenomenon. Students engaged in the practice of readiness behaviors, as they also received consistency, personal trust and support from Shannon. The ultimate outcome was students’ development on long-lasting readiness behaviors that would help them succeed in school. Students in Shannon’s classroom were for the most part evidencing these readiness skills. These were behaviors that were instituted and practiced in the classroom. I used double arrows to reflect the notion that the development of these behaviors was ongoing and happening over time.

As I attempted to understand the difference between the two behavioral outcomes, I noticed that Shannon reflected an overall consistency in her expectations and in her work in the classroom. Students’ ability to practice readiness behaviors, however, relied on their work with Shannon in the classroom. Students’ ability to prioritize and advocate for their learning needs, on the other hand, was vulnerable to additional factors stemming from outside the classroom.

**Prioritizing and Advocating for their Learning Needs.** Students’ ability to prioritize and advocate for their learning needs reflected dependence on students’ ability to overcome issues outside the classroom (e.g. personal challenges). The reasons why some students were able to overcome challenges outside the classroom over others are unknown. The data set provides evidence for two types of outcomes. Some students developed **ownership over their learning** while others showed **no apparent change.** I used double arrows for these outcomes to depict the notion of progress and ongoing development.

I end this section by revisiting some of Shannon’s students to describe their learning experiences through this model.

**Yadira.** Yadira had a good relationship with Shannon as indicated by the sculptures she gifted Shannon. During the observational period Yadira reflected inconsistencies in her level of
engagement. She was sometimes very quiet, not making contributions to class discussions. Sometimes she was more engaged, asking questions to Shannon or to a classmate. Upon discussions with Shannon, I found out of the many personal challenges that Yadira handled on a regular basis. She was practically alone in the United States with all her immediate family still living in her country of origin. Yadira lived with her aunt, and according to Shannon, she was in charge of the household while her aunt worked. Shannon noted that “sometimes she is on, and sometimes she is off” (check-in) emotionally, but that she has learned to give Yadira space whenever she recognized that Yadira was having a difficult time. When Yadira was not having a difficult time, Shannon said that she would read the situation in case Yadira “needed a push” (check-in) academically. In the case of Yadira, their built working relationship and the flexible lesson pace were key interventions that provided support for her. Shannon also added that in those difficult days, she sometimes placed her hand on Yadira’s shoulder as she walked around to check on everyone’s progress. Their interaction was also indicative of a trusting relationship. As with most students in Shannon’s classroom, Yadira demonstrated having readiness skills by coming prepared to class. Shannon recommended to have her exit the SLI program at the end of the year, because despite having challenging days, Yadira prioritized her learning by making up what she missed or by following up with Shannon outside the classroom after her absences.

**Grisselle.** Grisselle sat in the very last seat of her row. She was absent multiple times. When the observation period started, I did not know who Grisselle was. I made note that Shannon had asked the class if anyone had been in contact with her, because she had not taken her test yet for the previous unit. The day she returned back to school, Shannon explained that the quarter grades were due that day and that she needed to get a pass from her at the end of class
so that she could come in during a different period to take her test. In class, Grisselle was always engaged, taking notes and asking help from classmates when she had a question. Her behaviors indicated that she had **developed readiness behaviors**. She was, however, inconsistent about advocating for her learning because she was not making up work from her absences. She asked for help, but only in class. According to Shannon, her challenges were in coming for help outside of class time (check-in). Her math lab grade was low, which meant that she did get help from the tutor in the lab, but not as often as Shannon had expected her to do so. It also meant that Grisselle was not completing her homework because the math lab was required for students with a hw grade lower than 70%. Shannon noted that she reflected foundational gaps in her work, but that she thought she could overcome them if she was consistent about doing her homework and about getting help after her absences. Although Grisselle passed the course and moved on to Algebra 2, her behaviors did **not evidence an apparent change** in prioritizing her learning needs. There was some level of advocacy for her learning, but strictly during classroom time. I asked Shannon if she knew of any particular conditions outside the classroom that made it challenging for her to advocate for her learning. Shannon said that her sister, who was also in the SLI program, had recently had a baby and she suspected that Grisselle was involved in helping with the baby. She then turned around and said, “but her sister – she really is committed and does well, she is a whole different person” (check-in).

*Ana.* Ana sat in front of Grisselle. They worked together almost every time, unless one of them was absent. Although during the observational period Grisselle was absent more days than Ana, Shannon said that Ana was absent just as much during the year (check-in). I don’t have any evidence to suggest that Ana made contributions to whole class discussions. I do have evidence of other classmates stopping by Grisselle and Ana to help and answer questions
whenever Grisselle shared a question with them. My interpretation of their interactions is that Ana was struggling to keep up on her own. She was very quiet. Grisselle took on some of the advocating for her in the classroom. According to Shannon, Ana was also challenged by foundational gaps, but their effect in her ability to succeed where more impactful in Ana than in Grisselle’s case. In Ana’s case, the classroom norms to encourage group support and conversation in their first language provided her with support to get help from Grisselle and other classmates. There seemed to be a disconnect in her relationship with Shannon. Based on Shannon’s descriptions, their trust level was lower than it was with most of the other classmates. Still, Shannon attempted to provide help while they worked in groups through Grisselle. There were, however, no direct conversations between Shannon and Ana. Shannon approached her directly when she did homework checks and to ask her for her math lab log. Shannon suspected that Ana’s typical role as a leader in her household made her uncomfortable with her position in her class, requiring help from others (check-in). During the post-observation interview, Shannon explained that she regretted not having her repeat the pre-algebra course. She had hoped that her relationship with Grisselle would have helped them both support each other to continue learning while overcoming foundational gaps. Since Ana continued to evidence no apparent change in advocating for her needs and in making progress in learning, Shannon chose to have her repeat Algebra 1 the following year.
Case 3: Eddy – “I Will Not Stop”

The observation period was conducted for five weeks during the months of May and June of 2017. Eddy was on his third year of teaching at Mixville High School.

**Background information: Algebra 1 Academics course and their students’ learning paths.** Eddy had only taught Algebra 1 at the school, at two different levels of their three-tiered learning tracks. Eddy taught the lowest two levels. The observation period was conducted in two different course periods of the lowest track level, based on middle school performance. This level was called, academics. The two sections of academics Algebra 1 were taught in the last two periods of the day.

Eddy also taught sections of the second-tiered track course which was called Algebra 1 Accelerated. Both courses, Algebra 1 Academic and Accelerated were offered mainly to ninth graders. During the year of the study, all students that had repeated Algebra 1 had been put together in separate sections of the Algebra 1 Academics course. These courses were taught by their math department chair. The math department chair had shared with me that after teaching those sections this year, he did not think that creating separate course sections for all Algebra 1 repeating students was beneficial for their students. He was working to make changes for the following year so that all students were incorporated into the sections of their ninth grade Algebra 1 Academic course.

I describe these course assignments here, based on generalized descriptions, but I also note that special arrangements were made by student. For example, one tenth grader in the sixth period where I conducted observations had specifically requested to be placed in Eddy’s class. Eddy had been his ninth grade Algebra 1 teacher and he wanted to repeat the course with him. Figure 4.7 presents a general trend on students’ math placement, recognizing that it does not
describe all students’ experiences and/or learning paths. My primary focus was on understanding the learning paths for Eddy’s students from the observational period in relation to other existing learning opportunities within their school. In essence ninth graders were either placed on an Algebra 1 Academic section or an Algebra 1 Accelerated section. There were no other lower placements available for students in ninth grade, unless they were served for particular special needs (Eddy, check-in). This also means that students that failed their math course in middle school were placed directly into Algebra 1 Academic. Students in the third and highest math track (based on performance), completed Algebra 1 Accelerated in middle school. They started their ninth grade at Mixville High School in a Geometry Accelerated course. One afternoon, as we walked out of school together, Eddy said, “those are a whole different group, but I can’t talk about them. I never get to teach those”.

Figure 4.7. Students’ Math Course Learning Paths in Mixville High School
* Particular paths for Eddy’s Students in his Algebra 1 Academics course

Since this was Eddy’s third year at the school, his students from his first year of teaching were in eleventh grade. Anecdotally, Eddy said that most students in the academic track did not
tend to take a fourth math class in their senior year. The school used to require four years of math for graduation after the state had announced that it would be increasing requirements from three to four years for two disciplines – math and English. Eddy explained that because the state changed plans to only require the four years in English, their district changed policy to move back their graduation requirements in math to three years. The school offered multiple electives in math, including two tracks in computer programming for academic and accelerated students. There was an elective course titled Algebra 3 with Trigonometry, which seemed to match by description the typical Pre-Calculus content, but reduced in breadth of topics. There were multiple course options under Advanced Placement (AP), but they only add to the list of math learning options that the academic students did not typically have access to.

**Background information: Specifics on Eddy’s course.** Towards the end of his first year of teaching at Mixville High School, Eddy asked the administration to allow him to teach the course using a self-paced online platform. The school already provided laptops to each student, so they have the access to the technological requirements for implementing the platform. The size of the sections was modified to accommodate about fifteen students or fewer. The classroom was furnished to seat about six students per table. Although there was seating capacity for more students by table, Eddy typically used three of these tables. During the last two weeks of school, Eddy made use of an additional fourth table in his period six class in an attempt to help increase their productivity. Each class was started with a 10 minute activity called “Do Now’s”. These activities involved whole class discussions. During the fourth quarter Eddy used typical final exam questions for these Do Now’s to review different sections covered throughout the year in preparation for their final exam.
Eddy’s self-paced program encompassed 146 activities that covered all the topics in Algebra 1 that the district had required for all algebra classes. The topic selection had been aligned by the district to match the Common Core Standards. Table 4.10 describes the different types of activities that Eddy designed.

Table 4.10. Description of Lesson Activities in Eddy’s Self-Paced Program

<table>
<thead>
<tr>
<th>Activity Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Source Platform (Video Lessons)</td>
<td>This was an online site that housed video lessons made by Eddy. Each video was accompanied by a handout where students were expected to take notes. Eddy filled the notes as he spoke throughout the lessons. Notes in Eddy’s class counted 10% towards each quarter grade calculation. Although the site required administrative access, I was able to get all of Eddy’s videos through a separate site where he had originally uploaded them. I also had copies of all accompanying handouts.</td>
</tr>
<tr>
<td>Paper Class Activities</td>
<td>These were practice activities either designed by Eddy or taken from the state’s curriculum. Based on their design, Eddy’s activities could be completed well within one class period. The activities from the state’s curriculum would have required more than one day of class. During the observational period, only two class activities were from the state curriculum. Paper Class activities counted 15% towards each quarter grade calculation.</td>
</tr>
<tr>
<td>Math Assess** (Test Bank)</td>
<td>This was a separate online site that Eddy used to check for student mastery. Eddy used it for online assessments, each with a set of 5 questions. In order to demonstrate mastery, students needed to answer 4 out of the 5 questions correctly.</td>
</tr>
<tr>
<td>Tests**</td>
<td>Students were given one or two tests per quarter, depending on lesson coverage. Only one test was given in the fourth quarter (when the observational period was conducted). The tests often times took more than one period of time to complete.</td>
</tr>
</tbody>
</table>

Assessments from Math Assess and Tests constituted one combined grade for assessments. They counted 75% towards each quarter grade calculation. There was no midterm exam. There was one final exam, taken in June. It was worth 10% of the whole course’s final grade.

I have changed the names of the activities in cases where there were trademark names. Some of these activities were not designed by Eddy because the district was also using curricula...
specifically designed at the state level to support the teaching under the common core standards. Eddy’s tests, for example, were for the most part the same tests from the state level curriculum.

Due to the self-paced format, a student could be found working on different lesson numbers in comparison to other classmates. In one same class period, one student could be starting a unit, while another student could be taking the test for that same unit. This required students to compare the lesson number they were working on and the class’ target lesson (based on ideal pace). Each day, Eddy would place in the overhead projector a reminder about the lesson number they were expected to be at. His reminder read: “By the end of this week, you should be on lesson number X”. Because of this, discussions about students with Eddy tended to incorporate information about lesson progress. This included students’ grades and/or prospects of passing the course with a grade of 60. I have included grading criteria in table 4.10 within the description for the different types of lesson activities. Table 4.10 also makes reference to ‘quarters’. Mixville High School divided its academic year into four quarters. The school did not administer midterm exams, only a final exam at the completion of the fourth quarter in June.

The lessons used for the academic and accelerated sections were for the most part the same\textsuperscript{11}, but Eddy had the academic sections skip lessons during quarter four. His main concern was ensuring that students got to the last set of lessons that were dedicated to systems of linear equations. I summarize the lesson topics that were scheduled to be covered during the fourth quarter in table 4.11. I also indicate which lesson topics were skipped for academic students. The quarter actually started on lesson 89, but Eddy kept lessons open for the fourth quarter starting on lesson 86. He explained that he considered these sections very relevant to the course

\textsuperscript{11} Eddy described modifications that focused students’ attention on the problem objective (e.g. he provided academics with the scale for axes in some graphs, but the accelerated were required to calculate them).
and to the topics scheduled for quarter four. He wanted all students, regardless of their pace
during quarter three, to start the last quarter with a good foundation.

Table 4.11. Lesson Topics Assigned to Eddy’s Academic Students

<table>
<thead>
<tr>
<th>Lesson Numbers</th>
<th>Topic Description</th>
<th>Assigned or Skipped</th>
</tr>
</thead>
<tbody>
<tr>
<td>86 – 95</td>
<td>Graphing Linear Equations in Slope Intercept Form</td>
<td>Assigned</td>
</tr>
<tr>
<td></td>
<td>Writing Equations of Lines</td>
<td></td>
</tr>
<tr>
<td>96 – 97</td>
<td>Exploratory Activities on Slopes of Parallel and Perpendicular Lines</td>
<td>Skipped</td>
</tr>
<tr>
<td>98 – 106</td>
<td>Slopes of Parallel and Perpendicular Lines</td>
<td>Assigned</td>
</tr>
<tr>
<td></td>
<td>Applications of Linear Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equations of Horizontal and Vertical Lines</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit Review and Unite Test</td>
<td></td>
</tr>
<tr>
<td>107 – 114</td>
<td>Descriptive Statistics: Median, Mode, Box and Whiskers Plots</td>
<td>Skipped</td>
</tr>
<tr>
<td>115 – 119</td>
<td>Lesson on Scatter Plots with Desmos</td>
<td>Assigned</td>
</tr>
<tr>
<td></td>
<td>(One practice lesson skipped: 116)</td>
<td></td>
</tr>
<tr>
<td>120 – 130</td>
<td>Standard Deviation and Outliers</td>
<td>skipped</td>
</tr>
<tr>
<td>131 – 146</td>
<td>Systems of Linear Equations</td>
<td>Assigned</td>
</tr>
</tbody>
</table>

Aside from the lessons he asked academic students to skip, Eddy kept all other lessons
open during the year. He said that if a student needed to improve on a grade, all they had to do
was go back and complete any lessons missed. Any missed lesson had a grade of zero in the
gradebook. If the student completed it, he would change the gradebook to reflect the new grade.
According to Eddy, all students had plenty of time to complete the full set of 146 lessons within
180 days of school. For the academic students, this was actually a set of 123 lessons because of
the skipped lessons. Eddy explained that some students hardly did any work in class, but chose
to do all the work at home. Other students did not do any work at home, but instead, they did all
their work in the classroom (Eddy, post-observation interview). He wanted students to have
flexibility in how they completed their lessons. Students that completed the full set of lessons
before the semester ended had the option to either review more, start looking over geometry
sections, or complete work for other classes. Out of the two sections of Algebra 1 Academic
classes that I observed, only two students from period six completed the full set of lessons. They chose to play cards during the last week of school.

**Background information: Eddy’s students and classrooms.** The data were collected across two course periods over 5 weeks. Period six had a larger class with a total of 14 students, while period seven had 11 students. In period six, six students were female and eight students were male. Based on appearance and classroom interactions and comments, half of the students were of color (5 Hispanic, 1 Black, 1 other). In period seven, four students were female and seven students were male. Based on appearance and classroom interactions and comments, 10 out of the 11 students were of color (4 Hispanic, 6 Black).

Both periods were held in the same classroom. When the bell rang, Eddy would step into the hall way to monitor students. The overhead projector was already set up inside the room with a ‘Do now’. There was a small table at the door entrance with bins that stored copies for each of the lesson numbers that students could work on. There was also one bin that had a lesson progress sheet for each student. When students completed a lesson, they would ask Eddy to review what they had completed and to have him sign their lesson progress sheet for his approval. For each class, students would pick up a copy of the ‘Do now’ that was also projected on the board, a copy of the handout that accompanied the lesson they would work on that day, and their lesson progress sheet. Students brought in their computers to the classroom, but Eddy also had a few spare computers for students that did not bring in theirs that day. Students’ seats were assigned, but they also had flexibility with what they did during class time. I sat in a chair against the wall behind Eddy’s desk. This was also a very popular corner because it housed a charging station. Students charged their phones as they entered the room. They also checked on their phone’s battery charge progress during class. Eddy had an open policy about using the
phones to text, listen to music and/or go online during class time. Students moved around to ask each other questions on their work or to talk about non-class related topics. In most cases, when they needed help in class activities they looked for help from Eddy. Eddy answered questions and helped students think through their problems. He moved by table without any breaks during each class period, other than to put in the attendance in the computer and to read messages that he was asked to relate to the class from emails that were sent from the main office during class time.

The findings sections that follow make reference to some of Eddy’s students to provide select examples of their learning experiences. In an effort to assist the reading of the findings section, I provide in table 4.12 key characteristics about these students based on the full case data.

**Findings: Eddy’s teaching goals.** Eddy’s goals were first triangulated from interview and observational data. I then confirmed these goals through member checks in the post-observation interview. I list Eddy’s goals below, but I revisit them again when I describe the teaching interventions that Eddy implemented to support these goals.

1. To help students be comfortable learning and comfortable with working at something that they might not be good at, but that they can become good at later from working on it.
2. To help his students with life problem solving, that may not necessarily be math related
3. To help his students pass his course (Algebra 1) – “They do need 3 math credits [for graduation]. This should be one of them.” (Eddy, pre-observation interview)
4. To guide students’ mathematical thinking
Table 4.12. Eddy’s Referenced Students and Key Characteristics

<table>
<thead>
<tr>
<th>Students</th>
<th>Particular Characteristics Referenced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Victor</td>
<td>Victor was a student that was present on my first day in Eddy’s classroom. Although he was supposed to be absent for two weeks for a suspension, he missed the rest of the quarter for reasons not shared with me. Victor is referenced as an example of a student that was able to make up his work because of the self-paced platform used in Eddy’s classroom.</td>
</tr>
<tr>
<td>Samina</td>
<td>Samina was a student that reflected a high level of learning engagement. Her pace was very slow. She demonstrated having foundational gaps. She attempted to work individually, but she needed Eddy’s help constantly to help her make sense out of the problems she worked on.</td>
</tr>
<tr>
<td>Brenda</td>
<td>Eddy described Brenda as a success story on my first day of school because she had started the year with fears of learning algebra. Eddy said that she had made great improvements. During the last quarter, she had a lot of absences and she stopped working on her lessons. This placed her at risk of passing the course. Eddy said that Brenda’s case made him be more aware of issues with stamina. In his opinion, Brenda put a lot of effort for the first three quarters, but did not have enough energy at the end of the year to continue prioritizing work while also handling personal challenges.</td>
</tr>
<tr>
<td>Carmen</td>
<td>Carmen reflected low self-efficacy and low interest to engage in learning during class time. Her lessons progress was slow. She was off task through most of the observational period. Carmen is referenced here as an example of a student with low learning engagement that was not able to benefit from Eddy’s interventions.</td>
</tr>
<tr>
<td>Kegan</td>
<td>Kegan depended highly on Eddy to work on his lessons. He asked for help regularly. Kegan was often off-task when two other girls in his table (Lara and Carmen) teased him. Eddy said that he wanted Kegan to stop giving so much importance to what Carmen and Lara did. Kegan is referenced here in an example that showcases how Eddy used what he knew about his students in the classroom.</td>
</tr>
<tr>
<td>Daniel</td>
<td>Daniel was at risk of not passing the course. Out of concern for his grade, Eddy called the family. After that, Daniel showed a big change in learning engagement. Daniel is referenced here as an example of a student that turned around his behaviors. His work with Eddy on the unit test is showcased to demonstrate how Eddy attempted to use connections between a linear function’s graph, its ordered pairs and its equation.</td>
</tr>
<tr>
<td>Asante</td>
<td>Asante demonstrated a high engagement level. Due to foundational gaps (according to Eddy in check-ins), he required a lot of support from Eddy. A classroom snapshot is presented in Appendix J to showcase how Eddy made connections to familiar contexts for students to help them problem solve.</td>
</tr>
</tbody>
</table>
**Findings: Forms of knowledge of the student used by Eddy.** In table 4.13, I summarize the forms of knowledge of the student that Eddy used to advance his students’ learning. Although I did not quantify their frequency of use, the forms of knowledge that were most prominently used appear higher in the list. I also include their corresponding definitions formulated from and contextualized in Eddy’s case data.

<table>
<thead>
<tr>
<th>Form of Knowledge</th>
<th>MK or NON</th>
<th>Definition and Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relational[N]</td>
<td>NON</td>
<td>This form of knowledge refers to the ways that Eddy found to be able to relate this his students. Eddy self-reported being approachable (pre-observation interview). In class he used humor. Students used witty humor, especially when Eddy was redirecting them to do work, and so did Eddy. Many students, especially in period seven called him “Coach”. Eddy met with students that came in to do sports training at 6:30 am to work with them because they expressed being more comfortable working individually in that setting. Eddy also explained that his way to work with his students depended on each student. In some cases “a student needed a kick in the butt” while other students needed more “hand-holding and reaffirmation” (pre-observation interview).</td>
</tr>
<tr>
<td>Learning Attitudes: Fear and/or self-efficacy[N]</td>
<td>MK</td>
<td>Mathematical – Eddy found that many of his students were afraid of math or that they don’t like it (pre-observation interview). He works on building relationships with his students so that they will do the work for him, which is something he thinks they might not do for other teachers (pre-observation interview). Eddy also explained that “not afraid of getting it wrong and trying again” (recruitment interview) is an indicator of whether a student will be successful in upper level math courses. He also noted that his students in the academic level were in need of this skill because at some point in their lives someone had told them they are not good in math and they believe it. He described low self-efficacy in that he knew he students could do more than what they believed they could. This low self-efficacy was in Eddy’s opinion, “self-limiting” (recruitment interview).</td>
</tr>
<tr>
<td>Foundational Gaps&lt;sup&gt;O&lt;/sup&gt;</td>
<td>MK</td>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>This knowledge refers to students’ knowledge and application of pre-requisite concepts and skills. In Eddy’s case, students had difficulty with arithmetic, application of the order of operations and general operations and reasoning with fractions. Eddy also noted that there were conceptual gaps on the meaning of operations (addition, subtraction multiplication and division). For example, students could calculate the loss of height for a skydiver by repeatedly subtracting the same height values over an interval of time, but they could not make connections to the fact that they could have also made the same calculation by multiplying. Often times, students were not able to simplify basic fractional expressions (e.g. 9/3, 3/6, etc.). Eddy would ask them, “what does that simplify to?” or “can they [i.e. numerator and denominator] be divided by the same number?” If there was no response, he would ask students to check their numbers in the calculator. Eddy explained that at this stage, he prefers that they find a “coping mechanism to just get the calculations done” because he does not have time to teach these basic skills. (post-observation interview). Eddy called it “the trickle up theory. All those weak foundations keep being passed along and passed along and passed along until you get to a really shaky weak mathematical base and the kids that do get it, aren’t taking algebra in ninth grade” (Eddy, post-observation interview).</td>
<td></td>
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</tbody>
</table>

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<thead>
<tr>
<th>Mathematical Thinking&lt;sup&gt;N&lt;/sup&gt;</th>
<th>MK</th>
</tr>
</thead>
<tbody>
<tr>
<td>This form of knowledge is associated to students’ thinking process while they engaged in doing math. Students were constantly communicating their thinking as they worked through problems with Eddy. This form of knowledge was the most prominently used during classroom interactions between Eddy and his students. Often times Eddy would prompt them to re-consider their thinking with humor. He would also affirm their efforts through positive reinforcement. He also “translated” their interpretations of their calculations to help them make sense of their work. For example, a student was calculating the height of a skydiver after falling for 5 seconds. She said that the skydiver had lost 1,800 feet. Eddy told her to look back on how she was making that calculation and added, “you know you popped his ears out. What did he ever do to you?” Eddy also explained that his students think differently, and that he has learned from teaching his students that some make sense out of problems numerically, while others do it graphically. He has</td>
<td></td>
</tr>
</tbody>
</table>
learned from working with his students that he should try to make more connections with different representations.

**Learning Needs**

- **MK**

  This form of knowledge refers to the differences that Eddy noted in students’ learning. Eddy noted that certain approaches to think through a problem would work more for one student over another. Depending on the abilities demonstrated by a student, Eddy determined the approach he would suggest for them to apply and/or learn. He described these as different needs by learner (check-in).

  For example, in an exercise where students explored the slope of a line from the equation given in standard form, Eddy chose not have his students rewrite the equation into slope intercept form because he did not think they were able to do it themselves. He also said that the unit test did not require them to do it (pre-observation interview). Instead, he rewrote the equation in slope intercept for one student, so that she could compare it to the format \( y = mx + b \) in the poster of the classroom wall. Whereas for another student, he had him create ordered pairs from the equation and then use the ordered pairs to calculate the slope.

**Family Support**

- **NON**

  This form of knowledge refers to the type of support that Eddy found he could get from different families. He called parents to discuss progress, especially when students were at risk of not passing a quarter.

  Family support in Eddy’s case reflected association with follow through on efforts to ensure that students were doing their work. He considered a family supportive if when they agreed to have their child stay after school (or complete work at home), the child would actually do as agreed. In some cases, Eddie said that family was not “reachable or had a negligible effect”, but he said he also understood the fact that parents would explain that they were themselves having difficulty have an effect on their child to improve behaviors (recruitment interview).

**Perseverance**

- **Both**

  **Mathematical** – This knowledge refers to students’ ability to persevere through their struggle in math learning. Eddy has learned, however, that in general, his students have low perseverance. Although the self-paced program allowed him to individualize these experiences for his students, he has also been challenged in teaching because students often times stopped working when they were not getting his help (check-in). He shared this perspective when I asked him about the classroom dynamics during a check-in.

  Non-mathematical – mathematical perseverance, as described by Eddy, is just one area out of the many other where he wishes his students would grow in. Eddy considers this part of his role as
their teacher. When I asked him why he thought that, he said that he suspected that the message to persevere through frustration was not necessarily heard at their homes. He added, “I can only preach from the platforms that I am given” (Eddy, pre-observation).

| Cultural Dissonance<sup>N</sup> | NON | This form of knowledge refers to a difference in students’ lived experiences and Eddy’s (and/or the school).

Eddy noticed that students had difficulty understanding the context of many of the problems they had worked with. During the post observation interview, Eddy said, “we have a large number of students that have barely been out of the state, much less out of the country. It is a tiny, tiny world for them. They haven’t experienced a lot of these things. That skydiver problem – they don’t get that. And…so trying to tie that to something they do understand is tougher, or you try build context around that.” According to Eddy, this cultural dissonance also affected students’ ability to understand whether solution values for problems were reasonable or not.

Eddy also noted that some students “understand how to be a student”. According to Eddy, these students are the ones that get picked for advance courses early in seventh grade. He said that in his opinion, these students know how to persevere early and that having that skill is what gives them an advantage over other students. He explained that selection as early as seventh grade is not a measure of ability, but rather, habits of work (check-in).

The system favors some students over others when they “know how to do school” (recruitment interview).

| Language<sup>O</sup> | MK | Mathematical – This form of knowledge refers to students’ use of mathematical language throughout the unit of study, particularly: slope, intercepts, function, domain and range.

Outside the unit of study and in review for the final exam, Eddy would prompt students to use the name of properties that were applied for solving equations. He explained he stressed those because they would need to apply them in the final exam (check-in). Other times, Eddy explained that students needed to use the correct language because – “we can’t be point and grunt kids”, he wants them to tell him the meaning of what they are working with and in using the correct language he is also able to understand them better (check-in).

Eddy also frequently described terms that were not necessarily mathematical, but that were used in contexts that were not familiar to students. For example, in a problem where they were exploring
the profit from selling cookies, Eddy told them “that’s a special point, we call that the break-even point” (class observation).

| Learning Progress<sup>N</sup> | MK | This form of knowledge refers to Eddy’s understanding of the extent of students’ learning. Eddy explained that he is able to get a daily understanding of his students’ learning progress because he gets to work with them individually almost every day with the self-paced program. He was able to determine if a topic was learned or not, by tracking students’ progress in the mastery checks from Math Assess. For example, if he found a student taking a quiz more than three times, he considered that an indication to him that the student needs help in learning that topic (pre-observation interview).

Eddy also stated that students get themselves “instant feedback – as opposed to – I did my homework, I handed it in, two days later – the kids don’t look at the corrections” This form or knowledge also points to students’ ways to make use of feedback on their learning progress.

| Mathematical Structure and Symbolic Representation<sup>O</sup> | MK | Eddy noticed that students were challenged in understanding the structure used for rational numbers. For example, he noted that sometimes students would look at \( \frac{1}{2} \) and ask if that was the same as 2. This happened multiple times during the observational period. Eddy explained that they could not make connections to the meaning of “parts over the whole” (post-observation interview).

Algebraically, Eddy also found challenges with identifying slope and y-intercepts of lines. In most cases, Eddy gave them the equations in slope intercept form because he said that his students were not able to solve for y. He associated this challenge to foundational gaps. In the unit of observation, in particular, he expressed challenges with students understanding the difference between the equations for vertical and horizontal lines.

| Personal Conditions<sup>N</sup> | NON | Students, as individuals in society, were found to have characteristic personal conditions and/or life experiences that had an impact on how they learned or on how they engaged in learning. In Eddy’s classroom this included: single parent households, attention deficits (or suspicion for undiagnosed deficits), arrangements to maintain academic standing, etc.

| Personal Interests<sup>N</sup> | NON | Eddy made connections to contexts that he thought would be familiar to his students based on their personal interests. He knew many of his students outside the classroom in his role as the school’s football coach. In cases where students were
evidencing not understanding the context of a problem he would try to make connections to context that were familiar to them. During the recruitment interview, Eddy gave the example of making connections between ordering combos at McDonalds and the distributive property.

Life Choices\textsuperscript{N} & NON & Eddy has noticed that at this age (ninth graders) students struggle with general decision making on staying in school, not having a child, not having drugs or not drinking and driving. “Anything like that…” (pre-observation interview). Although he described these life choices as a typical experience of students’ age (check-ins and recruitment interview), he also stated that he wants to make a positive impact in his students so that they make the right life choices and that this type of outcome is not reflected on the grades they get at the end of the year – “You don’t get graded on that, especially in this district, unfortunately” (Eddy, pre-observation).

This category also includes academic and educational choices. On multiple occasions, Eddy commented that his students were not college bound. When I asked him how he knew, he said that his students at that age had already made up their mind about that, that some would go in the military and that others would help their parents in their work.

**Findings: Eddy’s teaching interventions.** Eddy exhibited four central teaching behaviors in using the forms of knowledge of the student from table 4.13. I describe these teaching behaviors as interventions going forward in this document because they represent a set of particular measures that Eddy implemented to advance his students’ learning based on what he knew about his students. These interventions were: (1) expands the boundaries of the learning space, (2) holds individual conversations on progress and lesson completion, (3) guides students’ mathematical thinking, and (4) seeks alternative approaches to explain math concepts. In general, Eddy’s teaching interventions were found to offer students flexibility, especially in their choices to engage in learning. Since this flexibility was evident throughout all his teaching behaviors, I did not consider it as a separate intervention. Rather, I best describe it as a pattern behavior that impacted how he implemented his interventions. I describe how this flexibility
impacted the overall teaching and learning phenomenon in Eddy’s classroom after describing all interventions.

In the subsections that follow, I provide an operational definition for each intervention. These definitions are based on Eddy’s context of instruction and on his descriptions of how he used what he knew about his students and why. For each intervention, I also report the forms of knowledge of the student that informed each interventions’ intended purpose in Eddy’s practice. These are summarized in Figure 4.8.

1. Expands the boundaries of the learning space: Definition. Eddy expanded the boundaries of the learning space by re-designing his Algebra 1 course to follow a self-paced format. When I asked him why he made this change, he said: “We have held the time constant and made the learning the variable. You’ve got from August to June to learn. We should make learning the constant and time the variable” (Eddy, check-in). Eddy also called his platform, “differentiation with respect to time” (check-in). In concept, this intervention represents an expansion of the boundaries of the learning space in that students were given the flexibility to complete lessons at their convenience. They could do lessons at different times of the day, at home, or at school.

This intervention yielded positive effects for some students based on Eddy’s teaching goals. For example, Eddy had the flexibility to bring in students during their free blocks to make progress in his room. Eddy also came in early before school opened and stayed late after school ended, to help students while they completed their lessons. He also reviewed the schedules for students that were bordering on not passing his course. He would then come up with plans to prioritize work for the classes these students had better chances at passing. In cases where the better chances were in Algebra 1, Eddy would work with the teachers to have the students come
do work in his classroom instead of having the students go to the class that they did not have a chance on passing. On multiple occasions throughout the study, Eddy referred back to the need for students to get their credits in ninth grade, so as to improve their likelihood of high school graduation. Work completion to accumulate grade score requirements in the self-paced platform was very important to Eddy.

Figure 4.8. Forms of Knowledge of the Student Informing Eddy’s Interventions
The number of lessons completed, however, were found to be different by track level. Whereas most students in the accelerated sections (higher track) completed their full set of lessons, only two students in the academic sections from the observational period completed the already reduced set of lessons (see table 4.11 for a list of assigned lessons). Eddy had asked his academic students to skip the chapter on descriptive statistics in an effort to get them closer to the last unit which was dedicated to systems of linear equations. According to Eddy, this section was extremely important because there were many problems from this topic in the SAT exam (check-in). He was also under discussions with his department chair to see if they would consider including systems of linear equations in the curriculum at the start of Algebra 2.

*Forms of knowledge informing ‘expands the boundaries of the learning space’.* Figure 4.8 above, depicts the forms of knowledge that reflected association with this intervention. I note, however, that this intervention was not originally implemented in response to Eddy’s knowledge of his particular students. The intervention was originally informed by a more general understanding that everyone learns at different paces (Eddy, check-in). Having stated this original intention, I also asked him why he thought it was helpful for his students. He explained that it allowed him to work individually with each student and get almost immediate feedback on their level of understanding and/or thinking approaches (*mathematical thinking*) and *learning progress*. Eddy’s efforts in guiding students’ mathematical thinking are described as a separate feature of Eddy’s practice below.

In addition to students’ mathematical thinking and their learning progress, the intervention on its own, was found to be in association with *family support, personal conditions and foundational gaps*. I provide Victor’s case as an example. I met Victor only once. He was a student that was absent for the rest of the observational period (one month). I
was told he was away for a two-week school suspension (personal condition), but he never returned to the classroom. I was really looking forward to seeing him on the last day of school after Eddy informed me that Victor would be there. But he was not there, and based on Eddy’s response, I understood that something else had happened that I could not be informed of. Because all lessons were accessible online and/or through handouts, Victor actually completed a great extent of his missed work from home during his absence. Going into the final exam, Victor had close to a passing grade. Eddy worked with his parents (family support) by giving them a list of all the lessons and handouts he needed to complete. Eddy also stayed with Victor after school to help him on graded activities Victor needed help with. Victor passed the course (post-observation interview).

Students struggled with foundational gaps regularly and on a daily basis. Eddy explained that the program Math Assess required all final answers to be in simplest form. Any answer in fraction form, for example, would be marked incorrectly if the student did not rewrite it in its equivalent simplified form. Eddy had already noted that most of his students had a lot of difficulty with fractions. During a check-in, Eddy said – “If I really wanted to get them, all I have to do is give them, even the accelerated students, a problem with decimals or fractions, I got you!” But Eddy’s point during this check-in was that he did not have time to teach and dedicate the time that learning fractions needed, if he also needed to teach algebra. He explained that instead, he needed to focus on students’ conceptual understanding of Algebra 1. In cases that he noticed that students took longer to complete unit 0 (the first unit in the course that served as review of arithmetic), he knew that meant that these students would “require more hand-holding during the year” (Eddy, post-observation interview). Eddy was not just referring to being able to enter answers in simplified form in Math Assess. He was referring to a hand-holding through
problem solving throughout the year. Samina, for example, needed to calculate the height of a falling skydiver after 5 seconds, given the rate in feet per second. Samina was not able to do this on her own. Having attempted different suggestions from Eddy, Eddy resorted to have Samina use a long wooden stick to mark out the loss in feet for each second. Although she succeeded with this approach, the exercise took Samina half of the class time to complete. Given the high level of occurrence of foundational gaps and their observed effect in problem solving with respect to the time students invested, I noticed an association between students’ learning progress and foundational gaps as I tried to understand the central teaching and learning phenomenon taking place in the classroom. The more the foundational gaps, the more the student needed Eddy’s support to complete each lesson. While Eddy moved around the room to help students without pause in this self-paced program, many students would stop working when it was Eddy’s turn to help someone else. The boundaries of the learning space were expanded, but not as much for some students with low self-efficacy and/or evidencing foundational gaps.

2. **Holds individual conversations on progress and lesson completion: Definition.**

Eddy spoke with each student about their progress. This progress was not limited to actual lesson completion. Eddy also held individual conversations related to personal choices in their needs to prioritize school and to make “the right life choices” (Eddy, pre-observation interview). Eddy held conversations one-on-one as he worked with students in the classroom. When the discussions were of a personal nature, he would step into the hallway with the student, or speak to the student after class. I noticed that students were very open about personal information that they shared with Eddy.

*Forms of knowledge informing ‘holds individual conversations on progress and lesson completion’.* Figure 4.8 above, depicts the forms of knowledge that reflected association with
this intervention. Eddy followed up on each student and he purposely held discussions around their need to persevere (perseverance) or their need to make better life choices (personal choices). Through his conversations with them, Eddy got to know his students, especially their personal interests to build relationships with them. I had noticed Eddy’s use of relational knowledge throughout the observational period. Eddy confirmed this when I asked him to describe the characteristics that a teacher should have in order to be able to help his students learn. He stated – “This is a relationship business” (post-observation interview). During the pre-observation interview Eddy had also explained that he gets to know his students and that in building their trust, the students do their work for him. He also said that students do more work for him than for other teachers because of their relationship.

Eddy tried to have a positive impact on students with low self-efficacy or that struggled with perseverance (recruitment interview). He was observed on multiple occasions prompting students to work and to overcome their expressed interests to give up. I provide Brenda’s example to illustrate Eddy’s interventions. According to Eddy, Brenda started the year expressing fear of failing his class. On the first day of observations, he described her as a success story, a student that he had worked with closely to support her learning. Although she did not have an A or a B average, she had demonstrated growth in overcoming her hesitations. Eddy said that she had “ups and downs”, but that on average, “she had set herself up to pass the course” (check-in). A few days later, Brenda started to skip class and stopped doing work during the last quarter. The first day she came back to class, Eddy thanked her for coming that day. After class he walked her out to the hallway to have a conversation with her. Their brief conversation is included in Appendix I. Brenda was absent multiple days after that. The second day she returned, Brenda acted frustrated in class and commented that she did not understand
what she was doing. Brenda continued to be absent. Eddy told me that he called her mother to let her know that Brenda was at risk of not passing. A few days later, Brenda came in after school with a young lady that she called her mentor. I was there because Eddy had scheduled to meet with me for a check-in. Eddy explained to both of them the work that Brenda needed to do. He also continued to remind her that she was very close to being done and that she should not let all her efforts during the year to go to waste. He reassured her that she was very capable and that all she had to do was apply herself. Brenda came back to one more class after that. She did turn in a few assignments, not all, but enough for her to pass the class (Eddy, post-observation interview).

Eddy also used personal interests about his students in different ways. For example, one student that had struggled with perseverance was involved in a theater play. Eddy attended the play. When they worked one-on-one in class, Eddy congratulated her on her involvement with the play. He explained during the check-in, that the student had gone through many personal challenges and that her participation in this event was giving her the motivation level to do better in his class. Eddy used personal interests regularly. I present the dialogue below, where Eddy used what he knew about Kegan for classroom management, to redirect him to do work, and to help him improve his personal choices. In this snapshot Lara, Kegan and Carmen had been teasing each other. Eddy had already asked Lara and Carmen to move away from Kegan’s table, but Kegan continued to pay attention to them across the room.

Eddy: Kegan you need little ears.
Kegan: I already have little ears.
Carmen: No he doesn’t. (from afar)
Lara: He’s got Dumbo ears. (from afar)
Eddy: Kegan, you play hockey, don’t you? Do you listen to what the other team says about you?
Kegan: Yeah, sometimes.
Eddy: Does it ever get you in trouble?
Kegan: I don’t know?
Eddy: Does it ever get you in trouble on the ice?
Kegan: Oh, maybe, yeah.
Eddy: Guess what’s happening here.
Kegan: I’m getting in trouble.
Eddy: Don’t let somebody else’s words affect you. They are just words.
Now, do yourself a favor, change your posture and get back to work.
Kegan: But I don’t know where to start!
Eddy: [Eddy leaned over to point to his screen] That’s your slope. That’s your y-intercept. Now write your equation.

3. Guides students’ mathematical thinking: Definition. This intervention refers to Eddy’s individual work with each student to guide their mathematical thinking as they completed their work in class. As I had noted in my description of expanding the boundaries of the learning space above, Eddy felt that the self-paced format was especially effective for his students because of the immediate feedback that they received when discussing their work and their thinking. Eddy asserted:

Being able to look at someone’s work and help them figure out where the mistake was, is so critical in this content area, versus assigning a homework set and either putting up the answers or collecting it, correcting it an giving it back. At that point is been too long from the action to the feedback to try to have any response to that feedback. If I can see the work early or real time, you can start nipping the problems early. I love it when I start seeing a student saying ‘Mister I’ve got my work right here, I know it’s wrong’. How do you know it’s wrong? Ok, because I am doing this and I am seeing that is not working. Ok, let’s start looking back. (Eddy, check-in)

Forms of knowledge of the student informing ‘guides students’ mathematical thinking’.

Figure 4.8 above, depicts the forms of knowledge that reflected association with this intervention. The most prominent forms of knowledge found through this intervention were: students’ own mathematical thinking, learning needs, self-efficacy, foundational gaps and cultural dissonance. Eddy discussed with each student how they considered attempting a problem. He sometimes gave them reminders or prompts to help them come up with a solution. My observations of students indicated that the students that chose to engage in learning through
the lessons and activities constantly communicated their **mathematical thinking**. Eddy also asked students to consider their answers to make sure they made sense to them. He used humor frequently to prompt his students to think. For example, in one class a student was modeling the height of a skydiver with respect to time. She said that the y-intercept was zero. Eddy listened to her response and said, “I think you ought to check that answer, your skydiver seems to be tripping”. The student smiled as she looked over her work again.

In guiding students’ **mathematical thinking**, Eddy considered their **learning needs** (Eddy, check-in). In helping students see that they could problem solve, Eddy noted that students overcame challenges with **self-efficacy** (Eddy, check-in). As described earlier, **foundational gaps** challenged students’ **mathematical thinking** in different ways. Students would “block” (Eddy, check-in) and choose not to attempt a problem when they saw fractions. They also could not make associations to basic operations (addition, subtraction, multiplication and division) when their conceptual application was needed. Eddy looked for what he described as “coping mechanisms” to get calculations done because he could not teach these basic foundations in his course. Students’ lack of familiarity with the contexts of problems also challenged their mathematical thinking (recruitment interview, check-ins, post-observation interview). I present a classroom snapshot in Appendix J to demonstrate how Eddy worked around this challenge which reflected association with **cultural dissonance**. In some cases Eddy would try to describe the context. In other cases, he would have students make associations to contexts that he knew were more familiar to them.

I observed a challenge with this intervention. Students needed Eddy’s support with their **mathematical thinking**, more than he could provide it in class. Eddy confirmed this observation by noting that he wished his students were able to work more independently because
he was only one person in the classroom. Students that had difficulty working on their own waited until it was Eddy’s turn to come back to their table. A concern with this challenge is that students were not necessarily evidencing full use of their learning time in class. Despite this challenge, it seemed that working with Eddy in this format had a positive effect on students. Samina for example, typically waited for Eddy to do her work with him. Her comments to Andrea in class best describe how she felt about her experience in Eddy’s class. Samina said – “I wonder how math is going to be next year, because he actually teaches you” (Samina, observational period).

I also noticed that students made mathematical connections in some types of activities more than others. When students used the lessons from the state level curriculum, they reflected more connections among mathematical concepts than when they worked on the video lessons. My personal interpretation is that this difference was associated with the particular design of these activities. Whereas the state curriculum activities required the application of concepts from linear functions in real life scenarios, the video lessons and their associated practice tended to present concepts as information for memorization. For example, in a lesson on horizontal and vertical lines, students were given the acronym VUX to know that vertical lines had an undefined slope and an equation of the form ‘x equals’. Similarly, they were given the acronym HOY to know that the slopes of horizontal lines were 0 and that their equations were of the form ‘y equals’. There were no connections provided to the actual calculation of slope, to different forms of linear equations, or to what it means to write the equation of a line. Thus, students’ questions on horizontal lines, for example, did not incite discussion of their mathematical thinking with Eddy. For example, when one student asked Eddy about the slope of a horizontal line, Eddy shaped a circle with his fingers. The students’ first response was the letter O. Eddy
then said, “what number looks like the letter O?” I present a sample activity from the video lessons as Appendix K. Sample questions from the state level curriculum can be seen in Appendix L, within Daniel’s case.

**4. Seeks multiple approaches to develop conceptual understanding: Definition.**

Through this intervention, Eddy sought to teach by helping students make connections to different forms of representations (algebraic, numeric and graphical). In some respects, this intervention was found to be associated with ‘guides students mathematical thinking’ because in explaining a concept, one can contend that this also involved guidance in thinking. But I found differences between these two interventions. Eddy had been guiding students’ mathematical thinking through his practice in the self-paced format since its inception in his first year of teaching at the high school. His intention was to support students’ thinking, so as to get an immediate feedback on their areas of mathematical need. This intervention, ‘seeks alternative approaches to develop conceptual understanding’ was found to be an outcome from his work with his students in the process of guiding their thinking. Eddy explained during the post-observation interview that he had found that this approach was effective from working with his students. Based on Eddy’s patterns in his work with his students, I present this intervention as an emerging intervention because Eddy did not exhibit it consistently.

Earlier in the observational period, I noticed some inconsistencies in Eddy’s efforts to teach using mathematical connections. The lesson description I presented earlier on horizontal and vertical lines exemplifies teaching without making mathematical connections. In a different incident from the observational period, I had noticed that Eddy rewrote the equation of a line from standard form into slope intercept form so that students could identify the slope. In that same lesson, he had another student find points using the equation, and then use those points and
the formula for slope to calculate the slope. Eddy explained that students had challenges manipulating the equation in standard form algebraically. He was not concerned for expecting students to rewrite the equation themselves into slope intercept form, because standard form was not included in the unit test (Eddy, check-in). In this case, due to foundational gaps, he used alternative approaches to find the slope of the line, but he avoided connections to the standard form of the equation of the line. This posed a concern, in my opinion, in that students would be eventually working with systems of linear equations where students are typically required to perform more algebraic manipulations and to write equations of lines that model real life scenarios in standard form. On the other hand, when Eddy worked with students on the state curriculum (see intervention 3), their discussions included rich connections between the graphs, the ordered pairs and the equation, as long as the equation was written in slope intercept form.

*Forms of knowledge of the student informing ‘Seeks multiple approaches to develop conceptual understanding’.* Figure 4.8 above, depicts the forms of knowledge that reflected association with this intervention. This intervention reflected a primary association to students’ mathematical thinking. It also reflected association to other forms of knowledge such as: mathematical structure, mathematical language and learning needs. Eddy used alternative approaches to help students problem solve especially when they were not comfortable with the mathematical language and/or structure that the problem involved. Eddy attempted different approaches based on what he had found that each student needed.

I present Daniel’s case to exemplify Eddy’s work in seeking multiple approaches to develop conceptual understanding. More specifics about Daniel’s case are included in Appendix L (his work and background triangulated from the data set). Daniel was a student that was at risk of passing the course. After contacting the family, Daniel was fully engaged attempting to
complete lessons in the last two weeks of school. He had moved out of his original seat location to work on the unit test (activity 106) next to Eddy. In a problem where they had to find the equation to model the sale of cookies (mathematical structure), Daniel was able to provide an equation by modeling the information with data points and a graph. Eddy told Daniel in that class, “getting data from the graph is a solid mathematical approach…tell you what, look at your patterns!” (mathematical thinking). During the check-in that day, Eddy walked me through Daniel’s test. I noticed that Daniel had found the equation in an earlier part of the same problem and that a goal in the problem was to use the equation to answer additional questions. I asked Eddy if Daniel could have used the equation. Eddy explained that he did not think that Daniel would have been able to use it. After checking all recordings, I confirmed that no other discussions took place to revisit the use of the equation. This was, in my opinion, a missed opportunity to help Daniel make connections to the equation. Daniel’s learning, however, was evident in that he successfully completed on his own the next question that modeled a falling skydiver. This was a question that many students had struggled with. Eddy had helped Daniel by drawing a small sketch of the skydiver (see Appendix L). Daniel modeled the problem graphically and with the ordered pairs to answer the questions (learning needs). When Eddy looked over Daniel’s work, he said – “I tell you what, you picked up on this very quickly with graphing” (class observation). I noticed that Daniel smiled and nodded. Towards the end of the period, Daniel looked at me and said, “best math teacher ever”. One minute before the bell rang, Daniel turned in his test. Eddy advised him to not rely on the final exam to pass the class. He told him to look over his grade that night, and to continue completing work.

During the post-observation interview, I asked again a question from the pre-observation interview. I asked Eddy what he had learned from his students that had an effect in his teaching.
Eddy explained that he was “a math person” and that understanding problems had always come easy to him. He learned from his students that he had to provide more descriptions and explanations than what he personally needed. He gave me the analogy of using directions to get to the mall. He explained that he can get there easily because it is a familiar place in his town, but that other people need more landmarks. In teaching math he has learned from his students that they need to get from him the “in-between steps” and a greater level of detail (post-observation interview). He also added that one of his goals is to go back and re-do many of his lessons to incorporate a more thorough understanding of concepts.

**Flexibility in choice to engage in learning.** Eddy offered a lot of flexibility in the tasks they engaged in within the classroom. My observations confirmed that many students spent time online watching videos that were unrelated to class and playing with each other. Many students (at least half of each class) were off-task or highly unproductive until Eddy got to their table. I asked Eddy on multiple check-ins about students’ behavioral patterns. His responses indicated an expectation for students to own their learning. In helping students understand problems and their own work, students were able to build some level of ownership, but Eddy’s feedback confirmed a generalized idea that students were either committed to their school learning of they were not (check-ins and post-observation interview). I pressed more through a cross case check on his approach to offer flexibility in the classroom because I had noticed that in Beth’s practice, not working was not an option. In Beth’s practice, however, she had social workers and counselors that were available to work with her students when they were not willing to be engaged in learning in her class (Beth, check-in). Eddy’s response confirmed Eddy’s need to maintain students in the classroom because the alternatives for him were different. Eddy stated,
“In the hallway, I have no chances of them learning math, in my classroom, the chances of learning are higher” (Eddy, check-in).

**Findings: Central phenomena in Eddy’s practice.** I now revisit Eddy’s interventions to take a closer look at central phenomena in Eddy’s practice. These phenomena include his perspective on learning, the interplay between his interventions and his teaching goals and his understanding of his students’ mathematical learning experiences.

**Eddy’s Interventions and his perspective on learning.** Eddy’s description of how his teaching in a diverse classroom has impacted his understanding of how students learn (e.g. pre-observation interview question), best reflected his perspective on learning. According to Eddy, all students can learn math. “Everyone can learn in some level or another” (Eddy, pre-observation) and because of this, he will not stop. He asserted:

> I like them not to stop. I think that sometimes they stop more than I do. And if that is the case, you keep pushing, you keep pushing and you pull and you tug. They can all do it to some extent. (Eddy, pre-observation interview).

My interpretation of Eddy’s perspective, based on his whole case data, is that Eddy considered it important for learning to address two dimensions of his students. These were their mathematical learning and their development as individuals. But students’ mathematical learning had been affected by earlier experiences where they had developed foundational gaps and/or low self-efficacy. These experiences in turn, impaired students’ efforts to engage in new math learning. Because Eddy believed that they can all do it “to some extent”, he made decisions on the types of tasks (and their depth) that his students could engage in, so that they could focus on what Eddy considered to be core and/or foundational of algebra.

**Interplay between Eddy’s interventions.** Eddy’s expansion of the boundaries of the learning space allowed Eddy to have almost daily individual discussions with his students.
These discussions focused for the most part on their work, which placed accountability on students to keep up with the lesson pace and to seek Eddy’s help. Eddy assisted their work by guiding their thinking and looking for ways to help them work around (where applicable) limitations due to foundational gaps. The self-paced format relied on weekly completion of work. Thus, Eddy addressed issues of motivation or self-efficacy through these same individual conversations that he used to guide student’s mathematical thinking. His intervention on individual conversations carried dual attention on mathematical on non-mathematical aspects of his students. When students had developed ownership or motivation about their learning, Eddy’s work was simplified to supporting their mathematical learning. But, the number of students with this ownership were low in his Algebra 1 classrooms. Eddy often times described the students with ownership as “the freshmen in the Geometry courses” (Eddy, check-in). On periods six and seven, Eddy’s conversations required his relational knowledge of his students to motivate them to engage in learning. When they did engage, then Eddy was able to support their thinking to show them that they were more capable in math than what they believed about themselves. As an emerging intervention, Eddy started to find mathematical ways to support his students’ learning more effectively by incorporating connections between different representations for relations and functions. His most common challenge, however, was incorporating algebraic connections.

**Model of the learning phenomena in Eddy’s case.** Up to this point I have presented and described the forms of knowledge of the student used by Eddy as well as how these have informed his interventions in the classroom. I now present a model for students’ learning experiences in Eddy’s classroom that is based on his teaching goals, his perspective on learning and the interventions implemented by Eddy. I used this model in Figure 4.9 as an analytical tool.
to further understand and explain the learning phenomena in Eddy’s practice. I describe the components in this model, as well as possible paths within the model. I end this section with examples from students’ learning experiences depicted through the model.

Figure 4.9  Model of Learning Phenomena in Eddy’s Case

**Interventions.** Eddy used different forms of knowledge of his students to inform his teaching interventions in his practice. These forms and their association to each intervention were depicted in Figure 4.8. All interventions were affected by a level of flexibility that students were given to engage in learning. For example, students had a choice of where and when to complete their lessons (see expands boundaries of the learning space). They had a choice as to whether to engage in learning in the classroom or not. Eddy’s interventions had the strongest potential to make an impact on students’ learning if the students engaged in the lessons and/or
worked individually with Eddy in the classroom as he guided their mathematical thinking, held individual conversations with them, and as he sought multiple approaches to develop their conceptual understanding. This distinction is important because, although students always responded to Eddy when he met with them individually, those that had built trust on Eddy or that had some level of buy-in in the process where observed asking questions to help them progress through the lesson schedule.

*Personal Trust or Buy-In.* Eddy demonstrated high use of relational knowledge about his students. He used humor and what he knew about them to encourage them to work on the lessons. Often times, students worked with him out of respect for his role as the school’s football coach. Eddy also had called teaching, “a relationship business” (post-observation interview) where students did work for him because they liked him, but that even liking him would not guarantee students working. Some students worked in the classroom out of their own buy-in and interest to do well. This buy-in was described by Eddy often times as a trait that some students possess because they “know how to play school” (recruitment interview). At the same time, Eddy demonstrated surprise and joy when he saw students that turned around their efforts and overcame learning attitudes so as to engage in learning (e.g. Daniel’s case).

Meaningful engagement in the lessons, however, rested on the students’ end. Students did not evidence buy-in or trust in Eddy and/or the learning process by going off-task as soon as Eddy moved on to another student or when they used petty excuses to explain why they had not made progress (i.e. “it is not loading”, “I could not find the page”, etc.).

*Engagement in Lesson Completion and Willingness to Work with Eddy.* Eddy made efforts to prompt students to work. Eddy also confronted lack of progress when students were not reflecting engagement. The students that reflected engagement and buy-in, worked on the
lessons at different paces. Based on observational data and on Eddy’s feedback during check-ins, students’ central behaviors reflected that they needed Eddy to support their learning engagement to overcome any of the following challenges: motivation, perseverance or foundational gaps.

**Outcomes: No change or Improved Confidence in Learning Math.** Regardless of the extent of foundational gaps or learning attitudes (e.g. self-efficacy or motivation), students that sustained consistent engagement in learning refelcted improved confidence in learning math. I used their comments in class, their willingness to continue progressing in lessons, and Eddy’s descriptions as indicatorss of this improved confidence. Students with less consistent engagement in learning did not evidence any apparent changes in learning attitudes (as indicated by classroom behavioral patterns and Eddy’s descriptions). This does not mean that the outcomes observed were absolute or final. Eddy described the outcome of his work with some students as “a start”. I used double arrows between these outcomes and students’ lesson engagement in the model to depict this aspect of progress.

I note here that few students demonstrated full independence in doing their work, regardless of the consistency in learning engagement. Based on Eddy’s feedback (post-observation interview), this level of student independence was observed in the accelerated classrooms, but not in the academic classrooms (the lowest performing track correponding to periods six and seven). Additionally, according to Eddy, none of his students (academic or acelarated) did well on the final exam. We looked over students’ exams while I obtained his opinion on the outcomes from the exam. I interpreted Eddy’s responses to indicate that he felt responsible for his students’ low exam grades. He compared his scores to a colleague’s, and suspected that because the other teacher used two days to do similar exam questions right before
the exam date, that the teacher had been more effective than him in preparing them for the exam. While there is no data to support causality, students’ high level of dependence to do daily work leads me to think that the stark difference between exam conditions and what students were accustomed to in class could have had some impact on students’ exam performance.

I end this section by revisiting some of Eddy’s students to describe their learning experiences through this model.

**Selena.** Selena was the first out of Eddy’s two students that completed the sets of assigned lessons. She worked independently most of the time. She asked Eddy questions when he came to her table. She also sometimes asked Eddy to come back to her table if she had questions before he was done rotating through the room. My original interpretation of Selena is that she was naturally driven to learn. Through check-ins, Eddy explained that he had done a lot of work with her. Eddy supported her challenges with *self-efficacy* through his **individual conversations and by guiding her mathematical thinking**. She needed a lot of support at the beginning of the year through “a hand-holding” that “paid off” (Eddy check-in). By the time I did observations in quarter 4, Selena had **consistently engaged in her lessons** for three full quarters. Eddy said in a check-in that Selena had “left her classmates in the dust”. She **demonstrated improvement in her confidence level** to work independently and to seek help when she needed it. This was so much so, that without Eddy’s descriptions from the check-ins, I would not have suspected her challenges with self-efficacy from my observations alone.

**Carmen.** Carmen demonstrated low engagement in learning. I noted from one class that she stated, “I hate this, I am not good at this” as she took her seat. Carmen sat just a few feet away from me. Carmen spent a lot of her time in class teasing Kegan. In one class alone, Carmen spent the full period watching videos on her phone. While I was conducting the post-
observation interview with Eddy, Carmen arrived to bring in late work. I was surprised about
this because the final exams were already graded. On a later interview session Eddy explained
that he had called the family the weekend before the exam to alert them about Carmen’s grade.
He gave them a list of the lessons that Carmen needed to complete because he was concerned
that her final exam grade could affect her course grade. Carmen did not do any of these lessons
that Eddy suggested and also failed the final exam, resulting in her grade being two points below
the passing grade of 60. Eddy called the home again and offered to take in late work up until
that day when the grades were due at the main office. That morning Eddy reviewed her work
and explained that it was very incomplete. It only helped her to get a 59. Eddy had her seat at his
desk and complete two more lessons with him so that she could obtain a 60. When Carmen was
done, Eddy asked her if she was happy. She said she was because she would be able to get her
cell phone back from her mom. Carmen was able to benefit from Eddy’s intervention on
“expands the boundaries of the learning space”, but there was not much buy-in. I suspect
that her buy-in was low as a result of low self-efficacy. Carmen’s engagement in learning did
not seem meaningful to me in that it focused on obtaining a score, not necessarily meaningfully
vesting in her mathematical thinking. Although Carmen completed enough lessons to pass her
class, I interpreted her learning experience as one with no apparent change.
Case 4: Dena – “Makes Sense? If not, come see me”

The observation period was conducted over a period of three weeks in a Quantitative Analysis course during the months of April and May of 2017. At the time of the study, Dena was on her ninth year of instruction at Beacon Community College.

Background information: Quantitative Analysis course and learning paths at the college. This was the third time that the Quantitative Analysis course had been offered at Beacon Community College. Dena taught the course all three times. She described it as “relatively new” to the college (pre-observation interview). Dena taught the only section of this course that was being offered this semester. She did not teach any courses, other than this course section, because she handled other administrative duties at the college.

The Quantitative Analysis course met graduation requirements for select associate degree programs at Beacon Community College. This course was not in the developmental or remedial math sequence, but it also did not meet math college level requirements at the public four-year universities in its state because the topics covered only required the application of content equivalent to a high school Algebra 1 course. The Algebra 1 courses offered at this college were called Elementary Algebra (or Math 95). The topics covered in this course included: principles of reasoning, problem solving techniques, basic statistics and every day mathematical models (Course Syllabus). These topics were covered through applications in personal finance, the arts, careers, and society in general (Course Syllabus).

Since discussions about students included past math course learning experiences as well as their math college level placement and learning trajectories, I include below a general schematic (Figure 4.10) of the math course electives at the college. I use arrows in this schematic to indicate when a course is considered a pre-requisite to other courses. Placement,
and consequently, the starting course for each student’s learning path was determined for the most part by scores from the college’s entrance placement exam – the Accuplacer – but there were other placement methods used, such as SAT scores. I also note that at the time of the study, all course offerings in Pre-Algebra had been eliminated at all the state colleges due to a recent state law that mandated the elimination of these remedial courses. Students that placed into Pre-Algebra were offered an Elementary Algebra course with embedded remediation. Since the state did not require Algebra 2 for high school graduation, the college’s equivalent course (Intermediate Algebra or Math 137) was not considered a remediation course. Math 137, however, did not meet collegiate level expectations at the public four year universities. All college level math courses at the public four year universities had Math 137 as a pre-requisite (or it’s equivalence in placement from Accuplacer scores).

Seven out of the eight students in Dena’s class had started their college experience in one of the developmental math courses. Two of Dena’s students were taking concurrently the Quantitative Analysis course and Intermediate Algebra. Their program of study required two courses above the 100 level for graduation, in which case, taking the two math courses would have met all of their math graduation requirements. There is one additional non-college level math course that was omitted from Figure 4.10 because none of Dena’s students had taken it. Its inclusion in Figure 4.10 would not have added understanding to students’ math learning paths in Dena’s class.

**Background information: Specifics on Dena’s course.** Dena’s class met once a week in the evening for fifteen weeks in the semester. Each class was three hours long, but she gave students a 15 minute break halfway through each class. The first half of the class was dedicated to instruction on the day’s topic which followed a lecture format. In the instructional component
of the class, she guided her lessons with power point slides. She provided each student with a hard copy of the slides that included space for notetaking within the same document. The lessons included examples where she demonstrated the application of the concepts taught. She followed these examples with additional problems that she completed with students seeking more input from them in the decision making process. The second half of the class was dedicated to practice and group work. In two out of the three weeks, Dena assigned group membership in advance. In one class, the students asked her to allow them to group themselves. She conceded to their request despite her intentions to mix them by ability (Dena, check-in).

Figure 4.10. Students’ Math Course Learning Paths at Beacon Community College

*Math 104 was Dena’s observed course

Each week, students were expected to complete and turn in for a grade the classwork that was worked on in groups during the second half of the lessons. This classwork was provided to students as handouts with room for them to write their solutions and the corresponding supporting work. Students used an online supplement program that included an electronic version of the textbook as well as online homework assignments. The program provided immediate feedback after each homework question was submitted, allowing students to
determine if their questions were correct or incorrect. The program also gave access to sample questions in association to those within the homework assignment and it included a special feature where students could request help as they worked on the homework online. The online help was provided through a series of guided questions that walked students through the solution process. For each unit in the semester, a project was assigned based on the topics covered within the particular unit. No projects were assigned during the observational period because the unit I observed was the last unit of the semester. Dena said that they would not have enough time to complete a unit project in addition to preparing for the final exam (Dena, lesson 2). There were discussions with students about their unit projects during the observational period because in some cases students discussed their graded assignments from earlier units and in other cases some students had requested extensions and had not turned in work that was due. According to the syllabus and as confirmed with Dena (check-ins and post-observation interview), students that turned in work late had one point taken off for each day that the assignment was turned in late from the due date.

The unit under observation was a unit on problem solving in probability. The topics by week are summarized in table 4.14.

Table 4.14. Topics Covered During Dena’s Observational Period

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic Taught</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>The Basics of Probability Theory</td>
</tr>
<tr>
<td>Week 2</td>
<td>Complements and Unions of Events</td>
</tr>
<tr>
<td>Week 3</td>
<td>Conditional Probability and Intersection of Events</td>
</tr>
</tbody>
</table>

Background information: Dena’s students and classroom. Dena’s class had a total of eight students. Six students were female and two students were male. Based on appearance and classroom interactions and comments, one female student was Hispanic (*Sonia*), one male
student was Black (*Omar*) and one female student was both Black and also an English Language Learner (ELL) (*Elsa*). The rest of the students appeared to be white. The classroom could easily accommodate 30 students. It was set up with four long tables on each side of the room, all parallel to the front wall that held three white boards. A pull down screen covered the middle wall when the overhead projector was on. This was the case through most of the first part of each lesson, when Dena held the lecture component of the class using power point slides. On the left side of the room, three students sat separately, using the front three tables and leaving the last (i.e., fourth) table in the back empty. On the right side of the room two students sat on each of the first two tables, leaving the eighth student, Omar, to occupy the third table away from the board. I sat on the fourth table on the right side of the room, behind everyone else.

The findings sections that follow make reference to some of Dena’s students to provide select examples of their learning experiences. In an effort to assist the reading of the findings section, I provide in table 4.15 key characteristics about these students based on the full case data.

### Table 4.15. Dena’s Referenced Students and Key Characteristics

<table>
<thead>
<tr>
<th>Student</th>
<th>Characteristics and Particular Situations Referenced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elsa</td>
<td>Elsa was a student identified by Dena as ELL. Dena expressed having difficulty identifying the nature of her challenges in learning. Elsa started to meet with Dena for help towards the end of the semester. Dena wished she had been able to engage her more outside the classroom so that she could better understand her needs. Through their work in test corrections, Dena was able to determine that Elsa did not know how to create a Venn Diagram which was a basic skill needed to do some of the problems on the test.</td>
</tr>
<tr>
<td>Omar</td>
<td>Omar worked full time and was also taking concurrently Dena’s course and the online Math 137 course. Dena expressed frustration in not being able to engage Omar. She attempted to invite him multiple times to come for help. He did not respond to her attempts. His test corrections and some of the take-home assignments were turned in at the end of the semester. Dena also noticed foundational gaps when she worked with Omar one-on-one within the classroom to solve linear equations. Dena was concerned that he was not engaging her as a result of cultural differences and/or gender difference that could affect his ability</td>
</tr>
</tbody>
</table>
to relate to her as Dena expected. She tried to get to know Omar through his Math 137 teacher, but he was also not turning in work in that class. Dena said that she discarded concerns for culture and/or gender differences when she found out that the teacher was male, and that Omar was also not turning in work for that teacher.

Sarah
Sarah had taken Math 95 (developmental math) already twice with Dena. This was the third time that Sarah had Dena as her teacher. Sarah shared this fact while in class. She frequently shared her thinking (mathematically and non-mathematically) in class. She used humor, which Dena sometimes responded to. Dena would also sometimes ignore her comments when the comments were given while Dena was trying to teach and/or when Sarah’s comments were unrelated to Dena’s questions. Dena described Sarah as a student that was inconsistent in her efforts. She ascribed this outcome to low self-efficacy. Dena explained that she knew herself that Sarah could do much more in math than what she gave herself credit for. Sarah worked full time with young children, she expressed being tired in class multiple times and even asked to be excused from group work one evening because of that. She made up the work, but late.

Sonia
Sonia had three children and worked full time. She was taking at least two classes that semester, Dena’s class and a Math 137 class online. She could not make every class because of child care conflicts and her husband’s work schedule. Dena allowed her to come to every other class with the agreement that she would complete the work on her own with Dena’s assistance. Sonia had succeeded earlier in a developmental math class that was taken with an adaptive online system. She frequently struggled with foundational gaps in math. She was interested in completing a B.S. degree in finance. Dena commented that she would most likely realize she would have to change degrees once she started her coursework at the university because even though she succeeded in the developmental math class, she still carried too many gaps. According to Dena she was a very responsible student that succeeded in her class, but she had low mathematical maturity (see table 4.16 for description).

Thomas
Thomas had started in a Math 95 course with Dena without success. He then took a course that included remediation in pre-algebra. While he did pass that developmental math course, Dena expressed concern for his foundational gaps. In one incident he turned in all his test corrections in the tutor’s handwriting. Dena noted that he was unaware about the fact that it was not an acceptable behavior. She had a similar incident in the Math 95 course with him where he took out his notes to take the test despite written expectations given by Dena the class before. Dena suspected that he was not given the opportunity in high school to think on his own. Based on his behavioral patterns, she concluded that this must have been the way he learned in school where he was assisted in his thinking, but not expected to work on his own.

Other students will be referenced in the findings, but the necessary background information will be referenced within the text. These are: Joyce, Linda, Marlee.
Findings: Dena’s teaching goals. Dena’s goals were first triangulated from interview and observational data. I then confirmed these goals through member checks in the post-observation interview. I list Dena’s goals below, but I revisit them again when I describe the teaching interventions that Dena implemented to support these goals.

(1) to develop students that are ready for college and/or their field of work
(2) to develop students’ problem solving skills and in doing so, to help students overcome preconceived notions about what it means to learn math and problem solve in math
(3) to create learning experiences that develop students’ critical thinking skills that in turn, develop ownership over their knowledge gain and over the learning process

Findings: Forms of knowledge of the student used by Dena. In table 4.16, I summarize the forms of knowledge of the student that Dena used to advance her students’ learning. I also include their corresponding definitions formulated from and contextualized in Dena’s case data.

Table 4.16. Forms of Knowledge of the Student Used by Dena Mathematical – MK and Non-Mathematical –NON

<table>
<thead>
<tr>
<th>Form of Knowledge</th>
<th>MK (math) vs. NON (non-math)</th>
<th>Definition and Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Thinking^N</td>
<td>MK</td>
<td>This form of knowledge is associated to students’ thinking process while they engage in doing math. Dena’s interventions reflected frequent patterns of using students’ mathematical thinking for various purposes, but with an ultimate goal to better understand her students’ needs. This form of knowledge is different from mathematical ownership in that thinking is associated to process, whereas ownership is associated to an outcome from the process of mathematical thinking.</td>
</tr>
<tr>
<td>Personal Conditions^N</td>
<td>NON</td>
<td>Students, as individuals in society, were found to have characteristics, personal conditions and/or life experiences</td>
</tr>
</tbody>
</table>
that had an impact on how they learned or on how they engaged in learning. Dena gave the example of a student that was a veteran soldier and how this life experience allowed him to have much more success in environments with structure (recruitment and post-observation interviews).

<table>
<thead>
<tr>
<th>Knowledge Area</th>
<th>Type</th>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SES – Life Priorities(^O)</td>
<td>NON</td>
<td></td>
<td>This knowledge is associated with life challenges, typically encountered as a result of low socioeconomic standing. It could be construed as a subset of “personal conditions”, but it is given its own category because of the high frequency and commonality across students. Examples include: holding multiple jobs, inability to afford childcare, prioritizing immediate living and survival needs over long-term goals such as education.</td>
</tr>
<tr>
<td>Relational(^R)</td>
<td>NON</td>
<td></td>
<td>This knowledge refers to particular ways in which students prefer to relate to the each other and to the teacher.</td>
</tr>
<tr>
<td>Foundational Gaps(^O)</td>
<td>MK</td>
<td></td>
<td>This knowledge refers to students’ knowledge and application of pre-requisite concepts and skills. Since the course observed only required Elementary Algebra (i.e. Algebra 1), the gaps were associated to any mathematical topics and concepts at that level or below. In Dena’s case, the most common occurring foundational gaps were associated to number sense – understanding equivalence between fractions and decimals, proportional reasoning and basics of algebra – like solving simple linear equations. General problem solving skills are included here as a subset of foundational gaps, based on Dena’s stated need for students to graduate high school with proficiency in this area in order to be successful in their collegiate math learning experiences (Dena, post-observation interview).</td>
</tr>
<tr>
<td>Anxiety(^N)</td>
<td>MK</td>
<td></td>
<td>Students have anxiety about learning math. Dena noted that more students exhibited anxiety than she had anticipated going into this course (recruitment interview). This anxiety, as described by Dena, causes students to “put road blocks for themselves” (recruitment interview), often choosing math courses with lower requirements that limit their access to other programs of study.</td>
</tr>
</tbody>
</table>
| Mathematical Maturity/“Muscle”\(^N\) | MK | | This is a student characteristic that Dena described as a long term outcome from thinking critically and problem solving. It is a combination of comfort with failure, but also an ease with breaking down a set of information and exploring ways to come up with a solution approach. Students with foundational gaps did not tend to have this mathematical maturity, but if mathematical maturity was
developed over time, the student could also overcome foundational gaps. Mathematical maturity was also described by Dena as the “hidden curriculum” that is not measured by objective assessments because it is related to thinking approaches, but is also the desired outcome for students to be successful in learning math (Dena, post-observation interview).

<table>
<thead>
<tr>
<th>Learning Attitudes:</th>
<th>Both</th>
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</table>
| Ownership | Dena described two different levels of ownership. Mathematical – Dena explained that students construct knowledge in their own way. Ownership over mathematical learning, was described by Dena as an individual outcome for each student which was very different from teaching. “Teachers teach so that students learn” (Dena, post observation interview), but what they learn is theirs and according to Dena, in order for teaching to be more effective, teachers need to seek an understanding of how students own their knowledge.

Non- Mathematical: Ownership over the learning process was more closely associated to responsibility and “follow through”. Students that have ownership are not passive learners. When material is not clear to them, they seek help and advocate for themselves and their learning.

<table>
<thead>
<tr>
<th>Language</th>
<th>Both</th>
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| Non-mathematical: Language encompasses different consequences in the learning process for the student whose first language is not English. Dena demonstrated awareness of the added cognitive demand in the learning process when all instruction and interactions were in English. Dena appreciated this particular aspect about her ELL student, Elsa (post-observation interview) and took it into consideration when reflecting on ways to best support her.

Mathematical: Dena used it in class to make distinctions on how elements in a set were related, to then make a decision on the type of probability calculation that was needed (e.g. complements, intersections). Dena used students’ mathematical language most frequently to help them make connections to pre-requisite knowledge (e.g. percent calculation vs. fraction form). In these instances she also made use of students’ mathematical structure, though Dena’s emphasis was on language.
<table>
<thead>
<tr>
<th>Culture and/or Cultural Dissonance&lt;sup&gt;O&lt;/sup&gt;</th>
<th>NON</th>
<th>Culture was found through its alignment to characteristic norms, practices and value systems that students hold. Dena used culture to interpret students’ value systems as associated to their ethnic heritage. Cultural Dissonance: In some cases Dena recognized cultural differences, without necessarily identifying and/or understanding particular cultural aspects of the students. This is described here as a cultural dissonance between Dena’s understanding or norms and her students’.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Efficacy&lt;sup&gt;N&lt;/sup&gt;</td>
<td>MK</td>
<td>Dena has observed that some students have a lower perception of their abilities and of the tasks they are able to accomplish than what Dena herself has noticed they are capable of (post-observation interview). Low self-efficacy caused students to be inconsistent in their efforts and to do “just enough to get by” (Dena, post-observation interview).</td>
</tr>
<tr>
<td>Learning Styles&lt;sup&gt;N&lt;/sup&gt;</td>
<td>Both</td>
<td>Dena described differences in how students learn. Mathematical – Dena explained that a student can be comfortable and make math connections by following an online lesson platform, while other students need the interactional components of the classroom. Although she expressed apprehension for learning math without interactional components, she also explained that some approaches like the online self-paced course formats work for some students over others because of their learning style. Non-mathematical – Dena described differences in which people think and learn. She explained having appreciation for this form of knowledge as a result of her background on students with special needs.</td>
</tr>
<tr>
<td>Mathematical Ability&lt;sup&gt;N&lt;/sup&gt;</td>
<td>MK</td>
<td>This is a student characteristic that Dena described with a close association to a natural ability and disposition to do math. This is different from “mathematical maturity” in that students could have an ability to do math, but not necessarily be comfortable with failure and/or struggle in the math learning process. Unlike mathematical maturity, math ability could be measured through objective assessments, but not necessarily guarantee success in a course.</td>
</tr>
</tbody>
</table>

**Findings: Dena’s teaching interventions.** Dena exhibited five central teaching behaviors in using the forms of knowledge of the student from table 4.16. I describe these
teaching behaviors as interventions going forward in this document because they represent a set of particular measures that Dena implemented to advance her students’ learning based on what she knew about her students. These interventions were: (1) flexibility in expectations, (2) holds students accountable for their mathematical learning and thinking, (3) extends individual invitations, (4) expands boundaries of the learning space, (5) mathematical and non-mathematical conversations, and (6) provides structure in mathematical thinking and in meeting expectations. Throughout the implementation of these interventions, Dena incorporated reflective practices. She described this as a core characteristic in her teaching (pre-observation interview), and she explained on multiple occasions how she reflected in her ongoing decision making. Because of this, I add reflective practices as a seventh intervention that permeates the implementation of all other six.

In the subsections that follow, I provide an operational definition for each intervention. These definitions are based on Dena’s context of instruction and on her descriptions of how she used what she knew about her students and why. For each intervention, I also report the forms of knowledge of the student that informed each interventions’ intended purpose in Dena’s practice. These are summarized in Figure 4.11.

1. **Flexibility in expectations: Definition.** Dena provided flexibility with expectations on due dates for graded work, on expectations in classroom attendance and in the content she expected her students to learn. This intervention best resembled a learning accommodation that was individualized because it responded to students’ particular needs. Setting aside her choices for content taught, she implemented these accommodations as part of a plan that she accorded with each student. Dena offered flexibility, but she also held students accountable for their learning. Dena’s plans to implement flexibility combined increased opportunities to learn while
also ensuring that students took responsibility for their work and for their learning. In some cases the students met their end of the plan, and in some cases they did not.

Figure 4.11. Forms of Knowledge of the Student Informing Dena’s Intervention
forms of knowledge informing ‘flexibility in expectations’. As shown in Figure 4.11, flexibility in expectations was found to be informed by five forms of knowledge of the student: anxiety, self-efficacy, language, personal conditions and SES priorities. Based on her experience, Dena stated that “this group of students has very high math anxiety, more than I anticipated, or more than I initially appreciated, and I had to let go of stuff” (recruitment interview). She used this new understanding to re-think the topics she taught. This required flexibility in her topic choices so that the content of instruction best served her students in terms of their goals and objectives. In using examples that had more applicability to them, Dena was able to overcome some of these anxieties and what she called their initial reaction where “they throw up their hands and they say, I don’t know” (recruitment interview). She had noticed that this anxiety affected their willingness to engage in problem solving. This was one of the reasons that she also gave for teaching for the first time the unit on problem solving that was based on probability during the observational period. It is also one of the reasons why she recommended the adaptive self-paced program that she instituted at her college for students in developmental math (i.e., algebra sequence). According to Dena, students with anxiety and with low self-efficacy evidence inconsistencies in their learning because they can’t necessarily sustain attention or process all information within one lesson. While she also stated that adaptive programs are not necessarily good for all students because of different learning styles, Dena explained that it gave the students that chose that instructional option the flexibility to complete lessons at their own pace (post-observation interview).

Dena personally arranged for new due dates when students expressed this need. Her syllabus did include penalties for late work, but she also listened to them and took into consideration their needs while still holding them accountable. This type of flexibility helped
students with competing priorities due to personal conditions or due to survival needs which are typically experienced by students from low SES (e.g. multiple jobs, family needs). Students with anxiety and low self-efficacy also benefited from this intervention because of the inconsistencies in effort mentioned above.

The forms of knowledge that I have described from Dena’s use up to this point have been particular to the group of students that Dena worked with in this class. That is, these were mostly students that placed originally in developmental math and that were in a terminal non-college level math course. Dena’s intervention, however, also used particular aspects of her students such as non-mathematical language. Elsa was an ELL student that was able to take advantage of flexibility and the additional time to process information. I followed up on Elsa after noticing that she left the first lesson early. When I asked Dena during the check-in about her, Dena explained that Elsa felt overwhelmed and she asked to be excused with the understanding that she would meet with Dena during the week for help. Dena explained that it took Elsa time to feel comfortable coming in for help outside of class. Dena was not sure why, but she suspected that either language or cultural differences posed initial discomfort for Elsa. In continuing to invite Elsa for help, she eventually overcame her apprehensions and started to come for help towards the end of the semester. Dena considered their work to be very productive outside class, but it was almost too late because it happened towards the end of the semester (post-observation interview).

A closer look at this intervention, reflected support of Dena’s teaching goals. Flexibility in expectations provided students with more access to their learning experience. Maintaining a rigid expectation on due work, would not have changed the realities that Dena’s students handled and that in different ways challenged their ability to fully engage in learning.
2. Holds students accountable for their learning: Definition. Dena held students accountable primarily for their mathematical thinking and learning. She did describe holding students accountable for turning in assignments, and for their presence in class because students needed to meet employer and college work expectations. But the accountability encompassed by this intervention was more closely associated to actual learning and to “owning their knowledge” (Dena, post-observation interview). Students were not held accountable for the sole act of turning in work, although that was a behavioral expectation in alignment with her teaching goals. Rather, students were being held accountable for engaging in mathematical thinking through these assignments, through discussions in class and through conversations in her office for individual help.

Forms of knowledge informing ‘holds students accountable for their mathematical thinking and learning’. Figure 4.11 above, depicts the forms of knowledge that reflected association with this intervention. Dena used three primary forms of knowledge to inform accountability in mathematical thinking and learning: their own mathematical thinking, foundational gaps, math ability, and cultural dissonance between some of her students’ and those of the mainstream. Their use was observed through patterns in the ways that Dena worked with her students. I then confirmed them through the post-observation interview. In her use of students’ mathematical thinking, I noticed that Dena did not necessarily answer questions, rather, she posed more questions. We spoke about this questioning approach originally in the pre-observation interview and she had described them as “guiding questions” (pre-observation interview). She would ask – “What’s this problem about?”, and as students answered, she would start circling components of the problems to confirm their input (observational period). During group work, she would walk and stand by quietly near each group. She seemed to me that she
was listening quite intently. Dena confirmed this as a purposeful behavior (check-in) because she wanted to hear her students have these conversations on how they “made their learning their own”. Although some of these descriptions will be revisited in her intervention on ‘conversations and discussions’, Dena’s self-reported purpose was to hold students accountable for their learning (post-observation interview). In some ways, holding students accountable for work was a form of accountability on learning, but the work was not necessarily reflective of her students’ learning if the students did not take the time to make it their own. For Dena, it was not about the act of turning in work, although she did incorporate test corrections as a graded component to support this accountability. For Dena, it was about the process of learning (recruitment interview post-observation interview). Throughout her lessons, she was found to pause often and ask her characteristic phrase – “Does it make sense?” She not only waited for an answer after her question, she also waited for them to show her how it made sense. This characteristic approach translated into her students’ interactions during group work. When Sarah and Linda helped her classmates, they were often heard asking the same question, “Does it make sense?”

Students’ foundational gaps in number sense and in algebraic skills often times were found to challenge students’ ability to fully grasp the meaning of a problem or to describe their thinking. Dena supported their thinking in these cases by reminding them of basic procedures and by helping them to make sense of the quantities they were working with. She still continued to hold them accountable in the process. In other cases, the students shared their own mathematical thinking to help others. Dena capitalized in their mathematical thinking often through classroom discussions and through group work. She also used her understanding of their math ability to use mixed-ability groups for classwork. Because the class size was so small, she
had limited options to change grouping assignments. Not all of Dena’s attempts to support students’ thinking necessarily resulted in their intended outcomes. The classroom snapshot, showcases at the end of this section a classroom learning incident where Dena tried to support students that were not familiar with the context of the problem used (e.g. students had never worked with a deck of cards). There was a cultural dissonance between the students’ lived experiences and those of the rest in the classroom. Since these students could not fully engage in the class exercise, Dena spent most of her time helping them sort the cards, or explaining the questions, but she was not able to get to their mathematical thinking. The snapshot also demonstrates how Dena worked with Sarah to address her foundational gaps in decimals and percent calculations.

3. Extends individual invitations: Definition. Dena invites her students to come see her individually for help. She learned that students did not necessarily respond to her invitations to come for help as whole class communications (verbally and/or in written form). Dena writes individual invitations for students to meet with her on the graded papers that she returns weekly. She also approaches students before class, during breaks and after class individually to ask them to come see her outside of class.

Forms of knowledge informing ‘extends individual invitations’. Figure 4.11 above, depicts the forms of knowledge that reflected association with this intervention. This is an intervention that reflected primary association with relational knowledge, but that maintained association with all other forms of knowledge. When Dena described what she learned from her students that has supported her efforts to advance their learning (pre-observation interview), she pointed that for some reason, unknown to her, her students best responded to individual invitations to see her for help. Students did not seem to take her up on the same invitations when
communicated as whole-class messages. In this regard, Dena used a new form of knowledge that was not included in the original list of codes. Based on her descriptions, I have denoted it as form of relational knowledge of her students.

Dena exhibited this intervention early on. I then followed through the observation period and through check-ins to better understand the role of these individual invitations to meet. These individual meetings took place throughout the three week period. In fact, in two out of the three weeks of the observation period, Dena spent twice the amount of time outside of class helping students than what she did in her weekly class time. We discussed why she met her students, what they discussed and what they accomplished through our check-ins. Based on this data, Dena made use of all forms of knowledge from table 4.16 to explain why it was important for her to meet with her students individually. For example, if Dena excused a student from class because of childcare and work conflicts (Sonia’s case – SES life priorities) or anxiousness in class (Elsa’s case), Dena also made the commitment to meet them outside of class at a time that worked for her and her students to re-teach and support their learning individually – in some cases two and three hours at a time. But these discussions on the importance of meeting her students led us to an even more central phenomenon in Dena’s practice, her perspective on learning. I dedicate a separate discussion to Dena’s perspective in a later subsection of the findings within this case.

4. Expands the boundaries of the learning space: Definition. This intervention refers to the multiple alternatives that Dena has come up with to arrange for instructional support for her students outside the classroom. These included: the online instructional program that supplements her textbook, the tutoring center, her individual sessions for help outside of class
(expounded as a separate category above) and her requirements to have students complete test corrections after receiving their graded tests back from her.

Dena credited the online instructional program that supplements her textbook and the tutoring center, as supports for her own practice. Although she is not too satisfied with the test bank of questions in the instructional program used for this course, Dena stated that “it is available to them 24 hours and I am not. And it makes them accountable” (post-observation interview). The program also provides links for each of the practice questions and students can send these links to her for additional support. Dena holds her office hours in the tutoring center so that she can reach out to as many students as possible (pre-observation interview). She has noticed that students are intimidated to come to her office. She holds her office hours in the center to increase interest in coming for help and to also help students become familiar with the center so that they can take advantage going forward. Test corrections seemed to me as a form of flexibility originally, but upon discussions with Dena, I confirmed that she expects learning to continue to take place even after assessments. Similarly, I include Dena’s individual meetings within this intervention because they focus on continued engagement in mathematical learning outside the classroom. Because of this, the interventions of flexibility in expectations and expands the boundaries of the learning space are interrelated. Dena is able to offer flexibility in expectations by expanding the boundaries of the learning space (and vice versa). My interpretation is that both interventions share one core tenet – learning does not have to be limited to the confines of the classroom and it can continue to take place over time throughout the course.

*Forms of knowledge informing ‘expands the boundaries of the learning space’*. Figure 4.11 above, depicts the forms of knowledge that reflected association with this intervention. All
forms of knowledge informed this intervention. Students with foundational gaps, for example, needed additional support outside classroom instruction. Test corrections demonstrated positive results for Sarah and Elsa “because they were more reflective” (Dena, pre-observation interview). Being more reflective, here, matches a personal characteristic for Sarah and Elsa, but also a learning attitude in their interest to learn and improve. The test corrections did not evidence the same results for Thomas because he was more concerned about turning in the work as a grade requirement than about learning from the experience (Dena, post-observation). This was a learning attitude.

5. Mathematical and non-mathematical conversations: Definition. Dena’s non-mathematical conversations with students revolved mainly around ways they could prioritize their learning and completion of work. She also had conversations that focused on their mathematical thinking and on ways to problem solve. Dena also designed her class so that during instructional time (the first half of the class) they held a whole class discussion. The second half of the class, students worked on problems together in groups. “Conversations” was an in-vivo code used in Dena’s case study data. It was a term that she used frequently because it was central to her practice. This was Dena’s primary way to get to know her students in order to determine how to best support them.

Forms of knowledge informing ‘mathematical and non-mathematical conversations’. Figure 4.11 above, depicts the forms of knowledge that reflected association with this intervention. Dena used all forms of knowledge through this intervention. While it can be argued that conversations are a basic form of communication, they were purposeful in Dena’s practice. I present dialogue from our post-observation interview. I was trying to understand how culture was evident in her interactions with Elsa. Based on her response, Dena was not really
clear about the reasons for Elsa’s needs. She wished she could have been able to engage her more through conversations to be able ascertain the nature of her challenges. This is an excerpt from our dialogue.

Dena: Yeah, some of it was cultural, but part of it was language based. I definitely did not have enough knowledge to know if there were language issues only, or if there were cognitive issues as well.

Ruth: What do you think would have allowed you to get to it?

Dena: Being able to engage her more?

Ruth: How would you have engaged her? I am being explicit, but I also want to make sure I have it.

Dena: Having conversations with her to better understand what she understood. Like, at the end of the semester she came over for help to go over her test corrections and I noticed that she had no idea how to make a Venn diagram.

Ruth: I remember she met with you to make that appointment.

Dena: But yet it was something that we had done in the group work and yet, she never approached me to ask me about it or tell me that she did not understand it. She wasn’t getting it. To me, that is, what is going on cognitively here that I am not getting through? Because I didn’t have a sense that it was an ESL issue, but I was not certain.

Conversations were a key activity in Dena’s classroom, because they allowed her to understand her students, but they also allowed her students to make sense of their own thinking.

During the post observation interview I had given Dena the scenario that she was in charge of training me as a new teacher. I asked her to “tell me why is it that I need to capitalize on these discussions and – Why is that important?” (Ruth, post-observation interview). This was Dena’s response:

Well I think that the discussion or the communication part really gets at what students understand or do not understand. If they are able to put it into their own language and it kind of helps structure their knowledge? Or give an idea of what they get to understand and what they don’t and you can clarify what they don’t. Um, because otherwise, I’m exposing the students to the material and I am teaching it, but it does not mean that they are learning it until they make it their own. In order to make it their own, that’s where they are putting it in their own language and then they get it. Then you know that they have it. If they can put it into their own language, then you know it’s theirs. (Dena, post-observation interview)
Thus, in Dena’s practice, conversations played an essential role in students’ learning. They helped students make sense of their own thinking and this in turn allowed them to make it their own (ownership). Our discussions during the post-observation interview about this intervention, along with the sixth intervention on providing structure, bring us even closer to Dena’s perspective on how her students learn. As reflected in Dena’ comments above, she believed that students structure knowledge differently.

6. Provides structure in mathematical thinking and in meeting expectations:

Definition. Dena described “structure” (also an in-vivo code) as a roadmap. Mathematically, this road map referred to helping students break down concepts and make sense out of them. But this breakdown was not necessarily the same for all students. Dena explained that students do not understand that there is not just one way to break down and make sense of a problem (post-observation), but that with a roadmap, “then they can take it from there” (post-observation). This intervention also made use of non-mathematical aspects of students in helping them find ways to organize themselves to ensure that they met weekly work expectations.

Dena’s ability to provide structure depended on her ability to listen to her students through conversations which took place inside and outside the classroom. Conversations outside the classroom were found to be highly dependent on students’ ability and willingness to meet with her. Conversations inside the classroom were not immune to challenges either. When a student was not able to engage in learning in class, Dena was not able to understand their thinking. This was the case with Omar (see classroom snapshot in Appendix M).

Forms of knowledge informing ‘provides structure in mathematical thinking and in expectations’. Figure 4.11 above, depicts the forms of knowledge that reflected association with this intervention. All forms of knowledge were found to be associated to this intervention.
Support with **mathematical structure**, according to Dena, helped students when they found themselves not knowing how to begin a problem. She described experiencing push back in that students wanted to be given a procedure – “Yeah, give me the steps, what do I have to do? I don’t know how to do this” (Dena, post-observation interview). Structure was not about drill and skill, rather, it was about supporting students with a guided approach to help them make sense out of a problem and the key concepts applied within the problem. Support with expectations, helped students understand the behaviors and tasks they needed to engage in to support their weekly learning and growth. In some cases, Dena had noticed that students at the community college carry a lot of **personal responsibilities** (e.g. work, families, etc.) and they do not know necessarily understand what it entails to be successful in college. Structure in expectations helped her students in this regard. Structure was not always effective. Dena noted that students that “only want to pass” do not fully take advantage of her support and they end up taking “calculated risks to do just enough” (Dena, post-observation interview).

**Reflective practices.** Dena described herself as reflective during the pre-observation interview, in how she taught, how she planned, in how she determined her students’ needs, in what she considered were the class’ standards, and in what she considered important for her students’ learning process. She indicated using this level of reflection in the ways she explained how she accommodated each student, what she had come to learn about each of them and how she had taken it into consideration for her interventions. Her reflection relied on the input she received from her students. As with all interventions described above, in cases where she was not able to engage her students, her reflection was based on her best understanding of the situation. For example, Dena told me that she suspected that Elsa and Omar were not familiar with decks of cards in the check-in after the first lesson. She commented that she would try to
figure out what to do. Having reflected on her concern, she brought in the cards as manipulatives for the whole class. While her new approach demonstrated an appreciation for Elsa’s and Omar’s challenge, her decision did not take into consideration that the mathematical connections required much more ease and familiarity with how the suits and the types of cards were related.

**Findings: Central phenomena in Dena’s practice.** I now revisit Dena’s interventions to take a closer look at central phenomena in Dena’s practice. These phenomena include her perspective on learning, the interplay between her interventions and her ultimate teaching goals and her understanding of her students’ mathematical learning experiences.

**Dena’s Interventions and her Perspective on Learning.** Dena’s descriptions on how she used students’ mathematical thinking and their communication of their thinking through conversations (see intervention 5 above), best reflected her perspective on learning. Students’ conversations played a dual role. They served as indicators of what students knew, but in the process of communicating what they knew, new knowledge was also constructed and “reaffirmed” (Dena, post-observation). This reaffirmation then turned into ownership of knowledge. Dena also noted that students construct knowledge differently, and that because of that, conversations enabled her to determine the students’ extent of their understanding of a problem.

**Interplay between Dena’s interventions.** Dena’s interventions were informed by both non-mathematical and mathematical forms of knowledge of her students. Some interventions, like flexibility and expanding the boundaries of the learning space supported students’ daily demands and competing priorities that affected their ability to prioritize their learning. In this regard, students were afforded accommodations that increased their access to the learning
experience. But the actual learning, whether it took place as a result of engagement or as a result of individual thinking, had to be internalized and owned by the student. Helping students have this type of added access to learning did not guarantee learning in Dena’s practice, though it did ‘even the plane’ for them. Students also had to overcome other more internal challenges, such as anxieties, preconceived notions about what it means to problem solve, low self-efficacy, and additional cognitive demands when the context of the problems were not familiar to some of them. This is where Dena’s central intervention, conversations, played an essential role in advancing her students’ learning. Through conversations Dena worked to “unblock” (Dena, post-observation) anything that was getting in the way of their learning. But because in Dena’s perspective, students construct knowledge differently, her conversations were customized to their particular needs. In this regard, her intervention of structure represented a form of cognitive support where she was not telling students how to do a problem, but she was guiding them so that they develop the problem solving and critical thinking skills that she intended for them to learn. In cases where they were able to experience successful learning (i.e. demonstrating ownership of knowledge and over the learning process) they were also able to combat some of the other more internal challenges mentioned above.

**Model of the learning phenomena in Dena’s case.** Up to this point, I have presented the forms of knowledge of the student that Dena used, as well as how she has used them to inform her interventions to advance her students’ learning. I now present a model for students’ learning experiences in Dena’s classroom. I used the model in Figure 4.12 as an analytical tool to further understand and explain the learning phenomena in Dena’s practice. Dena’s particular case, as an instructor in higher education, provides a long-term perspective for students’ educational paths
and their eventual college experiences. I will first describe the model and will then end this section with examples from students’ learning experiences depicted through the model.

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**Figure 4.12** Model of Learning Phenomena in Dena’s Case

**Interventions.** Dena used different forms of knowledge of her students to inform the teaching interventions she implemented in her practice. These forms of knowledge and their association to each intervention were depicted in Figure 4.11. As described above, the interplay between Dena’s interventions reflect Dena’s work on access to learning accommodating life demands and personal conditions that required prioritization over their course learning. At the
Dena’s interventions addressed access to learning with other components that were more internal to students (e.g., self-efficacy, anxiety, etc.). Dena wanted to have her students engage in critical thinking experiences that would help them overcome preconceived notions of what is math learning and anything else that Dena considered to be getting in the way of their ability to think critically. She was not always successful in her interventions. Our discussions of instances of success or non-success, led to Dena’s description of a form of knowledge of the student that more often than not, guaranteed their success in learning. This was mathematical maturity (in-vivo code). Dena also referred to mathematical maturity as mathematical muscle.

Although I have included a definition for this form of knowledge in table 4.16, I add here that it was found to be a combination of competencies that in Dena’s experience were developed over time. She described it as not just knowing how to ride a bike, but “knowing how to ride it in different terrains” (post-observation interview). Mathematically, Dena construed students with mathematical maturity to be comfortable with meeting failure and also able to consider alternatives within a problem and handle open ended situations. These students demonstrated critical thinking skills through their reasoning process (post-observation interview). From a non-mathematical standpoint, students demonstrated mathematical maturity through their buy-in and ownership over the learning process.

Since mathematical maturity was a characteristic that could only be developed over time, my interpretation of Dena’s work was that she used her interventions in an attempt to provide students with the experiences that would develop mathematical maturity, but also recognizing that its development could potentially be partial because of the duration of time she was able to work with her students. I tested this interpretation by asking her to describe the types of learning experiences that would have developed this maturity. Her responses confirmed my
understanding. Dena also expressed being challenged with not having enough time to offer her students open ended and exploratory types of activities that in her opinion would also help develop this maturity. She said that instead, she could only offer her students “guided discovery” (post-observation interview). Thus, having or not having this mathematical maturity paved two possible paths from Dena’s interventions. I have denoted these paths as Path 1 and Path 2 in the model. Students either followed path 1 or followed path 2, but not both at the same time.

Path 1: Path 1 in the model represents the experiences of students evidencing full development of mathematical maturity in Dena’s practice. These students needed Dena’s interventions to support them with life’s competing priorities, but they did not need support in mathematical thinking nor in developing buy-in. They were able to consistently engage in learning and successfully learn in Dena’s class and/or in other courses of their choice. Using Dena’s perspective, these students had mathematical muscle.

Path 2: In Dena’s class, at least six of her students followed path 2. These students had an incomplete inventory of skills that would not have indicated their full development of mathematical maturity. I note here that these students cannot be described “without mathematical maturity”. What I noticed was inconsistencies and partial development of the different skills that altogether represent Dena’s conception of mathematical maturity. Path 2 represents the learning experiences of students in the process of developing mathematical maturity. Because of this, their path starts with partial buy-in or ownership.

Students in path 2 needed support as those with mathematical maturity with regards to competing life priorities. Their access to learning, however, was not fully realized because their engagement needed additional supports with aspects that I have described before as more
internal in nature (e.g. anxieties, self-efficacies, overcoming foundational gaps, etc.). Dena’s descriptions of these students revolved around the different reasons why they were not able to fully have buy-in, reflecting an understanding that buy-in and their attempts to have ownership were earlier indicators of the long term outcome of mathematical maturity.

Students in Path 2 could take at any given point, one of the **two alternatives** shown in the model stemming from **partial buy-in or ownership**. They either successfully engaged in problem solving or they did not.

*Engagement in Problem Solving.* According to Dena, students’ engagement in problem solving helped develop their critical thinking skills. By supporting students through this type of learning engagement (using also all interventions), Dena helped students overcome internal aspects that represented self-imposed “blocks” (e.g., anxieties, etc.) to their learning access (Dena, recruitment interview check-in).

*Overcoming Challenges in Mathematical Learning.* As the model indicates, I used double arrows to connect the stage of ‘engagement in problem solving’ and ‘overcoming challenges in mathematical learning’ to reflect the notion of continuity and sustained practice. That is, challenges in mathematical learning could be overcome, but the process was described by Dena as one that takes time. Similarly, there isn’t any data to support the notion that students absolutely overcame challenges in mathematical learning. Because of this, I also used double arrows for the two possible outcomes. Some students worked “enough to get by” and pass the course. Other students developed **confidence and/or interest** to pursue additional math requirements. None of Dena’s students that followed path 2, had success in additional algebra courses, past Math 95 (equivalent to high school Algebra 1). I included the outcome of **interest in additional math requirements/algebra path** in the model, because Dena provided the
example of two students in her earlier class that started with low self-efficacy and actually enrolled in Math 137 after expressing to Dena that they felt more comfortable with learning math having taken the Dena’s Quantitative Analysis course.

_No Successful Learning Engagement._ As stated above, one same student, could evidence in some cases successful learning engagement. In other situations, the same student might not evidence successful learning engagement. Students did not evidence learning engagement for multiple reasons. These reasons were many. Some students experienced anxiety (Elsa took herself out of the classroom one night when she felt anxious), sometimes they were tired from a long day of work (e.g. Sarah’s case) or sometimes they chose to use someone else’s work (Thomas turned in his test corrections one night in the tutor’s handwriting). But sometimes, students did not have a choice in this engagement. For example, Elsa and Omar could not engage in one class because they did not understand the context used to learn. The outcome in these cases was one of _no apparent change_. Since this outcome cannot be construed as absolute (i.e. no data to support that), I have also used double arrows in the model for this outcome.

I now revisit some of Dena’s students’ situations to demonstrate their learning experiences.

_Linda._ Linda was described by Dena as a “Pre-Calculus” student. Dena did not use this descriptor because of course placement or past coursework. Dena noted that Linda had mathematical maturity and that she did not belong in that classroom because of the way Dena had designed it. Linda represents a student in path 1. Linda did not need support with her ability to think critically. According to Dena, Linda was in her class because she had the choice in courses. Although she had the preparation for Pre-Calculus and upper level math courses, her program did not require that course so she instead chose quantitative analysis. The classroom
snapshot in Appendix M reflects how Linda positioned herself to help her classmates when concepts were not clear. She demonstrated **consistent learning engagement**. Her explanations to her classmates and her ability to support them, also demonstrated **successful learning**.

**Sonia.** According to Dena, Sonia was responsible and made all attempts to meet with Dena to ask questions and to discuss problems that were not clear to her. I was present the night that she was making up a missed test. During the test, she asked Dena whether she should use the current month or the past month to calculate finance charges. Dena could not answer her question, but prompted her to think about the overall process to determine what would make sense. Two weeks later, I saw Sonia stay at the end of class to revisit this question with Dena, which reflected to me her interest to understand the problem even after it had been graded. During lessons, Sonia’s questions reflected foundational gaps, especially with the relationship between fraction form and decimal form. Sonia was able to have **some ownership over the learning process** as indicated by her perseverance and responsibility. Sonia represents a student in path 2. She was not able to have full ownership, because of foundational gaps that continued to challenge her understanding (Dena, post-observation interview). Dena explained that even though she had completed successfully an adaptive program for remediation, that the program could not fully remediate what one needs to learn over multiple years of math learning. In this model, Sonia represents a student that is able to **engage successfully in problem solving** through Dena’s supports. While she was not able to fully overcome all challenges (e.g. gaps), she showed **confidence** in asking questions and in supporting her own learning. Sonia, however, was also taking Math 137. She was not successful in that course because according to Dena, Sonia did not have mathematical maturity. She worked hard, but she was not capable of working with open ended situations. Because of this, Dena also said that she thought that Sonia would be
successful if she enrolled in Math 137 again using the adaptive model of instruction. This
reflects the partial development of skills associated with mathematical maturity.

Omar. Omar represents in this model, a student that was not able to fully engage in
learning successfully on multiple occasions. He missed the first half of class due to work
responsibilities, but according to Dena, he did not make up the missed time with her despite her
requests to do so. He also did not turn in his weekly graded assignments on time. Dena said that
he turned many of them at the end of the semester, but also stated that he did not know that he
would be getting deductions for lateness. There was some partial buy-in in Omar’s case. He
attempted to complete the work, but not in a way that would help him learn and prepare for tests.
The outcome in these case, was that of ‘no apparent change’. Dena’s learning interventions on
mathematical thinking did not evidence an impact on Omar.
Chapter 5: CROSS-CASE ANALYSIS

‘Your Thinking Can Empower You’

In this chapter I report my findings from the cross-case analysis of the four study participants. As I described in the Methods section (Chapter 3), I based my cross-case analysis on four primary comparison profiles: (1) the forms of knowledge of the student used across cases, (2) how teachers used that knowledge, (3) the factors that challenged or supported the teachers’ knowledge use in practice, and (4) the teachers’ learning goals. I end the chapter by describing an analytical model based on these analyses.

Knowledge of the Student

Teachers used a wide variety of knowledge of their students in their deliberations, ranging from mathematical aspects like their individual thinking and attitudes particular to their mathematical learning, to nonmathematical aspects like competing priorities outside the classroom and family support. Some of these forms were highly situated, like teachers’ understanding of students’ mathematical learning development. In total, the teachers used 23 different categories of knowledge of their students. I present these different categories in Appendix N, along with brief descriptions of re-consolidations of categories to assist comparisons for teachers’ use across cases.

Of this wide range of knowledge categories, ten knowledge categories were evident in all cases (see table 5.1). All teachers frequently described and used all of these forms of knowledge except for one – *mathematical strengths*. Eddy did not use this knowledge about his students as much as the other teachers, and when he did, he used it to support students’ individual learning.
Table 5.1. Common Forms of Knowledge of Student Used Across Cases

<table>
<thead>
<tr>
<th>Forms of Knowledge of the Student by Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical</td>
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<tr>
<td>Mathematical AND</td>
</tr>
<tr>
<td>Nonmathematical</td>
</tr>
<tr>
<td>Mathematical Thinking</td>
</tr>
<tr>
<td>Learning Attitudes</td>
</tr>
<tr>
<td>Cultural Dissonance</td>
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<tr>
<td>Foundational Gaps</td>
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<tr>
<td>Ownership Over the Learning</td>
</tr>
<tr>
<td>Competing Priorities and/or SES Challenges</td>
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<tr>
<td>Language and Structure of Math</td>
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<tr>
<td>Mathematical Strengths</td>
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</table>

Despite the teachers’ situated use of these common forms of knowledge, there were also common patterns across cases that provide us with a more cogent understanding of teachers’ knowledge of the student as a relevant resource in their practice. I describe these patterns below.

**Mathematical thinking.** Teachers highly prized students’ mathematical thinking. They engaged students in learning by exchanging their mathematical learning. Most of the classroom activities that teachers facilitated centered on students’ exchange of their mathematical thinking. In all cases, except for Eddy, students’ communication of their thinking was seen as a source or reaffirmation of learning. Teachers deliberately sought students’ partial understanding. They did not construe this partial understanding as a deficit, but rather, as part of the normal course of learning.

Teachers’ use of students’ mathematical thinking often also involved their understanding of students’ *foundational gaps* and their *mathematical language and structure*. As teachers attempted to address students’ partial understanding, they also addressed students’ foundational gaps that sometimes prevented their students from understanding new topics. Dena described them, for example, as “something that blocks their thinking”. The teachers facilitated discussions of mathematical language and structure as a way to support students’ communication of their mathematical thinking.
**Foundational gaps.** The teachers’ experienced challenges in helping their students make connections in learning because of foundational gaps. Teachers ascribed students’ difficulty with making connections to their low grasp on pre-requisite knowledge. The most common foundational gaps were associated with: understanding the meaning of operations, proportional reasoning, operations with fractions and decimals, and operations with signed numbers. All teachers looked for different ways to address these gaps, but with low success.

I noticed a pattern when teachers attended to students’ foundational gaps. Students’ with challenges in their motivation, interest or perseverance to learn also demonstrated foundational gaps. The teachers acknowledged this pattern. They considered students with high motivation, but with foundational gaps, to be able to succeed in learning. There were however, differences in the teachers’ perspectives of what that learning was and its extent.

**Language and structure of math.** Teachers used students’ interpretations of mathematical structure and/or students’ use of mathematical language to make connections to their meaning and to the operations represented by them. Despite differences in the units of study, when students experienced challenges in interpreting structure, they also exhibited foundational gaps. Students’ most common challenges interpreting the language and structure of math were associated with rational forms and/or effect of the negative sign (i.e., “the opposite”).

Teachers’ use of students’ language and structure of math varied depending on whether it helped or whether it deterred their learning. Teachers had to overcome students’ challenges with the language and structure of math when they were associated to foundational gaps. The teachers described the following common gaps associated with students’ challenges with the language and structure of math: students’ developed number sense and/or understanding of the number system, students’ interpretation of operations as represented by the structure, especially
with rational forms. Teachers, on the other hand, used the language and structure of math to build learning and support connections to meaning within the structure.

**Learning attitudes.** This form of knowledge includes students’ perseverance, self-efficacy, confidence and/or anxiety. The teachers placed a negative value on this form of knowledge when they challenged students’ learning. While all teachers facilitated classroom experiences that expected the development or the application of perseverance by their students, Beth was the only teacher that gave explicit instruction on how to persevere (e.g., she gave prompts on what to do if a problem was blank or if they found their efforts were not leading to a solution). The teachers pointed to the effect of other learning attitudes in students’ perseverance. Dena pointed in particular to issues with anxiety and low self-efficacy.

Dena’s conception of perseverance (a key component of her knowledge of students’ “mathematical muscle”) had two essential components – comfort with meeting failure and the ability to continue considering alternatives upon meeting failure. Dena’s conception places in perspective Beth’s use of this knowledge in relation to the other teachers. Beth incorporated practice on gaining awareness of alternatives that would sustain students’ efforts to persevere.

**Competing priorities and/or SES challenges.** All teachers, except for Eddy, communicated concern for their students’ competing life responsibilities as a deterrent to their ability to prioritize school responsibilities inside and outside the classroom.

**Cultural dissonance.** Teachers interpreted and responded to different forms of students’ cultural dissonance. They described students’ behaviors and practices that in their perspective, were not in alignment with the culture of school. The teachers frequently made reference to differences between their own personal experiences as students and their students’ school
preparation practices. The teachers sometimes construed this type of cultural dissonance as an indicator of students’ low ownership over their learning.

Dena and Eddy noticed cultural dissonance in students’ ability to understand the context of certain problems as a result of differences in students’ life experiences. They attempted in different ways to help students understand the contexts. Beth expressed challenges in being able to relate to her students as a result of differences between her life experiences and her students’. She stated, “I don’t have my backpack in my mom’s house and my shoes in my dad’s car. And this parent is fighting with this parent, so I slept in my brother’s car. I didn’t grow up that way” (post-observation interview). She shared that she understood to some extent, because her father had grown up with similar challenges, but that it was not something she had experienced herself.

Relational. Teachers capitalized on ways to relate to their students, but in different ways. Beth made connections to caring for her students and to have them care about their learning as something that was important in their lives that would support greater learning. Shannon looked for ways to build relationships with her students so that there would be trust and respect in their working relationships. Eddy used humor and discussed the importance of working on something they might not be good at. Dena learned from her students that they responded to individual invitations to meet her for help. She used this to increase her contact time with her students outside the classroom. She expected her students at that stage in their education, however, to want to learn.

Personal characteristics and/or conditions. Teachers used their students’ personal characteristics in different ways, depending on the types of additional resources available in their particular institutions. These resources ranged from technology access to access to information about their students to support services for socioemotional needs. All teachers reported having
learned from practice a form of situational awareness where they “read their students” and/or “read situations” to determine when to push their students to work and when to step back, and give their students space.

**Mathematical strengths.** Teachers construed mathematical strengths as an asset. All teachers, except for Eddy, used this strength for students’ collective learning gains. In Beth’s, Dena’s and Shannon’s classrooms, students seemed to position themselves to help others. These teachers also organized specific types of activities to make use of different student abilities. Eddy made some use of mathematical strengths through individualized discussions as he looked for ways to help students complete their lessons. In these cases, mathematical strengths seemed to serve also the role of *alternative mathematical approaches* for the purpose of completing a task.

**Ownership over their learning.** Each teacher used a particular set of indicators that in their perspective demonstrated students’ ownership over their learning. Common indicators included: responsibility in working outside the classroom to do homework and/or study and coming in for help outside the classroom times. In all cases, ownership over the learning process was construed as a form of self-advocacy. Beth and Dena shared a common understanding of ‘ownership of their learning’. They stressed the need for students to not just own the learning process, but to actually own the knowledge and/or what was learned.

**Other categories.** There are three additional forms of knowledge that were not used by all teachers, but that help us obtain a more complete picture of the forms of knowledge used across cases. First, in cases that involved ELL students, all teachers used knowledge of their students’ first language (*nonmathematical language*). Beth and Dena worked with these students individually to support their thinking. Shannon (perhaps due to her program) viewed
nonmathematical language as a form of capital that supported students’ collective discussions and group mathematical thinking, as long as students shared the same first language. Eddy did not use this form of knowledge because he did not identify any student as needing language support.

Second, all teachers, except for Dena, used students’ personal interests. The data does support a general trend on teachers’ use of students’ personal interests to find ways to relate to their students. -Dena, however, found other ways to relate to her students through her individual invitations for work outside the classroom.

Lastly, both Beth and Eddy used knowledge of family support. I would not have expected strong use of family support in Dena’s practice because her instruction took place in a college environment and communication with family for educational purposes would have required students’ consent. Shannon did not make use of family support. She mentioned on various occasions that this type of support was low. Ironically perhaps, it is worth noting that in Shannon’s classroom I received all parental signed consent forms for this study complete (without requiring follow up for incomplete signatures) and within days. In all other classrooms, the process took weeks to complete.

These common forms of knowledge help us understand what teachers used about their students. Teachers’ actual use of these common forms of knowledge, however, depended on teachers’ situational need which varied across contexts. The cross-case analysis surfaced patterns that were common across teachers’ use. I report these findings in the next section.

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12 Due to the protection of educational records through the Family Educational Rights and Privacy Act (FERPA)
Comparisons on Use of Knowledge of the Student

I used pattern codes (Miles & Huberman, 1994) to identify central behavioral patterns in each teacher’s practice. I named these behavioral patterns interventions because they helped explain general trends in which the teachers used what they knew about their students to advance their learning. This use was, however, highly situated. For example, Beth, Eddy and Dena all expanded the boundaries of the learning space (as an intervention) in different ways. To explore this in more depth, I compared the teachers’ intended purpose for their behavioral patterns in their use of a form of knowledge. For example, in Dena’s case, the patterns codes for expanding the boundaries of the learning space (i.e. EXP) were matched to different forms of knowledge to help explain the teachers’ purpose. The same intervention, depending on its association with a form of knowledge, exhibited different purposes. For example, Dena expanded the boundaries of the learning space with two different avowed purposes – “to get to students’ mathematical thinking” and also “to overcome foundational gaps in the classroom” through her individual work with students. Thus, the teachers reported multiple intended purposes for their behaviors based on what they claimed to know about their students. I extended this type of analysis across cases. This approach pointed to two central patterns that I discuss in more detail below.

“Good or bad, we are still using it”: Enablers vs. deterrents. Teachers’ ultimate goal was their students’ learning. Teachers were resourceful in that they used a wide range of knowledge of their students to support this ultimate goal. In some cases they ascribed a negative value or a positive value (or both) to what they knew about their students depending on whether they construed a particular aspect about their student to “enable” or “deter” their learning. For example, the teachers consistently positively valued their students’ mathematical thinking, because it enabled students’ learning. Students’ partial understanding as part of their
mathematical thinking was a form of untapped potential that led to learning. Students’ foundational gaps, on the other hand, were seen as an obstacle that they had to overcome to experience meaningful learning (Beth and Dena), or overcome through a “coping mechanism” (Eddy). Whether teachers ascribed a positive value (i.e., enabler) or a negative value (i.e., deterrent) to a form of knowledge, they still used it as an asset that led to learning. Also, depending on the learning situation, teachers valued students’ language and the structure of math positively or negatively. Teachers ascribed positive value when students’ language and structure of math enabled their mathematical meaning-making. When it deterred students’ meaning-making (typically as a result of foundational gaps), the teachers ascribed negative value to students’ language and structure of math (see table 5.2).

Table 5.2. Teachers’ Value of Common Forms of Knowledge of the Student

<table>
<thead>
<tr>
<th>Forms of Knowledge Listed Based on Teachers’ Value</th>
<th>Positive Value</th>
<th>Negative Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Thinking</td>
<td>Learning Attitudes</td>
<td></td>
</tr>
<tr>
<td>Relational</td>
<td>Foundational Gaps</td>
<td></td>
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<tr>
<td>Mathematical Strengths</td>
<td>Cultural Dissonance</td>
<td></td>
</tr>
<tr>
<td>Language and Structure of Math (BOTH)</td>
<td>Competing Priorities and/or SES Challenges</td>
<td></td>
</tr>
<tr>
<td>Ownership Over Their Learning</td>
<td>Personal Conditions or Characteristics</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Language and Structure of Math (BOTH)</td>
<td></td>
</tr>
</tbody>
</table>

Teachers’ understanding of students’ ownership over their learning was directly associated with their long-term goals for their students, as it was seen in each teacher’s analytical model. I interpreted teachers’ positive value of this form of knowledge through their ongoing purposeful attempts to monitor its development in their students.

Teachers’ did not just ascribe positive value to their students’ mathematical thinking. They prized it in ways that reflected a second pattern in their use of this form of knowledge.
“Show me your thinking”: Centrality of mathematical thinking. Teachers’ ongoing attempts to use students’ mathematical thinking and to facilitate experiences where students also exchanged their mathematical thinking underscored the centrality of this particular form of knowledge in the teaching and learning process. Dena and Eddy actively sought students’ mathematical thinking for the purpose of building strategies that would in turn support additional mathematical thinking (i.e., Dena’s intervention to provide structure in students’ mathematical thinking and Eddy’s intervention to guide students’ mathematical thinking). Students’ mathematical thinking informed five of Dena’s six interventions. Beth facilitated ‘meaningful conversations’ (an intervention) that also ultimately facilitated students’ sharing of their mathematical thinking through these conversations. When Beth expanded the boundaries of the learning space, she made use of students’ mathematical thinking. For example, through Steve’s case and his use of the phone app to ask Beth questions, it was evident that Steve was communicating his mathematical thinking to Beth outside the classroom. Beth also flipped the classroom to prioritize practice, but all practice relied on students’ sharing with Beth and with classmates, their mathematical thinking. When Beth worked out problems with Selena individually, they exchanged their mathematical thinking with each other.

Similarly, Shannon’s facilitation of conversations (an intervention), was also a way to facilitate the exchange and development of students’ mathematical thinking. Shannon’s intervention of ‘uses a flexible pace’ was highly informed by mathematical thinking. The flexible pace gave her students the opportunity to develop comfort as they communicated their mathematical thinking. As shown in Chapter 4, through a flexible pace students also communicated alternative mathematical approaches that Shannon appreciated as an indication
that her students had learned. These alternative thinking approaches were a form of mathematical thinking.

Thus, teachers prized their students’ mathematical thinking in way that it was both, a means and an end. They used students’ mathematical thinking as a resource from their students, but they also used it as a tool to engage students and induce more learning.

Teachers’ ability to use students’ mathematical thinking and any and all forms of knowledge, also depended on additional factors that supported or challenged the teachers’ practice. I present these in the next section.

Factors Supporting or Challenging Teachers’ Use of Knowledge of their Students

In this section I report findings on factors that either helped or challenged the teachers’ efforts to use what they knew about their students to advance their learning. The teachers reported supports and challenges that were sometimes common across cases, and other times very situated.

Supports. Beth and Shannon worked in the same school. They both identified their administration as supportive, but in different ways. Shannon felt supported by the administration because it facilitated a collaborative working environment. Beth felt supported by the administration because it reinforced the value of learning that she reinforced in her classroom. For example, as we were close to ending the post-observation interview, we were interrupted by the principal’s message over the school’s intercom. After reminding students that they needed to start collecting their belongings from their lockers, he ended his message with, “have a weekend of excellence as you study for your exams” (post-observation interview). Beth pointed to the classroom speaker and nodded in agreement. Beth also valued resources made available to her to support her own students (e.g., a dedicated vice-principal for each grade level and dedicated
behavioral counselors), reporting that she would not be able to maintain the classroom behavioral norms that she did without the help of a strong school security force: “As good as my classroom management is,….it is because it is supported by someone stronger than me” (post-observation interview).

Beth, Shannon and Dena also saw their colleagues as a source of support. Shannon’s sources of support revolved around her ELL students’ needs. Her responsibilities within the SLI team did not allow her to join the Algebra 1 team meetings. Shannon looked for support for her students that had left the SLI program by requesting they be placed with her “go to” teachers – teachers who had approached her for information about ELL needs and who had expressed an interest in learning how to support ELL students.

While Beth’s work with her Algebra 2 team supported her practice, she reported the Internet as a crucial resource, as it gave her access to the current work of other practicing teachers. Teaching blogs, for example, helped her see what different teachers had attempted, what had worked and not worked.

Dena identified work with her colleagues across other community college institutions as one of her biggest forms of support. When asked for an example, Dena said that her colleagues at another community college had come to work with her one-on-one at her college to help her pilot an online adaptive program for students that needed remediation in developmental math. These were the same colleagues who had also worked with Beth’s high school to institute the same online adaptive program for their Topics course.

The evidence on supports that Eddy drew upon was limited to the administration’s support of his implementation of the self-paced program. Direct support for his students in the areas that Beth and Shannon described seemed absent. Eddy confirmed this absence by noting
that his school had recently received funding to have behavioral counselors to support his students. He said these were positive changes that he looked forward to and that he thought could support his practice.

Finally, along with seeking support, all four teachers reported providing support to their colleagues and school. Shannon offered to help math teachers when they needed information on how to best support ELL students in the mainstream classroom. Beth was preparing to offer a professional development session to her colleagues on the flipped classroom. Dena positioned herself as a form of support, but her positioning was also associated to challenges in her practice. Dena took on the role of trainer for the programs she was instituting because of low support from publishing companies. Eddy created the school’s class schedule when they needed changes to accommodate special events like school assessments. He also took on additional duties in the cafeteria (or in other school areas) when there were concerns for student misbehavior. As the football coach, he felt was able to prevent behavioral incidents from escalating. These were some forms of support that were offered to Beth and Shannon.

**Challenges.** The most relevant finding associated with the teachers’ challenges was the interplay between two central themes, of *time* and *control*.

**“My 46 minutes”: Time and control.** The teachers highly prized their time to help their students learn because any other times were outside their control. Shannon, for example, kept an accurate count of how many lessons she could spend with her students under a flexible pace (time), so that her students learned as much as they could with her. If her pace lagged too far from that of the mainstream classroom, she knew she could catch up her students that remained in the program (within her control). Any student who exited the program would be outside her control.
The interplay between time and control helped place in perspective the teachers’ interventions. Expanding the boundaries of the learning space could be seen as a way to increase time space under a teacher’s control. Although Shannon did not use this strategy, she did work with her students before and after school, another ways of creating more learning space. Recall that Dena spent more time helping her students individually outside the classroom than inside class time. Beth’s explanation best captures, the interplay of time and control that all the teachers faced.

Beth: So, I can’t relate. My parents knew when I had a test. They knew when my exams were and if I didn’t clear my schedule, they did it for me, the weekend before finals. But that’s just something I have to acknowledge as my frame of reference. So if you practically laugh at my face because I say, you have to clear your schedule to study and say, ‘ha – ha, no because it’s 90’. I can’t get mad at that. We all have free will. I am recommending something, but my frame of reference is not theirs when it comes to something not being important.

Ruth: So, why does it challenge you? Is it internal, within yourself?

Beth: Because my want can’t make it so.

Ruth: Is it because it is outside your control?

Beth: Right. As of the minute you walk outside my door, whatever you choose to do for the final on Tuesday, is not in my control.

Ruth: But is interesting, because I would argue that it also informs what you do in the classroom.

Beth: Well, it comes full circle. Is why I build so much math into my forty six minutes as possible because I cannot control what you do outside my room, but I do control what you do within these forty six minutes. They are my forty six minutes and I will do the best I can with them…[Beth laughed]. [emphasis added].

Beth’s final comments on why she built so much math into her 46 minutes, additionally tie this need for control to her other interventions in the classroom, not just “expanding the boundaries of the learning space”. For example, Beth had also explained that she maximized practice and rigor in the classroom so that students would be better prepared to do their practice at home. These were purposeful attempts by Beth to extend her 46 minutes. These attempts were associated to Beth’s interventions on “maximizes engagement time” and “holds high expectations”.

251
The interplay between time and control also helped place into perspective the teachers’ perception on their students’ forms of knowledge. Anything that challenged the teachers’ reach to maximize learning time was deemed an obstacle. This helps further explain why some forms of knowledge were valued positively and others negatively, depending on the situation. My dialogue with Beth was part of her explanation on why she felt she could not relate to her students’ experiences as a result of cultural dissonance. Beth highly prized her 46 minutes because in her perspective the cultural dissonance posed a threat to her students’ preparation outside the classroom when she had less control. Consequently, Beth valued her students’ cultural dissonance negatively. This pattern was the same in teachers’ perspectives of their students’ foundational gaps. The teachers remediating foundational gaps took time, but they felt they could not spare the time. Compounding this challenge was the fact that when they attempted to make connections to foundational knowledge for new learning, they could see that their students were not making needed connections. This placed the teachers in a position that was outside their control. When teachers helped students overcome any deterrents to their learning, there was a shift in control in the learning process (from teacher to student). I revisit this observation when I describe the analytical model for all teachers at the end of this chapter.

I now describe additional challenges in the teachers’ use of knowledge of their students. These challenges were associated in different ways. The next challenge I present, for example, relates again time, but now, to teacher’s ability to support their long-term learning goals.

“It is hard to think about tomorrow if I have to first make it through today”: Time and learning goals. The teachers described time challenges with their ability to incorporate open-ended problems into their teaching. All teachers expressed valuing such problems but not being able to use them in their instruction because they considered them to take too much time, which
they preferred to spend on additional practice. According to Dena, these types of problems were key in helping students develop more long-term goals (in her case—“mathematical muscle”). Problems of this kind helped students become comfortable with failure while also being able to consider alternatives in problem solving. My interpretation, based on my discussions with the teachers, is that taking the time to work on these problems, which would have addressed a long-term learning goal, challenged the teachers’ ability to address more pressing short-term learning goals like maintaining the pace of their lessons to cover all projected content sections for the year, and increasing practice time to improve students’ performance in tests and/or assessments.

“You have to perform this trick now”: Assessment and accountability. All teachers described different challenges associated with the need for students’ to perform well in assessments. This need was associated with teachers’ understanding of their students’ learning progress (form of knowledge). I present some examples.

According to Eddy, none of his students did well on the final exam. Eddy was concerned about his students’ performance in relation to other teachers. His colleagues reviewed for the exam with students differently than he did. Whereas Eddy used sample exam questions daily throughout the last quarter, his colleagues used the last two days before the exams to do the same exam questions, but with different numbers. Eddy said that the following year, he would review as his colleagues did. I asked him if he thought that in doing so, that the exams would be representative of learning. Eddy said:

That is a great question. Is it representative of learning? My gut instinct is no. It gets into the short-term learning and the long-term learning. You know it now, versus mastery of it. Um…if you truly mastered something, you should be able to do it when you come to it at a later date. Maybe not at the same level of complexity, but you should be able to do it…The way the system is set up right now, you do not have to. (post-observation interview)
I then asked Eddy what he meant. He said, “You don’t have to master it. You just have to know it now. You have to perform this trick now” (post-observation interview). Eddy’s challenges with assessment reflected the effect of accountability measures in the system. Accountability measures that are based on a limited scope of what mathematical learning is about, have the potential to limit the scope of what mathematical learning is about in the classroom. While Beth did not have the same low grades, she did express a similar concern for how her test data compared to her colleagues’. Her colleagues had used the same test questions to review with their students, but with different numbers. Beth said that she would consider using some of those questions while she taught the unit in the future, but not for review. She said she did not think that her tests would represent her students learning if she used the same questions a few days before the test.

Dena’s challenges seemed to be more associated to course grades as a measure of performance. Dena felt pressure from administrators and state policy makers that measure teachers based on their grades. Compounding this challenge was also the fact that there was little consideration for the skillset that a student started a course with (Dena, post-observation interview). “Administrators don’t want to see those failure rates, and they blame us, but yet, what came into my class 15 weeks ago, it wasn’t reasonable” (Dena, post-observation interview). Dena’s assertion points back to issues of time and control because she felt she did not have enough time to both teach and remediate. It is a position that was outside her control, yet, she was held accountable for it.

Recall also the issue of students’ mathematical maturity. Dena asserted that teachers need to structure learning experiences to develop students’ ability to consider alternatives and used critical thinking skills, rather than just follow steps. Tests only measure defined outcomes,
she argued, like being able to solve an equation. She noted that two different students could solve the same equation while having very different learning experiences. One student could have solved the equation by following routine procedures while the other could have considered alternatives and really understood what he or she was trying to do. The latter case, according to Dena, was a student that developed mathematical maturity. Through this example, Dena pointed to what she called, “the hidden curriculum”. In her perspective, assessments are limited and should not dictate what and how math should be learned. She also recognized at the same time that everyone was under different pressures, especially with regards to performance.

In order to understand the central learning phenomena, we need to also understand all components that were found to affect teachers’ use of knowledge of their students. These components, are interconnected and, together, help explain the larger phenomena. Supports and challenges had an effect on teachers’ practice, and this in turn, had an effect on teachers’ use of knowledge of their students. Pressures for students’ to perform, for example, caused teachers to re-consider their own learning goals for their students. But teachers’ interventions, which is how they used their knowledge of their students, were implemented to advance their students’ learning and these were in alignment with teachers’ own learning goals. In order to fully capture the central learning phenomena, we must also understand the overall teachers’ learning goals.

**Comparisons of Teachers’ Learning Goals**

Teachers’ learning goals supported two objectives for their students. First, the teachers wanted to empower students to grow as learners and overcome challenging situations. Second, some goals worked to develop more long-term types of outcomes that were associated with students’ development of self-advocacy skills (e.g. ownership over the learning process). I describe these in more detail below.
Empowerment goals. Most of the teachers’ goals empowered students by preparing them in different ways for learning. These goals positioned students to continue growing in their learning. They also intended to provide students with the skills or tools to overcome factors of conditions that deterred or inhibited students’ learning. For example, Beth stated that it was important to challenge students and to teach with rigor in the classroom, so that students could do their homework at home. Beth knew that her students struggled with low confidence and with foundational gaps that inhibited their ability to do their homework. Beth’s teaching goal to maintain rigor empowered her students with the preparation in the classroom that would also develop self-confidence and/or help overcome foundational gaps. See table 5.3 for teacher’s learning goals indicating empowerment purpose or not.

Long-term goals. There were teaching goals that purposely developed different skills for ongoing and long-term learning. These types of goals were different from others that addressed more short-term needs. For example, Dena’s goal to develop students’ problem-solving skills helped her students in her class to overcome preconceived notions about what it means to learn math. This goal addressed a more short-term need to position students to learn within Dena’s class. Dena’s teaching goal on developing students’ critical thinking skills so that they could own the learning process, however, addressed a more long-term student need that empowered students for ongoing and future math learning. Similarly, Shannon’s teaching goal to have students advocate for their learning empowered students with particular skills that represented a form of self-support for their learning. For Shannon’s students in particular, these skills were a necessity as they moved out from the self-contained and highly structured support program of the SLI team to the mainstream classrooms.
<table>
<thead>
<tr>
<th>Teacher</th>
<th>Empowerment Type</th>
<th>Teaching Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beth</td>
<td>All Empowerment</td>
<td>To maintain rigor in students’ math learning experiences</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To help students learn to “make good choices” that will sustain their engagement in learning (i.e. prioritize their learning inside and outside the classroom)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To foster “genuine moments of learning” for her students</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To develop college and career readiness which included skills such as: responsibility, strong math foundations and ownership over their learning</td>
</tr>
<tr>
<td>Shannon</td>
<td>All Empowerment</td>
<td>To have students know the Algebra 1 content and develop the necessary mathematical skills to be prepared for their subsequent Algebra 2 course (within or outside the SLI team)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To have students demonstrate advocacy for their learning as indicated by their ability to: ask questions, come prepared to class, and seek help outside the classroom when needed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To develop students’ organizational skills to support their ongoing learning at the high school and in college or in their future careers</td>
</tr>
<tr>
<td>Eddy</td>
<td>NO Empowerment</td>
<td>To help his students pass his course (Algebra 1) – “They do need 3 math credits [for graduation]. This should be one of them.” (Eddy, pre-observation interview)&lt;sup&gt;13&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To help students be comfortable learning and comfortable with working at something that they might not be good at, but that they can become good at later from working on it&lt;sup&gt;14&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>Empowerment</td>
<td>To guide students’ mathematical thinking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To help his students with life problem solving, that may not necessarily be math related</td>
</tr>
<tr>
<td>Dena</td>
<td>All Empowerment</td>
<td>To develop students’ problem solving skills and in doing so, to help students overcome preconceived notions about what it means to learn math and problem solve in math</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To create learning experiences that develop students’ critical thinking skills that in turn, develop ownership over their knowledge gain and over the learning process</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To develop students that are ready for college and/or their field of work</td>
</tr>
</tbody>
</table>

Based on my comparisons, my interpretation is that the teachers deemed these long-term teaching goals essential for all students, just as much as Shannon did for students exiting the SLI

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<sup>13</sup> The scope of this goal is associated to the short-term objective of obtaining credit. The expectation is limited to a minimum requirement.

<sup>14</sup> The scope of this goal is limited to comfort with a condition. Preparation on how to actually “become good at math” is missing.
program. These goals empowered students with self-support in their ownership over their learning process. Students that exhibited ownership over the learning process did not demand the teachers’ concerns for maximizing their contact time or to look for ways to get to their mathematical thinking. These students shared their thinking and sought others’ thinking as a natural course of their learning progress. For example, Linda (in Dena’s case), or Elia and Yadira (in Shannon’s case) shared their thinking freely. Yadira, in particular, advocated for her learning after an absence or after overcoming a challenging time. In doing so, Yadira demonstrated a form of control over her learning.

I end this section by noting that the empowerment role from the teaching goals and the teachers’ implementation of interventions to advance their students’ learning did not necessarily result in successful learning. That is, the teachers’ goals and their efforts needed to target the particular areas where their students needed empowerment or support. For example, Yadira, one of Shannon’s students, demonstrated ownership over the learning process, but she also needed support and empowerment in other areas such as the flexible lesson pace. This intervention supported her in an area where she needed help. Selena, one of Beth’s students, struggled with anger management, but she exhibited an interest in learning. Beth’s work through individual attention was supplemented with work from the vice-principal’s office. The combination of both forms of support addressed Selena’s particular need.

I now present the analytical model that I constructed and used to test all cases and their students as part of the cross-case analysis. The model describes the teacher’s use of forms of knowledge of their students incorporating patterns noted from the process or comparing and contrasting the four profile areas I described above.
Cross Case Analysis Model

This cross-case model is presented in Figure 5.1. The model captures the central patterns across all cases. As such, the model helps explain the phenomena in each classroom. The model itself represents a “one-pass linear view” of the learning phenomena starting from knowledge of the student. I recognize that there are many other phenomena taking place in the classroom. The central phenomena captured here is strongly focused on teachers’ use of their knowledge of their students, which was the area of interest encompassed by the research questions. Consistent also with a situated practice framework, this model should be construed as embedded within a larger context. The green frame and green dotted lines in the model are meant to stress particular areas where the data reflected direct influence and/or association to larger contextual factors outside the classroom. I first present a brief description of the model. I then describe each component of the model in more detail.

Brief description of model’s components

Working from left to right, the model begins with common forms of knowledge of the student. The blue arrows go to teachers’ goals which mediate their interventions. The teachers’ interventions mediate and moderate students’ engagement and/or their mathematical thinking by empowering and/or supporting students’ learning. If the interventions’ target on factors that enable or deter student learning is low, student engagement is also low. When there is sufficient and targeted empowerment and/or support, the students’ engagement in mathematical thinking leads to their development of skills that empower them with ownership over the learning process. The development of these skills requires a sustained process (hence, the large colored arrows). In some cases, the development of skills was achieved, but not completely. The outcomes are understood to be met in the short-term and are in progress of more permanent development. In
other cases, the development of skills was more permanent. The double arrows reflect the notion of continuous development.

**Forms of knowledge of the student.** Starting at the far left side of this model, I list nine of the ten common forms of knowledge of the student. As described in my analysis on forms of knowledge, there was a tenth form of knowledge (i.e., *ownership over the learning process*) that was more strongly associated with long-term goals for all students. It reflected the teachers’ use in that teachers looked for progress in its development. *Ownership over the learning process*, however, was found to hold a central role as teachers’ ultimate goal for their students to develop. This form of knowledge is thus, incorporated in the model at the later stages of the process. I
revisit it again, as a separate category within the model. The forms of knowledge of the student reflected association with their context.

**Teacher’s goals.** My analysis of the teacher’s goals reflected the teachers’ need to empower their students in different ways. While there were some goals that were more passive in nature, the teacher’s goals seemed to correspond to aspects that the teachers knew about the students. This does not mean that the teachers’ goals were directly or strictly informed by what they knew about their students. The teachers’ goals corresponded to forms of knowledge of their students in that they took on a mediating role between what the teachers got to know about their students and how they used what they knew about their students. In some cases the goals addressed short-term needs to engage students for classroom learning. In other cases the goals addressed long-term needs for their students’ learning.

**Teachers’ interventions.** The teachers’ interventions represented how teachers ultimately used what they knew about their students. Their interventions were influenced by the teachers’ learning goals for their students. Teachers’ interventions can be best described as the experiences they created and/or facilitated to support their students’ learning. I found two relevant patterns associated with the teachers’ use of what they knew about their students. The first pattern was associated with how teachers perceived students’ forms of knowledge. Some forms of knowledge were more valued positively. Other forms of knowledge that seemed to challenge learning were construed negatively. The teachers’ interventions were thus, purposeful attempts by teachers to advance their students’ learning. The teachers’ use of some forms of knowledge, like relational knowledge, further enabled learning. Other forms of knowledge

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15 I used the terms mediator and moderator in this model to give more specificity to the relationship between the variables. Teaching goals here helped explain the interventions based on the forms of knowledge (i.e., mediator). The “strength” of the target of these interventions helped explain the teachers’ ability to engage students’ mathematical thinking (i.e., moderator).
needed to be “overcome”, thus construed as deterring learning. The second pattern was associated with teachers’ need to engage students and their mathematical thinking.

All teachers sought ways to engage their students in their learning process. Once they were able to engage their students, teachers and students exchanged their mathematical thinking as a central practice in the learning process. The exchange of mathematical thinking was a central practice that teachers wanted their students to engage in. The teachers facilitated this exchange. In some cases, like in Shannon’s and Beth’s cases, the teachers made strong use of collective practices to expand their facilitation of this exchange. In the case of Eddy, the exchange depended more on Eddy, explaining the challenges described with large spans of time where students waited to work or get help from Eddy. In Dena’s case there was a collective exchange in the classroom and an individual exchange outside the classroom. In Dena’s case, however, some students’ needs necessitated much more time outside the classroom.

The teaching interventions were highly dependent on context. The factors challenging and supporting the teachers’ practice impacted their ability to implement their interventions. For example, Beth’s and Shannon’s work with some students relied on collaborative efforts with specialists that could support students in their growth in other areas that were affecting their ability to engage in learning. These were supports that Eddy did not have at the time of the study.

My analysis of the teachers’ challenges pointed to an additional and phenomenon that impacted the teachers’ interventions. The teachers wanted to help their students, but they were limited by the factors they could control. They could only control what took place within the purview of their practice. While they attempted to expand the boundaries of their practice and the learning space, they also attempted to shift control of the learning process to their students.
through their development of self-sustaining skills that would empower them with self-support (e.g., ownership over the learning process).

**Empowerment or support targeting factors impeding or enabling access.** Even if the teaching goals were in perfect alignment with the teachers’ interventions, the overall effort to empower students needed to target the particular areas that the students needed empowerment with. For example, in Dena’s case, students that had already developed mathematical muscle (Dena’s ultimate long-term learning goal for her students) needed support with aspects like competing priorities and personal challenges. While Dena’s goal to change her students’ preconceptions of what it means to learn math was not applicable to students with mathematical muscle (in Dena’s conception of mathematical muscle), Dena’s interventions that targeted their competing priorities were still needed. Dena’s knowledge of her students in this particular facet allowed Dena to provide interventions that targeted her students’ particular need. Because of this, the teachers’ use of their knowledge of their students depended on how much the teacher knew about their students and also on how much the interventions targeted the students’ particular need. Teachers’ use of knowledge of their students both mediated the learning experience, but also moderated it. The model depicts this moderating role with two possible outcomes. The ultimate and central goal was to engage students in learning by exchanging their mathematical thinking. In cases that the teachers’ empowerment did not target (or the target was very low), the students were not able to engage in learning.

**Low target of factors deterring/enabling access.** If the factors that enabled or inhibited access were not targeted (or were targeted very low) students’ ability to engage in exchanging their mathematical thinking was affected. These are the students that Dena and Shannon described to me as the “students that fell through the cracks”. This was a situation that they
expressed wanting to avoid as much as possible. Sometimes the target was low, but enough to exhibit some level of student engagement. In these cases, the students eventually did not achieve their intended learning outcomes after some level of engagement.

**Engagement and/or mathematical thinking.** Mathematical thinking was a means and an end in learning. Teachers built on it to advance students’ learning. Teachers also facilitated its exchange among everyone, but the exchange was still dependent on student engagement. The teachers implemented interventions attempting to target factors that impeded or enabled students’ access to learning so that students could engage and exchange with them and their classmates their mathematical thinking. Engagement in this central practice mediated students’ development of more long-term outcomes that supported students with their ongoing learning.

**Ownership over the learning.** Each teacher provided particular descriptors and indicators for what they considered their students to ultimately develop. For example, in Beth’s case it was college and career readiness. In Dena’s case it was mathematical muscle. In Shannon’s case it was a combination of organizational skills and self-advocacy skills. In Eddy’s case it was just perseverance. The development of these skills, took place over time and required sustained engagement in the learning process and under the facilitation of learning experiences and conditions that also empowered students to sustain this engagement. I used circular arrows in the model to depict the notion that the process took place over time under a sustained cycle of engagement. In some cases the intended outcomes were found to be in the process of being developed, thus, in progress. In other cases, the intended outcomes were met, ultimately empowering students with a set of long-term, self-sustaining skills that shifted control over the learning process from the teachers to their students.
**Overall cross case description.** In the within case analysis I “walked” (figuratively speaking) students through their corresponding case models. I now walk the teachers’ models through this cross-case model.

**Beth.** Beth was probably the most complicated and detailed case of all. This cross-case model, however, helped me explain why in her case so much of the data seemed relevant and interrelated. Beth’s interventions were informed by a large number of forms of knowledge of the student. She also made use of outside resources that were not just supplied by the school, but also freely available through technology. Beth’s interventions addressed multiple knowledge aspects of her students. Her collective approaches in the classroom also intensified the effect of her interventions. The moderating role of the teachers’ interventions helped me explain why she exhibited so many examples of students meeting her intended learning outcomes. Beth also empowered students by providing them with skills that helped them overcome challenges. Instead of asking students to “just work”, she prompted them to consider why they were not working and gave explicit instruction on what they should be doing to support their learning. There were a few students in Beth’s classroom that exhibited low engagement. Beth described observing inconsistency in their efforts, but also not knowing the nature of their inconsistency. This model helps explain situations where teachers were not able to reach all students to understand the factors that impeded or that could have enabled their access to learning. Since Beth did not know the nature of the low engagement for some of these students, she also did not know if her interventions were targeting their particular needs.

**Shannon.** Shannon’s students in the SLI program needed support in areas that they might not necessarily overcome permanently themselves. For example, some of her students would continue to endure competing priorities and challenges. While Shannon’s students shared
an amazing trait as emergent bilinguals, the need to learn math in the language they were in the process of acquiring posed additional cognitive demand. Shannon was aware of their needs in these aspects of their learning. Her interventions that worked to develop skills to empower students despite ongoing challenges exhibited student engagement and their meeting of their intended outcomes. Three students in Shannon’s class, however, experienced challenges with foundational gaps and in learning attitudes. They needed additional types of learning experiences that could target these areas. Shannon’s descriptions seemed to indicate low buy-in, in the learning process. Yet, her interventions for these students (e.g. math lab grade) relied on their buy-in. In this regard, Shannon’s work was not targeting her students’ particular needs.

**Eddy.** Eddy’s goals reflected a strong alignment with his administrations’ goals. The school was working to increase the number of students that met all their credit requirements as freshmen so as to increase their likelihood of graduation. His interventions and goals relied on students’ possession of the same skills that he wanted his students to develop. That is, in order for students to persevere through the self-paced program, they needed to have buy-in and they also needed a lot of support with foundational gaps. This made it very difficult for Eddy to shift control to his students because he became the central form of support. While Eddy provided much support to his students, the focus was on sheer work. Thus, the target on factors impeding and/or enabling access was not high. There were positive outcomes in that students showed some level of confidence and learning.

**Dena.** Dena’s case, situated in higher education, provided a long-term perspective on the learning phenomena. Students that had developed long-term self-sustaining skills through earlier math learning experiences only needed support in areas such as competing priorities and personal challenges. But much of the students that had not developed these long-term self-
sustaining skills were also challenged by anxiety and foundational gaps. Dena’s target in this area required much more individualized attention and time. If students were able to meet Dena outside the classroom, they were able to sustain engagement in mathematical thinking with her and work to overcome challenges. If students did not engage Dena (like Omar), then Dena’s interventions operated without being informed on her students’ needs. Dena’s practice at the college level, towards the end of students’ math educational path, placed into perspective the effect of learning through years in an algebra path carrying foundational gaps. Foundational gaps and learning attitudes were highly confounded, placing into question whether at that stage in their education, Dena’s interventions were able to fully target her students’ needs.

**Summary**

This chapter presented findings from the cross-case analysis. Ten out of the 23 categories of knowledge of the student were used in common by all four teachers. The teachers used their knowledge of the students in different ways (i.e. interventions) to help their students either overcome challenges in their learning access or to enable students’ learning access. Teachers highly prized students’ mathematical thinking and used it to engage students’ in learning and to also build-in new learning. The more the teachers were able to target issues challenging or enabling students’ learning access through their interventions, the more the students were able to meet their intended learning outcomes. When teachers construed aspects about their students to deter their students’ learning and/or their ability to share their mathematical thinking, the teachers negatively valued these aspects about their students. When, on the other hand, teachers construed aspects about their students to enable their learning, teachers values these aspects positively. Teachers’ overall use of their knowledge of their students depended on their learning goals for their students. Their learning goals for their
students, for the most part, attempted to empower students to overcome challenges and/or to
grow as learners. Teachers’ long-term goals for their students, defined here as ‘ownership over
their learning’, were achieved by supporting students’ sustained engagement in: their
interventions and their facilitation of students’ exchange of their mathematical thinking. While
each teacher had his or her own conception of what this ownership over their learning entailed, it
represented an ultimate form of empowerment to support students’ present and future school
math learning. When students exhibited having ownership over their learning, teachers’ need to
control the learning time space was transferred from teachers to students.
CHAPTER 6

Discussion and Implications

This study originated from a basic question – What must teachers know and draw upon to prepare for equitable math teaching? A review of research work on practicing teachers shed light on core characteristics of the equitable math teacher. These characteristics were understood to be situated practices because they were grounded in their context of instruction. As such, they provide insight as guiding principles in equitable math teaching, but they also require at least some form of adaptation for their use in other contexts of instruction. The review also shed light on a gap in scholarship. That is, research in equitable math teaching had been guided by either of two distinct perspectives on learning (cognitivist vs. socioculturalist) – focusing on different aspects of the same learner. While research efforts aligned with sociocultural perspectives have brought attention to students’ lived experiences and issues of access, research efforts aligned with cognitivist perspectives have brought attention to students’ individual mathematical learning. Thus, our understanding of equitable math teaching was in need of addressing all aspects of the same learner – holistically.

I argued that teachers needed to gain knowledge on how to implement and/or adapt equitable teaching practices from “specific, well-documented” events from practice – thus building up our inventory of case knowledge (Shulman, 1986). This involved by turning to the practice field to seek a more in-depth, contextual, practice-based understanding of what teachers have found that works in advancing their students’ learning. I also argued that by focusing on what teachers know about their students, our case knowledge could incorporate a holistic understanding of students. I, used the term “forms of knowledge of the student” to describe what a teacher knows about a student, recognizing that this term represented a broad category of
different dimensions of the student that incorporated both mathematical and non-mathematical aspects of the same student. I incorporated this term in the problem statement in Chapter 1. As stated, our problem was that we do not have an integrative, cogent understanding of how forms of knowledge of the student are sought and capitalized on through instruction.

In summary, I positioned this study with a “proactive stance” (R. Gutiérrez, 2002) to advance the field by seeking understanding of teachers’ situated practice, while also positioning students as an essential knowledge resource for teachers’ efforts to advance students’ mathematical learning. I chose to focus on mathematical learning in algebra primarily because it helped maintain a consistent content of instruction throughout the study. The research questions that guided this study were:

- What forms of knowledge of the student (e.g. mathematically foundational, identity, community, etc.) do practicing algebra teachers use to leverage the learning of algebra for students from underserved populations?, (RQ.1)
- How are these multiple and diverse forms of knowledge of the student applied in practice to support the teachers’ algebra learning goals for their students? (RQ.2)
- In what ways do these teachers perceive their use of these forms of knowledge as helpful in supporting their students’ learning of algebra? (RQ.3)
- What models can be developed to understand these teachers’ practice as they attempt to advance their students’ learning within their situated context of instruction? (RQ.4)

The study findings helped answer the research questions. Particular sets of forms of knowledge of the student for each of the four participating teachers were generated (RQ.1). The teachers’ uses of these forms of knowledge of the student (i.e., “interventions”) were described (RQ.2), along with the teachers’ perceptions of these interventions (RQ.3). Analytical models of
the central learning phenomena for each teacher and across cases were created incorporating and triangulating the following: the forms of knowledge of the student used, the teachers’ interventions and their perceptions of this use, their teaching goals, their perspectives on learning and the full case data set (RQ.4).

In the sections that follow I discuss findings and implications from the cross-case analysis that I reported in Chapter 5. I then highlight aspects that stood out from each of the cases I reported in Chapter 4. I end this chapter by pointing out additional implications from the study and offering recommendations and final remarks.

**Discussion of Cross-Case Findings**

I discuss findings from the cross-case analysis recognizing that they are not transferrable to all classrooms or to all teachers’ practice. They can be used to understand other classrooms or teachers’ practice under similar circumstances and/or with particular characteristics as the ones that were used for the cross case analysis. I start by discussing forms of knowledge of the student that teachers used across cases. I then highlight implications of their use in light of the problem statement and the research questions. I end this section by pointing out aspects that stood out from the cross-case model.

**Forms of knowledge of the student.** By using ethnographic methods in this study, I opened myself to the possibility of letting the context and phenomena I encountered dictate the course of the data collection and its associated analysis. While I was not surprised to find that my start list of forms of knowledge of the student increased as the research process unfolded, I was surprised to find that out of the ten common forms of knowledge of the student used by all teachers, only three of them were not on the original start list. This finding reflects that research and practice have been noticing similar aspects from our diverse students.
The three common knowledge categories that were not on the original start list, are associated to other equity issues addressed by research. These three common categories were: mathematical thinking, personal characteristics/conditions, and ownership over their learning. Mathematical thinking was associated to another existing category in the start list (i.e., alternative thinking), but the teachers highly prized students’ mathematical thinking. They made use of many other aspects they knew about their students through interventions to further access students’ mathematical thinking. Students’ mathematical thinking was construed as core to each student, something that they (students and teachers) shared through their learning engagement. This category reflected much alignment with work on identity (e.g., Aguirre, Mayfield-Ingram & Martin, 2013; R. Gutiérrez, 2013; Varelas, Martin & Kane, 2012) and on positioning students to increase their access to classroom participation and discursive practices (e.g., Boaler & Staples, 2008; Louie, 2018; Turner, Dominguez, Maldonado & Empson, 2013). The relevance of students’ mathematical thinking in teachers’ practice is worth highlighting because it points to a potential area of preparation needed in equitable math instruction. Teachers must know ways to access their students’ mathematical thinking, and based on this study’s findings, how to make use of other aspects they know about their students to further access and/or facilitate the exchange of students’ mathematical thinking.

The second new common, personal characteristics and conditions (e.g. anger management, military service, personality, etc.) has not received as much attention in scholarship of equitable math teaching. This is probably because work has typically focused on issues of gender, race, culture, class, or disabilities (e.g., Lubienski, 2002b), which are areas that have been considered to address broader needs in discussions of equity. But these are not necessarily disjoint categories. Lubienski (2000b) found, for example, that little attention had been given in
math education to disabilities within the context of ethnicity and class (Lubienski, 2000b).
While Lubienski’s (2000b) concern at the time addressed only disabilities, her recommendation
to broaden the scope of attention to socioemotional factors, as well as class and ethnicity, is
applicable here to the form of knowledge of personal characteristics and/or conditions. Based on
these teachers’ practice, these personal characteristics and conditions required specialized
attention and administrative support in order for teachers to be able to implement their
interventions. The data from this study raises potential concerns for differential treatment for
these students as a result of low funding to prioritize support needs. Schools in districts with
higher representation of low SES students such as the ones in this case study, typically operate
with lower funds to be able to afford these supports. In Eddy’s practice these supports seemed
absent. Yet, it is worth highlighting that Beth pointed to this type of administrative support as
one of the biggest factors that supported her practice for all her students. This was not because
all her students needed the specialized support, but because it allowed her to focus more of her
attention on teaching math.

The third new common category, ownership over their learning, exhibited a highly
situated definition in this study. In this study, ownership over their learning represented a form
of empowerment for students because teachers worked to develop it as a form of self-advocacy.
For Beth and Dena in particular, ownership also reflected students’ actual knowledge gain (i.e.,
learning). In the cross-case model, this form of knowledge was represented as an ultimate
learning goal that teachers worked to develop in their students. Students developed ownership
over their learning through their sustained learning engagement and sustained exchange of
mathematical thinking. The more students sustained their learning engagement with the support
of teachers’ interventions, the more they developed ownership over their learning.
I noted that the definition of *ownership over their learning* was highly situated because the term has been used before in scholarship, not as a form of empowerment, but rather, as a form of disempowerment. For example, in Chapter 2, I had referenced Rousseau’s and Tate’s (2003) study where they found that teachers had passive behaviors towards their students and that they expected their students to have ownership over their learning. The issue that Rousseau and Tate (2003) pointed to was two-fold. First, the teachers in their study were passive and expected their students to address their learning needs in narrow-minded particular ways that did not pay attention to their students’ differences. Second, the teachers did not see that their practices were perpetuating inequalities in the classroom. In this study, the teachers were not passive about students’ ownership. Teachers’ interventions, however, led to their expected outcomes only when their interventions directly targeted what they knew about their students. When their interventions did not target what they knew about their students, the learning outcomes were not achieved. These outcomes, when obtained, disrupted issues of inequality on behalf of the students. I revisit this issue of target, in my discussion of the cross-case model.

**Forms of knowledge used: Cognitivist or socioculturalist?** Teachers’ particular use of their knowledge of students provide evidence supporting that principles from both cognitivist and sociocultural perspectives of learning played an important role in teachers’ practice. This has important implications on our understanding of how forms of knowledge of the student are capitalized on through instruction (i.e., the problem statement). Although I started the research process with a list of potential forms of knowledge that were holistic in nature (i.e., mathematical and nonmathematical), the teachers themselves used these forms holistically. The teachers’ holistic use was more evident in cases where the forms of knowledge were confounded, such as foundational gaps and learning attitudes. The teachers took on roles that required emotional and
psychological appreciation when addressing students’ learning attitudes that deterred their learning. The teachers also needed to know how to teach math in order to interpret students’ work and build on students’ partial understanding to create successful learning experiences that further helped them overcome the inhibiting effect of their learning attitudes.

Advancements to scholarship guided by both perspectives of learning together, were needed to support teachers’ interventions. For example, when Beth recognized that her students dealt with conflicting priorities and could not stay for help, she expanded the bounds of learning; this intervention is aligned with sociocultural perspectives related to issues of access. This intervention, however, would not have yielded its intended result, if Beth had not been able to interpret her students’ partial understanding from the work they shared from the pictures they sent through phone texts. Beth also needed to give students targeted answers that addressed their learning needs and that ensured that they continued working on their own. This component of her intervention reflected alignment with mathematical knowledge for teaching (MKT) (Ball et al., 2008) and cognitive perspectives of learning. More examples of this kind, in different classroom settings are needed to continue growing our knowledge base of how to advance our diverse students’ learning holistically.

**Implications from the Cross-Case Model.** One of the biggest implications from the cross-case model is that it provides empirical evidence that ties the teachers’ perspectives on how they use what they know about their students to their learning goals for their students. The findings also represent empirical evidence that suggest the necessity for teachers to target what they know about their students to achieve their learning goals. For example, the success of an intervention could be compromised if it relied on aspects about the students that the teachers
perceived negatively. I provide teachers’ work with students’ cultural dissonance as an example.

Students’ cultural dissonance was valued negatively by teachers because they construed it as deterring their efforts to advance their students’ learning. In some cases where the students exhibited low preparation outside the classroom, the teachers ascribed the dissonance to practices at the “home.” While all the teachers recognized the effect of school practices on their students’ foundational knowledge (e.g., low expectations, passing all students from eighth grade to Algebra 1 in ninth grade, even if they had failed math), only Beth and Dena recognized its effect on students’ developed practices (i.e., cultural dissonance) as a result of school itself. If we look at this from a student’s perspective – how and why will preparation matter in school? I argue that cultural dissonance was reinforced if not created by the culture of school. The issue of target for the teachers is that they implemented interventions with the purpose of changing students’ practices. When considering this change as a change in culture, it was also associated with a need to change students’ held beliefs and understandings that were outside the students’ cultural referent. When the teachers used interventions that relied in the culture of school itself, they were not successful in students’ development of new practices. Eddy attempted to capitalize on family support, but his discussions centered on lessons that needed to be completed. He used an enabling form of knowledge through family support, but missed the target by using the same culture of school as the reason to prepare outside the classroom. Beth, on the other hand, appealed to students’ interest to learn and to improve themselves as intelligent and capable individuals. Beth capitalized on family support (or as Beth told her students – “someone that loves and cares for you at home”) to open up discussions about the value of effort in learning in the classroom and at their homes, together.

276
The issue of preparation outside the classroom in this study is much more complex, but it reflects the importance in the model for teachers to target as many forms of knowledge of the student as possible. For example, the data set did not contain cases of just cultural dissonance. The data did show the co-presence of cultural dissonance with issues of foundational gaps and learning attitudes. The interventions that combined as many of these forms of knowledge as possible, exhibited the most successful outcomes. Beth appealed to students’ interest to learn while also orchestrating “meaningful” moments of learning. Capitalizing on students’ interest to learn addressed issues associated with cultural dissonance. Providing students with experiences where they could make sense of their thinking represented successful experiences that addressed foundational gaps and learning attitudes. This helps explain why the interventions’ target on forms of knowledge included a moderating effect in the model.

Although not directly shown in the model, teachers’ challenges with control were embedded throughout the central learning phenomena. This is worth noting because it is a reality of practice that I have not seen before in the literature related to equity. I have seen issues of control in the literature associated to power and hegemony (e.g., R. Gutiérrez, 2013; Ladson-Billings & Tate, 1995; Martin, 2013). These issues from the literature have brought to our understanding different ways that students’ equitable access can be challenged. In this study, control was embedded in the process of increasing access to learning. The teachers’ need to control the learning experiences was exacerbated when students’ learning access was threatened by factors deterring their learning. As a result, teachers felt that their control was limited to their time with their students which was guaranteed within their classroom. They attempted to expand their control by expanding the boundaries of the learning space. As students developed ownership, which was a form of self-advocacy for their students, the teachers’ transferred control
to their students. Control played a role in the teachers’ equitable teaching attempts. The effect of control in equitable math teaching merits more attention as we seek to bridge gaps between research and practice.

**Discussion of Case Findings**

Each teacher used what they knew about their students in situated ways. In this section, I discuss particular findings that stood out for me from each case. I remind readers that the teachers volunteered to participate in a descriptive case study, not an evaluative case study. I position my findings with that goal in mind – to describe what I have found in relation to what we know from research.

**Shannon.** Shannon’s case brought to light students’ life challenges from their status as recent immigrants. Shannon’s most pressing challenge was highly situated in relation to the other teacher cases. The number of students in Shannon’s classrooms continued to increase as she received new students throughout the year. Shannon expressed needing to “start all over” whenever she received a new student. Her challenge seems to match that of other teachers in similar programs that support English Language Learners (e.g., Horn, 2018). That is, their work and equitable efforts are not understood or valued, because the complexities in the classroom are also not easily understood. Although Shannon’s work was valued by her administration, she expressed feeling pressure from her evaluation process because it was based on students’ learning outcomes on yearly assessments that measured students’ math performance and their progress in English language acquisition.

A second aspect that stood out for me in Shannon’s case was the need for a stronger understanding in scholarship for math instruction in multi-lingual environments (Phakeng & Moschkovich, 2013). Shannon could not speak her students’ first languages, but she facilitated
ongoing group work and discussion on problems in their first language. This particular intervention reflected much alignment with scholarship in underscoring the value of students’ first language in the learning of math as a resource (Moschkovich, 2002). But this intervention could only benefit the students that were able to speak to others in their same first language. In Shannon’s classroom, nine students spoke Spanish and one spoke Russian. The one student that spoke Russian demonstrated high learning engagement through her contributions to class discussions and through her work with Shannon and other classmates. The case data points to the value of classroom norms that are inclusive and that position students as active contributors in the learning process.

**Eddy.** Although all teachers’ goals reflected in some ways their own institutions’ goals, Eddy’s case reflected it the most. Eddy’s stress on lesson completion was associated with his school’s work to increase the number of students passing their ninth grade so as to increase their likelihood of graduating from high school. Whether we agree with this teaching goal or not, it is a reality of the practice of many teachers in schools that are considered to be low performing by means of graduation data and/or students’ scores in their exit exams (in this case, the SAT). Eddy did note that his school was in the earlier stages of a multi-year plan to increase their institutional expectations of their students’ learning. Eddy suspected that the new criteria for higher learning expectations would be based on students’ grades.

Our discussions on student outcomes led me to understand an issue that actually applied to all the high school classrooms, not just Eddy’s. Their state used student SAT data as a measure of their schools’ educational quality. In math in particular, the high schools had adopted the Common Core which stipulated not just math content, but very specific student practices in mathematical learning. The alignment between their standard curriculum and the
SAT test was unclear to me, but it was used as a final measure of students’ math learning experiences in the high schools. While I recognize that I am “moving away” from the classroom in my discussion, Eddy’s case strongly reflected the effect of policy inside the classroom. These policies actually had an effect in all classrooms. All of the high school teachers expressed concerns for their students’ preparation for the SAT and changed many of their curriculum materials and the format of their tests to help students prepare for the SAT.

Eddy’s case data had the highest incidence of students’ struggle with foundational gaps, but also the richest descriptions on students’ individual mathematical thinking. While I do not have evidence to explain reasons for the high incidence of students’ struggle with foundational gaps, I suspect that the self-paced format of instruction itself, which was performance based and relied on individual efforts, made these challenges more evident. At the same time, since students relied on Eddy’s support to complete the lessons, they constantly described their thinking to Eddy.

I think it is worth noting that Eddy’s use of computer aided instruction is reflective of recent instructional trends in the field of practice (Kitchen & Berk, 2016). The findings from this study support some of the concerns that Kitchen and Berk (2016) expressed for diverse classrooms located in low income communities. Kitchen and Berk (2016) have brought to light issues with emphasis on remediation and reduced opportunities for mathematical thinking and discourse in complex tasks. Based on the case study data, much more discussions of mathematical thinking were seen when the students worked off the computer-based lessons. This had to do with differences in the design of the problems depending on the platform. The problems online required more the applications of set procedures, while the problems off-line incorporated connections across content topics and applications in real-life scenarios.
I end my discussion on aspects that stood out for me in Eddy’s case, by noting a very unexpected, but promising finding associated to specific knowledge needed to teach math. The learning interactions that relied on describing each other’s thinking (student to Eddy, and Eddy to student), helped Eddy recognize the importance of being able to explain and teach math. He acknowledged that he was in the process of developing this skillset. He acknowledged that math came easy to him, and that his students needed more “landmarks” (i.e., connections) to be able to understand math. Eddy’s case places in perspective the role of mathematical knowledge for teaching (MKT) (Ball et al., 2008) in advancing students’ learning. The role of MKT could have been taken for granted if all teachers had demonstrated use of MKT. By contrasting cases through the cross-case analysis, the role of MKT became more evident because of its lower stage of development in Eddy’s case. It is also worth noting that in this study, Eddy demonstrated and recognized developing MKT through his individual work with his students – an outcome from practice and particularly, from the process of exchanging mathematical thinking with his students. Although Eddy had a strong math knowledge base as a mechanical engineer, he was the only participant without a more formal teacher education base. He still needed to learn how to best teach math.

**Dena.** What stood out for me the most from Dena’s case –something I revisit again in my final remarks, is that Dena’s challenges were not that different from Beth’s or Eddy’s with regards to students’ struggles with foundational gaps and learning attitudes. But Dena’s case added a dimension of **anxiety** to the category of learning attitudes which was not present in the other cases. While I have no evidence to ascribe causality to the strong incidence for math anxiety in Dena’s students, it is also not too unexpected considering that from a long-term perspective students had endured more years of not being able to overcome foundational gaps
and challenging learning attitudes. Most of Dena’s students had also gone through the college’s developmental math program. Some students had taken these developmental math courses at least twice. I consider their efforts to continue enrolling in math coursework, despite these challenges, admirable. At the same time, I asked myself why. Why are we letting our students struggle in math for so long?

**Beth.** One of the aspects that stood out the most for me in Beth’s practice was her ongoing effort to keep looking for “what works and does not work” to advance her students’ learning. Just as the teachers in Plana’s and Civil’s (2009) study, Beth took into account everything that would allow her students to do a task. I suspect this is also why her case was the most complex and detailed. For each intervention, Beth used many forms of knowledge of her students. These same and multiple forms of knowledge, also informed other interventions; causing her case data to be highly interrelated within itself. I interpreted this phenomenon as an indication that Beth’s practice was strongly responsive to her students.

After revisiting Beth’s classroom norms for collective work and support, I could not help but notice that she re-defined notions of competence (Gresalfi, Martin, Hand & Greeno, 2009). This was a second aspect of Beth’s practice that stood out for me. Beth facilitated a classroom experience where students held each other accountable to Beth and to themselves. Based on the framework for competent participation from Gresalfi et al. (2009), we can assert that Beth positioned her students to not just help each other, but to hold each other accountable as they made sense of their thinking approaches. These types of interactions helped redefine notions of competence and proficiency, even though students struggled with their interpretation of the language and structure of math and foundational gaps, which were construed as deterrents to students’ thinking. This observation is worth highlighting, because Beth taught a class in
Algebra 2 on simplifying expressions with rational exponents, where the “solution” to a problem was one final expression that was deemed “in simplest form.” The students did not work on open-ended questions, but their work required the consideration of multiple alternatives as they worked to help each other overcome foundational gaps. I note here that I have not seen much work that re-frames classroom learning without the use of open-ended problems, in high school algebra. More recent work by Louie (2017) provides examples on students’ positioning based on inclusive practices in the study of Geometry. I recognize that algebra, in its reliance to symbolic representation and procedural approaches, may be more prone to the use of problems that tend to be “narrow” (Louie, 2017) in scope. But given the scale of adoption of the Common Core at the national level, which requires the study of Algebra 1 and Algebra 2, I think it would be helpful to the field of practice if more lessons, activities and/or curricula using less narrow approaches are made more accessible for the study of algebra.

A third aspect worth noting about Beth is that she was the only teacher that expressed having challenges with issues of ethnic culture. I will preface this discussion by noting that Beth described herself as half Puerto Rican. Beth wanted to be able to relate and understand her students more. She also considered music to be a very important aspect of a person’s culture. She felt she could relate to her students when they played music she grew up with (e.g., salsa and bachata), but she could not relate to rap music that used “foul language” and had lyrics that “were demeaning to women.” I considered it important to highlight this aspect from Beth’s case on ethnic culture for two reasons.

First, Beth was a minority teacher, but she still encountered challenges in being able to relate to her students because of their different lived experiences. This brings to light our need to have more diverse pools of teachers (Chern & Halpin, 2016). While it will not be feasibly
possible to represent all cultures in a teacher’s pool, it is important to seek diversity in teacher pools so that teachers can have colleagues to turn to and look for ways to understand their students. Beth would have been prepared to address a similar incident, had her students chosen music that was Latin@ with foul language or demeaning to women. She could have also been in the position of advising her colleagues if they had encountered this issue in their classroom.

Second, Beth’s challenge brings to the forefront a reality of practice that I have not seen captured in the literature yet. Not all aspects of a students’ culture need to be sustained. I believe that when Ladson-Billings (2014) argued for sustaining students’ cultural practices and experiences, she did not mean all of them. But it can be challenging to determine the ‘what’ to sustain or not sustain when the cultural referents are different. This once again, brings us back to the need for more diverse teacher pools. In the case of Beth, prioritizing her students’ growth as individuals, came at a price to her ability to relate to them. I contend that if the teacher had been black, or that if Beth had addressed her concerns with Latin@ students, the students would not have felt that part of their identity was being rejected, at least, not as strongly.

I end my discussion of Beth by noting that what stood out the most for me from her practice, was the high incidence of students observed meaning making. Perhaps this is what made Beth so successful. Across all teachers’ practice, the students’ smiles, their nods, and their explicit feedback saying, “this was a good class” or “best teacher ever”, were all seen and heard when their learning experiences were based on meaning making.

Additional Implications

One of the strengths of this study relies on the triangulation and comparison of data across all cases. But findings from this study are not transferrable to all classrooms. Additional case studies are needed to get a more comprehensive understanding of the forms of knowledge of
the student that are capitalized through instruction and how. For example, the teachers in this study worked to develop students’ ownership over their learning. This outcome may or may not be the same in classrooms with other characteristics. The high school classrooms in this study were tracked. Shannon’s classrooms were not tracked, but she said she aligned her pace to the pace in level 2 classrooms (i.e., the lower track). It would be helpful to understand the ways in which findings are different or similar if the classrooms corresponded to higher track levels, or if they were not tracked at all. I also think it would be worth conducting a case study with teacher participants that match Beth’s profile in her high use of knowledge of the student. All teachers in this study started with a similar participant selection requirement – to reflect a stance that recognized equity issues. This requirement was intentionally broad. After all, how can we create more specific criteria if we are in the process of understanding what is equitable teaching? But this study demonstrated in practice how the target of particular forms of knowledge actually yielded the advancement of learning engagement, and in Beth’s case in particular, learning as measured through classroom assessments. Additional case studies that purposely recruit teachers with a higher target on forms of knowledge would help increase our understanding of other forms of knowledge in use and/or how the same forms of knowledge have been used differently to advance students’ learning.

Despite limitations on the transferability of my findings, there are findings with implications for the broader field of math education (researchers and practitioners) to consider. The different cases that I reported on, demonstrate the situated nature of the practice of teachers. Not all work on equitable math teaching will necessarily apply to one classroom. With that noted, the more teachers seek to know their students, the better they will be able to determine how certain approaches to equitable math teaching are applicable to them, to their practice, and
to their students. Some of the basic principles from the cross-case model generated here can be used as a reflective tool to aid teachers in making this determination. For example, teachers can ask themselves what they know about their students and how they use it in contrast to their teaching goals. The teachers in this study worked to develop students’ ownership over their learning as a form of self-advocacy. Other teachers can consider what they are working to develop in their students, and the ways that their goals are consistent with their students’ equitable access to their learning.

Having spent close to a month in each of these four classrooms, I also note that in order for research work and professional development on equity to be consumable in these classrooms, it must also take into consideration the realities of daily practice. The high school classrooms, in particular, were fast-paced. The teachers multi-tasked in the classroom and offered any allocated “free” or “prep time” in their schedules to help their students. Beth, for example, used the internet to explore ways to improve her teaching. She explained that online resources provided her with the instantaneous access that she needed. More recent work that is seeking to increase connections between research and practice through data repositories are promising (Cai et al., 2018); however, teachers from diverse environments need to be involved with field-testing these resources.

**Recommendations and Final Remarks**

Dena’s concerns for her students’ challenges with foundational gaps and learning attitudes were depicted at the college level. Yet, Eddy’s challenges as early as ninth grade, were not that different from Dena’s. The high schools placed all graduating eighth graders into ninth grade Algebra 1 regardless of their progress in eighth grade. Whether we support tracking or not, tracking will not change the fact that students were placed into a course that relied on the
abstraction of foundational knowledge that the students did not have to begin with. Recent studies suggest that success in algebra learning is directly associated to foundational knowledge such as operations with fractions and decimals (Hurst & Cordes, 2018; Siegler et al., 2012). The criticality of building students’ strong foundations in their elementary math education cannot be overstated. But even if we were to improve all efforts in elementary math education, it would be unrealistic to presume that all students would have developed their foundations by seventh grade.

I learned from this study that the system is set up in a way that leaves little room for the further development of foundational knowledge after seventh grade. Students that started algebra with lower foundations were placed at a major disadvantage. Teachers like Beth, worked very hard to help their students overcome these foundational gaps. But we must keep in mind that despite all her arduous efforts, creating classroom norms that maintained high expectations, that were inclusive, and that focused on growth, were just a start. The case data is replete with examples of students doing all the “heavy lifting.” In the end, the students were the ones overcoming the gaps. From a learning demand standpoint, these students were learning Algebra 2 while they were also learning years of foundational knowledge that they did not acquire before.

**Recommendations.** In order to advance our efforts in equity, we need to incorporate these learning realities – both successes and challenges – that our students face, into our overall discussions of math teaching and learning. This includes re-framing our notions of mastery, to include students’ partial understanding, as part of the regular process of learning. In this study, partial understanding was found to be an opportunity to learn. Similarly, the teachers’ realities need to also be included. The teachers attempted to teach with connections, but they encountered much difficulty because the students could not make connections to a foundational knowledge that was practically absent. We certainly want to teach with connections. My
concern is that when our statement of what is ‘good teaching’ is reliant on making connections (NCTM, 2014), it is hard to see these teachers’ reality of practice included in our definition of good teaching. Our descriptions of what is good teaching need to acknowledge teaching a subject with the understanding that a student may have not developed full mastery. More research on how teachers can capitalize on partial understanding is also merited.

My final recommendation is given out of concern for the many years that students have been taking on high school math coursework without an opportunity to develop strong foundations. I have looked without success for curricula that re-engineers the teaching of algebra, so that it builds-in the development of foundational knowledge, instead of strictly relying on foundational knowledge to build-in new learning. I think we need to consider new ways to teach algebra. We can’t have students go so many years through high school without developing foundational knowledge. There are computer based remediation programs, such as the ones that Dena described using at their college. Many of these programs focus on routine procedures without connections, and they are typically individually paced. These types of programs do not allow for discussions and for the exchange of mathematical thinking which was found to play a central role in students’ learning in this study. I also recognize that the teaching of other math areas, aside from algebra, are being proposed (e.g., statistical analysis) so that students can have more participation in math. My concern is that we should not consider math alternatives if they dissuade our attention from ensuring that our students have a strong math foundation so that they can actually have choice in their math coursework.

**Final Remarks.** I end this chapter with final remarks about this study and its contributions to the fields of research and practice in equitable math teaching. One contribution from this study is that it provided realistic depictions of teachers’ successful and unsuccessful
attempts to advance their students’ learning. The case descriptions were highly contextualized, yet, they all worked together to explain one central phenomenon based on the teachers’ use of what they knew about their students. Across cases, students were found to be a source of capital to their teachers’ practice, with most promising effects depending on how well the teachers understood their students in order to target their particular needs. Moreover, teachers demonstrated the need to support students’ mathematical thinking and the need to foster experiences that depended on sense-making. This finding from practice was in much alignment with scholarship (e.g. Battey & Franke, 2015; Boaler & Staples, 2008; Gresalfi et al. 2009). But the same findings, obtained through the study of algebra, also demonstrated particular challenges inherent to the learning of algebra (e.g., high dependence on foundational knowledge) that placed students at a disadvantage in a system that limited their options to further grow and develop foundational needs.

Perhaps one of the most unexpected findings was the teachers’ long-term learning goals, re-framing conceptions of ownership over the learning, from a form of disempowerment to a form of empowerment for their students. The teachers looked for ways to prepare their students to succeed within the culture of school. This is the most significant finding. If we want to make an impact in the equitable learning of math in these classrooms, we cannot ignore that we are doing it through the structure of school. I do not advocate abiding by a structure. I do advocate that we understand a structure, so we can be strategic and best position our efforts going forward.
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http://dx.doi.org/10.1207/S15327671ESPR0601-2_7


doi:10.1080/10749039.2012.666317


doi: 10.1207/ S15327833MTL04023_2


APPENDIX A. Pre-Observation Interview Questions with Rationale

<table>
<thead>
<tr>
<th>Questions</th>
<th>Objective Related to the Research Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1. Tell me a little bit about your career as a teacher.</td>
<td>This question was written to get information on the teacher’s earlier experiences, years of teaching, types of school, contexts and extend of experience with highly diverse classrooms. Aside from alignment with the research questions, this question was also written to develop rapport with the participant. Informed from work by: R. Gutiérrez (2002); Tate, Jones, Thorne-Wallington &amp; Hogrebe (2012)</td>
</tr>
<tr>
<td>Possible prompts – How many years have you taught at this school and what courses have you taught at this school? Where have you taught before?</td>
<td></td>
</tr>
<tr>
<td>Q2. How would you describe yourself as a teacher?</td>
<td>This question was written to obtain the teachers’ self-description of their teaching, including particular teaching characteristics that they recognize about themselves. Informed from work by: Stigler &amp; Hiebert (1997; 2000)</td>
</tr>
<tr>
<td>Q.3. In your opinion, what is an effective math teacher?</td>
<td>This question is written to obtain the teacher’s perspective on what is a good teacher. Informed from work by: Ball et al. (2008); Darling-Hammond (2000); Lipman (1995); Civil (2014); Boaler (2002)</td>
</tr>
<tr>
<td>Q.4. What have you learned FROM your students that has impacted your teaching? and, How has it impacted your teaching?</td>
<td>This question was central in guiding my initial areas of focus during observations. This question provided information on the teacher’s perception of the forms of knowledge that he or she uses of the student and also, how this form of knowledge has made an impact in his or her teaching (i.e., what the teacher has learned from his or her students). Informed from work by: Boaler (2002); Civil (2014); Moschkovich (2002)</td>
</tr>
<tr>
<td>Possible prompts – In what ways has their teaching been impacted? Look for what types of form of knowledge: mathematical, outside the classroom, individual or collective knowledge, etc.</td>
<td></td>
</tr>
<tr>
<td>Q.5. How has teaching in a diverse classroom impacted your understanding of how students learn?</td>
<td>This question was specifically targeting the impact of classroom diversity on the teacher’s perception of how students learn. Informed from work by: Civil (2014); Vygotsky (1978); Murata (2013)</td>
</tr>
</tbody>
</table>
| Q.6. In what ways would you say that your teaching experience in this context (fill in based on participant) has helped you be more effective in helping students learn math? | This question targeted collective forms of knowledge of the student that are contextually tied to the teacher’s practice.  
Informed from work by: Cohen, Raudenbush & Ball (2003); Lampert (2001) |
|---|---|
| Q.7.a. Setting tests aside, how do you know when a student is having difficulty meeting your learning expectations? | This question helped give focus on what the teacher looked for, and/or used to inform his or her teaching to help students learn.  
Informed from work by: Bonner (2014); Bonner & Thomasenia (2012); Lubienski (2000a); Houssart (2002) |
| Q.7.b. Can you provide me some examples? | This question targeted instances when students were NOT meeting expectations.  
Informed from work by: Bonner (2014); Bonner & Thomasenia (2012); Lubienski (2000a); Houssart (2002) |
| Q.8.a. Setting tests aside, how do you know when a student is learning and is meeting your learning expectations? | This question helped give focus on what the teacher looked for, and/or used to inform his or her teaching to help students learn.  
Informed from work by: Boaler & Staples (2008); Bonner (2014); Bonner & Thomasenia (2012); Moschkovich (2002) |
| Q.8.b. Can you provide me some examples? | This question targeted instances when students WERE meeting expectations.  
Informed from work by: Boaler & Staples (2008); Bonner (2014); Bonner & Thomasenia (2012); Moschkovich (2002) |
| Q.9.a. How do you get to know your students? Prompts: look for mathematical and non-mathematical forms | This question helped give focus on HOW the teacher attempted to gain information from his or her students.  
Q.9.b was broader in nature. It sought an understanding of ways that knowledge of the student helped the teacher in his or her practice, but purposely not tied explicitly to student learning. The objective was to give the opportunity within the data collection to find ways that the teacher used knowledge of the student that may have had an association with student learning.  
Informed from work by: Battey (2013); Bonner & Thomasenia (20012); Tate (1995); Ladson-Billings (2014) |
| Q.9.b. In what ways does this help you teach your students? |  |
## APPENDIX B. Data Collection Instruments

### B.1. Common Data Collection Instruments

<table>
<thead>
<tr>
<th>Data Collection Instrument</th>
<th>BETH</th>
<th>SHANNON</th>
<th>EDDY</th>
<th>DENA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument</td>
<td>Algebra 2 Classroom at Sundryville High School</td>
<td>Algebra 1 Classroom in SLI Program in Sundryville High School</td>
<td>Algebra 1 Classroom in Mixville High School</td>
<td>Quantitative Analysis Course in Beacon Community College</td>
</tr>
</tbody>
</table>

#### Recruitment Interview Dates and Duration (total)

<table>
<thead>
<tr>
<th></th>
<th>Date:</th>
<th>Length:</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETH</td>
<td>3.30.17</td>
<td>39 min. 31 sec.</td>
</tr>
<tr>
<td>SHANNON</td>
<td>3.30.17</td>
<td>43 min. 24 sec.</td>
</tr>
<tr>
<td>EDDY</td>
<td>3.31.17</td>
<td>43 min. 24 sec.</td>
</tr>
<tr>
<td>DENA</td>
<td>3.16.17</td>
<td>37 min. 33 sec.</td>
</tr>
</tbody>
</table>

#### Pre-Observation Interview Date and Duration

<table>
<thead>
<tr>
<th></th>
<th>Date:</th>
<th>Length:</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETH</td>
<td>5.4.17</td>
<td>38 min. 58 sec.</td>
</tr>
<tr>
<td>SHANNON</td>
<td>4.3.17</td>
<td>1 hr. 1 min. 1 sec.</td>
</tr>
<tr>
<td>EDDY</td>
<td>5.3.17</td>
<td>1 hr. 5 min. 5 sec.</td>
</tr>
<tr>
<td>DENA</td>
<td>4.18.17</td>
<td>33 min. 28 sec.</td>
</tr>
</tbody>
</table>

#### Observational Period Length

|                       | 5.4.17 to 6.6.17 | 4.3.17 to 5.5.17 | 5.3.17 to 6.6.17 | 4.18.17 to 5.2.17 |

#### Number of Lessons Attended and Typical Duration

|                       | 46 min. lessons 14 lessons 2 lessons no recording | 46 min. lessons 14 lessons 3 lessons no recording | 48 min. lessons (2 classes) 21 lessons 5 lessons no recording | 3 hour lessons 2 lessons 1 lesson no recording |

#### Number of Lessons Strictly Audio Recorded

|                       | 2 lessons | 3 lessons | 3 lessons | none |

#### Check-Ins

|                       | 10 Check-ins Total estimate Time: 171 min | 8 Check-ins Total estimate Time: 210 min | 7 Check-ins Total estimate Time: 215 min | 4 Check-Ins Total estimate Time: 94 min |

#### Post-Observation Dates and Duration

### B.2 Artifacts by Participant

<table>
<thead>
<tr>
<th>Artifacts Used By Participant</th>
<th>BETH</th>
<th>SHANNON</th>
<th>EDDY</th>
<th>DENA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>Copy of Unit Handbook with worksheets and handouts for each lesson</strong></td>
<td>1. Worksheets and handouts for each lesson</td>
<td>1. Copies of all hard copies of lesson activities</td>
<td>1. Handouts used for each lesson</td>
<td></td>
</tr>
<tr>
<td>2. Copies of each unit quiz (3 quizzes)</td>
<td>2. List of course topics for the year</td>
<td>2. List of course topic for the year</td>
<td>2. Copies of handout for group work sessions from each lesson</td>
<td></td>
</tr>
<tr>
<td>3. Sample student work from quizzes</td>
<td>3. Copies of each unit quiz (3 quizzes)</td>
<td>3. Copy of Unit test</td>
<td>3. Access to, and Copy of her customized online supplement platform that included homework sets and learning supports</td>
<td></td>
</tr>
<tr>
<td>5. Sample student work from tests</td>
<td>5. Copy of units test</td>
<td>5. Links to all online video lessons made by Eddy</td>
<td>5. Copies of graded unit specific test questions by student that were part of the course final exam.</td>
<td></td>
</tr>
<tr>
<td>6. Print copies of conversation trails with students from REMIND phone app</td>
<td>6. Samples of student work from tests</td>
<td>6. Copies of sample signature lesson progress sheets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Copy of parent-communication handout &amp; sample parent responses</td>
<td>7. Samples of student work from final exams</td>
<td>7. Copies of graded student unit tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Website links to flipped videos</td>
<td>8. Copy of Pre-Algebra middle school exit test</td>
<td>8. Copies of sample graded activities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Pictures of student board work</td>
<td>10. Interactive Notebook sheets for all unit procedures</td>
<td></td>
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<tr>
<td>Participant</td>
<td>Sample Questions and Follow up Prompts</td>
<td>Purpose</td>
<td></td>
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</tr>
</tbody>
</table>
| BETH        | 1. So let me take you back to the characteristics that you said, that were really not content. I don’t want to put a label on them though, but you did say work ethic.  
Follow up: Yeah? What are those indicators? | First Objective: To capture Beth’s self-reported student characteristics that she wanted to develop in her students to support their long-term learning. She called them “indicators”.  
Second Objective: Cross-check – I had noticed a pattern in that the other teachers also had their own set of student skills that they wanted their students to develop. |
| SHANNON     | 1. You have said before that you are not there to be their friend. How would you define a good relationship?  
2. You had mentioned before that you have “go to” teachers for your students off-team. What characteristics do you look for in these teachers?  
Follow up: Why are these important? | First Objective: I was trying to understand Shannon’s perspective on how and why she thought it was important to relate to her students.  
Second Objective: I was trying to confirm my earlier observations on what she used about her students to relate to them.  
Objective: To understand the ways that Shannon thinks her students need support.  
Second Objective: To determine what aspects about her students this support is addressing. |
| EDDY        | 1. I am going to play devil’s advocate and you can push back. Some may argue that with a discovery lesson you can develop the concept stronger. Why is that the one that you chose to take out?  
2. What are the overall advantages of this program over a traditional approach based on your experiences in these two periods? | Objective: To confirm patterns in Eddy’s behaviors. To test my understanding of Eddy’s beliefs on what is important for his students to learn.  
Objective: To confirm Eddy’s perspective of the effectiveness of the self-paced program on the students in the lowest learning track. |
| DENA | Objective:  
|      | Member Check – We had discussed the role of test corrections in the interview. I had asked her to give me examples to help me understand how they were effective. I had also made a note from the observational period of an incident with a student where it did not seem to me that the intervention had worked.  
|      | The follow up was given to get a more descriptive understanding of the incident and about her instructional objective with test corrections.  
|      |  
| 1. I want to check with you, in my opinion this would be a student where the corrections were not effective. Am I correct?  
|      | Follow up: Tell me more about that.  
|      |  
| 2. If you had access to a national forum, and you could say to everyone, I have been teaching for these many years and this is what I have observed…what would you want your students graduating from high school with so that they could be prepared for any math class in college so that they could have choice?  
|      | Multiple Objectives:  
|      | Cross Check – I was trying to compare what Dena considered important for students to learn in high school with what the high school teacher participants considered to be important.  
|      | Second Objective – I was testing my understanding of Dena’s goals for her students [problem solving and critical thinking], and the notion that these goals needed to be developed over years of schooling.  
|      | Third Objective – I was trying to understand Dena’s perspective on why Linda [a high performer] had choice in coursework. |
APPENDIX D. Description of Initial Theoretical Codes

<table>
<thead>
<tr>
<th>Categories</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student Mathematical Knowledge</strong></td>
<td></td>
</tr>
<tr>
<td>MK: Typical Errors (e.g., MKT) Code: MK- ERR</td>
<td>This form of knowledge is associated with commonly encountered errors that student make. Teachers that possess this type of knowledge are able to anticipate these errors and modify their teaching approaches with a proactive stance. For example, teachers might forewarn students about the errors they have observed in the past and/or they might place additional emphasis on areas that they expect more student difficulty. Informed by work from: Ball et al. (2007)</td>
</tr>
<tr>
<td>MK: Foundational Gaps Code: MK – GAP</td>
<td>The teaching of math relies on previously taught concepts. For example, proportional reasoning relies on the understanding of fractions. When teachers recognize foundational gaps, they look for different ways to address these foundational gaps. Otherwise, new learning will be inhibited and/or reduced because students may lack the foundational concepts that are needed for an in-depth understanding. Informed by work from: Aguirre et al. (2013)</td>
</tr>
<tr>
<td>MK: Strength Areas Code: MK- STR</td>
<td>Teachers can accommodate the way they teach by capitalizing on students’ strength areas. For example, they may purposely point out to students that they have noticed these strength areas to help develop students’ self-awareness of their capabilities. They might also position students with certain strength areas in the classroom to help peers. Informed by work from: Aguirre et al. (2013)</td>
</tr>
<tr>
<td>MK: Alternative Approaches/Thinking Code: MK – Alt</td>
<td>Students might approach a problem very differently than others. For example, they might reason through a problem that requires division by making connections to multiplication, while other students might make connections to repeated subtraction. Teachers can purposely seek this form of knowledge of the student to ascertain whether the alternative approach is still reflecting understanding of the topic being taught. Informed by work from: Ball et al. (2007)</td>
</tr>
<tr>
<td>MK: Interests Code: MK – INT</td>
<td>Teachers can capitalize on students’ areas of interest in math to support new learning. For example, some students enjoy looking for patterns and engaging in inductive reasoning activities. Teachers can create activities and/or use alternative ways to develop a concept that capitalize on these areas of interest. Informed by work from: Aguirre et al. (2013)</td>
</tr>
<tr>
<td>MK: Language of Math Code: MK- LANG</td>
<td>Teachers need to closely pay attention to the ways that students communicate mathematically. Language in particular is used by students to communicate their mathematical thinking. Teacher’s use students’ mathematical language to better understand their reasoning</td>
</tr>
</tbody>
</table>

312
approaches. Teachers also purposely work to develop students’ mathematical language. This is a key component of students’ identity as mathematical learners.

Informed by work from: Aguirre & Zavala (2014); CCSSM (2010); Moschokovich (2002); Schleppegrell (2007)

| MK: Mathematical forms and structure | Teachers purposely work to help students understand mathematical forms and structures. For example, in algebra, students need to distinguish and identify terms when they are determining what type of polynomial they are working with. Teachers help develop this form of knowledge for their students. Teachers need to gauge when and how a student is correctly or incorrectly paying attention to mathematical forms and structure. |
| Code: MK – FRM |
| Informed by work from: CCSSM (2010) |

| MK: Unknown | This category is left open to help denote any other unpredicted forms of mathematical knowledge that are encountered. |
| Code: MK - UKN |
| Informed by work from: CCSSM (2010) |

| Student Non-Mathematical Knowledge |
| NON: Cultural/Ethnic | This type of knowledge is associated with cultural aspects outside of mathematics such as traditions, ideologies, social norms and social practices, etc. |
| Code: NON – CUL |

| NON: Language only | Although it can be argued that language is a cultural aspect, a distinct category is given to this form of knowledge. In order to advance learning, especially when teaching is conducted in a language that is different from the student’s first language, teachers use what they know about their students’ language to look for multiple ways to communicate and make conceptual connections. Teachers also need to know their students’ level of English language acquisition so that they know how to best support their learning. |
| Code: NON – LANG |
| Informed by work from: Aguirre & Zavala (2014); Celedón-Pattichis & Ramirez (2012); Cummings (1980); Moschkovich (2002) |

| NON: Learning Attitude | Teachers need to know how their students position themselves or become positioned towards learning. This category is associated with aspect such as interest, motivation and buy-in. |
| Code: NON- LAT |
| Informed by work from: R. Gutiérrez (1999); Hand (2010) |

<p>| NON: Accommodations needed | Many students need other forms of support to advance their learning. Although confidential aspects (e.g., physical, neurological and/or learning disabilities) will not be within the purview of this study, there are many other ways that students need accommodations. For example, students might have extended absences to visit family outside the country. Other students have work responsibilities, etc. |
| Code: NON- ACC |
| Informed by work from: R. Gutiérrez (1999) |</p>
<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NON-INT</td>
<td>This is a broad category associated with interests such as: sports, arts, extracurricular activities, clubs, hobbies, etc.</td>
</tr>
<tr>
<td>NON-COMM</td>
<td>This category is strongly associated with the community that the school serves. Sometimes this form of knowledge can be also associated with culture (see NON-CUL), depending on the cultural make-up of the community. This form of knowledge, however, merits its own category because some aspects may not be associated with a particular cultural/ethnic group. Sometimes, teachers need to know things like: people tend to gather at a certain park (regardless of particular cultural/ethnic association) on a Sunday afternoon, or there is a particular coffee shop or breakfast place that people in the community like to frequent on weekends, etc.</td>
</tr>
<tr>
<td>NON-FAM</td>
<td>This form of knowledge is associated with students’ families, their norms and practices. Some of these forms of knowledge may share components with culture and community. <strong>Consistent with this study’s theoretical framework, some of these categories will need to be best applied depending on their contextual reference.</strong></td>
</tr>
<tr>
<td>NON-SES</td>
<td>This category refers to aspects such associated to SES, such as poverty. Students with high SES may also experience challenges, such as understanding social issues (e.g., issues of social justice).</td>
</tr>
<tr>
<td>NON-UKN</td>
<td>This category is left open to help denote any other unpredicted forms of mathematical knowledge that are encountered.</td>
</tr>
</tbody>
</table>

Informed by work from: Aguirre et al. (2013); Moses & Cobb (2001); Aguirre et al. (2012); Civil, (2014); R. Gutiérrez (1999); R. Gutiérrez (1999).
APPENDIX E. Classroom Snapshot: The Bet

Background:

Students had been working in class to revisit the answers from a quiz. Beth chose this time not to write the answers on the board. She asked students to discuss their work in groups and then take turns to write an answer for each question on the board. They were expected to also discuss if their answer was different from the one shared by the classmate on the board.

On the background, student voices are heard asking each other to clarify what they wrote on the board. Beth was heard talking to students individually, looking over their notes and explaining what was considered enough work to support their answers. She was also heard asking students individually what they need to do to come prepared from class.

Incident: [27:40]

Beth walked over to Darnell and asked him why he was not doing his work. He said he did not need to review because he already had a 90. She told him that if that was the case, that he should not have any problems doing his work. She then added, “I am pretty sure it was not a 90”.

Darnell: Oh, are you sure? Are you sure?
T: Bet you coffee.
Darnell: So I get coffee if I can show you?
T: Let’s go.

Darnell looked over for his papers and found a paper with a 90. Beth continued to answer students’ questions. He approached Beth at her podium.

Darnell: Can I have the coffee now? You lost the bet. Can I have the coffee?
T: Let me see.

Beth looked over Darnell’s paper.

Darnell: You lost your coffee.

Darnell tried to grab Beth’s coffee.

T: No, not this one!
Darnell: What do you mean not this one?
T: So I hold on to this and we’ll see if you match the score.

Darnell: Nah – Nah –Nah that wasn’t it! [Darnell’s voice was now high pitched]

Students started watching, paying attention to their discussion.

T: I own that. I owe you the coffee. Match the score.
Darnell: No, no. Nah, Nah. That is not what you said.
T: Match this score.

Darnell: Match what? I already fixed that.

T: Right now, because this is from two weeks, and I will up it to a small coffee and a donut.

Darnell: Small coffee!

Many students started to laugh.

Darnell: Wow!

T: That’s called, a teacher’s budget.

Many students laughed. One student said: I’d like to see a sandwich and a donut.

T: I want to see if you can maintain the quality of the work. Cause’ what I have seen recently…hum.

Darnell turns to a classmate and asks- “Do you have a pencil?”

[31:20]

T: Can I see that?

Beth asked Darnell to show her his paper again.


Darnell: How?

T: Cause’ I put the wrong quiz on the back.

Darnell: Nah, miss, you can’t switch it around.

T: No, I hold to what I said.

Beth tells the class that she realized that she had given them the wrong quiz.

T: I am going to fix it.

Darnell: Man, she did not want to give me that coffee!

Beth left the room to get new copies.
APPENDIX F. Sample from Text Trail Between Steve and Beth Through the App

F.1 Trail for two questions on applying properties of exponents with rational exponents.

F.2 First Picture Attachment in Trail:
F.3 Second Attachment in Trail:

\[ 2\sqrt{x-1} + 8 = 2 \]
\[ \quad - 8 - 8 \]
\[ 2\sqrt{x-1} = 1 - 6 \]
\[ \quad 2 \]
\[ \left( \sqrt{x-1} \right)^2 = (1)^2 \]
\[ x - 1 = 1 \]
\[ x = 2 \]

\[ [x = 2] \]
APPENDIX G. The Guardian Incident

Background:
Date: 5/17/18

This is an incident where Beth was explaining to students that she had held back from putting in the grades of their quizzes in the gradebook.

T = Teacher

Dialogue:

T: Where is everybody today?

Bell rings

T: All right folks…Good morning…Good morning! [Repeated to get students’ attention]

Students: Good morning. [Various responses]

T: So I started looking at your quizzes from yesterday…and…guess what I saw?

Students: What? [Various responses]

T: How do you think you did in the quiz?

Students: [Different answers from ‘bad’ to ‘all right’]

T: So, here is the thing. You have been asking me to put in grades all from 8.1 and 8.2. Why do you think I held back? Because…Because…What do I genuinely care? What do I genuinely care about?

Students: [Different answers: Us/How we do…]

T: Right. So when I have people with beautifully accomplished homework…[pause]…but did 4 out of 17 questions on the quiz. How do I feel about that?

Student: You feel like they copied?

T: But, what do they care that gets in the gradebook?

Students: The homework [Same answer multiple students]

T: But then the quiz…[pause]…So I had asked this question in the B period and everybody said that I was old. But this is not an old movie. Anybody seen the movie The Guardian?

Students: [Different answers: What’?/What is it!]

T: I had this amazing point to make but nobody has seen this movie and is totally lost on all of you. Nobody has seen this movie? [Beth had the billboard projected on the smartboard]

Student: Miss, you are old!

Teacher: What? This is not even six years old. Nobody seen the movie? One person!

Students: [Different responses: What is it about?/Yeah]
T: So people are talking during the one moment that I am not speaking about the math and you are missing it.

Some students laughed.

Student: What’s the movie about?

T: It’s about the Coast Guard. It’s a really good movie. Like, there is a scene in the movie which I was going to show you, but you haven’t seen it and someday you’ll see it. There is a scene in the movie where all he cares about is beating the records, but if you are in the Coast Guard, what are you supposed to be caring about?

Student: Saving lives.

T: Right. But for him, it was all about records. Beating records, beating records. So there’s a day where his coach kind of turns the table on him and the entire day is about records. And he goes over the top to congratulate him, to the point that he kind of gets annoyed because he really cared about it, but did his trainer?

Students: No [Choir answer]

T: At all? No...At all. So the reason why that comes to mind is because that is how I feel when I am putting in your homework grades. Do I really care about homework grade?

Student: No

T: What do I genuinely care about?

Student: Answer the questions

T: Right. So when I look at your homework grades and then when I look at the quiz and more than half of it is blank…

Student: Then you know that we didn’t really…

T: Exactly. So, good movie, at some other point, I’ll show you that. So here is what we are going to do. I will put in your homework grades. I will put in the homework grades, but I started to look at these quizzes…and…left blank, left blank, left blank. Do I want to grade those?

Students: No [Choir answer]

T: No! No I don’t. So today’s type 3 is a great opportunity to ask questions if you have not asked questions. Go to pages 24 and 25. Do those questions look exactly like what was on the quiz yesterday? I will wait. Your workbooks…[waits for students to open up the workbook and look]. Work with folks.

A student did not have one.

T: So you left it at home. Grab a blank one, don’t write it in. Do your work on a separate piece of paper.

Student: Miss you didn’t put them in. [A student asked her about his missing grade]

T: I don’t have your workbook, so I do not have the grades to put them in. I put the quizzes in.

She looks at the workbook.
T: I will put the 10’s for the homework. But even right now, as I have this conversation with you, like I turn inside. Is like, I’ll put in these homework grades, but when I look at your quiz grade, your quiz grade is a 50.

Student: Even the quiz from yesterday?

T: I haven’t graded them because more than half the class left them blank. So this is what today is about. Today I am having people open up to the type 3 questions. Do those questions look like exactly what is on the quiz?

Students: Yes [Choir answer]

T: Yes! Yes! I am not going to grade yesterday’s quiz, till Friday. Because this is what I am going to do. All of D lunch tomorrow and all of D lunch on Friday, I am making myself available to you to help. And then Friday you can fix what’s blank. Then I will grade them. So you have all of today to ask questions and work on your type 3’s. So that we actually have a meaningful representation of your knowledge. If something is half blank, is that representing what you know?

S: No! [Multiple students answered]

T: Well for some of us no, and form some of us, frankly that is a yes. I am hoping that we fill in the blanks today and really practice. And in the future folks, for the homework credit, I need to change my practice because we care about what’s going in the books but a 10 out of 10, which does say 100% on your grade sheet, does that make a connection to a 20% on your quiz?

Student: No

T: So I am going to reflect on how I check in your homework because I need to create a system where – Is it done? Or do you actually know what you are doing? So that is something I am going to reflect on and I will have answer for you by tomorrow. Priority today is what?

Students: Type 3 [Choir answer]

T: The type 3 and is it just getting it done? It’s understanding, asking questions. You can work with me, you can work with a partner. That is what I care about. That is always what I care about, but we are sometimes so fixed into making something look done, and then we get to the quiz and we feel awful. You feel awful, how do you think I feel?

Student: Terrible

T: Even more awful. I want that fixed. Today is going to be an awesome day. I am going to come around, answer as many questions as I can between now and the bell.
APPENDIX H. Sample Classroom Conversations & Forms of Knowledge of the Student

H.1 Background for Snapshots on 4/26/17

Shannon reviewed any problems form the homework sheet that students pointed they needed help with. They had been working on solving systems of linear equations by the substitution method (i.e. an algebraic method). All of the homework questions already had one equation where the variable was already isolated. The objective in this class was to practice on problems where the equations were all in standard form. They needed to determine what variable to solve for. There were four problems in this worksheet. Shannon explained how to do the first two. She asked them to work on the other two problems for the rest of class. The snapshot share here was based on discussions on problem 11 with the following set of equations:

\[
\begin{align*}
4x - 8y &= -4 \\
-5x + y &= 5
\end{align*}
\]

H.2 Sample Classroom Snapshots Indicating Different Forms of Knowledge [4/26/17]

<table>
<thead>
<tr>
<th>Classroom Snapshots</th>
<th>Form of Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astrid: Antonio, dónde pusiste el 44?</td>
<td>Students worked on the following equation:</td>
</tr>
<tr>
<td>[Antonio, where did you put in the 44?]</td>
<td>4x -8(5 + 5x) = -4</td>
</tr>
<tr>
<td>Antonio: Cuál 44?</td>
<td>Astrid was mistakenly writing:</td>
</tr>
<tr>
<td>[What 44?]</td>
<td>4x - 40 + 40 x = -4</td>
</tr>
<tr>
<td>Astrid: Pues aquí</td>
<td>It should have been:</td>
</tr>
<tr>
<td>[Well, right here]</td>
<td>4x - 40 - 40x = -4</td>
</tr>
<tr>
<td>Antonio: Este 40 es negativo porque multiplicaaste por negativo 8.</td>
<td>Foundational gaps on operations with</td>
</tr>
<tr>
<td>Después estas sumando 4 y negative 40.</td>
<td></td>
</tr>
<tr>
<td>[This 40 is negative because you multiplied by negative 8. Then after that you are</td>
<td></td>
</tr>
<tr>
<td>adding a 4 and a negative 40]</td>
<td></td>
</tr>
<tr>
<td>Astríd: Ah, entonces cuatro y negativo 40 son negativo 36.</td>
<td></td>
</tr>
<tr>
<td>[Ah, so then 4 and negative 40 would be negative 36.]</td>
<td></td>
</tr>
<tr>
<td>[43:00]</td>
<td></td>
</tr>
</tbody>
</table>
Astrid is still working out her problem with Antonio’s help. Astrid has already found that the x-value in the solution is -1 and she is using this value of x to determine the value of the y-coordinate.

<table>
<thead>
<tr>
<th>Signed numbers (Astrid)</th>
<th>Knowledge of alternative approaches (Antonio)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antonio was helping Astrid with:</td>
<td></td>
</tr>
<tr>
<td>-5(-1) + y = 5</td>
<td></td>
</tr>
<tr>
<td>She made an error in multiplying -5 and -1 to get a positive 5.</td>
<td></td>
</tr>
<tr>
<td>Antonio advises Astrid to use the equation where she has isolated the variable instead.</td>
<td></td>
</tr>
</tbody>
</table>

**Astrid:** Ahora tengo negativo 5. [Now I have a negative 5.]

**Antonio:** Sí, es que le 5 es positivo. [Yes, but the 5 is positive.]

**Astrid:** No, mi 5 es negativo. [No, my 5 is negative]

Antonio looks over Astrid’s work.

**Antonio:** Pero mira, yo te recomiendo que major haces esto. Ese es el procedimiento largo y tienes que estar haciendo más trabajo para sacar el valor. [But look, I recommend that you instead do this. That is the longer procedure and you end up doing more work to get the value.]

From afar, Elia is heard telling Yadira: “No lo vas a poner en la misma, lo vas a poner en la otra, si no, no funciona.” [You are not going to put it in the same equation, you have to put it in the other one, if not, it will not work]

**Shannon** has announced a quiz on Friday. The bell is about to ring.

**Grisselle** asks Astrid: Y qué hiciste con el 44? [So what did you do with the 44?]

Astrid approaches Grisselle to show her her work.

The bell rings.

**Astrid:** Te da 36 porque el 40 es negativo. Cuatro menos 40 es negativo 36. [It gives you 36 because the 40 is negative. Four minus 40 is negative 36.]

**H.3 Background for Snapshots on 05/01/17**

Shannon started class by going over common errors that she noticed when she graded their quiz on substitution (an algebraic method to solve systems of linear equations). She pointed that some students had stated that a system did not have a solution when they obtained the value of zero for a variable. She explained that a variable can have the value of zero. She reminded them...
that if the variables cancelled out, that they would then state that there is no solution. She also spoke about what they should expect if the system had infinite many solutions and reminded them that graphically they had obtained the same graph for both equations. She said, “so, same variables, think of same graphs” (Shannon, observation). Today, her goal was to introduce the elimination method through what she described as a “discovery lesson”.

The first section in their worksheet asked them to “Combine the following terms if possible, if not, write not possible”. The pairs of terms were: (a) 9 and -9, (b) 3x and -3x, (c) –y and y and (d) 8x and -8y

H.4 Sample Classroom Snapshots Indicating Different Forms of Knowledge [5/1/17]

<table>
<thead>
<tr>
<th>Classroom Snapshots</th>
<th>Form of Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the start of class, Shannon had students look at their graded quizzes. She was reviewing common mistakes that she noticed. In the background, Antonio is heard talking to Hector:</td>
<td>Mathematical Structure – Antonio recognized that he did not write his solutions as ordered pairs</td>
</tr>
<tr>
<td></td>
<td>Shannon pointed out that she had noticed his need to write answers as ordered pairs.</td>
</tr>
<tr>
<td>[3:07] Antionio: Yo lo tenía todo bueno. Todas las respuestas estaban bien. Mira. Sabes qué, no los puse en el paréntesis. Van en el orden ese.</td>
<td>Shannon had students work on the first section of the worksheet that she had describes as a “discovery lesson”. She walked around for individual questions as students work on the worksheet.</td>
</tr>
</tbody>
</table>
Teacher: Yeah, put together

Shannon discussed answers with the whole class afterwards. Shannon reviewed the answers for parts (a).

[8:30]
Shannon: So look at a, can you add 9 and -9 together?
Choir answer: Yes
What do we get?
Choir answer: zero
How about 3x and negative 3x. Can we add those together?
Hector: Yes
Shannon: What to do we get?
Choir answer: zero
Shannon: zero. All right. How about c? Negative y and y, can we add those together? And you get…
Hector: Zero.
Teacher: So, do you notice a pattern here? What’s happening here?
Lexa: you are adding negative plus positive, so get zero.
Teacher: Ah…so Lexa says that she notices that one is positive and that the other is negative. And it does not matter in what order she adds them, so the first one could be positive and the second could be negative, or, the first one could be negative and the second one could be positive and when you add them together you get zero. But what do you notice about the numbers?
Antonio: They are the same.
Shannon: The same number, but what are they? They are the same, but they are not the same, What are they?
Lexa: Op…[noticeably difficulty pronouncing]…opposites.
Shannon: So it is the same number, but you are taking the opposite of each other.
Hector: Yes
Shannon: So you have 9 and then you have -9. That is why Lexa mentioned that when you add them you get zero. They are cancelling out.
Hector: Yes
Shannon: They are making zero.
Hector: Yes
Shannon: So let’s look at (d). 8x plus negative 8y. Can we put those together?
Antonio and Hector: No
Shannon: No! Do we know why?
Hector: They have different variables.
Shannon: Yeah. Perfect, cause’ they have different variable. So, not possible, they have different variables [she writes the whole statement. on the problem in the smartboard]
Right, just because the number in the front is opposite, doesn’t mean that they are going to cancel out because we are worried about that variable. We can’t put them together if that variable is not the same. So what happens when like terms have opposite coefficients? What does this word mean?

Elia: Numbers?
Shannon: Kind of, yes. Coefficients can be positive or negative? There is now silence in the room. Students look at each other. Shannon: Does anybody know what is that word?

Hectors looks back to his classmates: Oye, qué significa coeficientes? [Hey, what does coefficient mean?]
This now turned into a group discussion in Spanish as Shannon awaits for a response.

Antonio: Yo he oído esa palabra antes. [I have heard that word before]
Hector: Coeficientes…coefficientes [Coefficients, coefficients]
Monica: Lo que pasa es que yo sé lo que es, pero no en matemáticas. [What it is – is that I know what it is but not in Spanish]
Shannon [looking at Hector]: Do you know?
Hector looks down and shakes it to indicate he does not know.
Shannon: So coefficient is the word in front of the variable. Write that down.
Hector: It’s the number in front of that variable.
Shannon: Thank you Hector, you are saying exactly what I just said. Shannon: So, Lexa just said, that if the numbers, the coefficients are opposites they cancel out.
Hector: They cancel out.
Shannon: I think there is a parrot in the room.
Hector looks back smiling and said: Es que ella me lo dice primero [is more that she beats me to it]
APPENDIX I. Conversation on Lesson Progress [Eddy and Brenda]

[May, 11, 2017]

Eddy: Hey, thanks for coming here today. You are putting your grade in jeopardy, Ok? I am serious. You are on 94, you have done seven assignments this quarter. The unit test is coming up.

Brenda: I’ll do more stuff.
[Brenda walks away from Eddy]

Eddy: I can see you before school or after school. Hey – I wanna help you ok?
APPENDIX J. Asante’s Case: The Falling Skydiver

Problem: “A skydiver is in a plane 2,000 feet above the ground. He jumps out of the plane, releases his parachute and descends at a rate of 40 feet per second.”

(All problem parts are shown in the picture of Asante’s work below)

Classroom Snapshot:

Asante started his test on a Friday [May 12, 2107]. He skipped the first question on the test and proceeded to question two. Asante asks Eddy for help on a test question. He tells Eddy that the problem is about a skydiver on a plane, but he is not sure what the height when the time is 0. Based on his question, I could tell that Asante was on question 4.

This was their dialogue:

Eddy: So at zero, where is the skydiver?
Asante: At 0?
Eddy: So where is he? What is his altitude?
Asante: He’s on the plane.
Eddy: Ok, and where is the plane?
Asante: Uhm…2,000 feet [Asante keeps looking at his paper]
Eddy: Ok, he’s going to jump off from that plane. Where is he going to be after 5 seconds?
Asante: Out of the plane.
Eddy: Correct. There should be an altitude.
Brenda: I hate math
Eddy: Math loves you though!
Asante: Do I have to subtract? This from this? No?
Eddy: I agree there is some subtraction involved.
Asante: Oh, 2,000 minus 5. I don’t know.
Eddy: How far does he travel every second?
Asante: 40
Eddy: Ok, so how far did he travel in 5 seconds?
Asante: 5 seconds?
Eddy: So, he travels 40 feet in one second. How far does he travel in 5 seconds?
Asante: 5? I know he is descending and he is going down.

Eddy asked Asante to get up. He had him walk for 5 seconds. He told him that in those 5 seconds, he had walked 4 feet. “So now” – Eddy asked – “how far would you have gone if you
had walked for 10 seconds?” Asante answered that he would have walked 8 seconds. Eddy agreed with his answer and prompted him to think about how he calculated that distance. Asante moved back to his seat.

That same day Asante asked a question from what looked like a different problem. I did not have to check the recording to confirm that he had moved back to work on question 3 based of Eddy’s response – “that question is very similar to the teddy bear problem”. I assumed this meant that Asante had finished question 4.

During the check-in, Eddy said that Asante was having one of his best years in math. He explained that he had him get up to have him relate his movement to the problem. He then asked him to calculate the distance after doubling the time so that he could get a sense of the calculation.

On Monday [May 15, 2017], Asante approached Eddy again for a question on his test. Asante needed help again on question 4. This time, Eddy tried a different approach. He had him pretend that he was driving a car and told him to close his eye. He told him he was driving 60 miles per hour. He told him he had a steady rate and asked him how far he had gone in one hour. There was a long silence. Eddy now shifted his tactic:

Eddy: You are very good in Math. Let’s try this.
Asante: You are a running back, right?
Eddy: You are average 5 yards per carry. If I give you the ball, how many yards am I expecting you to carry?
Asante: 5
Eddy: Ok. If I give you the ball again, how many would you have had?
Asante: 10
Eddy: If I give you the ball three times
Asante: 15
Eddy: Ok, let’s say you are average 40 yards a carry, OK? How far are you after one carry?
Asante: 40
Eddy: After two carries?
Asante: 80
Eddy continued to ask him after three and four carries and Asante continued to give him the correct values, until he asked him about 5 carries. At this point Asante answered 20. Eddy asked him to explain how he calculated the 20, helping Asante realize that his answer should have been 200. Eddy now Asante back to the problem to ask him how far had the diver traveled after 1 second. Asante continued to answer correctly how far the diver had traveled after each additional second. Eddy then said – “Ok, tell me where he is now”. Eddy stopped to ask one of his students [who was on the phone] how he was doing. He asked the student if he had found the slope. Since the student said no, Eddy said – “oh, you must be texting someone to ask them how to find the slope, right?” Eddy now turned back to Asante. He told Asante that he had correctly calculated that the diver went down 200 feet and that the diver had started at 2,000. “Where is he going to be?” – asked Eddy.

Asante: thinks for a while. Ah, 2,000…
Eddy: Yeah, minus…
Asante: Oh, so I subtract the 200 from the 2,000
Eddy: Don’t give up on yourself. Don’t give up on yourself, you can do this. But next time I give you another context, I may throw you from up there, how about that.
Asante smiled…

Asante’s work is shown in the next page.
Asante’s work:

f. When will the class break-even (neither make nor lose money). Explain how you arrived at your answer. (3B4)

g. If the class sells 500 cookies, what will the profit be? Show your work. (3B4)

\[
\begin{align*}
500 \times \frac{\text{total earnings}}{\text{cost per cookie}} &= 2500 \\
\text{profit} &= 2500 - 1000 = 1500
\end{align*}
\]

4. A skydiver is in a plane 3000 feet above ground. He jumps out of the plane, releases his parachute, and descends at a rate of 40 feet per second.

a. What is the independent variable? \( t \) (time in seconds)

b. What is the dependent variable? \( h \) (height in feet)

c. Complete the following table:

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>0</th>
<th>15</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in feet)</td>
<td>3000</td>
<td>1960</td>
<td>800</td>
</tr>
</tbody>
</table>

d. Plot the points from your table and connect them with a straight line. Make sure to label your axes.

e. Find the slope (rate of change) and explain the real-world meaning of this value.

slope = \( \frac{3000 - 0}{30 - 0} = \frac{3000}{30} = 100 \) feet/second

f. Find the \( y \)-intercept and explain the real-world meaning of this value.

\( y \)-intercept = \( 3000 \) feet

5. Write an equation that represents this situation. (2F3) (2J4)

\( y = \frac{2000}{3} + 2000 \)

h. How long will it take the sky diver to reach the ground? Explain your answer. (2J4)

\( 200 \text{ seconds} \) I kept subtracting

200 each time
APPENDIX K. Sample Activity from Eddy’s Video Lessons: Graphing Lines in Slope Intercept Form

Handout for Lesson 89, page 1 (1 of 2)

#89 Writing Equations in SI Form Given a Graph

If you are given a ____________ of an equation, you can write the ____________ for the graph quite easily. You only need to identify the ____________ and the ____________.

Steps:

1.

2.

3.

Example: Write the equation of the line for the given graph:

Slope:

Y-intercept:

Equation:
Example: Write the equation of the line for the given graph:

\[ \text{Slope:} \]
\[ \text{Y-intercept:} \]
\[ \text{Equation:} \]

Example: Write the equation of the line for the given graph:

\[ \text{Slope:} \]
\[ \text{Y-intercept:} \]
\[ \text{Equation:} \]
APPENDIX L. Daniel’s Case

Case Description:

Daniel was a student in period 6. He sat on the third table, farthest away from the board. His table had three boys and one girl. At the start of the observational period Daniel did not evidence much work completion. When Eddy visited their table, Daniel would ask questions. When Eddy left the table Daniel would be off-task. Two weeks before the semester ended, Eddy received a list of names of students that were at risk of not passing the course. Daniel was in this list. Eddy shared this list with me during class time. He gave it to me right as the students entered the room. He told me that he would explain more at the check-in after school.

During the check-in, he explained that this was a school initiative to make sure that teachers were aware of students that were at risk of not passing their courses. We could only speak about students that had parental consent. Daniel was one of these students. Eddy explained that Daniel had him as his coach in the freshman team last semester. He knew that Daniel respected him because of that, and they had a good relationship. Eddy said, “there is not one bad bone in Daniel’s body; he is a good kid” (check-in). His family was very supportive of Daniel’s progress in school. Eddy had called the home before. His concern was not so much the family, but Daniel’s willingness to do as his family asked him to do. Eddy added that Daniel did not seem to have the maturity to understand the predicament he was in.

As with almost every student, Eddy knew exactly how many lessons Daniel needed to complete so that he would pass the course. Eddy noted that if Daniel did not get to take the unit test, that he would definitely not pass. He wanted Daniel to take the test, because that was the item with the most points in the gradebook. He also wondered if there were undiagnosed issues
with Daniel associated with attention. Eddy ended his comments about Daniel by noting that perhaps one more year of algebra would help him build the maturity.

After Eddy’s talk with Daniel’s family, Daniel was a completely different student in class. He moved himself out of table 3 and was now working next to Eddy on table 1. Most of the students that were bordering on a passing grade had moved to table 1. Eddy gave these students a lot more attention than the rest of the students at this time of the year. During a check-in, I asked Eddy to tell me his thoughts on Daniel’s progress. This time around Eddy described a changed perspective. He was surprised to see how Daniel turned around his efforts.

Mathematically, Eddy reflected a sense of awe at how well Daniel was understanding linear functions by working with the ordered pairs. He used them to graph the function and to inspect the patterns in the calculations. These patterns would have been evident from the equation, but Eddy explained that he did not think that Daniel was capable of making that additional connection.

Samples of Daniel’s Work:

Picture 1: The first picture shows Daniel’s work for the last part of question 3 of the test. This question models a scenario where cookies were purchased for a total of $160 for a fundraiser. The cookies were sold at $2.00 each. Daniel had found the equation, and was asked to find the break-even point.
4. A skydiver is in a plane 2000 feet above ground. He jumps out of the plane, releases his parachute, and descends at a rate of 40 feet per second.

a. What is the independent variable?

b. What is the dependent variable?

c. Complete the following table.

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in feet)</td>
<td>2000</td>
<td>1500</td>
<td>1000</td>
<td>500</td>
</tr>
</tbody>
</table>

d. Plot the points from your table and connect them with a straight line. Make sure to label your axes.

e. Find the slope (rate of change) and explain the real-world meaning of this value.

\[ \frac{-2000}{5} = -400 \]

The 2000 means a man jumping out 2000 feet above ground.

g. Write an equation that represents this situation.

\[ y = -\frac{2}{5}x + 2000 \]
Picture 2: Daniel calculated ordered pairs and graphed the line. He used his graph to find the break-even point.
APPENDIX M. Classroom Snapshot: Working with a deck of cards

Background:

I use narrative to present this snapshot from the second week of class. Dena was introducing probabilities of complements and unions of events. She had also applied probability calculations to examples on political party affiliations and on rolling two dice. This new example involved applications with a standard deck of card.

Snapshot:

Dena has moved on to question 3 in her handout for today’s examples. I could not help noticing that the question made reference to a deck of cards. In our last check-in Dena told me she had noticed that Elsa and Omar were not familiar with cards. I wondered how she would handle this problem today…Dena started by saying to the class – “I was kind of assuming that everybody was familiar with a standard deck of cards, which may not be the case, so I brought some cards”. As she spoke, she took out a shopping bag and dumped a pile of unopened packs of cards on the front table. She unwrapped the plastic cover from each package as she passed them along to each student starting with Omar. She asked Omar if he had worked before with cards, but he did not answer. Sarah was sitting a few feet from Dena as she ate her Taco Bell dinner. She ate and she stared, seeming amused with Dena’s struggles to get the packs opened. “Oops, dropped the king!” – Sarah told Dena. Dena acknowledged she dropped the king, as she picked it up to complete the deck for another student to use. “Those look brand new, did you just buy them?” – said Sarah. Dena told her that she made a stop at Target that afternoon. Joyce, who also sat in the front, now laughed and commented that Dena was going to do magic for them.
Dena asked “those that are familiar with a deck of cards” to describe them. Sarah and Thomas took the lead, listing the suits and the number of cards. Dena wrote the card options on the board, grouping the J, the Q and that K to denote the face cards. She called this summary an “orientation” and proceeded to ask the class to look at problem 3 with her. Sarah interrupted to ask if they still have jokers. Dena replied that it can be a wild card and that it depended on the game, but she immediately redirected Sarah to the problem by reading the first question without pause – “If you have a standard deck of cards, what is the probability of drawing a queen?”

Dena asked the class what they thought they needed to know in order to answer this question. Sarah and Marlee shared that they needed to know that there are 52 cards and four queens. I looked over Omar, since I was sitting just behind him. He had some face cards to the side. Having heard also that the probability was 4 out of 52, Dena asked them to tell her what that was equal to. Sarah answered – “seventy six point nine!” Dena stopped, unsure about what she had just heard – “No, what is it? What did you say?” Sarah repeated the same answer. Marlee tried to jump in – “seven point...”, but Dena interrupted her, still facing Sarah. “Wait for a second, does that make sense? I have 4 out of 52, that’s seventy six percent?” Sarah recognized in Dena’s response that she needed to reconsider her answer and asked Dena to wait. Joyce tried to help by murmuring that she needed to move the decimal. Dena was still intently working with Sarah despite classmates’ multiple attempts to rectify her answer. “No, it’s more. What did you get as a decimal value? Let me ask you that.” Sarah read her answer from the calculator, still not recognizing her error. Dena asked her how she should change a decimal into a percent. Sarah asked if she should move the period. Instead of answering her question, Dena asked her what allowed her to move the period. Sarah said she did not know. Having found the nature of Sarah’s error, Dena acknowledged Marlee’s explanation as to how to change the number into a
percent and used it to write on the board how they made the percent calculation. She turned back to Sarah and asked her if anything “went off in her head when she said 76%.” But Sarah simply said –“nope”. Dena tried to make connections to cents and dollars –“like three quarters out of a dollar?” But Sarah candidly continued to say no. Dena ended their discussion and moved on to the next question by restating that seven point 6 was much smaller.

They moved on to the next question which asked them to calculate the probability of drawing a queen or a heart. This question required a better understanding of how all the cards relate, so I looked over to Elsa and Omar. I tried not to be obvious about my intention to watch them. Elsa was taking notes of what was written on the board. Omar struggled to make sense out of the cards by continuing to sort them. At this point, Dena moved over near Omar to ask questions to the class while she tried to help Omar find the queens and sort out the hearts. Dena asked the class to think about how many queens and how many hearts they had. Sarah now jumped in –“but would you like…” She stopped and paused to think. After a few seconds of silence, Dena encouraged Sarah to keep going. Sarah noted that they were already counting the queen with the suit cards with the queen of hearts. She asked if instead of saying there were 13 hearts, if they should be saying 12. Dena remarked –“Good question!” She still did not answer it though. Dena affirmed that they had 4 queens and 13 hearts and she also asked about the queen of hearts. “How many times have I counted her so far?” – Dena asked. Sarah noted that it was twice. Dena now turned to the class and asked –“Does that make sense?” But Joyce said no. Linda now jumped in to explain. While she explained, Dena moved back to Elsa and Omar to help them sort their cards. Linda asked Joyce –“So if they ask you to go get a heart, how many you can get?” Joyce said, 13. Now Linda added –“Well, you need to also get queens, but you used one already in the 13 that you brought in, so how many more queens do you need?
Three.” Joyce nodded in appreciation. Dena agreed with Linda’s contribution and confirmed that they did not want to count her twice. She looked at Joyce as she said – “I get it”. Dena moved back to the front of the room to write an equation for their probability calculation.

**The Snapshot Revisited**

I picked this classroom snapshot because it portrays multiple findings from this study in practice – all happening within a 10 minute span of time. For example, the form of knowledge of ‘foundational gaps’ was evidenced through Sarah’s responses to Dena. Within the context of instruction, Sarah reflected foundational gaps in her inability to recognize her incorrect calculation of the percent. A closer look at the snapshot also demonstrates that as a result of these gaps, Sarah was not reaching a quantitative understanding of the probability, which was ultimately the objective for learning how to do the calculation. In this regard, the foundational gaps inhibited Sarah’s ability to fully engage critically. At the same time, despite foundational gaps, the snapshot also demonstrated Sarah’s successful engagement in a different aspect of the problem they were exploring. Sarah evidenced critical thinking in recognizing that they needed to change the count for the queen of hearts. Linda, on the other hand, waited to participate when she was needed by a classmate. This was Linda’s behavioral pattern during the observational period. Linda demonstrated her mathematical maturity in the ways she helped Joyce.

The snapshot also demonstrates the complexity of Dena’s work as she attempted to implement her interventions. She could have just rectified Sarah’s answer, but she prioritized Sarah’s meaning making of their calculation (mathematical conversations and holds students accountable). Dena’s patterns in teaching reflected a questioning approach and a need to make sense out of their discussions. In order for future calculations to make sense to Sarah, she also needed to review (or learn) how to do the percent calculation. At the same time, Dena was
facilitating the class conversation while also trying to open packs of cards and help Omar and Elsa sort their cards. There was a clear dissonance is Elsa’s and Omar’s understanding of the card context. The classroom learning experience was much more demanding for Elsa and Omar, and their lack of engagement in class made it a requirement for them to meet with Dena outside of class. Dena’s attempts to help them sort their cards reflected her awareness that they needed help. Elsa and Omar missed out on Sarah’s thinking about double counting the queen because the context was not familiar to them. Joyce was able to benefit from Linda’s question on, “how many more queens to you have left to bring in?” But that was not the case for Elsa and Omar because knowing the make-up of the cards was not information they could access as easily.
APPENDIX N. Common Forms of Knowledge

N.1. Descriptions of Consolidations of Forms of Knowledge of the Student for the Cross-Case Analysis.

1. **Learning Attitudes:** This was a form of knowledge from the original start list. *Learning attitudes* refers to students’ mental dispositions towards learning. For the within-case analysis, I had created sub-categories under learning attitudes to capture nuanced differences. These sub-categories were: *self-efficacy, confidence, perseverance* and *anxiety.* For the cross-case analysis, I have consolidated them all back under *learning attitudes.* They shared similar student indicators described by their teachers (e.g., inconsistency in completing work, apprehension towards risk to engage in learning, fear of failure, etc.).

2. **Personal Conditions:** I have eliminated this category because the data fit under the following two existing categories: *competing priorities and SES challenges* and *personal characteristics.* I re-distributed data from personal conditions to either of these categories depending on their association. *Competing priorities and SES challenges* represents outside of school priorities that challenges students’ ability to prioritize their school work and that are also associated to SES challenges. Personal Characteristics refers to student characteristics and/or conditions that have an impact on how students engaged in learning, without association to SES challenges.

3. **Foundational Gaps:** This category now also includes *mathematical errors.* Mathematical errors and foundational gaps were categories from the original start list. The aspects from mathematical errors found, also common across cases, were those associated with foundational gaps. I consolidated those aspects of *mathematical errors* under the category of *foundational gaps.*
### N.2. Consolidated Forms of Knowledge of the Student Used by Teacher Case

<table>
<thead>
<tr>
<th>Form of Knowledge of the Student</th>
<th>Teacher Case</th>
<th>BETH</th>
<th>SHANNON</th>
<th>EDDY</th>
<th>DENA</th>
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</thead>
<tbody>
<tr>
<td><strong>Culture (Dissonance Only)</strong></td>
<td>U</td>
<td>U</td>
<td>U</td>
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<td><strong>Foundational Gaps includes Math Errors</strong></td>
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<td><strong>Math Progress (Learning Progress)</strong></td>
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<td><strong>Family Support</strong></td>
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<td><strong>Language – NON</strong></td>
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<td><strong>Emotional Well-being</strong></td>
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<td>Partial Aspects</td>
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<tr>
<td><strong>Learning attitudes /Positive Interest</strong></td>
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