Life-Cycle Fatigue Performance of Coastal Slender Bridges Subject to Multi-Hazards

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Jin Zhu, PhD
University of Connecticut, 2018

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A general analytical VBWW platform is first established based on the finite element analysis (FEA) software ANSYS and programing software MATLAB. With the established VBWW platform: (1) global dynamic responses of the vehicle-bridge system subjected to various service and extreme wind and wave loads can be rationally predicted; (2) comprehensive vehicle driving safety and ride comfort evaluations are also carried out using current state-of-art evaluation criterion. As an extension of the VBWW platform, two probabilistic fatigue damage assessment schemes were developed based on machine learning algorithms. The first one is to integrate the multi-scale FEA and the support vector machine (SVM) for fatigue reliability evaluation considering life-cycle stochastic dynamic loads. The second one is to use the
dynamic Bayesian network (DBN) for fatigue damage diagnosis and prognosis of an OSD through integrating the physics-based model with field inspections while accounting for the associated uncertainties. Through the two established numerical schemes, the fatigue damage of the coastal bridge in the context of VBWW system can be evaluated.
Life-Cycle Fatigue Performance of Coastal Slender Bridges Subject to Multi-Hazards

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M.S., Southwest Jiaotong University, 2012
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Life-Cycle Fatigue Performance of Coastal Slender Bridges Subject to Multi-Hazards

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University of Connecticut
2018
DEDICATION

To my parents and my wife

For their unconditional love and endless support
ACKNOWLEDGEMENTS

I am greatly indebted to my advisor, Prof. Wei Zhang, for his trusts and encouragement, valuable guidance and advice, exceptional patience, and most importantly, dedication of his time throughout my study and preparation of the dissertation. I deeply appreciate his critical guidance to keep me on track while giving me sufficient freedom to pursue my research. His conscientious and meticulous work attitude not only have kept motivating and inspiring me inexhaustibly during the past four and half years, but will also definitely benefit me in my future career. My sincere gratitude to him, for all that I have learned, and looking forward to continuing this relationship for the rest of my professional career.

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1 Introduction

1.1 Overview

Serving as critical links in the transportation network for coastal regions, coastal slender bridges could constantly experience complex dynamic interactions with strong winds and/or high waves during extreme weather conditions, in addition to moving vehicles, such as cars, trucks, or trains. Continuously repeated stress cycles as well as corrosive coastal environments could cause significant fatigue damage accumulations at complicated weldments of the orthotropic steel deck (OSD) during lifetime, which could be critical and might affect structural safety and reliability. Nevertheless, fatigue is a damage accumulation process that is subjected to various aleatory (random) and epistemic (lack of knowledge) uncertainties from ambient environment, empirical justifications, model simplifications, inaccurate statistics of model parameters, measurement error, etc. Challenges, such as realistic load characterization, modeling and simulation of complex structures, model parameter identification and calibration as well as uncertainty quantification, exist when evaluating the dynamic performance and fatigue damage of structural details in the vehicle-bridge-wind-wave (VBWW) system. To address these challenges, this dissertation proposes a list of versatile and efficient numerical schemes to enable: (1) comprehensive dynamic performance analysis of coupled VBWW system; and (2) probabilistic assessment and prediction of fatigue damage of OSD accounting for various uncertainties.

Firstly, a general analytical VBWW platform is established based on the finite element analysis (FEA) software ANSYS and programming software MATLAB. Serving as the dynamic input for the VBWW system, the correlated wind and wave field around the bridge structure is first simulated with a novel simulation algorithm. Subsequently, the simulated wind and wave field facilitates the coupled VBWW dynamic analysis, in which the global dynamic responses of the bridge as well as each individual vehicle subjected to various service and extreme wind and wave loads can be rationally predicted in the time domain. Furthermore, comprehensive vehicle driving safety and ride comfort evaluations are also carried out using
current state-of-art evaluation criterion.

As an extension of the VBWW platform, two probabilistic fatigue damage assessment schemes were developed based on machine learning algorithms. The first one is to integrate the multi-scale FEA and the support vector machine (SVM) for fatigue reliability evaluation considering life-cycle stochastic dynamic loads. The multi-scale FEA enables computing the stress time history at the critical welded joints of OSD, while the SVM serves as a surrogate model to avoid time-consuming large numbers of FEA simulations. The second one is to use the dynamic Bayesian network (DBN) for fatigue damage diagnosis and prognosis of an OSD through integrating the physics-based model with field inspections while accounting for the associated uncertainties. Regarding diagnosis, the proposed numerical scheme enables tracking the fatigue damage evolution and calibrating time-invariant model parameters. Regarding prognosis, the proposed numerical scheme facilitates predicting the fatigue damage state in the future. The particle filter (PF) is implemented to perform the Bayesian inference of the DBN, and a Gaussian process (GP) surrogate model is established to construct the conditional probability distribution in the DBN model. Through the two established numerical schemes, the fatigue damage of the coastal bridge in the context of VBWW system can be evaluated.

The rest of this Chapter is organized as follows: after a brief overview in Section 1.1, a thorough literature review on the topics related to the dissertation research is provided in Section 1.2; after that, Section 1.3 proposes the research objectives in this dissertation; finally, Section 1.4 illustrates the general layout of this dissertation for each chapter.

1.2 Scientific Context: A Literature Review

In order to provide the necessary research background of this dissertation, a thorough literature review on the topics related to the dissertation research is presented in this section.

1.2.1 Hurricane Induced Wind and Wave Fields

Coastal infrastructures are extremely vulnerable to natural hazards such as hurricanes and the associated strong winds, storm surge, flooding, etc. For example, hurricane Katrina of 2005 caused more
than US $100 billion in losses and resulted in about 2,000 fatalities with the greatest coastal flood height ever recorded in the US [1]. Moreover, even larger human and economic losses are expected in the future, in recognizing the steady increase in population and wealth during the past decades [2]. Recently, there has been growing evidence showing that the global climate change may trigger more frequent and severe extreme events from natural hazards [3]. As reported by the Australian Greenhouse Office [4], the peak wind speed will increase by 2-5% by the year 2030 and 5-10% by the year 2070, respectively. Knutson et al. [5] concluded that the hurricane wind speed is likely to increase by 20% globally in the 21st century. IPCC (Intergovernmental Panel on Climate Change) also reported that both the hurricane intensity and frequency may be affected due to the increase of sea surface temperature [6].

For weather prediction or hurricane risk analysis, many efforts have been made to simulate hurricane associated wind field and wave field in a larger temporal and spatial scale. The research on hurricane near-surface wind field (i.e. time-varying mean) modeling can be traced back to Chow’s work in 1971 [7] and much improvement has been made since then [8–10]. Based on the planetary boundary layer (PBL), the hurricane wind field model uses a finite-difference scheme to solve for the steady-state wind field based on a set of nested rectangular grids. As an alternative approach, the parametric hurricane wind model is more frequently used for long-term wind and surge risk assessment and structural design due to its simplicity and efficiency [11]. In the parametric hurricane wind model, the near-surface time-varying mean wind is assumed as a vector summation of storm vortex associated with the hurricane itself and the environmental background wind vector related to the storm movement. However, none of the current hurricane wind models include wind turbulent fluctuations, which could introduce considerable dynamic response of structures [12].

Since wind is one of the major driving forces for waves, many methods were proposed, ranging from simple formulae for estimating wave field at a given site by using wind speed, fetch, and duration, to numerical models for wave field simulation covering large sea areas based on the input wind field time histories. Some sophisticated numerical wave prediction models, such as SWAN, WAM, WAVEWATCH models, were developed and have been widely used in various applications of weather prediction and ocean
dynamics in the past few decades [13]. However, due to their large spatiotemporal scales, the resolutions for the simulated wind and waves are still low and the wind and wave time histories are still not appreciable for the dynamic analysis of coastal infrastructures [14,15]. For example, the input wind data for the SWAN model [14] for wave simulation in the Black Sea has the spatial resolution of 0.25° in both longitude and latitude with a temporal resolution of 6 h, and the output wave data has the spatial resolution of 1.3 km × 1.83 km.

In addition, in the extreme weather conditions, both wind and wave could be nonstationary. The evolutionary power spectral density (EPSD) functions, which could include the energy distribution over both time and spectral domains [16], were used to characterize such transient features for wind fluctuations and wind driven waves [17,18]. As the frequency structures in normalized EPSDs of wind fluctuations of the downburst and typhoon were found to evolve very slightly with time, the nonstationary wind fluctuations, therefore, were assumed to be uniformly modulated processes [17,19,20]. Similarly, for the nonstationary wave, it is natural to extend the available stationary wave spectra to nonstationary ones, based on the slow-change assumption for the large-scale structure of the hurricane [9]. The power spectral density (PSD) function at each infinitely small interval, therefore, is treated to be stationary following the existing stationary wave spectrum. By combining these time-varying PSDs, a simplified nonstationary wave spectrum can be obtained and could be extended to describe the nonstationary wave after introducing the time-varying mean wind speed. With the prescribed PSD/EPSD, a number of approaches, such as Spectral Representation Method (SRM), linear filter method (e.g., Autoregressive-moving-average (ARMA)), wavelet based method, Proper Orthogonal Decomposition (POD), Empirical Mode Decomposition together with Hilbert Transform (EMD–HT) approach, etc., have been proposed to simulate the stationary/nonstationary Gaussian process [21].

1.2.2 Coupled Vehicle-Bridge-Wind-Wave (VBWW) System

In this section, the essential components of coupled VBWW system are introduced including bridge aerodynamics, bridge-wave interaction, vehicle-bridge-wind interaction, wind-wave-bridge tower interaction, respectively.
1.2.2.1 Bridge Aerodynamics

The ever increasing span length of the long-span bridges results in a remarkable decrease in their natural frequencies, which in turn makes them increasingly susceptible to the action of strong wind. Despite of periodic failures of wind-sensitive suspension bridges in the nineteenth century, it was not until the well-known failure of the first Tacoma Narrows suspension bridge under a relatively low (19 m/s) wind in 1940 that brought serious attentions to the bridge aerodynamics. Extensive investigations have been conducted on bridge aerodynamics by structural engineers and researchers since then [22–25]. Some important aspects of this subject include wind-induced vibrations (e.g., buffeting, flutter, galloping, and vortex shedding), non-stationary and non-linear flutter and buffeting, wind field simulation, wind vehicle-bridge interaction, wind-induced fatigue, wind-induced vibration control, and probabilistic analysis and reliability assessment [26]. The wind tunnel experiment approach, the analytical approach, and the computational fluid dynamics (CFD) approach are three major approaches currently adopted for the investigation of bridge aerodynamics [27].

Based on the assumption of stationary wind excitations, the wind induced aerodynamic forces acting on bluff bridge sections are commonly separated into three components: steady-state forces resulting from mean wind speed, self-excited forces resulting from the interaction between the wind and the bridge motion and buffeting forces resulting from unsteady wind velocity [28,29]. The aerostatic force components are formulated using static force coefficients, and the self-excited and buffeting force components are characterized by flutter derivatives and admittance functions in the frequency domain, and by aerodynamic impulse response functions in the time domain. Pioneering research of buffeting and flutter were made in the 1960s by Davenport [22,23] and in 1970s by Scanlan and his colleagues [24,30,31], and since then a number of analytical developments has been made in bridge aerodynamics/aeroelasticity by many other researchers. The traditional wind-induced bridge buffeting and flutter analyses are usually carried out in the frequency domain, primarily due to the aerodynamic forces can be conveniently expressed as the functions of frequency as well as due to computational efficiency. The major drawback of frequency domain approach is the inability to account for aerodynamic nonlinearities as it is in general restricted to linear structures.
subjected to stationary wind loads. To include the nonlinearities of structural and aerodynamic origins, the
time domain approach is more appropriate and has been adopted for analysis of flutter and buffeting
response by many researchers [32–34].

1.2.2.2 Bridge-Wave Interaction

Exposed to the harsh sea environment, the coastal bridges are often subjected to severe wave loads
which may influence the structural performance or cause fatigue issue. For example, a number of U.S.
coastal bridges along the Gulf Coast region have been damaged by the wind waves accompanied by the
storm surge generated by Hurricane Katrina in 2005 [35]. For coastal slender bridges, the foundations (i.e.
pile and cap) often bear considerable wave loads, while for coastal short span highway bridges, the entire
superstructure (i.e. bridge deck) may be subjected to wave load due to huge waves such as during hurricanes
[35–37]. To better understand the concept of wave loading mechanisms, the numerical (Computational
Fluid Dynamics) and laboratory studies are two major approaches adopted by engineers and researchers. A
brief review of previous theoretical and experimental research on wave forces on piles as well as coastal
bridges is summarized and discussed as follows.

Piles, usually designed as circular or rectangular cylinders, have been commonly used as substructure
elements of coastal structures such as bridge foundations, wind turbine foundations, offshore platform legs,
etc. Due to their importance for structural safety, numerous research studies, numerically or experimentally,
have been conducted to investigate the interactions between waves and piles [38–46]. For a single slender
pile with ratio of the diameter to wave length less than 0.15, the non-breaking wave force can be determined
by Morison equation as the sum of the quasi static inertia force and the drag force. Unlike the case of single
isolated slender piles, the presence of closely-spaced pile group or large-scale piles changes wave motion
locally that in turn causes the wave transformation (e.g., wave diffraction and wave breaking). As a result,
each pile in pile group is significantly affected by the neighboring piles. Therefore, wave force can be
accurately estimated only if the interactions between the waves and the pile group/single large-scale pile
are fully considered. An analytical solution is hardly feasible given the high complexity of the interaction
between waves and pile groups with various arrangements and laboratory experiments still represent the
most reliable alternative. In general, two methods have been mostly applied to analyze non-breaking wave loads on a cylindrical pile within pile groups, i.e. force coefficient approach (e.g. [47,48]) and wave force approach (e.g. [42,49]).

Many low-laying coastal bridges have suffered from critical damage during hurricane seasons 2004 and 2005 in the Gulf of Mexico, mainly due to the effects of storm surge and water wave loading [35–37,50–52]. The severe damage have begun to arouse public’s attention and many research have been conducted, experimentally [50,53–56] and numerically [57–61], on the effects of hurricane induced wave loads on coastal bridges since then. Since the effects of wave loads on the low-laying bridges are beyond the scope of current study, the research on wave-bridge deck interactions are not discussed here.

1.2.2.3 Vehicle-Bridge-Wind (VBW) System

The study of coupled vehicle-bridge (VB) system originated in the middle of 20th century. The earlier VB system as shown in Fig. 1.1 is relative simple in which the bridge is modeled as a simply supported beam and the moving vehicle is modeled as a constant moving load or a spring-mass (consider the inertial force). Due to the limitation of computer technology, the researchers sought to find the semi-analytical/analytical solution to this simply VB model [62–66]. However, the earlier VB model didn’t include the effects of uneven bridge surface, which is known to be the main cause of high-magnitude bridge vibrations [67]. In addition, the over simplification of the vehicle (e.g., single degree of freedom) makes it impossible to characterize the dynamic behavior of the vehicle. Despite of such drawbacks, these studies begin to reveal the coupled mechanisms of dynamic VB system and lay a foundation to more sophisticated VB system. Later, Guo and Xu [68] proposed a fully computerized approach for assembling equations of motions of coupled VB systems by considering the road surface roughness. In the dynamic system, the vehicles are idealized as a combination of several rigid bodies connected with several axle mass blocks, springs, and damping devices, while the bridge is modelled using the conventional finite element method. Shi et al. [69] developed a similar coupled VB system with focus on the effect of approach slab conditions on the dynamic behavior of short-span slab bridges. Various effects are considered including truck speeds, road surface conditions, and faulting conditions.
Figure 1.1 Moving mass over simply supported beam model

With the development in areas related to vehicle-bridge, vehicle-wind, and wind-bridge dynamics, the studies of the coupled VBW system become available and have drawn increasing attention over the last decade. The VBW system usually refers to the complex interactions between vehicles, long-span bridges and wind, which is especially true for long-span bridges built in wind-prone area while carrying a high volume of traffic. Chen and Cai [28] proposed a framework for the vehicle-bridge-wind aerodynamic analysis, which lays a very important foundation for vehicle accident analysis based on dynamic analysis results and facilitates the aerodynamic analysis of bridges considering vehicle-bridge-wind interaction. The framework built a general dynamic-mechanical model for VBW coupled system in which various types of vehicles are considered and the excitations such as wind, road roughness are included as well. Meanwhile, Guo [70] and Guo and Xu [71] proposed an approach to model and prediction of the safety and ride comfort of high-sided road vehicles running over a long-span cable-stayed bridge under cross winds, and the effects of road roughness, mean wind velocity, and vehicle speed were investigated.

Most existing vehicle-bridge-wind interaction models didn’t consider the actual traffic load in which the traffic load is simplified as only one or several uniformly distributed vehicles running with a constant speed on the bridge. Such an assumption differs significantly from reality due to the fact that the traffic flow on long-span bridges is typically stochastic such that the number, type, and speed of vehicle vary according to local traffic conditions. In the model proposed by Chen and Wu [72], the stochastic traffic flow simulation is incorporated in the bridge-traffic-wind interaction analysis to more realistically simulate the dynamic interactions of stochastic moving traffic on a long-span bridge in windy environment. Later, Zhou and Chen [29,73] developed a fully coupled bridge-traffic interaction model by coupling the mode-based bridge model and all individual moving vehicles of the simulated stochastic traffic flow.
1.2.2.4 Wind-Wave-Bridge Tower System

Recently, several studies have been conducted to investigate the dynamic behavior of bridge tower under wind and/or wave current loads numerically or experimentally [74–78]. Chen et al. [74] performed dynamic analysis of a bridge tower under wind and wave actions in the time domain based on boundary-element method and the FEM, and the results indicated both the wind and wave actions have significant effects on the dynamic responses of the bridge tower. In the model, the aerodynamic load is described by the sum of mean wind and buffeting forces, and the wave loads on the cap and piles are evaluated by the potential flow theory and the Morison equation. Later on, Guo et al. [75] conducted laboratory tests on the dynamic behavior of a freestanding bridge tower model under coupled wind-wave actions using wind tunnel and wave flume. During the tests, when the mean wind speed is low and the wave period is near the structural resonant frequency, coupled wind-wave effects were observed under which the structural displacement responses were suppressed. No significant coupled wind-wave effects were observed when the wave period is away from the structural resonant frequency. Similar experiments were also conducted by Wei et al. [78] with a main focus on the hydrodynamic effects on a free-standing bridge tower that consists of a diamond-type pylon and a large round-ended caisson foundation. No obvious coupled wave-current effects were observed. The base shear due to the wave current was approximately equal to the sum of shear force driven by wave and current separately. The dynamic amplifications from the forward current were not observed.

These studies have provided valuable insights into the aerodynamic and hydrodynamic characteristics of a freestanding bridge tower during the construction stage. During the service stage, the wind and wave effects on the coastal slender bridges could be even more complicated and the bridges are also expected to carry a high volume of traffic with additional wind loads on bridge superstructure (tower and deck) and wave loads on substructure (foundation). As a system, the interactions between the bridge, vehicle, wind and wave could become even more complicated, leading to a potential threat to the structural safety and reliability during the bridge’s lifetime. Consequently, coastal bridges may deteriorate over time that can increase the complexity for the bridge’s life cycle performance. Due to the complexity, neither the effective
experimental studies nor the numerical frameworks have been performed or proposed.

### 1.2.3 Bridge Fatigue

Fatigue is a progressive and localized process in which the structural damage accumulates continuously due to the repeated external loadings that may be well below the structural resistance capacity [79]. Fatigue is regarded as one of the most critical forms of damage and principal failure modes for steel structures, and about 80 to 90 percent of failures in metallic structures are related to fatigue fracture according to American Society of Civil Engineers (ASCE) Committee on Fatigue and Fracture Reliability. Fatigue can cause serviceability issue and damage to some local members or may eventually lead to complete failure of a structure, such as collapse and failure of the Point Pleasant Bridge (also known as Silver Bridge) in West Virginia (USA, 1967), Yellow Mill Pond Bridge in Connecticut (USA, 1976), and Sungsoo Grand Bridge (South Korea, 1994).

Among current methodologies for fatigue damage evaluation and life prediction, the $S$-$N$ curve approach has widely been used for various steel structures including high rise buildings, aircraft, offshore structures, and steel bridges, probably due to its simplicity [80]. For constant amplitude loading conditions, the relationship between the constant-amplitude stress range, $S$, and the number of cycles to failure, $N$, can be described by experimentally obtained $S$-$N$ curve. Nevertheless, the repeated dynamic loadings exerted on most bridges are random in nature thus the resultant stresses have variable amplitude ranges, which makes it difficult to directly use the $S$-$N$ curve approach. Later, Miner’s linear damage rule (LDR) is proposed [81] to extend the $S$-$N$ curve approach to variable-amplitude loadings by introducing the concept of equivalent stress range. Before applying LDR, the equivalent stress range and cycles are extracted from stress time history (either measured or simulated), by means of a suitable cycle counting algorithm (e.g., rain-flow counting method [82]). For engineering practice, the $S$-$N$ curves for different categories corresponding to commonly used bridge details are adopted for many design specifications, such as AASHTO [83], BS 5400 [84], and ECS [85]. Despite of such wide applications, the limitations of $S$-$N$ approach cannot be ignored due to the fact it neglects the effects of both load sequence and stress level, which have significant influence on the damage accumulation [86]. To overcome this, many other methods
have been proposed such as the fracture mechanics based approach which considers the complicated nonlinear damage propagation process. Comprehensive review of fatigue life prediction methods for metal structures can be found in [79,87].

Based on the knowledge of both fatigue resistance and load effects, it is possible to perform the fatigue assessment of bridges, either in a deterministic or a probabilistic manner. Due to the inherent randomness in both load and resistance, the probabilistic fatigue reliability assessment seems more appropriate in order to better understand the bridge fatigue behavior and guide decision-making regarding bridge maintenance and rehabilitation. It is beneficial to carry out the bridge fatigue evaluation using numerical framework, especially for bridges subjected to multiple loads, as the numerical simulation can take account into almost all the possible loading combinations during the bridges’ life-cycles. Several studies have been carried out on the fatigue reliability using numerical framework of coupled vehicle (train)-bridge system or vehicle (train)-bridge-wind system [88–91]. For example, Zhang et al. [88] proposed a framework for fatigue reliability analysis of coupled vehicle-bridge-wind dynamic system. In their work, the complicated structural details were modeled with equivalent orthotropic material and multiple parameters including vehicle speed, road roughness conditions, and wind velocity and direction were modeled probabilistically to generate values of the revised equivalent stress range. The results indicated that combined dynamic effects from winds and vehicles might result in serious fatigue problems for long-span bridges, while the effects from either winds or vehicles were not able to induce serious fatigue problems alone. Later, Li et al. [89] performed fatigue reliability assessment of railway bridges based on probabilistic dynamic stress analyses of a coupled train-bridge system, in which the train speeds and track irregularities were treated as random variables. The fatigue reliability is evaluated by solving a fatigue limit-state function established through the $S$-$N$ approach and the results indicated that both the train speed and track irregularities could significantly affect the fatigue reliability of railway bridges.

In addition to numerical simulation, many efforts have also been made to integrate the field measurement data with the numerical simulation for more reliable fatigue reliability evaluations of long-span bridges, with the aid of structural health monitoring systems (SHMSs). Over the past decade, many
SHMSs have been successfully implemented on a lot of newly built long-span bridges worldwide, in order to ensure the bridge safety and issue early warnings on possible damage or deterioration prior to costly repair or even catastrophic collapse [92]. Typical examples are the Tamar Bridge in UK [93], Tsing Ma Bridge [26] and Stonecutters Bridge [94] in Hong Kong, Great Belt Bridge in Demark [95], the Akashi Kaikyo Bridge in Japan [96], and Seohae Bridge in Korea [97]. Xu and his colleagues proposed a framework for fatigue reliability analysis of long-span bridges under various dynamic loadings by integrating numerical simulation with SHMSs [98–100]. In their work, the stress time histories at critical points of Tsing Ma Bridge under different loading combinations were obtained through long-term SHMSs, which were used subsequently to create a database for calculating fatigue damage accumulation and estimating fatigue reliability.

### 1.2.4 Bayes’ Theorem and Structural Reliability

Reliability-based method has been widely used by many researchers as well as design codes for evaluating the structural safety, in which the failure probability of structures is defined through the limit state functions. For the purpose of structural life-time management optimization, it is of critical importance to predict the structural reliability in the future based on the prior and present information that are obtained from various sources such as expert opinion, mathematical model, measurement data, existing laboratory and operational data, etc. Due to the large uncertainties that are inevitably associated with all those information, the probabilistic inference schemes are always more suitable for updating the time-dependent structural reliability. One such framework that enables the probabilistic inference in the context of reliability analysis is the Bayes’ theorem. In the past several decades, the Bayes’ theorem has become a widely used method due to its ability to learn and calibrate model by using the new information from the inspection, observations, etc.

#### 1.2.4.1 Bayes’ Theorem

The Bayes’ theorem has been widely used for a wide range of applications in various engineering fields such as structural monitoring, inspection, maintenance, and repair planning. Applications of Bayesian inference in civil engineering include structural reliability [101], structural identification [102], fatigue
crack growth prediction [103], strength degradation of deteriorating concrete bridges [104], structural health monitoring and damage detection [105]. Enright and Frangopol [104] predicted the time-variant system reliability of a highway bridge and then evaluated the bridge condition by incorporating inspection results through Bayesian updating. Later on, Estes and Frangopol [106] applied the visual inspection data from the bridge management system to update the lifetime bridge reliability assessment. Zhang and Mahadevan [107] proposed a Bayesian procedure to quantify the modeling uncertainty using nondestructive inspections. Maes [108] established a stochastic deterioration model based on a discrete empirical Bayesian method that allows for a probabilistic reliability assessment of a reinforced concrete slab subject to long-term chloride corrosion by using the available inspection data. Beck and Katafygiotis [102,109] presented a Bayesian probabilistic system identification framework for structural model updating and the associated uncertainties quantification. Later on, as an extension of their work, Yuen et al. [105] applied a Bayesian time-domain approach for damage detection, location and assessment of a 15-story building using noisy incomplete excitation and response data. The proposed approach allows for the calculation of the probability of damage of different severity levels in each substructure, based on the updated probability density functions (PDFs) from data in the undamaged state and in a possibly damaged state. Recently, Straub and Papaioannou [101] proposed a method, termed as Bayesian updating with structural Reliability methods (BUS), to enable robust and efficient Bayesian updating of mechanical and other computational models. To summarize, through the application of Bayesian framework, the uncertain and incomplete information from either inspection data or engineering judgment can be combined and used with existing model in a rational manner, which enables better predictions for future structural conditions [104].

1.2.4.2 Bayesian Network

In real engineering applications, the reliability analysis in the component level as well as in the system level are required, as complex structures/systems with multiple components or multiple failure mechanisms are often involved. Although efforts have been made to update the system-level reliability using Bayes’ theorem, the earlier Bayesian updating methods for system reliability are usually unable to provide information on component performance [110,111]. The Bayesian network (BN) methodology is a well-
suited framework for system reliability assessment, because it enables the uncertainty propagation and updating through nodes/components in the network that allows the transform of information from the system level to component level.

**Figure 1.2** Example of BN

A BN, also called a Bayesian belief network (BBN), is a probabilistic model based on directed acyclic graph (DAG) that represents a joint probability distribution among a set of variables. As shown in Fig. 1.2, BNs are graphically mathematical models, where each node denotes a stochastic variable or a deterministic parameter of interest, and the links denote informational or causal dependencies among the nodes. Pearl [112] proposed the BNs in 1988 and the BNs have been originally developed and applied most in the field of artificial intelligence as an efficient and robust framework for reasoning with uncertain knowledge. BNs have many appealing features such as semantic clarity, the ease of acquisition and incorporation of prior knowledge, the possibility of causal interpretation of learned models and the automatic handling of noisy and missing data [113,114]. Later on, due to the possibility of system reliability updating with newly provided evidence, the BNs have become a popular modeling framework for system reliability analysis in the field of mechanical and civil structures in early 1990s. Almond [115] applied the GRAPHICAL-BELIEF tool, one of the early attempts of using BNs as framework for reliability analysis, to calculate the reliability of a low pressure coolant injection system for a nuclear reactor. Mahadevan et al. [110] proposed a BN model that incorporates multiple failure sequences and correlations between component failures for structure system reliability assessment and validated it with traditional reliability analysis. Langseth and Portinale [116] reviewed the fundamental properties in using BN as a framework for engineering reliability applications and pointed out the present and future research trend. Meanwhile, Uusitalo [117] also
summarized the advantages and limitations of the BNs application in the field of environmental modelling and management. Extensive introduction to BN can also be found in the textbooks by Pearl [112] and Nielsen and Jensen [118]. Recently, BNs in the context of reliability analysis have also found applications in bridge [119] and marine platform [120], and the results show that BNs have significant advantages over the traditional reliability frameworks. Ma et al. [121] also proposed a BN-based framework for predicting the remaining strength of the entire bridge, by using measurements for individual components, including stiffness, corrosion damage, and load-deflection response. Franchin et al. [122] proposed a BN model to predict the seismic fragility curves of reinforced concrete girder bridges, which enables the performance assessment, upgrade/retrofit interventions and diagnosis of bridge damages in a seismic event. Other examples that are relevant to the proposed research is by Straub and his colleague. Straub [103] developed a dynamic Bayesian network (DBN) based reliability analysis framework which enables the efficient and robust reliability updating for a realistic deterioration due to fatigue crack growth. Later on, Luque and Straub [123] extended the approach to allow the system reliability updating considering the fatigue crack.

It should be mentioned that despite the popularity of using BN/DBN for system reliability analysis of civil structures, there are still some limitations for real engineering practice. For example, acquiring all conditional probabilities $P(X_i|pa(X_i))$ to get the conditional probability table (CPT) is sometimes not feasible even with domain experts’ experience. Also, it is difficult to build Bayesian network for a large complex structure/system and the inference of BN/DBN can become computationally complex even if such a complex BN/DBN (probably with a large number variables) is acquired. In addition, for the research in the field of civil engineering as well as many other fields, the data and parameters are often defined in a continuous space. However, for most applications of BN/DBN models, all the variables are usually discrete due to technicalities of the calculation scheme, which makes their applicability subject to great limitations in reliability analysis.

In recognition of the limitations of BN/DBN as discussed previously, the Bayesian network inference is briefly discussed herein. The BN/DBN enables the calculation of posterior distribution of a set of random variables when new observations are available. This task is also known as Bayesian inference. In general,
the Bayesian inference algorithms are divided into two categories, (a) exact inference algorithms (e.g., [124]), and (b) approximate inference algorithms which includes deterministic methods (e.g., [125,126]) and probabilistic-based sampling methods (e.g., [127–129]). Interested readers can refer to [118,130] for more details for these inference algorithms. The basic operations that are inherent in the exact inference are restriction, combination and elimination, regardless of which algorithm is implemented during the inference process [113]. One of the most popular algorithms for exact inference is proposed by Lauritzen and Spiegelhalter [124], which consists of two steps, i.e., the moralization of network structure and the triangulation of the resultant moral graph. Nevertheless, the exact inference is sometimes not feasible when dealing with hybrid BN/DBN containing both discrete and continuous variables, and approximate inference is often proposed for inference with hybrid BN/DBN. The most common approach for approximate inference is discretization, i.e., to replace the random variables that are defined in continuous space with equivalent variables that are defined in discrete space. By applying the discretization, the original continuous domain of a random variable divided into discrete intervals and the probability of each interval is calculated based on the conditional or the marginal PDF of the random variable. The choice of discretization intervals is critical in order to achieve high accuracy in the approximations without losing computational performance too much. Several algorithms have been proposed to obtain the optimal intervals in the context of reliability analysis where rare events are of interest [131–133]. Some commonly used probabilistic-based sampling methods include Markov Chain Monte Carlo (MCMC) [127], Gibbs sampling [128], Metropolis-Hastings sampling [129] and importance sampling [134]. Langseth et al. [113] applied and compared four different inference algorithms in a hybrid BN: discretization, mixtures of truncated exponentials (MTEs), variational methods, and MCMC. Based on the results, they concluded that discretization method (with moderate number of regions) is the fastest technique, while MCMC is comparably much slower than all the other three algorithms in order to obtain results of comparable quality. This is especially true that the computational efficiency of MCMC decreases significantly when the number of observations in the BN increases and/or the probability of failure (i.e., rare event in reliability analysis) of interest decreases.
During the past few decades, a number of BN tools have been developed for computing with BN/DBN such as, Hugin (http://www.hugin.com/), BayesiaLab (http://www.bayesia.com/), Netica (http://www.norsys.com/), BUGS (http://www.mrc-bsu.cam.ac.uk/bugs/), SamIam (http://reasoning.cs.ucla.edu/samiam/), etc. Most of the tools provide BN/DBN inference algorithms such as variable elimination [135,136], junction tree algorithm [118] and Gibbs sampling inference [137], etc.

1.3 Research Objectives

The overall goal of this dissertation is to evaluate the life-cycle dynamic characteristics and fatigue performance of the coastal slender bridges in the context of coupled VBWW dynamic system. The innovations of this dissertation are mainly related to the comprehensive dynamic analysis and probabilistic fatigue reliability evaluations of the complex VBWW system. Five objectives are pursued to achieve the overall goal, as illustrated as follows.

- **Objective 1:** To efficiently simulate correlated wind-wave filed in both normal conditions and extreme events such as hurricanes for coastal slender bridges. The simulated wind and wave fields will enter the VBWW system (objective 2) as the input excitations.

- **Objective 2:** To develop an analytical VBWW simulation platform that systematically incorporates the complicated interactions among bridge, running vehicles, wind and waves. Based upon the established platform, the global structural dynamic characteristics of the bridge under various combined vehicles, wind and wave loading scenarios are investigated.

- **Objective 3:** To evaluate the vehicle driving safety and ride comfort in the context of VBWW. In addition to the bridge structure, the vehicles, as an important component of the VBWW system, may also experience excessive vibrations due to their interactions with bridge and crosswind, which may affect both the comfortability of the drivers and the vehicle running safety.

- **Objective 4:** To evaluate the fatigue reliability of the OSD considering life-cycle stochastic dynamic loads by integrating the multi-scale FEA and the support vector machine (SVM). The multi-scale FEA enables computing the stress time history at the critical welded joints of OSD, while the SVM serves
as a surrogate model to avoid time-consuming large number of FEAs.

- **Objective 5:** To perform fatigue damage diagnosis and prognosis of the OSD through integrating the physics-based model with data from field inspections while accounting for the associated uncertainties, using the dynamic Bayesian network (DBN). Regarding diagnosis, this framework enables tracking the fatigue damage evolution and calibrating time-invariant model parameters. Regarding prognosis, this framework facilitates predicting the fatigue damage state in the future.

### 1.4 Organization of the Dissertation

The dissertation consists of 7 chapters, in which chapter 1 gives an overall introduction of the proposed research, chapters 2 ~ 6 are devoted to the five objectives proposed above, and chapter 7 summarizes this dissertation research and points out the limitations and future work.

**Chapter 1** – Introduction: an overview of the research background and motivation is presented, along with a thorough literature review on the topics related to the dissertation research. In addition, the research objectives and the dissertation layout are presented as well.

**Chapter 2** – Numerical simulation of wind and wave fields for coastal slender bridges: this chapter presents an efficient simulation framework of the nonstationary wind and wave fields around a coastal slender bridge during hurricane events. The wind is modeled as a time-varying mean component plus nonstationary fluctuation components, while the associated wave is modeled as a nonstationary random process. To include the non-stationarity, the nonstationary wind fluctuation is modeled as a uniformly modulated evolutionary vector stochastic process, and the evolutionary power spectral density (EPSD) for the nonstationary wave is obtained by directly extending from current stationary wave spectrum based on the assumption of slow-change of large-scale structure of the hurricane.

**Chapter 3** – Coupled dynamic analysis of vehicle-bridge-wind-wave system: in this chapter, a general analytical platform is developed to evaluate the dynamic performance of the coupled VBWW system. The dynamic system integrates the conventional buffeting analysis for the wind-bridge interaction, the quasi-static analysis for the wind-vehicle interaction, and dynamic interaction between the moving vehicles and
bridge based on the geometric and mechanical relationships between vehicle tires and the bridge deck. Additionally, the interaction between the wave and bridge pile group foundation is included in the system using Morison equation. For demonstration, the dynamic responses of a coastal slender cable-stayed bridge under different vehicle, wind and wave loading scenarios are analyzed.

**Chapter 4** – Evaluation of ride comfort and driving safety for moving vehicles on slender coastal bridges: this chapter proposes a comprehensive evaluation methodology on vehicle ride comfort and driving safety on the slender coastal bridges subject to vehicle, wind, and wave loads. The vehicle ride comfort is evaluated using the advanced procedures as recommended in the ISO 2631-1 standard based on the overall vibration total value (OVTV). The vehicle driving safety is analyzed based on two evaluation criteria, i.e., the roll safety criteria (RSC) and the sideslip safety criteria (SSC), through the vehicle contact force responses at the wheels. The proposed methodology is applied to a long-span cable-stayed bridge for the vehicle ride comfort and driving safety evaluation.

**Chapter 5** – Probabilistic fatigue damage assessment of coastal slender bridges under coupled dynamic loads: this chapter proposes an efficient probabilistic fatigue damage assessment framework for OSD considering life-cycle stochastic dynamic loads by integrating the multi-scale FEA and the support vector machine (SVM). The multi-scale FEA enables computing the stress time history at the critical welded joints, while the SVM serves as a surrogate model to avoid time-consuming large number of FEAs. A prototype cable-stayed bridge in a coastal region is presented to demonstrate the effectiveness of the proposed simulation framework. Finally, the impacts of the traffic growth including the traffic volume and the gross vehicle weight on the fatigue life of three welded joints are investigated and discussed, as well.

**Chapter 6** – Fatigue damage diagnosis and prognosis on orthotropic steel bridge deck subject to cyclic truck loads using dynamic Bayesian networks: this chapter proposes a fatigue damage diagnosis and prognosis framework for the OSD under cyclic truck load through integrating the physics model with field inspections while accounting for the associated uncertainties, using the dynamic Bayesian network (DBN). The proposed framework aims to fulfill two interdependent tasks (1) diagnosis: track the evolution of the fatigue damage and calibrate the time-invariant model parameters; and (2) prognosis: predict the fatigue
damage state in the future. The particle filter (PF) is implemented to perform the Bayesian inference of the established non-Gaussian DBN of arbitrary topology. In addition, a Gaussian process (GP) surrogate model is established to construct the conditional probability distribution (CPD) in the DBN model.

Chapter 7 – Summary of the dissertation and future studies: in this chapter, the accomplishments and innovations of this dissertation are summarized, and the possible improvements of the dissertation and several potential future works are discussed as well.
2 Numerical Simulation of Wind and Wave Fields for Coastal Slender Bridges*

2.1 Background

The coastal communities from the Gulf States to the Carolinas have witnessed destructive damages on coastal infrastructures, including coastal bridges and residential home buildings, etc., from hurricane induced strong winds and high waves in several hurricane events [36,138]. With the rapid increase of the bridge’s span length to cross large bodies of water, the reduction of fundamental frequency of the coastal bridges due to the increasing slenderness has increased the damaging fluid-structure interactions since large portions of the energies in winds and waves are concentrated in the low frequency ranges. Such interactions between the coastal slender bridges and winds and waves could possibly lead to progressive damage accumulations or catastrophic failures in extreme hurricane events or other coastal natural hazards events. Meanwhile, as the transient nonstationary features of strong winds and high waves are often observed during hurricane events [18,19], the interactions of winds, waves and coastal slender bridges could be complicated due to the nonlinearity of the structural system and fluid-structure interactions. To better evaluate structural safety and reliability of coastal slender bridges, it is important to accurately predict the dynamic responses and possible fatigue damage accumulations of the coastal slender bridges when subjected to strong winds and high waves during hurricane events. With many existing studies focusing on the dynamic responses of coastal bridges subjected to either strong winds for long-span bridges [28,88] or solitary waves for short-span low-laid bridges [53,59], few studies have been focused on the dynamic interactions of coastal slender bridges with both wind and wave loads. Serving as the important inputs for the coupled bridge-wind-wave (BWW) dynamic system, realistic simulation of the wind and wave fields has been essential to quantify the system performance for the coupled dynamic system under extreme

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weather conditions.

For weather prediction or hurricane risk analysis, many efforts have been made to simulate hurricane associated wind field and wave field in a larger temporal and spatial scale. The research on hurricane near-surface wind field (i.e. time-varying mean) modeling can be traced back to Chow’s work in 1971 [7] and much improvement has been made since then [8–10]. Based on the planetary boundary layer (PBL), the hurricane wind field model uses a finite-difference scheme to solve for the steady-state wind field based on a set of nested rectangular grids. As an alternative approach, the parametric hurricane wind model is more frequently used for long-term wind and surge risk assessment and structural design due to its simplicity and efficiency [11]. In the parametric hurricane wind model, the near-surface time-varying mean wind is assumed as a vector summation of storm vortex associated with the hurricane itself and the environmental background wind vector related to the storm movement. However, none of the current hurricane wind models include wind turbulent fluctuations, which could introduce considerable dynamic response of structures [12].

Since wind is one of the major driving forces for waves, many methods were proposed, ranging from simple formulae for estimating wave field at a given site by using wind speed, fetch, and duration, to numerical models for wave field simulation covering large sea areas based on the input wind field time histories. Some sophisticated numerical wave prediction models, such as SWAN, WAM, WAVEWATCH models, were developed and have been widely used in various applications of weather prediction and ocean dynamics in the past few decades [13]. However, due to their large spatiotemporal scales, the resolutions for the simulated wind and waves are still low and the wind and wave time histories are still not appreciable for the dynamic analysis of coastal infrastructures [14,15]. For example, the input wind data for the SWAN model [14] for wave simulation in the Black Sea has the spatial resolution of 0.25° in both longitude and latitude with a temporal resolution of 6 h, and the output wave data has the spatial resolution of 1.3 km \( \times 1.83 \) km.

In addition, in the extreme weather conditions, both wind and wave could be nonstationary. The evolutionary power spectral density (EPSD) functions, which could include the energy distribution over
both time and spectral domains [16], were used to characterize such transient features for wind fluctuations and wind driven waves [17,18]. As the frequency structures in normalized EPSDs of wind fluctuations of the downburst and typhoon were found to evolve very slightly with time, the nonstationary wind fluctuations, therefore, were assumed to be uniformly modulated processes [17,19,20]. Similarly, for the nonstationary wave, it is natural to extend the available stationary wave spectra to nonstationary ones, based on the slow-change assumption for the large-scale structure of the hurricane [9]. The power spectral density (PSD) function at each infinitely small interval, therefore, is treated to be stationary following the existing stationary wave spectrum. By combining these time-varying PSDs, a simplified nonstationary wave spectrum can be obtained and could be extended to describe the nonstationary wave after introducing the time-varying mean wind speed. With the prescribed PSD/EPSD, a number of approaches, such as Spectral Representation Method (SRM), linear filter method (e.g., Autoregressive-moving-average (ARMA)), wavelet based method, Proper Orthogonal Decomposition (POD), Empirical Mode Decomposition together with Hilbert Transform (EMD–HT) approach, etc., have been proposed to simulate the stationary/nonstationary Gaussian process [21]. In the present study, SRM is used to generate the sample functions for the nonstationary wind fluctuations and waves due to its simplicity and efficiency [139,140].

In the present study, a numerical scheme is proposed to simulate the nonstationary wind and wave fields around a coastal slender bridge during hurricane events that can be further used in the coupled bridge-wind-wave dynamic analyses. The wind field is simulated by adding the time-varying mean wind speed generated by parametric hurricane wind model and the nonstationary wind fluctuations generated by spectral representation model. The wave field is simulated based on spectral component method through the use of a proposed nonstationary wave spectrum. Both of the nonstationary wind fluctuations and waves are characterized in terms of their evolutionary power spectral density (EPSD) functions. This chapter is organized as follows. First, the wind field is simulated including modeling of deterministic time-varying mean by using parametric hurricane wind model and modeling of wind fluctuation as a uniformly modulated process. For comparison, four different gradient wind profile models under two synthetic storms with different intensities, are used for simulation. The time-varying mean winds are used to estimate the
EPSDs of nonstationary wind and waves. Then, the stationary directional wave spectrum is extended to nonstationary one, based on the slow-change assumption of the large-scale structure of the hurricane. After the coupled bridge-wind-wave dynamic system is briefly introduced, simulation of the hurricane associated nonstationary wind and wave fields around a coastal slender bridge will be carried out. A brief summary and concluding remarks are provided at the end of this chapter.

### 2.2 Wind Field Simulation

Due to the three-dimensional vortex structure of hurricane, the hurricane winds might not impact the bridge in a perfect normal direction to the bridge longitudinal axis [26]. To consider such skew wind condition, the three-dimensional hurricane induced wind velocity field (Fig. 2.1) in Cartesian coordinate system, of a typical slender cable-stayed bridge under mean yaw wind (in horizontal plane), can be defined as follows [26,141],

\[
U(z,t) = \bar{U}(z,t) + u(t) \quad (2.1a)
\]

\[
V_n(z, t) = v(t) \quad (2.1b)
\]

\[
V_w(z, t) = w(t) \quad (2.1c)
\]

As shown in Fig. 2.1, XYZ is the global structural coordinate system used to describe the bridge structural model and the overall dynamic equilibrium conditions. X-axis denotes the longitudinal direction that is along the deck axis, Y-axis denotes the cross-deck direction that is normal to the deck axis and within the horizontal plane, and Z-axis denotes the vertical direction. xyz is the global wind coordinate system used to define wind direction and fluctuating wind components. x-axis denotes the along-wind direction that is along the direction of the mean wind \( \bar{U}(z,t) \), y-axis denotes the lateral wind direction that is in the horizontal plane and normal to the mean wind direction, and z-axis coincides with Z-axis and is defined as the vertical wind direction.
In Eq. (2.1), $\overline{U}(z,t)$ is the time-varying mean at height $z$ and time $t$ and regarded as a deterministic function; and the along-wind fluctuation $u(t)$ along $x$-axis, lateral wind fluctuation $v(t)$ along $y$-axis, and vertical wind fluctuation $w(t)$ along $z$-axis, respectively, are the three components of wind turbulence. The three components of the wind turbulence $u(t)$, $v(t)$, and $w(t)$ can be characterized as nonstationary evolutionary processes. In the present study, the correlations among each turbulence component $u(t)$, $v(t)$, and $w(t)$ are not considered, due to their weak correlations [141,142]. The simulation of time-varying mean and the wind fluctuations will be presented in this section.

2.2.1 **Parametric Hurricane Wind Model**

As discussed earlier, the near-surface (i.e. 10 m height) time-varying mean, can be estimated as the vector summation of a storm-wind component associated with the hurricane itself and a background-wind component related to the storm translation velocity. The storm-wind component is determined by the gradient radial profile and the effect of surface friction, which is represented by the empirical surface wind reduction factor (SWRF) and surface inflow angle ($\alpha$) [11]. Similarly, due to the effect of surface friction, modification is also applied for the background-wind component. However, disagreements still remain regarding both for the reduction factor in magnitude and for the direction of the translation velocity when converting to surface background-wind. In some applications, the storm’s translation wind speed is
incorporated fully into the storm’s wind field [9,143], while in other applications, only partial translation
wind speed is included by implementing a reduction factor [144–146] and/or rotated in a direction [11]. In
the present study, in order to better present the wind field, a reduction factor, $\alpha_{t}=0.55$, and a counter-
clockwise (in the Northern Hemisphere) rotation angle, $\beta=20^\circ$, is adopted to obtain surface background-
wind component [11]. In addition, the storm-wind component is estimated by calculating the gradient wind
with radial profile and converting to surface level with SWRF, $\alpha_{r}=0.85$ [147], and NWS’s expression of inflow angle ($\alpha$) given by Eq. (2.2) [146].

$$\alpha = \begin{cases} 
10^\circ (1 + r/R_m), & 0 \leq r \leq R_m \\
20^\circ + 25^\circ (r/R_m - 1), & R_m \leq r \leq 1.2R_m \\
25^\circ, & r \geq 1.2R_m 
\end{cases} \tag{2.2}$$

where $r$ is radial distance from the storm’s pressure center; and $R_m$ is the radius of maximum wind.

\[ \] Figure 2.2 Schematic of parametric hurricane wind model

The schematic of calculating the near-surface time-varying mean with parametric hurricane wind
model is illustrated in Fig. 2.2. As shown in the figure, the storm track is assumed to be aligned with the
horizontal axis and the coordinate system is fixed to the center of the moving storm. The observation point
$P$ is assumed to be located in the right front quadrant of the storm, an area where strongest wind may occur
for a tropical cyclone in the Northern Hemisphere. Initially at time $t=0$, the coordinate of the observing
point is $(d_0, d_e)$, where $d_e$ represents the offset distance of point $P$ from the storm track, and $d_0$, the initial
horizontal distance of vector \( r \), is assumed as the distance where the storm starts to impact point \( P \). As the storm moves forward along its track with a translation speed of \( V \), the coordinate of observation point can then be expressed by the vector \( r \) given as \( r = (d_0 - Vt, -dl) \). Note that the modification is applied when calculating the storm-wind and background-wind components to account for the effect of surface friction. Finally, the near-surface time-varying mean wind experienced at the point \( P \) as storm passes, could be estimated using Eq. (2.3) given by,

\[
\bar{U}(z_s, t) = \alpha_r V_r(z_s, t) + \alpha_t V_t(z_s, t)
\]

where \( \alpha_r V_r(z_s, t) \) and \( \alpha_t V_t(z_s, t) \) are storm-wind component and background-wind component, respectively, where \( \alpha_r \) and \( \alpha_t \) are the corresponding reduction factor, and \( z_s \) is standard 10 m elevation. It is noteworthy that the resulting wind speed from Eq. (2.3) corresponds to the sustained wind, which is regarded as having an averaging time of 8 to 10 minutes [146]. Since the parametric hurricane model requires 1-s gust wind speed, a gust factor of 1.4 is adopted in the present study to consider the timescale differences [148]. Meanwhile, the storm-wind component \( \alpha_r V_r(z_s, t) \) will be determined by the gradient radial profile (also a function of vector \( |r| \)), which is introduced as follows.

### 2.2.2 Radial Profile of Hurricane Wind

Many radial profiles for hurricane wind were proposed to analytically reconstruct the axisymmetric wind (or tangential wind) field by fitting available observations near hurricane with a few parameters such as symmetrical maximum wind speed at the gradient level \( (V_m) \), the radius of maximum wind \( (R_m) \), central pressure deficit \( (\Delta P) \), etc. Four typical radial profiles are introduced below. Firstly, the Holland [149] (hereafter H80) wind profile has been mostly used for reconstructing hurricane wind field in various applications. The H80 is based on an empirical radial distribution of the storm pressure and the assumption of gradient wind balance. One drawback of H80 is the inability to accurately represent the eyewall and external wind structure simultaneously. Based on H80, the gradient wind velocity, \( V(r) \), at a radius \( r \) is expressed as,
\[ V(r) = \left( \frac{R_m}{r} \right)^{\frac{n}{2}} \left[ \frac{(R_m/r)^n}{\rho} + \frac{r^2 f_c^2}{4} \right]^{1/2} - \frac{f_c r}{2} \] (2.4)

where \( e \) is the base of natural logarithms; \( \rho \) is the air density; \( f_c = 2\Omega \sin \varphi \) is the Coriolis parameter, where \( \Omega = 7.292 \times 10^{-5} \) and \( \varphi \) is the latitude; and \( B \) is the Holland parameter (usually ranges from 0.5 to 2.5), and given by considering the Coriolis effect,

\[
B = \frac{V_e^2 \rho + f V_m R_m \rho}{100 \Delta \rho}
\] (2.5)

Secondly, as the basis for a number of simple descriptions of hurricane wind fields, the two-parameter Rankine combined vortex model [150] has been used in many applications and it is expressed as,

\[
V(r) = \begin{cases} 
V_m (r/R_m), & r < R_m \\
V_m (r/R_m)^{\alpha_0}, & r \geq R_m 
\end{cases}
\] (2.6)

where \( \alpha_0 \) is a scaling parameter that controls the profile shape, and here a pure vortex is adopted in which \( \alpha_0 \) is set as 1.

Thirdly, based on assumption that the hurricane’s structure is determined by different mechanisms in different regions, the Emanuel model [151] (hereafter E04) was developed to obtain a wind profile for the entire storm structure, and the radial wind profile is given by,

\[
V(r)^2 = V_m^2 \left( \frac{r_0 - r}{r_0 - R_m} \right)^2 \left( \frac{r}{R_m} \right)^{2m_0} \left[ \frac{(1 - b_0)(n_0 + m_0)}{n_0 + m_0 (r/R_m)^{2(n_0 + m_0)}} + \frac{b_0 (1 + 2m_0)}{1 + 2m_0 (r/R_m)^{2(n_0 + m_0)}} \right] \] (2.7)

where \( r_0 \) is a large radius where the vortex perturbation drops to zero; and \( b_0, m_0, n_0 \) are scaling parameters such that: \( b_0 = 0.25, m_0 = 1.6, \) and \( n_0 = 0.9 \) [144].

Later, Emanuel and Rotunno [152] (hereafter E11) improved E04 model to produce physically realistic results regarding radial structure of the storm outside its radius of maximum wind, which is given by,

\[
V(r) = \frac{2r (R_m V_m + f_c R_m^2/2)}{R_m^2 + r^2} - \frac{f_c r}{2}
\] (2.8)
Fig. 2.3 shows the comparison of all the four parametric radial profiles, with two different sets of hurricane parameters as input: the most and least intense hurricanes at landfall among the synthetic storms for Tampa, Florida [11]. It is observed that, for both two storms, the storm structure within $R_m$ is similar among all radial profiles while the storm structure varies greatly outside $R_m$. In the outer region of storm’s eye wall for intense storm, wind speed decreases more rapidly in the H80 and Rankine profiles compared with E04 and E11 profiles. For the weak storm, the velocity in the H80, E04 and E11 profiles decay more slowly with the increase of the radius than that in the Rankine profile.

**Figure 2.3** Comparison of radial profile models of tangential hurricane wind, using two different sets of parameters (Table 2.1): (a) an intense storm; and (b) a weak storm

By using the parameters from the literature that are also listed in Table 2.1 [11,144,146], the time histories of the simulated near-surface time-varying mean wind speed and direction using parametric hurricane wind model can be obtained and are shown in Fig. 2.4. It is noteworthy that the wind direction angle of 0° is defined for north (Fig. 2.2), and 90° is for east.
Figure 2.4 Simulated near-surface time-varying mean wind speed and direction under intense storm and weak storm using: (a) H80 radial profile; and (b) E11 radial profile

Table 2.1 Parameters for the simulation of near-surface time-varying mean wind speed

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Parameter set 1 (intense storm)</th>
<th>Parameter set 2 (weak storm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td>latitude (°)</td>
<td>28</td>
<td>28.1</td>
</tr>
<tr>
<td>Δp</td>
<td>storm central pressure deficit (mb)</td>
<td>88.3</td>
<td>30.3</td>
</tr>
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<td>V_m</td>
<td>symmetrical maximum wind speed (m/s)</td>
<td>80.2</td>
<td>39.5</td>
</tr>
<tr>
<td>R_m</td>
<td>radius to maximum winds (km)</td>
<td>20.5</td>
<td>31.6</td>
</tr>
<tr>
<td>r_0</td>
<td>outer radius (km)</td>
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<td>400</td>
</tr>
<tr>
<td>V_t</td>
<td>translation velocity (m/s)</td>
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<td>12</td>
</tr>
<tr>
<td>β</td>
<td>rotation angle (°)</td>
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<td>20</td>
</tr>
<tr>
<td>α_t</td>
<td>reduction factor of translation velocity</td>
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</tr>
<tr>
<td>α_r</td>
<td>surface wind reduction factor (SWRF)</td>
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<td>0.85</td>
</tr>
<tr>
<td>ρ</td>
<td>air density (kg/m³)</td>
<td>1.225</td>
<td>1.225</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Parameter set 1 (intense storm)</td>
<td>Parameter set 2 (weak storm)</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>---------------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>SWRF</td>
<td>surface wind reduction factor</td>
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<td>0.85</td>
</tr>
<tr>
<td>(G_t)</td>
<td>gust wind factor</td>
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<td>1.4</td>
</tr>
<tr>
<td>(d_c)</td>
<td>offset distance (km)</td>
<td>10.25</td>
<td>15.8</td>
</tr>
<tr>
<td>(d_0)</td>
<td>initial horizontal distance between storm center and observation point (km)</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

### 2.2.3 Vertical Mean Wind Profile

Note that the simulated near-surface time-varying mean wind is at 10 m height, and a vertical mean wind profile is needed to describe the mean wind distribution along the height for hurricanes. Different from the downbursts and boundary layer wind, the mean wind profile within the atmospheric boundary layer due to hurricanes can be well represented by “logarithmic law” [153]. The logarithmic law was originally derived for the turbulent boundary layer on a flat plate, but it has been found to be valid in an unmodified form for strong winds in the atmospheric boundary layer near the ground. By observing the vertical mean wind profiles from dropsonde data during hurricanes, Powell et al. [153] and Vickery et al. [9] found that: (1) logarithmic mean wind profiles exist in hurricanes; and (2) in the lower few 100 m (e.g. over a height range of ~20-300 m), the mean wind profile is well described using logarithmic law, which has the following expression,

\[
\frac{\bar{U}(z_1, t)}{\bar{U}(z_2, t)} = \frac{\ln(z_1/z_0)}{\ln(z_2/z_0)} \tag{2.9}
\]

where \(z_0\) is the sea surface roughness; and \(\bar{U}(z_1, t)\) and \(\bar{U}(z_2, t)\) are the time-varying mean wind speeds corresponding to heights, \(z_1\) and \(z_2\). The sea surface roughness \(z_0\) is estimated using Charnock expression [154] given by,

\[
z_0 = \alpha_0 \frac{u_*^2}{g} \tag{2.10}
\]

where \(\alpha_0\) is a constant in the range of 0.015-0.035; \(u_*\) is the friction velocity which depends on the surface shear stress expressed as,

\[
\tau = \rho u_*^2 = \rho C_d \bar{U}_{10}^2 \tag{2.11}
\]
where $C_d$ is the surface drag coefficient, $\rho$ is the density of air, and $\bar{U}_{10}$ is the mean wind speed at a height of 10 m. The study conducted by Vickery et al. [9] shows that, on average, the drag coefficient $C_d$ first increases with the wind speed up to about 24-30 m/s, and then starts to level off or slightly decrease for a higher wind speed. In the present study, the Large and Pond [155] drag coefficient model, modified to be consistent with the observed data by Vickery et al. [9] is used as follows,

$$C_d = \begin{cases} 
(0.5 + 0.064\bar{U}_{10}) \times 10^{-3} & 0 \leq \bar{U}_{10} \leq 25 \\
0.0021 & 25 \leq \bar{U}_{10}
\end{cases}$$

(2.12)

2.2.4 Wind Fluctuation

In the present study, the nonstationary wind fluctuation is treated as a uniformly modulated evolutionary vector stochastic process, and the simulation of along-wind fluctuation $u(t)$ is presented. Consider a vector-valued zero mean evolutionary process $\{u(t)\} = (u(z_1,t), u(z_2,t), ..., u(z_n,t))^T$, which is comprised of the fluctuation processes $u(z_k,t)$ at $n$ heights $z_k$ ($k=1, 2, ..., n$). The $k$th component $u(z_k,t)$ and the corresponding stationary process $\bar{u}(z_k,t)$ have the following spectral representations [16],

$$u(z_k,t) = \int_{-\infty}^{+\infty} A(z_k,t) e^{j\omega t} dZ(z_k,\omega)$$

(2.13)

$$\bar{u}(z_k,t) = \int_{-\infty}^{+\infty} e^{j\omega t} dZ(z_k,\omega)$$

(2.14)

where $A(z_k,t)$ is slowly varying modulation function given by $A(z_k,t) = I \bar{U}(z_k,t) = 0.2\bar{U}(z_k,t)$, where $I$ is constant turbulence intensity; $\omega$ is circular frequency; $dZ(z_k,\omega)$ is zero mean orthogonal-incremental process with following property,

$$E\left\{dZ(z_j,\omega_1) dZ(z_k,\omega_2)\right\} = \delta(j-k) \delta(\omega_1-\omega_2) S(z_j,z_k,\omega_1) d\omega_1 d\omega_2$$

(2.15)

where $\delta(\ldots)$ is Dirac delta function; and $\tilde{S}(z_j,z_k,\omega)$ is auto PSD of $\bar{u}(z_k,t)$ when $j = k$ and cross PSD between $\bar{u}(z_j,t)$ and $\bar{u}(z_k,t)$ when $j \neq k$. 
The EPSD of \( u(z_k,t) \) and the cross EPSD between \( u(z_j,t) \) and \( u(z_k,t) \) are then expressed as,

\[
S(z_k,\omega,t)=|A(z_k,t)|^2\tilde{S}(z_k,\omega) \tag{2.16}
\]

\[
S(z_j,z_k,\omega,t)=A(z_j,t)\overline{A(z_k,t)}\tilde{S}(z_j,z_k,\omega) \tag{2.17}
\]

where the cross PSD \( \tilde{S}(z_j,z_k,\omega) \) is given by \( \tilde{S}(z_j,z_k,\omega) = \text{Coh}(z_j,z_k,\omega)\sqrt{S(z_j,\omega)S(z_k,\omega)} \). The coherence function suggested by Davenport [156] is adopted in the present study, which can be expressed by the following equation,

\[
\text{Coh}(z_j,z_k,\omega) = \exp\left(-\frac{\omega |z_j-z_k|}{2\pi \left(\bar{U}_j(z) + \bar{U}_k(z)\right)/2}\right) \tag{2.18}
\]

where \( \lambda \) is non-dimensional decay constant, ranging from 7 to 10.

As a result, the above presented fluctuation model has the following EPSD matrix,

\[
S(t,\omega)=A(t)S(\omega)\overline{A^T(t)} \tag{2.19}
\]

where \( A(t)=[A(z_1,t),A(z_2,t),...,A(z_n,t)] ; \overline{A^T(t)} \) is the conjugate and transpose of \( A(t) \); and \( S(\omega) \) is the XPSD matrix of the vector process \( \{\bar{u}(t)\} = (\bar{u}(z_1,t),\bar{u}(z_2,t),...,\bar{u}(z_n,t))^T \) which is given by,

\[
S(\omega) = \begin{bmatrix}
S_{11}(\omega) & S_{12}(\omega) & \cdots & S_{1n}(\omega) \\
S_{21}(\omega) & S_{22}(\omega) & \cdots & S_{2n}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
S_{n1}(\omega) & S_{n2}(\omega) & \cdots & S_{nn}(\omega)
\end{bmatrix} \tag{2.20}
\]

where \( S_{kk}(\omega)=\tilde{S}(z_k,\omega) (k=1,2,\ldots,n) \); and \( S_{jk}(\omega)=\text{Coh}(z_j-z_k,\omega)\sqrt{S(z_j,\omega)S(z_k,\omega)} \) (j, k=1, 2, ..., n, j ≠ k). Note that \( \tilde{S}(z_k,\omega) \) (k=1, 2, ..., n) are normalized wind spectrum (e.g. Kaimal spectrum).

To summarize, the wind fluctuations at \( n \) heights along the pylon (or at the same level for bridge deck) are assumed as a uniformly modulated vector-valued evolutionary process \( \{u(t)\} \), characterized by its EPSD matrix \( S(t,\omega) \). The sample functions of \( \{u(t)\} = (u(z_1,t),u(z_2,t),...,u(z_n,t))^T \) are generated by
amplitude-modulating the sample functions of \( \{ \vec{u}(t) \} = (\vec{u}(z_1,t), \vec{u}(z_2,t), ..., \vec{u}(z_n,t))^T \), which is a stationary vector stochastic process completely characterized by its cross-spectral density matrix \( S(\omega) \) in Eq. (2.20). The SRM approach is used to simulate the sample functions of \( \{ \vec{u}(t) \} \). The main formulas of SRM are listed as follows and more details could be found in the literature [140].

By setting \( t = p\Delta t \) and \( \omega_m = (l-1)\Delta \omega + (m/n)\Delta \omega \), where \( m=1,2,...,n \); \( l=1,2,...,N \), the stochastic process \( \vec{u}(z_j,t) \) can be simulated by the following equation as \( N \to \infty \),

\[
\vec{u}(z_j, p\Delta t) = \Re \left\{ \sum_{m=1}^{N} g^{(i)}_{jm}(q\Delta t) \exp\left[i(m\Delta \omega/n)(p\Delta t)\right] \right\}
\]

(2.21)

where \( \Delta \omega = \omega_u / N \); \( j = 1, 2, ..., n \); \( p=0,1,...,nM-1 \); \( q \) is the remainder of \( p/M \); \( M=2N \); \( \omega_u \) is an upper cutoff frequency beyond which the elements of \( S(\omega) \) in Eq. (2.20) is assumed to be zero; and

\[
g^{(i)}_{jm}(q\Delta t) = \sum_{l=0}^{M-1} B_{jm}(l\Delta \omega) \exp[i(l\Delta \omega)(p\Delta t)]
\]

(2.22)

where \( q=0,1,...,M-1 \) and

\[
B_{jm}(l\Delta \omega) = \begin{cases} 2H_{jm}(l\Delta \omega + m\Delta \omega/n) \cdot \sqrt{\Delta \omega} \cdot \exp\left[-i\theta_{jm}(l\Delta \omega + m\Delta \omega/n)\right] \cdot \exp(i\Phi_{jm}) , & 0 \leq l \leq N \\ 0 , & N < l < m \end{cases}
\]

(2.23)

As shown in the equation, \( B_{jm}(l\Delta \omega) \) and \( g^{(i)}_{jm}(q\Delta t) \) \( (l, q=0,1,2,...,M-1) \) are a FFT pair for given \( j \) and \( m \).

### 2.3 Nonstationary Wave

Based on the assumption that the wave field consists of the superposition of a finite number of sinusoidal wave components with individual amplitudes, phases and directions of propagation, the random wave field is simulated based on spectral component method in the present study. Two basic numerical simulation methods are often used, including the double summation and single summation methods [139]. Since the double summation method may result neither ergodic nor spatial homogenous wave field due to artificial phase-locking, i.e. wave components travelling in different directions with identical frequency [157], the single summation method, which has been widely used for wave simulations in numerical
prediction models and lab experiments, is adopted in the present study.

2.3.1 Simplified Model for Nonstationary Wave Field

The directional wave spectrum describes how the wave energy is distributed over the ranges of the frequency $f$ and directional angle $\theta$, which has the following expression [158],

$$S_{\text{wave}}(\omega, \theta) = S_{\text{TMA}}(\omega)G(\omega, \theta)$$  \hspace{1cm} (2.24)

where $S_{\text{wave}}(\omega, \theta)$ is the directional wave spectrum; $G(\omega, \theta)$ is the normalized directional spreading function satisfying $\int_\omega G(\omega, \theta)d\theta = 1$; and $S_{\text{TMA}}(\omega)$ is frequency spectrum [159], which is a finite-depth equivalent of the JONSWAP spectrum $S_J(\omega)$ [160]. The TMA spectrum is given below,

$$S_{\text{TMA}}(\omega) = S_j(\omega)\phi(k_0h)$$

$$= \left\{\alpha_s g^2 \frac{1}{\omega^3} \exp\left[\frac{-5}{4}(\omega_p/\omega)^4\right] \exp\left[-\left(\omega - \omega_p\right)^2/(2\sigma^2)\right]\right\} \phi(k_0h)$$  \hspace{1cm} (2.25)

where $g$ is acceleration of gravity; the coefficient $\alpha_s$ is given by $\alpha_s = 0.076(U(z_s)/Fg)^{0.22}$, where $F$ is the fetch distance and $U(z_s)$ is near-surface mean wind (10 m height); $\sigma$=0.07 when $\omega \leq \omega_p$, else $\sigma$=0.09 when $\omega > \omega_p$; $\omega_p$ is the peak circular frequency given by $\omega_p = 22\left(g^2/U(z_s)F\right)^{1/3}$; $\gamma$ is the peak enhancement factor; and $\phi(k_0h) = \left(\tanh k_0h/\left(1 + 2k_0h/\sinh 2k_0h\right)\right)$ is a factor accounting for the effect of shallow water with finite depth $h$ on the spectrum, where $k_0$ is the wave number determined by $k_0g \tanh k_0h = \omega^2$.

Fig. 2.5 shows the change of TMA spectrum corresponding to different values of water depth, peak enhancement factor, fetch distance, and near-surface mean wind, respectively. As shown in the figure, all of the four parameters could greatly influence the shape of TMA spectrum.

The Mitsuyasu-type spreading function [158,161] is expressed as,

$$G(f, \theta) = G_0 \left|\cos\left(\frac{\pi(\theta - \theta_0)}{2\theta_{\text{max}}}\right)\right|^{2s}$$  \hspace{1cm} (2.26)

where $G_0$ is constant to satisfy the normalization condition expressed as $G_0 = \frac{\sqrt{\pi}}{2\theta_{\text{max}}} \frac{\Gamma(s+1)}{\Gamma(s+\frac{1}{2})}$, where $\Gamma$ is
gamma function; \( \theta_0 \) denotes the principal wave direction, assumed as 0; \( \theta \) is the azimuth from the principal wave direction within the range of \(-\pi/2 \leq \theta \leq \pi/2\); and the spreading parameter \( s \) is a function of frequency,

\[
s = \begin{cases} 
  s_{\text{max}} \left( \frac{f}{f_p} \right)^5, & f \leq f_p \\
  s_{\text{max}} \left( \frac{f}{f_p} \right)^{2.5}, & f > f_p 
\end{cases}
\tag{2.27}
\]

Mitsuyasu et al. [161] obtained the value of \( s_{\text{max}} \) in the range of 5 to 30 with the mean of about 10 for wind waves and expressed \( s_{\text{max}} \) as a function of a dimensionless wind speed. For engineering application, Goda and Suzuki [162] suggested to use a fix value for wind waves and swell: \( s_{\text{max}} = 10, 25, \) and 75 for wind waves, swell with short decay distance, and swell with long decay distance, respectively.

**Figure 2.5** TMA spectrum with: (a) varying water depth \((\bar{U}(Z_s)=25 \text{ m/s}, F=100 \text{ km}, \gamma=3.3)\); (b) varying peak enhancement factor \((\bar{U}(Z_s)=25 \text{ m/s}, F=100 \text{ km}, h=40 \text{ m})\); (c) varying fetch distance \((\bar{U}(Z_s)=25 \text{ m/s}, h=40 \text{ m}, \gamma=3.3)\); and (d) varying mean wind speed \((F=100 \text{ km}, h=40 \text{ m}, \gamma=3.3)\)

Fig. 2.6(a) shows a 3D illustration of directional wave spectrum and Fig. 2.6(b) shows the 2D contour for Fig. 2.6(a) to better depict spectral content in details. As shown in the figure, most of the energy of the
wave spectrum is concentrated within the region centered by the peak frequency ($\omega_p = 0.74 \text{ rad/s}$) and principal propagation direction ($\theta = 0$).

Figure 2.6 Directional spectrum $S_{wave} (\omega, \theta)$ with parameters: $\bar{U}(z_s)=25 \text{ m/s}$, $F=100 \text{ km}$, $\gamma=3.3$, $h=40 \text{ m}$, $s_{max}=10$, $\theta_0=0$, $\theta_{max}=\pi/2$. (a) 3D plot; and (b) 2D contour plot

Since neither any empirical models nor such treatment as wind fluctuation (i.e. uniformly modulated process) is available for nonstationary wave, the nonstationary wave spectra can be extended from the available stationary wave spectra based on the slow-change assumption of the large-scale structure of the hurricane [9]. By introducing the time-varying mean wind speed, the stationary directional wave spectrum could be extended to describe the nonstationary wave and the time-varying wave spectrum is expressed as the product of time-varying JONSWAP spectrum, water depth coefficient and directional spreading function, which is given by,

$$S_{wave} (\omega, t, \theta) = S_{MA} (\omega, t) G(\omega, \theta, t) = S_{J} (\omega, t) \phi(k_{ij}, h,t) G(\omega, \theta, t)$$

(2.28)

2.3.2 Single Summation Method

The random wave field is assumed as the superposition of a finite number of sinusoidal wave components with individual amplitudes, phases and directions of propagation. As discussed earlier, the single summation method is often used among many mathematical representations and given by,

$$\eta(x, y, t) = \sum_{i=1}^{N_f \cdot N_\theta} a_i \cos(k_i (x \cos \theta_i + y \sin \theta_i) - \omega_i t + \xi_i)$$

(2.29)

where $\eta(x, y, t)$ is the surface elevation at a given point $(x, y)$ on still water plane at time $t$; $N_f$ and $N_\theta$ are the
number of frequency and directional spectra components; \( \omega_i = i(\Delta \omega/N_\Theta) \), where \( \Delta \omega \) is the frequency resolution of the directional spectrum denoted as \( \Delta \omega = (\omega_{\text{max}} - \omega_{\text{min}})/N_f \), and \( \omega_{\text{max}} \) and \( \omega_{\text{min}} \) are, respectively, the lower and upper cutoff frequencies; \( \theta_i = \theta_{\text{min}} + i \Delta \theta = \theta_{\text{min}} + (\theta_{\text{max}} - \theta_{\text{min}})/N_\Theta \); \( k_i \) is the wave number satisfying the dispersion relationship with frequency \( \omega_i \) in water depth \( h \) such that \( \omega^2_i = k_i g \tanh(k_i h) \); \( \epsilon_i \) is \( N_f \times N_\Theta \) sequences of independent random phase angles distributed uniformly over the interval \([0, 2\pi]\); and \( a_i \) is wave amplitude of each individual wave component expressed as

\[
a_i = \sqrt{2S_{w_{\text{wave}}}(\omega_i, t, \theta_i) \Delta \omega \Delta \theta}.
\]

### 2.4 Coupled Bridge-Wind-Wave Dynamic System

This section introduces the quantification of wind and wave excitation loads exerted on the bridge based on the simulated nonstationary wind and wave time histories. The governing equations of motion of the coupled bridge-wind-wave (BWW) dynamic system is established as well.

#### 2.4.1 Modeling of Aerodynamic Wind Forces and Wave Forces

Based on the assumption of stationary wind excitations, the wind induced aerodynamic forces acting on bluff bridge sections are commonly separated into three components: steady-state forces resulting from mean wind speed, self-excited forces resulting from the interaction between the wind and the bridge motion and buffeting forces resulting from unsteady wind velocity [163,164]. For nonstationary wind excitations, it is assumed that the steady-state forces and self-excited and buffeting forces can be given in the similar formulations as those in traditional stationary wind, but with a further consideration of the time-varying mean wind speed and evolutionary wind fluctuations, under the assumption of low variation rate of mean wind speed [165]. That is to say, the aerostatic force components and the self-excited and buffeting forces can be formulated using static force coefficients and aerodynamic impulse response functions determined in a wind tunnel under stationary wind but with consideration of nonstationary wind characteristics in a quasi-steady manner [165].

In addition, the bridge could also experience skew wind condition as shown in Fig. 2.4. However, the
bridge dynamic analysis under skew wind condition is still challenging currently. As a result, there is no general accepted analysis schemes for quantifying the aerodynamic wind loads for skew wind even though some progress has been made by some researchers [166–169]. In the present study, the mean wind decomposition approach [166,167] is adopted and the simulated mean yaw wind is decomposed into two components in the horizontal plane: one is normal to the bridge longitudinal axis and the other is along the bridge longitudinal axis. However, the longitudinal component is often neglected [167]. Referring to the coordinate systems shown in Fig. 2.1 and assuming that the mean wind has a yaw angle $\beta_0$, the cross-deck and longitudinal velocity components $U_N$ and $U_P$ (Fig. 2.7) can be expressed as,

\[
U_N = \bar{U}_N(z, t) + u_N(t) = \bar{U}(z, t)\cos(\beta_0) + [u(t)\cos(\beta_0) + v(t)\sin(\beta_0)]
\]

(2.30a)

\[
U_P = \bar{U}_P(z, t) + u_P(t) = \bar{U}(z, t)\sin(\beta_0) + [u(t)\sin(\beta_0) - v(t)\cos(\beta_0)]
\]

(2.30b)

where $\bar{U}_N(t)$, $\bar{U}_P(t)$ are the cross-deck and longitudinal mean wind velocity and $u_N(t)$, $v_P(t)$ are the corresponding wind fluctuations. The transformed wind fluctuation in the vertical direction remains the same as $w(t)$. The transformed mean wind velocity and the corresponding fluctuating wind components will be used to derive the aerodynamic wind forces for the bridge deck and pylon as follows.

**Figure 2.7** Plan view of the mean wind decomposition approach

The time-varying aerostatic forces have three components, namely, the drag force, lift force and twist moment, which are defined in per unit length as,

\[
D_u(t) = 0.5\rho\bar{U}_N^2 B_u C_D; \quad L_u(t) = 0.5\rho\bar{U}_N^2 B_u C_L; \quad M_u(t) = 0.5\rho\bar{U}_N^2 B_u^2 C_M
\]

(2.31)
where \( \bar{U}_N \) is cross-deck mean wind velocity; \( B_d \) is the bridge deck width; \( C_L, C_D, \) and \( C_M \) are mean lift, drag, and pitching moment coefficients determined from section model wind tunnel tests.

The resultant drag, lift, and twist component of the aerostatic force over the entire beam element
\[
\mathbf{\Pi}^e = \left\{ \mathbf{D}^e, \mathbf{L}^e, \mathbf{M}^e \right\}^T
\]
with length \( L_e \) is given as,
\[
D^e(t) = \int_0^{L_e} D_a(t) \, dx
\]
(2.32a)
\[
L^e(t) = \int_0^{L_e} L_a(t) \, dx
\]
(2.32b)
\[
M^e(t) = \int_0^{L_e} M_a(t) \, dx
\]
(2.32c)

The self-excited wind forces also have drag, lift, and twist force components, which can be expressed as convolution integral between the arbitrary bridge deck motion and the associated impulse functions [25],
\[
D_{se}(t) = D_{sep}(t) + D_{sel}(t) + D_{seq}(t)
\]
(2.33a)
\[
= 0.5 \rho \bar{U}_N^2 \int_{-\infty}^{t} \left( f_{Dp}(t-\tau)p(\tau) + f_{Dh}(t-\tau)h(\tau) + f_{D\alpha}(t-\tau)\alpha(\tau) \right) d\tau
\]
\[
L_{se}(t) = L_{sep}(t) + L_{sel}(t) + L_{seq}(t)
\]
(2.33b)
\[
= 0.5 \rho \bar{U}_N^2 \int_{-\infty}^{t} \left( f_{Lp}(t-\tau)p(\tau) + f_{Lh}(t-\tau)h(\tau) + f_{L\alpha}(t-\tau)\alpha(\tau) \right) d\tau
\]
\[
M_{se}(t) = M_{sep}(t) + M_{sel}(t) + M_{seq}(t)
\]
(2.33c)
\[
= 0.5 \rho \bar{U}_N^2 \int_{-\infty}^{t} \left( f_{Mp}(t-\tau)p(\tau) + f_{Mh}(t-\tau)h(\tau) + f_{M\alpha}(t-\tau)\alpha(\tau) \right) d\tau
\]
where \( p(t), h(t), \) and \( \alpha(t) \) are lateral, vertical, and torsional displacement responses of the bridge deck; \( f_{ij} \) \((i=D, L, M; j=p, h, \alpha)\) are the impulse response functions of the self-excited wind force components, where the first subscripts \( D, L, M \) represent the impulse response functions corresponding to the self-excited drag, lift and twist forces, and the second subscripts \( p, h, \alpha \) represent the impulse functions with respect to lateral, vertical and torsional unit impulse displacement, respectively. The impulse response functions can be obtained with the experimentally determined flutter derivatives through the use of rational approximation approach [25].

As an illustration of rational approximation approach, the derivation of the lift force \( L_{se}(t) \) is presented here. The lift component of self-excited force per unit span can also be expressed in terms of flutter
derivatives as,
\[
L_{se}(t) = \rho \ddot{U}_N^2 B_d \left[ K H_1^* \frac{\dot{h}(t)}{U_N} + K H_2^* \frac{B_d \dot{\alpha}(t)}{U_N} + K^2 H_3^* \alpha(t) + K^2 H_4^* \frac{\dot{h}(t)}{B_d} + K H_5^* \frac{p(t)}{U_N} + K^2 H_6^* \frac{p(t)}{B_d} \right] \tag{2.34}
\]
where \( K = B_d \omega / U_N \) is the reduced circular frequency; \( H_i^* \ (i=1\sim6) \) is the flutter derivatives; and the dot on the cap denotes the derivative with respect to the time.

The relationship between the aerodynamic impulse functions and flutter derivatives can be established through taking the Fourier transform of Eq. (2.33b) and comparing to the corresponding terms in Eq. (2.34),
\[
\bar{f}_1 = 2K^2 (H_6^* + iH_5^*) \tag{2.35a}
\]
\[
\bar{f}_L = 2K^2 (H_4^* + iH_1^*) \tag{2.35b}
\]
\[
\bar{f}_L = 2K^2 B_d (H_3^* + iH_2^*) \tag{2.35c}
\]
where the overbar represents the Fourier transform operator and \( i \) denotes imaginary parts.

Note that the flutter derivatives are normally experimentally determined at discrete values of the reduced frequency \( K \), thus the approximate expressions are often applied to express these as continuous functions of \( K \) for future analysis. Take the lift induced by the vertical motion \( L_{seh}(t) \) as an example, and the rational function approximation approach is then used to rewrite the aerodynamic transfer function as \([170]\),
\[
\bar{f}_L = 2K^2 (H_4^* + iH_1^*) = A_1 + A_2 \left( \frac{i\omega B_d}{U_N} \right) + A_3 \left( \frac{i\omega B_d}{U_N} \right)^2 + \sum_{i=4}^{m} A_3 \left( \frac{i\omega B_d}{U_N} \right)^2 \tag{2.36}
\]
where \( A_1, A_2, A_3, A_{l+3}, \) and \( d_l \ (d_l \geq 0; \ l=1 \text{ to } m) \) are frequency dependent coefficients, which are obtained through the linear or nonlinear least-squares methods using the experimentally determined flutter derivatives at different reduced frequencies \([170]\).

In Eq. (2.36), the rational function representation of the aerodynamic transfer functions can be first expressed in Laplace domain by replacing the \( i\omega \) with \( s \), where \( s = (\zeta + i)\omega \) and \( \zeta \) is the damping ration of the motion. After applying the inversed Laplace transform, the aerodynamic response impulse function \( f_{Lh}(t) \) is then given by,
\[ f_{lh}(t) = A_1 \delta(t) + A_2 \frac{B_d}{U_N^2} \delta(t) + A_3 \frac{B_d^2}{U_N^4} \delta(t) + \sum_{i=1}^{m} \int_{-\infty}^{t} A_{1+i} \exp \left( -\frac{d U_N}{B_d} (t - \tau) \right) \delta(\tau) \, d\tau \]  \hspace{1cm} (2.37)

where \( \delta(t) \) is Dirac delta function.

Thus, the self-excited lift induced by the arbitrary vertical motion is rewritten as,

\[ L_{seh}(t) = 0.5 \rho U_N^2 \left( A_1 h(t) + A_2 \frac{B_d}{U_N} \dot{h}(t) + A_3 \frac{B_d^2}{U_N^2} \ddot{h}(t) + \sum_{i=1}^{m} \phi_i(t) \right) \]  \hspace{1cm} (2.38)

where the additional variables \( \phi_i(t) \) are introduced to express the aerodynamic phase lag which satisfy the following equations,

\[ \phi_i(t) = -\frac{d U_N}{B_d} \phi_i(t) + A_{1+i} \dot{h}(t) \] \hspace{1cm} (l=1 to \( m \))  \hspace{1cm} (2.39)

Following the same procedure, the other self-excited force components can be obtained. Finally, the resultant drag, lift, and twist component of the self-excited force over the entire beam element \( \mathbf{F}_{se}^e = \{ D_{se}^e \ L_{se}^e \ M_{se}^e \}^T \) with length \( L_e \) is given as,

\[ D_{se}^e(t) = \int_0^{L_e} D_{se}(t) \, dx \] \hspace{1cm} (2.40a)

\[ L_{se}^e(t) = \int_0^{L_e} L_{se}(t) \, dx \] \hspace{1cm} (2.40b)

\[ M_{se}^e(t) = \int_0^{L_e} M_{se}(t) \, dx \] \hspace{1cm} (2.40c)

Using the similar way of formulating the self-excited forces, the buffeting forces on a unit span can be expressed as the convolution integral between the wind fluctuations and the associated impulse response functions in both cross-deck and vertical directions,

\[ D_{s}(t) = D_{sw}(t) + D_{sv}(t) \]
\[ = 0.5 \rho U_N^2 \int_{-\infty}^{t} \left( f_{Du}(t - \tau) \frac{U_N(\tau)}{U_N} + f_{Du}(t - \tau) \frac{w(\tau)}{U_N} \right) \, d\tau \] \hspace{1cm} (2.41a)

\[ L_{s}(t) = L_{sw}(t) + L_{sv}(t) \]
\[ = 0.5 \rho U_N^2 \int_{-\infty}^{t} \left( f_{Lu}(t - \tau) \frac{U_N(\tau)}{U_N} + f_{Lu}(t - \tau) \frac{w(\tau)}{U_N} \right) \, d\tau \] \hspace{1cm} (2.41b)
\[ M_b(t) = M_{bu}(t) + M_{bw}(t) \]
\[ = 0.5 \rho \frac{U_N}{B_d} \int_{-\infty}^{\infty} \left[ f_{Mu}(t-\tau) \frac{u_N(\tau)}{U_N} + f_{Mw}(t-\tau) \frac{w(t)}{U_N} \right] d\tau \]  

(2.41c)

where \( u_N(t) \) and \( w(t) \) are the cross-deck and vertical turbulent wind velocities; \( f_{ij} \) (\( i=\text{D, L, M}; j=u, w \)) are the impulse response functions, where the second subscripts \( u, w \) represent the impulse functions with respect to cross-deck and vertical turbulent wind velocities, respectively.

Similar to the approach used in the self-excited forces, the buffeting forces can be formulated in the time domain by means of the frequency dependent aerodynamic admittance functions. As an illustration, the derivation of the lift force \( L_b(t) \) is presented here. The lift component of self-excited force per unit span can also be expressed in terms of aerodynamic admittance functions as,

\[ L_b(t) = 0.5 \rho \frac{U_N}{B_d} B_d \left( 2C_L \chi_{Lu} \frac{u(t)}{U_N} + (C'_{L} + C_D) \chi_{Lw} \frac{w(t)}{U_N} \right) \]  

(2.42)

where \( C'_L \) is the first derivative of the lift coefficient with respect to the wind attack angle; and the frequency dependent aerodynamic admittance functions \( \chi_{Lu} \) and \( \chi_{Lw} \) are dependent on deck configuration [170].

Similar to the approach used for self-excited forces, after applying the Fourier transform of Eq. (2.41b) and comparing to Eq. (2.42), the relationship between aerodynamic impulse functions and the aerodynamic admittance functions is given by,

\[ \bar{f}_{Lu} = 2B_d C_L \chi_{Lu} \]  

(2.43a)

\[ \bar{f}_{Lw} = B_d \left( C'_{L} + C_D \right) \chi_{Lw} \]  

(2.43b)

Accordingly, rational functions are then applied to rewrite the aerodynamic impulse function, e.g., the term with regard to the lift induced by the vertical fluctuation \( L_{bw}(t) \) can be expressed as,

\[ \bar{f}_{Lw}(i\omega) = A_{w,1} + \sum_{l=1}^{m} A_{w,l+1} i\omega \frac{A_{w,l+1} i\omega}{i\omega + d_{w,l} \frac{\bar{U}_N}{B_d}} \]  

(2.44)

and the buffeting lift due to the arbitrary vertical wind fluctuation is given by,
\[
L_{w} (t) = 0.5 \rho \bar{U}_{N}^{2} \left( A_{w,1} + A_{w,1}U_{N} \right) \left( \frac{w(t)}{U_{N}} - \sum_{l=1}^{m_{w}} A_{d,w,l} \frac{\bar{U}_{N}}{B_{d}} \phi_{w,l}(t) \right) \tag{2.45}
\]

\[
\dot{\phi}_{w,l}(t) = - \frac{d_{w,l} \bar{U}_{N}}{B_{d}} \phi_{w,l}(t) + A_{w,l} \frac{w(t)}{U_{N}} \quad (l=1 \text{ to } m_{w}) \tag{2.46}
\]

where \( A_{w,1} \), \( A_{w,1} \), and \( d_{w,l} \ (l=1 \text{ to } m_{w}) \) are frequency independent coefficients; and \( \phi_{w,l}(t) \ (l=1 \text{ to } m_{w}) \) are additional variables.

Following the same procedure, the other buffeting force components can be obtained. Finally, the resultant drag, lift, and twist component of the buffeting force over the entire beam element \( \mathbf{F}_{b}^{e} = \{ D_{b}^{e} \ L_{b}^{e} \ M_{b}^{e} \}^{T} \) with length \( L_{e} \) is given as,

\[
D_{b}^{e}(t) = \int_{0}^{L_{e}} D_{b}(t) dx \tag{2.47a}
\]

\[
L_{b}^{e}(t) = \int_{0}^{L_{e}} L_{b}(t) dx \tag{2.47b}
\]

\[
M_{b}^{e}(t) = \int_{0}^{L_{e}} M_{b}(t) dx \tag{2.47c}
\]

For bridge pylon, the self-excited forces are often neglected due to the large stiffness of the concrete pylon. The static forces and buffeting forces can be formulated with the same fashion as the bridge deck (Eq. (2.31) and Eqs. (2.41a – c)), but with consideration of the longitudinal wind.

In addition to the wind-bridge interactions, the wave-bridge interaction is considered by taking into account the interaction between the water flow and bridge foundations (i.e. bridge piles). The wave forces exerted on the bridge piles, with ratio of the diameter to wave length less than 0.15, are estimated by using Morison equation [43] in which the wave forces acting on per unit length of a pile is assumed to consist of the quasi static inertia and drag forces,

\[
F_{w} = \frac{1}{2} \rho_{w} C_{wD} D_{p} (u_{w} - u_{b}) \left[ (u_{w} - u_{b}) \right] + \rho_{w} A \ddot{u}_{w} + (C_{wM} - 1) \rho_{w} A (\ddot{u}_{w} - \ddot{u}_{b}) \tag{2.48}
\]

where \( C_{wD} \) and \( C_{wM} \) are the drag coefficient and inertia coefficient, respectively, which depends in general on the Keulegan Carpenter (KC) number, Reynolds number, and surface roughness; \( \rho_{w} \) is the mass density
of water; \( D_p \) is the pile diameter, \( A \) is the section area of the pile, and \( u_w \) is the water particle velocity and \( u_b \) is the velocity of the bridge pile. The direction of wave force is the same as the wave propagation direction. The wave propagation direction is assumed the same as the mean wind direction, considering the wind is the driving force for wave. The input horizontal velocity and acceleration of water particle for Morison equation are determined by the first and second derivatives of wave surface elevation \( \eta \) (Eq. (2.29)), respectively, as

\[
 u_w(z,t) = \sum_{i=1}^{N_fN_d} a_i \omega_i \frac{\cosh k_i z}{\sinh k_i d} \cos \left( k_i (x \cos \theta_i + y \sin \theta_i) - \omega t + \epsilon_i \right)
\]

(2.49a)

\[
 \ddot{u}_w(z,t) = -\sum_{i=1}^{N_fN_d} a_i \omega_i^2 \frac{\cosh k_i z}{\sinh k_i d} \sin \left( k_i (x \cos \theta_i + y \sin \theta_i) - \omega t + \epsilon_i \right)
\]

(2.49b)

Finally, the resultant wave force over the entire beam element \( F_w \) with length \( L_e \) is given as,

\[
 F_w(t) = \int_0^{L_e} F_w(t) \, dx
\]

(2.50)

### 2.4.2 Equations of Motion for the Bridge-Wind-Wave Dynamic System

The governing equations of motion with respect to the static equilibrium position of a bridge excited by aerodynamic forces and wave forces are given in a matrix form by,

\[
 \begin{bmatrix} M \end{bmatrix} \ddot{\mathbf{d}} + \begin{bmatrix} C \end{bmatrix} \dot{\mathbf{d}} + \begin{bmatrix} K \end{bmatrix} \mathbf{d} = \begin{bmatrix} F \end{bmatrix} = \begin{bmatrix} F_{st} \\ F_{se} \\ F_b \\ F_w \end{bmatrix}
\]

(2.51)

where \([M]\), \([C]\) and \([K]\) are the mass, damping, and stiffness matrices, respectively; \([d]\) is the nodal displacement vector; \([F]\) is the nodal force vector due to the wind and wave excitations; and superscripts “st”, “se”, “b”, “w” represent the aerodynamic static forces, self-excited forces, buffeting forces and wave forces. The solution of the equations of motion can be obtained by the Newmark beta step-by-step integration method. The procedure for the total nodal force vectors \( F_{st} \), \( F_{se} \), \( F_b \) and \( F_w \) is as follows. The aerodynamic wind forces and wave forces acting on each element of the bridge, i.e., \( F_{st} \), \( F_{se} \), \( F_b \) and \( F_w \), are first calculated with the preceding formulations using the simulated nonstationary wind and wave time histories, and then are assembled to form the total nodal force vectors. Since the aerodynamic self-excited forces and the wave forces are motion-dependent which are unknown at the beginning of each time step,
several nonlinear iterations are required in order to reach prescribed convergence at each time step. The influence of the structural nonlinearities (i.e. geometry nonlinearity) can also be readily included in the analysis.

2.5 Numerical Example

Based on the proposed model, hurricane associated wind and wave fields around a typical coastal slender bridge are simulated in the present study for a demonstration purpose. The total length of the bridge deck is 1172 m with a span arrangement of 60+176+700+176+60 m, and the height of the deck is 45 m above the water. Also, the bridge has two H-shaped concrete pylons with a height of 183 m. The simulation points of wind and wave fields associated with hurricane is shown in Fig. 2.8. All the necessary parameters for the simulation have been provided in the earlier discussions. For the wind fluctuations, the cutoff frequency is $\pi$ rad/s, and $N$ is equal to 2048 (Eq. (2.21)), while for wave surface elevation, the cutoff frequency is $\pi$ rad/s, and $N_f$ and $N_\theta$ are set as 1024 and 256, respectively (Eq. (2.29)). The sampling frequencies of both simulated wind and wave fields are 1 Hz.

![Figure 2.8 Simulation points of hurricane induced wind and wave fields (unit: cm)](image)

2.5.1 Fluctuating Wind Velocity Field

As shown in Fig. 2.8, totally 83 (denoted as D1~D83) wind speed simulation points are assigned to the bridge deck, which are evenly distributed with a distance of 14 m, while the wind field on each pylon consists of 11 (denoted as P1~P11) distributed wind points with an even spacing of 16.8 m. Also, the following forms are selected as the target wind spectra for along-wind fluctuation $u(t)$, lateral wind fluctuation $v(t)$, and vertical wind fluctuation $w(t)$, which are given by,
\[ S_u(\omega) = \frac{50}{\pi} u_*^2 \frac{z}{U(z)} \left[ 1 + (50\omega z)/(2\pi U(z)) \right]^{5/3} \] (Kaimal’s spectrum) \hfill (2.52a)

\[ S_v(\omega) = \frac{15}{4\pi} u_*^2 \frac{z}{U(z)} \left[ 1 + (9.5\omega z)/(2\pi U(z)) \right]^{5/3} \] (Kaimal’s spectrum) \hfill (2.52b)

\[ S_w(\omega) = \frac{3.36}{4\pi} u_*^2 \frac{z}{U(z)} \left[ 1 + (10\omega z)/(2\pi U(z)) \right]^{5/3} \] (Lumley-Panofsky’s spectrum) \hfill (2.52c)

where \( u_* \) is the shear velocity of the flow given by \( u_*=K\bar{U}(z)/[\ln(z/z_0)] \) and \( K=0.4 \) is Von Karman’s constant. Without loss of generality, the proposed wind spectra and the Davenport’s coherence function (Eq. (2.18)) are adopted for a demonstration purpose, which can be readily substituted with other more accurate models in the future study once these models have been proposed and validated.

**Figure 2.9** Simulated hurricane along-wind turbulence \( u(t) \) for the pylon at points P1, P2 and P11 for (a) weak storm (H80 model); and (b) intense storm (H80 model)

Fig. 2.9(a) and (b) show the sample functions of along-wind turbulence \( u(t) \) at points P1, P2 and P11 along the pylon, respectively. The time-varying mean wind used for simulation is derived from parametric hurricane wind model with hurricane parameter set 1 and set 2 (Table 2.1) and H80 radial profile. Figs. 2.10(a) – (f) show the correlation functions \( R_{ij,k}(\tau) \) of the simulated along-wind turbulence at points P1, P2 and P11 versus the corresponding targets \( R^0_{ij,k}(\tau) \). It is clearly shown in Fig. 2.10 that the correlation functions of simulated turbulence and the targets agree well with each other.
Figure 2.10 Temporal auto-/cross-correlation functions $R_{j,k}(\tau)$ of simulated along-wind turbulence $u(t)$ at points P1, P2 and P11 versus the corresponding targets $R_{j,k}^0(\tau)$ (weak storm (H80 model)).

Figs. 2.11(a) ~ (c) show the segments of sample functions of along-wind fluctuation $u(t)$, lateral wind fluctuation $v(t)$, and vertical wind fluctuation $w(t)$ at points D1, D2 and D83 along the bridge deck, respectively. It is found that for each turbulence component, there exists a strong correlation between points D1 and D2 due to their close locations, whereas the correlation between points D1 (or D2) and D83 becomes much weaker due to the fact that the two points are more than 1000 m apart. To better demonstrate their correlations, the auto-/cross-correlation functions of the simulated along-wind turbulence $u(t)$ at points D1, D2 and D83 are presented, as well, in Figs. 2.12(a) ~ (d). It can be observed from Fig. 2.12 that the simulated wind turbulences represent well with the required auto-/cross-correlations characteristics.
Figure 2.11 Segments of simulated hurricane along-wind turbulence $u(t)$, lateral wind turbulence $v(t)$, and vertical wind turbulence $w(t)$, for the deck at points D1, D2 and D83 for intense storm (H80 model).

Finally, the combined along-wind velocity is estimated by combining of the time-varying mean and along-wind turbulence. Fig. 2.13 shows the simulated time histories of hurricane induced along-wind velocity for the bridge deck at points D1, D2 and D3.
Figure 2.12 Temporal auto-/cross-correlation functions $R_{j,k}(\tau)$ of simulated along-wind turbulence $u(t)$ at points D1, D2 and D83 versus the corresponding targets $R_{j,k}^0(\tau)$ (intense storm (H80 model))

Figure 2.13 Simulated time histories of hurricane induced along-wind velocity for the bridge deck at points D1, D2 and D83 (the red line is the slowly time-varying mean during intense storm with H80 model)
2.5.2 Wave Field Simulation

As shown in Fig. 2.8, the wave field at mean water level is simulated independently at each bridge pylon foundation since the correlation between the wave fields around each pylon foundation with 700 m apart is assumed to be negligible. The width of the pylon foundation is about 15 m and 3 simulation points (denoted as W1~W3) with an equal spacing of 5 m are selected to show the results. Fig. 2.14 shows the simulated wave surface elevation time histories around left pylon foundation at points W1, W2 and W3.

![Figure 2.14](image-url) Simulated wave surface elevation time histories around left pylon foundation at points W1, W2 and W3 for (a) weak storm; and (b) intense storm. The other parameters are same for both figures such that: H80 radial profile, $F=100$ km, $\gamma=3.3$, $h=40$ m, $s_{\text{max}}=10$

To illustrate the accuracy of single summation method for wave field simulation, the EPSDs of simulated sample functions are estimated and compared with the corresponding target EPSDs. Many time-frequency analysis methods have been proposed to describe the time-dependent characteristics of nonstationary process, including short-time transform (STFT) [171], wavelet transform (WT) [172,173], EMD+HT [174], and wavelet+HT [175,176]. In comparison with the traditional window based scheme with a fixed window size, WT can adjust the window functions to achieve high resolutions at both high and low frequencies, which makes the WT a popular tool for analyzing and processing nonstationary processes [177]. In the present study, the WT method is used to estimate the EPSDs of the sample functions of nonstationary wave, and the details of this method are available in literature [172].
The target and estimated EPSDs for nonstationary wave with principal wave direction (i.e. \( \theta=0 \)) at simulation point W1 are shown in Figs. 2.15(a) and (b). Fig. 2.15(a) reveals that the majority of energy of wave is distributed in the lower frequency region (i.e. 0.05~0.15 Hz). As shown in the figure, the dominant energy is concentrated at the intervals of 7000~9000 s and 11000~13000 s, due to higher wave elevation during those two time segments (Fig. 2.14(a)). The time-frequency of energy distribution of the estimated EPSD in Fig. 2.15(b) matches quite well with the target one, indicating the effectiveness of the proposed simulation method. Similarly, the target and estimated EPSDs of for nonstationary wave at wave direction \( \theta=15^\circ \) are shown in Figs. 2.16(a) and (b), respectively. In both of the target and estimated EPSDs, high energies are mainly concentrated at low frequencies of 0.05~0.15 Hz, especially around time intervals 7000~9000 s and 11000~13000 s. By comparing Fig. 2.15(a) and Fig. 2.16(a), one can also find that the energy at the principal wave propagation direction is much larger (almost twice as large) than that at the propagation direction of \( \theta=15^\circ \), which is consistent with the directional wave spectral content shown in Fig. 2.6.

Figure 2.15 Comparison between the target and estimated EPSDs for simulation point W1 at principal wave direction (\( \theta=0 \)): (a) target EPSD; and (b) estimated EPSD. (The parameters are: weak storm, H80 profile, \( F=100 \) km, \( \gamma=3.3 \), \( h=40 \) m, \( s_{\text{max}}=10 \))
Figure 2.16 Comparison between the target and estimated EPSDs for simulation point W1 at wave
direction equals to $\theta=15^\circ$: (a) target EPSD; and (b) estimated EPSD. (The parameters are: weak storm,
H80 profile, $F=100$ km, $\gamma=3.3$, $h=40$ m, $s_{\text{max}}=10$)

2.6 Summary

During extreme weather scenarios with strong winds and high waves, fatigue damages could possibly
accumulate and lead to catastrophic failures in the bridge’s design life. Due to the nonlinear nature of the
structural dynamic system and the complex fluid-structure interactions, the interactions of wind, wave,
vehicle, and bridge dynamic system could be very complicated. In the present study, an effective simulation
scheme for hurricane induced wind and wave fields around a coastal slender bridge is presented and will
serve as the input for the coupled bridge-wind-wave dynamic system. The wind and wave simulation
procedure involves two consecutive steps: (1) the simulation of the near-surface time-varying mean wind;
and (2) the simulation of nonstationary wind fluctuations and wave field. The time-varying mean wind,
regarded as a deterministic function, is modeled through the use of parametric hurricane wind model, which
assumes the near-surface wind field to be the sum of a storm-wind component determined by storm gradient
wind profile, and a background-wind component related to storm translation velocity. Due to the effect of
surface friction, the modification of the magnitude and the direction of the two wind components are applied.
Hereafter, the modeled time-varying mean wind is used to derive the EPSDs for the simulation of stochastic
wind fluctuation and wave processes. The nonstationary wind fluctuation is modeled as a uniformly
modulated evolutionary vector stochastic process. For nonstationary wave, the EPSD of nonstationary wave is obtained by directly extending from current stationary wave spectrum. With the EPSD, both of the time histories of nonstationary wind fluctuation and wave surface elevation can be generated by SRM. Finally, the proposed scheme is used to generate wind and wave fields for a coastal slender bridge and wavelet transform (WT) method is applied to check the similarity of the time-frequency of energy distribution for the target and estimated EPSDs. Based on the present study, the proposed approach is found to be effective to simulate the hurricane associated wind and wave fields for coastal slender bridges. Meanwhile, the proposed approach could also be applied to any other large scale structures, such as offshore wind farms, coastal power transmission line networks, etc. In addition, the proposed simulation scheme could also be combined with different hurricane tracking models, varied vertical and radial wind profiles, different coherences functions, and PSD functions for wind fluctuations and waves. Serving as the input for the coupled dynamic bridge-wind-wave system and any other large complex infrastructure system, further research will be carried out on the dynamic performance of coastal slender bridges and other coastal infrastructures and/or systems under extreme hurricane wind and waves.
3 Coupled Dynamic Analysis of Vehicle-Bridge-Wind-Wave System*

3.1 Background

With continuous economy development especially in coastal regions that reside more than 50% of the entire population in the Earth, the span length of coastal bridges continue to increase to cross over large water bodies and many long-span bridges have been built worldwide such as Oresund Crossing Denmark-Sweden, Hong Kong-Zhuhai-Macau Bridge, and the planned Qiongzhou Strait Bridge and Çanakkale 1915 Bridge. Compared with bridges with short or medium spans, long-span bridges are much more flexible and more susceptible to experience large amplitudes of vibrations from combined dynamic excitations from traffic, winds and waves. Under the corrosive coastal environments with continuous dynamic impacts especially under extreme scenarios, concerns on the safety and functionality have been raised up for the coastal slender bridges during their lifetime.

During the past two decades, significant efforts have been made by many researchers regarding the coupled bridge-environment dynamic system, such as the bridge dynamic performance and fatigue reliability assessment, running safety of vehicles, and ride comfort of passengers [28,29,88,178–180]. Unlike inland bridges that are mainly subject to traffic and/or wind loads, coastal bridges could sit in deep water and experience strong wave actions. Waves in some extreme weather conditions such as hurricanes were found to be the primary cause of structural failure of coastal bridges [181]. During Hurricane Katrina in 2005, over 44 bridges along the Gulf Coast were reported in damaged conditions due to excessive wave loads [182]. Similar bridge damages were also observed in Escambia Bay, Florida during Hurricane Ivan (2004) [183] as well as in the earlier decades during Hurricane Camille (1969) [184] and Hurricane Frederic (1979) [52]. Typical failure modes for these low laid bridges are the deck damages or extensive scour near

* This chapter is adapted from a paper (with permission from ASCE) that is accepted by the ASCE Journal of Bridge Engineering [223].
foundation [36,138]. Therefore, the wave load effects on coastal low-laid bridges were investigated numerically and experimentally in recent years [53,59,181,185,186]. Recent studies [75,78] showed that the hydrodynamic load may have significant effects on the dynamic behavior of a freestanding bridge tower in coastal environment. However, dynamic characteristics of the high coastal long-span bridges as a system subject to extreme wave actions are still unknown.

Coastal slender bridges usually use a large pile group or caisson as their foundation in the deep water and estimation of wave loads on such foundation could be very complicated. Unlike the single isolated slender piles, where the wave-induced force can be approximated by the Morison equation, an analytical solution is hardly feasible for a pile group given the high complexity of the interaction between the wave and the pile group. The laboratory experiments still represent the most reliable alternative. To simplify the modeling procedure of the wave loading, the wave load of a slender pile within a pile group is usually calculated by multiplying the wave load on an individual pile with an experimentally determined force coefficient. Many research indicated that the force coefficient is related to $KC (KC=\frac{u_{\text{max}} T}{D})$ number and relative spacing $S_{CG}/D$ [40,42,49]; in which $u_{\text{max}}$ is the maximum horizontal wave-induced flow velocity, $T$ is the wave period, $D$ is the pile diameter, and $S_{CG}$ is the gap between the surfaces of two neighboring piles in a pile group. Recently, based on a series of laboratory data, Bonakdar et al. [49] proposed wave load formulae to predict the wave-induced force on a slender pile within a pile group with various configurations, as a function of $KC$ number and the relative spacing. The proposed formulae were found to agree well with the experimental data in the literature [187,188]. These studies provided useful guidelines to determine the wave force of a slender pile within a pile group in a simple way. It is noted that the Morison equation is used under condition $D/L \leq 0.2$ ($L$ is the wave length) such that the effect of the wave diffraction can be ignored. However, for large-diameter cylinder/structure such as caisson foundation with $D/L > 0.2$, the wave diffraction is dominant and the wave diffraction theory is usually used. Great efforts have been made to simulate the wave-large cylinder interactions using analytical solution [189] and experiments [190–192], based on linear and second-order wave theories. Meanwhile, numerical simulations of nonlinear interaction between three-dimensional wave and a vertical cylinder/cylinder array have also been performed by many
research using finite element method (FEM) [193] or volume-of-fluid (VOF) method [44,194,195].

Recently, several studies have been conducted to investigate the dynamic behavior of bridge tower under wind and/or wave current loads numerically or experimentally [74–78]. Liu et al. [77] carried out experiments to investigate the group effects of the group-pile foundation for the East Sea Bridge under wave current load. Four types of oblique piles were systematically analyzed. Chen et al. [74] performed dynamic analysis of a bridge tower under wind and wave actions in the time domain based on boundary-element method and the FEM, and the results indicated both the wind and wave actions have significant effects on the dynamic responses of the bridge tower. In the model, the aerodynamic load is described by the sum of mean wind and buffeting forces, and the wave loads on the cap and piles are evaluated by the potential flow theory and the Morison equation. Later on, Guo et al. [75] conducted laboratory tests on the dynamic behavior of a freestanding bridge tower model under coupled wind-wave actions using wind tunnel and wave flume. During the tests, when the mean wind speed is low and the wave period is near the structural resonant frequency, coupled wind-wave effects were observed under which the structural displacement responses were suppressed. No significant coupled wind-wave effects were observed when the wave period is away from the structural resonant frequency. Similar experiments were also conducted by Wei et al. [78] with a main focus on the hydrodynamic effects on a free-standing bridge tower that consists of a diamond-type pylon and a large round-ended caisson foundation. No obvious coupled wave-current effects were observed. The base shear due to the wave current was approximately equal to the sum of shear force driven by wave and current separately. The dynamic amplifications from the forward current were not observed.

These studies have provided valuable insights into the aerodynamic and hydrodynamic characteristics of a freestanding bridge tower during the construction stage. During the service stage, the wind and wave effects on the coastal slender bridges could be even more complicated and the bridges are also expected to carry a high volume of traffic with additional wind loads on bridge superstructure (tower and deck) and wave loads on substructure (foundation). As a system, the interactions between the bridge, vehicle, wind and wave could become even more complicated, leading to a potential threat to the structural safety and
reliability during the bridge’s lifetime. Consequently, coastal bridges may deteriorate over time that can increase the complexity for the bridge’s life cycle performance. Due to the complexity, neither the effective experimental studies nor the numerical frameworks have been performed or proposed. In the present study, an integrated numerical simulation framework for the dynamic analysis of the coupled vehicle-bridge-wind-wave (VBWW) system is proposed to understand the complex system interactions. In a nutshell, each part of the coupled dynamic system, such as vehicle-bridge, wind-bridge, wave-bridge, etc. could be used separately for different types of bridges under different scenarios.

3.2 Modeling of VBWW System

3.2.1 Stochastic Wind Velocity Field

Natural wind is usually decomposed as a mean wind speed and three wind turbulent components for easy implementation in numerical simulations. As shown in Fig. 3.1, the mean wind $U(z)$ is normal to the bridge deck axis, and the three wind turbulent components $u, v, w$, are normal to the deck axis (i.e., $y$), along the longitudinal bridge axis (i.e., $x$), and along the vertical direction (i.e., $z$), respectively. Correspondingly, the three-dimensional wind field of the bridge can be defined as follows,

$$
\begin{align*}
U_z &= U(z) + u(x, z, t) \\
v &= v(x, z, t) \\
w &= w(x, z, t)
\end{align*}
$$

For engineering applications, the three-dimensional correlated wind field is simplified as several one-dimensional independent wind fields along the bridge tower and deck [141]. Each one-dimensional wind component is treated as a stationary Gaussian stochastic process which can be simulated using spectral representation method [196].

58
3.2.2 Irregular Wave Field

Based on irregular wave theory, the irregular wave field is assumed as the summation of a series of independent regular waves. Given the standard wave energy spectrum of irregular wave trains, the desirable irregular wave field can be simulated through a linear superposition of a finite number of regular waves with varied amplitudes, phases and periods, as follows,

\[ \eta(y, t) = \sum_{i=1}^{N_f} a_i \cos(k_i y - \omega_i t + \varepsilon_i) \]  

(3.2)

where \( \eta(y, t) \) is the surface elevation along the wave propagation direction (i.e., \( y \)) on a still water plane as a function of \( y \) and time \( t \); \( N_f \) is the number of frequencies; \( a_i = \sqrt{2S_\eta(\omega_i) \Delta \omega} \) is the wave amplitude of each individual wave component; \( S_\eta \) is the wave spectrum; \( \omega_i = [i \Delta \omega + (i-1) \Delta \omega]/2 \), where \( \Delta \omega = (\omega_{\text{max}} - \omega_{\text{min}})/N_f \) is the frequency resolution, and \( \omega_{\text{min}} \) and \( \omega_{\text{max}} \) are the lower and upper cutoff frequencies; \( \omega_i \) is the frequency of the \( i \)th wave, which is a random number in the range of \( \omega_{i-1} \) and \( \omega_i \); \( k_i = 2\pi/\lambda_i \) is the wave number satisfying the dispersion relationship with frequency \( \omega_i \) in water depth \( h \) such that \( \omega_i^2 = k_i g \tanh(k_i h) \) and \( \lambda_i \) is the wave length of \( i \)th wave and \( g \) is acceleration of gravity; \( \varepsilon_i \) is \( N_f \) sequences of independent random phase angles distributed uniformly over the interval \([0, 2\pi]\).

Fig. 3.2 shows wave action on a single pile. The horizontal components of wave orbital velocity and acceleration along the pile vertical axis are represented in a similar manner as the wave surface elevation.
given by,

\[ u_x(y, z, t) = \sum_{i=1}^{N_i} a_i \tilde{\omega}_i \frac{\cosh k_i (h + z)}{\sinh k_i h} \cos \left( k_i y - \tilde{\omega}_i t + \varepsilon_i \right) \]  \hspace{1cm} (3.3)

\[ \dot{u}_x(y, z, t) = \sum_{i=1}^{N_i} a_i \tilde{\omega}_i^2 \frac{\cosh k_i (h + z)}{\sinh k_i h} \sin \left( k_i y - \tilde{\omega}_i t + \varepsilon_i \right) \]  \hspace{1cm} (3.4)

where \( u_x(y, z, t) \) and \( \dot{u}_x(y, z, t) \) are the components of water particle velocity and acceleration in the wave propagation direction at time \( t \); \((h+z)\) is the vertical distance of any point along the pile height from the sea bed; and \( y \) is the horizontal distance of the point in the wave from a reference point. The wave velocity and acceleration time histories will be used to calculate the wave force on the bridge foundation.

\[ \text{Figure 3.2 Sketch of wave action on a single pile} \]

It is worth noting that the preliminary study indicated that the wind and wave correlation at the bridge site is not significant during normal weather conditions. Considering the main focus of the present study is to investigate the influence of various combinations of wind, wave and traffic on the bridge performance during normal operational conditions, the wind and wave correlation is therefore not considered.

3.2.3 Modeling of Bridge

In the present study, the cable-stayed bridge, which mainly consists of bridge deck, bridge tower, stayed cables, and pile foundations, is modeled as a three-dimensional finite element model in ANSYS. The detailed modeling process is introduced later on.

3.2.4 Modeling of the Vehicles

The vehicles are idealized as a combination of several rigid bodies connected with several axle mass...
blocks, springs, and dampers. As an example, Fig. 3.3 shows a 17-DOF (degrees of freedom) high-sided road vehicle model, which consists of nine rigid bodies: one for the vehicle body, four for the front and rear axle sets, and four for the tires. The displacement vector \( \{d_v\} \) for the 17-DOF vehicle is expressed as,

\[
\{d_v\} = \{Z_v, Y_v, \theta_v, \phi_v, Z_{s1}, Y_{s1}, Z_{s2}, Y_{s2}, Z_{s3}, Y_{s3}, Z_{s4}, Y_{s4}, Y_{c1}, Y_{c2}, Y_{c3}, Y_{c4}\}
\]

(3.5)

where the first five degrees of freedom are assigned for the vehicle rigid body with respect to its gravity center. Each rigid body in either the front or rear axle set is assigned two degrees of freedom, i.e., the vertical displacement \( Z_{si} \) and the lateral displacement \( Y_{si} \). Point contact is assumed for the vehicle and the bridge and sideslip is allowed. Each rigid body for one tire is assigned one degree of freedom \( Y_{ci} \) in the \( y \)-direction. As shown in Fig. 3.3, the spring stiffness coefficient \( K \) and damping coefficient \( C \) are labeled with their axle number. The subscripts “\( u \)” and “\( l \)” represent the upper and lower position of the springs or dampers, “\( y \)” and “\( z \)” represent the lateral and vertical direction of the movement, and “1”~“4” represent the axle number. Based on Lagrange method, the equation of motions for the vehicle can be obtained in local vehicle coordinate system [28].

**Figure 3.3** Numerical model of a high-sided road vehicle: (a) elevation view; (b) side view

### 3.3 Dynamic Interactions within the VBWW System

#### 3.3.1 Bridge Aerodynamics

The wind induced forces acting on bridge decks include three components: steady-state forces resulting from the mean wind speed, self-excited forces resulting from the wind-bridge interactions, and buffeting forces resulting from the wind turbulence. The three wind force components are usually called
lift, drag and torsional moment. Therefore, the wind forces on bridge deck, \( F_{bw} \), can be obtained,

\[
\begin{bmatrix}
L_w^u(x,t) \\
D_w^u(x,t) \\
M_b^u(x,t)
\end{bmatrix} =
\begin{bmatrix}
L_{st} + L_{se}(x,t) + L_b(x,t) \\
D_{st} + D_{se}(x,t) + D_b(x,t) \\
M_{st} + M_{se}(x,t) + M_b(x,t)
\end{bmatrix}
\]

(3.6)

where the subscripts \( st, se \), and \( b \) refer to the static, self-exited, and buffeting wind force components; and \( L, D, M \) refer to the lift, drag and torsional moment, respectively.

The static wind forces on the bridge deck per unit length are expressed as,

\[
L_{st} = 0.5\rho U^2 B \cdot C_L; \quad D_{st} = 0.5\rho U^2 B \cdot C_D; \quad M_{st} = 0.5\rho U^2 B^2 \cdot C_M
\]

(3.7)

where \( \rho \) is the air density; \( B \) is the bridge deck width; and \( C_L, C_D \), and \( C_M \) are the mean lift, drag, and pitching moment coefficients.

The self-excited wind forces can be expressed as convolution integrals between the arbitrary bridge deck motion and the associated impulse functions [25]. For brevity, only the equation for the lift component of the self-excited force is given below,

\[
L_{se}(t) = L_{sep}(t) + L_{seh}(t) + L_{sea}(t)
\]

\[
= 0.5\rho U^2 \int_{-\infty}^{t} \left( f_{Lp}(t-\tau) p(\tau) + f_{Lh}(t-\tau) h(\tau) + f_{Lw}(t-\tau) \alpha(\tau) \right) d\tau
\]

(3.8)

where \( f \) are the response impulse functions evaluated by experimentally determined flutter derivatives using the rational approximation approach; and the subscripts “p”, “h” and “a” are the lateral, vertical, and torsional displacements of the bridge deck.

Similarly, the lift buffeting forces per unit length can be expressed as,

\[
L_b(t) = L_{bas}(t) + L_{bwa}(t)
\]

\[
= 0.5\rho U^2 \int_{-\infty}^{t} \left( f_{Lb}(t-\tau) \frac{u(\tau)}{U} + f_{Lw}(t-\tau) \frac{w(\tau)}{U} \right) d\tau
\]

(3.9)

Different from bridge decks, bridge towers for cable-stayed bridges are usually much stiffer. With a typical bluff shape, the self-excited wind forces on the tower are usually ignored. Therefore, in the present study, only the static and buffeting wind forces are considered and they are modeled in a similar way as those for the bridge decks using Eq. (3.7) and Eq. (3.9). The drag and lift coefficients of the three typical
cross sections of the bridge tower are obtained using CFD (computational fluid dynamics) simulations [197]. Detailed modeling of wind forces on the bridge deck and tower can be found in the reference [198].

### 3.3.2 Wind Loads on Vehicles

The wind loading vector $\mathbf{F}_{vw}$ on the running vehicles is evaluated using a quasi-static approach proposed by Baker [199],

$$
\begin{align*}
F_S &= 0.5 \rho U_r^2(t) C_S(\psi) A_0; & F_L &= 0.5 \rho U_r^2(t) C_L(\psi) A_0; & F_D &= 0.5 \rho U_r^2(t) C_D(\psi) A_0; \\
M_p &= 0.5 \rho U_r^2(t) C_p(\psi) A_0 h_v; & M_L &= 0.5 \rho U_r^2(t) C_L(\psi) A_0 h_v; & M_R &= 0.5 \rho U_r^2(t) C_R(\psi) A_0 h_v,
\end{align*}
$$

(3.10)

where $F_S$, $F_L$, $F_D$, $M_p$, $M_L$, $M_R$ are the side force, lift force, drag force, pitching moment, yawing moment, and rolling moment acting on the vehicle, respectively, with their sign conventions shown in Fig. 3.4(a); $C_S(\psi)$, $C_L(\psi)$, $C_D(\psi)$, $C_p(\psi)$, $C_y(\psi)$ and $C_R(\psi)$ are the corresponding wind aerodynamic coefficients; $A_0$ is the frontal area of the vehicle; $h_v$ is the distance from the vehicle gravity center to the road surface; $U_r$ is the relative wind velocity to the vehicle which is obtained, as shown in Fig. 3.4(b), from the vector subtraction of vehicle driving speed, $V$, from the wind speed, $U$; and $\psi$ is the yaw angle defined as the angle between the direction of the incoming wind and the vehicle driving direction.

![Figure 3.4](image)

**Figure 3.4** (a) Sign convention of wind force components on vehicle; (b) relative wind speed

In the present study, the wind aerodynamic coefficients for both the vehicles and the bridge are experimentally determined considering their aerodynamic interference [200,201], in order to better predict their dynamic responses.

### 3.3.3 Interactions between Vehicle and Bridge

#### 3.3.3.1 Road Surface Roughness

The road surface roughness profile is assumed as a stationary Gaussian random process, and it is
simulated through an inverse Fourier transformation [202].

### 3.3.3.2 Geometric Compatibility and Force Equilibrium between Vehicle and Bridge

In the vehicle-bridge system, the motion of a running vehicle is restricted by the bridge deck vibration, namely, the bridge provides the boundary condition for the vehicle. As a result, both the geometric compatibility and the force equilibrium conditions at the tire-deck contact point (hereafter referred to contact point for brevity) should be satisfied, as shown in Fig. 3.5. For an illustration purpose, the 2-axel vehicle shown in Fig. 3.2 is used in the vehicle-bridge interaction analysis.

![Diagram showing geometric relationship and interaction forces between the tire and bridge deck](image)

**Figure 3.5** (a) Geometric relationship; (b) interaction forces between the tire and bridge deck

Based on the point-contact assumption and constant contact between the tire and the deck, the equivalent road roughness $Z_{ci}$ at $i$th contact point considering the bridge deformation can be expressed as [28],

$$Z_{ci} = r_{ci}(x) + w_b + e_i \theta_{sb} \quad (i=1\sim4)$$  \hspace{1cm} (3.11)

where $r_{ci}(x)$ is the road surface roughness under $i$th contact point; $w_b$ and $\theta_{sb}$ are the vertical and torsional displacements of the deck center; and $e_i$ is the horizontal distance from $i$th contact point to the deck center.

In the finite element model, the contact points are not necessarily coincident with elements nodes as the vehicle travels along the bridge. The shape functions (also referred as Hermitian cubic polynomials) are usually employed as interpolation functions to transfer the nodal displacements to the contact points as follows [68],

$$\mathbf{u}'(x,t) = \mathbf{N}(x)\delta^v$$  \hspace{1cm} (3.12)

where $\mathbf{u}' = \{v_b, w_b, \theta_{sb}\}$ is the displacement vector of the contact point; and $\mathbf{N}(x)$ and $\delta^v$ are the shape function
and the nodal displacement vector for the bridge deck element in contact. If the vehicle travels at a constant speed $V$ on the bridge, the velocity vector at the contact point can be derived as,

$$\mathbf{u}'(x,t) = \frac{\partial \mathbf{u}'}{\partial t} + \frac{\partial \mathbf{u}'}{\partial x} \frac{dx}{dt} = \mathbf{N}(x)\dot{x} + \frac{\partial \mathbf{N}(x)}{\partial x} \delta_x - V$$  \hfill (3.13)

Now it is clear from Eqs. (3.11) ~ (3.13) that the equivalent roughness are the source of excitations of vehicle system and based on the equilibrium condition of the tire accounting for the gravity of the vehicle, the vertical contact force at $i$th contact point is expressed as [26],

$$f_{bci} = - f_{vbi} = C_{t\ell}(\dot{Z}_{si} - \dot{Z}_{ei}) + K_{t\ell}(Z_{si} - Z_{ei}) + F_{Gi} \quad (i=1\sim4)$$  \hfill (3.14)

where the vertical contact forces $f_{bci}$ and $f_{vbi}$ are the action and reaction forces at the $i$th contact point applied on the vehicle and the bridge, respectively; $Z_{si}$ is the vertical displacement of the $i$th tire; $F_{Gi}$ is the force on the $i$th tire due to the gravity of the vehicle.

Meanwhile, the lateral contact force, i.e., the tire sideslip force, at the $i$th contact point can also be obtained [199],

$$f_{bcl} = - f_{vbi} = -m \left( \frac{\dot{\lambda}_{sci}}{V} + \dot{\delta} \right) f_{bci} \quad (i=1, 3)$$  \hfill (3.15)

$$f_{bci} = - f_{vbi} = -m \frac{\dot{\lambda}_{sci}}{V} f_{bci} \quad (i=2, 4)$$  \hfill (3.16)

$$\dot{\lambda}_{sci} = \dot{Y}_{ci} - \dot{Y}_{bci} \cos(\phi_i) \quad (i=1\sim4)$$  \hfill (3.17)

where $m=0.7$ or 0.5 is the sideslip friction coefficient of tire for a dry or wet road surface condition; $\dot{\lambda}_{sci}$ is the relative lateral speed between the tire and bridge deck at the contact point and $\dot{Y}_{bci}$ is the bridge lateral speed at the contact point; and $\delta$ is the steering angle, i.e., the angle of the front tires to the vehicle axis. The introduction of the steering angle of the front tires is to include driver behavior for course correction, which will be discussed in the followed section.

Putting the vertical and lateral components together, the force vectors applied on vehicle and bridge, i.e., $\mathbf{F}_{bv}$ and $\mathbf{F}_{vb}$, due to vehicle-bridge interactions can be written as follows,
\[ \mathbf{F}_{bv} = \{ f_{bv1}, f_{bv2} \}^T \quad (i = 1 \sim 4) \quad (3.18) \]

\[ \mathbf{F}_{sb} = \{ f_{sb1}, f_{sb2}, f_{sbr} \}^T \quad (i = 1 \sim 4) \quad (3.19) \]

where \( f_{sbr} \) is the resultant moment from \( f_{sb1} \) and \( f_{sb2} \) given by \( f_{sbr} = e_i \cdot f_{sb1} + h_i \cdot f_{sbr} \). After \( \mathbf{F}_{sb} \) is obtained, the equivalent nodal force can be obtained using the shape function of the bridge deck element that is in contact with the vehicle.

### 3.3.3.3 Driver Behavior Model

As shown in Fig. 3.6, when a vehicle is running on a bridge subject to crosswind, the vehicle trajectory actually fluctuates around the longitudinal axis of bridge (i.e., \( X \) axis) instead of a straight line, due to the lateral and rotational vibrations. To prevent the vehicle from blown laterally and rotationally across the bridge, the drivers usually adjust their driving behavior accordingly, which requires a driver behavior model.

In many related studies in the field of automobile engineering, the driver’s steering angle is usually adopted to simulate the driver’s adjustment of driving when there are constant lateral impacts from gusting winds. In the present study, the steering angle model proposed by Baker [203] is adopted to consider the driver behavior,

\[ \delta = -\lambda_1 \Delta_y - \lambda_2 \dot{\Delta}_y \quad (3.20) \]

where \( \lambda_1 \) and \( \lambda_2 \) are the parameters related to the driver behavior; and \( \Delta_y \) and \( \dot{\Delta}_y \) are the relative lateral displacement and velocity between the vehicle center and the bridge given by,

\[ \dot{\Delta}_y (t) = \dot{Y}_v (t) \cos(\phi_y (t)) + V \sin(\phi_y (t)) - \dot{Y}_b (t) \quad (3.21) \]

\[ \Delta_y (t) = \int_0^t (\dot{Y}_v (\tau) \cos(\phi_y (\tau)) + V \sin(\phi_y (\tau))) \, d\tau - Y_b (t) \quad (3.22) \]

where \( Y_b (t) \) and \( \dot{Y}_b (t) \) are the bridge lateral displacement and lateral velocity at the vehicle center.
3.3.4 Wave-Bridge Interaction

Due to the high elevation of coastal slender bridges, water level usually could not reach the level of bridge decks, which are different from low-laid coastal bridges. In the present study, the wave-bridge interaction only includes the interactions between the wave and the group-pile foundations, which can be evaluated using the Morison equations [74,78].

Referring to Fig. 3.2, the inline wave force per unit length at the depth $z$ for a single slender pile can be evaluated by the Morison equations as the summation of velocity-dependent drag force and acceleration-dependent inertia force,

$$F_{wave} = \frac{1}{2} \rho_w C_{wD} D (u_w - u_b) (u_w - u_b) + \rho_w A \ddot{u}_w + (C_{wM} - 1) \rho_w A (\dot{u}_w - \dot{u}_b)$$

(3.23)

where $C_{wD}$ and $C_{wM}$ are the drag and inertia coefficients, which depend in general on the $KC$ number, Reynolds number, and surface roughness; in the present study, the Morison coefficients are taken as $C_{wD} = 1.2$ and $C_{wM} = 1.5$, according to AASHTO [83]; $\rho_w$ is the water density; $D$ and $A$ are the diameter and section area of the pile; $u_w$ and $\dot{u}_w$ are the water particle velocity and acceleration obtained using Eq. (3.3) and Eq. (3.4); and $u_b$ and $\dot{u}_b$ are the velocity and acceleration of the pile.

As discussed earlier, the group effect of multiple piles needs to be considered for the wave-induced force, and an experimentally determined force coefficient is usually adopted to consider the pile group effects. Mindao et al. [40] introduced the interference and shelter coefficients to consider the group pile effect for pile groups with side by side and tandem arrangement, as shown in Fig. 3.7. Later on, Bonakdar et al. [49] and Bonakdar and Oumeraci [42] systematically studied the wave-pile group interactions based
on extensive large-scale laboratory tests, and the wave load formulae for wave-induced force on a slender pile within a pile group were developed as a function of \( KC \) number and relative spacing, \( S_G/D \). For all of the observed pile group arrangements, no significant shelter effect were observed, i.e., \( K_z = 0.9 \sim 1.1 \), when the relative spacing \( S_G/D \) is in the range of \( 0.75 \sim 2 \) regardless of the \( KC \) number. In addition, the interference coefficient \( K_z \) (large than 1) was also found to be dependent on both of \( KC \) number and the \( S_G/D \), with a larger amplification of the wave load on the middle pile than the side pile due to the influence of two neighboring piles from both sides. The results agreed well with other experiments in the literature [187,188]. Therefore, in the present study, the wave force coefficients proposed by Bonakdar et al. [49] and Bonakdar and Oumeraci [42] are adopted to calculate the wave force for the slender pile within pile group with the arrangement shown in Fig. 3.10,

\[
K_z = 1 \\
K_g = \begin{cases} 
1.265 - 0.225 \ln \left( \frac{S_G}{D} \right) & \text{(middle pile)} \\
1.0 & \text{(side pile)}
\end{cases}
\]  

(3.24) (3.25)

where \( K_z \) and \( K_g \) are shelter and interference coefficients defined as the force ratio, \( F_{\text{Group}}/F_{\text{Single}} \), and \( F_{\text{Group}} \) is the wave force on a slender pile within a pile group and \( F_{\text{Single}} \) is the wave force on a single isolated pile.

![Figure 3.7 The tandem and side by side pile group arrangements](image)

### 3.4 Governing Equations of Motion for the VBWW System

With all the pair interactions between the vehicle, bridge, wind and wave included in the VBWW
The governing equations of motion can be obtained as follows,

\[
M_i \ddot{\mathbf{d}}_i + C_i \dot{\mathbf{d}}_i + K_i \mathbf{d}_i = F_{ivG}^i + F_{iwv}^i + F_{ibv}^i 
\]

(3.26a)

\[
M_i \ddot{\mathbf{d}}_b + C_i \dot{\mathbf{d}}_b + K_i \mathbf{d}_b = \sum_{i=1}^{n} F_{ib}^i + F_{ibw}^i + F_{ibwave}^i 
\]

(3.26b)

where \( i (i=1,2,\ldots,n) \) represents the \( i \)th vehicle on the bridge; the subscripts \( b \) and \( v \) represent the bridge and vehicle; \( M, C, K \) are the mass, damping, and stiffness matrices, respectively; \( \mathbf{d}, \dot{\mathbf{d}}, \ddot{\mathbf{d}} \) are the nodal displacement, velocity and acceleration vectors, respectively; \( F_{ib}^i \) and \( F_{ibv}^i \) are the interaction forces between \( i \)th vehicle and bridge in the vehicle-bridge system given by Eqs. (3.18) and (3.19); \( F_{ivG}^i \) is the vehicle’s weight; \( F_{iw}^i \) represents the wind forces on \( i \)th vehicle given by Eq. (3.10); \( F_{ibw} \) represents the wind forces on the bridge given by Eq. (3.6); \( F_{ibwave} \) represents the wave forces on the pile foundation obtained using Eq. (3.23).

Note that the force vectors \( F_{ib}^i, F_{ibv}^i, F_{ibw} \) and \( F_{ibwave} \) are motion-dependent which are unknown at the beginning of each time step. In addition, a large number of DOFs are involved in the governing equations of motion for the VBWW system. Therefore, it has great advantages to solve Eq. (3.26) independently using separation iterative method, and then to achieve the solution through equilibrium iterations based on the coupling relationships of the two systems [28,178]. The iteration between the vehicle and bridge systems is performed until the balance of vertical and lateral contact forces are satisfied. The simulation procedures for the VBWW system as discussed earlier are established in MATLAB and the general flow chart is shown in Fig. 3.8. The entire simulation process consists of three steps: firstly, both of the numerical models for the bridge and the vehicle are defined and the initial coefficient matrices are obtained; secondly, the time-dependent dynamic excitation sources, i.e., road surface roughness, turbulent wind speed, and wave surface elevation, are simulated and used as the input for the VBWW system; thirdly, the dynamic analysis for the vehicle and bridge is performed independently and iterations are carried out at each time step to meet the convergence criteria between the two coupled systems. The Newmark-\( \beta \) method is adopted to solve the coupled equations. A common choice of time step is to make sure that the time step is one order of magnitude less than the smallest significant natural period. For long-span bridges, the vibration energy usually concentrates primarily on the first few modes. According to the preliminary sensitivity analyses, a
time step of 0.01 s is selected for integration in the simulation to provide accurate response results for both the bridge and vehicles with affordable computational costs.

Figure 3.8 The simulation procedure of VBWW dynamic system
3.5 Numerical Example

3.5.1 Prototype Bridge and Vehicle Model

As shown in Fig. 3.9(a), the prototype coastal slender cable-stayed bridge in the present study has a main span, two inner side spans, and two outer side spans, with a span arrangement of 60 + 176 + 700 + 176 + 60 m. The steel box girder is adopted for the main span and inner side spans that are close to the bridge towers, and concrete box girder is adopted for outer side spans. The bridge girder has a cross-section with 40 m in width and 3.5 m in height, which supports a two-way traffic. A total of six lanes with three lanes in each driving direction is shown in Fig. 3.9(b). The bridge deck is 45 m above the still water level (SWL) and the two H-shaped concrete bridge towers are 186 m in height. The bridge superstructure is mainly supported by two group-pile foundations. Each tower foundation has 30 piles with a radius of 3.0 m and a length of 32 m in the water, as shown in Fig. 3.10. A concrete mass pile cap with 73.0 m in the transverse direction, 24.50 in the longitudinal direction, and 6.5 m of thickness is used to connect all the piles. The elevation of the bottom of the pile cap is +1.3 m, which is 8 m above SWL.

![Figure 3.9](image-url)

**Figure 3.9** The prototype coastal slender cable-stayed bridge (unit: m): (a) elevation view; (b) typical deck cross section of bridge with vehicle lane layout
A three-dimensional dynamic finite element model is established for the cable-stayed bridge, in which the bridge deck is simplified as a three-dimensional spine beam and the stay cables are modeled as cable elements. The bridge tower, pier and foundation are also idealized as three dimensional beam elements. The bearings implemented at all the six supports of the bridge deck are modeled by swing rigid links and horizontal rigid links so as to allow free longitudinal motion of the bridge deck. A total of 176 cables are used in the bridge and each cable is regarded as one bar element with the modified modulus of elasticity accounting for the cable sag effect. The initial stain is also applied on each cable element according to the designed pre-tension force. The piers and pile foundations are fixed at the bottom. The soil surrounding the pile foundations is simplified as spring elements. The Rayleigh damping is adopted to construct the bridge damping matrix as a function of stiffness and mass matrices, and two structural damping ratios associated with two specific vibration modes of the bridge. The first ten natural frequencies and the associated modes of the bridge are listed in Table 3.1.

Figure 3.10 Main design parameters of the tower and the tower foundation (unit: m)
Table 3.1 First ten structural natural frequencies and modes

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Natural frequency (Hz)</th>
<th>Mode shape description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.085</td>
<td>Longitudinal translation (deck)</td>
</tr>
<tr>
<td>2</td>
<td>0.231</td>
<td>1st symmetric lateral (deck)</td>
</tr>
<tr>
<td>3</td>
<td>0.287</td>
<td>1st asymmetric lateral (tower)</td>
</tr>
<tr>
<td>4</td>
<td>0.295</td>
<td>1st symmetric lateral (deck and tower)</td>
</tr>
<tr>
<td>5</td>
<td>0.296</td>
<td>1st symmetric vertical (deck)</td>
</tr>
<tr>
<td>6</td>
<td>0.363</td>
<td>1st asymmetric vertical (deck)</td>
</tr>
<tr>
<td>7</td>
<td>0.458</td>
<td>2nd symmetric vertical (deck)</td>
</tr>
<tr>
<td>8</td>
<td>0.490</td>
<td>2nd asymmetric vertical (deck)</td>
</tr>
<tr>
<td>9</td>
<td>0.534</td>
<td>1st asymmetric lateral (deck)</td>
</tr>
<tr>
<td>10</td>
<td>0.561</td>
<td>3rd symmetric vertical (deck)</td>
</tr>
</tbody>
</table>

According to the hydrologic conditions at the bridge site, the wave height and the corresponding wave period for a 100-yr return period are 5.6 m and 9.6 s, and for a 25-yr return period are 4.7 m and 7.0 s. Based on the hydrologic condition, the wave loads are assumed to apply on the piles only and four wave conditions are adopted for the numerical simulation, as shown in Table 3.2. In addition, the wave field at each bridge foundation is simulated independently, since the two foundations are 700 m apart and the correlations between the wave fields at the two foundations are weak.

Table 3.2 Wave conditions for the bridge

<table>
<thead>
<tr>
<th>Load case</th>
<th>$H_s$ (m)</th>
<th>$T_s$ (s)</th>
<th>$h$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>4.7</td>
<td>7.0</td>
<td>32</td>
</tr>
<tr>
<td>W2</td>
<td>5.6</td>
<td>9.6</td>
<td>32</td>
</tr>
<tr>
<td>W3</td>
<td>6.2</td>
<td>9.9</td>
<td>32</td>
</tr>
<tr>
<td>W4</td>
<td>6.8</td>
<td>9.9</td>
<td>32</td>
</tr>
</tbody>
</table>

With a focus of establishing the VBWW framework, only one type of vehicle, i.e., the light truck, is adopted in the present study for a demonstration purpose and the main parameters are listed in Table 3.3 [28,73]. During the simulation, the entire travel path for the traffic flow includes three parts, which correspond to the bridge segment of 1172 m and two road segments with a length of 100 m at the two ends of the bridge segment. When the vehicles are travelling on the road segments, the external excitations on vehicles are only from the road surface roughness and the wind.
Table 3.3 The main properties and dimensions of the representative vehicle

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Light truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the rigid body 1</td>
<td>kg</td>
<td>6500</td>
</tr>
<tr>
<td>Pitching moment of inertia of rigid body 1</td>
<td>kg m²</td>
<td>9550</td>
</tr>
<tr>
<td>Rolling moment of inertia of rigid body 1</td>
<td>kg m²</td>
<td>3030</td>
</tr>
<tr>
<td>Yawing moment of inertia of rigid body 1</td>
<td>kg m²</td>
<td>100000</td>
</tr>
<tr>
<td>Mass of axle block 1</td>
<td>kg</td>
<td>800</td>
</tr>
<tr>
<td>Mass of axle block 2</td>
<td>kg</td>
<td>800</td>
</tr>
<tr>
<td>Upper vertical spring stiffness</td>
<td>kN/m</td>
<td>250</td>
</tr>
<tr>
<td>Upper vertical spring stiffness</td>
<td>kN/m</td>
<td>250</td>
</tr>
<tr>
<td>Lower vertical spring stiffness</td>
<td>kN/m</td>
<td>175</td>
</tr>
<tr>
<td>Lower vertical spring stiffness</td>
<td>kN/m</td>
<td>175</td>
</tr>
<tr>
<td>Upper lateral spring stiffness</td>
<td>kN/m</td>
<td>187.5</td>
</tr>
<tr>
<td>Upper lateral spring stiffness</td>
<td>kN/m</td>
<td>187.5</td>
</tr>
<tr>
<td>Lower lateral spring stiffness</td>
<td>kN/m</td>
<td>100</td>
</tr>
<tr>
<td>Lower lateral spring stiffness</td>
<td>kN/m</td>
<td>100</td>
</tr>
<tr>
<td>Upper vertical/lateral damping coefficient</td>
<td>kN s/m</td>
<td>2.5</td>
</tr>
<tr>
<td>Upper vertical/lateral damping coefficient</td>
<td>kN s/m</td>
<td>2.5</td>
</tr>
<tr>
<td>Lower vertical/lateral damping coefficient</td>
<td>kN s/m</td>
<td>1</td>
</tr>
<tr>
<td>Lower vertical/lateral damping coefficient</td>
<td>kN s/m</td>
<td>1</td>
</tr>
<tr>
<td>Distance between axle 1 and rigid body 1</td>
<td>m</td>
<td>1.8</td>
</tr>
<tr>
<td>Distance between axle 2 and rigid body 1</td>
<td>m</td>
<td>2</td>
</tr>
<tr>
<td>Distance between axle and rigid body</td>
<td>m</td>
<td>1</td>
</tr>
<tr>
<td>Frontal area $A_0$</td>
<td>m²</td>
<td>6.5</td>
</tr>
<tr>
<td>Reference height $h_v$</td>
<td>m</td>
<td>1.65</td>
</tr>
</tbody>
</table>

3.5.2 Simulation of Excitation Sources

In practice, the incident wind and wave could be in various directions, and the wind/wave direction misalignment may occur as well. Some preliminary studies have been conducted to investigate the influence of wind and wave actions with different incident angles on the bridge responses and the results indicated that the case with both the wind and wave applied in the lateral direction (i.e., $Y$-axis) generates the largest bridge dynamic responses, compared to the cases with other directions. Therefore, both the incident wave and dominant mean wind are assumed to act on bridge in the lateral direction, and the influence of wind and wave directionality on the structural performance will be explored in our future study. As shown in Fig. 3.9(a), for the wind field simulation, a total number of 83 and 11 uniformly distributed wind points are assigned along the bridge deck axis and the height of each bridge tower. The vertical mean wind profile is described by logarithmic law given by,
\[
\frac{U(z_1, t)}{U(z_2, t)} = \frac{\ln(z_1/z_0)}{\ln(z_2/z_0)}
\]

(3.27)

where \(U(z_1, t)\) and \(U(z_2, t)\) are the mean wind speeds at heights \(z_1\) and \(z_2\), respectively; the surface roughness \(z_0\) is estimated by the Charnock expression [154] as \(z_0 = (\alpha_0 g)(KU_{10}\ln(10/z_0))^2\), where \(U_{10}\) is the mean wind speed at 10 m height, \(K=0.4\) is Von Karman constant, and \(\alpha_0\) is an empirical constant in the range of 0.01-0.02.

The horizontal and vertical wind spectra proposed by Panofsky and McCormick [204] and Simiu and Scanlan [142] are adopted for the wind turbulence simulation, which are expressed as,

\[
nS_n(n) = \frac{200f}{(1 + 50f)^{3/3}} \quad (3.28)
\]

\[
nS_n(n) = \frac{6f}{(1 + 4f)^{3/3}} \quad (3.29)
\]

where \(f = nz/U(z)\) is the dimensionless normalized frequency; \(n\) is the frequency; \(u^* = KU(z)/\ln(z/z_0)\) is the shearing wind velocity; \(U(z)\) is the mean wind speed at the height of \(z\); and \(z\) is the height of wind points above the SWL. In addition, the Davenport coherence function with a dimensionless decay factor of 10 is adopted for the wind turbulence simulation [156].

As shown in Fig. 3.9(a), irregular wave fields are also simulated at each pile foundation using the TMA spectrum \(S_{TMA}(\omega)\), which is an equivalent finite-depth of JONSWAP spectrum \(S_J(\omega)\) [160] given by,

\[
S_J(\omega) = S_{TMA}(\omega) = S_{TMA}(\omega)\phi(k, h)
\]

\[
= \left[ \beta J H_s^2 \omega_p^4 \exp \left[ -\frac{5}{4} (\omega/\omega_p)^4 \right] \right] \left[ \exp \left[ -(\omega/\omega_p)^2 (2\gamma) \right] \right] \phi(k, h)
\]

(3.30)

where \(\beta_J = 0.0638(1.094-0.01915\ln\gamma)[0.230+0.0336(\gamma-0.185(1.9+\gamma)^{-1})]\); the wave peak circular frequency \(\omega_p = 2\pi/T_p\) and \(T_p = T_s[1-0.132(\gamma+0.2) - 0.559]\); \(H_s\) and \(T_s\) are the significant wave height and the period; \(\sigma = 0.07\) when \(\omega \leq \omega_p\), else \(\sigma = 0.09\) when \(\omega > \omega_p\); \(\gamma = 3.3\) is the peak enhancement factor; and \(\phi(k, h) = (\tanh^2 k_0 h)/(1 + 2k_0 h/\sinh 2k_0 h)\) is a factor accounting for the effect of the shallow water with a finite depth \(h\) on the spectrum, where \(k_0\) is the wave number.
Figure 3.11 Example of the simulated wind and wave fields: (a) wind turbulence $u(t)$ and $w(t)$ at the bridge deck wind point D1 ($U=20$ m/s); (b) wave surface elevation $\eta(t)$ at the location of one pile, and the corresponding water particle velocity $u_x(t)$ and acceleration $a_x(t)$ along wave propagation direction at the water depth $h=24$ m ($H_s=5.6$ m, $T_s=9.6$ s).

For the simulation of wind and wave fields, the time interval and time duration are 0.01 s and 1024 s, and the upper cutoff frequency and the frequency interval are 2 Hz and 0.002 Hz, respectively. Fig. 3.11(a) shows the time histories of the wind turbulences at deck wind point D1 with a mean wind velocity $U=20$ m/s, and Fig. 3.11(b) shows the time histories of wave surface elevation, water particle velocity, and acceleration along the wave propagative direction for one pile.

The power spectral density (PSD) function $\phi(n)$ used to simulate the road surface roughness is given by [202],

$$
\phi(n) = \phi(n_0)(n/n_0)^2
$$

where $n = 2048$ is the number of points in the inverse Fourier transform; $n_0$ is the discontinuity frequency given as $0.5/\pi$ (cycle/m); and $\phi(n_0)$ is the road roughness coefficient which is chosen depending on the road roughness condition, and $\phi(n_0) = 80\times10^{-6}$, $20\times10^{-6}$, $5\times10^{-6}$ m$^3$/cycle are for the average, good and very good surface, respectively [205]. The road roughness coefficient for a good road surface condition is adopted for road surface roughness simulation, and the transverse difference of the roughness is not included.

3.5.3 Bridge Responses under Turbulent Wind and Wave Loads

To understand the bridge responses, a mean wind velocity of 20 m/s, wave height of 4.7 m and wave...
period of 7.0 s, are adopted first for the VBWW system. The dynamic responses are obtained and compared for three types of loading scenarios: (1) wind only; (2) wave only; (3) combined wind and wave. It is noted that only the dynamic effects from the wind and wave are considered in the present study, as the governing equations are built upon the equilibrium state and the mean displacement due to mean wind is not reflected in the dynamic analyses. Fig. 3.12 shows the time histories of the lateral and vertical dynamic responses at the midpoint of the bridge deck. As shown in Fig. 3.12(a), the lateral wave action seems not to affect the vertical vibration of the bridge deck and the wind turbulence is the major dynamic source of the vertical dynamic response of the bridge. In addition, the vertical response due to the combined wind and wave loads is slightly smaller than that cased by wind load alone, indicating the presence of the random wave load could possibly suppress the vertical response. Differently, the wave-induced lateral response is comparable to that generated by the turbulent wind. However, the lateral response due to the combined wind and wave loads could be considerably smaller than the simple superposition of the lateral response induced by wind load and wave load individually.

![Dynamic displacement histories](image)

**Figure 3.12** Dynamic displacement histories at the midpoint of the bridge deck considering turbulent wind and/or wave loads: (a) vertical displacement; (b) lateral displacement

Similar trends could also be observed for the displacement responses of the tower, which is shown in
Fig. 3.13. As shown in Fig. 3.13(a) and (b), the lateral incident wave only excites the lateral vibration of the tower, and it has a significant contribution to the entire lateral displacement response from the combined wind and wave loads. Both Fig. 3.12 and Fig. 3.13 suggest that there exists coupling effects between the wind and wave. In the present study, smaller bridge dynamic responses were observed for the combined effects than those from a simply superposition of the dynamic responses from the wind and wave load only. However, with different phase angles between the wind and waves, possible larger responses could also be expected, which deserves a separate further study to investigate.

![Figure 3.13](image)

**Figure 3.13** Left tower-top displacement considering turbulent wind and/or wave loads: (a) longitudinal displacement; (b) lateral displacement

For a better illustration of the structural dynamic characteristics, the response spectra of the midpoint of the bridge deck in vertical and lateral directions are shown in Fig. 3.14(a) and (b). As shown in Fig. 3.14(a), for the vertical displacement under wind only, and combined wind and wave, three notable peak values are observed around 0.296 Hz, 0.458 Hz and 0.561 Hz, corresponding to the first, second, and third symmetric vertical modes of the bridge, respectively. When only the lateral incident wave is applied, the amplitude of the vertical response spectrum is much smaller than those from the other two loading scenarios, indicating the weak resonance excited by the wave in the vertical direction. It is shown in Fig. 3.14(b),
however, the lateral wave is able to excite the lateral response at the frequencies around 0.231 Hz and 0.295 Hz, corresponding to the first lateral modes of deck, and deck and tower. Both Fig. 3.14(a) and Fig. 3.14(b) show that the amplitude of the response spectrum due to the combined wind and wave loads is slightly smaller than that in wind only scenario, indicating that the coupling wind-wave effect does not significantly amplify the dynamic response and the wind load contributes to a larger portion of the bridge dynamic response.

**Figure 3.14** Response spectra of the midpoint of bridge deck considering turbulent wind and/or wave loads: (a) vertical displacement; (b) lateral displacement

The wave force on the pile group is obtained through the separate calculation for each pile in the group, and Fig. 3.15 shows the total wave force histories on some piles in the left pile foundation under the wave only scenario. Pile 7 is the middle pile and piles 1, 15, and 30 are the side piles. It is shown in Fig. 3.15 that the amplitude of the total wave force on pile 7 is larger than those on the other 3 piles, due to the interference effects from the side piles. It is also shown in Fig. 3.15 that the phases of wave forces on all four piles are different, indicating the maximum wave force on each pile does not occur simultaneously.
Figure 3.15 Total wave force histories on some piles in the left pile group foundation (refer to Fig. 3.10 for pile # in the left pile group foundation)

3.5.4 Statistical Analysis of Bridge Dynamic Responses

As discussed earlier, the lateral incident wave does not excite the vertical vibration of the bridge. In order to further investigate the lateral structural dynamic response due to the wind and wave loads, a statistical analysis is conducted considering various combinations of wind and wave loads. The preliminary analyses indicated that the bridge dynamic responses from only one sample may not represent the dynamic characteristics reasonably due to large uncertainties from the input stochastic wind, wave and road roughness. In order to obtain reasonable results, multiple simulations are performed to get more samples of bridge dynamic responses under each loading scenario for the statistical analysis in this section. The number of simulation is selected as 100 to ensure the statistical analysis is reasonable with affordable computational costs. For the combined wind and wave loads, it is assumed that the wind speed varies from 2.5 m/s to 50 m/s while the wave condition remains as W1, i.e., $H=4.7m$, $T=7s$. It is noteworthy that such loading scenarios may not represent the real scenarios at the bridge site. Such sensitivity analysis is to help identify the possible dynamic effects on bridges subjected to different wind and wave loads. Fig. 3.16 shows the standard deviation of the lateral displacement response (hereafter referred to $\sigma_{\text{Disp}}$ for brevity) at both the midpoint of the bridge deck and the left tower-top under combinations of various wind speed and wave load W1. For comparison, $\sigma_{\text{Disp}}$ under wave only is also included in the figure. It is shown in Fig. 3.16(a) that $\sigma_{\text{Disp}}$ at the midpoint of the bridge deck in the combined wind and wave scenario with relatively small wind
speed, i.e., under 12.5 m/s, is almost the same with the results in the wave-only scenario, which suggests the negligeable effect of wind load on the structural dynamic response compared with wave load effect. This indicates that the wave load dominates the lateral structural dynamic response as the wind speed is less than 12.5 m/s. As the wind speed increases to approximately 22.5 m/s, the \( \sigma_{\text{Disp}} \) in the combined wind and wave case and the wave only case are comparable, indicating that both the wind and wave loads have influences on the lateral structural dynamic response when the wind speed is in the range of 12.5 m/s~22.5 m/s. Furthermore, as the wind speed continues to increase, i.e., larger than 22.5 m/s, the wind load begins to dominate the lateral structural dynamic response and the \( \sigma_{\text{Disp}} \) under combined wind and wave loads is the same with that under wind load only. Similar trends are also found for the \( \sigma_{\text{Disp}} \) at the left tower-top in Fig. 3.16(b).

Figure 3.16 Comparison of the standard deviation of the lateral displacement responses under combinations of various wind speed and wave load W1: (a) midpoint of the bridge deck; (b) left tower-top

In addition, the lateral structural responses under four different wave loading scenarios (see Table 3.2) are obtained and compared to investigate the effects of various wave height on the bridge’s dynamic responses. Fig. 3.17 shows \( \sigma_{\text{Disp}} \) at both the midpoint of the bridge deck and the left tower-top under W1~W4 wave loading scenarios. The lateral structural displacement response is found to increase with the wave height. As the return period of the wave condition increases from a 25-yr (i.e., W1) to a 100-yr (i.e., W2),
\( \sigma_{\text{Disp}} \) at the midpoint of the bridge deck increases from 0.0146 m to 0.0176 m by 20.5\%, and \( \sigma_{\text{Disp}} \) at the left tower-top increases from 0.0103 m to 0.0117 m by 13.6\%, respectively.

Figure 3.17 Standard deviation of the lateral displacement responses at both the midpoint of the bridge deck and the left tower-top under four wave loading scenarios

Finally, the influence of the wave period on the lateral structural dynamic responses is also investigated. The wave height remains constant here as shown in Fig. 3.18. The constant wave height is adopted as \( H_s = 4.7 \) m while the wave periods of 2s, 3s, 4.3s, 5s, 6s, 7s, 8s, and 9s are adopted. As shown in Fig. 3.18, the peak lateral structural displacement response occurs when the wave period is near the structural resonant frequency. As shown in Fig. 3.18, \( \sigma_{\text{Disp}} \) at the midpoint of the bridge deck increases 76.7\% from 0.0146 m to 0.0258 m, when the wave period is shifted from 7s to 4.3s, which corresponds to the period for the first lateral mode of the deck (see Table 3.1). Similarly, \( \sigma_{\text{Disp}} \) at the left tower-top increases 82.4\% from 0.0102 m to 0.0186 m when the wave period changes from 7s to 3s, which is around the period for the first lateral mode of the deck and tower. As the wave period moves away from the resonant frequency, i.e., \( T_s \geq 6s \), the \( \sigma_{\text{Disp}} \) drops quickly and then tends to remain constant. Therefore, the lateral incident wave with the wave period near the period of the first lateral mode of vibration seems to introduce considerably larger lateral structural responses compared with the cases when the wave period is away from the structural basic period. Similar observations are also found in the experimental study by Guo et al. [75]. Based on Fig. 3.17 and Fig. 3.18, the lateral structural dynamic responses are found to be more sensitive to the wave period than the wave height.
Figure 3.18 Standard deviation of the lateral displacement responses at both the midpoint of the bridge deck and the left tower-top under various wave period while wave height remains constant as $H_s=4.7$ m

3.5.5 Bridge Responses under Service Traffic, Turbulent Wind and Wave Loads

As discussed earlier, normal traffic might still remain on the bridge with existing wind and wave loadings during hurricane evacuations or extreme scenarios. To assess the sensitivity of each dynamic impact sources for the VBWW system, four loading scenarios are considered here, i.e., (1) traffic only; (2) traffic and wave; (3) traffic and wind; and (4) traffic, wind and wave. The moderate wind and wave conditions are adopted, i.e., mean wind $U=20$ m/s, and wave height $H_s=4.7$ m and wave period $T_s=7.0$ s. For a moderate traffic condition, 30 vehicles are considered in the vehicle fleet on each lane. The vehicles travels at a constant speed of 20 m/s (i.e., 44.7 mph), and the distance between two adjacent vehicles is set as 30 m. With zero initial conditions, the entire simulation starts as the vehicle fleet enters one road segment, continues as the vehicle fleet goes across the bridge segment, and ends as the vehicle fleet exits the other road segment. When the vehicle fleet is traveling on the road, the vehicle response under wind only and the bridge response under combined wind and wave are simulated separately, while the coupled VBWW is simulated after the vehicle fleet enters the bridge. The length of each road segment is 800 m to ensure the vehicles and the bridge have enough time to be excited with stable responses, before the vehicle fleet enters the bridge. Only the dynamic responses of the bridge and the vehicles during the period when the vehicle fleet is traveling on the bridge are presented hereafter.
Figure 3.19 Dynamic response histories at the midpoint of the bridge deck under dynamic excitations from wind, wave, and six lanes’ traffic: (a) vertical displacement; (b) lateral displacement

Fig. 3.19 shows the vertical and lateral displacements at the midpoint of the bridge deck, assuming all the six lanes are occupied by the vehicle fleets. It is shown in Fig. 3.19(a) that the vertical displacement under the traffic only scenario is identical to that under the combined traffic and wave scenario, and the vertical displacement due to the combined traffic and wind loads is also the same as that due to the combined traffic, wind and wave loads. This indicates that the presence of lateral incident wave has negligible effects on the bridge vertical vibration. In addition, as the vehicle fleet is far away from the bridge deck midpoint, the vertical vibration is mainly due to the wind turbulence. As the vehicle fleet is approaching close to the bridge deck midpoint, both the vehicle and wind turbulence contribute to the vertical responses. Different from the vertical displacement under wind turbulence only, which has a mean value approximately equal to 0, the vertical displacement under additional traffic load is below zero due to the vehicle self-weight. It is also found that for the vertical response under combined traffic and wind loads, the traffic mainly contributes to the mean trend part while the wind mainly contributes to the fluctuation part. As far as the bridge lateral vibration is concerned, it is shown in Fig. 3.19(b) that the traffic has very limited influence on the lateral response, while the wind turbulence significantly increases the lateral response. The lateral
response under wind turbulence is far larger than those in the cases without wind turbulence. In addition, the mean value of lateral response under both the traffic and wind is not equal to 0, due to the lateral vehicle-bridge interaction forces. Moreover, compared with the case under combined traffic and wind loads, the presence of wave actions does not introduce remarkable increase of the lateral response. For comparisons, Fig. 3.20 also shows the dynamic response at the midpoint of the bridge deck assuming only lane 3 is carrying traffic, and the wind and wave conditions remain the same as the previous case. Similar findings can be found as those shown in Fig. 3.20. Therefore, as demonstrated in Figs. 3.19 and 3.20, the traffic and wind loads contribute a large portion of the bridge vertical and lateral dynamic responses, while the wave load only has slight effects on the bridge lateral dynamic response.

![Figure 3.20 Dynamic response histories at the midpoint of the bridge deck under dynamic excitations from wind, wave, and one lane’s traffic: (a) vertical displacement; (b) lateral displacement](image)

### 3.5.6 Vehicle Responses under Different Combinations of Dynamic Loading

Vehicle dynamics in the coupled VBWW system could also help to evaluate the driver ride comfort and traffic safety. The vehicle dynamic response under moderate wind ($U=20\text{m/s}$), moderate wave ($H_s=4.7\text{m}, T_s=4.3\text{s}$) and six lanes’ traffic is investigated. Fig. 3.21 shows the vertical and lateral accelerations of the vehicle rigid body of the first vehicle under four different loading scenarios, i.e., traffic
only, traffic and wave, traffic and wind, and traffic, wind and wave. As shown in Fig. 3.21(a), the vertical responses in cases with and without wave load are almost identical, indicating the lateral incident wave load has little influence on the vehicle vertical response. It is also shown in Fig. 3.21(a) that the additional wind turbulence can significantly enhance the vehicle vertical vibrations. It is also shown in Fig. 3.21(b) that the vehicle lateral acceleration is dominated by the wind load, and the contribution of the wave load to the overall lateral acceleration is negligible. Therefore, it is concluded from the Fig. 3.21 that in the coupled VBWW dynamic system, the wave load has negligible influence on the vehicle dynamic response, while the wind turbulence contributes significantly to the vehicle dynamic response.

Figure 3.21 Vehicle acceleration responses under various loading scenarios: (a) vertical acceleration; (b) lateral acceleration

In order to show the influence of driver behavior on the vehicle responses, the lateral, yawing responses and the corresponding steering angle of the first vehicle with and without considering the driver behavior are compared, as shown in Fig. 3.22. It is shown in Fig. 3.22(a) that when no driver behavior is considered (i.e., steering angle $\delta=0$), both the lateral and yawing responses under the crosswind increase linearly with the time, indicating that the off-lane or “sway the tail” will occur without the driver interference. In contrast, when the driver’s steering angle is applied on the driving vehicle, both the lateral and yawing angle responses are well suppressed and limited to a much lower value, as shown in Fig. 3.22(b). Therefore, it is
necessary to include the driver behavior in the VBWW analysis.

**Figure 3.22** Vehicle responses under dynamic excitations from wind, wave, and six lanes’ traffic: (a) without steering angle input; (b) with driver steering ($\lambda_1=\lambda_2=0.3$)

### 3.6 Summary

In the present study, a numerical framework of the coupled VBWW system is established to investigate the dynamic characteristics of a coastal slender cable-stayed bridge under different vehicle, wind and wave loading scenarios. The results from the case study considering only the wave load indicate that the lateral incident wave with wave period near the bridge resonant period can excite the bridge lateral vibrations significantly, while almost no dynamic effects from waves are observed for the bridge vertical vibrations. In addition, depending on the wind speed, the bridge lateral responses under the combined wind and wave excitations can be divided into the wave dominant region, the wind-wave dominant region, and the wind dominant region, respectively. Furthermore, based on the results from the fully coupled VBWW analysis, the traffic and wind turbulence are found to contribute mostly to the bridge vertical vibrations, while the wind turbulence controls the bridge lateral vibrations. Finally, the results also indicate that the wind turbulence dominates both the vertical and lateral vibrations of the vehicle, while the effects from the wave actions are negligible.
4 Evaluation of Ride Comfort and Driving Safety for Moving Vehicles on Slender Coastal Bridges*

4.1 Background

In recent decades, an increasing number of slender bridges have been built worldwide with long spans in many coastal areas to cross straits or seas, linking cities or islands. These long-span bridges are usually highly flexible with low structural damping, which in turn makes them more vulnerable to vibrations due to the ambient environmental excitations. Nevertheless, serving as the backbone of the transportation system in coastal areas, these long-span bridges usually could carry a high volume of traffic on a daily basis. During extreme weather related events, such as hurricanes, associated with strong winds, flooding or storm surges, the safety and reliability of the bridges as well as the moving vehicles in evacuations on the bridges are of great concern [88,206]. The complex dynamic coupling effects among the flexible supporting long-span bridge, running vehicles, and wind and/or wave were found to have significant effects not only on the bridge performance, but also on the vehicle ride comfort and driving safety [29,73].

The vehicle ride comfort is directly associated with the vehicle vibrations that are transmitted to occupational drivers in various directions as a result of their contact with the seat, back and footrest. Exposure to excessive whole-body vibration from the vehicles may lead to short-term discomfort and long-term physical damage such as the musculoskeletal pain and back pain, especially for the drivers with long-distance driving or with higher responsibilities, e.g., for public transportation and large cargo trucks [207,208]. Therefore, as one of the important performance of vehicle, the vehicle ride comfort plays a critical role in determining the driving satisfaction as well as the driving safety and long-term health of the drivers. In addition, when vehicles are travelling through long-span bridges, the vehicle accident risks are

* This chapter is adapted from a paper (with permission from ASME) that is accepted by the *ASME Journal of Vibration and Acoustics* [311].
found to increase considerably due to the complex dynamic interactions among the wind, vehicles, and the flexible supporting bridge structures [71]. There have been several reports about the accidents for various types of moving vehicles on bridge due to the strong lateral winds. For example, on August 11, 2004, seven high-sided road vehicles on the Humen suspension bridge in China were blown over due to the strong wind gust right before a strong typhoon landing [209]. Similar accidents were also reported for a semi-truck on the Mackinac Bridge in U.S. on July 18, 2013, at the wind speed over 65 miles per hour during a severe storm [210]. To ensure the safety of the drivers and passengers, it is essential to perform the vehicle ride comfort and safety evaluation.

Active research has been carried out regarding the vehicle ride comfort and driving safety assessment based on vehicle-bridge-wind coupled system in the last decade [29,73,163,211]. The current existing studies on ride comfort in terms of evaluating human exposure to whole-body vibration and repeated shock are primarily based on several existing standards [212–214]. Xu and Guo [215] investigated the ride comfort of heavy vehicles on a long-span cable-stayed bridge under crosswind, based on the root-mean-square (RMS) value with respect to one-third octave-band frequency recommended by ISO 2631/1 [213]. This specification was later updated to a newer version [214], in which the frequency-weighted RMS values are evaluated for ride comfort based on multi-axis whole-body vibrations. Later on, this newer version was adopted by Yin et al. [216] to evaluate the ride comfort of a single truck on a high pier, multi-span continuous bridge based on lateral vehicle responses. Most recently, Zhou and Chen [73] evaluated the ride comfort based on vehicle multi-axis vibration responses by incorporating the stochastic traffic flow and wind effects. In addition to the ride comfort, the vehicle driving safety issue may also arise for highway vehicles under hazardous driving environments, e.g., strong crosswind and/or slippery road surface. Guo and Xu [71] conducted vehicle safety analyses of high-sided road vehicles based on coupled vehicle-bridge-wind interactions, in which the vehicle accidents including the overturning, excessive sideslip, and exaggerated rotation were investigated. Chen and Chen [217] conducted accident risk assessment under comprehensive hazardous driving conditions by combing a local single-vehicle accident model with established accident criteria. Later on, Chen and Chen [218] further improved the efficiency of the proposed
accident risk assessment framework by using the response surface method. Recently, Zhou and Chen [29] incorporated the stochastic traffic flow simulation into the vehicle-bridge-wind system for more advanced traffic safety assessment. Chen et al. [211] further investigated the influence of wind barrier on the vehicle driving safety based on wind tunnel experiments and numerical simulations.

The above research on the vehicle ride comfort and driving safety assessment are all based on the vehicle-bridge-wind system. Nevertheless, for slender coastal bridges, in addition to the wind actions, the wave actions could also contribute to the bridge dynamic responses significantly [75], which may in turn have influences on the vehicle ride comfort and driving safety. However, the investigations of wave effects on the vehicle ride comfort and driving safety in the context of VBWW dynamic interactions are very limited in the literature. In the present study, the vehicle ride comfort and driving safety evaluation are performed for slender coastal bridges considering the dynamic interactions between the bridges and the surrounding environmental loads, such as wind and wave loads. First, an analytical numerical framework of the VBWW system is introduced, which can be used to obtain the dynamic responses of each individual vehicle under various wind and wave conditions. Based on the vehicle dynamic responses, the ride comfort is evaluated based on the frequency-weighted RMS values of the vehicle whole-body vibration responses through the frequency weighting and averaging techniques. Meanwhile, the vehicle driving safety analysis is also evaluated based on two predefined safety criteria, i.e., the roll safety criteria (RSC) and the sideslip safety criteria (SSC). To demonstrate the methodology, a coastal slender bridge is modeled to form the coupled VBWW system. Parametric studies are carried out to investigate the influence of multiple variables on the vehicle ride comfort and driving safety, e.g., the wind and wave load combinations, the vehicle moving speeds, traffic lane locations and vehicle types, etc.
4.2 Coupled VBWW Dynamic System

The coupled VBWW dynamic system consists of bridge and vehicles as well as the external wind and wave loads that are depending upon the vibrational states of bridges or vehicles. Because the complicated dynamic coupling effects among the bridge, vehicles, wind and wave cannot be modeled appropriately using the existing finite-element (FE) software, an analytical framework of the coupled VBWW dynamic system is established using the commercial FE software ANSYS [219] and the computer programming language MATLAB [220]. The general flowchart of the entire simulation process consists of three steps, as summarized in Fig. 4.1. Firstly, the numerical models for both the bridge and vehicles are developed to extract the initial coefficient matrices. Second, the stochastic wind and wave fields are simulated which are
used to calculate the wind and wave forces on the bridge/vehicle. Thirdly, the governing equations of the coupled VBWW system are developed to facilitate the dynamic analysis. Subsequently, the dynamic responses of the running vehicles subject to various wind and wave loading conditions can be predicted, which are used for further ride comfort evaluation and safety analysis. The details of these procedures are elaborated in this section.

4.2.1 Stochastic Wind and Wave Fields

Wind and wave could be correlated as the wind is one of the major driving forces for the surface wave generation. During the past few decades, many wind-wave models, ranging from simple formulas to more sophisticated numerical models (e.g., SWAN, WAM, SLOSH, etc.), have been proposed for various applications. Nevertheless, the sophisticated numerical models are developed and orientated for applications from ocean to coastal scales in general, in which the spatiotemporal resolutions of the output wind and wave fields are not appreciable for the dynamic analysis of coastal infrastructures [13]. As an alternative, some spectrum-based methods, such as spectral representation method (SRM), are still used widely for structural engineering field, which utilizes only a few parameters as input (e.g., wind speed, fetch, etc.) for generating the wind and wave fields with small spatiotemporal scale [198]. In the present study, the correlated wind and wave fields are generated by using the SRM, which is presented in details in this section.

4.2.1.1 Mean Wind

The three-dimensional wind field near a coastal bridge site can be decomposed into a mean wind and three random fluctuating components. The mean wind speed $U(z)$ at elevation $z$ above the still water level (SWL) can be described by the logarithmic law as [26],

$$\frac{U(z)}{U_{10}} = \frac{\ln(z/z_0)}{\ln(10/z_0)}$$

(4.1)

where $U_{10}$ is the mean wind speed at elevation of 10 m above the SWL; the surface roughness $z_0$ is estimated by the Charnock expression [154] as $z_0 = (\alpha_0 g)(KU_{10}/\ln(10/z_0))^2$, where $K=0.4$ is Von Karman constant and $\alpha_0$ is an empirical constant ranging from 0.01 to 0.02. The mean wind speed is used to obtain the wind and
wave spectra, which will be further used to simulate the wind and wave fields based on the SRM [196].

4.2.1.2 Stochastic Wind Field

The three wind turbulence components, i.e., \( u(t) \), \( v(t) \) and \( w(t) \), are usually treated as stationary Gaussian stochastic processes which can be generated using the fast SRM [221]. The time history of the wind component \( u(t) \) at the \( j \)th \((j=1,2,\ldots,n)\) point along the bridge span is simulated as,

\[
 u_j(t) = \sqrt{2\Delta \omega} \sum_{m=1}^{N} \sum_{l=1}^{j} S(\omega_m) G_m(\omega_m) \cos(\omega_m t + \phi_m) \tag{4.2}
\]

where \( \Delta \omega = \omega_u/N \) is the frequency interval; \( \omega_u \) is upper cutoff frequency and \( N \) is a sufficient large number of frequency intervals; \( \phi_m \) is random phase uniformly distributed between 0 and \( 2\pi \); \( \omega_m=(l-1)\Delta \omega + \Delta \omega \cdot m/n \); \( S(\omega) \) is the wind spectrum; and

\[
 G_m(\omega_m) = \begin{cases} 
 0, & \text{when } 1 \leq j < m \leq n \\
 C^{j-m}, & \text{when } m=1, m \leq j \leq n \\
 C^{j-m} \sqrt{1-C^2}, & \text{when } 2 \leq m \leq j \leq n 
\end{cases} \tag{4.3}
\]

in which \( C=\exp(-\lambda \omega A/2\pi U) \), where \( \lambda=10 \) is the exponential decay coefficient; \( A \) is the distance between two adjacent wind points; and \( C^{j-m} \) is the coherence function between wind points \( j \) and \( m \), which was proposed by Davenport [156].

The wind spectrum \( S(\omega) \) for each wind turbulence component can be expressed as [26],

\[
 S_u(\omega) = \frac{50}{\pi} u_*^2 \frac{\omega}{U(z)} \left[ \frac{1}{1 + \left(50 \omega z / (2\pi U(z))\right)^{5/3}} \right] \tag{4.4}
\]

\[
 S_v(\omega) = \frac{15}{4\pi} u_*^2 \frac{\omega}{U(z)} \left[ \frac{1}{1 + \left(9.5 \omega z / (2\pi U(z))\right)^{5/3}} \right] \tag{4.5}
\]

\[
 S_w(\omega) = \frac{3.36}{4\pi} u_*^2 \frac{\omega}{U(z)} \left[ \frac{1}{1 + \left(10 \omega z / (2\pi U(z))\right)^{5/3}} \right] \tag{4.6}
\]

where \( u_* \) is the shear velocity of the flow given by \( u_*=KU(z)/\ln(z/z_0) \).

4.2.1.3 Stochastic Wave Field

Based on the irregular wave theory, the irregular wave field is assumed as the summation of a series
of independent regular waves given by,

\[ \eta(y,t) = \sum_{i=1}^{N_f} a_i \cos(k_i y - \phi_i t + \varepsilon_i) \]  

(4.7)

\[ u_i(y,z,t) = \sum_{i=1}^{N_f} a_i \omega_i \frac{\cosh k_i (h+z)}{\sinh k_i h} \cos(k_i y - \phi_i t + \varepsilon_i) \]  

(4.8)

\[ \hat{u}_i(y,z,t) = \sum_{i=1}^{N_f} a_i \omega_i \frac{\cosh k_i (h+z)}{\sinh k_i h} \sin(k_i y - \phi_i t + \varepsilon_i) \]  

(4.9)

where \( \eta(y,t) \) is the surface elevation along the wave propagation direction (i.e., \( y \)) on a still water plane as a function of \( y \) and time \( t \), as shown in Fig. 4.2; \( a_i = \sqrt{2S_i(\omega_i)\Delta\omega} \) is the wave amplitude of each individual wave component; \( N_f \) is the number of frequencies; \( S_i \) is the wave spectrum; \( \omega_i = [i\Delta\omega + (i-1)\Delta\omega]/2 \), where \( \Delta\omega=(\omega_u-\omega_0)/N_f \) is the frequency resolution, and \( \omega_u \) and \( \omega_0 \) are the upper and lower cutoff frequencies; \( \phi_i \) is a random number between \( \omega_u \) and \( \omega_0 \); \( \varepsilon_i \) is \( N_f \) sequences of random phases distributed uniformly between 0 and 2\( \pi \); \( k_i = 2\pi/\lambda_i \) is the wave number determined from the dispersion relationship \( \omega_i^2 = k_i g \tanh(k_i h) \); \( \lambda_i \) is the wave length of \( i \)th wave; \( g \) denotes the acceleration of gravity; \( u_i(y,z,t) \) and \( \hat{u}_i(y,z,t) \) are the components of water particle velocity and acceleration in the wave propagation direction; \( h+z \) is the vertical distance of any point along the pile height from the sea bed; and \( y \) is the horizontal distance of the point in the wave from a reference point.

**Figure 4.2** Wave action on a single pile

In the present study, the TMA spectrum is adopted for the wave simulation. The TMA spectrum was original proposed by Kitaigorskii et al. [222] for the use of shallow water waves [222] and was later
verified by Bouws et al. [159]. The TMA is given by,

\[
S_n(\omega) = \left\{ \alpha_s g^2 \frac{1}{\omega^2} \exp\left[ -\frac{5}{4} \left( \omega_p/\omega \right)^4 \right] \exp\left[ -\left( \omega-\omega_p \right)^2 / \left( 2\sigma^2 \omega_p \right) \right] \right\} \varphi(k_0 h) \tag{4.10}
\]

where \( \alpha_s = 0.076(U_{10}/F_g)^{0.22} \) is the coefficient; \( \gamma \) is the peak enhancement factor; \( F \) is the fetch distance; \( \sigma = 0.07 \) when \( \omega \leq \omega_p \), else \( \sigma = 0.09 \) when \( \omega > \omega_p \); \( \omega_p = 22(g^2/U_{10}F)^{1/3} \) is the peak frequency; and \( \varphi(k_0 h) = \left( \tanh^2 k_0 h \right) / \left( 1 + 2k_0 h / \sinh 2k_0 h \right) \) is a factor considering the shallow water effect on the spectrum and \( k_0 \) is the wave number.

4.2.2 Modeling of the Bridge and the Vehicles

4.2.2.1 Modeling of the Coastal Slender Cable-Stayed Bridge

In the present study, a prototype coastal slender cable-stayed bridge (see Figs. 4.6 and 4.7) is modeled in 3D using finite element method. Bridge deck, tower, pier and pile foundation are modeled with three-dimensional spatial beam elements based on Timoshenko beam theory. Cables are modeled as link elements with the modified modulus of elasticity accounting for the cable sag effect. The initial stain is also applied on each cable element according to the designed pre-tension force. All the six bearings between the bridge deck and the pier/tower are modeled by swing rigid links and horizontal rigid links to allow free longitudinal motion of the bridge deck. The piers and pile foundations are fixed at the bottom. The soil surrounding the pile foundations is simplified as spring elements. Rayleigh damping is adopted to construct the bridge damping matrix as a function of the stiffness and mass matrices, and two structural damping ratios associated with two specific vibration modes of the bridge.

4.2.2.2 Modeling of Vehicles

In studying the interaction between the vehicles and structures, the vehicle model is usually simplified but maintaining all the relevant essential information [26,28,206]. Following their work, in the present study, the vehicles are modeled as a couple of rigid bodies, suspension systems and tires that are connected by a series of springs and dampers. The vehicle bodies and the tires are modeled as the rigid bodies, whereas the elasticity and dissipation capacities of both the suspension system and the tires are idealized as springs and
viscous dampers, respectively. For example, a typical two-axle four-wheel road vehicle is modeled with 5 rigid bodies and 16 sets of springs and dampers, as shown in Fig. 4.3 [26,28]. As shown in Fig. 4.3, 13 DOFs are assigned for the road vehicle, among which 5 DOFs are assigned for the vehicle body including 2 translation DOFs (i.e., vertical $Z_v$ and lateral $Y_v$) and 3 rotational DOFs (i.e., rolling $\phi_v$, yawing $\psi_v$, and pitching $\theta_v$), and the remaining 8 DOFs are assigned for the four tires (i.e., vertical $Z_s(i=1~4)$ and lateral $Y_s(i=1~4)$).

In addition, the bridge deck and the tires of the vehicles are assumed to be point-contact.

![Figure 4.3](image)

**Figure 4.3** Numerical model of a high-sided vehicle: (a) elevation view; (b) side view

### 4.2.3 Modeling of Wind Loads and Wave Loads

The stochastic wind and wave fields provides the time histories of the wind speed and wave velocity/acceleration, which will be used to simulate the dynamic interactions among the vehicles, bridge, wind and wave. The modeling of wind forces on the bridge and vehicles, and the modeling of the wave forces on the bridge pile-group foundation are elaborated in this section.

#### 4.2.3.1 Modeling of Wind Loads

The wind loading effects on the dynamic system consists of two parts. One part is the wind effects on the bridge and the other part is the wind effects on the vehicles. The wind forces acting on the bridge deck are commonly separated into three components: steady-state forces resulting from the mean wind speed, self-excited forces resulting from the wind-bridge interactions, and buffeting forces resulting from the wind turbulence. Each of the three wind force components are usually discretized as the lift force, the drag force
and the torsional moment. Therefore, the wind forces on bridge deck, $F_{bw}$, can be obtained as,

$$
F_{bw} = \begin{bmatrix}
L_b^w(x,t) \\
D_b^w(x,t) \\
M_b^w(x,t)
\end{bmatrix} = \begin{bmatrix}
L_{st} + L_{se}(x,t) + L_b(x,t) \\
D_{st} + D_{se}(x,t) + D_b(x,t) \\
M_{st} + M_{se}(x,t) + M_b(x,t)
\end{bmatrix}
$$

(4.11)

where the subscripts $st$, $se$, and $b$ refer to the static, self-excited, and buffeting wind force components; and $L, D, M$ refer to the lift, drag and torsional moment, respectively. Different from the bridge deck, the wind effects on the bridge tower mainly consist of the static and buffeting wind forces, as the bridge towers are usually much stiffer and the self-excited wind forces can be ignored. The formulations of the lift force, drag force and torsional moment of each wind force component in Eq. (4.11) are not presented for the sake of brevity. Interested readers are referred to literature [26,28] for more details.

The wind loading vector $F_{vw}$ on the running vehicles is evaluated using a quasi-static approach [28],

$$
\begin{align*}
F_S &= 0.5 \rho U_r^2(t)C_S(\psi)A_0; \\
F_L &= 0.5 \rho U_r^2(t)C_L(\psi)A_0; \\
F_D &= 0.5 \rho U_r^2(t)C_D(\psi)A_0; \\
M_P &= 0.5 \rho U_r^2(t)C_P(\psi)A_0 h_v; \\
M_Y &= 0.5 \rho U_r^2(t)C_Y(\psi)A_0 h_v; \\
M_R &= 0.5 \rho U_r^2(t)C_R(\psi)A_0 h_v;
\end{align*}
$$

(4.12)

where $F_S$, $F_L$, $F_D$, $M_P$, $M_Y$, $M_R$ are the side force, lift force, drag force, pitching moment, yawing moment, and rolling moment acting on the vehicle, respectively; $C_S(\psi)$, $C_L(\psi)$, $C_D(\psi)$, $C_P(\psi)$, $C_Y(\psi)$ and $C_R(\psi)$ are the corresponding wind aerodynamic coefficients; $A_0$ is the frontal area of the vehicle; $h_v$ is the distance from the vehicle gravity center to the road surface; $U_r$ is the relative wind velocity to the vehicle; and $\psi$ is the yaw angle defined as the angle between the mean wind direction and the vehicle driving direction.

### 4.2.3.2 Modeling of Wave Forces on Bridge

The wave-bridge interaction only confines within the interactions between the wave and the pile-group foundation, since the water level usually could not reach the deck level of these slender coastal bridges (see Fig. 4.7). For a single slender pile shown in Fig. 4.2, the wave force per unit length at the depth $z$ can be evaluated by the Morison equations as the summation of velocity-dependent drag force and acceleration-dependent inertia force [43],

$$
F_{\text{wave}} = \frac{1}{2} \rho_w C_{wD} D (u_w - u_p) \left| (u_w - u_p) \right| + \rho_w A \ddot{u}_w + (C_{wM} - 1) \rho_w A (\ddot{u}_w - \ddot{u}_p)
$$

(4.13)
where $C_{wD}$ and $C_{wM}$ are the drag and inertia coefficients which are taken as $C_{wD} = 1.2$ and $C_{wM} = 1.5$ [83]; $\rho_w$ is the water density; $D_0$ and $A$ are the diameter and section area of the pile; $u_w$ and $\dot{u}_w$ are the water particle velocity and acceleration obtained using Eq. (4.8) and Eq. (4.9); and $u_b$ and $\dot{u}_b$ are the velocity and acceleration of the pile.

The Morison equation was initially proposed to estimate the wave-induced forces on a single isolated slender pile, which may not be directly used for pile-group given the high complexity of the interaction between the wave and the pile-group. In order to predict the wave loads on the pile-group, the laboratory approaches are usually adopted in which the wave load on a slender pile within a pile-group is calculated by multiplying the wave load on an individual pile with an experimentally determined force coefficient. Recently, Bonakdar et al. [49] proposed the wave force coefficients to modify the Morison equation to predict the wave-induced force on a slender pile within a pile-group with various configurations, which were found to agree well with the experimental data in the literature [187, 188]. In the present study, the wave force coefficients proposed by Bonakdar et al. [49] are adopted to calculate the wave force for the slender pile within pile-group with the arrangement shown in Fig. 4.7,

$$K_z = 1$$

$$K_g = \begin{cases} 
1.265 - 0.225 \ln \left( \frac{S_G}{D} \right) & \text{(middle pile)} \\
1.0 & \text{(side pile)}
\end{cases}$$

(4.14)

(4.15)

where $K_z$ and $K_g$ are shelter and interference coefficients defined as the force ratio, $F_{Group}/F_{Single}$, and $F_{Group}$ is the wave force on a slender pile within a pile-group and $F_{Single}$ is the wave force on a single isolated pile determined using Eq. (4.13); $S_G/D_0$ is relative spacing in which $D_0$ is the pile diameter, and $S_G$ is the gap between the surfaces of two neighboring piles in a pile-group. It is worth mentioning that the effects of the currents are not considered in the present study as the currents are found to be weak near the prototype bridge foundation.

### 4.2.4 Equations of Motions for VBWW System

Considering the interactions among vehicle, bridge, wind and wave loads, the governing equations of
the VBWW system can be expressed as follows [198,223],

\[
M_i \ddot{d}_i + C_i \dot{d}_i + K_i d_i = F_{vG}^i + F_{vw}^i + F_{bv}^i \tag{4.16a}
\]

\[
M_i \ddot{d}_b + C_i \dot{d}_b + K_i d_b = \sum_{i=1}^{n} F_{vb}^i + F_{bw} + F_{bwave} \tag{4.16b}
\]

where \( \mathbf{d} \) refers to the displacement vector, in which the superscript \( i \) represents the \( i \)th vehicle, and the subscripts \( b \) and \( v \) represent the bridge and vehicle subsystems; \( \mathbf{M}, \mathbf{C}, \mathbf{K} \) are the mass, damping, and stiffness matrices, respectively; \( F_{vG}^i \) is the weight of the \( i \)th vehicle; \( F_{bw} \) is the wind force vector on the bridge deck and the tower evaluated by Eq. (4.11); \( F_{vw} \) is the wind force vector on the \( i \)th vehicle evaluated by Eq. (4.12); \( F_{bwave} \) is the wave force vector on the pile-group foundation evaluated by Eqs. (4.13) ~ (4.15); \( F_{bv}^i \) and \( F_{vb}^i \) are the interaction forces between the bridge and the \( i \)th vehicle, which are action and reaction forces existing at the contact points of the two systems as a function of deformation of the vehicle’s lower spring [28],

\[
F_{bv}^i = -F_{vb}^i = K_l \{ Z_a - Z_b - r(x) \} + C_l \{ \dot{Z}_a - \dot{Z}_b - \dot{r}(x) \} \tag{4.17}
\]

where \( K_l \) and \( C_l \) are the coefficients of the vehicle’s lower spring and damper; \( Z_a \) is the vehicle-axle-suspension displacement; \( Z_b \) is the displacement of the bridge at road-tire contact points; \( r(x) \) is the road surface profile; and \( \dot{r}(x) = (dr(x)/dx)(dx/dt) = (dr(x)/dx)V(t) \) and \( V(t) \) is the vehicle velocity.

In recognizing that the governing equations given in Eq. (4.16) contain a large number of DOFs and motion-dependent load vectors, i.e., \( F_{vG}^i, F_{bwave}, F_{bv}^i \) and \( F_{vb}^i \), the separation iterative method is applied to solve the two equations separately at each time step, and then to achieve the solution through the equilibrium iterations based on the coupling relationship of the two subsystems [28,178]. The Newmark-\( \beta \) method is adopted to solve the differential equations, in which an integration time step of 0.01 s is selected to provide accurate dynamic responses, according to the preliminary sensitivity analyses.

4.2.4.1 Driver Behavior Model

As an important factor of the vehicle safety analysis, the driver behavior models describe the movement of vehicles under various traffic conditions. The past several decades have witnessed remarkable
progresses in developing and modeling the driver behavior from a simple car-following model to more advanced integrated driver models by integrating the drivers’ planning capacities and decision making strategies, e.g., acceleration, lane changing and gap acceptance. The integrated driver models are usually developed in the context of embodied cognition, as the driving is a highly complex task that involves dynamic interleaving and execution of multiple critical subtasks [224]. In the present study, a simple steering angle model [163] is adopted to consider the drive behavior for simplicity,

\[
\delta = -\lambda_1 \Delta_y - \lambda_2 \dot{\Delta}_y
\]  

(4.18)

where \(\lambda_1\) and \(\lambda_2\) are the parameters related to the driver behavior, and \(\lambda_1 = \lambda_2 = 0.3\) are adopted in the present study as suggested by Chen and Cai [28]; \(\Delta_y\) and \(\dot{\Delta}_y\) are the relative lateral displacement and velocity between the vehicle center and the bridge; the \(\delta\) is the driver’s steering angle, which is usually adopted to simulate the driver’s adjustment of driving when there are constant lateral impacts from gusting winds; the \(\delta\) is incorporated into the vehicle governing equations, i.e., Eq. (4.16a) during the simulation.

4.3 Vehicle Ride Comfort Evaluation

Due to the differences of individual’s tolerance of accelerations that could be affected by individual’s age, health or psychological conditions, setting the ride comfort criterion could be complicated despite of many efforts that have been made in recently years. In the present study, the whole-body vibration measures recommended by ISO 2631-1 [214], which has been widely used as the criterion for health, comfort, perception and motion sickness in practice, is adopted for vehicle ride comfort evaluation.

4.3.1 Whole-body Vibration Measures

According to ISO 2631-1 [214] standard (hereafter defined as “standard”), the whole-body vibration measures, usually referring to accelerations, should take into account all the motions transmitted to a human body through the supporting surfaces, for instance, the buttocks, back and feet of a seated person, the feet of a standing person, or the supporting area of a recumbent person. Only the seated position accounting for three supporting surfaces is considered in the present study for an illustration purpose, and the latter two positions are less complicated which can be easily evaluated by the proposed method. Fig. 4.4 illustrates
the axes and locations where vibrations are measured for a seated person for the drive comfort evaluation. A total of 8 types of vibrations are considered for comfort evaluation: one vertical and one lateral vibration at all three supporting surfaces, i.e. seat, backrest, and floor locations, and one pitching and one rolling vibration at the seat location. The fore-and-aft DOF and yawing DOF are not considered here due to their insignificant influences on the ride comfort results. The 8 vibration responses at the seat, backrest, and floor of the vehicle can be calculated from the vehicle centroid responses based on their relative positions with respect to the vehicle centroids [73],

\[
\begin{align*}
    a_{vs} &= a_{sb} = a_{sf} = \ddot{Z}_v \\
    a_{ls} &= a_{lb} = a_{lf} = \ddot{Y}_v \\
    a_{ps} &= \ddot{\phi}_v \cdot d_s \\
    a_{ns} &= \ddot{\phi}_v \cdot y_s + \frac{1}{2} \ddot{\phi}_v \cdot h_s
\end{align*}
\] (4.19a-d)

where \( a_{ij} \) (i=\( v, l, p, r \); j=\( s, b, f \)) denotes the acceleration: the first subscript indicates the response direction such that “\( v \)”, “\( l \)”, “\( p \)” and “\( r \)” represent vertical, lateral, pitching, and rolling direction; and the second subscript describes the axis location such that “\( s \)”, “\( b \)” and “\( f \)” represent seat, backrest, and floor; \( \ddot{Z}_v \), \( \ddot{Y}_v \), \( \ddot{\phi}_v \), and \( \ddot{\phi}_v \) are the vertical, lateral, pitching, and rolling accelerations at the vehicle centroid; \( d_s, y_s, h_s \) are the longitudinal, transverse, and vertical distances between the vehicle centroid and the seat.
4.3.2 Frequency Weighting

As suggested by the standard, the accelerations should be frequency-weighted in order to model the vibrating frequencies of the human body more realistically [214]. First, the time histories of the original accelerations at all participating axes are transformed into frequency domain using the Discrete Fourier Transform (DFT) and then multiplied by the corresponding frequency weighting factor. After frequency weighting, those frequency-weighted responses in the frequency domain are converted back into the corresponding frequency-weighted responses in the time domain by applying the inverse DFT. Fig. 4.5 shows the frequency weighting factor \( W_i (i=k, d, e, c) \) as recommended by the standard, in which the subscript “k” represents the vertical vibration at the seat and both vertical and lateral vibrations at the floor; the subscript “d” represents lateral vibration at the seat and floor; the subscript “e” represents the rotational vibrations at the seat; and the subscript “c” represents the vertical vibration at the backrest. Those weighting factors serve as filters to the original responses such that the effects of low and high frequency contents of the original responses are reduced. The frequency range of the frequency-weighted responses is set as 0.5 ~ 80 Hz for the ride comfort analysis.
4.3.3 Ride Comfort Criteria

As recommended by ISO 2631-1 [214] standard, when the ride comfort is affected by vibrations in more than one direction/axis, which is the case for a seated person in the present study, the overall vibration total value (OVTV) that takes into account varying vibrating effects should be used for ride comfort evaluation. The OVTV is determined from the root-sum-of-squares summation of all the frequency-weighted vibrations transmitted to the human body [214],

\[
OVTV = \sqrt{M_k^2 \left( RMS_{vs}^2 + RMS_{lf}^2 + RMS_{ps}^2 \right) + M_d^2 \left( RMS_{vl}^2 + M_e^2 RMS_{sb}^2 \right) + M_c^2 \left( RMS_{vl}^2 + RMS_{rs}^2 \right) + M_e^2 RMS_{sc}^2}
\]

(4.20)

where \( RMS_{ij} \) (i=v, l, p, r, j=s, b, f) denotes RMS values of the frequency-weighted acceleration response and the subscripts “i” and “j” have the same definitions as those of the acceleration responses; and \( M_i \) (i=k, d, e, c) are the multiplying factors for each frequency-weighted response to compensate the varying vibrating effects due to different locations and directions, as shown in Table 4.1. The subscripts of the multiplying factors have the same definitions as those of the subscripts for the frequency weighting factor.
Table 4.1 Multiplying factors for each vibration measures

<table>
<thead>
<tr>
<th>Multiplying factor</th>
<th>Value</th>
<th>Location</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_k$</td>
<td>1.00</td>
<td>Seat</td>
<td>Vertical</td>
</tr>
<tr>
<td>$M_d$</td>
<td>1.00</td>
<td>Seat</td>
<td>Lateral</td>
</tr>
<tr>
<td>$M_e$</td>
<td>0.40</td>
<td>Seat</td>
<td>Pitching</td>
</tr>
<tr>
<td>$M_r$</td>
<td>0.20</td>
<td>Seat</td>
<td>Rolling</td>
</tr>
<tr>
<td>$M_c$</td>
<td>0.40</td>
<td>Backrest</td>
<td>Vertical</td>
</tr>
<tr>
<td>$M_d$</td>
<td>0.50</td>
<td>Backrest</td>
<td>Lateral</td>
</tr>
<tr>
<td>$M_k$</td>
<td>0.40</td>
<td>Floor</td>
<td>Vertical</td>
</tr>
<tr>
<td>$M_k$</td>
<td>0.25</td>
<td>Floor</td>
<td>Lateral</td>
</tr>
</tbody>
</table>

Table 4.2 Comfort criteria based on OVTV values

<table>
<thead>
<tr>
<th>OVTV value (m/s²)</th>
<th>Comfort level</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.315</td>
<td>Not uncomfortable</td>
</tr>
<tr>
<td>0.315 ~ 0.63</td>
<td>A little uncomfortable</td>
</tr>
<tr>
<td>0.5 ~ 1.0</td>
<td>Fairly uncomfortable</td>
</tr>
<tr>
<td>0.8 ~ 1.6</td>
<td>Uncomfortable</td>
</tr>
<tr>
<td>1.25 ~ 2.5</td>
<td>Very uncomfortable</td>
</tr>
<tr>
<td>&gt; 2.0</td>
<td>Extremely uncomfortable</td>
</tr>
</tbody>
</table>

To facilitate the ride comfort evaluation, the comfort criteria is also provided in which six different comfort levels are defined based on various ranges of OVTV values, as shown in Table 4.2.

### 4.4 Vehicle Safety Analysis

The vehicle safety analysis is evaluated based on the contact forces at the vehicle wheels, since the contact forces are good indicators of the stability status of the road vehicles. Accordingly, two safety criteria can be defined based on the contact force, i.e., the RSC (Roll Safety Criteria) and the SSC (Sideslip Safety Criteria). The RSC was first proposed by Liu [225] as a rollover indicator of road vehicles, which is derived based on the load transfer ratio of the axles that experience wheel lift-off. Later on, Chen et al. [211] modified the RSC to be more conservative for rolling accident analysis given by,

$$\text{RSC} = \min \left[ \frac{\sum_{i=1}^{k} (F_{vli} + F_{vri})}{\sum_{i=1}^{k} (F_{vli} - F_{vri})} \right] \geq 1.2 \quad (4.21)$$

where $F_{vli}$ and $F_{vri}$ denote the vertical contact forces at the left and right wheels of the $i$th axle, and $k$ is the number of the axles ($k \geq 2$). Eq. (4.21) indicates that the rolling accident will occur as the value of RSC is
less than the threshold value of 1.2.

When a wheel of the vehicle remains contact with the ground, a sideslip accident will occur if the lateral friction cannot prevent the wheels from sliding laterally, i.e., the total lateral contact force exceeds the total static friction force. Similar to RSC, the SSC can be expressed as [211],

\[ \text{SSC} = \frac{\bar{F}_{SR} - 1.645\sigma_{SR}}{0.2\mu_s G_a} \geq 1 \]  \hspace{1cm} (4.22)

where \( \mu_s \) is the sideslip friction coefficient of tire and \( \mu_s = 0.7 \) is adopted in the present study for dry road surface condition; \( G_a \) is the gravity of the lightest axle; \( \sigma_{SR} \) and \( \bar{F}_{SR} \) are, respectively, the mean square root and the mean value of the sideslip resistance \( F_{SR} \) given by,

\[ F_{SR} = \mu_s \left( F_{vl} + F_{vr} \right) - (F_{hl} + F_{hr}) \]  \hspace{1cm} (4.23)

where \( F_{vl} \) and \( F_{vr} \) are the vertical contact forces at the left and right wheels of an axle; \( F_{hl} \) and \( F_{hr} \) are the lateral contact forces at the left and right wheels of an axle.

4.5 Numerical Simulation

4.5.1 Analytical Parameters

One cable-stayed bridge that consists of five spans with a span arrangement of 60 + 176 + 700 + 176 + 60 m is used as the simulation example in the present study, as shown in Fig. 4.6(a). The main span and two inner approach spans of the bridge are made of steel, and the two outer approach spans are made of prestressed concrete. The streamlined box girder is 40 m wide and 3.5 m high, which supports 6 traffic lanes as shown in Fig. 4.6(b). The bridge girder is supported by 176 stayed cables along the bridge span and 6 bearings implemented at the towers and the piers. The bridge deck is 45 m above the SWL. The two H-shaped concrete bridge towers are 186 m tall, supported by the pile foundation that consists of 30 piles with a radius of 3.0 m and a length of 32 m in the water. A concrete mass pile cap is adopted to connect all the piles, which is 73.05 m in the transverse direction and 24.5 m in the longitudinal direction. The elevation of the bottom of the pile cap is 8 m above the SWL. The layout of the tower along with the pile-group foundation is shown in Fig. 4.7. The wind field is simulated for both the bridge deck and the tower such
that a total number of 83 and 11 uniformly distributed wind points are assigned along the bridge deck axis and the height of each bridge tower, respectively. Meanwhile, the wave fields at the two pile foundations with 700 m apart are simulated independently due to their weak correlations.

![Diagram of the prototype coastal slender cable-stayed bridge](image)

**Figure 4.6** (a) Configuration of the prototype coastal slender cable-stayed bridge (unit: m); (b) cross section of the bridge deck (unit: m)

For the simulation of the correlated wind and wave fields, the time interval is selected as 0.01s in accordance with the integration time step used to solve the governing equations. The upper cutoff frequency and the frequency interval are set as 2 Hz and 0.002 Hz, respectively, to ensure the simulated wind and wave fields can accurately represent their characteristics [196]. As an illustration, Fig. 4.8 shows the time histories of the correlated wind and wave fields at selected points with the mean wind velocity \( U = 10 \) m/s and \( U = 20 \) m/s (here and hereafter, the mean wind refers to the mean wind at the deck level, unless otherwise noted). As shown in Fig. 4.8, the higher mean wind speed can generate relatively larger wind and wave time histories compared with the lower mean wind speed. The mean wind is assumed to act on the bridge deck and the tower and the wave is assumed to act on the pile foundation. Both of them are in the lateral direction, which are believed to represent a worse scenario than the other wind or wave directions. However,
the effects of wind-wave loading directionality will not be discussed in the present study.

Figure 4.7 Layout of the tower and the tower foundation (unit: m)

Two types of vehicles are adopted in the analysis, i.e., the sedan car and light truck, with the corresponding parameters shown in Table 4.3. For the traffic simulation, the number of vehicles in each traffic lane is set as 30 with an equal distance of 30 m. The 15th vehicle is selected as the representative vehicle and the corresponding dynamic responses as the vehicle travels on the bridge are used for the following vehicle ride comfort and safety evaluation. All the analyses are based on the VBWW system, i.e., the correlated wind and wave fields are applied on the vehicle-bridge system, unless otherwise noted.
Figure 4.8 Correlated wind and wave fields with different input wind speed ($z_0=0.01$ m, $\gamma=3.3$, $h=32$ m, $F=200$ km): (a) wind turbulence $u(t)$ and $w(t)$ at the midpoint of the bridge deck; (b) wave surface elevation $\eta(t)$, and water particle velocity $u_x(t)$ and acceleration $\dot{u}_x(t)$ at the location of one pile.

Table 4.3 The main properties and dimensions of the representative vehicles

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Light truck</th>
<th>Sedan car</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the rigid body 1</td>
<td>kg</td>
<td>6500</td>
<td>1600</td>
</tr>
<tr>
<td>Pitching moment of inertia of rigid body 1</td>
<td>kg m$^2$</td>
<td>9550</td>
<td>1850</td>
</tr>
<tr>
<td>Rolling moment of inertia of rigid body 1</td>
<td>kg m$^2$</td>
<td>3030</td>
<td>506</td>
</tr>
<tr>
<td>Yawing moment of inertia of rigid body 1</td>
<td>kg m$^2$</td>
<td>100000</td>
<td>10000</td>
</tr>
<tr>
<td>Mass of axle block 1</td>
<td>kg</td>
<td>800</td>
<td>39.5</td>
</tr>
<tr>
<td>Mass of axle block 2</td>
<td>kg</td>
<td>800</td>
<td>39.5</td>
</tr>
<tr>
<td>Upper vertical spring stiffness</td>
<td>kN/m</td>
<td>250</td>
<td>109</td>
</tr>
<tr>
<td>Lower vertical spring stiffness</td>
<td>kN/m</td>
<td>175</td>
<td>176</td>
</tr>
<tr>
<td>Upper lateral spring stiffness</td>
<td>kN/m</td>
<td>187.5</td>
<td>79.5</td>
</tr>
<tr>
<td>Lower lateral spring stiffness</td>
<td>kN/m</td>
<td>100</td>
<td>58.7</td>
</tr>
<tr>
<td>Parameter</td>
<td>Unit</td>
<td>Light truck</td>
<td>Sedan car</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>------</td>
<td>-------------</td>
<td>-----------</td>
</tr>
<tr>
<td>Upper vertical/lateral damping coefficient</td>
<td>kN/s/m</td>
<td>2.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Lower vertical/lateral damping coefficient</td>
<td>kN/s/m</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>Distance between axle 1 and rigid body 1</td>
<td>m</td>
<td>1.8</td>
<td>1.34</td>
</tr>
<tr>
<td>Distance between axle 2 and rigid body 1</td>
<td>m</td>
<td>2</td>
<td>1.34</td>
</tr>
<tr>
<td>Distance between axle and rigid body</td>
<td>m</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>Frontal area $A_0$</td>
<td>m²</td>
<td>6.5</td>
<td>1.96</td>
</tr>
<tr>
<td>Reference height $h_r$</td>
<td>m</td>
<td>1.65</td>
<td>1.1</td>
</tr>
<tr>
<td>$d_s$ between centroid and seat location</td>
<td>m</td>
<td>1.5</td>
<td>0.9</td>
</tr>
<tr>
<td>$y_s$ between centroid and seat location</td>
<td>m</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>$h_s$ between centroid and seat location</td>
<td>m</td>
<td>0.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### 4.5.2 Vehicle Ride Comfort Evaluation

#### 4.5.2.1 Vehicle Ride Comfort Analysis for a Typical Vehicle

Note that the original vehicle accelerations corresponding to each axis of consideration need to be frequency-weighted with corresponding frequency weighting curves before calculating the OVTV and performing the ride comfort evaluation. The frequency-weighting and averaging procedures are illustrated in details using a case study, in which a vehicle fleet consisting of 30 light trucks moves at a speed of 15 m/s (54 km/h) on the third traffic lane with a good road condition. The necessary parameters for the correlated wind and wave fields simulation are given as $U=10$ m/s, $\gamma=3.3$, $h=32$ m, and $F=200$ km, respectively (here and hereafter, same $\gamma$, $h$, and $F$ are used for the wave field simulation, unless otherwise noted). For brevity, only the vertical acceleration at the seat position for the 15th vehicle, i.e. $a_{vs}$, is presented.

Fig. 4.9 shows the power spectrum density (PSD) of the original vertical acceleration response and the frequency-weighted PSD after applying frequency weighting curve $W_k$. Through the comparison between Fig. 4.9(a) and Fig. 4.9(b), the amplitudes of the original PSD at the frequency range below 2 Hz are significantly reduced, while the amplitudes of the original PSD at the frequency range from 3 Hz to 6 Hz are not affected too much. This is due to that the value of the filter $W_k$ is very small for smaller frequency, i.e., $W_k < 0.25$ for $f < 2$ Hz, and then $W_k$ quickly reaches to a large value with higher frequency, i.e., $W_k$ remains around 1.0 at the frequency range 3 ~ 6 Hz, as shown in Fig. 4.5. The updated time history for the acceleration, therefore, can be obtained based on the inverse DFT using the frequency-weighted PSD. Fig. 4.10 compares the vertical acceleration before and after the frequency weighting. The comparison results
indicate that the original acceleration is significantly reduced after frequency weighting, i.e., the corresponding RMS is reduced from 0.257 to 0.077, with a reduction of 70.2%.

![Figure 4.9](image)

**Figure 4.9** One-sided PSD of the vertical acceleration response of the vehicle: (a) PSD of the original response; (b) PSD of the frequency-weighted response

![Figure 4.10](image)

**Figure 4.10** Comparison between the time segment of the original and frequency-weighted vertical acceleration response at seat location

Following the same procedure, the RMS for the original and frequency-weighted accelerations at all participated axes can be obtained which are shown in Table 4.4 for comparison. As shown in Table 4.4, the RMS of the accelerations are all reduced at various degrees after applying frequency weighting, except for the vertical acceleration at the backrest, which has only a reduction of 1.0%. All the other accelerations have a reduction rate ranging from 33.3% to 73.9%. The reason for only a 1.0% reduction for $a_{vb}$ is that the frequency weighting curve $W_c$ remains constant at around 1.0 in the frequency range of 1 ~ 7 Hz, which covers the dominant frequency of the original PSD, as shown in Fig. 4.5. Finally, the frequency-weighted RMS for all axes are modified by applying the multiplying factors to obtain the OVTV in order to evaluate
the ride comfort level. The OVTV of the representative light truck in the case study is obtained as 0.165 m/s², and the corresponding comfort level is classified as “not uncomfortable” based on comfort criteria in Table 4.2.

Table 4.4 The RMS of the original and frequency-weighted responses at all participated axes for the representative 15th light truck (Unit: m/s²)

<table>
<thead>
<tr>
<th>Response</th>
<th>RMS_{x}</th>
<th>RMS_{y}</th>
<th>RMS_{zh}</th>
<th>RMS_{zh}</th>
<th>RMS_{zb}</th>
<th>RMS_{zb}</th>
<th>RMS_{zf}</th>
<th>RMS_{zf}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>0.257</td>
<td>0.075</td>
<td>0.326</td>
<td>0.139</td>
<td>0.257</td>
<td>0.075</td>
<td>0.257</td>
<td>0.075</td>
</tr>
<tr>
<td>Frequency-weighted</td>
<td>0.077</td>
<td>0.050</td>
<td>0.134</td>
<td>0.066</td>
<td>0.255</td>
<td>0.050</td>
<td>0.077</td>
<td>0.019</td>
</tr>
<tr>
<td>Reduction</td>
<td>70.2%</td>
<td>33.3%</td>
<td>58.9%</td>
<td>52.7%</td>
<td>0.8%</td>
<td>33.3%</td>
<td>70.2%</td>
<td>73.9%</td>
</tr>
</tbody>
</table>

4.5.2.2 Influence of Wave Loads and Wind Speed

Fig. 4.11 shows the OVTVs for both of the light truck and sedan car driven on lane 1 at a speed of 25 m/s (90 km/h) with and without considering the wave loads. Fig. 4.11 shows that, for the same type of vehicle under the same wind speed, the difference of the OVTVs for the cases with and without considering the wave loads are all within 7%, indicating that the effects of wave on the vehicle ride comfort are much less than those from the wind. It is also shown in Fig. 4.11 that the OVTVs increase with the wind speed for both types of vehicles. The OVTVs for the two types of vehicles first increase with a steady rate when the wind speed is lower than 20 m/s, and then increase faster as the wind speed continues to increase. In addition, the light truck has a higher OVTV than that of the sedan car under the same wind speed, indicating the light truck is more prone to the potential ride comfort issue. A threshold value of 0.315 based on the ISO ride comfort criteria is also shown in Fig. 4.11. The value below the threshold line indicates that the driver will not feel uncomfortable based on the defined criteria. Accordingly, the critical wind speed at which the driver will began to feel uncomfortable for the light truck and the sedan car is 12.5 m/s (45 km/h) and 22 m/s (79.2 km/h), respectively.
Figure 4.11 The OVTVs for both the light truck and sedan car at different wind speeds with and without the wave loads (\(V=25\) m/s; lane 1; good road condition)

4.5.2.3 Influence of Vehicle Speed

Figure 4.12 The OVTVs for the light truck at different vehicle speed (lane 1; good road condition)

In order to show the influence of vehicle speed on the ride comfort, Fig. 4.12 shows the OVTVs for the light truck with various vehicle speed driven on lane 1. Again, it should be noted that all the case studies in the present study are based on the VBWW system, unless otherwise noted. It is shown in Fig. 4.12 that the OVTVs increase as the vehicle speed increases under the same wind speed. Fig. 4.12 also shows that the critical wind speed for the three vehicle moving speeds are 22.5 m/s for \(V=15\) m/s (54 km/h), 20 m/s for \(V=20\) m/s (72 km/h), and 12.5 m/s for \(V=25\) m/s (90 km/h), indicating the higher vehicle speed, the higher value of OVTV. The same trend can also be observed for the sedan car. For the sake of brevity, the results of the sedan car are not presented here.
4.5.2.4 Influence of Traffic Lane

It is noted that the eccentricity of the traffic lane has effects on the bridge dynamic characteristics, which may in turn affect the vehicle dynamic characteristics. Fig. 4.13 compares the OVTVs for the light truck driven at a speed of 25 m/s (90 km/h) on three different traffic lanes. As shown in Fig. 4.13, lane 3 is the most unfavorable lane for the light truck. For example, the driver may feel a little uncomfortable on lane 3 at a wind speed of 10 m/s, while the driver will not experience uncomfortable under the same wind speed when the driver drives on lane 1 or lane 2. Again, the same trend is also found for the sedan car.

![Figure 4.13 The OVTVs for the light truck at different traffic lane (V=25 m/s; good road condition)](image)

4.5.2.5 Influence of the Presence of Multiple Vehicles

It should be noted that the previous results are all based on a representative vehicle travelling on the bridge with the simultaneous presence of multiple other vehicles, which is the most common scenario on the bridge. As mentioned before, the coupling vehicle-bridge effects have large influences on the vehicle dynamics. Consequently, the ride comfort condition for a vehicle in the presence of multiple vehicles may be different from the single vehicle scenario. Fig. 4.14 compares the OVTVs for both the light truck and sedan car with and without considering the presence of other vehicles. The case considering the traffic flow uses the same traffic fleet as described earlier and the moving speed is taken as 25 m/s (90 km/h). As shown in Fig. 4.14, the OVTVs for both the two types of vehicles in a single vehicle scenario are smaller than those when considering the traffic flow. For instance, under the wind speed of 15 m/s, the reduction ratio of OVTVs from traffic flow scenario to single vehicle scenario is 25.6% and 17.6% for the light truck and
the sedan car, respectively. Accordingly, the critical wind speed increases from 12.5 m/s to 20 m/s for the light truck, and from 22 m/s to 23.8 m/s for the sedan car. The results also indicate that the light truck is more likely to be affected by the presence of multiple other vehicles.

![Graph showing OVTVs with and without considering the traffic flow.](image)

**Figure 4.14** OVTVs with and without considering the traffic flow (V=25 m/s; lane 1; good road condition)

### 4.5.3 Vehicle Safety Analysis

#### 4.5.3.1 Influence of Wave Loads

Similar to the ride comfort evaluation, a parametric study is also performed to study various factors on the vehicle driving safety. Fig. 4.15 compares the safety indicators, i.e., RSC and SSC, of the vehicle driven on lane 1 at a speed of 25 m/s (90 km/h) with and without the wave loads for both the two types of vehicles. It is shown in Fig. 4.15 that the difference of the safety indicators in the cases with and without considering the wave loads are less than 5% under all the wind speeds for both the light truck and sedan car. This suggests that the wave load has minor effects on the vehicle driving safety compared with the wind load. Based upon the simulation results from this prototype bridge, the evaluation results of both the ride comfort and driving safety considering the wind and wave loads suggest that the wind loads have more significant effect than the wave loads. Two reasons may explain why wave loads have negligible influences on the vehicle dynamics. First, the wave surface elevation could not reach the pile cap in general, since the still water level is 6.7 m below the bottom surface of the pile cap, as shown in Fig. 4.7. In addition, considering that the water depth at the bridge foundation is 32 m, the wave-induced forces on these piles
may not be sufficiently large enough to cause appreciably bridge dynamic responses. The other reason is that a large rigid concrete mass pile cap is adopted to connect all the 30 piles as a whole, which makes the pile-group very stiff. Consequently, the wave-induced dynamic effects on the bridge deck of this prototype bridge are not significant, which may in turn have negligible effects on the running vehicles.

Figure 4.15 Vehicle safety indicators with and without considering the wave loads (V=25 m/s; lane 1; good road condition): (a) RSC; (b) SSC

4.5.3.2 Influence of Vehicle Speed

The vehicle safety indicators for both the light truck and sedan car varying with the vehicle speed and the wind speed as the vehicle fleet moving on the first traffic lane are shown in Fig. 4.16. It is shown in Fig. 4.16(a) and 4.16(b) that the driving safety indicators of the vehicles decrease with the increase of the vehicle speed. Fig. 4.16(a) shows that the RSC of the light truck at a moving speed of 25 m/s (90 km/h) approaches the threshold value of 1.2 under the crosswind \( U=25 \) m/s and quickly drops below the threshold line as the wind speed continues to increase. However, the RSC for the sedan car remains above the threshold value for all the simulation cases, indicating that the wind speed less than 35 m/s may not cause rolling accident for the sedan car. The relative low critical wind speed for the rolling accident of the light truck compared with the sedan car may be due to the substantial lateral wind loads applied on the light truck due to its large side area. Different from the RSC, sedan car is more prone to have sideslip accidents in comparison with the light truck under the same moving speed and the same wind and wave load conditions, as shown in Fig. 4.16(b). For instance, for a moving speed of 25 m/s (90 km/h), the critical wind speed for the sedan car and
the light truck is obtained as around 25 m/s and 32 m/s, by interpolating the SSC curves at the threshold value of 1.

![Graphs of Rolling Safety Criteria (RSC) and Sideslip Safety Criteria (SSC)](image)

Figure 4.16 Vehicle safety indicators with traffic speed (— light truck; -- sedan car; lane 1; good road condition): (a) RSC; (b) SSC

4.5.3.3 Influence of Traffic Lane

Fig. 4.17 compares the safety indicators of the vehicle driven on three different traffic lanes at a moving speed of 25 m/s (90 km/h). Similar to the ride comfort analysis, Fig. 4.17 also shows that lane 3 is the most unfavorable lane for the vehicle driving safety. For instance, when the traffic lane shifts from lane 1 to lane 3, the critical SSC wind speed for the light truck and sedan car drops from 31.5 m/s to 27.5 m/s and from 25 m/s to 20 m/s, respectively, as shown in Fig. 4.17(b). Therefore, in the windy conditions, the drivers are suggested to drive their vehicles in lane 1 other than lane 3. Similar findings are also found by Chen et al. [211] that the outer traffic lane can pose more threats to vehicle sideslip risks than inner lanes under strong crosswind. Similarly, the critical RSC wind speed for both the two types of vehicles also drops when the traffic lane shifts from lane 1 to lane 3, as shown in Fig. 4.17(a). This suggests that the coupling effects between the bridge and the moving vehicles have significant influence on the vehicle driving safety.
As an important factor for the vehicle-bridge interaction, the road surface roughness can also affect the vehicle dynamic characteristics. The road surface roughness can be generated by the inverse Fourier Transformation. In order to investigate the influence of various road condition on the vehicle driving safety, three different road conditions, i.e., very good, average and very poor, are adopted [205]. Fig. 4.18 shows the safety indicators of the vehicle driven at a speed of 15 m/s (54 km/h) on lane 3 with various road conditions. It is shown in Fig. 4.18(a) and 4.18(b) that the driving stability of vehicles decreases with worse road surface conditions. In particular, the very poor road condition has significant influence on the driving stability of the sedan car. For instance, under the mean wind of 20 m/s, the RSC of the sedan car reduces...
from 4.1 for very good road surface condition to 2.7 for very poor road surface condition by 34%, while the SSC of the sedan car drops from 2.0 for very good road condition to 1.2 for very poor road condition by 40%.

4.6 Summary

This chapter presents a comprehensive study on the vehicle ride comfort and driving safety evaluation for two types of vehicles based on the coupled vehicle-bridge-wind-wave (VBWW) dynamic system. Based on the VBWW framework, the effects of the correlated wind and wave loads on the vehicle performance are evaluated for the first time. After the evaluation criteria are defined, the influence of various factors, e.g., wind and wave excitations, vehicle moving speed, location of the traffic lane, etc., on the vehicle ride comfort and driving safety are investigated. The evaluation results show that the wind loads, compared with the wave loads, affect the ride comfort and driving safety for the road vehicles more significantly. In addition, both the vehicle ride comfort condition and driving stability will reduce or decrease with the wind speed and/or the vehicle moving speed. Under the same vehicle moving speed and load conditions, the light truck is more prone to the ride comfort issue and rolling accidents, while the sedan car is more prone to have sideslip accidents. Finally, the traffic lane and the road conditions are also found to have significant effects on the vehicle ride comfort and driving safety, indicating that the coupling vehicle-bridge effects significantly affect the vehicle ride comfort and driving safety as well.

The proposed analytical numerical framework can be easily applied on other coastal bridges to predict the dynamic characteristics of its running vehicles, which may further provide guidance for vehicle ride comfort improvement or accident mitigation strategies under hazardous driving environments. In addition, further efforts are also needed to improve the vehicle driver behavior to integrate the drivers’ planning capacities and decision making strategies for more comprehensive vehicle ride comfort and safety evaluations.
5 Probabilistic Fatigue Damage Assessment of Coastal Slender Bridges under Coupled Dynamic Loads

5.1 Background

Serving as critical links in the transportation network for coastal regions, coastal slender bridges could constantly experience complex dynamic interactions with strong winds and/or high waves during extreme weather conditions, in addition to moving vehicles, such as cars, trucks, or trains. Continuously repeated stress cycles as well as corrosive coastal environments could cause significant fatigue damage accumulations through the complex interactions of vehicle-bridge-wind-wave systems during the bridge’s lifetime [226]. Many approaches have been proposed for fatigue damage evaluation of existing long-span bridges, which can be mainly categorized in two groups: finite-element analysis (FEA)-based approach [206,227] and structural health monitoring (SHM)-oriented approach [228]. Recently, increasing attentions have been paid to the hybrid approach that integrates the FEA and the SHM [229], aiming to seek more reliable and effective methodologies for fatigue performance evaluation. The FEA can be used to pinpoint the critical structural details, while the SHM serves as an essential supplement for validation as well as to provide site-specific loading information.

For the FEA approach in particular, challenge still remains in modeling the large-scale coastal slender bridges. Due to the complexity of the structural details, the length scale of the local structural details in the FEA, where fatigue damages are usually initiated, are much smaller compared with that of the entire structure. A high-fidelity FEA model that includes all structural details is usually computational prohibitive due to a huge number of degrees of freedom involved if not impossible. To this end, many multi-scale/multi-level modeling schemes that include a refined FE model built with detailed geometry or substructure modeling schemes with homogenized material properties considering mesoscale or microscale material

* This chapter is adapted from a paper published in the Engineering Structures [312], with permission from ELSEVIER.
properties, are proposed [227,230–232]. Nevertheless, these modeling schemes usually only have deterministic parameters for the analysis to save the calculation cost. Uncertainties associated with the fatigue damage accumulation process, therefore, could not be considered. However, uncertainties from the ambient environment, such as the stochastic dynamic loads from vehicles, wind and waves as well as the other environmental parameters, such as the temperature, humidity, chloride density, etc., could affect the fatigue damage prediction significantly. As a result, there are strong needs for the reliability based approaches for fatigue damage assessment of a coastal slender bridge.

To include the aforementioned stochastic loads, the conventional reliability based approach usually involves with a large-scale FEA runs, or Monte-Carlo simulations (MCS), which could be very computationally expensive, if not inhibitive. To this end, several approaches have been proposed to avoid the exhaustive-computational MCS. Kwon et al. [233] proposed a probabilistic approach for modeling the equivalent stress range for a ship, in which the continuous domain of loading parameters are discretized into a series of representative blocks with associated probabilities of occurrence. Similar approach was also adopted for bridges by Zhang et al. [227], in which lifetime wind and traffic loads are partitioned into several representative blocks. Later on, Yan et al. [234] proposed a fatigue stress prediction strategy for OSD under cyclic truck loads, in which the random truck load parameters such as axle weight and location are considered by integrating the influence surfaces with the regression model. Recently, Lu et al. [235] proposed a machine learning algorithm to develop a regression model between the input traffic load and the output stress with limited number of FEAs, to enable efficient fatigue performance evaluation.

Although many works have been carried out for fatigue reliability analysis of steel bridges, research on the combined effects of vehicles, wind, and waves on the coastal slender bridges are still limited due to the complexity of the coupled structural dynamic system. Based on the established VBWW system [223], this study proposes a framework for probabilistic fatigue damage assessment of coastal slender bridges by combining the multi-scale FEA with SHM. Firstly, the stochastic load models are established using the site-specific SHM data: the truck load model is characterized by the vehicle type, vehicle-occupied lane, and the vehicle gross weight; the correlated wind and wave model is parameterized with the wind speed, wave
height, and the wave period. Secondly, multi-scale FEA is performed using the established load models as the dynamic input to compute the stress responses at critical welded joints, which are further transformed into the daily equivalent fatigue damage accumulation based on the Miner’s law. To overcome the time-consuming issue in dealing with the stochastic loads, a machine learning algorithm, i.e., support vector regression (SVR) model, is implemented to substitute large-scale FEA simulations to improve the computational efficiency. Finally, the efficiency and accuracy of the proposed framework is illustrated through a case study on a coastal cable-stayed bridge. The effects of the traffic, wind and wave loads on the fatigue damage of the OSD is discussed. The impact of an increase in the traffic volume and the vehicle weight on the fatigue reliability is investigated, as well.

5.2 Modeling of Stochastic Dynamic Loadings

Primary structural loads on coastal slender bridges include those from the traffic, wind, and wave. Since the structural loads are stochastic in nature and can be affected by many factors, it is vital to parameterize these loads using parameters that contribute most to the structural fatigue damage. In the present study, the following fatigue-related parameters will be included: the traffic related ones, such as the vehicle types, vehicle weights, and driving lanes [235]; and wind and wave related ones, such as the wind speed, wind direction, wave height, wave period, and wave direction [198]. In addition, the traffic related parameters are assumed to be independent with the wind and wave related parameters. These parameters for modeling the coupled VBWW system based on long-term field measurements will be elaborated below.

5.2.1 Vehicle Load Model

For fatigue design in many codes or specifications, such as AASHTO [236] and Eurocode 1 [237], fatigue trucks are typically defined to represent the truck traffic. Several fatigue truck models with various deterministic gross vehicle weights (GVW) and configurations are defined in these design codes. However, the actual site-specific truck loads could be different. In a long term, possible increase or pattern change of local traffic, such as over-loaded heavy trucks, could also introduce different fatigue truck loadings. Therefore, a realistic fatigue truck load model is needed to account for the actual traffic condition, especially
for the probabilistic bridge fatigue damage evaluation.

In the present study, the fatigue truck load model is established based on the site-specific WIM data including the vehicle types, GVW, and vehicle moving lanes. A coastal slender cable-stayed bridge with WIM system, which supports two-way six traffic lanes, is selected as a prototype. The traffic data was collected over one month, during which a total of 307,200 vehicles were passing through the bridge. Among the total monitored vehicles, 62% vehicles with GVW larger than 30 kN are considered to have contributions to the fatigue damage, which can be further classified into 5 categories, as summarized in Table 5.1. The reason to exclude the vehicles with GVW less than 30 kN from the fatigue analysis is due to their negligible contributions to the fatigue damage, according to the preliminary analysis. As shown in Table 5.1, among the effective traffic volume, the vehicle type 2 with two axles accounts for 56.29%, followed by vehicle type 6 with six axles and vehicle type 5 with five axles, occupying 22.26% and 15.35%, respectively. The remaining two types of vehicles, i.e., type 3 and type 4, in the effective traffic volume is 6.10%. This indicates that there is a large amount of heavy trucks, i.e., type 5 and type 6, among the effective traffic volume. The majority of these loads are in the slow lane and middle lane. In addition, the GVW of each type of vehicle were also recorded and modeled with an appropriate probability density function (PDF). It is worth noting that the actual vehicle weight may exhibit multi-peaks feature rather than a typical Gaussian distribution, considering multiple loading conditions of the truck being empty loading, fully loading or in between. Hence, finite mixed normal distributions (also referred to as Gaussian mixture model or GMM) are used to describe the multiple peak distributions [238]. Fig. 5.1 shows the weight for type 6 truck and the fitted PDF. It is observed that the fitted GMM could well represent the weight of the vehicle obtained from WIM. Similarly, the GVWs of the other four types of vehicles in Table 5.1 can be obtained and modeled using the appropriate GMMs. Since the axle spacing for each type of vehicle has small variations, only the mean values of the axle spacing are used in this study, as illustrated in Table 5.1. The transverse distance between two wheels in each axle is taken as 2.0 m. It should be noted that the transverse locations of the vehicle tires (or truck axles) are not recorded in the SHM system. For conservative purpose, the effects of the transverse positions distribution of the vehicle tires on the accumulated fatigue damage
are not considered in the present study.

### Table 5.1 Classifications of effective fatigue vehicles

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Axle spacing (m)</th>
<th>Total occupancy rate (%)</th>
<th>Occupancy rate in each lane (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_2$ (Two-axle truck)</td>
<td>5.0</td>
<td>56.29</td>
<td>19.76</td>
</tr>
<tr>
<td>$V_3$ (Three-axle truck)</td>
<td>4.0 3.0</td>
<td>2.90</td>
<td>1.50</td>
</tr>
<tr>
<td>$V_4$ (Four-axle truck)</td>
<td>3.3 6.6 1.3</td>
<td>3.20</td>
<td>1.81</td>
</tr>
<tr>
<td>$V_5$ (Five-axle truck)</td>
<td>3.2 6.9 1.6 1.3</td>
<td>15.35</td>
<td>8.80</td>
</tr>
<tr>
<td>$V_6$ (Six-axle truck)</td>
<td>2.9 1.5 6.9 1.3</td>
<td>22.26</td>
<td>13.18</td>
</tr>
</tbody>
</table>

To enable the fatigue stress analysis in the FEA, the traffic flow is simulated by considering the parameters with higher contributions to the structural fatigue stress. In the present study, the effects of vehicle spacing is not considered because of the insignificant impact of simultaneous truck loads on the fatigue stress on the welded joints. This is due to the fact that the vehicle gap for highway bridges is extremely large compared to the length of the influence lines of the welded details [234,235]. In addition, the vehicle speed can be considered as constant for long-span bridges [88]. Accordingly, the traffic flow is simulated by incorporating the three aforementioned most influential parameters, i.e., the vehicle type,
vehicle-occupied lane, and the GVW, in a daily basis. The simulation process mainly consists of three steps as shown below:

(1) Firstly, the number of trucks for each vehicle type, i.e., \( V_2 \sim V_6 \), is determined by multiplying the corresponding total occupancy rate (in Table 5.1) with the effective daily traffic volume.

(2) Secondly, for each vehicle type, a set of GVWs is randomly sampled from the corresponding established GMM, which is assigned to each individual truck of that vehicle type.

(3) Finally, after all the trucks of each vehicle type are assigned with GVW, those trucks are further divided into three groups, each corresponding to a certain traffic lane. The number of trucks of each vehicle type occupied in three traffic lanes is determined by the corresponding occupancy rate as shown in Table 5.1.

Figure 5.2 Simulated stochastic fatigue truck loads in a 2-hour duration: (a) slow lane; and (b) middle lane

As a demonstration, Fig. 5.2 displays the simulated fatigue truck loads with different arrival time and GVW on both slow lane and middle lane in a 2-hour duration in a typical day. As illustrated in Fig. 5.2, each vehicle type is labeled with a different symbol, \( x \)-axis represents the arrival time, and \( y \)-axis represents the individual GVW. Each truck has distinctive characteristics in terms of GVW and arrival time. It is also observed that the traffic pattern is obviously different for the slow lane and the middle lane: the heavy-loaded trucks have a higher probability of occurrence in the slow lane, whereas the light trucks have an opposite trend. Therefore, the simulated stochastic fatigue truck loads can accurately reflect the site-specific
traffic statistics.

5.2.2 Wind and Wave Copula Model

Figure 5.3 Dependence among the observed data ($\rho$ is the linear correlation coefficient): (a) $\theta_w$ and $\theta_s$; (b) $H_s$ and $T_p$; and (c) $V_w$ and $H_s$

Statistical analysis is performed for the wind and wave data, recorded by the Meteorological Observatory near the bridge site from 1980 to 2012 [239]. The wind and wave data include wind speed $V_w$ (m/s), wind direction $\theta_w$ (°), significant wave height $H_s$ (m), wave direction $\theta_s$ (°), and peak wave period $T_p$ (s). Both $\theta_w$ and $\theta_s$ refer to the directions measured in degrees clockwise from the North (i.e., 0°), as indicated in Fig. 5.4. The recorded wind speed refers to 10-minute wind speed at the height of 10 m above the mean sea level. To illustrate the dependencies among the observed data, the scatter diagrams of three data sets, i.e., $\theta_w$ and $\theta_s$, $H_s$ and $T_p$, and $V_w$ and $H_s$, are displayed in Fig. 5.3. The corresponding linear correlation coefficients for the three observed data sets, denoted as $\rho$, are computed as 0.6232, 0.4769, and 0.6852,
respectively. The relatively large values of the correlation coefficient indicate there exists relative strong linear dependence between the paired data points $\theta_w$ and $\theta_s$, $H_s$, and $T_p$, and $V_w$ and $H_s$. It should be noted that a single linear correlation coefficient may be inadequate to uncover the underlying complex structure among the observed wind and wave data as shown in Fig. 5.3. To better capture the characteristics of the observed wind and wave, the copula model is proposed for the subsequent statistical analysis as discussed later on.

For better characterizing the complex data, a pretreatment on the observed wind and wave data is performed before the statistical analysis. Fig. 5.4(a) shows the wind rose map indicating the frequency of occurrence of the various wind speeds at 32 different wind directions. The prevailing wind directions are within the wind angle range of $135^\circ$ to $180^\circ$, which also have higher wind speeds than those at other wind angles. As shown in Fig. 5.4(a), the prevailing mean winds can be assumed to be normal to the bridge axis. Therefore, in the present study, only the wind characteristics within this directional sector are adopted in the subsequent analysis. To maintain the data consistency of the observed wind and wave data, the wave data paired with the wind data in the selected wind directional sector, $\theta_w \in [135^\circ, 180^\circ]$, are adopted for better characterizing their correlations.

Wind-wave misalignment is a very common phenomenon that could occur at all wind speeds. Studies show that a range of physical effects could attribute to the wind-wave misalignment, such as spatial and temporal variations in the wind-wave dynamic system, refraction by spatially varying depth and/or currents, and upwind fetch restrictions [240]. Because the wind-wave misalignment is of practical importance for the fatigue performance of the coastal infrastructures such as bridges, the wind-wave misalignment is first checked through the angular histogram of the difference between the wind direction and the wave direction, $\theta_w - \theta_w$, as shown in Fig. 5.4(b). It is observed that the mean value of the $\theta_w - \theta_w$ is $2.43^\circ$, among which 81% are within $30^\circ$, indicating the wind-wave misalignment is not significant. Since only small differences between $\theta_w$ and $\theta_w$ are observed, the wind and wave, therefore, are assumed to be in the same direction in this study for the sake of simplicity. Based on the above analysis, only three parameters, i.e., $V_w$, $H_s$, and $T_p$, of the selected wind and wave data are investigated subsequently.
In probability theory and statistics, a copula is a multivariate probability distribution in which the marginal probability distribution of each variable is uniform. The copula has been widely used for modeling the dependence among random variables in recent decades. The copula approach is rooted in Sklar’s theorem that any multivariate joint distribution can be described by univariate marginal distribution functions together with a copula that describes the dependence structure between the variables [241]. The copula allows the marginal univariate distributions to vary with different forms to provide best fit for variables. Such flexibility makes the copula approach highly desirable in modeling the environmental parameters with inherent non-obvious inter-dependencies, such as in hydrological engineering and ocean engineering [242]. With co-existence of wind and wave at the bridge site of slender coastal bridges, the copula could be a possible fit for characterizing the correlated wind and wave.

For a bivariate case in particular, the joint cumulative distribution function (CDF) $H(x_1, x_2)$ for any pair of random variables $(x_1, x_2)$ can be constructed by combining the marginal distributions with specific dependence structure through copula function as,

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2), \theta) \quad (5.1)$$

where $F_1(x_1)$ and $F_2(x_2)$ = cumulative marginal distribution function for variable $x_1$ and $x_2$, respectively; $C:[0, 1]^2 \rightarrow [0, 1]$ = 2-dimensional copula; $\theta$ = dependence parameter.
Before applying the copula approach for the two selected data sets \((H_s, V_w)\) and \((H_s, T_p)\), the marginal distribution for each variable needs to be obtained as indicated in Eq. (5.1). Three commonly used marginal distributions, i.e., Weibull, Gamma, and Lognormal distributions, are adopted to fit the observed \(V_w, H_s,\) and \(T_p\), with model parameters estimated from the maximum likelihood method [243]. The suitability of the three proposed distribution models for each variable is examined through comparison of their maximum log-likelihood values, as summarized in Table 5.2. The model with the maximum log-likelihood value will be selected as a best fit. As a result, the Weibull distribution is selected for \(V_w\), whereas the Gamma distribution is selected for both \(H_s\) and \(T_p\).

### Table 5.2 Marginal distribution model parameter estimates

<table>
<thead>
<tr>
<th>Marginal PDF</th>
<th>Wind velocity ((V_w))</th>
<th>Significant wave height ((H_s))</th>
<th>Wave peak period ((T_p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lognormal model</td>
<td>(\mu = 1.8612)</td>
<td>(\mu = 0.9013)</td>
<td>(\mu = 1.9636)</td>
</tr>
<tr>
<td></td>
<td>(\sigma = 0.4602)</td>
<td>(\sigma = 0.4036)</td>
<td>(\sigma = 0.2012)</td>
</tr>
<tr>
<td></td>
<td>Log-likelihood = -107,719</td>
<td>Log-likelihood = -98,271</td>
<td>Log-likelihood = -132,532</td>
</tr>
<tr>
<td>Gamma model</td>
<td>(a = 5.3860)</td>
<td>(a = 6.2907)</td>
<td>(a = 25.1008)</td>
</tr>
<tr>
<td></td>
<td>(b = 1.3140)</td>
<td>(b = 0.4248)</td>
<td>(b = 0.2896)</td>
</tr>
<tr>
<td></td>
<td>Log-likelihood = -100,399</td>
<td>Log-likelihood = -92,710(^a)</td>
<td>Log-likelihood = -115,490(^a)</td>
</tr>
<tr>
<td>Weibull model</td>
<td>(k = 2.6022)</td>
<td>(k = 2.4729)</td>
<td>(k = 4.9226)</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 7.9762)</td>
<td>(\lambda = 3.0146)</td>
<td>(\lambda = 7.8713)</td>
</tr>
<tr>
<td></td>
<td>Log-likelihood = -100,139(^a)</td>
<td>Log-likelihood = -97,178</td>
<td>Log-likelihood = -120,829</td>
</tr>
</tbody>
</table>

Note: \(^a\) Maximum log-likelihood value indicates the best model.

After the marginal distribution for each random variable is obtained, an appropriate bivariate copula model can be selected to correlate each pair of the random variables. A parametric family of a single parameter copula, i.e., the Gumbel, Gaussian, Frank and Clayton copulas, is adopted in this study. More details about these copulas can be found in [244]. The dependent parameter \(\theta\) for each copula is also estimated through the maximum likelihood method. Two information Criterion, i.e., Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), are utilized to evaluate the goodness of fit amongst the proposed copulas. A lower value suggests a better model [244].

\[
\text{AIC} = -2l(\theta) + 2p
\]

(5.2)
where \( l(\theta) \) = maximum log-likelihood value of the copula model; \( p \) = number of parameters used in the copula model; \( Q \) = total number of observations of the random variable.

**Table 5.3** Evaluation of proposed copulas for two observed data sets

<table>
<thead>
<tr>
<th>Data set</th>
<th>Copula</th>
<th>Dependent parameter (( \theta ))</th>
<th>Log-likelihood</th>
<th>No. of parameters</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>((V_w, H_s))</td>
<td>Gumbel</td>
<td>1.7827</td>
<td>-233851</td>
<td>5</td>
<td>467711*</td>
<td>467757*</td>
</tr>
<tr>
<td></td>
<td>Gaussian</td>
<td>0.6481</td>
<td>-235161</td>
<td>5</td>
<td>470332</td>
<td>470378</td>
</tr>
<tr>
<td></td>
<td>Frank</td>
<td>5.0743</td>
<td>-236234</td>
<td>5</td>
<td>472477</td>
<td>472523</td>
</tr>
<tr>
<td></td>
<td>Clayton</td>
<td>1.0033</td>
<td>-242682</td>
<td>5</td>
<td>485374</td>
<td>485420</td>
</tr>
<tr>
<td>((H_s, T_p))</td>
<td>Gumbel</td>
<td>1.2624</td>
<td>-203815</td>
<td>5</td>
<td>407640</td>
<td>407686</td>
</tr>
<tr>
<td></td>
<td>Gaussian</td>
<td>0.3986</td>
<td>-202579</td>
<td>5</td>
<td>405168*</td>
<td>405213*</td>
</tr>
<tr>
<td></td>
<td>Frank</td>
<td>2.6711</td>
<td>-202903</td>
<td>5</td>
<td>405817</td>
<td>405862</td>
</tr>
<tr>
<td></td>
<td>Clayton</td>
<td>0.4758</td>
<td>-204684</td>
<td>5</td>
<td>409379</td>
<td>409424</td>
</tr>
</tbody>
</table>

Note: *The lowest AIC and BIC indicate the best copula model.

**Figure 5.5** Comparison of contour plot of PDF between the fitted copula model and the observed data: (a) data set \((V_w, H_s)\); and (b) data set \((H_s, T_p)\)

The estimated dependent parameter, log-likelihood and the associated AIC and BIC values for the proposed copulas are summarized in Table 5.3. Both the AIC and BIC values suggest that the Gumbel copula family is the best fit to \((V_w, H_s)\), and the Gaussian copula family is the best fit to \((H_s, T_p)\). To further show the goodness of fit, the contour plots of the PDF of the fitted copula models are compared with those of the observed data sets \((V_w, H_s)\) and \((H_s, T_p)\), as shown in Fig. 5.5. Good agreement is found between the fitted copula and the observed data for both the data sets \((V_w, H_s)\) and \((H_s, T_p)\), indicating the proposed
A copula approach can be effectively used for modeling the correlated wind and wave field.

### 5.3 Probabilistic Modeling for Numerical Simulations

To consider the uncertainties for fatigue damage accumulations, a large number of FEAs need to be performed to cover all loading conditions for conventional MCS, which could be computationally exhaustive. Therefore, an efficient probabilistic modeling scheme is needed to manage the uncertainties of the VBWW system. In the present study, a machine learning algorithm is developed to correlate the multiple stochastic inputs of environmental loadings (vehicle, wind, and wave) and single output of structural response, as elaborated in this section.

The proposed numerical framework consists of 4 steps and the flowchart is shown in Fig. 5.6. In step I, the stochastic load models are established based on the long-term field measurements. As discussed earlier, the gross vehicle weight (GVW), wind speed ($V_w$), significant wave height ($H_s$), and the peak wave period ($T_p$) are selected as the four most influential parameters for developing the stochastic load models. To efficiently predict fatigue damage probabilistically, a machine learning algorithm integrating the uniform design sampling (UDS) method and the support vector regression (SVR) is proposed to avoid the time consuming MCS, as illustrated in step II, i.e., deterministic simulation, and step III, i.e., probabilistic modeling. The deterministic simulation aims to develop a regression model between the input dynamic loads and the output equivalent fatigue damage accumulation using limited number of dynamic loading samples; whereas the probabilistic modeling aims to establish the probabilistic model for fatigue damage at critical welded joints under stochastic dynamic loads. Steps II and III are described in details as below.

The deterministic simulation in Step II consists of three sub-steps, i.e., uniform design sampling (UDS), multi-scale FEA, and response-surface approximation. The UDS is a spacing filling method, which aims to seek representative points that are orthogonal and uniformly scattered in the entire design domain [245,246]. Compared with the conventional Latin hypercube sampling, the UDS has better uniformity and spacing filling in the design domain especially for multiple random variables, which could dramatically reduce the number of training samples required for the machine learning scheme. After a small number of samples for the loading parameters are created by UDS, the multi-scale FEAs are performed using these loading samples.
to obtain the stress time histories at the critical welded joints. With the extracted stress time responses, the equivalent stress range $S_{eq}$, number of cycles $n_t$, and the equivalent fatigue damage accumulation $D$ are calculated based on rain-flow counting method and Miner’s linear fatigue damage accumulation rule. Subsequently, the SVR is adopted to learn the input-output training data set. Because each vehicle configuration varies, five SVR models are developed corresponding to five types of vehicle configurations. After successful training, the five SVR models can be used to substitute the FEA to predict the $D$ at critical welded joints under all possible loading conditions using the stochastic dynamic loads, as illustrated in step III. In step III, the PDF of the predicted $D$ is also modeled with appropriate distribution model, which will provide statistical basis for the subsequent fatigue reliability analysis based on limit-state function in step IV. The limit-state function and fatigue damage accumulation are presented in the next section.

The most essential aspect of the deterministic analysis is to establish a regression model to correlate the dynamic loads and the equivalent fatigue damage accumulation. The SVR formulation is briefly presented as follows. SVR is a learning method that generates input-output mapping function $f(x)$ from available training data set $D_T = \{(x_i, y_i), i=1,2,\ldots,N\}$, where $x$=multidimensional input vector, $y$=observed output, and $N$=total number of data. In the present study, $x=[\text{GVW}, \text{V_w}, \text{H_s}, \text{T_p}]$ and $y=D$. The regression function $f(x)$ can be expressed as,

$$f(x) = \sum_{i=1}^{l} \alpha_i \kappa(x, x_i) + b \tag{5.4}$$

in which $\kappa(x, x_i)$=kernel function such as linear, Gaussian, polynomial, and sigmoid kernels; the Gaussian kernel function is adopted in this study and given by $\kappa(x_i, x_j) = \exp(-\gamma ||x_i - x_j||^2)$, where $\gamma$=kernel parameter; $\alpha_i$=weight of the $i$th kernel function; $b$ = bias; and $l$=number of kernel functions. These parameters can be determined from the structural risk-minimizing principle and the Lagrange multiplier optimal programming method [247]. More details about SVR can be found in [248].
Step I: Establish stochastic dynamic load model
1. Vehicle load model
2. Wind & wave load model

\[ x = [G_{VW}, V_w, H_s, T_p] \]

Generate small-scale loading sample:
1. Generate small amounts of loading samples, \( x \), with UDS.

Generate the training sample:
1. Compute the output \( D \) with multi-scale FEA using \( x \) as input;
2. Construct the training sample \( \{(D_i, x_i)\} \).

Fit the SVR model (surrogate model):
1. Establish the SVR model, \( f(x) \), based on the training sample \( \{(D_i, x_i)\} \).

Generate large-scale loading sample:
1. Generate large amounts of loading samples to simulate daily traffic flow, and wind and wave.

Predict \( D \) under the large sample:
1. Substitute the time-consuming FEAs with SVR model to predict \( D \) under all loading conditions.

Probabilistic modeling:
1. Approximate the PDF of the \( D \) utilizing Weibull distribution.

Step II: Deterministic simulation

Step III: Probabilistic simulation

Step IV: Fatigue life estimation:
1. Construct the limit-state function (LSF)
2. Estimate the fatigue life based on \( D \) and other random variables

Figure 5.6 Flow chart of the proposed reliability-based numerical framework

5.4 Fatigue Limit State Function

The Miner’s linear fatigue damage accumulation theory together with the \( S-N \) curve (stress-life) is the most commonly used approach for predicting the structural fatigue damage accumulation. A set of \( S-N \) curves corresponding to various detail categories is defined in different design specifications, such as AASHTO [236] and Eurocode 3 [249]. Each curve is derived based on the nominal stress range versus life in cycles. In this study, the AASHTO specification is adopted, in which the expression of \( S-N \) curves is given by,

\[
S = \left( \frac{A}{N} \right)^{1/m}
\]

(5.5)

where \( S \) = constant stress range; \( N \) = fatigue life in cycles of a detail; \( A \) = detail category constant; \( m = 3 \) represents the slope of the \( S-N \) curve. These parameters for three types of welded joints of interest, i.e., U-rib butt joint, rib-to-diaphragm joint, and rib-to-deck joint, are listed in Table 5.4.
### Table 5.4 Parameters of S-N curves in the AASHTO specification [236]

<table>
<thead>
<tr>
<th>Welded Joint</th>
<th>Detail category</th>
<th>Constant A (×10^{11} MPa^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-rib butt</td>
<td>D</td>
<td>7.21</td>
</tr>
<tr>
<td>Rib-to-diaphragm (diaphragm)</td>
<td>C</td>
<td>14.4</td>
</tr>
<tr>
<td>Rib-to-deck (deck)</td>
<td>C</td>
<td>14.4</td>
</tr>
</tbody>
</table>

It is noted that the S-N curve is appropriate to compute the fatigue damage with constant-amplitude fatigue stresses. Nevertheless, the fatigue stresses of the welded joints caused by the stochastic loads are random variables. To this end, an equivalent fatigue damage accumulation rule is introduced to substitute the variable stress ranges based on the Miner’s linear fatigue damage accumulation theory [81],

\[
D_a = \sum_i \frac{n_i S_{ri}^m}{A} \quad (5.6)
\]

where \( D_a \) = fatigue damage accumulation; \( S_{ri} \) = ith stress range; \( n_i \) = corresponding number of cycles for \( S_{ri} \).

With the Miner’s rule, the equivalent fatigue stress range and corresponding number of cycles are expressed as,

\[
S_{re} = \left( \sum_i \frac{n_i}{n_i} \cdot S_{ri}^m \right)^{1/m} \quad (5.7)
\]

where \( S_{re} \) = equivalent fatigue stress range; and \( n_i = \sum n_i \) is the corresponding number of stress cycles. As a result, the variable-amplitude stresses are transformed to constant-amplitude stresses, which can be utilized to compute the fatigue damage based on the S-N curves in the ASSHTO specification subsequently.

The previous derivations provide the fatigue damage expressions caused by the fatigue stress cycles due to an individual truck passage with simultaneous presence of the combined wind and wave loads. In practice, the \( S_{re} \) mainly dependents on the traffic, wind and wave loads that are random in nature and should be regarded as a random variable. In addition, the additional parameter, i.e., the average daily truck traffic (ADTT), should also be incorporated in the limit-state function (LSF) of the fatigue damage. Consequently, the LSF can be expressed as,

\[
g(X) = D_a - \sum_{i=1}^{6} D_i(X) = D_a - \sum_{i=1}^{6} (365 \cdot D) \quad (5.8)
\]
where $D_{c}$=critical fatigue damage; $D_{i}$= fatigue damage accumulation in the $i^{th}$ year of life cycle; $n$=design life of the bridge; and $D$=daily equivalent fatigue damage accumulation. In the LSF, the critical variable associated with the stochastic loads is $D$, which can be computed by linearly superposing the accumulated fatigue damage caused by each individual truck passage with simultaneous presence of the combined wind and wave loads, according to Miner’s rule.

$$D = \sum_{j=1}^{N_{ADTT}} \frac{S_{j,\text{re}} n_{j,t}}{A}$$

where $N_{ADTT}$ = amount of current ADTT; $j$=jth truck; $S_{j,\text{re}}$ = equivalent fatigue stress range caused by the $j$th truck passage with simultaneous presence of the combined wind and wave loads; and $n_{j,t}$ = the corresponding number of stress cycles. The calculation of $S_{j,\text{re}}$, $n_{j,t}$, and $D$, have been already discussed in details in the previous sections.

## 5.5 Results and Analysis

In the present study, a cable-stayed bridge located in southern China coastal regions, is selected as a prototype to demonstrate the application of the proposed numerical framework. The cable-stayed bridge has a span arrangement of 60+176+700+176+60 m, which supports six lanes with three traffic lanes traveling in the same direction. The concrete box girder is adopted in the two outer side spans, while steel box girder is adopted for the two inner side spans and the main span. The streamlined steel box girder is 3.5 m high and 40 m wide, with an orthotropic steel bridge deck. More details about this bridge and the corresponding global FE model with beam and link elements can be found in [198,223]. In this section, the efficiency and accuracy of the proposed framework are demonstrated, and the influences of the dynamic loads on the fatigue reliability are discussed as well.

### 5.5.1 Deterministic Analysis for Multi-Scale FEA

As discussed earlier, the multi-scale FEA, i.e., the global-local modeling is adopted for fatigue-related stress analysis. The global-local modeling aims to capture the accurate stress responses at critical welded joints through independent local refined FE model with 3-D shell/solid elements, whereas the boundary conditions are adopted from the corresponding global less-refined FE model. The global-local FE modeling
approach has been used in many applications including the fatigue analysis of long-span bridges, which confirms the effectiveness for large-scale simulation [80,250,251]. The procedure for the multi-scale FEA of the OSD is as follows. Firstly, the global structural dynamic analysis of the bridge under loading samples created by UDS (see Fig. 5.6) is performed based on a coupled vehicle-bridge-wind-wave (VBWW) analytical platform developed by the authors [198]. The global dynamic analysis takes into account the complex interactions among the bridge, traffic, wind and wave. Next, a separate local model is developed using refined shell elements (see Fig. 5.7) with detailed geometry. It is noted that the length of the local refined model should be simulated to have more than four times of its height, to avoid the end effect according to the Saint-Venant’s Principle [252]. After applying both the boundary conditions and loading conditions extracted from the global VBWW analysis, the stress time history at the structural details can be obtained.

Based on the simulation results from the global analysis, the deck segment around the bridge mid-span is identified as the critical region. Hence, a local model of 18m OSD at the bridge mid-span is built using refined Shell63 element with detailed geometry, as shown in Fig. 5.7. The segment length of the local refined model is 6 times of the diaphragm spacing in order to eliminate the potential influence of the rigid region on the cutting boundary based on Saint-Venant’s principle. The dimensions of the cross section and the U-rib are shown in Fig. 5.8. Since a majority of heavy-loaded trucks are travelling on the slow lane as indicated in Table 5.1 and Fig. 5.2, the structural members under the slow lane is more likely prone to potential severe fatigue damage than those under the other two lanes. As a result, the present study will focus only on the structural members and details near the slow lane. In addition, since the pavement is not included in the FE model, a spreading angle of 45° for a vertical uniformly distributed wheel load is applied in the bridge deck. Considering the thickness of the pavement (5.5 cm) and the dimensions of the front and back wheels (contact area is 30cm×20cm and 60cm×20cm), the updated distribution load areas for the front and back wheels are calculated as 41cm×31cm and 71cm×31cm.
Firstly, we will discuss the results based on the multi-scale FEA under one representative loading case from the UDS samples. The loading parameters are: GVW=920 kN (V\text{6} truck), V\text{w}=15.6 m/s, H\text{s}=3.7 m, T\text{p}=7.4 s. As discussed earlier, both the wind and wave loads are applied laterally on the bridge without considering the wind and wave directions. The linear wave model is used to generate the random waves using the shallow water TMA spectrum for a given H\text{s} and T\text{p} [253]. The spectral representation method is adopted to simulate the stochastic wind fluctuations using the Kaimal spectrum and Lumley-Panofsky spectrum [26]. The time histories of wind and wave are used to calculate the structural loads applied on the bridge [198]. The traffic flow for each type of vehicle is simulated in accordance with the vehicle statistics as discussed previously. In addition, a constant vehicle speed of 20 m/s is adopted for all the simulations.
Fig. 5.8 Dimensions of OSD: (a) cross section; and (b) U-rib with typical fatigue-prone details

Fig. 5.9 compares the stress time histories for the U-rib butt joint under different combinations of the loading parameters, in order to clearly show their contributions to the stress histories. As shown in Figs. 5.9(a) ~ (c), the stress response due to the coupled wind and wave loads is larger than those caused by individual wind load or wave load. To observe the stress time history from the coupled VBWW dynamic analysis, the stress response including two $V_s$ truck passages is shown in Fig. 5.9(d). By comparing the Figs. 5.9(c) and (d), it is observed that when the additional truck loads are included, the stress level has been increased dramatically, i.e., the stress peak value increases from 11.9 MPa under coupled wind and wave loads to 62.9 MPa for coupled VBWW system. In addition, the stress response due to truck load only is also shown in Fig. 5.9(d) for comparison. It is shown in Fig. 5.9(d) that both the amplitude and the shape of the vehicle-induced stress time history are affected when the additional wind and waves are included in the analysis. For example, the stress peak value increases from 51.6 MPa under vehicle only loading scenario to 62.9 MPa for coupled VBWW system by 21.8%. For trucks with smaller GVWs, the wind and wave loads are found to have more profound influences on the vehicle-induced stress responses. The above observations indicate that all the external loads from the traffic, wind and wave can contribute to the
structural dynamic responses and it is necessary to include the coupled VBWW analysis for the fatigue evaluation.

Figure 5.9 Stress time history segments of butt joint of U-rib: (a) wind only; (b) wave only; (c) coupled wind and wave; (d) vehicle only and coupled vehicle-bridge-wind-wave

After the stress time histories are obtained, the fatigue stress ranges and the corresponding number of cycles can be calculated using the rain-flow counting method. Table 5.5 summarizes the equivalent stress range \( S_{re} \) and the number of stress cycles \( n_t \) in 10-minute duration with 10-\( V_6 \) truck passages. \( S_{re} \) and \( n_t \) calculated from the vehicle alone, wind alone, wave alone loading scenarios are also listed for comparison. It is observed that for all the three welded joints, the truck loads prone to induce large \( S_{re} \) with relative small \( n_t \), whereas the wind and wave loads are likely to cause small \( S_{re} \) with relative large \( n_t \). Since each individual load has different effects on the \( S_{re} \) and \( n_t \), the equivalent fatigue damage accumulation \( D \) has a better indication for the structural fatigue damage which combines the two essential parameters into one. By further comparing the \( D \) under each loading scenario, it is clear to quantify the contribution from each load to the structural fatigue damage. In this case study in particular, the truck load contributes most to the fatigue damage accumulation while the wave load contributes the least. The equivalent fatigue damage accumulation is utilized in the subsequent analysis.
Table 5.5 Summary of $S_{re}$, $n_t$ and $D$ in 10-minute duration with 10-$V_6$ truck passages ($S_{re}$: MPa)

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>U-rib but joint</th>
<th>Rib-to-diaphragm joint</th>
<th>Rib-to-deck joint</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{re}$</td>
<td>$n_t$</td>
<td>$D$</td>
</tr>
<tr>
<td>VBWW</td>
<td>26.47</td>
<td>240</td>
<td>6.17E-06</td>
</tr>
<tr>
<td>Vehicle alone</td>
<td>40.61</td>
<td>55</td>
<td>5.11E-06</td>
</tr>
<tr>
<td>Wind alone</td>
<td>13.32</td>
<td>210</td>
<td>6.88E-07</td>
</tr>
<tr>
<td>Wave alone</td>
<td>10.57</td>
<td>162</td>
<td>2.65E-07</td>
</tr>
</tbody>
</table>

5.5.2 Probabilistic Analysis

The probabilistic modeling of the equivalent fatigue damage accumulation is illustrated using the stochastic load models proposed in the earlier sections. In the present study, a total number of 300 training samples are adopted to construct the five SVR models, i.e., 60 samples for each SVR model that corresponds to one specific vehicle type. Taking the $V_6$ truck as an example, 60 training samples (denoted as U60) are first generated with the UDS scheme, which are uniformly distributed in the design space determined by the four influential parameters, i.e., GVW, $V_w$, $H_s$ and $T_p$. After performing the necessary FEAs 60 times, the SVR model can be established for approximating the response surface between the input loading parameters and the output daily equivalent fatigue damage accumulation. The SVR model of the U-rib butt joint under the $V_6$ truck, wind and wave loads is shown in Fig. 5.10. Fig. 5.10(a) shows the response surface of the GVW and $V_w$ when the wave loading parameters are supposed as $H_s=2.0$ and $T_p=6.5s$. It is observed that the response surface is nonlinear, i.e., the $D$ increases nonlinearly with the $V_w$ and GVW. The $D$ first increases slowly when both GVW and $V_w$ are small and then increases quickly as both GVW and $V_w$ go higher. In addition, the extreme wind alone loading condition can induce significant fatigue damage accumulation, e.g., the $D$ can reach $2.0\times 10^{-5}$ under wind speed of 25 m/s, which is comparable to that due to heavy truck load alone when GVW=600 kN. However, for even heavier truck load such that GVW $\geq$800 kN, the additional wind speed does not amplify the $D$ significantly.
In order to study the effects of the coupled wind and wave loads on the daily equivalent fatigue damage accumulation, the response surface of $V_w$ and $H_s$ is also presented by assuming GVW=100 kN and $T_p=6.5s$, as shown in Fig. 5.10(b). It is also found that the $D$ increases slowly when both the $V_w$ and $H_s$ are small, and then increases much faster with higher $V_w$ and $H_s$. It is also worth mentioning that the extreme wave alone can introduce large fatigue damage accumulation, i.e, $D$ reaches $2.1 \times 10^{-5}$ under $H_s=6.0m$. Furthermore, Fig. 5.10(c) also illustrates the response surface of the two wave loading parameters $H_s$ and $T_p$, when the wind speed and truck load are assumed as $V_w=2$ m/s and GVW= 50 kN. Different from the $H_s$ that large wave heights induce large fatigue damage accumulation, it is interesting to see the $D$ does not vary too much for small $H_s$. For large $H_s$, however, small $T_p$ can cause relatively large $D$ in general.
To quantitatively evaluate the prediction performance of the SVR model, three performance indices, i.e., root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE), are utilized [254]. These three commonly used performance indices are proposed to measure the deviation between the predicted and observed (refers to FEA results in the present study) values, and smaller indices usually indicate better prediction performance. These three performance indices are defined as,

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}
\]  
(5.10)

\[
MAE = \frac{1}{N} \sum_{i=1}^{N} |y_i - \hat{y}_i|
\]  
(5.11)

\[
MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\%
\]  
(5.12)

where \(y_i\) and \(\hat{y}_i\) (\(i=1\sim N\)) are the \(i\)th observed and predicted outputs and \(N\) is the number of predicted data.

It is noted that multivariate inputs are involved in the SVR model. For more clear and thorough presentation of the evaluation results in 3D domain, the validation case study takes two parameters as variables while the remaining two are assumed to be deterministic. Without loss of generality, the input parameters GVW (\(V_6\) vehicle) and \(V_w\) are treated as variables, and the input parameters \(H_s\) and \(T_p\) are assigned with constant values, i.e., \(H_s=2.0\) m, \(T_p=6.5\)s. To further investigate the influence of the number of training samples on the prediction performance of the SVR model, three different sets of training samples that consist of 30 samples (U30), 60 samples (U60), and 90 samples (U90), respectively, are adopted for comparison purpose. Subsequently, a new testing sample comprised of 60 samples is employed to evaluate the three established SVR models, in which the corresponding error measures are tabulated in Table 5.6. It is observed that compared with the SVR models with U60 and U90 training samples, the SVR model with U30 training samples generates the largest statistical errors in terms of all the three indices, indicating that the SVR model with U30 data has the worst performance. The reason is due to that 30 training samples are too sparse in the entire design domain to construct a SVR model with desirable prediction accuracy. In contrast, both U60 and U90 training samples are sufficient as the corresponding SVR models can generate
much lower statistical errors. Since the SVR models with U60 and U90 training samples have similar good performance, as all the three performance indices for both models are very low and close, U60 is selected in the present study to save computational cost. Fig. 5.11 displays the fitted SVR model as well as the training and testing samples computed from the FEAs. It is observed that the SVR response surface is close to all the samples with maximum absolute difference less than 1.3%, showing that the nonlinear characteristics of the output daily equivalent fatigue damage accumulation is well captured by the SVR model.

Table 5.6 Performance evaluations of SVR models ($V_6$ vehicle) with three different sets of training samples

<table>
<thead>
<tr>
<th>SVR model</th>
<th>RMSE ($\times 10^{-7}$)</th>
<th>MAE ($\times 10^{-7}$)</th>
<th>MAPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U30</td>
<td>6.33</td>
<td>3.59</td>
<td>2.58</td>
</tr>
<tr>
<td>U60</td>
<td>4.46</td>
<td>2.58</td>
<td>0.86</td>
</tr>
<tr>
<td>U90</td>
<td>4.45</td>
<td>2.58</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Figure 5.11 Fitted SVR model (for $V_6$ vehicle) and the FEA-based results

After a total number of 300 FEA simulations, five SVR models for five types of vehicles are established and utilized for probabilistic modeling of the daily equivalent fatigue damage accumulation, $D$,
with a large number of samples obtained from the established stochastic truck, wind and wave load models. It is found from the preliminary sensitivity analysis that the PDF of the predicted $D$ with 1 million samples is almost identical to that with 5 million samples, and therefore, the sampling number is selected as 1 million to provide accurate distribution of the predicted $D$. The PDF of the predicted daily equivalent fatigue damage in U-rib butt joint is show in Fig. 5.12, which is fitted by three proposed distribution models. It is found that the Weibull distribution fits best, which has a maximum log-likelihood value in the three models. The estimated Weibull parameters for all the three welded joints are shown in Table 5.7. Compared with the conventional MCS that requires a large number of samples for the FEA simulations that are extremely time consuming, the proposed numerical framework provides a more efficient way for probabilistic modeling of the daily equivalent fatigue damage accumulation.

![Figure 5.12 PDF of daily equivalent fatigue damage accumulation in U-rib butt joint](image)

**Table 5.7** Parameters in Weibull distributions of $D$ for three welded joints

<table>
<thead>
<tr>
<th>Welded joint</th>
<th>Scale parameter $\lambda$ ($\times 10^5$)</th>
<th>Shape parameter $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-rib butt joint</td>
<td>1.00</td>
<td>2.35</td>
</tr>
<tr>
<td>Rib-to-diaphragm joint</td>
<td>0.82</td>
<td>2.27</td>
</tr>
<tr>
<td>Rib-to-deck joint</td>
<td>0.73</td>
<td>2.32</td>
</tr>
</tbody>
</table>

### 5.5.3 Fatigue Life Estimation

The approximated PDF of the $D$ will provide a reasonable basis for the fatigue reliability evaluation of the critical welded joints. In addition, the other random variables and constants that contribute to the
fatigue damage are summarized in Table 5.8 [88,235]. With the limit-state function shown in Eq. (5.8) and the statistics of the random variables shown in Table 5.8, the fatigue reliability index of each welded joint is calculated, as shown in Fig. 5.13. It is observed that, among three welded joints, the U-rib butt joint and the Rib-to-deck joint is most and least prone to the fatigue damage. If the service life of the bridge is 100 years, the corresponding reliability indices for the U-rib butt joint, Rib-diaphragm joint, and Rib-to-deck joint in the 100th year are reduced to 2.4, 2.9 and 3.2, respectively.

Table 5.8 Statistics of random variables in limit-state function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Distribution</th>
<th>Mean</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\Delta}$</td>
<td>Critical fatigue damage</td>
<td>Lognormal</td>
<td>1.0</td>
<td>0.30</td>
</tr>
<tr>
<td>$A$</td>
<td>Fatigue detail constant</td>
<td>Lognormal</td>
<td>See Table 5.4</td>
<td>0.34</td>
</tr>
<tr>
<td>$D$</td>
<td>Daily equivalent fatigue damage accumulation</td>
<td>Weibull</td>
<td>See Table 5.7</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>Slope constant</td>
<td>Deterministic</td>
<td>3.0</td>
<td></td>
</tr>
</tbody>
</table>

Fatigue life can be estimated when a target reliability index ($\beta_{\text{target}}$) is provided. In the present study, $\beta_{\text{target}}$ is determined to be 2.0, which corresponds to a failure probability of 2.3% [255]. In addition, the traffic growth rates in terms of the traffic volume and the vehicle weight will grow in practice, which should also be considered for fatigue life estimation of the coastal bridges. Nevertheless, the traffic growth rates can be varied significantly depending on the geographic area, the proximity of growth rates as well as the socioeconomic conditions. Because no traffic growth analysis is available on the bridge site, a simple linear traffic growth rate is adopted in the present study for fatigue life estimation. The linear traffic growth rate model can be readily replaced in the future once the site-specific traffic growth rate model is developed. For demonstration purpose, the traffic volume is assumed to increase linearly with an annual growth rate of 1% and 2%, while the gross vehicle weight is assumed to increase linearly with an annual growth rate of 0.3% and 0.6%, respectively. As a result, the fatigue life of the three welded joints under $\beta_{\text{target}}$ with and without considering the influence of traffic growth are shown in Table 5.9. It is shown that the fatigue life for all three welded joints decreases noticeably after taking into account the traffic growth. Take the U-rib but joint as an example, the fatigue life drops from 119 year to 98 year with 1% annual traffic volume growth, and drops from 119 year to 96 year with 0.3% annual GVW growth.

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Figure 5.13 Fatigue reliability indices of three welded joints

Table 5.9 Fatigue life estimation ($a =$ traffic volume growth rate; $b =$ GVW growth rate)

<table>
<thead>
<tr>
<th>Welded joint</th>
<th>Fatigue life estimation (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a=b=0$</td>
</tr>
<tr>
<td>U-rib butt joint</td>
<td>119</td>
</tr>
<tr>
<td>Rib-to-diaphragm</td>
<td>141</td>
</tr>
<tr>
<td>Rib-to-deck</td>
<td>157</td>
</tr>
</tbody>
</table>

5.6 Summary

A numerical framework is developed for probabilistic fatigue damage evaluation of coastal slender bridges based on deterministic multi-scale FEA under stochastic vehicle, wind and wave loads. To efficiently predict fatigue damage probabilistically, a machine learning algorithm integrating the uniform design sampling method and the support vector regression (SVR) is proposed to avoid the time consuming MCS. A prototype cable-stayed bridge is presented to demonstrate the feasibility of the proposed framework.

The results from the deterministic FEA indicate that each load can contribute to the fatigue damage accumulation in various degrees and it is necessary to include the coupled VBWW analysis into the fatigue reliability analysis. After performing the deterministic FEAs with limited loading samples, the response surface between the multiple input dynamic loads and the structural equivalent fatigue damage accumulation is established using the machine leaning algorithm. With the established response surface, the fatigue damage accumulation under stochastic dynamic loads can be predicted and probabilistically
modeled, which are used for the subsequent fatigue reliability analysis. The results show that, among three critical welded joints, the U-rib butt joint and the Rib-to-deck joint is most and least prone to the fatigue damage. Further investigations on the impact of the traffic growth indicate that the increase in the traffic volume or the vehicle weight can result in significant reduction of the fatigue life of the welded steel bridge. With the proposed machine-learning based approach, the complex dynamic system can be simplified parametrically as response surfaces for multiple stochastic input parameters.

The present study has made important efforts on the fatigue damage evaluation of a coastal slender bridge in the context of complex vehicle-bridge-wind-wave interaction. One potential application of the proposed framework is that it enables real-time fatigue damage assessment with established learn functions to make effective decision making especially during or shortly after extreme human or natural disasters. Further efforts may be pursued in the future to improve the proposed framework in the following four aspects. First, this study utilizes only three influential parameters that contribute most to the fatigue damage, i.e., vehicle type, vehicle-occupied lane, GVW, for the traffic flow simulation. Future studies can be carried out to develop a more sophisticated stochastic traffic flow model, which is capable of simulating the vehicle behavior in a more rigorously way, e.g., accelerating/decelerating, lane changing and braking operations, to fully reflect the site-specific traffic condition. In addition, uncertainties associated with the structural parameters such as geometry, material and mechanical properties could be incorporated as well. Furthermore, the proposed framework can be extended to investigate the fatigue crack propagation by utilizing appropriate FEA and a fracture lime-state function. Finally, the copula-based concept can be extended to higher dimension to include the wind-wave misalignment in cases when the wind-wave misalignment cannot be ignored.
6 Fatigue Damage Diagnosis and Prognosis on Orthotropic Steel Bridge Deck Subject to Cyclic Truck Loads using Dynamic Bayesian Networks

6.1 Background

Civil infrastructures are subjected to continuous deterioration and fatigue damage accumulation under constant or variable stress cycles in their life-cycle operations. In recent decades, prognostics and health management (PHM) has emerged as a popular multidisciplinary engineering approach to provide early warning of failure, by integrating various technologies such as sensing, reliability, machine learning, failure physics and statistics. An effective PHM is expected to fulfill two interdependent tasks [256]: (1) Diagnosis: To identify and determine the relationship between the cause and the effect; and (2) Prognosis: To evaluate real-time system performance as well as predict the system’s future behavior and health state. Current prognostic approaches can be mainly categorized into two groups, physics-based approach and data-driven oriented approach. Recently, the hybrid prognostics approach that fuses the outputs from physical model with data-driven methodology is claimed to be more reliable and accurate, which has drawn increasing attention.

On the other hand, a crucial step to construct an effective PHM is to select an appropriate algorithm depending on algorithm suitability, system properties, and data characteristics. Numerous algorithms have been developed, each of which has its own applicability and limitations. In particular, the Bayesian network (BN) has been widely deployed for diagnosis and prognosis in PHM for representation, inference, and learning under uncertainty. BN is a directed acyclic graph (DAG) model that provides a compact and visual representation of joint probability distribution over a set of random variables using a set of nodes (or vertices) and edges (or arcs). In a BN, the nodes denote random variables, while the edges indicate their conditional dependence relationships. BN not only allows different types of random variables (discrete or continuous)
of various distribution types, but also enables incorporation of disparate sources of information such as laboratory data, operational data, reliability data, expert opinion as well as physical and empirical models [257]. A BN is updated through Bayesian inference upon the arrival of new data from any child node, and consequently, the uncertainty in the state variables can be reduced. To track the evolution of the health state of a time-dependent system, the static BN can be readily extended to a dynamic Bayesian network (DBN). A DBN consists of a sequence of discrete time steps (BNs) that are connected by additional edges between variables in adjacent BNs. The underlying first-order Markov assumption of DBN indicates that the BN in current time step depends only on the BN at the previous time step.

The ability to evaluate the time-dependent system with evolving state makes the DBN particular desirable for modeling deteriorated structures surrounding by various sources of uncertainties (e.g., material properties, environmental conditions) in component-, system-, and system-of-system-level. Straub [103] proposed a DBN-based framework for probabilistic predicting the fatigue crack with given inspection results. Later on, Zhu and Collette [258] improved the Straub’s model in terms of the modeling accuracy and efficiency through the use of a novel iterative discretization algorithm. Ma et al. [121] predicted the remaining bridge strength subject to corrosion damage by integrating the BN with in-situ load testing. Recently, Li et al. [257] proposed a framework for probabilistic health diagnosis and prognosis of aircraft components using a DBN, in which a particle filter algorithm is utilized for inference of the nonlinear and non-Gaussian hybrid DBN. Later on, Li and Mahadevan [259] developed a fast inference algorithm to enable converting the original complex multi-layer DBN of continuous variables into an equivalent simpler two-layer DBN. In addition to the applications in structural component level, increasing efforts have also been devoted to extend the current BN/DBN methodology to structural system- and system-of-system-level [110,123,260–263]. For example, Mahadevan et al. [110] proposed a BN-based approach to facilitate the structural system reliability assessment, in which the multiple failure sequences and correlations among component failures were considered based on the component deterioration limit states. Later on, Luque and Straub [123] proposed a DBN-based numerical framework to predict the fatigue reliability of deteriorating structural system, in which the dependence among deterioration at different components was modeled.
through a hierarchical model. Following the similar idea, a DBN-based framework integrating the physical model and the inspection results was applied for assessing the system reliability of a concrete bridge subject to corrosion [260,264].

Although the BN/DBN has many applications in modeling the deteriorated structures subject to fatigue and corrosion, very limited research have been focused on the fatigue damage diagnosis and prognosis on the orthotropic steel deck (OSD) subject to truck loads. The OSD has been widely employed in the girder structures of long-span bridges. As the top structure of bridge girder, the OSD contains lots of complex welded joints, which are very vulnerable to fatigue damage due to directly applied cyclic truck loads. There are strong needs to track the evaluation of its damage state in a timely manner in order to provide decision makers with strategies for a maintenance purpose.

This study proposes a DBN-based fatigue damage diagnosis and prognosis framework for OSD subject to cyclic truck loads aiming at: (1) diagnosis: track the evolution of the time-variant variables, i.e., fatigue crack growth, and calibrate the time-invariant (or deterministic) variables, i.e., plate thickness and multiplier for the crack shape factor; and (2) prognosis: predict the future crack growth. This chapter is organized as follows. First, the diagnostic and prognostic in the DBN is introduced including the implementation of the inference algorithm, i.e., particle filter. Next, a DBN model of the fatigue crack growth is established by including various sources of uncertainties. As a key step to construct the DBN model, the conditional probability distribution (CPD) of output stress response for the OSD under the input truck load and model parameters is needed. A Gaussian process (GP) surrogate model is then established to construct the CPD which is implemented in the DBN subsequently. Finally, the proposed framework is illustrated by a numerical examples of fatigue crack growth on the OSD under cyclic truck load. The diagnosis and prognosis results are presented and discussed.

6.2 Diagnosis and Prognosis in the DBN

Fig. 6.1 shows a DBN representation of a generic deterioration process, in which the state variables of a system evolve with time and the measurement data are obtained via noisy measurement made at each time step. The evolution of the system state can be expressed as,
\[
X_t = f(X_{t-1}, v_{t-1})
\]

(6.1)

in which \(X_t \in \mathbb{R}^m\) denotes the state variables at time step \(t\); \(f\) is a possibly nonlinear function of the state \(X_{t-1}\); \(v_{t-1} \in \mathbb{R}^m\) denotes the noise vector in the state function; and \(Z_t \in \mathbb{R}^m\) denotes the measurement that is obtained through measurement function,

\[
Z_t = h(X_t, n_t)
\]

(6.2)

where \(n_t \in \mathbb{R}^m\) is the measurement noise vector; and \(h\) is a possibly nonlinear function of measurement process.

**Figure 6.1** DBN representation of a generic deterioration process

In order to track the evaluation of \(X_t\) and \(Z_t\) in Fig. 6.1, Bayesian inference is required to fulfill the following two tasks: (1) forward propagation, i.e., predict the system state at current time step, \(X_t\), depending on the system state at previous time step, \(X_{t-1}\), and the CPD between adjacent networks; and (2) backward inference, i.e., update the joint distribution of the network. In this study, the prognosis and diagnosis are defined as:

(1) Prognosis step: a time step with purely forward propagation inference, which occurs when no observation is available or all the observations are for the root nodes;

(2) Diagnosis step: a time step involves with both forward propagation and backward inference, which occurs only when any child node is observed, i.e., whenever measurement is obtained.

Both the exact and approximate inference algorithms have been developed to achieve the aforementioned two interdependent tasks, each of which has its limitations. Exact inference algorithm, such as Kalman filter, is only applicable in a restrictive set of cases when the state function and/or the measurement function are linear (Eqs. (6.1) and (6.2)). To track the nonlinear system, several filter-based approximation algorithms are developed including the extended Kalman filter, unscented Kalman filter,
particle filter (PF), and approximate grid-based filter [265]. Among these filters, the PF is a general Mont Carlo (sampling) based method for DBN inference. PF is also known as “survival of the fittest”, in which a particle with higher weight is prone to be duplicated and a particle with lower weight is prone to be discarded [259]. One advantage of PF over the other algorithms is that the PF can handle both discrete and continuous DBNs of various DBN topology or CPD format. Thus, the PF is adopted as the Bayesian inference for DBN in the present study due to its flexibility. A brief introduction to PF and its implementation in DBN is presented in the subsequent sections.

6.2.1 Particle Filter (PF)

The basic idea of PF is to approximate the posterior density function of the state through a set of weighted particles or samples. The sequential importance sampling (SIS) is the most commonly used PF algorithm, in which the full joint posterior distribution at time step \( t, p(X_0|Z_{1:t}) \), can be approximated using a weighted set of particles \( \{x_{i,t}^j, \omega_i^j\}_{j=1}^N \) as,

\[
p(X_0|Z_{1:t}) \approx \sum_{j=1}^{N} \omega_i^j \delta_{x_{i,t}^j}
\]

where the lower case letter \( x \) denotes the particle, in which the superscript \( i \) represents \( i \)th particle and the subscript \( t \) represents time step \( t \); \( \omega_i^j \) is the weight of \( x_{i,t}^j \); \( \delta_{x_{i,t}^j} \) is a delta function at \( x_{i,t}^j \); and \( N \) is the number of particles.

The \( i \)th particle of new state \( X_i \) at time step \( t \), denoted as \( x_{i,t}^j \), is sampled from a proposal distribution using the current state \( X_{0:t-1}^i \) and the observation \( Z_{1:t} \) as parameters,

\[
x_{i,t}^j \sim q(X_i^j|X_{0:t-1}, Z_{1:t})
\]

and the weight at time step \( t, \omega_i^j \), can be computed recursively as [266],

\[
\omega_i^j \propto \omega_i^{j-1} \frac{p(Z_t|X_i^j)p(X_i^j|X_{i-1}^j)}{q(X_i^j|X_{i-1}^j, Z_t)}
\]

Note that the initial states \( X_0^i \) are sampled from the joint prior distribution of the state variables, with
an equal initial weight of $\omega_i^0 = 1/N$.

A common issue with SIS is the particle degeneracy phenomenon, i.e., after several iterations (Eqs. (6.4) and (6.5)), only a few particles have significant weights while the remaining particles have negligible weights. This will compromise the computational efficiency by devoting much computational efforts to update the particles with negligible contributions to the posterior distribution. The degeneracy issue can be resolved by performing resampling. The basic idea of resampling is to eliminate particles with small weights and to concentrate the particles with large weights. The resampling procedure involves generating a new set of equally-weighted $N$ particles by resampling (with replacement) $N$ times from the discrete approximations shown in Eq. (6.3). The newly sampled particles has the same posterior distribution as the old particles. The PF with resampling procedure is also named as sequential importance sampling with resampling (SIR). In the SIR, the prior (or transition) distribution is adopted as the proposal distribution,

$$q\left( X_i | X_{0:t-1}^i, Z_t \right) = p\left( X_i | X_{t-1}^i \right) \quad (6.6)$$

and consequently, the Eqs. (6.4) and (6.5) reduce to,

$$X_i^t \sim p\left( X_i | X_{t-1}^i \right) \quad (6.7)$$

$$\omega_i^t \propto p\left( Z_i | X_i^t \right) \quad (6.8)$$

Note that the resampling procedure is performed after computations of Eqs. (6.7) and (6.8) at each time step, in which new particles are generated and the associated weights are reset as $1/N$.

Although the prior distribution in Eq. (6.6) is not the optimal proposal distribution, it is intuitive and straightforward to implement the SIR algorithm, which only requires sampling from the prior distribution $p\left( X_i | X_{t-1}^i \right)$ and evaluating the likelihood $p\left( Z_i | X_i^t \right)$. However, the aforementioned resampling strategy introduces a new problem of particle impoverishment: the particles of larger weights are duplicated while the particles of smaller weights are discarded, which leads to loss of diversity among the new sampled particles. This is due to the fact that resampling procedure is performed according to the discrete distribution as shown in Eq. (6.3), rather than a continuous one. A potential solution to reduce the sampling
impoverishment is to replace the discrete approximation in Eq. (6.3) with a continuous approximation in the resampling stage. This modified SIR filter is the so-called regularized particle filter (RPF) algorithm. Accordingly, the Eq. (6.3) is modified as,

\[ p(X_t | Z_{1:t}) \approx \sum_{i=1}^{N} \alpha_i \kappa \delta_{\tilde{X}_t^i} \]  

(6.9)

in which the \( \kappa \) is a kernel function. In the case that all the particles are equally-weighted, the Epanechnikov kernel [267] is the optimal kernel as suggested by [265].

In summary, for a diagnosis step that involves with both forward propagation and backward inference, the SIR algorithm is adopted in the PF to update the weights of the particles according to Eqs. (6.7) and (6.8), followed by a resampling procedure to generate new particles according to Eq. (6.9). However, for a prognosis step with purely forward propagation, only Eq. (6.7) is required. The implementation procedure of PF for a complex DBN is presented as follows.

6.2.2 Implementing PF in DBN

The implantation of the PF algorithm to a DBN involves with two challenges. First, in addition to the time-variant dynamic nodes, there exists time-invariant static nodes that are shared by all the time steps such as the static node \( \theta \) as shown in Fig. 6.2(a), which violates the prerequisite assumption of the DBN, i.e., one separate BN for each time step. This challenge can be resolved by introducing additional identical static node \( \theta \) in the DBN. As shown in Fig. 6.2(b), an arrow directed from \( \theta_{t-1} \) to \( \theta_t \) between two adjacent BNs represents the deterministic relationship \( \theta_{t-1} = \theta_t \). The additional identical static node will not bring extra computational efforts, yet it fulfills the requirements of one BN for each time step and ensures that the same static node is shared by each time step.
Figure 6.2 Implementation of particle filter for a typical DBN

The other challenge is associated with the states of some dynamic nodes. In the current time step, their states depend on not only their states in the previous time step, but also some other variables in the current time step. For instance, as illustrated in Fig. 6.2(a), both dynamic nodes E and F are facing this challenge. Specifically, node $E_t$ depends on $E_{t-1}$ and $C_t$, and node $F_t$ depends on $F_{t-1}$ and $D_t$. As a result, $C_t$ ($D_t$) must be sampled before $E_t$ ($F_t$), indicating that the parents nodes of each state variables in $X_t$ needs to be identified before implementing Eq. (6.7). To realize Eq. (6.7), the state variables are divided into 5 groups as follows,

1. The first group, denoted as $\tilde{X}_{t-1}$, contains all the nodes in BN at time step $t-1$ that are the parent nodes of state variables in BN at time step $t$. As a result, Eq. (6.7) reduces to $X_t' = p(X_t' | \tilde{X}_{t-1}')$. As shown in the Fig. 6.2(a), only nodes $\theta_{t-1}$, $E_{t-1}$ and $F_{t-1}$ have arrows directed to nodes in the $X_t$ and therefore, $\tilde{X}_{t-1} = \{\theta_{t-1}, E_{t-1}, F_{t-1}\}$.

2. The second group, denoted as $\alpha_t$, contains the child nodes of $\tilde{X}_{t-1}$ in the BN at time step $t$. The sampling of $\alpha_t$ is based on the $\tilde{X}_{t-1}$ in the previous BN. According to Fig. 6.2(a), $\alpha_t = \{\theta_t, E_t, F_t\}$.

3. The third group, denoted as $\beta_t$, contains the intermediate nodes of $\alpha_t$, i.e., a node in $\beta_t$ has both the ancestor and the descendant nodes in $\alpha_t$. According to Fig. 6.2(a), $\beta_t = \{C_t\}$.

4. The fourth group, denoted as $\gamma_t$, contains the ancestor nodes of $\alpha_t$ or $\beta_t$ in the BN at time step $t$. The distribution of $\gamma_t$ is given by $p(\gamma_t)$. The sampling of $\alpha_t$ and $\beta_t$ is based on both the $\tilde{X}_{t-1}$ and $\gamma_t$, i.e., a CPD
\( p(\alpha_i, \beta_i | \bar{X}_{i-1}, \gamma_i) \). According to Fig. 6.2(a), \( \gamma_i = \{B_i, D_i\} \).

(5) The fifth group, denoted as \( \tau_i \), contains the descendant nodes of \( \alpha_i \) or \( \beta_i \) in the BN at time step \( t \). Thus, the sampling of \( \tau_i \), is based on \( \alpha_i \) or \( \beta_i \) through a CPD \( p(\tau_i | \alpha'_i, \beta'_i) \). According to Fig. 6.2(a), \( \tau_i = \{G_i, H_i\} \).

After node classifications, the state variables at time step \( t \) can be expressed as \( X_i = \{\alpha_i, \beta_i, \gamma_i, \tau_i\} \) and the sampling of \( X_i \) in Eq. (6.7) is performed sequentially by,

\[
\gamma_i \sim p(\gamma) \quad (6.10)
\]
\[
\alpha'_i, \beta'_i \sim p(\alpha_i, \beta_i | \bar{X}_{i-1}, \gamma_i) \quad (6.11)
\]
\[
\tau_i \sim p(\tau_i | \alpha'_i, \beta'_i) \quad (6.12)
\]

Following the aforementioned procedures and taking the DBN in Fig. 6.2(b) as an example, the generation of new particles \( X'_i = \{\theta'_i, B'_i, C'_i, D'_i, E'_i, F'_i, G'_i, H'_i\} \) based on \( \bar{X}_{i-1} = \{\theta'_{i-1}, E'_{i-1}, F'_{i-1}\} \) is as follows:

(1) Firstly, generate new particles \( \gamma'_i = \{B'_i, D'_i\} \) from \( p(\gamma_i) = p(B_i, D_i) = p(B_i)p(D_i | B'_i) \);

(2) Secondly, generate new particles \( \alpha'_i = \{\theta'_i, E'_i, F'_i\} \) and \( \beta'_i = \{C'_i\} \) from

\[
p(\alpha_i, \beta_i | \bar{X}_{i-1}, \gamma_i) = p(\theta | \theta'_{i-1})p(C_i | C'_i)p(E_i | E_i')p(F_i | F_i')p(D_i | D'_i).
\]

(3) Finally, generate new particles \( \tau'_i = \{G'_i, H'_i\} \) from \( p(\tau_i | \alpha'_i, \beta'_i) = p(G_i | E_i')p(H_i | F'_i) \).

### 6.3 DBN for Crack Growth on Orthotropic Steel Bridge Deck

The fatigue life is commonly predicted using either the conventional \( S-N \) approach or the fracture mechanics-based approach. The \( S-N \) approach employs the accumulation fatigue damage as the indicator for likelihood of crack occurrence. Unlike the \( S-N \) approach, the fracture mechanics-based approach is able to predict the evolution of the crack size, which is more suitable for evaluating the impact of crack on the structural fatigue performance especially when the crack measurements are available. Various fracture mechanics-based fatigue crack growth models have been proposed for predicting the long crack propagation.
such as the Paris’ law [268], the modified Paris’ law [269], and the Wheeler’s retardation model [270]. These models need to determine the stress intensity factor, which is often computed from the finite element analysis (FEA) that falls into two categories: (1) compute the stress intensity factor directly from the FEA that is capable of updating the time-variate crack geometry; and (2) compute the stress intensity factor using the nominal stress at the crack from the FEA without considering the crack geometry. The former method involves complex meshing around the crack location and rebuilding the FEA models to facilitate crack geometry updating, which is computational more expensive than the latter method. For higher computational efficiency, the latter method is utilized in the present study to establish the DBN-based fatigue crack growth model. Through the DBN, various uncertainty sources as well as the inspection results can be incorporated, which are introduced subsequently.

6.3.1 Uncertainty Sources

6.3.1.1 Uncertainty Source in FEA Model and Surrogate Model

Fig. 6.3 displays a FEA model of 18m-long orthotropic steel deck (OSD) that is established with shell63 element with detailed geometry in ANSYS. A five-axle fatigue truck load model is utilized to simulate the loading condition, and both ends of FEA model are fixed to simulate the boundary condition. The five-axle fatigue truck load is established to reflect the site-specific traffic conditions based on the weight-in-motion (WIM) measurements [271], as shown in the Fig. 6.4. Seven geometric parameters, denoted as $T_1$~$T_7$ and listed in Table 6.3, are assumed as the random variables in the FEA model, which correspond to the thickness of different plate of the OSD. All the seven geometric parameters have deterministic but unknown true values, which bring epistemic uncertainty. The epistemic uncertainty is due to the lack of data and/or knowledge and can be reduced. The prior distributions are assigned to them based on the experts’ judgment and the proposed DBN-based fatigue damage diagnosis and prognosis model aims to reduce their uncertainty through Bayesian inference.
Figure 6.3 Local FE model: (a) 18m-long steel deck segment; and (b) refined portion of the slow lane between the two diaphragms.

Figure 6.4 Fatigue truck-load model with gross vehicle weight (GVW) of 550 kN
The seven geometric parameters $T_1$~$T_7$ and the fatigue truck load GVW are the inputs to the FEA, and the output is the stress time history by performing the transient analysis to simulate the passage of the fatigue truck model. Subsequently, the computed stress response at the crack under the truck passage is used to calculate the equivalent nominal fatigue stress range $\Delta S$, based on Miner’s linear fatigue damage accumulation theory [81]. Because both the probabilistic prediction and the Bayesian inference require a large number of FEAs, a Gaussian process (GP) surrogate model [272] is established to substitute the FEA model to achieve computational efficiency. The training points for the GP model are obtained though repeatedly running the FEAs provided with a sufficient number of the input parameter combinations. After successful training, the predicted outputs $\Delta S$ at given inputs follow the Gaussian distribution, i.e., $\Delta S \sim N(\mu_{GP}, \sigma_{GP}^2)$, which represents the surrogate model uncertainty in computing the equivalent stress range at given inputs. In fact, GP surrogate model constructs the conditional probability distribution (CPD), i.e., $p(\Delta S | T_i, GVW) \ (i = 1 \sim 7)$, in the DBN, in which the seven geometric parameters $T_1$~$T_7$ and the truck load GVW are the parent nodes of equivalent stress range $\Delta S$. The GP surrogate model will be introduced in the section “Gaussian Process Regression”.

### 6.3.1.2 Crack Growth Model Uncertainty

According to the FEA, the U-rib butt joint at the middle of the slow lane is the most critical welded joint, where the fatigue crack initiation and propagation are likely to occur. Due to the fact that the U-rib butt welded joint is mainly subjected to tensile stress under the fatigue truck load, model I crack, i.e., opening mode that a tensile stress normal to the plane of the crack, is assumed for the sake of illustration. Therefore, the range of stress intensity factor in one time step can be computed as,

$$\Delta K = 1.2 F \Delta S \sqrt{\pi a^0} \quad (6.13)$$

where $\Delta S$ is the equivalent stress range as discussed previously; $a^0$ is the initial crack length in the current time step; $1.2F$ is the crack shape factor; and multiplier $F$ is adopted to introduce the uncertainty in the shape factor.

The calculated $\Delta K$ is then used to compute the long crack growth $\Delta a$ in each time step, based on the
Paris’ law in each time step,

\[
\frac{da}{dN} = C\Delta K^m
\]  

(6.14)

where \( C \) and \( m \) are the experimentally derived Paris’ constants that are adopted as \( C = 1.2 \times 10^{-9} \) and \( m = 3.0 \) as recommended by BS 7910 [273]; in the present study, \( C \) and \( m \) are treated as known constants but can be considered as random variables of unknown true values and included in the DBN for calibration purpose as desired; \( da/dN = \Delta a \) denotes the crack grow rate in one time step, which is used to calculate the crack length after the current time step \( a = a^0 + \Delta a \).

**6.3.1.3 Load Uncertainty and Crack Length Data Uncertainty**

In the present study, both the load and the crack length are measureable, i.e., the truck load model can be established through the WIM system, and the crack length data can be obtained from the in-situ inspections of the structural members. As such, both the load and crack length uncertainties are associated with the measurement errors, which are generally assumed to follow zero mean Gaussian distributions, i.e.,

\[
\varepsilon_{GVW} \sim N(0, \sigma^2_{GVW}), \quad \text{and} \quad \varepsilon_a \sim N(0, \sigma^2_a);
\]

the subscript GVW denotes the truck load and the subscript \( a \) denotes the crack length. It should also be noted that both the load measurement and crack length data are only available at sparse time steps, due to the fact that the WIM measurements and inspections are usually carried out in certain time intervals in practice.

**6.3.2 Construction of DBN Model**

Based upon the crack growth model and the various associated uncertainties as introduced previously, a DBN-based crack growth model is established as shown in Fig. 6.5. In the DBN model, all the essential variables, deterministic or stochastic, are represented by nodes, and the arrows among the nodes represent their CPDs or deterministic functional relations. Three types of nodes are used to construct the DBN: (1) an elliptical node with solid lines denotes a continuous random variable and the arrows towards it represent a CPD; (2) a triangular node (or functional node) denotes a deterministic variable and the arrows towards it represent a deterministic function; and (3) a rectangular node denotes an observed variable, i.e., load and the crack length, and an arrow towards it represents a CPD. In addition, solid arrows are adopted within
each BN slice, while dashed arrows are adopted to connect the nodes at two adjacent BNs. The symbols in Fig. 6.5 are summarized in Table 6.1. Note that the seven geometric properties should be represented using seven separate nodes in the DBN, and they are denoted as a single node $\theta$ only for illustration. It is also worth mentioning that each time step in Fig. 6.5 refers to one truck passage, in which the equivalent fatigue stress range $\Delta S$ and the corresponding number of cycles $n$ are used to predict the fatigue crack length after the current time step. The preliminary FEA results show that the number of cycles $n$ remains constant as $n=9$. Therefore, $n=9$ is used to calculate the crack growth at the current time step using Eq. (6.14). In cases when the $n$ varies under the input $\theta$ and GVW, the Gaussian classifier can be used to predict the distribution of $n$, which is similar to the GP surrogate model of $\Delta S$ as discussed in the next section.

Figure 6.5 DBN-based crack growth model
Table 6.1 Parameters in the DBN-based fatigue crack growth model

<table>
<thead>
<tr>
<th>Value</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>GVW</td>
<td>Truck load</td>
</tr>
<tr>
<td>GVW_{obs}</td>
<td>Truck load observation</td>
</tr>
<tr>
<td>ΔS</td>
<td>Equivalent stress range</td>
</tr>
<tr>
<td>a^0</td>
<td>Crack length before current time step</td>
</tr>
<tr>
<td>F</td>
<td>Multiplier for the crack shape factor</td>
</tr>
<tr>
<td>ΔK</td>
<td>Stress intensity factor range</td>
</tr>
<tr>
<td>Δa</td>
<td>Crack growth in current time step</td>
</tr>
<tr>
<td>a</td>
<td>Crack length after current time step</td>
</tr>
<tr>
<td>a_{obs}</td>
<td>Crack length observation</td>
</tr>
<tr>
<td>θ</td>
<td>Geometric properties</td>
</tr>
</tbody>
</table>

At any time step, all the root nodes in each BN are first assigned with prior distributions, followed by the subsequent uncertainty propagation or Bayesian inference. Except for the first time step, in which the prior distribution is predefined, the prior distribution of BN at time step \( t \) is computed through propagating the posterior distribution of previous BN at time step \( t-1 \), based on the state transition function between the two adjacent BNs. The prior distribution for the root nodes and the state transition function will be discussed in the section “Diagnosis and Prognosis Results and Discussion”.

6.4 Gaussian Process Regression

As discussed in the previous sections, one crucial step to construct the DBN-based fatigue crack growth model is to obtain the CPD, i.e., \( ΔS \sim N(\mu_{GP}, \sigma^2_{GP}) \), through a GP surrogate model. In this section, a brief introduction to the GP regression is presented.

A GP regression adopts a Gaussian Process as a prior to represent the distribution over the lateral functions. Suppose that a training data set \( D = \{(y_i, X_i)\}_{i=1,2,...,m} \) consists of \( m \) samples with \( d \)-dimensional feature space, for any set of input points \( X = \{x_1, x_2, ..., x_m\}^T (m \times d \text{ matrix}) \), the corresponding observed outputs \( y = \{y_1, y_2, ..., y_m\}^T (m \times 1 \text{ vector}) \) is assumed to follow a multivariate Gaussian distribution. A GP is completely specified by its mean function and covariance function, in which the function \( y = f(x) \) is written as \( f(x) \sim \mathcal{GP}(m(x), k(x, x')) \). The mean function \( m(x) \) encodes the central tendency of the function
that is usually regarded as zero, and the covariance function \( k(x, x') \) encodes the information about the shape and the structure of the function. Considering the observed data are associated with error or noise, the relationship between the observed output, denoted as \( y \), and the true output, denoted as \( f(x) \), of the system is assumed to be,

\[
y = f(x) + \varepsilon
\]  

(6.15)

where the noise term \( \varepsilon \) is introduced to represent the errors in numerical results or the noise in experimental data, which is assumed to be an independent, identically distributed Gaussian distribution, i.e., \( \varepsilon \sim N(0, \sigma_n^2) \) and \( \sigma_n^2 \) is variance. According to Eq. (6.15), the likelihood is given by,

\[
p(y|f) = N(y|f, \sigma_n^2 I)
\]  

(6.16)

where \( f = \{f(x_1), f(x_2), \ldots, f(x_m)\}^T \); and \( I \) denotes the \( m \times m \) unit matrix. The marginal distribution \( p(f) \) admits a zero mean Gaussian process with covariance determined by Gram matrix \( K \) [272],

\[
p(f) = N(f|0, K)
\]  

(6.17)

where \( K_{ij} = k(x_i, x_j) \). Since both the likelihood (Eq. (6.16) and the prior (Eq. (6.17)) follow the Gaussian distribution, the marginal distribution \( p(y) \) is written as,

\[
p(y) = \int p(y|f) p(f) df = N(f|0, K_y)
\]  

(6.18)

where \( K_y = K + \sigma_n^2 I \).

The fundamental assumption of a GP is that any set of the outputs follows the multivariate Gaussian distribution, as defined in Eq. (6.18). Accordingly, the joint probability distribution of the predictive vector \( y_* \) at target input vector \( x_* \), and the observed vector \( y \) at the training input vector \( x \), can be expressed as,

\[
\begin{bmatrix} y \\ y_* \end{bmatrix} = \begin{bmatrix} f \\ f_* \end{bmatrix} + \begin{bmatrix} \varepsilon \\ \varepsilon_* \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ k_* \end{bmatrix}, \begin{bmatrix} K_y & k_* \\ k_*^T & k_* + \sigma_n^2 \end{bmatrix} \right)
\]  

(6.19)

where \( y_* = f(x_*) \) is the latent function for the target input vector \( x_* \), and \( \varepsilon_* \) is the corresponding noise term; and \( k_* = [k(x_*, x_1), \ldots, k(x_*, x_m)]^T \) and \( k_{**} = k(x_*, x_*) \). Based on conditioning Gaussians, the predictive distribution \( p(y_*|y) \) is also a Gaussian process with its mean and covariance given by,
\[
m(x_*) = k^T_x K_y^{-1} y
\]

(6.20)

\[
\sigma^2(x_*) = k_{xx} - k^T_x K_y^{-1} k_x + \sigma_n^2
\]

(6.21)

As discussed earlier, the covariance function (or kernel function) encodes all the generalization properties of a GP model such that (1) it specifies the prior on the latent function we wish to learn such as smoothness; and (2) it also measures similarity between data points, i.e., the training points near a test point are informative for the prediction at that point [254]. Various types of covariance functions are available in the literature. To better capture the various characteristics of the data, a set of automatic relevance determination (ARD)-based covariance functions are used in the present study to construct the GPR model, as shown in Table 6.2 [272]. ARD refers to the inclusion of a separate length-scale for each input feature, which can provide more flexibility of the GP model. The length-scale can also reflect the relative importance of different input variable such that the input variable with a small length scale has more significant impact on the predictions. The covariance functions are typically defined by the so-called hyper-parameters, denoted as \( \Theta \), which are listed in Table 6.2 as well. In Table 6.2, the \( l_q (q=1\sim d) \) is the length-scale in all dimensions, \( \sigma_f^2 \) is the variance of the (noise free) signal, \( \sigma_n^2 \) is the noise variance, and \( \alpha \) is the scale parameter. The hyper-parameters can be determined through maximum long-likelihood estimations based on the training data.

\[
\log p(y|\Theta) = -\frac{1}{2} y^T K_y^{-1} y - \frac{1}{2} \log |K_y| - \frac{d}{2} \log (2\pi)
\]

(6.22)

where \( d \) is the dimension of the input variables; \( K_y = K + \sigma_n^2 I \) and \( K_{ij} = k(x_i, x_j) \) (see Table 6.2).

One advantage of the GP over other learning machines such as artificial neural network and support vector machine is that the GP model can quantitatively provide the uncertainties associated with the predictions [272], as specified in Eqs. (6.20) and (6.21). The uncertainty due to the GP surrogate model was discussed in the previous section “Uncertainty Sources”. As such, the prediction uncertainty due to GP surrogate model can be propagated in the DBN for fatigue diagnosis and prognosis.
Table 6.2 Automatic relevance determination (ARD)-based covariance functions

<table>
<thead>
<tr>
<th>ARD-based covariance function</th>
<th>Formula</th>
<th>Hyperparameters (Θ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARD-Squared Exponential (ARD-SE)</td>
<td>( K(x_i, x_j</td>
<td>\Theta) = \sigma_f^2 \exp\left(-\frac{1}{2} r^2\right) )</td>
</tr>
<tr>
<td>ARD-Rational Quadratic (ARD-RQ)</td>
<td>( K(x_i, x_j</td>
<td>\Theta) = \sigma_f^2 \left(1 + \frac{1}{2\alpha} r^2\right)^{-\alpha} )</td>
</tr>
<tr>
<td>^2AR-D-Matérn3</td>
<td>( K(x_i, x_j</td>
<td>\Theta) = \sigma_f^2 \left(1 + \sqrt{3} r\right) \exp\left(-\sqrt{3} r\right) )</td>
</tr>
<tr>
<td>^3AR-D-Matérn5</td>
<td>( K(x_i, x_j</td>
<td>\Theta) = \sigma_f^2 \left(1 + \frac{\sqrt{5} r + \frac{5}{3} r^2}{2\alpha^2\sigma_n^2}\right) \exp\left(-\frac{\sqrt{5}}{\alpha} r\right) )</td>
</tr>
</tbody>
</table>

^1The r in kernel functions is expressed as \( r = \sqrt{\sum_{q=1}^{d} \left(\frac{(x_{iq} - x_{jq})^2}{l_q^2}\right)} \);  
^2Matérn3 denotes the Matérn kernel with degrees of freedom \( \nu = 3/2 \);  
^3Matérn5 denotes the Matérn kernel with degrees of freedom \( \nu = 5/2 \).

### 6.5 Numerical Demonstration

A numerical example of crack growth on the OSD subject to cyclic truck load is used to demonstrate the proposed DBN-based fatigue damage prognosis and diagnosis framework. The GP surrogate model is first introduced, followed by the diagnosis and prognosis results and discussion.

#### 6.5.1 GP Surrogate Model

The OSD and the truck load model were explained in section “Uncertainty Source in FEA model and Surrogate Model”. Table 6.3 provides the true value (or nominal value) of the seven geometric variables. The procedures to construct the GP surrogate model are as follows,

1. First, 160 combinations of the seven geometric variables and the truck load are generated to constitute the input loading samples, which are uniformly distributed in the design domain. The possible range of each input parameter is its nominal value bounded with ±25% variations.

2. Second, for each loading combination, transient analysis is performed to calculate the stress response, which is further transformed to the equivalent fatigue stress range Δ\( S \) using Miner’s law. As an illustration, Fig. 6 displays the stress time history of the most critical welded joint using the nominal values of the input parameters. Again, it is noted that the corresponding number of cycles remains unchanged under FEAs, i.e.,
n = 9. Therefore, n is constant and included in the functional node Δa in the DBN, while the output response ΔS is considered as a random variable in the DBN.

(3) Finally, the 160 samples obtained from the FEAs are partitioned into two parts: the first 120 samples (75%) for training and the remaining 40 samples (25%) for testing. The training samples are used to construct the predictive GP surrogate model, and the testing samples are adopted for validation as well as to identify the most suitable covariance function.

Table 6.3 Geometric variables of the FEA model

<table>
<thead>
<tr>
<th>Geometric variable</th>
<th>Plate type</th>
<th>Nominal thickness (×10^{-3} m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₁</td>
<td>U-rib</td>
<td>8</td>
</tr>
<tr>
<td>T₂</td>
<td>Deck (slow lane)</td>
<td>18</td>
</tr>
<tr>
<td>T₃</td>
<td>Deck (fast &amp; middle lane)</td>
<td>16</td>
</tr>
<tr>
<td>T₄</td>
<td>Diaphragm</td>
<td>14</td>
</tr>
<tr>
<td>T₅</td>
<td>¹Bottom plate (type 1)</td>
<td>16</td>
</tr>
<tr>
<td>T₆</td>
<td>Bottom plate (type 2)</td>
<td>12</td>
</tr>
<tr>
<td>T₇</td>
<td>Stiffener at bottom plate</td>
<td>6</td>
</tr>
</tbody>
</table>

¹The bottom plate is comprised of two types of plates in the OSD.

Figure 6.6 Stress-time history for the critical U-rib butt joint under one passage of fatigue truck using nominal values of the input parameters (vehicle speed V=20m/s)

To quantitatively evaluate the performance of the proposed covariance functions, the root mean square error (RMSE) is utilized as the performance index [254]. The RMSE is proposed to measure the deviation...
between the GP surrogate model predictions (predictive values) and the FEA results (observed values), and a smaller RMSE usually indicates better predictive performance.

\[
\text{RMSE} = \sqrt{\frac{1}{N_0} \sum_{i=1}^{N_0} (y_i - \hat{y}_i)^2}
\]

(6.23)

where \(y_i\) and \(\hat{y}_i\) \((i=1\sim N_0)\) are the \(i\)th observed and predicted output and \(N_0\) is the number of predicted data.

The GPML (Gaussian processes for machine learning) toolbox developed by Rasmussen and Nickisch [274] is used to establish the GP surrogate model in MATLAB. The corresponding error measures of the four proposed ARD-based kernels are tabulated in table 4. It is observed that the ARD-Matérn3 kernel outperforms the other three kernels, as it can generate the lowest RMSE. As a result, the ARD-Matérn3 kernel is selected for the subsequent analysis.

**Table 6.4** Performance evaluations of different covariance functions

<table>
<thead>
<tr>
<th>Covariance function</th>
<th>RMSE (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARD-SE</td>
<td>0.6817</td>
</tr>
<tr>
<td>ARD-RQ</td>
<td>0.4724</td>
</tr>
<tr>
<td>ARD-Matérn3</td>
<td>0.4676</td>
</tr>
<tr>
<td>ARD-Matérn5</td>
<td>0.4948</td>
</tr>
</tbody>
</table>

**Figure 6.7** Natural logarithm of length-scale for the input geometric variables

Not all the input parameters, i.e., \(T_i (i=1\sim7)\) and GVW, are equally important to the output equivalent fatigue stress \(\Delta S\), and the resultant crack growth. The length-scale can be utilized to evaluate the contribution of each input parameter to the uncertainty in the output \(\Delta S\), and smaller length-scales indicate
higher contributions. The natural logarithm values of the characteristic length-scales for the input variables are displayed in Fig. 6.7. It can be observed that the fatigue truck load (GVW), the thickness of the U-rib ($T_1$), and the thickness of the deck plate ($T_2$) are much smaller than the other five parameters, indicating that they have more significant impact on the output $\Delta S$. Sensitivity analysis results show that the GP model considering only the randomness of GVW, $T_1$ and $T_2$ while fixing the $T_3$ to $T_7$ at their nominal values has a very similar prediction performance to the GP surrogate model that treats GVW and all $T_1$-$T_7$ as random variables (differences are within 1.5%). As a result, the parameters of low sensitivity, i.e., $T_3$ to $T_7$, can thus be fixed at their nominal values, to save the computational cost in the subsequent DBN prognosis and diagnosis in the present study. In addition, Fig. 6.8 shows the fitted GP model as well as the training and testing samples computed from the FEAs. In Fig. 6.8, the GVW is treated as deterministic for better demonstration in 3D domain. It is observed that the GP response surface is close to all the samples with maximum difference less than 1.6 %, showing that the nonlinear characteristics of the output response $\Delta S$ is well captured by the GP surrogate model with ARD-Matérn3 kernel.

**Figure 6.8** Comparison of the fitted GP surrogate model and the FEA results
6.5.2 Diagnosis and Prognosis Results and Discussion

The main purpose of the proposed DBN-based fatigue damage diagnosis and prognosis framework is to track the evaluation of the fatigue crack growth in 12,000 steps and calibrate the true values of the two geometric parameters, $T_1$ and $T_2$, as well as the multiplier for the crack shape factor, $F$. As discussed previously, the prior distributions of the root nodes and the state transitions need to be defined first before performing the Bayesian inference. In the present study, the prior distributions of the root nodes are assumed to be: $T_{1, n=1} \sim U (0.006, 0.009), T_{2, n=1} \sim U (0.014, 0.020), F_{n=1} \sim N (0.8, 0.08^2), a^0_{n=1} \sim N (0.01, 0.0001^2)$ and $\text{GVW}_{n=1} \sim N (550, 20^2)$. The transition functions from $t-1$ to $t$ of the crack length and the truck load are defined as follows. As shown in Fig. 6.5, $a^0_t$ is the initial crack length at current time step $t$, which depends on the crack length at previous time step, $a_{t-1}$, and the measurement at previous time step, $a_{obs, t-1}$ (if available). Thus, $a^0_t$ is defined as: $a^0_t = a_{t-1}$, if $a_t$ is not measured; and $a^0_t = a_{obs, t-1} + N (0, \sigma^2)$, if $a_t$ is measured. The transition function of the truck load at time $t$ is assumed to follow $p(GVW_t | GVW_{t-1}) = GVW_{t-1} + N (0, 20^2)$.

In addition, the observed truck load data, $\text{GVW}_{obs}$, are assumed to be available at the first 6,000 steps; and the crack length data, $a_{obs}$, are assumed to be available at five specific time steps, $t=2,000, 4,000, 5,600, 7,200$, and $8,400$. All the observed data are synthetic which are calculated based on the DBN in Fig. 6.5 using the assumed true values (nominal values). The nominal values of $T_1 \sim T_7$ are listed in Table 6.3, the nominal values of $F$ is $F=0.75$, and the nominal value for $a^0_t$ is $a^0_t = 0.01$, respectively. Also note that the observed crack length is calculated by adding the measurement noise (a zero mean Gaussian noise) to the true crack length, i.e., $a_{obs} = N (a, \sigma^2)$. In addition, the number of particles in the present study is selected as $N=1.5 \times 10^4$ and the overall computational time is about 6.6 h. The diagnosis and prognosis results based on these predefined prior distributions, transition functions as well as the synthetic observations are shown in Figs. (6.9) ~ (6.12), which are discussed as follows.

Fig. 6.9(a) shows the diagnosis and prognosis of the $T_1$ at each time step. Due to its high sensitivity in
the resultant stress and the crack growth, the uncertainty of \( T_1 \) is reduced significantly just after first inspection of the crack length at time step \( t=2,000 \). The final posterior distribution of the \( T_1 \) after the fifth inspection at time step \( t=8,400 \) is \( T_1 \sim N(0.00803, 0.0002^2) \). Recall that the prior distribution of \( T_1 \) is \( U(0.0006, 0.0009) \) and the true mean is 0.0008. Therefore, the posterior mean is very close to the true value, and the standard deviation is also be reduced significantly. In addition, the posterior distribution of \( T_1 \) at each time step of inspection is also depicted in Fig. 6.9(b). Similar findings are also observed that after 5 inspections of crack length, the posterior mean is very close to the true value with small variation.

![Diagram](https://via.placeholder.com/150)

**Figure 6.9** Updating of \( T_1 \): (a) Diagnosis and prognosis of \( T_1 \); and (b) PDF of \( T_1 \) at each time step of inspection

It should be noted that the distribution of \( T_1 \) is only updated at the time steps when the observed crack length data are available, due to the following two reasons. First, in the prognosis step, only propagation is performed which does not change the distribution of \( T_1 \), as indicated by the state function \( T_{1,t} = T_{1,t-1} \) (\( T_1 \) is in the static node \( \theta \)) in Fig. 6.5. Second, the observed load data \( GVW_{obs} \) cannot be used to update the distribution of \( T_1 \), because the node \( GVW_{obs} \) is independent of \( T_1 \) due to the rules of d-separation [112].
The \textit{d-separation} is due to the fact that the trail between GVW_{obs} and \( T_1 \) has a \( v \)-structure, i.e., \( \text{GVW}_{\text{obs}} \rightarrow \ldots \rightarrow a_{\text{obs}} \leftarrow \ldots \leftarrow T_1 \). Similar to \( T_1 \), the trail between node GVW_{obs} and the node \( T_2 \) or node \( F \) also has a \( v \)-structure, and therefore, the \( T_2 \) and \( F \) can only be updated at the time steps when crack length data are available, as well.

Similarly, Figs. 6.10(a) and (b) depict the diagnosis and prognosis, and the posterior PDF of the \( T_2 \), respectively. As shown in Fig. 6.10(a), the uncertainty of \( T_2 \) is reduced significantly at the first inspection of time step \( t=2,000 \). According to Fig. 6.10(b), the final posterior distribution after the fifth inspection of crack length is obtained as \( T_2 \sim N(0.01803, 0.00037^2) \), indicating the posterior mean is very close to the true mean value of 0.018 with a small variation.

Figure 6.10 Updating of \( T_2 \): (a) Diagnosis and prognosis of \( T_2 \); and (b) PDF of \( T_2 \) at each time step of inspection

In addition to geometric parameters \( T_1 \) and \( T_2 \), the diagnosis and prognosis of the multiplier of the crack shape factor is also performed, and the results are displayed in Figs. 6.11(a) and (b). Recall that the prior distribution of the \( F \) is \( N(0.8, 0.08^2) \) and the true mean is 0.75. According to Fig. 6.11(b), the posterior
distribution of $F$ is obtained as $N(0.7509, 0.024^2)$, showing that after the diagnosis and prognosis, the posterior mean is very close to the true mean value and the standard deviation of the prior distribution is reduced by 70.0%.

![Graph](image)

**Figure 6.11** Updating of $F$: (a) Diagnosis and prognosis of $F$; and (b) PDF of $F$ at each time step of inspection

Finally, the diagnosis and prognosis results of the crack length is displayed in Fig. 6.12. It is observed that the uncertainty in the crack length accumulates gradually during each inspection interval due to the uncertainty propagation, and is reduced to measurement error upon arrival of each inspection. It is also observed that the uncertainty in crack length during the first inspection interval has the highest accumulation rate (widest bounds) compared with other inspection intervals. In addition, the predicted mean also deviates from the true value in the first inspection interval. This is due to the large uncertainties in geometric parameters $T_1$, $T_2$, and the multiplier $F$, as shown in Figs. (6.9) – (6.11). After the first inspection, however, the predicted mean crack length agrees well with the true value, and the prediction uncertainty accumulates much slower in the subsequent inspection intervals compared with the first inspection interval. The smaller
uncertainty with slow accumulation rate is due to the following two reasons: (1) the uncertainties in $T_1$, $T_2$, and $F$ are reduced through the diagnosis using the inspection results and the corresponding posteriors are close to the true values, as shown in Figs. (6.9) ~ (6.11); (2) whenever the observed crack length is available, it will be used to construct the prior distribution of the initial crack length in the next time step with assumed measurement error, i.e., $a_{s1} \sim N\left(a_{obs}, \sigma_a^2\right)$. After the fifth inspection at time step $t=8,400$, no further inspection is performed and the time steps beyond $t=8,400$ are prognosis steps with purely uncertainty propagation. Nevertheless, the accumulation rate is slow and the predicted mean value agrees well with the true mean, due to the significant reduction of the uncertainties associated with $T_1$, $T_2$, and $F$ in the diagnosis stage during time steps $t=[2,000, 8,400]$.

![Crack growth vs. Time step](image)

**Figure 6.12** Diagnosis and prognosis of crack length

### 6.6 Summary

As one important component of long-span bridges, the OSD often suffers from severe fatigue damage accumulations due to cyclic truck loads. Nevertheless, the damage accumulation process is random in nature due to various aleatory and epistemic uncertainties, which could affect the diagnosis of the existing damages and prognosis of future health conditions. The present study proposes a dynamic Bayesian network (DBN)-based fatigue damage diagnosis and prognosis framework for the OSD under cyclic truck load. The particle filter (PF) is implemented to perform the Bayesian inference of the established non-Gaussian DBN
of arbitrary topology. The PF is a sample-based inference algorithm which is particularly suitable for Bayesian inference of DBN comprised of both discrete and continuous variables with various distribution types, as well as the nonlinear functional relationships among them. In addition, a Gaussian process (GP) surrogate model is established to relate the output stress response of the physics model (OSD) under the input truck load and model parameters, which will be implemented as the conditional probability distribution (CPD) in the DBN model. Meanwhile, various uncertainties involved in the FEA model, GP surrogate model, crack growth model as well as the load and crack length inspections are incorporated in the DBN model as well. Finally, the proposed framework is illustrated by a numerical example of fatigue crack growth on the OSD. The results indicate that the proposed framework is capable of (1) diagnosis: track the evaluation of the crack growth and reduce the uncertainty in time-invariant geometric parameter \( T_1, T_2 \), and the crack shape multiplier \( F \); and (2) prognosis: predict the time-variant crack growth in the future.
7 Summary of the Dissertation and Future Studies

This dissertation has developed a list of versatile and efficient numerical schemes to enable dynamic performance and fatigue damage evaluations for coastal slender bridges subject to traffic and correlated wind and wave loadings during the life cycle. The dynamic performance evaluations of bridges along with the travelling vehicles are approached by developing a coupled vehicle-bridge-wind-wave (VBWW) dynamic system that systematically integrates the complex interactions among the vehicle, bridge, wind, and wave. To facilitate fatigue damage evaluation on the orthotropic steel deck (OSD), two probabilistic fatigue damage assessment schemes are developed based on machine learning algorithms. Firstly, fatigue reliability evaluation of an OSD is proposed considering life-cycle stochastic dynamic loads by integrating the multi-scale FEA and the support vector machine (SVM). Secondly, fatigue damage diagnosis and prognosis of an OSD are proposed by integrating the physics-based model with data from field inspections while accounting for the associated uncertainties, using the dynamic Bayesian network (DBN). The main accomplishments and innovations of the dissertation, and the possible improvements in future studies are summarized as follows.

7.1 Main Accomplishments and Innovations of the Dissertation

7.1.1 Numerical Simulation of Correlated Wind and Wave Fields to Facilitate the Structural Dynamic Analysis

Chapter 2 developed a numerical scheme to simulate the nonstationary wind and wave fields around a coastal slender bridge during hurricane events with sufficient spatial-temporal resolution, which can be directly employed as the dynamic input for the coupled structural dynamic analysis. The correlated wind and wave simulation procedure involves with two consecutive steps: (1) the simulation of the near-surface time-varying mean wind; and (2) the simulation of nonstationary wind fluctuations and wave field. The time-varying mean wind, regarded as a deterministic function, is modeled through the use of parametric
hurricane wind model, which assumes the near-surface wind field to be the sum of a storm-wind component determined by storm gradient wind profile, and a background-wind component related to storm translation velocity. Due to the effect of surface friction, the modification of the magnitude and the direction of the two wind components are applied. Subsequently, the modeled time-varying mean wind is used to derive the EPSDs for the simulation of stochastic wind fluctuation and wave processes. The nonstationary wind fluctuation is modeled as a uniformly modulated evolutionary vector stochastic process. For the nonstationary wave, its EPSD is obtained by directly extending from current stationary wave spectrum. With the EPSD, both the time histories of nonstationary wind fluctuation and wave surface elevation can be generated by SRM. The proposed numerical scheme could be applied to any other large structures, such as offshore wind farms, coastal power transmission line networks, etc. In addition, the proposed simulation scheme could also be combined with different hurricane tracking models, varied vertical and radial wind profiles, different coherences functions, and PSD functions for wind fluctuations and waves.

7.1.2 Coupled Dynamic Analysis of Vehicle-Bridge-Wind-Wave System

Chapter 3 developed a general analytical VBWW platform that systematically incorporates the complicated interactions among bridge structures, running vehicles, and wind and wave dynamic excitations. Firstly, the bridge is discretized using finite element method and vehicles are modeled as mass-spring-damper systems in order to build the equations for the dynamic equilibrium. The time histories of the wind and wave around the bridge site are simulated as stochastic random processes and generated using SRM. The dynamic system integrates the conventional buffeting analysis for the wind-bridge interaction, the quasi-static analysis for the wind-vehicle interaction, and dynamic interaction between the moving vehicles and bridge based on the geometric and mechanical relationships between vehicle tires and the bridge deck. Additionally, the interaction between the wave and bridge pile group foundation is included in the system using Morison equation. Based upon the established VBWW system, comprehensive analyses are performed to investigate the dynamic characteristics of the coastal slender cable-stayed bridge under various combinations of vehicle, wind and wave loadings.
7.1.3 **Evaluation Methodology for Vehicle Ride Comfort and Driving Safety Analysis**

Chapter 4 proposed a comprehensive evaluation methodology on vehicle ride comfort and driving safety on the slender coastal bridges based on the VBWW platform developed in Chapter 3. Different with many existing studies on the vehicle ride comfort and driving safety, the effect of the wave loads is discussed. Based upon the guidelines as recommended in the ISO 2631-1 standard in the context of VBWW system, several essential evaluation criterions, such as the whole-body vibration response, the frequency weighting the original response, and the OVTV (overall vibration total value), are used for vehicle ride comfort evaluation. In addition, to facilitate the vehicle driving safety analysis, two evaluation criteria, i.e., the roll safety criteria (RSC) and the sideslip safety criteria (SSC), are developed based on the vehicle contact force responses at the wheels. The proposed methodology is applied to a long-span cable-stayed bridge for the vehicle ride comfort and driving safety evaluation under various loading scenarios.

7.1.4 **Fatigue Reliability Evaluation of OSD Considering Life-Cycle Stochastic Dynamic Loads**

Chapter 5 proposed a probabilistic fatigue damage evaluation framework for coastal slender bridges based on deterministic multi-scale FEA under stochastic vehicle, wind and wave loads. To efficiently predict fatigue damage, a machine learning algorithm integrating the uniform design sampling method and the support vector regression (SVR) is proposed to avoid the time consuming Monte-Carlo simulations (MSC). Firstly, stochastic load models are developed based on the long-term field measurements for realistic modeling the truck load and the correlated wind and wave load, which serve as the input for the VBWW system to extract the stress time histories at critical structural details using multi-scale FEA. After calculating the equivalent stress range and the corresponding number of cycles using the rain-flow counting method, the daily equivalent fatigue damage is obtained using the linear fatigue damage rule. To reduce the calculation cost, a machine learning algorithm is utilized for probabilistic modeling of the daily equivalent fatigue damage by integrating uniform design and support vector regression to link the multiple random inputs of environmental loadings with the single output of the stress time history. The fatigue life of critical
structural details, therefore, can be obtained using the established limit-state function with a target reliability index. With the proposed machine-learning based approach, the complex dynamic system can be simplified parametrically as response surfaces for multiple stochastic input parameters. With established learned functions, the real-time fatigue damage assessment can be achieved for complex structures or system to make effective decision making especially during or shortly after extreme human or natural disasters.

7.1.5 Fatigue Damage Diagnosis and Prognosis on OSD Subject to Cyclic Truck Loads using Dynamic Bayesian Networks

Chapter 6 proposed a framework for fatigue damage diagnosis and prognosis of an OSD through integrating the physics model with field inspections while accounting for the associated uncertainties, using the dynamic Bayesian network (DBN). The DBN is suitable for representation and reasoning under uncertainty in various fields. The proposed framework aims to fulfill two interdependent tasks (1) diagnosis: track the evolution of the time-variante crack growth and calibrate the time-invariant geometric parameters; and (2) prognosis: predict the crack growth in the future. The particle filter (PF) is employed as the Bayesian inference algorithm for the established non-Gaussian DBN composed of continuous variables of various distribution types, with nonlinear conditional dependence relationships among them. In addition, a Gaussian process (GP) surrogate model is established to relate the output stress response of the physics model (OSD) under the input truck load and model parameters, which will be implemented as the conditional probability distribution (CPD) in the DBN model. Finally, the proposed framework is illustrated by a numerical example of fatigue crack growth on the OSD subject to cyclic truck loads.

7.2 Possible Improvements of the Dissertation and Future Studies

The dissertation has made some promising progress on the dynamic performance and fatigue damage evaluation of the coastal slender bridges during life cycle under the traffic, wind and wave loads. Several potential future works worth being pursued are discussed as follows.

7.2.1 Sophisticated Wind-Wave Correlation Model

In Chapter 2, a numerical framework is developed to enable the simulation of correlated wind and
wave fields in hurricane event. Several assumptions are made to simplify the simulation: (1) the conventional PSDs derived from the normal conditions are adopted to develop the ESPDs for the correlated wind and wave in hurricanes; (2) Davenport’s coherence function is adopted to consider the spatial correlation of the wind field; (3) the wind-wave directionality is ignored. To release these constraints, further efforts may be devoted to develop more sophisticated numerical models by integrating with advanced SHM system instrumented in the bridge and/or Buoy system in coastal regions. The newly developed numerical models are expected to be able to reveal the energy transfer and spatial-temporal correlations between the wind and wave in both normal and extreme events more realistically and accurately.

7.2.2 Structural Dynamic Analysis of Coastal Bridges under Various Hurricane Events

In chapter 3, the dynamic characteristics of a prototype coastal cable-stayed bridge under vehicle, wind and wave loads are investigated. During the analysis, the wind and wave loads are assumed to apply on the bridge laterally, i.e., normal to the axis of the bridge deck, to present the least favorable loading scenarios. In additions, only limited loading scenarios with various combinations of vehicle load, wind speed, wave height are considered. For more thorough structural dynamic analysis considering all the possible hurricane events, further efforts may be devoted to: (1) incorporate the wind-wave directionality effects into the analysis. This can be achieved using sophisticated numerical model in Section 7.2.1; (2) perform bridge-wind-wave coupled dynamic analysis using synthetic or realistic hurricane tracks to generate the structural response surface. The hurricane inventory can be obtained using advanced atmospheric model, while the structural response surface can be achieved using statistical models such as machine learning, Bayesian network, etc.; (3) investigate the dynamic behavior of various types of bridges (e.g., mid-span and short-span bridges) in response to same loading scenarios; (4) in addition, the aerodynamic characteristic of the slender bridges during hurricane induced strong wind field should also be addressed.

7.2.3 More Refined Vehicle Model and More Well-established Vehicle Driving Behavior Model

In chapter 4, the vehicle ride comfort and running safety are evaluated by integrating the VBWW platform with the state-of-art vehicle evaluation criterion. During the simulation, (1) the road vehicles are
idealized as a combination of rigid bodies and mass axles connected by a series of springs and dampers; (2) a simple vehicle driver behavior is adopted; (3) a deterministic traffic flow is adopted. For more comprehensive evaluation, further efforts may be devoted to: (1) develop more sophisticated vehicle dynamic models or detailed finite element models alternatively, to obtain more accurate vehicle response; (2) develop more realistic vehicle driver behavior model by integrating the drivers’ planning capacities and decision making strategies; (3) develop a stochastic traffic flow framework which is capable of simultaneously including various vehicle types with different configuration, speed, position along the deck, etc., to reflect the traffic condition more realistically.

7.2.4 Further Improvements on the Proposed Probabilistic Fatigue Damage Evaluation Framework on the OSD Subject to Stochastic Dynamic Loadings

In chapter 5, important efforts have been made on the fatigue damage evaluation of a coastal slender bridge in the context of VBWW system. Further efforts may be pursued in the future to improve the proposed framework in the following four aspects. (1) First, this study utilizes only three influential parameters that contribute most to the fatigue damage, i.e., vehicle type, vehicle-occupied lane, gross vehicle weight, for the traffic flow simulation. Future studies can be carried out to develop a more sophisticated stochastic traffic flow model, which is capable of simulating the vehicle behavior in a more rigorously way, e.g., accelerating/decelerating, lane changing and braking operations, to fully reflect the site-specific traffic condition. (2) In addition, uncertainties associated with the structural parameters such as geometry, material and mechanical properties could be incorporated as well. (3) Furthermore, the proposed framework can be extended to investigate the fatigue crack propagation by utilizing appropriate FEA and a fracture lime-state function. (4) Finally, the copula-based concept can be extended to higher dimension to include the wind-wave misalignment in cases when the wind-wave misalignment cannot be ignored.

7.2.5 Further Improvements on the DBN-based Fatigue Diagnosis and Prognosis Framework

In chapter 6, a probabilistic DBN-based fatigue diagnosis and prognosis framework is proposed to
track the fatigue damage evolution of OSD under various sources of uncertainties. Further efforts may be pursued in the future to improve the proposed framework in the following three aspects. (1) First, the particle filter (PF) used for Bayesian inference is a sample-based algorithm, which has computational issues. For a DBN comprised of large number of nodes, the computational cost will increase exponentially as more particles are required to cover the sampling space of the system state. Future efforts may be devoted to develop more efficient inference algorithms with the same capability as PF to handle non-linear and/or non-Gaussian hybrid DBNs comprised of both continuous and discrete variables of various distribution types. (2) Second, the DBN-based framework is established based on the FEA without considering the crack geometry. Future efforts may be pursued to incorporate the crack geometry updating in the current framework using more sophisticated FEA. (3) Finally, the proposed framework is applicable at structural component level. Nevertheless, the fatigue reliability analysis in system- or system-of-systems are also required, as complex structures/systems with multiple components or multiple failure mechanisms are often involved. Therefore, the extension of current DBN-based framework from component-level to system- or system-of-systems could be included in the future studies as well.

7.2.6 Multi-Hazards Analysis by Incorporating More Types of Natural Hazards.

In this dissertation, the hurricane associated wind and wave are the major hazardous events been considered. Nevertheless, during the life cycle, the coastal bridges are also highly likely to encounter other types of hazards, such as earthquakes, tsunamis, storm surge, floods, etc. Typical natural hazard modes associated with dynamic loading for structural reliability analysis can be categorized into three groups, i.e., independently occurring hazards, concurrently interacting hazards, and hazard chains. Further efforts may be devoted to extend the current single-hazard numerical framework to multi-hazards numerical framework, aiming to shed the light about the vulnerability and resilience of coastal bridge system performance subjected to various typical hazards. This could be achieved by first characterizing each type of hazard individually, and then incorporating it into the entire numerical framework while taking into account its interactions with other hazards.
7.3 Concluding Remarks

The main goal of this dissertation is to propose a numerical framework to evaluate the life-cycle dynamic characteristics and fatigue performance of the coastal slender bridges in the context of coupled VBWW dynamic system. A list of versatile and efficient numerical schemes have been developed to achieve the main goal, in which several challenges are addressed, including: (1) correlated wind and wave fields simulation for coastal bridges; (2) complex vehicle-bridge-wind-wave interactions; (3) comprehensive vehicle ride comfort and safety evaluations; (4) bridge fatigue reliability under life-time stochastic traffic load, wind and wave load; (5) effective fatigue damage diagnosis and prognosis under various sources of uncertainties.

The proposed numerical framework, as discussed in details in the Introduction Section, has made important contributions to the current understanding of the dynamic characteristics as well as the fatigue performance of the coastal slender bridges in the context of VBWW system. An important feature of the developed framework is that it enables incorporating various aleatory and epistemic uncertainties involved in the VBWW system, e.g., stochastic environments, empirical justifications, model simplifications, inaccurate statistics of model parameter, measurement error, through a list of machine learning algorithms.
APPENDIX A: Coupled Analysis of Multi-impact Energy Harvesting from Low-Frequency Wind Induced Vibrations

A.1 Background

With recent advancement in wireless sensing technique and control system, power demand in an off-grid location, such as remote inland, offshore, or subsea locations, has been critical for effective real-time monitoring and control. However, replacement and recharge of batteries and associated large cost on time and money for these devices in the remote locations have greatly restrained their applications. As multiple renewable energy, such as solar power, heat energy, and wind, wave, and tidal energy are widely available at various remote locations, harvesting these renewable energy from ambient environment to enable energy-autonomous electronic devices has attracted continuously growing attention in the past few decades in order to find clean, regenerative, and potentially sustainable power sources in the remote locations [275–277].

Meanwhile, for remote sensors or devices with low power demand, piezoelectric-based energy harvesting, especially on scavenging the wind energy with piezoelectric materials based on aerodynamic instability, has received greater attention to generate energy compared with magnetic-based energy harvesting [275,278,279]. Some research focused on harvesting wind energy from vortex-induced vibrations by placing a flexible piezoelectric cantilever beams inside turbulent boundary layers and wakes of circular cylinders [280–282]. Some researchers proposed energy harvesters based on the galloping of a prismatic structure attached to a flexible piezoelectric cantilever beam [283–285]. In addition, flutter instability, which was accused for the failure of the 1st Tacoma Narrows Bridge in 1940, was widely used, as well [286,287]. For example, McKinney and Delaurier first proposed a device called wingmill, which utilized flutter of a wing [288]. Bryant and Garcia proposed, modeled, prototyped, and experimentally validated a piezoelectric energy harvesting device driven by aeroelastic flutter vibrations of a simple pin-

* This Appendix is adapted from a paper published in the Smart Materials and Structures [314] with permission from ELSEVIER.
connected flap and beam [289]. Following the concept of flutter-based piezoaeroelastic airfoil energy harvesting proposed by [290], Abdelkefi et al. [291] proposed a piezoaeroelastic system consists of a rigid airfoil supported by nonlinear torsional and flexural springs to generate energy at low free stream velocities through limit cycle oscillations (LCO).

However, for civil infrastructures, the frequency of wind induced vibrations is usually in the range of several hertz. To achieve higher energy output and higher energy transfer efficiency, the piezoelectric energy harvester needs to be designed to have a resonant frequency to match the frequencies of the ambient vibrations. As the resonant frequency of the piezoelectric patch or devices are usually in the range of tens to hundreds of hertz, the efficiency of the traditional piezoelectric energy harvester is greatly reduced when harvesting the low frequency vibration energy from civil infrastructures. Therefore, many strategies, such as using more efficient piezoelectric materials, distinct piezoelectric mode couplings, optimization of the power conditioning circuitry and the device configuration, have been proposed [292]. Recently, Zhang and Cai [293] and Zhang et al. [294] designed and experimentally validated a piezoelectric multi-impact energy harvester to achieve higher energy transfer efficiency by triggering high frequency vibrations of cantilever beams with a series of sequential impacts from a hung mass.

In the present study, based on the recent development on wind energy harvesting and multi-impact energy harvesting, a novel piezoelectric multi-impact wind energy harvesting device is proposed to scavenge the wind energy with a considerable high efficiency and high energy output. The device consists of an H-shape beam and four bimorph piezoelectric cantilever beams. The H-shape beam, which can be easily triggered to vibrate in a wide range of wind speed, is originated from the 1st Tacoma Narrows Bridge. The outline of this paper is as follows. After the introduction, Section A.2 introduces the modeling scheme of the multi-impact wind energy harvester including the design concept and modeling of the piezoelectric cantilever beam and the H shape beam. Section A.3 focuses on the coupled structural, aerodynamic and electrical analysis. Meanwhile, the procedures to solve the coupled equations numerically are presented, as well. The numerical simulation results are presented and a parametric study is carried out to explore the effect of parameters of the system in Section A.4, followed by concluding remarks in Section A.5.
A.2 Modeling of Multi-impact Wind Energy Harvester

In this section, details of the design concept of the proposed multi-impact wind energy harvester and the mathematical modeling of both the piezoelectric bimorph cantilever beam and the H-shape beam are presented.

**Figure A.1.** Schematic drawing of the piezoelectric multi-impact wind energy harvester

**A.2.1 Design Concept**

After the failure of the 1st Tacoma Narrows Bridge at a wind speed of 18.8m/s (42 mph) in 1940, continuous efforts were made to explain how wind induced large amplitude of vibrations and finally led to a catastrophic failure. The slender H-shaped deck design was believed as the main reason for the disaster. However, such an undesirable and destructive bridge design could possibly be favorable for energy harvesting. In the present study, the proposed new wind energy harvester is based on this H-shaped deck of the bridge and it is expected to extract energy more effectively from the wind field.

The schematic drawing of the proposed piezoelectric multi-impact wind energy harvester
(30cm×15cm×10cm) is shown in Fig. A.1. The harvester has a spring-supported H-shape beam and four clamped flexible bimorph cantilever beams which are symmetrically positioned besides the H-shape beam. The H-shape beam is supported by eight vertical springs and the stiffnesses of the springs are scaled proportionally from the first bending and torsional frequencies of the 1st Tacoma Narrows Bridge, which could be easily excited by wind even at a low wind speed. Both vertical edges of the two flanges for the H-shaped beam are mounted with a series of parallel and equally spaced teeth and the teeth are very close to the cantilever beams. As shown in Fig. A.2, the bimorph cantilever beam is composed of one aluminum beam in the middle and two bonded piezoelectric patches on each side of the aluminum beam. The piezoelectric patches are connected in series. Each bimorph cantilever beam has a bulge at its tip. As the bulge is very small in comparison with the dimension of the cantilever beam itself, the influence of the tip bulge on the overall dynamic response of the cantilever beam is insignificant thus ignored and the cantilever beam is assumed to be perfectly symmetric with respect to its neutral axis. The geometric and the material properties of the bimorph cantilever beam and the H-shape beam used for the proposed multi-impact harvester are summarized in Table A.1.

**Figure A.2.** (a) Configuration of bimorph piezoelectric cantilever beam with series connection of piezoelectric patches, (b) cross-sectional view of the bimorph cantilever, (c) dimension of the tip bulge
Table A.1. Geometric and material properties of the proposed wind energy harvester†

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_s$</td>
<td>Young’s modulus of the aluminum (GN m$^{-2}$)</td>
<td>70</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Young’s modulus of the piezoelectric material (GN m$^{-2}$)</td>
<td>23.3</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Mass density of the aluminum (g cm$^{-3}$)</td>
<td>2.7</td>
</tr>
<tr>
<td>$m_p$</td>
<td>Mass of the piezoelectric patch (g)</td>
<td>3.5</td>
</tr>
<tr>
<td>$L_0$</td>
<td>Length of the aluminum beam (mm)</td>
<td>61</td>
</tr>
<tr>
<td>$L_p$</td>
<td>Length of the piezoelectric patch (mm)</td>
<td>55</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Length of the H-shape beam (mm)</td>
<td>300</td>
</tr>
<tr>
<td>$b_s$</td>
<td>Width of the aluminum beam (mm)</td>
<td>35</td>
</tr>
<tr>
<td>$b_p$</td>
<td>Width of the piezoelectric patch (mm)</td>
<td>35</td>
</tr>
<tr>
<td>$B$</td>
<td>Width of the H-shape beam (mm)</td>
<td>150</td>
</tr>
<tr>
<td>$H$</td>
<td>Height of the H-shape beam (mm)</td>
<td>50</td>
</tr>
<tr>
<td>$h_s$</td>
<td>The thickness of the aluminum beam (mm)</td>
<td>1</td>
</tr>
<tr>
<td>$h_p$</td>
<td>The thickness of the piezoelectric patch (mm)</td>
<td>0.2</td>
</tr>
<tr>
<td>$t_h$</td>
<td>The thickness of the H-shape beam (mm)</td>
<td>10</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of the tip bulge (°)</td>
<td>30</td>
</tr>
<tr>
<td>$b$</td>
<td>Length of the upper tip bulge (mm)</td>
<td>5</td>
</tr>
<tr>
<td>$a$</td>
<td>Length of the lower tip bulge (mm)</td>
<td>3</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between teeth (mm)</td>
<td>5</td>
</tr>
<tr>
<td>$\omega_{0h}$</td>
<td>The natural frequency of first vertical mode of the H-shape beam (Hz)</td>
<td>0.13</td>
</tr>
<tr>
<td>$\omega_{0t}$</td>
<td>The natural frequency of first torsional mode of the H-shape beam (Hz)</td>
<td>0.2</td>
</tr>
<tr>
<td>$e_{31}$</td>
<td>Piezoelectric stress coefficient (C m$^{-2}$)</td>
<td>11.2</td>
</tr>
<tr>
<td>$\varepsilon_{33}^{s}$</td>
<td>Permittivity component at constant strain (nF m$^{-1}$)</td>
<td>15.93</td>
</tr>
<tr>
<td>$\varepsilon_{33}^{s}/\varepsilon_0$</td>
<td>Relative permittivity</td>
<td>1800</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Absolute permittivity (pF m$^{-1}$)</td>
<td>8.85</td>
</tr>
<tr>
<td>$R$</td>
<td>External resistance in the circuit (kΩ)</td>
<td>15</td>
</tr>
</tbody>
</table>

†The natural frequencies of the first vertical and torsional mode of the H-shape beam used for the proposed energy harvester are the same as those of the 1st Tacoma Narrows Bridge [295].

As discussed earlier, the H-shape beam can be easily excited by the wind flow, which is z direction as shown in Fig. A.1. In addition to the static wind loads, the wind turbulent component can add buffeting forces and the self-excited forces can be generated due to the wind-structure interactions. With the increase of the wind speed, the self-excited forces could be dominant. As the system damping turns from positive to negative, the flutter occurs and leads to a divergent vibration [296].

Before the divergent vibration occurs, the H-shape beam moves both randomly with limited amplitude in torsional and vertical directions. Since the length of the teeth is designed to contact and hit the bulges (Fig. A.2), the vertical movements of the teeth will hit the bulges and push the bulges away from their
neutral axis in the horizontal direction. After pushing the cantilever away, the teeth pass through the bulges and allow another impact from the next teeth. The impact process is shown in Fig. A.3. A roller is mounted at the tip of each tooth to reduce the friction during the impact between the teeth and the bulges and to prevent the two from sticking together. As a result, during each vibration cycle of the H-shape beam, the cantilever beams are triggered to vibrate by the impact of the teeth for multiple times and vibrate freely between each impact. Therefore, the low-frequency wind induced vibrations of the H-shape beam are up-converted to the high frequency vibrations of the cantilever beams by several impacts in one vibration cycle. This could generate more energy with high energy harvesting efficiency.

![Figure A.3. The impact process. (a) Teeth move upward, (b) teeth move downward](image)

### A.2.2 Modeling of Bimorph Cantilever Beam

To simulate and predict the vibration of the bimorph cantilever beam with piezoelectric patches, a distributed-parametric electromechanical model is used, which incorporates fully electromechanical coupling effect of the whole dynamic system [277]. In this model, the symmetric bimorph cantilever is modeled as a uniform composite beam based on Euler-Bernoulli beam theory and the governing equation of motion of the electromechanical system considering only the external excitation force is expressed as:

\[
EI \frac{\partial^4 w_{rel}(x,t)}{\partial x^4} + c_s I \frac{\partial^5 w_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x,t)}{\partial t} + m \frac{\partial^2 w_{rel}(x,t)}{\partial t^2} + \left( \frac{d \delta(x)}{dx} - \frac{d \delta(x-L_0)}{dx} \right) \delta V(t) = F_{np}(t) \delta(x-L_0)
\]

(A.1)

where \( w_{rel}(x,t) \) is the transverse displacement of the beam along neutral axis relative to its base; \( c_s \) is the strain rate damping coefficient; \( c_a \) is the viscous air damping coefficient; \( \delta(x) \) is the Dirac delta function;
$F_{tip}(t)$ is excitation force applied on the beam tip bulge due to impact; $L_0$ is the length of the cantilever beam; $EI$ and $m$ are the bending stiffness and the mass per unit length; $V(t)$ is the generated voltage; and $\vartheta_s$ is the coefficient of piezoelectric backward coupling term. The piezoelectric backward coupling term includes the inverse piezoelectric effect on the dynamic response of the cantilever beam, which is given by $\vartheta_s = -e_{31}b_p(h_p + h_i)/2$, where $e_{31}$ is the effective piezoelectric stress coefficient, $b$ and $h$ are width and thickness, and the subscripts $s$ and $p$ represent the aluminum and piezoelectric material.

**A.2.3 Coupled Electromechanical Equations**

According to the integral form of the Gauss law [277], the current $i$ delivered by a pair of electrodes in an admittance circuit (resistor $R$) is:

$$i(t) = \frac{dQ(t)}{dt} = \frac{d}{dt} \int D_s(x,t) dA = \frac{V(t)}{R} \tag{A.2}$$

where $D_s$, the electric displacement component along $x$-axis due to bending, has the following form:

$$D_s(x,t) = e_{31} \epsilon_{xx} + \epsilon_{33}^p E_3 = -e_{31} h \frac{\partial^2 w_{rel}(x,t)}{\partial x^2} - \frac{\epsilon_{33}^s}{2h_p} V(t) \tag{A.3}$$

where $\epsilon_{xx}$ is the axial strain in the $x$-direction; $h$ is the distance between the neutral axis of the piezoelectric patch and the center of the composite beam which is $h = (h_p + h_i)/2$; $\epsilon_{33}^s$ is the permittivity component at constant strain; and $E_3$ is the electric field component in the $y$-direction within the patch.

After substituting Eq. (A.3) into Eq. (A.2), the Eq. (A.2) is rewritten as:

$$-\frac{e_{31} b_p}{2} (h_p + h_i) \int_0^h \frac{\partial^2 w_{rel}(x,t)}{\partial t \partial x^2} dx - \frac{\epsilon_{33}^p b_p L_0}{2h_p} dV(t) = \frac{V(t)}{R} \tag{A.4}$$

The vibration response relative to the base for the bimorph cantilever beam can be represented by a series of eigenfunctions:

$$w_{rel}(x,t) = \sum_{r=1}^\infty \phi_r(x) \eta_r(t) \tag{A.5}$$

where $\phi_r(x)$ and $\eta_r(t)$ are the mass normalized eigenfunction and modal coordinate of the $r$th vibration.
mode. The mass normalized eigenfunction \( \phi_r(x) \) is calculated from the undamped free vibration problem of Eq. (A.1) along with the following clamped-free boundary conditions:

\[
\left. w_{rel}(x,t) \right|_{x=0} = 0, \quad \left. \frac{\partial^2 w_{rel}(x,t)}{\partial x^2} \right|_{x=L_0} = 0, \quad \left. \frac{\partial^3 w_{rel}(x,t)}{\partial x^3} \right|_{x=L_0} = 0 \quad (A.6)
\]

After applying the boundary conditions, the resulting mass normalized eigenfunction is expressed as:

\[
\phi_r(x) = \sqrt{\frac{1}{mL_0}} \left[ \cosh \frac{\lambda_r}{L_0} x - \cos \frac{\lambda_r}{L_0} x + \varsigma_r \left( \sinh \frac{\lambda_r}{L_0} x - \sin \frac{\lambda_r}{L_0} x \right) \right] \quad (A.7)
\]

where \( \varsigma_r \) is given by \( \varsigma_r = (\sinh \frac{\lambda_r}{L_0} - \sin \frac{\lambda_r}{L_0}) / (\cosh \frac{\lambda_r}{L_0} + \cos \frac{\lambda_r}{L_0}) \) and \( \lambda_r \) is the dimensionless frequency parameter of the \( r \)th vibration mode which is obtained from the characteristic equation \( 1 + \cos \lambda \cosh \lambda = 0 \).

The eigenfunction given by Eq. (A.5) satisfies the following orthogonality conditions:

\[
\int_0^{L_0} m \phi_r(x) \phi_s(x) dx = \delta_{rs}, \quad \int_0^{L_0} EI \phi_r(x) \frac{d^4 \phi_r(x)}{dx^4} dx = \omega_r^2 \delta_{rs} \quad (A.8)
\]

where \( \delta_{rs} \) is the Kronecker delta, defined as unity when \( s \) is equal to \( r \) and zero otherwise, and \( \omega_r \) is the undamped natural frequency of the \( r \)th vibration mode given by \( \omega_r = \lambda_r^2 \sqrt{EI / (mL_0^4)} \).

Assuming the first beam mode contributes most of the response and ignoring the air damping, after substituting Eq. (A.5) into Eq. (A.1) and Eq. (A.4) and applying the orthogonality conditions of the eigenfunctions, the governing equations of motion for the bimorph cantilever beam considering electromechanical coupling effect are as follows:

\[
\frac{d^2 \eta_i(t)}{dt^2} + 2\omega_0 \dot{\eta}_i(t) + \omega_0^2 \eta_i(t) + \chi V(t) = f(t) \quad (A.9)
\]

\[
\frac{V(t)}{R} + C \frac{dV(t)}{dt} = i(t) \quad (A.10)
\]

where \( \phi_i(x) \) and \( \eta_i(t) \) are the first modal shape and modal coordinate of the cantilever beam; \( \omega_0 \) is the fundamental natural frequency; \( f(t) \) is the first mode of the excitation force given by \( f(t) = F_{tip}(t) \phi_1(L_0) \);
the internal capacitance $C$ and current source $i(t)$ of the piezoelectric patch are given by

$$C = \varepsilon_0 b \frac{L_0}{2h_p}$$

and

$$i(t) = \chi \dot{h}(t) ;$$

$\chi$ is the modal electromechanical coupling term which is expressed as

$$\chi = \oint_\ell \frac{d\phi(x)}{dx} \mid_{\ell_0} ;$$

and $\xi_0$ is the first modal damping ratio which is given by

$$\xi_0 = c_I \omega_0 / 2EI.$$ It is worth mentioning that the viscous air damping is assumed to be negligible compared to strain-rate damping and therefore the damping ratio $\xi_0$ includes the effect of strain-rate damping only.

### A.2.4 Modeling Wind Induced Vibrations of the H-shape Beam

As the H-shape beam is considered as a rigid body motion with only two modes of vertical and torsional vibrations, the governing equations of motion for the 2D H-shape beam can be written as [296]:

$$M_0(y) \left( \ddot{h}(y,t) + 2\zeta_0 \omega_0 \dot{h}(y,t) + \omega_0^2 h(y,t) \right) = L_{se}(y,t) + L_b(y,t)$$

(A.11)

$$I_0(y) \left( \ddot{\alpha}(y,t) + 2\zeta_0 \omega_0 \dot{\alpha}(y,t) + \omega_0^2 \alpha(y,t) \right) = M_{se}(y,t) + M_b(y,t)$$

(A.12)

where $h$ and $\alpha$ are the vertical and torsional displacements of the H-shape beam; $M_0$ and $I_0$ are the mass and mass moment of inertia per unit span; the subscripts $0h$ and $0\alpha$ stand for the first vertical and first torsional modes; $\zeta$ and $\omega$ are the damping ratio and natural frequency of the corresponding vibration mode; $L$ and $M$ are the aerodynamic lift and moment per unit span; and the subscripts $se$ and $b$ stand for self-excited and buffeting force.

The self-excited lift and moment are non-linear functions of the vertical and torsional displacements and their flutter derivatives which are expressed as [296]:

$$L_{se} = \frac{1}{2} \rho U^2 B \left[ K_0 H_1^* \frac{\dot{h}(y,t)}{U} + K_0 H_2^* \frac{B \dot{\alpha}(y,t)}{U} + K_0^2 H_3^* \alpha(y,t) + K_0^2 H_4^* \frac{h(y,t)}{B} \right]$$

(A.13)

$$M_{se} = \frac{1}{2} \rho U^2 B^2 \left[ K_0 A_1^* \frac{\dot{h}(y,t)}{U} + K_0 A_2^* \frac{B \dot{\alpha}(y,t)}{U} + K_0^2 A_3^* \alpha(y,t) + K_0^2 A_4^* \frac{h(y,t)}{B} \right]$$

(A.14)

where $\rho$ is the air density; $U$ is the mean wind speed; $B$ is the width of H-shape beam cross section; $K_0$ is the reduced frequency given by $K_0 = B\bar{\omega} / U$; $\bar{\omega}$ is the oscillation frequency; and $H_i^*$ and $A_i^*$ ($i=1-4$) are the flutter derivatives which are functions of the reduced frequency $K_0$. These flutter derivatives can be
obtained either from wind tunnel tests or computational fluid dynamics (CFD) simulation. Matsumoto et al. [297] investigated the flutter characteristics of 2D H-shape beams with various side-ratios based on the wind tunnel tests. In the present study, the flutter derivatives $H_i^*$ and $A_i^*$ are derived from their work.

The buffeting lift force and moment which are associated with the wind turbulent components are expressed as:

$$L_b = \frac{1}{2} \rho U^2 B \left[ C_L \left( \frac{2 u(t)}{U} \right) + \left( C_l + C_D \right) \frac{w(t)}{U} \right]$$  \hspace{1cm} (A.15)

$$M_b = \frac{1}{2} \rho U^2 B \left[ C_m \left( \frac{2 u(t)}{U} \right) + C_M \frac{w(t)}{U} \right]$$  \hspace{1cm} (A.16)

where $C_L$, $C_D$, $C_M$ are the static coefficient for lift, drag and pitch moment which are from the wind tunnel tests; a “prime” over the coefficients represents a derivative with respect to the wind attack angle; and $u(t)$ and $w(t)$ are the wind turbulent components in the lateral (z-direction) and the vertical directions.

Based on modal analysis, the two displacement variables $h$ and $\alpha$ are expressed as follows:

$$h(y,t) = h_{v1}(y) \xi_{v1}(t), \quad \alpha(y,t) = \alpha_{t1}(y) \gamma_{t1}(t)$$  \hspace{1cm} (A.17)

where $h_{v1}(y)$ and $\alpha_{t1}(y)$ are the first vertical and torsional vibration modes; and $\xi_{v1}(t)$ and $\gamma_{t1}(t)$ are the corresponding modal coordinates. It is worth mentioning that the H-shape beam along with the teeth as a whole is assumed to be a rigid body in which the deformation of itself is neglected. Therefore, the vibration mode $h_{v1}(y)$ and $\alpha_{t1}(y)$ are equal to 1 and the modal coordinates $\xi_{v1}(t)$ and $\gamma_{t1}(t)$ are actually the vertical and torsional displacements of the H-shape beam.

Substituting Eqs. (A.13), (A.14) and (A.17) into Eqs. (A.11) and (A.12), the governing equations of motion of the H-shape beam for 2-D buffeting and flutter analysis can be rewritten in a 2×2 matrix form:

$$\begin{bmatrix} I \end{bmatrix} \{ \dot{X} \} + \begin{bmatrix} C \end{bmatrix} \{ \dot{X} \} + \begin{bmatrix} K \end{bmatrix} \{ X \} = \{ Q_b \}$$  \hspace{1cm} (A.18)

where $\{ X \} = (\xi_{v1}(t) \gamma_{t1}(t))^T$, $[I]$ is the unit matrix, the $[C]$, $[K]$ and $\{ Q_b \}$ are, respectively, the aerodynamic damping, stiffness, and force matrices of the coupled wind-structure system which are expressed as:
\[
[C] = \begin{pmatrix}
A_1 & A_2 \\
A_3 & A_4
\end{pmatrix} = \begin{pmatrix}
2\zeta_{0h}\omega_{0h} - \rho B^2 \bar{\omega}_{0h} H_{hy} / (2M_h) & -\rho B^2 \bar{\omega}_{0a} H_{ya} / (2I_a) \\
-\rho B^2 \bar{\omega}_{0h} A_h G_{ah} / (2I_a) & 2\zeta_{0a}\omega_{0a} - \rho B^2 \bar{\omega}_{0a} A_a G_{aa} / (2I_a)
\end{pmatrix}
\]  (A.18a)

\[
[K] = \begin{pmatrix}
B_1 & B_2 \\
B_3 & B_4
\end{pmatrix} = \begin{pmatrix}
\omega_{0h}^2 - \rho B^2 \bar{\omega}_{0h}^2 H_{hy} / 2M_h & -\rho B^2 \bar{\omega}_{0a}^2 H_{ya} / 2M_h \\
-\rho B^2 \bar{\omega}_{0h}^2 A_h G_{ah} / 2I_a & \omega_{0a}^2 - \rho B^2 \bar{\omega}_{0a}^2 A_a G_{aa} / 2I_a
\end{pmatrix}
\]  (A.18b)

\[
\begin{bmatrix}
\{Q_0\} = \begin{pmatrix}
\int_0^L h_i(y)L_h dy \\
\int_0^L \alpha_{i1}(y)M_{a} dy
\end{pmatrix}
\end{bmatrix}
\]  (A.18c)

where \( G_{sh} = \int_0^L h_i(y)dL_h \); \( G_{ha} = \int_0^L h_i(y)\alpha_{i1}(y)dL_h \); \( G_{aa} = \int_0^L \alpha_{i1}^2(y)dL_h \); \( L_i \) is the span of the H-shape beam; and \( M_i = \int_0^L M_i(y)\alpha_{i1}^2(y)dy \) and \( I_i = \int_0^L I_i(y)\alpha_{i1}^2(y)dy \) are the generalized mass and mass moment of inertia per unit span.

Since the aerodynamic damping and stiffness matrices have frequency-dependent terms, namely, \( K_0 \), \( H_i^* \) and \( A_i^* \), which are functions of the oscillation frequency \( \bar{\omega} \), the governing Eq. (A.18) is both frequency- and time dependent [23]. Due to the wind structure interactions, the oscillation frequencies \( \bar{\omega}_{0h} \) and \( \bar{\omega}_{0a} \), though associated with vibration modes, are different from the natural frequencies \( \omega_{0h} \) and \( \omega_{0a} \).

To eliminate the frequency-dependence of the equations of motions and obtain the dynamic responses of the H-shape beam in time domain, iterative complex eigenvalue analysis is carried out in this study. Based on the approach, the complex eigenvalue analysis is used to obtain the oscillation frequency of each vibration mode (first vertical and first torsional mode) under each desired wind speed. Due to the page limit, the details of iterative complex eigenvalue analysis are not included in the present study and can be found in the literature [26,28,298].

Through an iterative eigenvalue analysis, the oscillation frequency \( \bar{\omega} \) and reduced frequency \( K_0 \) at any given wind speed can be calculated. Consequently, the frequency-dependent component of flutter derivatives and \( K_0 \) are resolved from the damping and stiffness matrices \( [C] \) and \( [K] \) and the Eq. (A.18) will be time-dependent only and ready for time domain analysis if buffeting force is known. The flutter wind speed and the corresponding flutter frequency can also be identified from the eigenvalue solutions at the
conditions when the oscillation damping approaches to zero.

A.3. Coupled Analysis for Wind Energy Harvesting

In this section, nine possible impact statuses of the system during the operation of the multi-impact harvester and the corresponding coupled structural, aerodynamic and electrical governing equations are introduced. Numerical simulation is followed to solve the coupled equations and obtain the dynamic responses and the power output of the harvester.

A.3.1 Impact Status

As discussed in Section A.2, the wind-induced vibrations of the H-shape beam can lead to a series of impacts between its attached teeth and the cantilever beams. During the impact, the impact forces are developed at the interface between the teeth and the tip bulges, which enables the coupled effects of the cantilever beams and the H-shape beam in the coupled dynamic system.

As shown in Fig. A.3, three possible impact statuses between the tooth and the bulge can be defined depending on where the impact occurs, namely, “upper face impact”, “bottom face impact”, and “no impact”. As shown in Fig. A.1, either the two upstream cantilever beams or the two downstream ones are always in the same impact status. However, since both the vertical and torsional displacements are involved during the vibration of the H-shape beam, the movement of the teeth at the upstream and that of the teeth at downstream will be different. Noting that each one of the three possible statuses may occur for both the upstream and downstream cantilever beams, the whole system may experience a total of nine impact statuses during the vibration, which are shown in Table A.2.

As shown in Table A.2, the ninth status is the one during the interval of each two impacts, which indicates that the H-shape beam and the cantilever beams vibrate freely and independently. While for other statuses, either each upstream or each downstream cantilever beam or both of them are in contact with the H-shape beam. The ninth impact status and all the other statuses can be categorized as “without impact” and “with impact”, respectively, and each category will be discussed in the following two sections.
Table A.2. Possible impact statuses for the proposed wind energy harvester through the whole impact process††

<table>
<thead>
<tr>
<th>Status</th>
<th>Upstream cantilever beam</th>
<th>Downstream cantilever beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>upper face impact</td>
<td>upper face impact</td>
</tr>
<tr>
<td>2</td>
<td>upper face impact</td>
<td>bottom face impact</td>
</tr>
<tr>
<td>3</td>
<td>upper face impact</td>
<td>no impact</td>
</tr>
<tr>
<td>4</td>
<td>bottom face impact</td>
<td>upper face impact</td>
</tr>
<tr>
<td>5</td>
<td>bottom face impact</td>
<td>bottom face impact</td>
</tr>
<tr>
<td>6</td>
<td>bottom face impact</td>
<td>no impact</td>
</tr>
<tr>
<td>7</td>
<td>no impact</td>
<td>upper face impact</td>
</tr>
<tr>
<td>8</td>
<td>no impact</td>
<td>bottom face impact</td>
</tr>
<tr>
<td>9</td>
<td>no impact</td>
<td>no impact</td>
</tr>
</tbody>
</table>

††The “upper face impact”, “bottom face impact” and “no impact” represent the three possible impact status between the tooth and the bulge of the cantilever beam, namely, the tooth is in collision with the upper face, in collision with the bottom face and in no collision with the bulge.

A.3.2 Modeling the System without Impact

During the interval of two impacts, the H-shape beam and the cantilever beams vibrate independently. Therefore, based on the discussion in Section A.2, the governing equations of motion for the cantilever beams without external excitation force can be derived by dropping the force term on the right side of the Eq. (A.9):

\[
\frac{d^2 \eta_u(t)}{dt^2} + 2\omega_0 \xi_0 \frac{d \eta_u(t)}{dt} + \eta_u(t) \omega_0^2 + \chi V_u(t) = 0
\]  

(A.19)

\[
\frac{d^2 \eta_d(t)}{dt^2} + 2\omega_0 \xi_0 \frac{d \eta_d(t)}{dt} + \eta_d(t) \omega_0^2 + \chi V_d(t) = 0
\]  

(A.20)

where the subscripts \(u\) and \(d\) denote the upstream and downstream cantilever beams, respectively.

The governing equation of motion of the H-shape beam without impact is exact the Eq. (A.18) we derived in Section A.2.4. Therefore, the vibrations of the H-shape beam and the cantilever beams can be obtained by solving Eqs. (A.18 ~ A.20), respectively.

A.3.3 Modeling the System with Impact

The impact between the H-shape beam and each cantilever beam is most relevant to the transverse impact of a rigid body on a flexible element, which is called low-velocity impact [299]. The impact process
consists of two separate phases, namely, the compression development phase and the separation phase [300]. In phase one, when the H-shape beam approaches and contacts the cantilever beam, they tend to interpenetrate each other and a local impact force $F$ develops in the interface between the tip bulge and the roller as shown in Figs. A.4(b) and A.4(c). Since the surface of the bulge is assumed to be perfectly smooth, the friction is neglected and the impact force is perpendicular to the surface of the bulge. Therefore, the impact force is actually a pair of forces in the opposite direction and could be applied to the tooth and the cantilever, respectively. As illustrated by the blue dash line in Figs. A.4(b) and A.4(c), when the tooth keeps sliding on the surface of the bulge and pushing it away horizontally, both the bending of the cantilever beam and the impact force continue to increase. In phase two, with the increase of the impact force, the tooth and the bulge could repulse each other. Otherwise, the tooth could pass through the bulge. Hereafter, in all cases, the H-shape beam and the cantilever beam will separate and vibrate independently.

![Figure A.4](image)

**Figure A.4.** (a) Dimension of the cantilever tip bulge, (b) force diagram when tooth hits upper bulge, (c) force diagram when tooth hits bottom bulge

For impact status 1, the H-shape beam is in collision with the upper face of the bulges of both the upstream and downstream cantilever beams. As shown in the force diagram for the upper face impact in Fig. A.4(b), the impact force applied to the bulge has two components, namely, the vertical component $F\sin\theta$ and the horizontal one $F\cos\theta$. Noting that the axial stiffness in the vertical direction of the cantilever beam is much larger than that in the horizontal direction, which means the axial displacement of the cantilever beams can be ignored and only the horizontal component $F\cos\theta$ has an effect on the motion of
the cantilever beam. Therefore, by substituting \( F_{\text{tip}} \) with \( F \cos \theta \) into Eq. (A.9), the governing equations of motion for the upstream and downstream cantilever beams considering impact force are expressed as:

\[
\frac{d^2 \eta_u(t)}{dt^2} + 2\alpha_0 \xi_0 \frac{d \eta_u(t)}{dt} + \eta_u(t)\omega_0^2 + \chi V_u(t) = \cos \theta F_u(t)\phi(L_0) \tag{A.21}
\]

\[
\frac{d^2 \eta_d(t)}{dt^2} + 2\alpha_0 \xi_0 \frac{d \eta_d(t)}{dt} + \eta_d(t)\omega_0^2 + \chi V_d(t) = \cos \theta F_d(t)\phi(L_0) \tag{A.22}
\]

where \( F_u \) and \( F_d \) denote the impact force applied to the upstream and downstream cantilever beams, respectively.

**Figure A.5.** Force diagram of the H-shape beam for impact status 1

As mentioned earlier, all the four cantilever beams are in collision with the H-shape beam and therefore the H-shape beam is subjected to the counterforces from these four cantilever beams at their collision points. As shown in the force diagram in Fig. A.5, since the teeth are symmetrically deployed on both sides of the flange, the horizontal component \( F_u \cos \theta \) and \( F_d \cos \theta \) of the two upstream and downstream cantilever beams cancel each other. Therefore, each counterforce applied to the H-shape beam can be reduced to its vertical component, namely, \(-\sin \theta F_u\) for each upstream cantilever beam and \(-\sin \theta F_d\) for each downstream one. These resultant vertical forces can be further simplified to a single vertical force \( F_z \) \((F_z = -2\sin \theta F_u - 2\sin \theta F_d)\) and bending moment \( M_z \) \((M_z = B \sin \theta (F_u - F_d))\), after converting them to the axis of the H-shape beam. Therefore, the governing equation of motion for the H-shape beam is derived by only updating the force term on the right side of Eq. (A.18):
$$\begin{bmatrix} I \end{bmatrix} \begin{bmatrix} \ddot{X} \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \dot{X} \end{bmatrix} + \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} X \end{bmatrix} = \{Q'\}$$

(A.23)

where $\{Q'\}$ is expressed as follows,

$$\{Q'\} = \begin{bmatrix} \frac{1}{M_h} \left[ \int_0^{h_i} h(y)L_b \, dy - 2 \sin \theta (F_u(t) + F_d(t)) \right] \\ \frac{1}{I_u} \left[ \int_0^{h_i} \alpha(y)M_b \, dy + B \sin \theta (F_u(t) - F_d(t)) \right] \end{bmatrix}$$

(A.24)

It is noted that Eqs. (A.21) and (A.22) are coupled with Eq. (A.23) due to the impact force $F_u$ and $F_d$.

In order to solve these coupled equations, the impact conditions for impact status 1 is introduced as follows.

In the proposed wind energy harvester, 10 teeth are mounted at each vertical edge of the flange of the H-shape beam and all the bulges are initially located between the fifth and sixth tooth as shown in Fig. A.4(a). A simple geometric relationship can be derived for the upper face impact shown in Fig. A.4(b):

$$d_v(t) = \tan \theta d_t(t)$$

(A.25)

where $d_t$ denotes the transverse displacement of the cantilever tip bulge, and $d_v$ denotes the vertical distance between the initial impact point and the tooth as it slides along the upper surface. $d_v$ is associated with the displacements of the H-shape beam and the initial impact point is the point where the collision begins.

If the $i$th tooth in the upstream and the $j$th tooth in the downstream contacts the bulges of the upstream and the downstream cantilever beams, respectively, the relationship between the displacements of the H-shape beam and the transverse tip displacement of the upstream and downstream cantilever beams can be described by Eqs. (A.26) and (A.27).

$$\phi_i(L_o)\eta_u(t) = \tan \theta \left[ \xi_{v_i}(t) - \gamma_{v_i}(t) \cdot \frac{B}{2} - \Delta - (i - 6)d \right]$$

(A.26)

$$\phi_i(L_o)\eta_d(t) = \tan \theta \left[ \xi_{v_i}(t) + \gamma_{v_i}(t) \cdot \frac{B}{2} - \Delta - (j - 6)d \right]$$

(A.27)

where $\Delta$ is the vertical distance between the tooth and the surface (or initial impact point) of the bulge when both of them are in their neutral positions, and $d$ is the distance between two adjacent teeth.

From Eq. (A.26) and Eq. (A.27), the velocity and acceleration have the following relationship:
Substituting Eqs. (A.26) ~ (A.28) into Eqs. (A.21) ~ (A.23) and eliminating the impact forces \( F_{u} \) and \( F_{v} \), the governing equation of motion for impact status 1 of the coupled vibration system is written as a 2x2 matrix form:

\[
[I] \{ \ddot{X} \} + [C_{\text{status1}}] \{ \dot{X} \} + [K_{\text{status1}}] \{ X \} + [N_{\text{status1}}] \{ V_{u}(t) \} = \{ Q_{\text{status1}} \}
\]  

where:

\[
[C_{\text{status1}}] = \begin{bmatrix}
1 & \left( A_{L}M_{h}^{*} \right) \\
\cos \theta \phi_{0}^{2} \left( L_{o} \right) & \left( A_{L}M_{h}^{*} \right) \\
\cos \theta \phi_{0}^{2} \left( L_{o} \right) & \left( A_{L}M_{h}^{*} \right)
\end{bmatrix}
\]

\[
[K_{\text{status1}}] = \begin{bmatrix}
1 & \left( B_{L}M_{h}^{*} \right) \\
\cos \theta \phi_{0}^{2} \left( L_{o} \right) & \left( B_{L}M_{h}^{*} \right) \\
\cos \theta \phi_{0}^{2} \left( L_{o} \right) & \left( B_{L}M_{h}^{*} \right)
\end{bmatrix}
\]

\[
[N_{\text{status1}}] = \begin{bmatrix}
\frac{X}{C_{1}} & \frac{X}{C_{1}} \\
-\frac{X}{C_{2}} & \frac{X}{C_{2}}
\end{bmatrix}
\]

\[
Q_{\text{status1}} = \left\{ \begin{array}{l}
\frac{1}{C_{1}} \left( \frac{2\Delta + (i + j - 12)d}{\cos \theta \phi_{0}^{2} \left( L_{o} \right)} \cos \theta \phi_{0}^{2} \left( L_{o} \right) \right) + \frac{L_{b}L_{b}^{*}}{2 \sin \theta} \\
\frac{1}{C_{2}} \left( \frac{(j - i)}{\cos \theta \phi_{0}^{2} \left( L_{o} \right)} \cos \theta \phi_{0}^{2} \left( L_{o} \right) + \frac{B_{L}M_{h}^{*}}{B_{L}M_{h}^{*}} \right)
\end{array} \right\}
\]

The impact status 1 is defined when \( i \)th tooth (upstream) contacts the upstream cantilever beam and
jth tooth (downstream) contacts the downstream cantilever beam. It occurs only if the following conditions are satisfied:

\[
\Delta + \frac{\phi(L_v)\eta_j(t)}{\tan \theta} + (i-6)d \leq \left( \xi, \gamma_{i} \right) - \left( \eta_{i} \right) \cdot \frac{B}{2} \leq \frac{d}{2} + (i-6)d
\]

(A.30a)

\[
\Delta + \frac{\phi(L_v)\eta_j(t)}{\tan \theta} + (j-6)d \leq \left( \xi, \gamma_{i} \right) + \left( \eta_{i} \right) \cdot \frac{B}{2} \leq \frac{d}{2} + (j-6)d
\]

(A.30b)

\[
F_u(t) > 0, \quad F_d(t) > 0
\]

(A.30c)

Condition (A.30a) and (A.30b) require that the tooth and the tip bulge coincide spatially and condition (A.30c) requires that they have interactions. The impact conditions are used to select the right governing equations for its corresponding impact status.

So far, we have derived the governing equations of motion and the corresponding impact conditions for two impact statuses of the vibration system, namely, impact status 1 and impact status 9. Eqs. (A.18) ~ (A.20) are the governing equations for impact status 9 and Eq. (A.29) is the governing equation for impact status 1. Similarly, the governing equations and the corresponding impact conditions for the impact statuses 2-8 can be obtained by following the same procedure as discussed in this section. For the sake of brevity here, the governing equations for the other impact statuses are not presented here.

**A.3.4 Solving the Coupled Equations of the Multi-impact Harvester**

To solve the coupled equations and predict the responses of the multi-impact harvester system, a numerical simulation is used and the flow chart is shown in Fig. A.6. The complex eigenvalue analysis is carried out to eliminate the frequency-dependence of the governing equations for wind-structure interactions, which has been discussed in Section A.2.4. The random lateral and vertical component of the wind velocity are simulated using Kaimal’s spectrum [301] and Lumley and Panofsky’s spectrum [302], respectively. After this, it is possible to solve the coupled governing equations for the dynamic system in time domain, which is detailed as following.

In the numerical simulation, a very short time interval \(dt (dt=0.0002s)\) is used. Within each time interval, the coupling terms \(\chi V_u(t)\) and \(\chi V_d(t)\) are assumed to be unchanged and are calculated by substituting
the voltage at the beginning of the time interval into the governing equations. Using the results from the last time step as the initial condition, the impact status is determined and the corresponding governing equations for the current time step can be selected. The selected governing equations are then updated by the calculated coupling term $\chi V_u(t)$ and $\chi V_d(t)$, and wind velocity components $u(t)$ and $w(t)$. Based on the assumption that the impact conditions do not change during the time interval, the responses of the system are then obtained by solving the updated equations and the results can be used as the initial condition for the next time interval.

Perform iterative complex eigenvalue analysis to get the corresponding oscillation frequencies for the H-shape beam: $V_u(t_n)$, $V_d(t_n)$, $P_u(t_n)$, $P_d(t_n)$.

At $t_n=n\times dt$: use the impact conditions to select the right governing equations and update them by inputting the precalculated coupling term $\chi V_u(t_{n-1})$ and $\chi V_d(t_{n-1})$, and the wind velocity components $u(t_{n-1})$ and $w(t_{n-1})$.

Solve the equations to obtain the vibration responses at the end of this time interval: $\eta_u(t_n)$, $\eta_d(t_n)$, $\xi_u(t_n)$, $\xi_d(t_n)$ and their derivatives.

Obtain the voltage and the power responses of the bimorph cantilever beams: $V_u(t_n)$, $V_d(t_n)$, $P_u(t_n)$, $P_d(t_n)$.

At the initial status ($t=0$), the structural system including the H-shape beam and all the cantilever beams.

![Figure A.6. Flow chart of the numerical procedure for predicting the responses of the proposed wind energy harvester.](image-url)

At the initial status ($t=0$), the structural system including the H-shape beam and all the cantilever beams...
are at rest on their neutral positions with zero displacement and velocity. Therefore, one can have:

\[ V_d(0) = \dot{V}_d(0) = 0, \quad \dot{\xi}_v(0) = \gamma_v(0) = 0, \quad \eta_u(0) = \eta_d(0) = 0, \quad \frac{d\xi_v(0)}{dt} = \frac{d\gamma_v(0)}{dt} = 0, \quad \text{and} \quad \frac{d\eta_u(0)}{dt} = \frac{d\eta_d(0)}{dt} = 0 \]

\[ (A.31) \]

During the first time interval, by checking the impact conditions, no impact is triggered in the system and therefore, impact status 9 is used for current time interval and the control Eqs. (A.18) ~ (A.20) are used as the governing equations. The governing equations are then updated with the coupling term \( \chi V_d(0) \), \( \chi V_d(0) \), and wind velocity components \( u(0) \) and \( w(0) \). By solving the updated coupled equations, the dynamic responses of the system at the end of the first time interval, including \( \eta_d(1 \times dt) \), \( \eta_d(1 \times dt) \), \( \xi_v(1 \times dt) \), \( \gamma_v(1 \times dt) \) and their derivatives, can be obtained.

Since the derivation of the voltage and power response of the upstream and downstream cantilever beams are exactly the same, only the derivation for the upstream cantilever beams are presented here for a demonstration purpose. From Eq. (A.10), at the end of the first time interval \( t = dt \), the voltage response of the upstream cantilever beam equals to the voltage change during the first time interval:

\[ dV_u \bigg|_{t=dt} = V_u \bigg|_{t=dt} = R \left( i \bigg|_{t=dt} - C \frac{dV_u \bigg|_{t=dt}}{dt} \right) = R \left( \chi \eta_u \bigg|_{t=dt} - C \frac{dV_u \bigg|_{t=dt}}{dt} \right) \]

\[ (A.32) \]

where \( dV_u \bigg|_{t=dt} \) is the voltage change during the first time interval, and \( V_u \bigg|_{t=dt} \), \( i \bigg|_{t=dt} \) and \( \eta_u \bigg|_{t=dt} \) are the voltage, current and the first derivative of the tip displacement at the end of the first time interval, respectively.

From Eq. (A.32), the voltage and the power at the end of the first time interval can be expressed as:

\[ V_u \bigg|_{t=dt} = \frac{R \chi \eta_u \bigg|_{t=dt}}{1 + \frac{RC}{dt}} \]

\[ (A.33) \]

\[ P_u \bigg|_{t=dt} = \frac{(V_u \bigg|_{t=dt})^2}{R} \]

\[ (A.34) \]

where \( P_u \bigg|_{t=dt} \) is the power at the end of the first time interval.
Similarly, at any specified time \( t=n\times dt \), with the results from the last time interval \((t=(n-1)\times dt)\) as the initial conditions, the vibration responses \( \eta_u(n\times dt) \), \( \eta_d(n\times dt) \), \( \xi_1(n\times dt) \) and \( \gamma_1(n\times dt) \) and their derivatives can be obtained following the same procedure described earlier. The voltage at the end of the time interval is the summation of the voltage at the end of the last time interval and the voltage increment during current interval:

\[
V_u\big|_{t=n\times dt} = V_u\big|_{t=(n-1)\times dt} + dV_u\big|_{t=(n-1)\times dt} \\
= R \left( i\big|_{t=n\times dt} - C \frac{dV_u\big|_{t=(n-1)\times dt}}{dt} \right) = R \left( \chi \hat{\eta}_u\big|_{t=n\times dt} - C \frac{dV_u\big|_{t=(n-1)\times dt}}{dt} \right)
\]  

(A.35)

where the subscripts \((n-1)\) and \(n\) denote the \((n-1)\)th and \(n\)th time interval, respectively.

From Eq. (A.35), the voltage change during the \(n\)th time interval can be expressed as:

\[
dV_u\big|_{t=(n-1)\times dt} = \frac{R \chi \hat{\eta}_u\big|_{t=n\times dt} - V_u\big|_{t=(n-1)\times dt}}{1 + \frac{RC}{dt}}
\]  

(A.36)

Finally, the voltage and power at the end of the \(n\)th time interval are expressed as:

\[
V_u\big|_{t=n\times dt} = V_u\big|_{t=(n-1)\times dt} + \frac{R \chi \hat{\eta}_u\big|_{t=n\times dt} - V_u\big|_{t=(n-1)\times dt}}{1 + \frac{RC}{dt}} \quad (n = 2, 3, 4, \ldots) 
\]  

(A.37)

\[
P_u\big|_{t=n\times dt} = \frac{(V_u\big|_{t=n\times dt})^2}{R} \quad (n = 2, 3, 4, \ldots)
\]  

(A.38)

Therefore, with the results from the previous time interval as its initial condition, the vibration responses together with the voltage and power response of the system for each time interval under any given wind speed can be calculated. The whole process of the step-by-step calculation is realized using a Matlab program. It is noted one assumption made in the proposed methodology is that the critical flutter speed is the cut-out wind speed for the design. When the wind speed is over critical flutter speed, the vibrations of the H-shape beam will be restrained by the stoppers mounted at the top and bottom end of the harvester to dissipate excessive energies in order to ensure the safety of the system. Therefore, the proposed methodology is performed under the critical flutter speed and the wind speed beyond that is not considered.
in present study.

**A.4. Numerical Simulation Results and Parametric Study**

In this section, the numerical simulation results are presented and a parametric study is carried out as well to explore the effects of several parameters on the output of the proposed harvester including external resistance, wind speed, mass of the H-shape beam, and excitation frequency.

**A.4.1 Vibration of the H-shape Beam and Harvested Power**

As discussed in Section A.3, the coupled analysis of the proposed wind energy harvester can be simulated using Matlab ODE method. The geometric and material properties of the cantilever beam and the H-shape beam are shown in Table A.1. The resistance for the external circuit is 15kΩ. The mean wind speed of the incoming flow is set as 10m/s and the wind velocity components are randomly generated. In the initial status, the whole system including the H-shape beam and all the cantilever beams remain still with zero displacement, velocity and acceleration.

Fig. A.7(a) shows the vibration of the system without the cantilever beams, i.e., no impact involved during the vibration of the H-shape beam. The amplitudes for both vertical and torsional displacements of the H-shape beam are 0.14m and 0.16rad, respectively. Fig. A.7(b) shows the coupled vibration of the system and the transverse tip displacement of the upstream cantilever beam is shown in solid green line. It can be seen that the vertical and torsional displacements of the H-shape beam considering impacts are reduced to 0.05m and 0.04rad, which are only one third of the corresponding displacements if no impact is considered. Therefore, a large portion of energy goes into the cantilever beams. It also shows that the cantilever beam is hit several times during one vibration cycle of the H-shape beam. Therefore, the cantilever beam vibrates at a much higher frequency than the H-shape beam and significant frequency up-converting is achieved.
Figure A.7. Displacement versus the time (U=10m/s). (a) H-shape beam without the cantilever beams, (b) H-shape beam with the cantilever beams ($d_{tip,u}$ represents the transverse tip displacement of the upstream cantilever beam)

Figure A.8. Voltage and harvested power versus time (U=10m/s, R=15kΩ). (a) Voltage in each upstream cantilever beam, (b) voltage in each downstream cantilever beam, (c) harvested power from each upstream cantilever beam, (d) harvested power from each downstream cantilever beam

The voltage and harvested power of the cantilever beams are plotted in Fig. A.8. As shown in Figs.
A.8(a) and A.8 (b), the peak values of generated voltage in both the upstream and downstream cantilever beam reach as high as 42V and 46V, respectively. Figs. A.8(c) and A.8(d) shows the average harvested power for the upstream and downstream cantilever beam are 11.77mW and 9.45mW, which are higher than or comparable with that of most of the wind energy harvesters shown in Table A.3 (Section A.4.3). However, it is worth mentioning that sizes of the wind energy harvesters are different. Therefore, it is better to use power density to compare their effectiveness. In the present study, the corresponding power densities for the upstream and downstream cantilever beam are 6.1 mW/cm$^3$ and 4.9 mW/cm$^3$, which is much higher than the other devices even before the wind speed increases to critical flutter speed. Due to the series of sequent impacts between the teeth and the bulges, high energy keeps transferring from the considerably large vibration amplitude of the H-shape beam to the cantilever beams with high power generation efficiency. The effects of several key parameters of the proposed energy harvester are discussed hereafter.

**A.4.2 Effect of External Resistance**

![Figure A.9](image)

**Figure A.9.** Average harvested power with different external resistance for each upstream and downstream cantilever beam, respectively (U=10m/s)

The external resistance can be optimized in order to maximum the harvested power of the proposed energy harvester. Fig. A.9 shows the average harvested power for the wind energy harvester when the external resistance varies from 1 Ω to 150 kΩ. Therefore, for the cantilever beam, an optimum resistance that gives the maximum harvested power can be found. This result agrees well with the experimental results
reported by Zhang et al. [294]. The optimum external resistances for both the upstream and downstream cantilever beams are very close to 15 kΩ and the corresponding maximum harvested power are 11.77mW and 9.45mW, respectively. It can also be observed from Fig. A.9 that the influence of the external resistance on the harvested power is small when it is between 9 kΩ to 24 kΩ, while the harvested power drops significantly as the resistance approaches to zero. The optimum external resistance of 15 kΩ is used in the following sections.

### A.4.3 Effect of Wind Speed

The wind speed is a critical design parameter for the proposed wind energy harvester. Fig. A.10 shows the average harvested power for the first 20s of each upstream and downstream cantilever beam with respect to different wind speeds. All the parameters for the modeling are shown in Table A.1. The harvested power is found to be less than 2.0 mW as the wind speed is less than 4m/s. As the displacement of the H-shape beam is very small under very lower wind speed, it is hard or even impossible for the teeth to pass through the bulge of the cantilever beams. Consequently, the teeth can only hit the bulge twice in one vibration cycle and the performance of the harvester is the same as a single-impact harvester. It is noteworthy that the results are in good agreement with the experimental results for the single-impact harvester which gives a lower harvested power at a lower frequency [303]. However, the harvested power increases significantly when the wind speed is larger than 4m/s. Under such a higher wind speed, the vibrational amplitude of the H-shape beam is large enough for the teeth to pass through the bulge, which trigger the multi-impacts. Much more impacts can get involved during each vibration cycle of the H-shape beam. Under the wind speed of 14m/s, the average harvested power can be reached up to around 45mW for each cantilever beam. It’s worth noting that the critical flutter wind speed is 14.5m/s. Table A.3 shows the power density comparison of the proposed wind energy harvesting device with several other ones. As shown in Table A.3, the power density of the proposed energy harvester shows a significant increase from 4.9mW/cm³ to 23.4mW/cm³ as the wind speed varies from 10m/s to 14m/s, which implies that higher power goes into the cantilever beam with high transfer efficiency.
Figure A.10. Average harvested power for the first 20s of each upstream and downstream cantilever beam with respect to different wind speed

Table A.3. Comparison of power density for piezoelectric wind energy harvesting devices†††

<table>
<thead>
<tr>
<th>Device</th>
<th>dimension of the device (mm)</th>
<th>Average Power (mW)</th>
<th>Power density (mW/cm³)</th>
<th>Operation wind Speed (m/s)</th>
<th>Wind induced vibrations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric windmill [304]</td>
<td>----</td>
<td>7.5</td>
<td>0.9</td>
<td>4.5</td>
<td>----</td>
</tr>
<tr>
<td>Small scale windmill [305]</td>
<td>76×102×127</td>
<td>5.0</td>
<td>0.4</td>
<td>4.5</td>
<td>----</td>
</tr>
<tr>
<td>Piezoaeroelastic airfoil [290]</td>
<td>500×125×30</td>
<td>10.7</td>
<td>----</td>
<td>9.3</td>
<td>Flutter</td>
</tr>
<tr>
<td>Li’s harvester [306]</td>
<td>172×100×1</td>
<td>0.6</td>
<td>1.3</td>
<td>4.0</td>
<td>Flutter</td>
</tr>
<tr>
<td>Wu’s harvester [307]</td>
<td>1200×150×12.9</td>
<td>1020</td>
<td>3.2</td>
<td>9.0-10.0</td>
<td>Vortex</td>
</tr>
<tr>
<td>Hobeck’s harvester [308]</td>
<td>95.3×25.5×0.39</td>
<td>0.46</td>
<td>3.3</td>
<td>8.1</td>
<td>Vortex</td>
</tr>
<tr>
<td>Felix’s harvester [309]</td>
<td>300×130×50</td>
<td>13.0</td>
<td>2.3</td>
<td>8.2</td>
<td>Galloping</td>
</tr>
<tr>
<td>Proposed wind energy harvester</td>
<td>300×150×100</td>
<td>9.45-45.0</td>
<td>4.9-23.4</td>
<td>10.0-14.0</td>
<td>Flutter</td>
</tr>
</tbody>
</table>

†††The volumes are of piezoelectric patch used in the device

A.4.4 Effect of Mass for the H-shape Beam

The average harvested power of the cantilever beam with respect to different mass of the H-shape
beam under the wind speed $U=10\text{m/s}$ is shown in Fig. A.11. The mass moment of inertia is adjusted accordingly with respect to the change of the mass. Meanwhile, the stiffnesses of the eight supported springs are adjusted to maintain the original natural frequencies (same frequencies as those of the 1st Tacoma Narrows Bridge). All the other parameters are kept the same as those in Table A.1. As shown in Fig. A.11, when the mass is less than 0.40kg, the average harvested power of the upstream and downstream cantilevers are less than 2mW, which indicates that the harvester works more like a single-impact harvester as discussed earlier. As the mass increases from 0.40kg to 0.52kg, the harvested power shows a significant increase and the harvested power of the upstream and downstream cantilever beam are 10.27mW and 8.45mW, respectively. Since the large mass momentum makes the teeth easier to pass through the bulge, multi-impacts are triggered in one vibration cycle and higher vibration energy is transferred from the H-shape beam to the cantilever beam. As shown in the figure, when the mass is larger than 0.52kg, the harvested power increases only slightly with the mass. When the mass increases from 0.52 kg to 3.0kg, the increased harvested power of the upstream and downstream cantilever beams are 2.53mW and 2.58mW. Therefore, the harvested power is not sensitive to the mass and it remains at the same level regardless of the change of the mass, if the mass is large enough to make the teeth to pass through the bulge and trigger multi impacts for frequency up-converting.

![Figure A.11.](image)

**Figure A.11.** Average harvested power for the first 20 seconds of each upstream and downstream cantilever beam with respect to different mass of the H-shape beam ($U=10\text{m/s}$)
A.4.5 Effect of Excitation Frequency

The natural frequencies of the first vertical and torsional mode of the H-shape beam are originally 0.13 Hz and 0.2 Hz, which are the same as those of 1st Tacoma Narrows Bridge discussed earlier in the present study. A factor $k_\omega$ is introduced as the ratio of the modified natural frequency to the original one, in order to study the influence of the natural frequency of the H-shape beam on the harvested power. Assuming that both the first vertical and torsional frequencies increase with the same factor for a demonstrate purpose. Following the same procedure as discussed earlier, the stiffness of the eight supported springs are adjusted to achieve the desired natural frequencies with the same frequency ratio with respect to the original ones. Fig. A.12 shows the average harvested power of the cantilever beam with respect to different natural frequencies of the H-shape beam. The peak average harvested power occurs when the factor $k_\omega$ equals to 12.5 and the corresponding natural frequencies of the first vertical and torsional mode are 1.625 Hz and 2.5 Hz, respectively. Also, the average harvested power is almost invariant with the frequency factor $k_\omega$ when the $k_\omega$ is within the range 1-50, which indicates that the harvester can operate over a relative larger excitation frequency range.

![Figure A.12. Average harvested power for upstream and downstream cantilever beam](image)

A.5. Summary

To effectively harvest energy from wind-induced vibrations, a new piezoelectric multi-impact wind energy harvesting device is introduced, which can effectively up-convert low-frequency wind induced vibrations into high-frequency ones. Impact mechanism is incorporated in the wind energy harvester in
order to convert more vibrational energy into usable electrical power with high energy harvesting efficiency. The governing equations for different impact status are derived and a numerical simulation is carried out to estimate the dynamic responses and the power output of the system. A parametric study is also conducted to find the effects of several parameters on the power output of the proposed harvester and provide guidance for future design and manufacture. The simulation results show that the cut in wind speed of the proposed wind energy harvester is 4m/s, which suggests that the device can operate under a wide range of wind speed. The average output power for the piezoelectric cantilever beam reaches 11.77mW with a power density of 6.11mW/cm³ under the wind speed of 10m/s, which is sufficient enough to power small sensors. A high level of average output power (larger than 9mW) is achieved under a relatively wide band of excitation frequency (0.13Hz ~ 10Hz), which shows a very promising aspect of the proposed wind energy harvester in terms of the application in civil infrastructures.

Even though experimental study have been carried out and can be used to validate the components of the system [287,293,294,310], it is of great importance to conduct a proof of concept by carrying out either experimental study or fluid-structure-interaction (FSI) modeling or perhaps both to validate the proposed wind energy harvester. Nevertheless, the proposed energy harvester is based on vibrations and multi-impacts. When flutter occurs, the whole system could have large energy input from wind with series of large impacts between the piezoelectric cantilever beams and H-shape beam. As a result, the safety of the whole system including the H-shape beam, the piezoelectric materials, and the teeth, in the long run is of a great concern. To avoid possible failures and increase the safety of the system, two strategies will be applied for the current device design: (1) to increase the critical flutter speed by adjusting the geometry and properties of the harvester; (2) to implement stoppers at both top and bottom end of the harvester to dissipate excessive energies from the H-shape beam and reduce the vibration amplitude of the beam. Therefore, it is necessary to carry out more research for the proposed harvester especially the experimental study of the proposed harvester using wind tunnel to validate the theoretical modeling, evaluate the system reliability, and examine the feasibility of its application.
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Education Background

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Southwest Jiaotong University, Chengdu, China
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B.S. Civil Engineering 2006.09 ~ 2010.07
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Publications

Referred Journal Papers


**Referred Conference Papers or Presentations**


May. 12-15, Tokyo, Japan (Full paper).


