Seismic Transverse Response of Multi-Frame Bridges

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ABSTRACT

Long concrete bridges are often constructed in multiple frames separated by in-span hinges. Shear key elements at in-span hinges preserve the transverse integrity of adjacent frames. This method of construction facilitates post-tensioning and lowers adverse effects of creep and thermal deformations. The transverse response of multi-frame system has not been comprehensively studied. In addition, no rational method is available for estimating seismic design forces of shear keys. In the course of this study, approximately 9,400 nonlinear response history analyses were performed on the high-fidelity models of realistic prototype bridges. The seismic demands on columns, abutments, and in-span hinge shear keys were studied. Statistical methods such as the analysis of variance (ANOVA) and factorial analysis were implemented to understand the effect of independent factors including the number of frames, substructure system, valley shape, soil type, intensity of ground motions, etc.

It was found that in multi-frame system seismic demands on columns are smaller than those in equivalent continuous system where the heights of columns vary along the length of a bridge. The interaction of transverse and longitudinal displacement increases the probability of seismic unseating in in-span hinges. In multi-frame bridges, considerable damage to the abutment is expected. Contribution of the higher modes of vibrations to shear key forces is significant. In multi-frame bridges, higher modes may
generate large plastic deformations in the columns. An easy-to-implement formulation was developed for seismic design of shear keys using the concept of spectral analysis. In this method, modal shear key forces are modified separately by their corresponding modal displacement ductility before the modal combination. This concept was developed and proposed as a new analysis methodology. The proposed method accounts for the effect of impact on shear keys due to transverse gaps, and the effect of non-uniform base excitations.

**KEYWORDS**

Multi-Frame Bridge, Seismic Transverse Response, In-Span Hinge Shear Key, Modal Displacement Ductility, Modified Elastic Dynamic Analysis (MEDA)
APPROVAL PAGE

Doctor of Philosophy Dissertation

Seismic Transverse Response of Multi-Frame Bridges

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GLOSSARY

ARS: Acceleration Response Spectra
CQC: Complete Quadratic Combination
Two-Column: A bridge substructure system composed of two columns in a bent
EDA: Elastic Dynamic Analysis
In-Span Hinge: A joint between adjacent frames in multi-frame bridges
Multi-Frame Bridge: A bridge composed of multiple frames connecting at hinges by shear keys
MEDA: Modified EDA method
MCE: Maximum Considered Earthquake
NTH: Nonlinear Time History
NUBE: Non-Uniform Base Excitation
Pipe Seat Extender: A steel pipe connecting adjacent frames at in-span hinges
PS: Post tensioned concrete
PGA: Peak Ground Acceleration
PGV: Peak Ground Velocity
RC: Reinforced Concrete
SDC: Caltrans Seismic Design Criteria
SDOF: Single Degree-of-Freedom
Shear Key: A structural member connecting adjacent frames in transverse direction
Single-Column: A bridge substructure system made of a single extended pile-shaft.
SRSS: Square Root of Sum of Squares
UBE: Uniform Base Excitation
1 Introduction

1.1 MULTI-FRAME BRIDGES WITH IN-SPAN HINGES

A frame bridge is an integrated column-superstructure system (Figure 1.1a) which offers advantages including less construction and maintenance cost due to elimination of bearings at columns. It also provides more redundancy for seismic response. On the other hand in long concrete bridges the creep and thermal deformations may impose large demands on columns and expansion joints in a frame bridge system. Therefore, long cast-in-place post-tensioned (CIP/PS) and reinforced concrete (RC) box girder bridges are often constructed in multiple frames separated by in-span hinges in their superstructure (Figure 1.1b). The multi-frame construction facilitates post-tensioning of the superstructure and lowers the adverse effects of creep deformations in long bridges.
It also allows for the longitudinal thermal expansion and contraction of the superstructure without inducing large forces in supporting columns (Hube & Mosalam, 2008).

![Schematics of a) Continuous Frame Bridge, and b) Multi-Frame Bridge](image)

**Figure 1.1** Schematics of a) Continuous Frame Bridge, and b) Multi-Frame Bridge

A multi-frame bridge system is composed of the following components: 1) frames that act individually under service loads; 2) intermediate or in-span hinges performing as longitudinal expansion joints that allow for longitudinal expansion and contraction of the superstructure; and 3) abutments that support the reaction of the end spans (DesRoches & Fenves, 1998).

An in-span hinge detail is composed of end diaphragms, a concrete seat, bearings, and shear key/s. In-span hinges allow for the relative longitudinal movement of adjacent frames; at the same time, the transverse seismic integrity of the bridge needs to be preserved using one or more shear keys within the hinge to transfer lateral loads. These shear keys are typically constructed in the form of concrete blocks (Figure 1.2a). As a new practice in construction of concrete bridges in California, xx-Strong steel pipes (Figure 1.2b) are being used as in-span shear keys and as a measure to prevent seismic unseating (Yashinsky, 2013).
Figure 1.2  Typical Components of In-Span Hinges a) with Concrete Block Shear Key b) with Pipe Seat Shear Key

Figure 1.3  Left) In-Span Hinge in a Highway Interchange, Oakland, California (Courtesy of Godden), top right) Interstate 580 connector, San Rafael (Hube & Mosalam, 2008), bottom right) Interstate 80 connector, Albany (Hube & Mosalam, 2008)
The in-span hinges are typically positioned at or near the point of contraflexure of a continuous span superstructure under dead loads and, occasionally, in other places along the length (Figure 1.3).

According to National Bridge Inventory (FHWA, 2013), 25,000 (4%) of the bridges in the United State are located in the State of California. Figure 1.4 shows the demography of California’s bridges. Approximately 8,000 (32%) of California’s bridges are concrete box girder bridges. The number of multi-frame bridges was not found in the inventory; however it can be assumed that long box girder bridges longer have been constructed as multi-frame.

![Diagram of Bridge Types](Figure 1.4 California Bridge Distribution Based on Super Structural Type (FHWA, 2013))

**1.2 PROBLEM STATEMENT AND BACKGROUND**

There is an acute interest in using multi-frame systems for the construction of long concrete bridges especially in California, because of the advantages this system offers. However, the seismic response of multi-frame bridges is more complex than continuous frames due to discontinuity in superstructure. Several bridges collapsed due to hinge
unseating in past earthquakes. Estimation of seismic demand on columns in a discontinuous superstructure could be more complex comparing to a continuous system. In addition, estimation of seismic shear demand on in-span hinges is a complex problem. Section 7.2.5 of the Seismic Design Criteria (SDC 1.7) (Caltrans, 2013) indicates that in-span hinges are expected to transmit the lateral shear forces under small earthquakes and service loads. It also suggests that determining the seismic force demand on shear keys is a complex task because shear key forces are dependent on the magnitude of the relative displacement of the adjacent frames. There is no simple analytical method for the calculation of in-span shear key force. It is understood that the elastic dynamic analysis (EDA) method (i.e. response spectrum analysis) leads to a significant overestimation of shear key force demands and shall not be used to size shear keys.

The Caltrans SDC allows for shear transfer between adjacent frames if the ratio of the fundamental periods of vibration of the stiffer frame to that of the more flexible frame, in transverse direction, is larger than 0.7, assuming a synchronized vibration of frames. While no specific design force for in-span shear keys is recommended, the force demand is limited to the sum of the columns overstrength shear (typically defined as $M_{0, col}/L$, SDC Sec. 2.3.2) in the weaker of the adjacent frames. In cases where the ratio of the period of adjacent frames is smaller than 0.7, it is suggested that the capacity of in-span shear keys is limited to prevent transfer of large lateral forces to the stiffer frame. In a simplistic approach, shear keys are designed as a capacity-protected member for a portion of the maximum of the overstrength shear in the neighboring bents (Caltrans project committee, personal communication, 2012).
The lack of a reliable method for finding the force demand on in-span shear keys may lead to unrealistic and cost-prohibitive shear key designs. Although a similar natural frequency of the adjacent frames support the in-phase longitudinal vibration of the frames, it does not ensure a synchronous transverse seismic response of the frame. The transverse dynamic response of multi-frame bridges is more complicated than its longitudinal equivalent. This is because the transverse vibration of individual frames is compounded by the rotational modes of the frames' vibration and the in-plane vibration of the superstructure (Priestly, et al., 1996). Likewise, non-simultaneous lateral yielding of bents and abutment shear keys, as well as the transverse pounding of the frames, adds further complexity to the problem.

1.3 LITERATURE REVIEW

After the collapse of several bridges in the 1971 San Fernando earthquake due to hinge unseating, the application of cable restrainers was considered for the seismic retrofitting of existing bridges (DesRoches & Fenves, 1998). The failure of some of these restrainers in the 1989 Loma Prieta and the 1994 Northridge earthquakes led studies to focus mainly on the longitudinal seismic response of multi-frame bridges. The goal of these studies was to determine the minimum gap size and seat width required to avoid significant pounding and unseating due to the out-of-phase movement of adjacent frames (Fenves & Ellery, 1998; Hao & Chouw, 2008; Singh, 1994; DesRoches & Muthukumar, 2004; Shrestha, et al., 2013). Some others focused on the design of hinge restrainers as a retrofit measure for bridges with inadequate seat width.

A dissertation titled “Earthquake Analysis of Multi-Frame Bridges” (Singh, 1994) includes the basic results of the elastic transverse response analysis of multi-frame bridges with no assessment of shear key demands. This work presents that the gap size between adjacent frames affects the transverse response and the coupling of longitudinal and transverse responses. Concerning the transverse seismic analysis of a multi-frame bridge, Priestly et al., 1996 state that it is essential for hinges to be modeled by their exact geometry, because the hinges open and close in a non-uniform fashion. They also suggest that the deck of the bridge is assumed rigid in plane (Figure 1.5). The in-plane rigidity of the superstructure allows the system to be statically determinate, making the shear key force easily obtainable (Priestly, et al., 1996).

![Figure 1.5](image)

*Figure 1.5*  The Concept of a Simple Model for Calculating the Force Demand of In-Span Shear Key (Priestly, et al., 1996)
Bozorgzadeh (2007) showed that concrete shear keys are stronger than anticipated, so a new modular shear key has been designed to enable the capacity protection of bridge abutments (Bozorgzadeh, et al., 2007).

A review of the technical literature reveals that there is a lack of understanding of the transverse seismic response of multi-frame bridges and the parameters affecting that response. In addition, the role of in-span shear keys in the seismic performance of multi-frame bridges is unknown, and the current bridge seismic design codes do not include detailed provisions for determining the lateral force demands on in-span hinge shear keys. This study will address this gap by investigating the transverse response of multi-frame bridges, with a particular focus on developing a method that enables the shear force demands in in-span hinge shear key force to be assessed.

1.4 OBJECTIVES AND OUTREACHES

The main goals of this study are to understand seismic response of multi-frame bridges and to develop a simple rational method for determining reliable design force demands for sizing in-span hinge shear keys. The specific objectives of this study are presented below:

- Demonstrate the dynamic transverse response characteristics of a multi-frame bridges system making use of the governing equation of motion and simplified analytical models.
- Develop high-fidelity three-dimensional nonlinear models for a suite of prototype bridges that enables performing a large number of nonlinear dynamic analyses.
• Examine the effects of several geometric, ground motion and design parameters on the seismic response of multi-frame bridges and force demands on in-span shear keys.

• Develop a rational method for the reliable estimation of in-span shear key force demands as well as a realistic upper bound design force.

• Investigate the significance of in-span shear keys to the seismic performance of multi-frame bridges by studying bridge systems with ductile shear keys.

The results of this study are expected to expand the application of multi-frame bridges by addressing the lack of knowledge regarding the seismic response of these systems. It also benefits the safety of the design of in-span shear keys by offering an easy-to-implement guideline. This method eliminates the need for conducting sophisticated and computationally expensive nonlinear response history analyses for design purposes.

1.5 METHODOLOGY

Figure 1.6 presents the overall design of the study. This study is consisted of two components: 1) understanding the dynamic characteristics of multi-frame bridge systems in transverse direction, and 2) conducting extensive analytical study on prototype bridges to generate the analytical data needed for the study of several seismic responses. First part will benefit the second part by implementing the findings and perceptions on multi-frame system.
Figure 1.6 Research Map

1 Introduction
1 Introduction

1) Understanding the Dynamic Characteristics of Multi-Frame Bridge Systems:
Dynamic equation of transverse motion for a generic multi-frame system is developed. The dynamic response characteristics, unique to this bridge system, are investigated by performing steady state and modal analyses on two elastic bridge models. Nonlinear properties are assigned to the same models, and a limited number of time history analyses are performed to quantify the isolated effects of different system properties, such as period ratios of adjacent frames, on the seismic response of this bridge system.

2) Extensive Analytical Study:

The main body of this research is comprised of extensive analytical approach to achieve the aforementioned objectives. Approximately 9400 nonlinear time history (NTH) analyses were conducted on prototype bridges which designed in accordance with Caltrans’ seismic design practices. The following describes detailed mythology for this component of the research:

a) Development of Prototype Bridges: In consultation with bridge design experts at Caltrans, a set of 56 box-girder prototype bridges with a span length of 110 and 200 ft was defined. The prototype models are comprised of multi-frame bridges with single-column of extended pile shaft and pinned-base two-column bents. To study the effect of the number of frames, sets of two-, three-, four-, and five-frame bridges are considered. The frame lengths vary from 440 to 720 ft as a practical length for post-tensioned superstructures. Prototype models with different realistic valley shapes are included in each set.
\textit{b) Defining Seismic Hazard Levels and Ground Motions:} Three seismic hazard levels of moderate, large, and severe are defined for design purpose. The corresponding acceleration response spectrums (ARS) are obtained from Appendix B of SDC (Caltrans, 2013) for four soil types of B, C, D, and E for the hazard levels. Then, the Pacific Earthquake Engineering Research Center (PEER) NGA ground motion database (PEER, 2011) is used to select and scale a suite of eleven sets of three biaxial ground acceleration histories (total thirty-three). Each set is selected from sites with a soil type, moment magnitude, and intensity level compatible with one ARS. SeismoMatch software (Seismosoft, 2011) is used to match the ground motions to the target ARS.

\textit{c) Developing Refined Analytical Models:} The spine modeling method (Caltrans, 2014; Aviram, et al., 2008) is used to develop the three-dimensional analytical model of each prototype bridge in OpenSees 2.4.4 (McKenna & Fenves, 2014). These analytical models are used in the design of the prototype bridges and for conducting the response history analyses. The elastic frame elements are used for the deck, diaphragms, and bent caps. The columns are modeled by the inelastic beam-column elements with fiber sections. For the single-column bridges, soil parameters are considered to locate the depth of fixity and the location of the plastic hinge. For the Two column bents, plastic hinges are assigned to the top of the column elements. Abutments and in-span hinges are explicitly modeled. Their assembly is composed of elements representing diaphragms, bearings, shear keys, gaps and impact, backwall, and backfill soil.
The inelastic response of an individual column model is verified using the test data from shake table experiments previously performed in UNR (Zaghi & Saiidi, 2010). The system level response of a sample OpenSees model was compared to that of the same model obtained from SAP2000 v15.1 (CSI, 2011). A series of sensitivity analyses are performed to define the modeling parameters of shear keys and transverse impact.

A robust OpenSees script with about 10000 lines was developed in this research to generate and design the analytical models. This OpenSees code is used to perform gravity loading; modal response spectrum analysis (EDA) to find displacement demand; moment curvature analysis to find the effective section property; pushover analysis of subsystem column bents to calculate yield displacement and local ductility capacity; and pushover analysis of the entire structure to check the global displacement capacity.

d) **Designing the Prototype Models:** The selected hazard levels and corresponding ARSs are used for the seismic design of each prototype model according to Caltrans SDC v1.7 (Caltrans, 2013). The following five criteria are considered in designing each prototype: 1) the minimum local displacement ductility capacity of the bents; 2) the maximum displacement ductility demands of the bents; 3) the global displacement capacity of bridge; 4) the minimum lateral load capacity of the bents; and 5) the maximum permissible P-Delta effects.

e) **Performing Nonlinear Time History (NTH) Analysis on the Prototype Models:** Each prototype model is subjected to ground motion accelerations that is corresponded to its design ARS. Because each set of the ground accelerations are compatible with the ARS used to design of the prototype bridges, the nonlinear response history analyses
are expected to be proportional to the design and result in a realistic estimation of the force demands in in-span shear key. In addition to the “main analyses”, a subset of analyses are conducted to investigate the effects of the following parameters on shear keys forces: gap closure and pounding of adjacent frames, yielding of abutment shear keys, and spatial variations in the base excitation along the length of the bridge. Responses of the system with ductile in-span shear keys - as opposed to elastic shear keys - on the seismic performance of multi-frame bridge system is also investigated.

In addition, equivalent continuous frame bridges (elimination of in-span hinges) are analyzed in order to compare the responses between multi-frame and continuous frame bridges.

f) Data Processing and Analysis of Variance (ANOVA): Approximately 260GB of raw data was generated in this study. Multiple post-processing codes are developed using MATLAB 2012a (Mathworks, 2012) to develop the statistical distributions of the different responses, as well as the relationships between the shear key forces and the other response parameters. Responses such as the maximum displacement and acceleration at hinges, base shear, column ductility, period ratios of standalone frames, EDA forces, and pushover forces among several others are studied. Correlations between different response parameters and the relationships between the maximum shear key forces and multiple possible analysis methods are presented and discussed. The analysis of variance (ANOVA) is used for comparing the seismic responses between two groups (multi-frame and continuous frame) with respect to six independent factors:
number of frames (or bridge length), substructure system, valley shape (columns stiffness
distribution), soil type, ground motion intensity, and columns overstrength.

\textit{g) Developing a Rational Analysis Method and Presenting the Design Examples:}
The maximum shear key forces, obtained from the NTH analyses, are considered as
the reference to develop a rational method for determining the design force demands for
in-span hinge shear keys. In the proposed method, forces obtained from spectral
analysis (i.e. EDA) are modified to account for the transverse yielding of the bridge.
Factors are proposed to adjust the base force for the effects of the transverse pounding
of frames and non-uniform base excitation. In addition, a rational upper bound force is
defined to cap the shear key design force demand. Finally, a set of examples are
presented to facilitate the implementation of the proposed method by bridge designers.

\textbf{1.6 ORGANIZATION OF DISSERTATION}
This dissertation comprises eight chapters and three appendices: Chapter 2
presents the transverse dynamics of this bridge system and provides the results of
steady state analyses of elastic systems and response history analyses of simplified
inelastic systems.

Chapter 3 provides information on the geometry of prototype bridges, seismic
hazard levels, and the selection and spectrum matching of input ground motions.

Chapter 4 is allocated to the detailed description of the analytical modeling method.
The assumptions used for developing elastic and inelastic models in OpenSees are
presented in this chapter. This chapter also includes the results of the sensitivity analyses on modeling parameters.

In Chapter 5, the procedure for the seismic design of the prototype bridges based on SDC v1.7 is discussed in detail.

Chapter 6 presents the results obtained from the analyses of the prototype bridges. It presents the correlations, statistical distributions, observations, analysis of variance (ANOVA) and interpretations of the results. Correlations between different response parameters and the relationships between the maximum shear key forces and multiple possible analysis methods are also presented and discussed in this chapter.

Chapter 7 describes the proposed rational method for prediction of in-span shear key demand forces. This is followed by some design examples.

A summary of this dissertation and a list of observations and important conclusions are presented in Chapter 8.

Three appendices (A, B, and C) are included in the document to present the design of the ground motions and the spectrums, a summary of the design results of the prototype bridges, and graphs presenting the important analytical results, respectively.
2 Transverse Dynamics of Multi-Frames

A generalized equation of motion for the transverse vibration of a multi-frame bridge system is discussed in this chapter. The response of elastic systems is studied through steady state analyses of simple elastic models. Subsequently, the dynamic response of inelastic systems is investigated by performing nonlinear time history (NTH) analyses of the same models after implementing nonlinear elements. This chapter also investigates the effects of stiffness and the capacity ratios of adjacent frames on the dynamic response of the system. Finally, the results of the elastic dynamic analysis (EDA) are compared to the nonlinear results for the simple model.

2.1 EQUATION OF MOTION FOR THE MULTI-FRAME BRIDGE SYSTEM

Figure 2.1 provides a schematic of a three-frame bridge plan, as well as a free-body diagram of the effective forces on each frame in transverse direction. In this system,
the superstructure is assumed to be composed of an articulated and flexible superstructure that is supported by a series of springs representing the transverse stiffness of the columns. The forces acting on the bridge superstructure are shown through a consideration of the shear key and column forces as external loads on each frame and the inertial forces as internal actions.

![Bridge Plan](image)

**Figure 2.1** Schematic of a Multi-Frame Bridge Plan and Dynamic Free Body Diagram of Frames in Transverse Direction

The inertial forces on the middle frame are shown in Figure 2.2 with a nonuniform distribution because of the flexibility of the superstructure and a nonuniform distribution of accelerations. The effects of the flexibility of the deck will be discussed in detail later in this chapter. Damping forces are not shown in this free-body diagram.
The equations of dynamic equilibrium for this frame are presented as Eq. 2.1 and 2.2. The equations are split into frames while they are coupled via shear key force. For each frame, two equations govern the equilibrium of transverse forces and the equilibrium of moments about the left end (in-span hinges).

\[
\sum F = 0; \quad \int_0^l F_{i2}(x,t) \, dx + F_{S3}(t) + F_{S4}(t) + V_1(t) + V_2(t) = 0 \quad \text{Eq. 2.1}
\]

\[
\sum M = 0; \quad \int_0^l F_{i2}(x,t) \cdot x \, dx + F_{S3}(t) \cdot l_1 + F_{S4}(t) \cdot l_2 + V_2(t) \cdot L = 0 \quad \text{Eq. 2.2}
\]

where \( F_{i2}(x,t) \) is inertial force profile along the second frame, \( F_{S3}(t) \) and \( F_{S4}(t) \) are column reactions, and \( V_1(t) \) and \( V_2(t) \) are left and right shear key forces, respectively. \( L, l_1 \) and \( l_2 \) are frame length, first column and second column distance from the left hinge, respectively. By observing the equations of motion, it is clear that the shear key force stems from the difference between inertial and stiffness forces within individual frames. These forces are also affected by the interaction of frames with each other due to the continuity of displacements (the continuity of rotations does not exist because of the in-span hinges). Applying compatibility of displacement or acceleration between
adjacent frames at the hinges enables for solving the equation of equilibrium for the entire system. It is concluded that in large ground motions the column forces are nonlinear; hence elastic dynamic analysis (EDA) results in over predicted force. The inertial force along the frame is a complex function thus the static analysis methods such as pushover fail to estimate the shear key force.

2.2 MULTI-FRAME BRIDGES AND “PERIODIC STRUCTURES”

From a different point of view, long multi-frame bridges may be considered a finite periodic structure. An example of a periodic structure is shown in Figure 2.3. A periodic structure is a structure that is made up of similar sub-systems, which are connected end to end (Mead, 1996). Each sub-system is made up of two parts: The first is a group of point masses connected one after the other by massless springs; in the case of multi-frame bridges, the tributary mass is set on the column nodes. The second consists of elements that hold up a group of connected masses; for bridges, this is presented by the flexural stiffness of the superstructure.

![Figure 2.3](image)

*Figure 2.3* A Typical Periodic Structure (Zhang, et al., 2012)

A periodic structure undergoing a forcing function experiences wave propagation within its length (Mead, 1996). These waves move through each frame of a periodic structure. Based on the theory of wave propagation, a wave is reflected and transferred when it reaches a discontinuity. In multi-frame bridges, the presence of in-span hinges
causes discontinuity in the rotation of the superstructures about the vertical axis. Thus, the in-plane flexural waves are reflected once they reach an in-span hinge. Due to the presence of these reflections, a form of explicit time history analysis capable of simulating the wave propagations is needed to obtain the exact response of the superstructure. Wave propagation deals with a transient response at a local level. Thus, the modal analysis method may provide slightly different results as the time-dependent effects are not captured.

Modal analysis is based on the theory that the response of a structure can be modeled by a combination of a set of harmonic modes referred to as the natural modes of vibration. The natural modes of vibration are inherent to the structure and depend only on physical characteristics such as stiffness, mass, and damping as well as the spatial coordinates of each characteristic (He & Fu, 2001). Because the modal analysis cannot capture the transient (time-dependent) local effects, it ignores the presence of a wave’s reflection. One should be mindful of this limitation of analysis methods - that time varying local responses are not explicitly simulated.

2.3 DYNAMIC CHARACTERISTICS OF ELASTIC SYSTEMS

2.3.1 Modal Characteristics of Multi-Frame Bridge Models

In order to understand the dynamic characteristics of multi-frame bridges, a series of modal and steady state analyses are performed on two simple elastic multi-frame bridge models shown in Figure 2.4. The shear key force, in-span hinge displacements, and in-span hinge accelerations are selected as structural responses. The excitation was applied as a base acceleration. For this purpose, two-dimensional stick models are
developed in SAP2000 v15.1.0 (CSI, 2011) for the two- and four-frame bridges shown in Figure 2.4. The span length is selected as 200 ft and the in-span hinges are located at one-fifth of the span. The superstructure is modeled by elastic frame elements with a realistic cross section shown in Figure 2.4. Each span is divided into five segments to capture the effect of the higher modes. To be able to read the shear key force responses, a 5-ft frame is used to connect the adjacent frames. The moment at one end of the connector element is released to account for the hinge action. The columns are simply modeled by linear springs with an elastic stiffness value of 100 kip/in. Abutments are modeled as pinned supports. Translational masses and rotational masses are assigned to each node. These masses are calculated based on the tributary length for each node.

The rationale for defining two- and four-frame models is to study the effects of the number of frames on the responses of the hinge/s. As such, comparing the results between hinge H1 in the two-frame model and hinges H1, H2, and H3 in the four-frame model allows for the consideration of different frame numbers and different frame
boundary conditions. The first ten transverse mode shapes of the two- and four-frame models and their corresponding natural frequencies are presented in Table 2.1 and Table 2.2, respectively. The dashed line shows the locations of in-span hinges. The mode shapes are obtained from two-dimensional models with only transverse and rotational DOFs. In a three-dimensional model of a bridge, the longitudinal and vertical modes may fall between the transverse modes.

The modal shapes with spread out frequency values indicate the dynamic effects of the in-plane flexibility of the superstructure. The first and the first three modes of vibration in the two- and four-frame models, respectively, may be considered as “rigid superstructure” modes since the flexural deformations of the superstructure do not appear to be defining the modal shapes. Another important observation from Table 2.3 is the close frequencies of the first three natural modes of the four-frame model.

It is notable that individual frames experience the same modal shapes one at a time but at different frequencies. The number of zero-crossings (or the number of points of contraflexure) within one frame is an identifier of the modal shape. Therefore, a multi-frame bridge with $n$ frames is composed of $n$ dynamic subsystems. As an example, for the two-frame model, mode shapes #2 and #3 differ because Frame 1 goes from its first inter-frame mode to the second one. Similarly, in mode shapes #3 and #4, Frame 2 goes from its second to third inter-frame modes. This is a well-understood phenomenon in periodic structures, which is discussed in Sec. 2.2. For the same reason, the rule that “the mode number is equal to the number of zero-crossings + 1” can be violated in
multi-frame bridge systems. As an example, the 9th and 10th natural mode of the four-frame model have 9 and 10 zero-crossings, respectively.

The significance of this phenomenon, with respect to the forces in the shear keys at different hinges, is that each shear key is influenced by the mode shapes of the adjacent frames. There will be mode shapes that do not contribute to the shear key forces. This distinct modal characteristic of multi-frame bridges necessitates the inclusion of the participation of a large number of modes in order to accurately estimate the maximum shear key forces at all the hinges.
Table 2.1 Two-Frame Transverse Modal Properties

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. (Hz)</th>
<th>Mode Shape</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Frame-1</td>
</tr>
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</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>5</td>
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<td><img src="#" alt="Graph" /></td>
</tr>
<tr>
<td>10</td>
<td>7.15</td>
<td><img src="#" alt="Graph" /></td>
</tr>
</tbody>
</table>
### Table 2.2: Four-Frame Transverse Modal Properties

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. (Hz)</th>
<th>Mode Shape</th>
</tr>
</thead>
<tbody>
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<td></td>
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<tr>
<td>2</td>
<td>0.58</td>
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</tr>
<tr>
<td>3</td>
<td>0.63</td>
<td><img src="image9" alt="Mode 3" /></td>
</tr>
<tr>
<td>4</td>
<td>0.73</td>
<td><img src="image13" alt="Mode 4" /></td>
</tr>
<tr>
<td>5</td>
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<td><img src="image17" alt="Mode 5" /></td>
</tr>
<tr>
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</tr>
<tr>
<td>10</td>
<td>2.10</td>
<td><img src="image37" alt="Mode 10" /></td>
</tr>
</tbody>
</table>
2.3.2 The Concept of Steady State Response

When a harmonic force is applied to a structural system, it will typically reach a steady state after going through some transient behavior. The steady state analysis is a common method used for studying the response of elastic structures to harmonic excitations at different frequencies (Rao, 2010). If the support of a spring-mass-damper system with mass, stiffness, and damping coefficients of $m$, $k$, and $c$ undergoes harmonic displacement excitation at the base (as shown in Figure 2.5a), the equation of motion (Eq. 2.3) can be obtained from the free body diagram shown in Figure 2.5b.

![Figure 2.5 Base Excitation on a Mass-Spring System](image)

The steady state displacement response of the mass, $x_p(t)$, can be expressed as Eq. 2.4.

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \quad \text{Eq. 2.3}$$

$$x_p(t) = X \sin(\omega t - \phi) \quad \text{Eq. 2.4}$$

Where $\omega$ and $\phi$ are the frequency of the base excitation and phase angle respectively, and $X$ is the amplitude of the response. The ratio of the amplitude of the response, $x_p(t)$, to that of the base motion, $y(t)$, i.e. $X/Y$, is called the displacement transmissibility (named as steady state response). The magnitude and phase of the steady state response can be presented as a function of $\omega$ as expressed in Eq. 2.5 and 2.6, respectively.
\[
\frac{X}{Y} = \left[\frac{1 + (2\zeta \omega / \omega_n)^2}{1 - (\omega / \omega_n)^2 + [2\zeta \omega / \omega_n]^2}\right]^{\frac{1}{2}} \tag{Eq. 2.5}
\]

\[
\phi = \tan^{-1}\left[\frac{2\zeta (\omega / \omega_n)^3}{1 + (4\zeta^2 - 1)(\omega / \omega_n)^2}\right] \tag{Eq. 2.6}
\]

where \(\omega_n\) is the natural frequency of the dynamic system and \(\zeta\) is the critical damping ratio. The amplitude of the steady state response and phase angle can be plotted for different frequencies of excitation as shown in Figure 2.6.

Steady state relationships can be obtained for any given excitation and response measures of a structure, such as displacements, accelerations, and forces. The steady state analyses can be performed on any complex multi-degree of freedom structural system using structural analysis software packages, such as SAP2000 (CSI, 2013). In a more complex multi-support system under non-uniform base excitation, the excitation frequencies must be kept the same.
2.3.3 Steady State Response Analysis of Simple Bridge Models

The two models introduced in Sec. 2.3.1 are used for the steady state analyses. The steady state analysis is performed using a base displacement with unit magnitude and with frequencies ranging from 0.01Hz to 100Hz. The steady state responses of the in-span hinges are obtained for the key forces, in-span hinge transverse absolute displacements, and in-span hinge transverse absolute accelerations. The unit of a steady state response curve is equivalent to that of the response divided by that of the excitation (in this case Y=1.0). Yet, in order to be able to compare the effects excitations with different frequencies on the response of the system, the magnitudes of the response curves are normalized with their maximum values. The steady state relationships for in-span hinges are shown in Figure 2.7 for the two-frame system. The response of hinges H1, H2, and H3 of the four-frame bridge model are shown in Figure 2.8, Figure 2.9, and Figure 2.10, respectively. The natural frequencies and the mode numbers are presented by the vertical dot lines.
Figure 2.7  The Normalized Steady State Responses of the In-Span Hinge H1, Two-Frame Model

Figure 2.8  The Normalized Steady State Responses of the In-Span Hinge H1, Four-Frame Model

Figure 2.9  The Normalized Steady State Responses of the In-Span Hinge H2, Four-Frame Model
A study of these curves indicates that the displacement responses at in-span hinges are not significantly affected by the higher modes. In contrast, the higher natural modes contribute significantly to the accelerations and shear key forces. In general, the shear key force responses often follow a pattern similar to that of the acceleration for entire range of frequencies. The input frequencies that have an impact on the acceleration response of the hinges are those that contribute to the shear key force responses too.

For the two-frame model, the first mode of vibration has the largest contribution to the displacement while the shear key force is maximized with the 11th mode. The hinges in the four-frame model do not respond similarly to the excitation of the different modes. In this model, hinges H1 and H2 are controlled by the second mode of vibration, while the response of hinge H3 is governed by the third mode. The shear key force and acceleration are maximized by mode numbers higher than 50. This confirms the multi-subsystem response characteristics of multi-frame bridges as discussed in Section 2.3.1.
2.3.4 Analysis of Subsystem Models

To investigate the effect of the dynamic responses of the far frames on the force responses of an in-span hinge, a separate set of analyses are conducted on the four-frame model. In these analyses, three subsystems, each comprised of only two of the adjacent frames of the four-frame model, are analyzed. These subsystem models include Frame 1-Frame 2, Frame 2-Frame 3, and Frame 3-Frame 4 to study the H1, H2, and H3 hinges, respectively. Three steady state shear key force response curves are generated as shown in Figure 2.11 to Figure 2.13. These figures show that an approximate shear key force response may be obtained from the analysis of a subsystem of the two adjacent frames in isolation. This is more evident for the case of H3. This may be a result of the isolated dynamic response of this hinge, while the responses of the other two hinges are closely coupled because of their close natural frequencies. This can be observed by comparing the modal shape of the four-frame bridge in Table 2.2.

---

**Figure 2.11** Steady State Responses of Hinge H1 from the First Subsystem of Four-Frame Model
Figure 2.12  Steady State Responses of Hinge H2 from the Second Subsystem of Four-Frame Model

Figure 2.13  Steady State Responses of Hinge H3 from the Third Subsystem of Four-Frame Model

2.3.5 Effect of the Rigidity of the Superstructure

Finally, to study the significance of the flexibility of the superstructure on the force response of the shear key, the two-frame model is analyzed after the superstructure was made 100 times stiffer. Figure 2.14 shows the steady state responses obtained from the models with flexible and stiff superstructures. It is evident that only the effect of the first mode of vibration can be captured accurately, which may result in an under prediction of the shear key forces. The model shows some higher mode effects because the superstructure is not completely rigid. This is a clear indication that, even in the case of unrealistically stiff superstructures (100 times stiffer), the higher modes may form and contribute to the force due to the large length of the superstructure. Thus, to obtain the
maximum shear key forces, the modal analyses shall include modes with frequencies that are typically larger than those sufficiently used for finding the maximum displacements.

2.4 DYNAMIC RESPONSE CHARACTERISTICS OF INELASTIC SYSTEMS

2.4.1 Modeling Assumptions

To investigate the dynamic response characteristics of an inelastic multi-frame system and study the effects of basic system properties, a number of response history analyses are performed on the same bridge models illustrated in Figure 2.4. This time, an elasto-plastic force-displacement relationship was assigned to the support springs. For nonlinear response history analyses, OpenSees 2.4.3 (McKenna & Fenves, 2014) is employed in place of SAP2000 because of the simplicity it offers for the execution of the parametric studies that will be presented later in this chapter.

The deck is modeled using “elasticBeamColumn” elements with the transverse stiffness of the cross section shown in Figure 2.4. Each span is broken into five segments and the same masses used in the elastic models are assigned to the nodes.
For modeling the in-span hinge, “zeroLength” elements with uniaxial material “Elastic” with large shear stiffness are used to connect the end nodes of adjacent frames. The support springs are modeled by “zeroLength” elements with the inelastic material “Steel02” assigned to them (OpenSees Wiki, 2014). This material has a bilinear force-displacement relationship. The elastic stiffness is set equal to 100 kip/in, the yield capacity was taken to be 300 kips, and the post-elastic stiffness of the springs was assumed to be 3% of the elastic stiffness. The yield force is based on an approximate seismic design for the acceleration response spectrum (ARS) for soil type C, a magnitude of 7.0-7.5, and a PGA of 0.4g per Caltrans SDC (Caltrans, 2013). A damping ratio of 5% is assigned to the first and third modes using the Rayleigh formulation (Chopra, 2001).

The ground motion number NGA#782, which was obtained from the PEER database (PEER, 2011), is utilized as the base excitation. The ground motion is matched to the design spectrum. Figure 2.15 shows the design ARS and the acceleration spectrum of the matched acceleration history.
2.4.2 Responses and Observations

The response histories of the shear key force, in-span hinge displacement, and acceleration responses are normalized with their maximum values and plotted in the same graphs as shown in Figure 2.16 for the two-frame model. Figure 2.17 to Figure 2.19 are the same graphs but for H1, H2, and H3, respectively, for the four frame model. Only the portion of the response that is comprised of the large intensity responses is presented.

![Figure 2.16 Normalized Response Histories of Hinge in Two-Frame Model](image)

![Figure 2.17 Normalized Response Histories of Hinge H1 in Four-Frame Model](image)
The general observation drawn from the figures shown above is that the shear key force profile approximately follows that of the hinge acceleration. Careful comparison of the shear key force histories, the hinge acceleration histories, and the hinge displacement histories confirms that the first two responses have larger high frequency contents. The maximum shear key force may not happen simultaneously with the maximum displacement or acceleration. These observations are consistent with the findings of the steady state analyses, thus demonstrating that the dynamic transmissibility of the system is similar to the shear key force and the hinge acceleration responses. The steady state graphs and response history results clearly indicate that higher modes play a significant role in maximizing the shear key force. On the other
hand, these higher modes do not contribute significantly in the displacement response of a multi-frame bridge system.

Figure 2.20 to Figure 2.23 show the relationship between shear key force and hinge displacement, velocity, acceleration, and the base shear values. No clear relationship exists between displacement and shear key force.

**Figure 2.20** Two-Frame Model, Relationship of Shear Key Force with a) Hinge Displacement, b) Hinge Velocity, c) Hinge Acceleration, and d) Base Shear
Figure 2.21  Four-Frame Model, Hinge H1, Relationship of Shear Key Force with a) Hinge Displacement, b) Hinge Velocity, c) Hinge Acceleration, and d) Base Shear

Figure 2.22  Four-Frame Model, Hinge H2, Relationship of Shear Key Force with a) Hinge Displacement, b) Hinge Velocity, c) Hinge Acceleration, and d) Base Shear
The snapshots of displacement and acceleration profiles along the length of the two-frame and four-frame prototypes are shown in Figure 2.24 through Figure 2.27. The in-span shear key with the maximum shear force is indicated by a solid black dot. These Figures show that, when the maximum shear key force happens, the superstructure is highly deformed transversely. It reveals that the transverse flexibility of the superstructure plays a substantial role in the force response of the shear keys. Consequently, assuming a rigid body transverse displacement and rotation of the individual frames may lead to significant errors in the estimation of shear key forces.

Figure 2.23  Four-Frame Model, Hinge H3, Relationship of Shear Key Force with a) Hinge Displacement, b) Hinge Velocity, c) Hinge Acceleration, and d) Base Shear
2 Transverse Dynamics of Multi-Frames

Figure 2.24  Two-Frame Model, System State at the Time of Maximum Shear Key Force

Figure 2.25  Four-Frame Model, System State at the Time of Maximum Shear Key Force at H1

Figure 2.26  Four-Frame Model, System State at the Time of Maximum Shear Key Force at H2

Figure 2.27  Four-Frame Model, System State at the Time of Maximum Shear Key Force at H3
2.5  EFFECTS OF THE ADJACENT FRAMES’ RELATIVE PROPERTIES

To examine the effect of adjacent frames’ stiffness and capacity on shear key force demand, three cases of parametric analysis are performed (indicated in Table 2.3). The K1 and C1 represent the stiffness and the capacity of the nonlinear springs of the frame with the short cantilever, respectively, while K2 and C2 represent that of the frames with the long cantilever. The ratios are varied between 0.1-1.0. In Case A, only stiffness varies while the capacities of the frames are kept constant. Case B is the opposite of Case A; and, in Case C, both stiffness and capacity vary at the same rate.

<table>
<thead>
<tr>
<th>Case</th>
<th>Stiffness Ratio</th>
<th>Capacity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.1 to 10</td>
<td>1.0</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>0.1 to 10</td>
</tr>
<tr>
<td>C</td>
<td>0.1 to 10</td>
<td>0.1 to 10</td>
</tr>
</tbody>
</table>

Although stiffness and capacity usually change relatively, three cases are studied to isolate different effects. In the four-frame model, only the properties of the adjacent frames are considered. The same inelastic models introduced in Sec. 2.4 are used. They have been analyzed under the Loma Prieta ground motion at three PGA intensity levels: 0.18g, 0.36g, and 0.72g to achieve different levels of nonlinear response. The results are shown in Figure 2.28 to Figure 2.30. For the hinge in the two-frame model and hinge H1 in the four-frame, when either of these ratios increases from 0.1 to approximately 0.6, the shear key force decreases. After that point, it starts increasing. Shear key force is not very sensitive to the ratios of frame properties larger than 4. The minimum forces are not correlated with the same ratio. In other words, adjacent frames
2 Transverse Dynamics of Multi-Frames

Figure 2.28  Frame Property Ratio Effect, Two-Frame Models, a) Case A, b) Case B, and c) Case C

Figure 2.29  Frame Property Ratio Effect, Four-Frame Models, Hinge1, a) Case A, b) Case B, and c) Case C

Figure 2.30  Frame Property Ratio Effect, Four-Frame Models, Hinge2, a) Case A, b) Case B, and c) Case C
with similar stiffness or capacities do not necessarily generate the smallest shear key values.

2.6 ELASTIC DYNAMIC ANALYSIS (EDA)

According to SDC, the design displacement demand may be estimated using EDA. For this purpose, a spectral analysis is typically used. However, forces are overpredicted in the EDA method. The EDA analysis results were compared with that of the nonlinear time history (NTH) analysis to examine their difference in terms of shear key force. This was done by using the same models in each of the relative property ratios presented in Table 2.3. Different graphs comparing the shear key force from EDA with NTH are presented in Figure 2.31 to Figure 2.36. Dashed lines show the EDA results, and solid lines show NTH ones. The EDA shear key force is constant for all C1/C2 ratios because the capacity does not change the modal properties.

Figure 2.31 Comparing EDA with NTH Shear Key Force, Two-Frame Model a) Case A, b) Case B, and c) Case C
Figure 2.32  Ratio of EDA/NTH Shear Key Force, Two-Frame Model a) Case A, b) Case B, and c) Case C

Figure 2.33  Comparing EDA with NTH Shear Key Force, Four-Frame Model, Hinge H1 a) Case A, b) Case B, and c) Case C

Figure 2.34  Ratio of EDA/NTH Shear Key Force, Four-Frame Model, Hinge H1 a) Case A, b) Case B, and c) Case C
The EDA results follow the pattern of those from NTH. However, its accuracy is highly variable for different frame property ratios. As expected, three intensity levels of motion show different behaviors due to nonlinear behavior. In the four-frame model, the EDA is underestimating the force by up to 50%.

Surprisingly, the maximum discrepancy of the results from EDA and NTH methods belongs to the models with comparable frame stiffness. This may be due to the fact that the system loses its symmetry after yielding. It can be concluded that the difference of
the EDA and NTH forces not only depends on the ground motion intensity, but also on
the stiffness and capacity ratio of the frames adjacent to the hinge.
3 Prototype Bridges and Input Motions

This chapter includes information regarding the selection of prototype bridges and the input motions used for the nonlinear response history analyses. Earthquake hazard levels used for the design and analysis of prototypes are also introduced in this chapter. In addition, the variables that are considered for the parametric study analyses are introduced.

3.1 BRIDGE PROTOTYPES

3.1.1 Geometry of the Prototype Bridges

In consultation with bridge design professionals at Caltrans, a suite of fifty-six multi-frame bridges including two-, three-, four-, and five-frame models are selected. Each bridge is considered with two substructure systems: single-column extended pile-shaft (single-column) and two-column bents. Two span lengths of 110-ft and 200-ft are selected as short-span and long-span constructions, respectively. However, this study
is mainly focused on long-span prototypes. The span length of 200 ft is selected as a practical span length for post-tensioned box girder bridges. The short span bridges with the span length of 110 ft may represent superstructures without post-tensioning. The prototypes are expected to represent a wide variety of realistic multi-frame bridge geometries. Only straight bridges with no skew are included in this study. These prototypes were carefully designed according to the Caltrans Seismic Design Criteria (SDC 1.7) (Caltrans, 2013) for three hazard levels and four soil types. The seismic design of these prototypes will be discussed further in Chapter 5.

The configurations of the long- and short-span prototype bridges are shown in Figure 3.1 and Figure 3.2, respectively. The total length of two-, three-, four-, and five-frame bridges are 1400, 2000, 2600, and 3200 ft for the long-span and 1210, 1760, 2310, and 2860 ft for the short-span prototypes, respectively. The in-span hinges are positioned at one-fifth of the span lengths (indicated by a circle in the figures), which is a distance of 40 ft and 22 ft from the adjacent column in the long- and short-span prototypes, respectively. This location is near the point of contraflexure under the dead loads.

The average frame length for the long-span bridges is approximately 650 ft., and the frame length is limited to 760 ft. Each frame is supported by three columns. In the short-span bridges, the average frame length is approximately 550 ft; thus, each frame has four or five columns.
Figure 3.1  Long-Span Prototype Bridges (200-ft Span Length)
For long-span bridges diverse valley shapes are included to capture the possible effects of asymmetric geometries. The number of valley shapes in two-, three-, four-, and five-frame prototypes are four, five, seven and ten, respectively. For short-span bridges, only uniform valley shapes are considered. One bridge from each group has a non-uniform column height within its frame/s to account for effects of large eccentricity of the center of stiffness and mass. The shallow, intermediate, and deep valleys are 20 ft, 30 ft and 40 ft, respectively. These heights are the distance between the ground and the bottom the superstructure.

A labeling scheme in the form of $F_x-V_y$ is used to name these prototypes. The letters “F” and “V” stand for Frame and Valley, respectively. Variables “x” and “y” define the number of frames and an arbitrary valley shape tag, respectively. For example, $F_3-V_4$ is a three-frame prototype with valley tag 4. The array of numbers in curly braces presents the valley depth for each frame. Using “Var.” indicates that the length of the
columns is linearly varied within that frame. Symmetric shapes are excluded (F2-V2 and F2-V3 are not the same because the location of the hinge).

3.1.2 Superstructure System

The type of superstructure commonly used in multi-frame bridge construction is a cast-in-place, post-tensioned, multi-cell, concrete box-girder. This superstructure system is also used for the prototype bridges. Two superstructure widths of 40 ft and 64 ft are used that correspond to three- and five-lane bridges, respectively (Figure 3.3a, b). For the short-span prototypes, only a 40-ft wide superstructure is used (Figure 3.3c). The geometric proportions of each superstructure and the thickness of the deck slab, web, and soffit are determined according to MTD-10-20 (Caltrans, 2008) for the two span lengths of 110 ft and 200 ft. The total dead weight including overlay and barriers per linear foot of the superstructures are 11.18, 13.56, and 20.78 kip/ft for cross sections shown in Figure 3.3a, b, and c, respectively. In the same way, the transverse moments of inertia of these cross sections are 1.493e8, 1.979e8, and 7.934e8 in$^4$.

![Figure 3.3](image.png)

**Figure 3.3** Superstructure Sections, a) Long-Span, Three-Lane, b) Long-Span, Five-Lane, and c) Short-Span, Three-Lane
The components of a standard in-span hinge are shown in Figure 3.4. The diaphragm width is 6.5 ft on each side. Four and six “pipe extender/shear key” elements are assumed to serve as in-span hinge shear keys in the 40-ft and 64-ft superstructures, respectively. The “pipe extender/shear key” detail is shown in Figure 3.5.

The size of the longitudinal gaps at in-span hinges and at the abutments is assumed 2.0 in. The gap size is calculated using a spreadsheet that Caltrans provided for the research team, which was developed according to AASHTO provisions (AASHTO, 2012). This gap size accommodates thermal movements for a temperature variation of 70ºF. The weight of a 2.0-in asphalt overlay and two standard barriers of Type 732 (AASHTO, 2012) are added to the weight of the superstructure.

Figure 3.4 In-Span Hinge Detail
3.1.3 Substructure System

Two substructure systems are studied: the single-column (single extended pile-shaft) that is used for short- and long-span bridges (shown in Figure 3.6a and b) and the pinned base two-column bent (shown in Figure 3.6c). The two-column bent substructure is only used for long-span prototypes. In both systems, the columns are circular. The diameter of the columns is defined in the design. For the two-column bent, the height is extended by 5 ft below the ground surface.
Columns are integrated into the superstructure through the solid cap-beams shown in Figure 3.7. The width of the cap-beam is determined after the diameter of the column is defined. This cap-beam extends along the width of the soffit.

![Solid Concrete Cap-Beam (Caltrans, 2013)](image)

The reason for studying both single- and two-column substructure systems is that, under seismic loading, they deflect in different ways, as demonstrated in Figure 3.8. The single-column system allows for horizontal translation and torsion of the superstructure about the longitudinal axis of the bridge. In contrast, a superstructure supported by a two-column bent only allows horizontal translations. Because of the different ways the superstructures displace under seismic loading, the inertial forces developed in these two bridge systems are different. For a single-column bridge, both torsional and translational masses participate in the seismic response, while, for a two-column bridge, only translational masses work. In addition, bridges that are supported on single-column bents are in general softer than the two-column ones. Other substructure systems such as multiple-column bents, fixed-base columns, or multiple extended pile-shafts are also practical in bridge design. However, it is expected that the results from the studied prototype is applicable to other types.
3.1.4 Abutment System

A seat-type abutment system is used in prototype bridges models. Figure 3.9 shows the main components of this abutment system. A weak backwall with a shear-off joint is commonly used in this type of abutment to avoid transferring large forces to the abutment piles (Caltrans, 2013). In the transverse direction, two exterior ductile shear keys are used. These shear keys are designed to fail in shear, or yield in flexure (Bozorgzadeh, et al., 2007). In this study, a flexural behavior is assumed for these shear keys. Gap sizes of 2.0 in and 1.0 in are used between the superstructure and the abutments in longitudinal and transverse directions, respectively.
3.1.5 Continuous Frame Bridges

A set of long-span continuous frame bridges equivalent to multi-frame bridges (same geometry) is defined in this study. The specifications of these bridges are similar to that of multi-frame with two modifications: 1) elimination of in-span hinges and associated diaphragms, and 2) increasing the abutment gap sizes. The abutment gap sizes are determined based on AASHTO provisions (AASHTO, 2012) as 3.5, 5.0, 6.0, and 7.5 in. for 1400, 2000, 2600, and 3200 ft long bridges, respectively.

3.2 HAZARD LEVELS AND DESIGN SPECTRUMS

According to Section 2.1 of SDC 1.7 (Caltrans, 2013) for structural design applications, seismic demand should be determined using an elastic 5%-damped acceleration response spectrum (ARS) corresponding to the largest of:

1) A probabilistic spectrum for a hazard level of 5% in 50 years (975-year return period);

2) A deterministic spectrum based on the largest median response resulting from the maximum rupture of any fault in the vicinity of the bridge site (corresponding to $M_{Max}$);

3) A statewide minimum spectrum, defined as the median spectrum, that is generated by $M=6.5$ earthquake on a strike-slip fault located 12 kilometers from the bridge site.

Beginning in 2013, Caltrans launched a web-based calculator for generating acceleration response spectrum (ARS) (Caltrans, 2014). This source calculates both deterministic and probabilistic acceleration response spectra for any location in
California based on the criteria mentioned earlier. In addition to the online ARS calculator, SDC contains a wide range of ARS’s for soil types B, C, D, and E, each for three magnitude ranges ($M_w$) and different peak ground acceleration (PGA) levels ranging from 0.1g to 0.7g. SDC is one of few seismic design codes to provide different design response spectra for different magnitudes. Mohraz (1978) showed that ground acceleration amplification, for earthquakes of magnitudes $6.0 < M < 7.0$ is more than that for earthquakes with a magnitude within $5.0 < M < 6.0$. The peak ground acceleration also depends on the earthquake magnitude and epicentral distance (Mohraz, 1978). The effect of the duration of strong motion on the shapes of spectrums was studied by (Peng, et al., 1989). They showed that ground accelerations of larger earthquakes have longer period contents.

To ensure that the findings of this study are applicable to different hazard levels and site specifications, a diverse set of ARS’s are selected. This set includes three hazards levels of moderate, large, and severe, each for soil types B, C, and D. Only two hazard levels of moderate and large are considered for soil type E. Thus, the number of spectrums used in this study totals $3 \times 3 + 2 = 11$. The shear wave velocity for soil types B, C, D, and E are defined as 1500-760, 760-360, 360-180, <180 m/sec., respectively (Caltrans, 2013).

Table 3.1 summarizes the magnitude-intensity-soil type combinations selected for this study. A labeling scheme in the form of “Xyz” is used to name the selected ARS’s. The first letter indicates the soil type. The second variable, “y”, is 1, 2, or 3 and specifies a magnitude level of 6.25-6.75, 7.0-7.5, and 7.75-8.25, respectively. The third
variable, “z”, takes the value of 1, 2, or 3 representing the intensity levels of 0.2g, 0.4g and 0.6g, respectively. For example, B22 refers to the ARS for soil type B, magnitude level 2 (7.0-7.5), and PGA level 2 (0.4g).

The corresponding pseudo spectral accelerations of the selected spectrums are shown in Figure 3.10. These spectrums served two purposes: to design the prototype bridge models, and to spectrum match the ground accelerations used in nonlinear time history (NTH) analyses of the prototype models.

### Table 3.1 Selected Hazard Levels and ARS Labels

<table>
<thead>
<tr>
<th>Magnitude Level (M&lt;sub&gt;W&lt;/sub&gt;)</th>
<th>Intensity Level (PGA)</th>
<th>ARS Labels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (6.25-6.75)</td>
<td>1 (0.2g)</td>
<td>B11, C11, D11, E11 ( Moderate Hazard)</td>
</tr>
<tr>
<td>2 (7.00-7.75)</td>
<td>2 (0.4g)</td>
<td>B22, C22, D22, E22 ( Large Hazard)</td>
</tr>
<tr>
<td>3 (7.75-8.25)</td>
<td>3 (0.6g)</td>
<td>B33, C33, D33 ( Severe Hazard)</td>
</tr>
</tbody>
</table>
3.3 GROUND MOTIONS

3.3.1 Selection of the Ground Motions

The NGA ground motion database of the Pacific Earthquake Engineering Research Center (PEER, 2011) is used to select 11 sets of three ground motions compatible with each ARS presented in Table 3.2. Each set contains three biaxial ground motions with a soil type, magnitude, and intensity level that are “approximately” similar to that of the corresponding ARS. In addition, special care is taken to select motions with spectral shapes comparable to the target spectrum in the period range of 0.1-2.0 sec. These motions are scaled using the PEER-NGA search engine such that the difference of the ground motion spectrum with the target ARS is minimized. The ground motion spectrum is taken as geometric mean of two horizontal components (PEER, 2010). The scale factor was limited to 0.5 and 2.0. Several combinations of motions are considered.
to ensure each set includes one or two pulse motions, with the exception of moderate
hazard level (i.e. B11, C11, D11, and E11 spectrums).

3.3.2 Spectral Matching

SeismoMatch software (Seismosoft, 2011) is used to generate a library of 33 ARS-
matched biaxial acceleration histories. SeismoMatch software utilizes a wavelet-based
algorithm to match acceleration histories to a target response spectrum. This method
conserves the time-energy characteristics of acceleration histories, while their spectral
accelerations are matched to target spectrums. To avoid distorting the non-stationary
characteristics of the ground motions, the number of iterations was limited to five. For
the same reason, the spectrum misfit tolerance between periods 0.1 and 2.0 sec. is
defined as large as 30%. The upper bound period value of 2.0 sec. is defined to
preserve the pulse characteristics of the motions. Two samples original and matched
ground motions for non-pulse and pulse-like motions are shown in Figure 3.11 and
Figure 3.12, respectively. It is evident in both figures that the distortions of acceleration
and velocity histories are minimal, while the shape of the spectrum of the matched
motions resembles the target spectrum.

Figure 3.12 shows that the velocity pulse is preserved during the matching.
Comparing the spectrums of the matched and unmatched motions demonstrates that
the long-period effect of the pulse remains unchanged.
Figure 3.11 Original and Matched Acceleration History, Velocity History, and Acceleration Spectrum for ARS-C22 (Non-Pulse Motion)

Detailed information on the final ground motions used for the nonlinear time history analyses is summarized in Table 3.2. PGA-1, PGA-2, PGV-1, and PGV-2 define the peak ground accelerations and velocities of the two normal components of the ground motion.
3.3.3 Application of Ground Motions into Analysis

The horizontal components of the motions are randomly assigned to the longitudinal and transverse directions of the bridge models. The sets of three motions developed for each ARS include both non-pulse and pulse motions. In near fault sites, bridges are designed for 1.2 times ARS (Caltrans, 2013). To avoid multiple prototype designs for each ARS, instead of magnifying the design ARS, a reduction factor of 1/1.2 is applied.
to pulse motions that are used for the NTH analyses. The acceleration spectrums of the entire set of matched ground motions are presented in Figures A.1 to A.33 of Appendix A. Each graph includes the target ARS and two spectrums for the components of matched ground motions.

**Table 3.2** Matched Ground Motions Properties

<table>
<thead>
<tr>
<th>No.</th>
<th>ARS</th>
<th>NGA #</th>
<th>Event</th>
<th>Year</th>
<th>Station</th>
<th>MW</th>
<th>V30  (m/s)</th>
<th>Rupture Mech.</th>
<th>Pulse</th>
<th>Dist. (km)</th>
<th>PGA-1 (g)</th>
<th>PGA-2 (g)</th>
<th>PGV-1 (cm/s)</th>
<th>PGV-2 (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1091</td>
<td>Northridge-01</td>
<td>1994</td>
<td>Vasquez Rocks Park</td>
<td>6.69</td>
<td>996.4</td>
<td>Reverse</td>
<td>23.6</td>
<td>0.183</td>
<td>0.221</td>
<td>14.9</td>
<td>11.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1011</td>
<td>Northridge-01</td>
<td>1994</td>
<td>LA - Wonderland</td>
<td>6.69</td>
<td>1222.5</td>
<td>Reverse</td>
<td>0.0</td>
<td>20.3</td>
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<td>0.171</td>
<td>15.8</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>957</td>
<td>Northridge-01</td>
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<td>Burbank - Howard Rd.</td>
<td>6.69</td>
<td>821.7</td>
<td>Reverse</td>
<td>0.0</td>
<td>16.9</td>
<td>0.178</td>
<td>0.199</td>
<td>15.3</td>
<td>13.7</td>
<td></td>
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<tr>
<td>4</td>
<td>143</td>
<td>Tabas - Iran</td>
<td>1978</td>
<td>Tabas</td>
<td>7.35</td>
<td>766.8</td>
<td>Reverse</td>
<td>0.0</td>
<td>2.0</td>
<td>0.426</td>
<td>0.337</td>
<td>44.7</td>
<td>56.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>292</td>
<td>Irpinia- Italy-01</td>
<td>1980</td>
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**Dist.**: Closest distance to rupture plane, **Pulse**: 0=Non-pulse, 1=Pulse, related to Fault Normal and Fault Parallel Components.
4 Development of Analytical Models

This chapter describes the analytical models (previously defined in Chapter 3) that were developed for the prototype bridges. Modeling provisions established by the Caltrans Seismic Design Criteria (SDC 1.7) (Caltrans, 2013), Bridge Design Practice Manual, Chapter 4 (Caltrans, 2014), and the Guidelines for Nonlinear Analysis of Bridge Structures in California (Aviram, et al., 2008) are presented as general modeling considerations. Refinements that are made to these guidelines are discussed. This chapter addresses the following topics: material models, fibers section, modeling assumptions for elastic elements and inelastic elements, impact simulations, and mass assignments.
4.1 SCOPE OF THE ANALYTICAL MODELING

Two analytical models are created for each prototype bridge: 1) an elastic (linear) model for seismic design purposes (as outlined in SDC 1.7) and for elastic dynamic analyses (modal analyses), and 2) an inelastic (nonlinear) model for pushover and time history analyses. Because the configurations of both models are identical, the nonlinear model is explained first, and then differences between the linear and the nonlinear models are addressed.

4.2 ANALYTICAL SIMULATION PLATFORM

OpenSees v2.4.4 (McKenna & Fenves, 2014) is used for the analytical simulations. OpenSees is a software framework used to simulate the performance of structural and geotechnical systems subjected to earthquakes. This software was developed for computational simulation in earthquake engineering. OpenSees has advanced capabilities to model and analyze the nonlinear response of systems using a wide range of material models, elements, and solution algorithms. The software is designed for parallel computing to allow for scalable simulations on high-end computers or for parametric studies. This platform utilizes the capabilities of Tcl (Tool Command Language), a very powerful yet easy to use programming language. In this study, the basic control structures (if, while, and loops) enable the development of programing scripts to automatize the geometric modeling and design of prototypes, as well as parametric studies and iterative analyses.
4.3 GENERAL MODELING INFORMATION

4.3.1 Coordinate System

There are three global directions defined in the bridge model (Figure 4.1). The global 1-axis is in the direction of the chord connecting the abutments, which is denoted as the longitudinal direction, while the global 2-axis is orthogonal to the chord in the horizontal plane, representing the transverse direction. Finally, the global 3-axis defines the vertical direction of the bridge.

4.3.2 Modeling Scheme

A three-dimensional spine model with line elements located at the centroid of the cross section following the alignment of the bridge is implemented. This geometric modeling scheme is shown in Figure 4.1. The spine models provide a good balance between computational efficiency and accuracy. Detailed multi-component models are developed for the abutment and the in-span hinges. The superstructure, diaphragm, and cap beams are modeled as elastic frame elements, while the columns are modeled by nonlinear beam-column frame elements.

For all the prototype models, the superstructure is discretized to equal length segments for better precision. This discretization helps approximate the distributed mass of the bridge components with lumped masses at the nodes between segments. The additional assignment of rotational mass of the superstructure as well as of the columns is required when a global torsional mode is excited under certain dynamic conditions (Aviram, et al., 2008). The use of fewer elements, even for the linear elastic superstructure element, could result in a loss of accuracy in capturing the effects of
higher modes. Masses are assigned to nodes for all the six degrees-of-freedoms (DOFs), corresponding to the tributary mass associated with each node.

![Generic Spine Model Developed for Multi-Frame Bridges](image)

**Figure 4.1** Generic Spine Model Developed for Multi-Frame Bridges

### 4.3.3 Nodal Masses

The dead load includes the self-weight of the superstructure, the weight of 2.0 in. of asphalt overlay with a density of 35 lb/ft², and two barriers with a density of 0.53 lb/ft each. No live load is included in masses, which is a common practice for seismic modeling and analyses of highway bridges. Masses are calculated by dividing the dead weights by the gravitational constant, \( g = 386 \text{ in/sec}^2 \).

The translational mass of all the line elements in the three global directions of the bridge (longitudinal, transverse, and vertical) are calculated based on the weight of the tributary lengths, and are assigned as a lumped mass to each node.

Rotational masses (mass moment of inertia) are assigned to each node. The assignment of rotational mass helps the model capture the accurate dynamic responses, especially for single-column bridges. Ignoring rotational masses about the
vertical axis leads to an inaccurate calculation of higher modes. In the spine models, the rotational mass about the longitudinal axis \((m_{r1})\) must be added because the width of the superstructure is not explicitly modeled. The effects of the other two rotational masses \((m_{r2}, m_{r3})\) can be accurately captured in the spine models only if a large number of segments exist in the model of the superstructure. Rotational masses can be calculated from Eqs. 4.1 to 4.3:

\[
m_{r1} = \frac{m (w^2 + d^2)}{12} \quad \text{(Rotational Mass about Longitudinal Axis-1)} \quad \text{Eq. 4.1}
\]

\[
m_{r2} = \frac{m (l^2 + d^2)}{12} \quad \text{(Rotational Mass about Transverse Axis-2)} \quad \text{Eq. 4.2}
\]

\[
m_{r3} = \frac{m (w^2 + l^2)}{12} \quad \text{(Rotational Mass about Vertical Axis-3)} \quad \text{Eq. 4.3}
\]

where \(m\) is the translational mass of the segment, \(w\) is the superstructure width, \(d\) is the superstructure depth, and \(l\) is the segment of the superstructure length. The translational and rotational masses of cap-beams, diaphragms, and columns are added to their corresponding nodes. For the abutments, the masses of the backwall and a 45° wedge of backfill soil are assigned to the frame element representing the backwall (abutment model is discussed in Section 4.4.3).

### 4.3.4 Pounding Effects (Impact)

The presence of gaps in structures causes a sudden change in stiffness and exerts a significant force upon the gap’s closure at the contact surface, which is due to the momentum of the colliding masses. The impact phenomenon is complex and depends on mass, speed, and the contact surface of the two objects (Muthukumar & DesRoches, 2006; He, 2010). The pounding effects have a high intensity but a short duration. In a
multi-frame bridge system, closure of the gaps may affect the global response of the structure (Figure 4.2). However, the impact forces do not directly affect the overall response of the system due to their localized effect and short duration.

Figure 4.2  The Exaggerated Transverse Displacement of a Multi-Frame Bridge Showing Gap Opening and Gap Closure in an In-Span Hinge

Effects of the closure of longitudinal and transverse gaps should be considered for an accurate response analysis of multi-frame bridge models. It is necessary to capture the impact effects in this study in order to estimate the impact forces on shear keys. In the multi-frame bridge system, the longitudinal gaps include the spacing between the superstructures of adjacent frames at in-span hinges, as well as the gap between the abutment backwall and the superstructure (Figure 4.1). Transverse gap is the gap between the male and female portions of the in-span hinge shear key, as well as the gap between the abutment shear key and the end diaphragm. These gaps are explicitly simulated in the model provided below (Figure 4.3).

OpenSees v2.4.4 (McKenna & Fenves, 2014) is capable of simulating the gap closure and of capturing impact effects. It includes an impact material object, named “ImpactMaterial”, which implements the inelastic collision and the Hertz theory of impact (Muthukumar & DesRoches, 2006). To provide a better estimation of the pounding force, the impact material object is implemented as a compression-only gap, which accounts for the dissipated energy at the contact surface during impact. Figure 4.3 shows the load-displacement relationship defined for this material. This differentiates it
from the simple gap element, "ElasticPPGap", which does not realistically model the energy dissipation. The forming parameters of "ImpactMaterial" are defined in Eqs. 4.4 through 4.8 (OpenSees Wiki, 2014):

Figure 4.3  Impact Material Model

\[ K_1 = K_{eff} + \frac{E}{0.1 \times \delta_m^2} \] (Initial Stiffness)  \hspace{1cm} \text{Eq. 4.4}

\[ K_2 = K_{eff} - \frac{E}{0.9 \times \delta_m^2} \] (Secondary Stiffness)  \hspace{1cm} \text{Eq. 4.5}

\[ E = \frac{K_h}{2.5} \delta_m^{2.5} (1 - e^2) \] (Dissipated Energy)  \hspace{1cm} \text{Eq. 4.6}

\[ K_{eff} = K_h \sqrt{\delta_m} \] (Effective Stiffness)  \hspace{1cm} \text{Eq. 4.7}

\[ \delta_y = 0.1 \delta_m \] (Yield Penetration Displacement)  \hspace{1cm} \text{Eq. 4.8}

\[ e = \text{Coefficient of Restitution with typical values from 0.6-0.8} \]

where \( \delta_m \) is the maximum penetration during the pounding event and \( K_h \) is the impact stiffness parameter. In this study, \( K_h \) was assumed the larger of the stiffnesses of colliding elements. Combining the above relationships yields the following:

\[ K_1 = 3.0 \sqrt{\delta_m} K_h \]  \hspace{1cm} \text{Eq. 4.9}

\[ K_2 = 0.25 K_1 \]  \hspace{1cm} \text{Eq. 4.10}
These parameters are uniquely set for modeling different gaps in the analytical model and are discussed in the in-span hinge modeling and abutment modeling sections of this chapter.

4.4 SUPERSTRUCTURE

4.4.1 Box-Girder Superstructure

The box-girder superstructures introduced in Chapter 3 are modeled by an elastic beam column element using “elasticBeamColumn” in OpenSees. It is expected that the superstructure elements remain elastic. Other elements such as the columns and abutments are designed to undergo inelastic excursions, while the superstructure is protected by a capacity design and is expected to remain in the elastic range.

Given the significant effects of the higher dynamic modes of the superstructure and of the in-span hinge shear key forces (demonstrated in Chapter 2), the section properties of the superstructures must be precisely calculated. Incorrect specification of the moment of inertia (about the vertical axis), shear area, and torsional constant (for the superstructure) will significantly alter the modes of deformation of the superstructure.

The values of the cross-sectional area, $A_g$; torsional constant, $J_g$; moment of inertia about the strong and weak axes, $I_g$; and the corresponding shear areas, $A_v$, are determined using the “superstructure Section” module of CSi Bridge software v15.2.0 (CSI, 2013). Figure 4.4 shows the built-in generic concrete box-girder section in CSi Bridge. Figure 4.5a, b, and c show the cross-section properties calculated for the
superstructures of long-span prototypes with single-column, long-span prototypes with two-column bents, and short-span prototypes with single-column, respectively.

The value of the torsional constant, $J_g$, is compared to the value obtained using BRIDGE STRUDL Manual, Appendix C (Caltrans, 1973).

![Figure 4.4](image)

Figure 4.4 Generic Concrete Box-Girder in CSi Bridge

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Figure 4.5 Superstructure Section Properties, a) Long-Span Prototypes with Single-Column Bent, b) Long-Span Prototypes with Two-Column Bents, and c) Short-Span Prototypes with Single-Column

The stiffness values are obtained based on the properties of un-cracked cross sections of the superstructure. Multi-modal analyses are incapable of capturing the variations in flexural stiffness caused by moment reversal (Aviram, et al., 2008). Therefore, no flexural stiffness reduction is recommended for post-tensioned concrete.
box-girder sections \(l_{\text{eff}} = l_g\), as specified by Section 5.6.1.2 of SDC (Caltrans, 2013). Per Section 5.6.2 of SDC, a reduction of the torsional moment of inertia is not required for bridge superstructures because the prototype models meet the Ordinary Bridge requirements in Section 1.1 of SDC.

To obtain the module of elasticity of concrete, a compressive strength, \(f'_{c}\), equal to 5.0 ksi is assumed for the superstructures. The weight of normal concrete, \(w_c\), is specified by SDC Section 3.2.6 to be approximately 144 lb/ft\(^3\). Equation 3.2.6-1 in SDC 1.7 (Caltrans, 2013) is used to determine the modulus of elasticity of concrete, \(E_c\), which was obtained as approximately 4000 ksi.

**4.4.2 Cap-Beam**

In both the single-column and the two-column bent models, the cap-beam is explicitly modeled by an elastic beam column element at each side of the superstructure axis with a concrete solid rectangular section. The cap-beam width is \(D_{\text{col}} + 2\) ft, and the depth is equal to the depth of the superstructure. \(D_{\text{col}}\) is the diameter of the column as determined in the design of the prototypes.

For the single-column models, the cap-beam elements are modeled for the accurate placement of mass (or weight). However, in the two-column bent models, it is important to model the cap-beam with accurate stiffness values. Since the concrete superstructure and cap-beam are integrated, the superstructure’s flexural stiffness enhances the torsional stiffness of the cap-beam. The actual dimensions of the cap-beam-superstructure system resisting torsion and in-plane bending are larger than the cross-sectional dimensions of the cap-beam element exclusively. Because the spine
model does not make this realization, the bridge stiffness in the longitudinal direction is underestimated, as shown in Figure 4.6. To handle this issue, the effective torsional constant, \( J_{\text{eff}} \), and effective moment of inertia about the vertical axis, \( I_{z\text{-eff}} \), were increased by a factor of 100 and 5, respectively, as indicated in Eqs. 4.11 and 4.12 (Aviram, et al., 2008):

\[
J_{\text{eff}} = 100 \times J_g \quad \text{Eq. 4.11}
\]
\[
I_{z\text{-eff}} = 5 \times I_{z-g} \quad \text{Eq. 4.12}
\]

where \( J_g \) is the gross section torsional constant and \( I_{z-g} \) is the gross section moment of inertia about the vertical axis. It should be noted that using equal constraints or slave nodes for this purpose might lead to unrealistic results because the nodes are constrained in the global axis. In the transverse direction, the cap-beam interacts with a portion of the superstructure and soffit of the superstructure; therefore, the cap-beam’s bending stiffness increases. However, cracking due to bending will reduce its stiffness. These two effects approximately negate each other; therefore, no modification was applied on the out of plain bending stiffness of the cap-beam elements.

![Figure 4.6 Twisting and In-Plane Bending of Cap-Beam under Longitudinal Loads](image-url)
4.4.3 Seat-Type Abutment

A refined form of the abutment model proposed by Mackie and Stojadinovic (2006) is used in this study (Aviram, et al., 2008). Figure 4.7 shows the schematic of the abutment model and its components. The end diaphragm and backwall are modeled using elastic-beam column element objects, “elasticBeamColumn”, with solid rectangular sections of $D_s \times 5$ ft and $D_s \times 1$ ft, respectively where $D_s$ is the depth of the superstructure. The effective flexural moment of inertia of the end diaphragm, about the vertical axis ($I_{z-\text{eff}}$) was increased by a factor of five because of the presence of the superstructure on one side. Nodes connecting the backwall elements were restrained in the vertical direction. For the transverse direction, only the middle node was fixed to avoid binding when the backwall rotates about the vertical axis. A set of zero length element objects “zeroLength” are used to model the backfill, impact, abutment shear keys, and bearing as discussed below.

![Figure 4.7](image-url)
I. Backfill

The longitudinal response of the backfill at the abutments is modeled using the uniaxial hyperbolic gap material, "HyperbolicGapMaterial". This is a compression only material that is developed specifically for modeling abutment backfill because it accumulates plastic damage in unloading-reloading cycles as shown in Figure 4.8. The values of initial stiffness, $K_{Abut}$, and the ultimate passive force, $P_{bw}$, are calculated according to SDC 1.7 Sec. 7.8.1 for seat-type abutments (Eq.4-13 and 4-14):

$$K_{Abut} = K_i \frac{k/in}{ft} \times w \times \frac{h}{5.5\ ft} \ \text{(kip/in)} \quad \text{Eq. 4.13}$$

where $K_i = 50$ kip/in/ft, $w$ and $h$ denote the backwall’s width and height, respectively. $w$ is the superstructure’s width, and $h$ is assumed to be 13 ft:

$$P_{bw} = A_e \times 5.0 \ \text{ksf} \times \frac{h_{bw}}{5.5} \ \text{kip} \quad \text{Eq. 4.14}$$

$$A_e = h_{bw} \times W_{bw} \ \text{(in$^2$)} \quad \text{Eq. 4.15}$$

where $h_{bw}$ is the effective height of the backwall and $W_{bw}$ is the width of the backwall. These parameters are assumed to be equal to the superstructure’s depth and the superstructure’s soffit width, respectively.

![Figure 4.8 Abutment Backfill Material Model](image-url)
The initial stiffness and the ultimate passive force calculated for the entire width of the backwall are proportionally assigned to the middle and side nodes based on their tributary widths. The initial gap was set to zero for the backfill elements because the gap was modeled in the impact element introduced below.

II. Longitudinal Gap (Impact)

The impact material introduced in Sec. 4.3.4 is used to model the gap between the end diaphragm and the backwall. In accordance with (Muthukumar & DesRoches, 2006) study on the effects of longitudinal impact on the response of multi-frame bridges, the maximum penetration displacement at the contact surface, $\delta_m$, is assumed to be 0.5 in. The impact stiffness parameter, $K_h$, is assumed equal to the longitudinal (axial) stiffness of the superstructure as defined in Eq. 4.16:

$$K_h = \frac{E_c A_{sup.}}{L}$$  \hspace{1cm} Eq. 4.16

where $E_c$ is the concrete modulus of elasticity, $A_{sup.}$ is the cross-section area of the superstructure, and $L$ is the average length of the frames in the models, which is taken to be 600 ft.

For the main analysis, the longitudinal gaps were opened in order to avoid the noises in the shear key forces due to the longitudinal impacts. A set of supplemental analyses is conducted, specifically to study the impact effects. In those models, a 2.0-in. gap was implemented.
III. Abutment Shear Keys

According to SDC 1.7, the abutment shear key should be designed as a fuse (either sacrificial or ductile) to limit the force demands on the foundation. Its capacity is prescribed to be equal to $\alpha P_{dl}$ where $P_{dl}$ is the superstructure’s dead load reaction at the abutment and $0.5 < \alpha < 1.0$. The exterior shear key shows a ductile load-displacement behavior when it yields in flexure (Bozorgzadeh, et al., 2007). In this study, an elastoplastic model is used for the force-displacement relationship. To this end, the “Steel02” material model shown in Figure 4.9 is used in series with the impact material (Figure 4.3) to simulate the behavior of the shear key. One shear key assembly is modeled on each side of the end diaphragm to replicate the two exterior abutment shear keys.

The yield force is $\alpha P_{dl}$ and the elastic stiffness is approximately $K_e=2000\alpha^{1.5}$ kip/in, which is found through a moment curvature analysis of the generic shear key shown in Figure 4.9. For the Main Analyses, $\alpha$ is assumed 1.0. Upon the yielding of the abutment, shear key(s), the end boundary condition of the model changes. To examine this effect on the force demands for in-span shear keys, two other values of 0.5 and 2.0 are also considered for $\alpha$ (only for uniform valley shape prototypes). This parametric analysis allows studying the effect of the yielding of abutment shear key(s) on in-span shear keys.
To model the transverse gap between the superstructure and the abutment shear key, the impact element is used. This is modeled by implementing an axillary node that connects the shear key and impact elements. The approximate mass of the shear key (0.01 kip.s²/in) is assigned to this auxiliary node to help with the numerical stability of the model. This makes it possible to assign mass-proportional damping to the element.

To find the impact parameters (similar to the backwall), $\delta_m$ was assumed 0.5 in. The impact stiffness parameter ($K_h$) is taken to be equal to the elastic stiffness of the shear key. The transverse gaps were considered closed in the main analysis. A gap size of 1.0 in. is used in the models to study the effects of impact.

IV. Bearings

Steel-reinforced elastomeric bearing type is used for the prototype bridges. The bearings are modeled by an elastic material in both the longitudinal and transverse directions. In the vertical direction, the bearings are modeled using an elastic no-tension material called “ENT,” which allowed for corner uplift. The bearings’ sizes, axial stiffness, and shear stiffness are determined according to the Caltrans Bridge Design Specifications, Sec.14.6.5.3 (Caltrans, 2008). The required bearing area, $A_b$, was determined using Eq. 4.17a, and the bearing shear stiffness, $K_s$, was calculated using Eq. 4.17b:
\[ A_b = \frac{P}{\sigma_{all}} \quad \text{Eq. 4.17a} \]
\[ K_s = \frac{G_r A_b}{t_r} \quad \text{Eq.4-17b} \]

where \( P \) is the dead load reaction on each bearing, \( \sigma_{all} \) is the allowable compressive stress of the bearing, which was assumed to be 1.6 ksi. \( G_r \) is the shear modulus of the elastomer, which was taken to be 0.12 ksi. \( t_r \) is the total thickness of the rubber layers, which was taken to be two times the longitudinal gap size to accommodate for the transverse displacements of the superstructure.

### 4.4.4 In-Span Hinges and Shear Keys

The in-span hinge model is developed using a concept similar to the one used for modeling the abutments. The in-span hinge model is shown in Figure 4.10. The in-span diaphragms were modeled using the elastic beam column elements “elasticBeamColumn” with a solid L-shape cross section. The effective flexural moment of inertia of the diaphragms about the vertical local axis (\( I_{z\text{eff}} \)) was increased by a factor of five because of the presence of the superstructure. This is because the spine model does not incorporate the rigidity of the superstructure on the diaphragm. Bearings and longitudinal impact elements were modeled with properties similar to those of the abutments. One shear key model is incorporated in the in-span hinge assembly. This element is placed at the center, which is on the same line as the superstructure elements. To model the in-span shear keys, a uniaxial elastic material “Elastic” is defined and assigned to the shear direction of a zero length element “zeroLength”. To model the transverse gap (clearance around the shear key) and impact, two zero length elements with impact material “ImpactMaterial” are defined in series on the two sides of
the shear key element. An axillary node is implemented to connect the shear key and the impact elements. An infinitesimal translational mass of 0.01 kip.s²/in is assigned to the auxiliary node.

The concrete block shear key, shown in Figure 4.10a, is the standard type of in-span shear key for modeling purposes. As in Section 3.1.2, Caltrans has developed a new detail called the “pipe extender/shear key” which is available in Bridge Standard Detail Sheets, xs-7-80 (Caltrans, 2014). This detail is being used in new multi-frame bridges as an alternative for concrete shear keys. The shear key type only reflects itself in the value of the shear stiffness of the shear key element. The stiffness calculation of the in-span shear keys is explained in the following section. The effect of varying the shear key stiffness based on its force response is examined through sensitivity analysis, which is discussed later in Section 4.8.

![Figure 4.10](image_url)

**Figure 4.10**  a) In-Span Hinge Configuration, b) In-Span Hinge Modeling Scheme, and c) Zero Length Elements
**Stiffness of Concrete Block Shear Keys:** Megalley proposed an empirical force-shear deformation relationship for interior concrete block shear keys, which is shown in Figure 4.11 and presented in Eqs. 4.18 and 4.19 (Megalley, et al., 2002).

![Figure 4.11 Interior Concrete Shear keys Model (Bozorgzadeh, et al., 2007)](image)

Assuming a 30 in x 45 in \((b \times d)\) concrete block with \(f'_c=4\) ksi and a 1.0-in gap size, the elastic stiffness is determined to be 22500 kip/in:

\[
V_{max} = 11.3 \sqrt{f'_c \times b \times d} = 11.3 \sqrt{4000 \times 45 \times 35} = 1125 \text{ (kip)}
\]
\[
K_1 = \frac{V_{max}}{0.05 \text{ Gap}} = \frac{1125}{0.05 \times 10} = 22500 \text{ (kip/in)}
\]

**Stiffness of Pipe Extender/Shear Key:** There is no technical reference on the capacity and stiffness of steel pipe shear keys. Thus, the behavior of this element is studied as part of this study through refined finite element (FE) simulations using ABAQUS 6.11-1 (Dassault Systems, 2011). The details of these FE simulations are explained in Part 2 of this dissertation. The maximum elastic stiffness of one pipe when the longitudinal gap is closed is found to be approximately 1000 kip/in, as shown in Figure 4.12. This value corresponds to zero longitudinal gap size. Because the pounding on shear keys may happen at other longitudinal gap sizes as well, 80% of this value is used (800 kip/in). Four and six pipe extender/shear keys are assumed for the single-column prototypes (40-ft wide superstructure) and two-column bent prototypes.
(64-ft wide superstructure), respectively. The total shear stiffness for these two models is calculated accordingly as 3200 kip/in and 4800 kip/in, respectively.

![Force-Displacement Relationship for One Standard Pipe Extender/Shear Key Detail with Zero Longitudinal Gap from Finite Element Analysis](image)

**Figure 4.12** Force-Displacement Relationship for One Standard Pipe Extender/Shear Key Detail with Zero Longitudinal Gap from Finite Element Analysis

The results show that a concrete shear key is stiffer than a set of multiple pipe extender/shear keys. To understand the effect of shear key stiffness on maximum shear key force, a set of sensitivity analyses is performed and their results are discussed in Section 4.8. The stiffness value of the pipe extender/shear key is used in the models since the pipe extender/shear key offers several construction benefits over the concrete block shear key.

One of the objectives of this research is to study the seismic performance of multi-frame bridges with ductile in-span shear keys. In the separate set of models used for this purpose, the “Elastic” material model was replaced with “Steel02” material to simulate the yielding of the shear keys. Figure 4.13 shows the force-shear deformation represented by the inelastic material. The elastic stiffness of the pipe seat extenders is used. The post-elastic hardening is 0.5% of the elastic stiffness. The yield capacity
was assumed equal to the lateral load capacity of the adjacent bent with a 20% over
strength factor.

![Graph](image)

**Figure 4.13** Material Model for Ductile In-Span Shear Key

For the two impact elements, the maximum penetration displacement ($\delta_m$) was
assumed 0.5 in. The impact stiffness parameter ($K_i$) was taken as the transverse
stiffness of the shorter superstructure acting as a cantilever ($3EI/L^3$). For the main
analyses, the transverse gaps are closed as they are for the abutments. For the models
used to study the effects of impact, a 1.0 in. gap is considered on both sides.

### 4.5 COLUMNS

Unlike building structures, bridge columns are designed to yield under strong ground
motions. The columns of the prototype bridges are carefully modeled to capture the
accurate inelastic responses.

The columns in the two-column bent prototypes are modeled as pinned base
cantilevers. For the single-column bent, two modeling methods have been developed
and used by researchers. In the first approach, linear or nonlinear soil is explicitly
modeled using a series of springs, called p-y springs. In the second method, an
equivalent depth of fixity is defined under the ground level to replicate the lateral
stiffness of the soil-pile system. In the latter method, soil effects are indirectly accounted for. The first approach is not used in this study for the following reasons: 1) to avoid the complexities and uncertainties involved in selecting the proper p-y relationships (values of the equivalent depth of fixity are simply presented in design standards like Sec. 10.7.3.13.4 of AASHTO LRFD (AASHTO, 2012)), and 2) because explicit modeling of a nonlinear p-y spring is computationally demanding. With the large number of prototype models and ground motions, using the second approach is more practical.

The plastic hinging of the columns is modeled using the fiber section definition. The following sections present the modeling details of the columns, including the definition of inelastic material, the fiber section properties, the moment-curvature analyses, and the distribution and location of inelastic behavior along the length.

### 4.5.1 Material Properties and Models

The characteristics of concrete and reinforcing steel are defined in accordance with SDC1.7 Sec. 3.2 (Caltrans, 2013).

**Unconfined Concrete:** The specified and expected strengths of unconfined concrete are assumed to be $f'_c = 5.0$ ksi and $f'_{ce} = 6.5$ ksi, respectively. The strain at the maximum strength and the ultimate strain are $\varepsilon_{co}=0.002$ and $\varepsilon_{sp}=0.005$, respectively. The tensile strength of concrete and the modulus of elasticity are $f_t = 0.6$ ksi and $E_c = 57000 \times \sqrt{f'_{ce}}$ for normal weight concrete. The constitutive model for unconfined concrete is shown in Figure 4.14a.
**Confined Concrete:** The strength (strain at maximum stress) and the ultimate strength of the confined concrete core are determined based on Mander model, which is defined in Eq. 4.20 to 4.22 (Mander, et al., 1988):

\[
f'_{cc} = f'_c [-1.254 + 2.254 \sqrt{1 + \frac{7.94f'^r}{f'_c} - \frac{2f'^r}{f'_c}}]
\]

Eq. 4.20

\[
\varepsilon_{cc} = \varepsilon_{c0}[1 + 5 \left(\frac{f'_{cc}}{f'_c} - 1\right)]
\]

Eq. 4.21

\[
\varepsilon_{cu} = \varepsilon_{sp} + 1.4 \frac{\rho_s f_{yh} \varepsilon_{suh}}{f'_{cc}}
\]

Eq. 4.22

where \(f'_i\) is the effective confining pressure and \(\rho_s\) is the transverse steel ratio. \(f_{yh}\) is the strength of transverse reinforcement and \(\varepsilon_{suh}\) is the ultimate strain of the transverse hoops, which are assumed 60 ksi and 0.12, respectively.

This formulation is implemented in the OpenSees script in order to automatically update the material properties based on the design results for each column diameter and transverse reinforcement ratio. Both confined and unconfined concrete were modeled by using “Concrete04” uniaxial material, as shown in Figure 4.14b. This uniaxial material model is based on Popovics concrete material object with degraded linear unloading/reloading stiffness and tensile strength with exponential decay. This is all according to the work of Karsan-Jirsa (OpenSees Wiki, 2014).

**Longitudinal Reinforcing Steel:** The longitudinal reinforcing bar was considered as A706 (Grade 60) with a module of elasticity \(E_s = 29000\) ksi, expected yield stress \(f_{ye} = 68\) ksi, ultimate stress \(f_{su} = 95\) ksi, ultimate strain \(\varepsilon_{su} = 0.09\), and tangent at initial strain hardening \(E_{sh} = 0.04E_s = 11600\) ksi. The “ReinforcingSteel” material, shown in Figure 4.14c, is used to model the longitudinal steel.
4.5.2 Element Type and Geometric Specifications

Each column is modeled by a single nonlinear force-based beam column element “forceBeamColumn” with seven and six integration points for the single-column and two-column respectively (shown in Figure 4.15). The formulation with mid-distance integration points is selected because this option allows for the user-specified location of the integration points and the integration weights (Scott, 2011). The associated integration weights are defined as being equal to the distance between the adjacent integration points. The force-deformation response at each integration point is defined by different section properties.
To define the proper location of the integration points for each column, the following parameters are defined.

**Column Height:** The total column height \((H_t)\) is defined as clearance height \((H_c)\) plus the depth of fixity \((D_f)\) and the superstructure centroid height \((H_s)\), as defined in Figure 4.15. \(H_s\) is one half of the superstructure depth. For Two-column bents, \(D_f\) is assumed to be a constant value of 5 ft, while, for the single-column, the equivalent depth of fixity is varied based on soil stiffness and column diameter. According to AASHTO LRFD 2012 Sec. 10.7.3.13.4 (AASHTO, 2012), the equivalent depth of fixity, \(D_f\), for sandy soils is defined as:

\[
D_f = 1.8 \times \left( \frac{\sqrt[5]{E_p I_{eff}}}{n_h} \right)
\]

Eq. 4.23

where \(E_p\) is the modulus of elasticity of the pile in ksi, \(I_{eff}\) is the effective moment of inertia for the pile in ft\(^4\), and \(n_h\) is the rate of increase of soil modulus with depth in ksi/ft. Table C10.4.6.3-2 of AASHTO (AASHTO, 2012) presents the values of 0.471, 1.11 and
2.78 for loose, medium, and dense sand, respectively. As discussed in Section 3.2, the design acceleration response spectrums (ARSs) used in this study are defined for the four soil types of B, C, D, and E. To find the proper value for $n_h$, a correlation is developed between the seismic soil type, soil density, and the value of $n_h$. These values are presented in Table 4.1.

**Plastic Hinge Location:** Figure 4.16 to Figure 4.18 show the distribution of bending forces and the expected locations for plastic hinges. In the Two-column bents, the plastic hinge forms only at the top of the columns because of the inverse cantilever behavior of the columns (Figure 4.16). The location of the center of the plastic hinge region can be assumed to be at half of the plastic hinge length below the superstructure soffit. For the single-column, however, two plastic hinges are permissible. In the transverse direction, the location of the point of contraflexure is typically close to the superstructure (Figure 4.17). Consequently, the magnitude of the moment at the top of the column is smaller than that generated at the base. This being said, one plastic hinge may form below the ground’s surface. In the longitudinal direction, two plastic hinges may form, one at each end of the column, due to the frame action of the column-superstructure (Figure 4.18). The location of the center of the top plastic hinge is similar to that of the two-column bents. The bottom plastic hinge forms at the point of maximum bending moment, which is between the depth of fixity and the ground surface. It should be noted that, in some cases, the torsional restraining of the superstructure provided by the abutments and large torsional stiffness of the superstructure might cause double curvature bending. In such cases, under transverse loading, plastic hinges may form at both the top and the bottom of single-column (Aviram, et al., 2008).
Figure 4.16  Plastic Hinge Locations for Two-Column Bents

Figure 4.17  Plastic Hinge Location in Single-Column under Transverse Loads

Figure 4.18  Plastic Hinge Location in Single-Column under Longitudinal Loads
An analytical approach is developed in this study to find the depth of the maximum moment ($D_p$) under the ground surface for a single-column, based on the equilibrium of the passive soil pressure, internal moment, and lateral load. The moment equilibrium at the point of maximum moment gives:

$$M_{max} = V(H_c + H_s) - \frac{1}{3} V_p D_p$$  \hspace{1cm} \text{Eq. 4.24}

where $V$ is the shear force and $V_p$ is the passive soil pressure resultant force. $H_c$ and $H_s$ are column length and half of the superstructure depth, respectively. At the point of maximum moment, the internal shear force is zero. Thus:

$$V - V_p = 0$$  \hspace{1cm} \text{Eq. 4.25}

Combining Eqs. 4.24 and 4.25 results in Eq. 4.26:

$$M_{max} = V_p \left( H_c + H_s + \frac{2}{3} D_p \right)$$  \hspace{1cm} \text{Eq. 4.26}

Assuming the linear distribution for passive soil pressure drives the resultant passive force as demonstrated in Eq. 4.27. The ultimate passive pressure for piles is assumed to be three times the passive pressure (Hutchinson, et al., 2002):

$$V_p = \frac{3}{2} \times \gamma_s \times K_p \times D_{col} \times D_p^2$$  \hspace{1cm} \text{Eq. 4.27}

where $\gamma_s$ is the soil density and $K_p$ is the passive pressure coefficient. These values are presented in Table 4-2 for the four ARS soil types. $D_{col}$ is the diameter of the column. Combining Eqs. 4.26 and 4.27 gives the maximum moment:

$$M_{max} = \frac{3}{2} \gamma_s K_p D_{col} \left( D_p^2 (H_c + H_s) + \frac{2}{3} D_p^3 \right)$$  \hspace{1cm} \text{Eq. 4.28}

It can be assumed that $M_{max}$ is equal to the plastic moment of the column ($M_p$). Finally, Eq. 4.28 can be solved for $D_p$. 

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Table 4.1 Assumed Soil Properties

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<td>45</td>
<td>5.8</td>
<td>2.7</td>
</tr>
<tr>
<td>C</td>
<td>1850 (557)</td>
<td>120</td>
<td>40</td>
<td>4.6</td>
<td>1.5</td>
</tr>
<tr>
<td>D</td>
<td>900 (271)</td>
<td>112</td>
<td>35</td>
<td>3.7</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>500 (150)</td>
<td>105</td>
<td>30</td>
<td>3.0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

$V_s$: Average Shear Wave Velocity  
$K_p$: Soil Passive Pressure Coefficient  
$n_h$: rate of increase of soil modulus with depth

**Plastic Hinge Length:** Plastic hinge length is found according to SDC 1.7 Sec. 7.6.2 (Eq. 4.29). The length of the top plastic hinge for the single-column and the two-column bent is:

$$L_{pt} = 0.08 L + 0.15 f_{ye} d_{bl} \geq 0.3 f_{ye} d_{bl} \text{ (in, ksi)} \quad \text{Eq. 4.29}$$

where $d_{bl}$ is the longitudinal bar diameter and was taken to be 1.25 in. (bar #10) for all cases, and $L$ is the length from the point of maximum moment to the point of contraflexure. The length, $L$, for the longitudinal action of a single-column and a two-column bent are defined in Eq. 4.30 and Eq. 4.31, respectively:

$$L = \frac{1}{2} (H_c + D_p) \quad \text{(For single-column, under longitudinal loads)} \quad \text{Eq. 4.30}$$

$$L = H_c + D_f \quad \text{(For two-column bents)} \quad \text{Eq. 4.31}$$

Because the pile-shafts have frame action in the longitudinal direction, the contraflexure point is located at the mid-distance of the top and bottom plastic hinges.

The length of the bottom plastic hinge for the single-column under transverse loads is indicated by Eq. 4.32 based on (Caltrans, 2013):

$$L_{pb} = D_{col} + 0.08 H_{o-max} \quad \text{(For single-column under transverse loads)} \quad \text{Eq. 4.32}$$
where $H_{o-max}$ is the length of the column from the point of the maximum moment to the point of contraflexure above the ground (Eq.4.33):

$$H_{o-max} = H_s + H_c + D_p \quad \text{Eq. 4.33}$$

It should be noted that $H_{o-max}$ is approximately half of the length of that in Eq. 4.33 because of the double curvature action in the longitudinal direction. Because of the limitations in the assignment of modeling parameters of the columns, this difference between plastic hinge lengths in orthogonal directions is ignored, and Eq. 4.29 is used as a basis for the calculation of the bottom plastic hinge length in the single-column bent.

**Location and Weight of Integration Points:** Figure 4.15 schematically shows the distribution and the numbering of the integration points along the length of the columns. The location and the weight of the integration points are set according to the plastic hinge location and length discussed earlier.

For the single-column, seven integration points are used. This includes two points for plastic hinges (#2 and #7), four points that are uniformly distributed between the two plastic hinges (#3 to #6), and one integration point that is defined below the bottom plastic hinge (#1). For the two-column piers, six integration points are implemented. This includes one point for the plastic hinge at the top (#6) and five points equally distributed over the remaining length (#1 to #5). The weight assigned to the integration points representing the plastic hinges is equal to the ratio of the plastic hinge length to the total length of the element. For the intermediate integration points, the assigned weight is equal to the ratio of the tributary length to the total length of the element. In all
cases, the summation of the integration weights must be equal to one. The rigid end zone at the column-cap beam connection was considered using the joint offset option with a length equal to half of the superstructure depth. The flexibility of the joint is not considered in the models.

### 4.5.3 Section Properties

A nonlinear fiber section is used for all the integration points except for the first integration point of the single-column. The fiber section configuration used in this study is shown in Figure 4.19. The section is divided into twelve circumferential and five radial subdivisions (fibers). The number of circumferential subdivisions in the central part is reduced to eight, while the size of the radial fibers is increased. This helps with the analysis time without adversely affecting the accuracy because the stresses and plastic strains, at the center of the section, are smaller than those of the exterior portions. The unconfined concrete material, confined concrete material, and steel material discussed earlier, are assigned to the concrete cover, the concrete core, and the longitudinal bar fibers, respectively.

Because the single-column bents are modeled with an equivalent depth of fixity, the maximum moment occurs at the base of the column; however, Eq. 4.28 shows that the bottom plastic hinge forms at a location between the base and the ground surface. To cope with this issue, an elastic fiber section is assigned to the first integration point (Figure 4.15). The configuration of the elastic fiber section is the same as that of the nonlinear section, but elastic material is used for steel, while an elastic compression only material is used for concrete. The moduli of elasticity of these materials are taken
as the elastic moduli used in the constitutive material models. This elastic fiber section is assigned to the lowest integration point in order to capture the cracked stiffness of the section in nonlinear response history analyses. At the same time, it must remain elastic and preserve the formation of the plastic hinge at the expected location.

![Figure 4.19 The Column Fiber Section](image)

For the inelastic and elastic fiber sections, the section aggregator command “Aggregator” is used to combine the torsional stiffness of the column section with the fiber section. An elastic section with a torsional stiffness of $0.2xG_c \times J_{col}$ was aggregated with the fiber sections, where $G_c$ and $J_{col}$ are the concrete shear modulus and the torsional constant, respectively. The reduction factor of 0.2 is based on Sec. 5.6.2 of SDC 1.7 and accounts for the torsional cracking of the columns section.

### 4.5.4 Verification of Column Model

To ensure that the definition of the material, fiber section, and integration points allow for an accurate estimation of the hysteresis behavior of the reinforced columns (RC), data from a shake table experiment on a 1:5 scaled RC column performed at the University of Nevada, Reno (Zaghi & Saiidi, 2010) are used for validation. Figure 4.20 shows a comparison of the experimental results and those obtained from a model of a
single cantilever column constructed in this study using the modeling assumptions discussed in Section 4.5. A reasonable agreement between the analytical and experimental data confirms the accuracy of the modeling assumptions.

Figure 4.20  a) Shake Table Test Specimen, b) Lateral Force-Displacement Hysteresis (Zaghi & Saiidi, 2010)

4.6  LOADING AND ANALYSIS

4.6.1  Gravity Loading

The self-weight of the line elements plus the additional superstructure dead loads (as mentioned in Sec. 4.3.3) are applied on the frame elements as a linearly distributed load. Static analysis with a load control integrator was used for the gravity analysis.

4.6.2  P-Delta Effects

P-Delta effects are considered using the “PDelta” coordinate transformation object for all the frame elements. This object performs a linear geometric transformation of the beam stiffness and the resisting force from the basic system to the global coordinate system, considering second-order P-Delta effects. This transformation only considers P
large Delta (P-Δ) effects and does not include P small delta (P-δ) effects due to inter-element deformation (OpenSees Wiki, 2014). Because each column was modeled by a single element, P-δ effects are not captured in the analyses.

4.6.3 Damping

Mass and stiffness proportional damping is utilized in the model according to the Rayleigh method that is formulated in Eqs. 4.34 to 4.36 (Chopra, 2001). The Rayleigh equations specify the classical damping matrix \( C \) (uniform distribution of damping) as the linear combination of proportioned mass, \( M \), and current (tangent) stiffness, \( K \):

\[
C = \alpha M + \beta K \quad \text{Eq. 4.34}
\]

\[
\alpha = \frac{2\xi \omega_i \omega_j}{\omega_i + \omega_j} \quad \text{Eq. 4.35}
\]

\[
\beta = \frac{2\xi}{\omega_i + \omega_j} \quad \text{Eq. 4.36}
\]

In these equations, \( \xi \) is the critical damping ratio, and \( \omega_i \) and \( \omega_j \) are the two separate frequencies at which the damping value is set. The Rayleigh damping command of OpenSees defines the damping based on parameters \( \alpha \) and \( \beta \). Special caution is needed when the proportional damping is used to avoid artificial damping (Charney, 2008). In this study, the ratio of critical damping \( \xi = 5\% \) at the periods of 2.0 and 0.5 sec. (corresponding to the use of \( \omega_i = 2.51 \) and \( \omega_j = 12.56 \text{ rad/sec} \)). These periods are selected such that approximately uniform damping is obtained over a wide range of periods of vibration, as shown in Figure 4.21. The reason for targeting the periods of higher modes is that the effects of the local modes of vibration of the superstructure are not artificially damped. It is demonstrated in Chapter 2 that these significantly contribute to the in-span shear key force.
4.6.4 Time History Analysis

A step-by-step direct integration method was used for the nonlinear time history analyses. The Newmark’s time integration method with $\gamma = 0.5$ and $\beta = 0.25$ is implemented. In the case of a convergence problem, the Central Difference, Hilber-Hughes-Taylor, and TRBDF2 methods are automatically tried. The maximum time step is set as 0.005 sec. and, in the case of convergence problems, the time step is reduced by a factor of 0.5. This is done until convergence is achieved, while other integration methods were tried instantaneously. Once a stable converged step is achieved, the integration method and the time step are automatically reset to their original settings.

4.7 SYSTEM-LEVEL VERIFICATION OF THE MODEL

In addition to the element level verification of the column model using experimental data, the modeling method is verified using the results of an analogous model developed in SAP2000 Ver. 15.1 (CSI, 2011). The prototype model F3-V1 single-
column (Section 3.1) is used for this purpose. Figure 4.22 shows a comparison of the lateral and longitudinal displacements of the midpoint of the bridge and the shear key force obtained from these two models under the input motion D22 (Section 3.3). In spite of using a lumped plasticity formulation to model the nonlinear behavior of columns in the SAP2000 model, an acceptable agreement is achieved between the OpenSees and SAP2000 simulation results. The minor differences may be related to the different hysteresis models incorporated in OpenSees and SAP2000.

![Graphs showing comparison of SAP2000 and OpenSees results for system-level modeling verification.](image)

**Figure 4.22** Comparison of SAP2000 and OpenSees Results for System-Level Modeling Verification
The agreement between the system-level responses obtained from the two different structural analysis software platforms confirms the correctness of the analytical model in OpenSees.

### 4.8 SENSITIVITY TO MODELING PARAMETERS

To validate the modeling assumptions for the in-span shear key, a brief sensitivity analysis is performed on the parameters with uncertain values. The designated parameters are the elastic stiffness of the shear key element and the yield penetration displacement ($\delta_y$) used for the impact modeling (Section 4.3.4). Note that $\delta_y = 0.1 \delta_m$. These parameters are varied one-at-a-time to investigate the sensitivity of the shear key force response to the variation of these parameters. For this purpose, the value of each parameter is first doubled and then halved from the initial parameter. The single-column prototype model F3-V4 is subjected to the suite of B33 and D22 ground motions (six motions).

The effect of the shear key stiffness is examined in two cases: 1) one with no transverse gap and 2) one with a 1.0-in transverse gap. Figure 4.23 shows that shear key stiffness does not have a pronounced effect on maximum shear key force when the transverse gap is closed. If a transverse gap exists, then shear key stiffness affects the impact forces. Figure 4.24 shows that a 100% change in stiffness yields - on average - a 15% change in the maximum shear key force. This implies that the impact forces are not too sensitive to shear key stiffness values. Thus, the assumption of shear key stiffness being 3200 kip/in is appropriate.
The effect of yield penetration displacement ($\delta_y$) for the impact material used in modeling the transverse gaps is shown in Figure 4.25. It is shown that, on average, a 100% variation in the parameter leads to less than an 8% change in the shear key force, rendering the initial assumption of 0.05 inappropriate.

**Figure 4.23** Effect of Shear Key Stiffness on the Maximum Shear Key Force (with Closed Transverse Gap)

**Figure 4.24** Effect of Shear Key Stiffness on the Maximum Shear Key Force (with Transverse Gap)
4.9 LINEAR MODEL

4.9.1 Complete Model

A set of linear models are built for the purposes of seismic design, in accordance with the modeling criteria of Sec. 5.2.2, SDC 1.7. The configuration of the linear model is similar to that of the nonlinear model with a few exceptions stated below:

- Columns are modeled by the elastic beam column element with an effective moment of inertia over the entire height of the column.
- Each column is divided into four segments.
- All transverse gaps are closed.
- All longitudinal gaps, including abutments, are released.
- End diaphragms were fixed in the transverse direction.

4.9.2 Stand-Alone Model

The frames of the prototype bridges are also modeled individually. These models are used to find the transverse period of vibration and stiffness of the stand-alone
frames and are not used for the purposes of design. Each model was extracted from a complete model, and, for the side frames, the abutment was considered as a transverse fixed support.

4.10 CONTINUOUS FRAME MODEL

The purpose and definition of continues frame prototypes were mentioned in Sec. 3.1.5. The modeling assumptions for continuous frame bridges are similar to multi-frame bridges with elimination of in-span hinges (continuous superstructure) and larger gap size at abutments.
5 Seismic Design of Prototype Bridges

This chapter presents the seismic design procedure for both multi-frame and continuous frame prototype bridges. The implementation of related SDC provisions to the prototypes’ design is described here.

5.1 SDC V1.7 SEISMIC DESIGN PHILOSOPHY

The SDC classifies bridges into two categories for seismic design purposes: standard and non-standard bridges. Some of the standard bridges in SDC Sec.1.1 are listed as bridges with spans smaller than 300 ft., bridges with regular geometry, and bridges with a fundamental period larger than 0.7 sec. All other bridges with irregular
geometry, unusual framing, and unusual geological conditions are classified as non-standard (MTD-20-1) (Caltrans, 2010).

In addition, SDC categorizes bridges as ordinary and important. An important bridge is defined by the following criteria: 1) it is required to provide post-earthquake life safety, 2) the time taken for the restoration of its functionality after closure would be a major economic impact, and 3) it is formally designated as critical by local emergency plans (Caltrans, 2010). All other bridges are considered “ordinary.” Table 5.1 lists the seismic performance criteria for ordinary and important bridges.

![Table 5.1 Seismic Performance Criteria, (Caltrans, 2010)](image)

The scope of SDC V1.7 is limited to ordinary standard bridges. It requires that project-specific criteria for ordinary non-standard bridges be defined to address non-standard features. These criteria shall be approved by the Caltrans Office of Structural Design. Standard ordinary prototypes were studied in this research; therefore, the prototypes were designed according to SDC V1.7 provisions.

The seismic design philosophy of the SDC is based on the equivalent displacement method. According to this method, elastic analysis is used to estimate the expected displacement demand on the bridge due to design seismic hazards (Figure 5.1). Later, the displacement demand is compared to the displacement capacity of the structure.
The bridge design is satisfactory if the displacement capacity exceeds the displacement demand and several other criteria are met (Chen & Duan, 2014). The design will be a process of trial and error since displacement demand depends on member properties.

Structural members are identified as either ductile or capacity-protected in ordinary bridges. A ductile member can deform inelastically for several cycles without a significant degradation of strength or stiffness under the design earthquake. Columns are the most critical ductile members in ordinary bridges. The capacity-protected members of a bridge are expected to remain essentially elastic.

![Displacement Based Design Method](image)

**Figure 5.1** Displacement Based Design Method

The SDC defines two types of component performances: local and global. Individual local components or subsystems perform independently from adjacent components, subsystems, or boundary conditions. The term “global” describes the overall behavior of the component, subsystem, or bridge system and includes the effects of adjacent
components, subsystems, or boundary conditions (Caltrans, 2013). Further details of the displacement-based method are explained extensively in Sections 5.3 to 5.5.

5.2 ASSUMPTIONS FOR THE DESIGN OF PROTOTYPE BRIDGES

California’s seismic design criteria were used to design the prototype bridges in this study. In this study, the following assumptions are made in the prototype design process:

- The column diameter is the same in each bridge.
- The column reinforcement ratio is the same in each frame. The column reinforcement ratios may be variable in different frames where the column’s height varies due to the shape of valley. Variable reinforcement ratios in different frames allows for the realistic distribution of demand and capacity along the bridge.
- Prototypes are designed only in the transverse direction of the bridge. It is expected that the bridge details will be the same when it is designed either in the transverse direction or in the longitudinal direction.

5.3 DESIGN FLOWCHART

A design flowchart is developed, as shown in Figure 5.2. This flowchart is programed in OpenSees to design each prototype model automatically.
Figure 5.2 Seismic Design Flowchart
5.4 DESIGN DISPLACEMENT DEMAND

Elastic static analysis (ESA) or elastic dynamic analysis (EDA) can be utilized to determine the global displacement demand ($\Delta_D$) of bridges using the effective section properties for columns (Caltrans, 2013). Components attributed to the flexibility of the foundation, bent cap, and ductile members should be considered in the global displacement demand.

5.4.1 Modeling

The linear model explained in Sec. 4.9 was used to determine the prototypes' displacement demand. This model includes detailed frames including effective section properties.

5.4.2 Initial Design Parameters

An initial column diameter and reinforcement ratio was selected to start the analysis and to determine the displacement demand. An Axial Load Index (ALI) of 0.1 was assumed to calculate the initial column section diameter (Eq. 5.1).

$$ALI = \frac{P_{dead}}{A_g \times f'c}$$

Eq. 5.1

The longitudinal reinforcement ratio ($\rho$) (Eq. 5.2) and the transverse reinforcement volumetric ratio ($\rho_s$) (Eq. 5-3) were 1% in the initial design:

$$\rho = \frac{A_s}{A_g} ; \quad 0.01 \leq \rho \leq 0.03$$

Eq. 5.2

$$\rho_s = \frac{4 A_b}{D'S} ; \quad 0.01 \leq \rho_s \leq 0.016$$

Eq. 5.3
where $A_s$ and $A_g$ are the longitudinal steel area and the column gross section area, respectively. $A_b$ is the transverse reinforcement area of an individual hoop or spiral. $D'$ and $S$ are the outer diameter of the hoop or spiral and spacing of transverse reinforcement, respectively. An increment of 0.002 was considered for $\rho$ and $\rho_s$ up to the maximum amounts indicated in Eq. 5-2 and 5-3.

### 5.4.3 Effective Section Properties

Eq. 5.4 is used to determine the column effective moment of inertia SDC Sec. 5.6.1 (Caltrans, 2013). A moment curvature analysis of the fiber section is conducted to determine the yielding moment and the yielding curvature. The average axial load of the columns is applied to the section in the moment curvature analysis:

$$I_{eff} = \frac{M_y}{E_c \times \varphi_y}$$

Eq. 5.4

$M_y$ is the moment capacity of the section at the first yielding of the reinforcing steel, and $\varphi_y$ is the corresponding first yield curvature (Figure 5.3). $E_c$ is the concrete modulus of elasticity. The ratio of the effective torsional constant of the columns section to the gross sections is 0.2.
5.4.4 Elastic Dynamic Analysis (EDA)

The EDA method is used to calculate the displacement demands. OpenSees is able to solve eigenvalue problems to find the natural frequencies and mode shapes; however, it does not perform EDA. Thus, a code is developed in OpenSees to perform the matrix operations for spectral analysis. These operations are shown in Eqs. 5.5 to 5.10:

\[ M^* = \Phi^T M \Phi \]  
\[ L = M \Phi \]  
\[ \Gamma = \frac{L}{M^*} \]  
\[ F = \Gamma M \Phi S_a \]  
\[ \gamma = \Gamma L / \text{Total Mass} \]  
\[ \Delta_D = \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} \Delta_{D,i} \Delta_{D,j} \right]^{1/2} \]
where

\( \Phi \) = Mode shape matrix.

\( M \) = Mass matrix.

\( M^* \) = Modal mass matrix.

\( \Gamma \) = Modal participation factor.

\( F \) = Equivalent force vector.

\( S_a \) = Spectral acceleration obtained from ARS.

\( \Upsilon \) = Modal mass participation ratio.

\( \Delta D,i \) and \( \Delta D,j \) = Global displacement of the \( i^{th} \) and \( j^{th} \) mode.

\( C_{ij} \) = Modal response correlation coefficient between modes \( i \) and \( j \), which is used for CQC method. In the SRSS method, \( C_{ij}=0 \) for \( i \neq j \) and \( C_{ij}=1 \) for \( i = j \).

\( N \) = Number of total modes.

The CQC method is used in this study. The displacement obtained for each node is considered as the global displacement demand (\( \Delta D \)). Dynamic modes mobilizing 97% of the total mass are included in the EDA analysis.

### 5.4.5 Displacement Ductility Demands

The displacement ductility demand (\( \mu_D \)) is a measure of the imposed post-elastic deformation on a member, which is indicated by Eq. 5.11.

\[
\mu_D = \frac{\Delta_D}{\Delta_Y} \tag{Eq. 5.11}
\]

where \( \Delta_D \) is the estimated global displacement demand at subsystem (bent) discussed in section 5.4.4. \( \Delta_Y \) is the idealized yield displacement of the subsystem (bent) that can be calculated using the following two methods:
1) Estimating $\Delta_Y$ from the yield curvature of the section using the Priestley method (Eq. 5.12):

$$\Delta_Y = \varphi_Y \times \frac{L_{col}^2}{3}$$  \hspace{1cm} \text{Eq. 5.12}

2) Estimating $\Delta_Y$ from the bi-linear force-displacement curve achieved from the pushover analysis of the subsystem (Figure 5.4).

The second method is used to calculate yield displacement in this study. The pushover analysis is performed for individual models of a representative bent within each frame. The modeling assumption of the bent is similar to the global model explained in Sec. 4.5. The individual bent models are subjected to axial dead loads and are pushed in the bridge’s transverse direction until they collapse. This analysis does not include the P-Delta effect. The P-Delta effect will be considered as an additional provision for local member capacity later in the analysis. SDC Sec.3.1.3 allows this assumption with the first simplified method (Caltrans, 2013).

![Figure 5.4 Pushover of Individual Bent Models for Single-Column and Two-Column Bents](image)

**5.5 DESIGN LIMIT STATES**

SDC defines local member capacity and global system capacity in the design procedure. The local member displacement capacity ($\Delta_c$) is the member displacement
capacity prior to collapse, considering the member’s individual elastic and plastic flexibility regardless of the flexibility of other adjacent members. A global system’s displacement capacity ($\Delta_C$) is the reliable capacity of a bridge or subsystem as it approaches a collapse limit state. In global systems, the flexibility of all the members, such as the foundation and the cap beam, shall be included.

### 5.5.1 Local Member Displacement Ductility Capacity

The local displacement ductility ($\mu_c$) of a member is calculated by Eq. 5.13 for the cantilever columns shown in Figure 5.5. $\mu_c$ should be equal or larger than 3.0 (Eq. 5.14):

$$\mu_c = \frac{\Delta_c}{\Delta_y}$$  \hspace{1cm} Eq. 5.13

$$\mu_c \geq 3.0$$  \hspace{1cm} Eq. 5.14

$\Delta_c$ is the local displacement capacity before collapse. The collapse limit state occurs when the core of the concrete reaches its ultimate compressive strain or its reinforcement reaches its ultimate tensile strain. Whichever is smaller controls the capacity. The local displacement capacity is independent of the displacement demand and is only related to member specifications. The confinement by transverse reinforcements can effectively increase the displacement capacity of the columns.
A pushover analysis of individual bent models is conducted to calculate and verify the displacement ductility capacity. The yield displacement is estimated from the bilinearized force-displacement curve. The capacity displacement is determined upon reaching collapse. The transverse reinforcement ($\rho_s$) is increased to satisfy the minimum displacement ductility capacity. It should be noted that, by increasing the longitudinal reinforcement ratio, there is an adverse effect on the displacement ductility capacity. That is, it increases the yield displacement, $\Delta_Y$, while the capacity displacement ($\Delta_C$) remains the same.

### 5.5.2 Global Displacement Capacity

Per the SDC, bridges shall satisfy global displacement capacity (Eq. 5.15):

\[ \Delta_D < \Delta_C \]  \hspace{1cm} \text{Eq. 5.15}
where $\Delta_C$ is the bridge or frame displacement capacity when the first ultimate capacity is reached by any plastic hinge, and $\Delta_D$ is the displacement demand along the local principal axes of a ductile member.

To check this criterion, a pushover analysis of bridge model was performed using the nonlinear model explained in Chapter 4. Every single node of the bridge model was pushed to the corresponding displacement demand by the displacement control method. To satisfy Eq. 5.15, transverse reinforcement was increased, if needed, and, if still not satisfied, the column diameter was increased.

### 5.5.3 Controlling for Target Displacement Ductility

The SDC limits the maximum displacement ductility demand of columns to the target values listed below:

- For Single-Column Bents $\mu_D \leq 4.0$
- For Multiple-Column Bents $\mu_D \leq 5.0$

where $\mu_D$ is defined in Eq. 5.11. To satisfy this criterion, longitudinal reinforcement was increased if needed and, if still not satisfied, the column’s diameter was increased.

### 5.5.4 Checking the Minimum Lateral Capacity

The SDC defines a minimum lateral flexural capacity based on expected material properties by Eq. 5.16:

$$M_P \geq 0.1 \times P_{dl} \times L_c$$

Eq. 5.16

where

$M_P = \text{section’s idealized moment capacity (Figure 5.3)}$
$L_c =$ column length

$P_{dl} =$ tributary dead load applied at the center of gravity of the superstructure.

To satisfy this criterion, longitudinal reinforcement was increased if needed.

### 5.5.5 Consideration of P-Delta Effects

The SDC defines a criterion to account for the P-Delta effect by controlling Eq. 5.17. This criterion should be checked only if a nonlinear time history analysis is not performed for the design.

\[ P_{dl} \times \Delta_r \leq 0.20 \times M_P \]  

Eq. 5.17

where

- $M_P =$ section’s idealized moment capacity (Figure 5.3)
- $P_{dl} =$ tributary dead load applied at the center of gravity of the superstructure
- $\Delta_r =$ the relative offset between the point of contraflexure and the base of the plastic hinge (Figure 5.6).

For a single-column bent $\Delta_r = \Delta_D - \Delta_s$ and can be approximately equal to $\Delta_D$.

Longitudinal reinforcement is increased to satisfy Eq. 5.17, if needed. This conservative criterion mostly governs the design.

---

![Figure 5.6 P-Delta Effect on Bridge Columns (Caltrans, 2013)](image-url)
5.5.6 Capacity Protected Components

Non-ductile members/actions, including bent beams, joints, foundations, and the shear in columns, should have a nominal expected capacity larger than the capacity of the connecting ductile members’ considering a 20% over strength. This criterion was not controlled for the studied prototype bridges because all components were assumed elastic as explained in Chapter 4.

5.5.7 Design Requirements for Stand-Alone Frames

A stand-alone analysis of frames shall be performed to quantify the strength and ductility capacity of individual frames (SDC reference). This criterion was not checked in the design of this study’s prototype bridges, but, in a few analysis cases, it did not govern the design. Meanwhile, frame stand-alone analyses were performed only to find individual periods of frames, as noted in Sec. 4.9.2.

5.6 DESIGN RESULTS

The prototype bridges’ design results - including column diameter and reinforcement ratio are presented in Appendix B in the form of bar graphs. Figure 5.7 shows an example from Appendix B, which belongs to single-column, F3-V4 prototype. The first graph shows the design of the prototype bridge for the eleven ARSs. The wide bars show the column diameter on the left axis. The solid bars show the longitudinal reinforcement of the column on the right axis. A transverse reinforcement ratio of 0.01 is sufficient for all the prototypes and is not presented in the results. The second graph in each figure shows the design ductility demand of the frame for the eleven ARSs.
Figure 5.7  Seismic Design Results of Prototype F3-V4, Single-Column
6 Analysis Results and Observations

The results obtained from the analyses of the prototypes introduced in Chapter 3 are presented and discussed in this chapter. The results of the NTH analyses, EDA, and pushover analyses are presented. These data are summarized in the form of scatter plots, bar charts, distributions of responses along the bridge length, and hysteresis relationships. The correlations of the in-span hinge shear key force demands with other response parameters are presented and discussed as well. The current and proposed analysis methods to estimate the shear key forces are explained and evaluated. This chapter presents the background information that is the basis for the rational analysis method presented in the next chapter. The effect of parameters such as the yielding of abutment shear keys, gap closure and impact, non-uniform base excitation, and the
yielding of the in-span shear keys are also investigated in this chapter. The data set from the NTH analyses are presented in Appendix-C.

6.1 ANALYSIS CATEGORIES

Prototype bridges are analyzed in the eight categories listed in Table 6.1, which reflect the study’s objectives. The specific considerations for each category are indicated in this table. The purpose of the analysis category labeled as “Main Analyses” is to find the maximum shear key forces from NTH analyses, when the effects such as impacts due to longitudinal and transverse gap closure under biaxial motions are excluded. The Reference Shear Key Forces from NTH analysis are the bases for the development of the rational analysis method. In the “Main Analyses”, all transverse gaps are set to be closed, the size of the longitudinal gap in the “ZeroLength” elements is defined as 20.0 in to avoid impact, and the bridge is subjected to only transverse motions.

Each prototype is subjected to the thirty-three ground motions introduced in Section 3.3. The total number of analyses for all the categories is 7,656 as indicated in Table 6.1.
Table 6.1  Analysis Categories

<table>
<thead>
<tr>
<th>Description</th>
<th>Remarks</th>
<th>Single-Column Models (Analyses)</th>
<th>Two-Column Models (Analyses)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Main Analyses</td>
<td>Zero Transverse Gap, Only Transverse Ground Motion</td>
<td>26 (858)</td>
<td>26 (858)</td>
</tr>
<tr>
<td>2 Effect of Abutment Shear Key Capacity on In-Span Shear Key Force</td>
<td>Same as Main Analyses</td>
<td>8 (264) Uniform Valleys</td>
<td>8 (264) Uniform Valleys</td>
</tr>
<tr>
<td>3 Impact Analyses</td>
<td>1.0-in Transverse Gap, 2.0-in Longitudinal Gap, Biaxial Ground Motion</td>
<td>26 (858)</td>
<td>26 (858)</td>
</tr>
<tr>
<td>4 Ductile In-Span Shear Key Analyses</td>
<td>Similar to ImpactAnalyses</td>
<td>26 (858)</td>
<td>26 (858)</td>
</tr>
<tr>
<td>5 Non-Uniform Base Excitation Analyses</td>
<td>Similar to Main Analyses</td>
<td>26 (858)</td>
<td>26 (858)</td>
</tr>
<tr>
<td>6 Short-Span Prototypes</td>
<td>Similar to Main Analyses</td>
<td>4 (132) Uniform Valleys</td>
<td>-</td>
</tr>
<tr>
<td>7 Impact Analyses of Short-Span Prototypes</td>
<td>Similar to Impact Analyses</td>
<td>4 (132) Uniform Valleys</td>
<td>-</td>
</tr>
<tr>
<td>8 Continuous Frame Prototypes</td>
<td>Biaxial Ground Motion</td>
<td>26 (858)</td>
<td>26 (858)</td>
</tr>
<tr>
<td>Total Number of Analyses</td>
<td></td>
<td>284 (9,372)</td>
<td></td>
</tr>
</tbody>
</table>

6.2  DATA COLLECTED FROM THE ANALYSES

6.2.1 Nonlinear Time History (NTH) Analyses

The data recorded from the nonlinear time history (NTH) analyses includes: base accelerations and column base reactions in the transverse, longitudinal, and vertical directions; longitudinal and transverse displacements at the top of the columns; transverse accelerations on the top of the columns; longitudinal and transverse displacements on the diaphragm nodes; in-span hinge transverse accelerations; in-span shear key forces and in-span shear key deformations; transverse and longitudinal impact forces; and longitudinal and transverse abutment forces and deformations.
6.2.2 Elastic Dynamic Analyses (EDA)

The data collected from the elastic dynamic analyses (EDA) includes: modal periods; three-dimensional mode shapes; spectral accelerations for each mode; modal participation factors; modal mass participation; modal base shears; modal displacements; modal shear key forces; and modal periods of standalone frames.

6.2.3 Pushover Analyses

The data collected from the pushover analyses includes base reactions of the columns; displacements at the top of the columns; force reactions at the hinges; and shear forces in the superstructure elements next to the in-span hinges.

6.3 DATA PROCESSING AND PRESENTATION OF RESULTS

MATLAB R2012a/7.14.0.739 (Mathworks, 2012) is used for the processing of 260GB of raw data by extracting peak values from response histories, generating plots, and calculating the statistical values and analysis of variance (ANOVA). The general algorithm and MATLAB scripts that are used for summarizing the data are presented in Appendix-D. In addition, Excel 2010 (Microsoft, 2010) is used in places to generate some of the graphs. The mass data presented in the form of bar charts, time history responses, and hysteresis responses are incorporated in Appendix-C for the completeness of the dissertation. Appendix-C includes 446 figures.

6.3.1 Presentation Formats

Processed data is presented in one of the following formats:

**Histograms:** Histograms are made up of bar charts. The bars are positioned over a range of values, or definitive parameters. The height of the bars indicates the size of
the group. The values for the “median” and “mean” of data are presented on the graph. The “median” is the numerical value separating the higher half of the data from the lower half, while the “mean” is the arithmetical mean, also known as the average. The total number of data points is also indicated on the graph. A theoretical lognormal distribution curve is fitted to bar charts, using Eq. 6.1 (Mathworks, 2012). The theoretical and data driven lognormal cumulative distribution functions (CDF) are plotted next to the bar charts. The CDF plots give the probability of exceeding a given value, such as displacement, force, etc. These graphs are intended to provide an illustrative presentation of the statistical distributions of a specific parameter.

\[ f(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right]; x > 0 \quad \text{Eq. 6.1} \]

**Bar Charts:** The bar charts present mean value of a desired response for a specific parameter or factor. The standard deviation and median are also indicated on top of the bars by error bars and diamond marker, respectively.

**Scatter Plots:** These plots present the degree of correlation between two different data sets in a summarized format. To quantify the correlation, the Mean Absolute Percentage Error (MAPE) (Montgomery, et al., 2010) as a statistical measure (Eq. 6.2) is indicated on the scatter plots. The smaller MAPE reflects more accurate method.

\[ MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y_i - X_i}{Y_i} \right| \times 100 \quad \text{Eq. 6.2} \]

where, \( Y \) is the reference response from NTH analysis and \( X \) is the predicted response using a rational method.

The scatter data points are separated for the three hazard levels of moderate, large and severe, which are shown using cyan, red and black, respectively.
corresponding correlation coefficients are also indicated separately on the graphs. In addition, the total number of data points is indicated on the graph.

**Hysteresis Plots:** These plots present a pictorial representation of the relationship between two response parameters, such as shear key forces and hinge displacements, through the duration of the base excitation. If the two presented parameters are perfectly synchronized, a hysteresis plot simply shows a line in the first and third, or second and third quadrants. Any other form occurs purely because of a time lag between the two responses parameters.

**System State Graphs:** These graphs show a snapshot of the distribution of a parameter along the length of the bridge. They show the transverse displacements and transverse accelerations of the superstructure at the occurrence of the maximum force. These graphs illustrate what the value of accelerations and displacements are at different locations on the bridge when either one of the shear keys is at its maximum force.

### 6.3.2 Overview of ANOVA

**Factorial Design:** The multi-frame and continuous frame prototypes are defined as two groups to be compared. The intent of this approach is to understanding the response of multi-frames by comparing to a benchmark (continuous frame). Six independent factors each one with two levels are defined as Table 6.2. For valley shape (Factor 3), coefficient of variation of columns stiffness (CVCS) is defined as the ratio of standard deviation to mean of the columns stiffness in a bridge. The smaller CVCS denotes for uniform column stiffness and vice versa. CVCS=0.0 means perfect uniform valley. For column
over strength (Factor 6) OS is defined as the average of the ratio of columns lateral capacity to lateral demand force in a bridge. OS=1.0 means perfect balanced design. Factors 1, 2 and 4 are categorical variables while other factors are continuous. Thus the continuous factors are converted to categorical with two levels by their medians.

### Table 6.2  Factorial Design of Independent Variables for both Multi-Frame and Continuous Frame

<table>
<thead>
<tr>
<th>Factor Name</th>
<th>Factor Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Bridge Length</td>
<td>Short (1400, 2000 ft)</td>
</tr>
<tr>
<td></td>
<td>Long (2600, 3200 ft)</td>
</tr>
<tr>
<td>2 Substructure Type</td>
<td>Single-Column Bent</td>
</tr>
<tr>
<td></td>
<td>Two-Column Bent</td>
</tr>
<tr>
<td>3 Valley Shape</td>
<td>Uniform (CVCS &lt; 0.35)</td>
</tr>
<tr>
<td></td>
<td>Non-Uniform (CVCS &gt; 0.35)</td>
</tr>
<tr>
<td>4 Soil Type</td>
<td>Stiff (Type B, C)</td>
</tr>
<tr>
<td></td>
<td>Soft (Type D, E)</td>
</tr>
<tr>
<td>5 Ground Motion</td>
<td>Low (PGA &lt; 0.36)</td>
</tr>
<tr>
<td>Intensity</td>
<td>High (PGA &gt; 0.36)</td>
</tr>
<tr>
<td>6 Column Overstrength</td>
<td>Low (OS &lt; 1.6)</td>
</tr>
<tr>
<td></td>
<td>High (OS &gt; 1.6)</td>
</tr>
</tbody>
</table>

**ANOVA**: Analysis of Variance (ANOVA) is a particular form of statistical hypothesis testing as a reliable method of making decisions using data. ANOVA in its simplest form tests whether or not there is a statistically meaningful difference between means of several groups. For example a hypothesis could be the peak column drifts in multi-frame and continuous frame bridges are the same for a given soil type. In the ANOVA setting, the observed variance in a particular variable is partitioned into components attributable to different sources of variation (Montgomery, 2001). In the ANOVA logic, the ratio of variance between groups to variance within groups, known as F, is found, then the probability (p-value) of value of F greater than or equal to the observed value is calculated. When p-value is less than a threshold (significance level shown as alpha...
usually 0.05), justifies the statistically significant or meaningful difference between groups. In the aforementioned example, if p-value is less than 0.05 it means that the hypothesis is not true or the peak column drift in multi-frame statistically is different than continuous frame system for the given soil type. The smaller p-value than 0.05 demonstrates stronger evidence on difference between groups. MATLAB (Mathworks, 2012) was used for running a six-way ANOVA in order to test significance of bridge system (multi-frame and continuous frame) on several seismic responses with respect to independent factors introduced in Table-1. In addition, the dual interaction of these factors with bridge system was assessed. Interaction means that a specific factor has different effects on different groups. For example a hypothesis could be the soil type has similar effect on columns drift response in multi-frame and continuous frame bridges (no interaction of soil type with bridge system). If the p-value is less than 0.05 it can be concluded that hypothesis is not true or there is a statistically interaction between bridge system and soil type affecting columns drift.

6.4 PROTOTYPES IDENTIFICATION

6.4.1 Columns Diameter and Reinforcement

All prototypes were designed according to SDC 1.7 to satisfy seismic design requirements as explained in Chapter 5 explicitly. The column’s diameter in all single column bent is 96 in. for both multi-frame and continuous frame with a few exceptions of 102 in. diameter in multi-frame with irregular valley shapes and severe hazard levels. The column’s diameter in a two-column bent is 86 in. in both multi-frame and continuous frame. In only a few cases, the column’s reinforcement in a multi-frame is slightly more
than in a continuous frame. Therefore, the multi-frame system in general does not increase columns materials compared to continuous frame systems.

### 6.4.2 Fundamental Period

Modal analysis was performed to determine the fundamental periods of prototype bridges as shown in Figure 6.1. In this figure, bars show mean, median (diamond marker) and standard deviation (error bar) for multi-frames and continuous frames with respect to six independent factors. The p-value from ANOVA for testing the difference between multi-frame and continuous frame for an individual factor is indicated in the first row. The p-value for testing interaction between a factor and bridge system (multi-frame and continuous frame) is indicated in the second row. P-values less than 0.05 are shown in bold format to emphasize statistically significant difference or interaction.

Multi-frame bridges show consistently longer periods than continuous frames, as expected (0.2 sec longer in average). This difference is maximized in bridges with two-column bent. In other word there is a significant interaction between bridge system and substructure system affecting the fundamental period. Multi-frame bridges with single-column bent have a similar period to that of continuous frames. It should be noted that a small p-value in this case shows meaningful difference from a statistical point of view, however the absolute difference (0.06 sec) is negligible from an engineering point of view. Valley shape also has a considerable interaction with the bridge system. In non-uniform valleys, the multi-frame system has a longer period than the continuous frame. This is due to vibration of the softest standalone frame in longitudinal direction. Another important observation is that bridges with lower overstrength have longer period than those with higher overstrength. This is valid for both multi-frame and continuous frame
systems. The reason for this behavior could be related to the seismic design method which will be discussed in the next section.

![Figure 6.1 Fundamental Period of Prototype Bridges](image)

### 6.5 GLOBAL RESPONSES

#### 6.5.1 Maximum Displacements and Verification of Equal Displacement Method

Current Caltrans and AASHTO codes implement the Equal Displacement method to determine the seismic displacement demand at the system level. This method assumes that the maximum lateral displacement of a nonlinear structural system is approximately equal to the maximum displacement of the same system behaving elastically with unlimited strength as shown in Figure 5.1. This assumption is valid for medium- to long-period structures with minimal strength and stiffness degradation and insignificant P-delta effect (NCHRP, 2013). The elastic displacement could be found through equivalent static analysis (ESA) if dynamic analysis will not add significantly more insight into behavior. Elastic dynamic analysis (EDA) shall be used for other bridges (Caltrans, 2013). It should be noted that the Equal Displacement method is different
than Direct Displacement. In Equal Displacement, the bridge is analyzed with cracked stiffness (secant to yield point) of columns. In Direct Displacement, the effective stiffness (secant to target displacement) is used. A Multi-frame bridge is a discrete system, but is the Equal Displacement method reliable for this system? This question is investigated through a correlation study between NTH results and EDA in transverse direction. For EDA, a modal response spectrum analysis using real ground motions pseudo-acceleration spectrums is performed. Further details for EDA were explained in Chapter 5. The maximum transverse displacements of entire columns from NTH analysis in correlation with the Equal Displacement method are presented in Figure 6.2.

The results for continuous frame and multi-frame prototypes are shown separately. It can be observed that both continuous frame and multi-frame show almost similar trends. In both systems, for low intensity ground motions, scatter in data is low but the Equal Displacement has a slight tendency to over predict. For ground motions with high intensity, the data is more scattered but the average response is satisfactory in both multi-frame and continuous frame bridges.

**Figure 6.2** Correlation between Equal Displacement Method and Nonlinear Time History Analysis
To quantify these results, the ratio of transverse displacement from the Equal Displacement method (EDA) to that of NTH analysis is shown in Figure 6.3. The ratio of 1.0 with a small standard deviation represents ideal reliability of the Equal Displacement method. It can be observed that on average, the Equal Displacement method shows slight over prediction for multi-frames and slight under prediction for continuous frames. However, dispersion (standard deviation) for continuous frames is slightly less than multi-frames. Bridge length (number of frames) and soil type do not have considerable effects on reliability of Equal Displacement method. However, substructure systems show considerable effect on both systems. Single-column bent shows more reliable results than two-column bent. This could be related to a longer fundamental period of bridges with single-column. Two-columns show under prediction especially for continuous frames. The intensity of ground motion also has significant effect. For low intensity, multi-frame shows 13% over prediction on average while continuous frames are close to ideal. For high intensity however, both systems show about 10% under prediction on average.

The Equal Displacement method shows more reliable results for bridges with lower overstrength in both systems. In the previous section it was observed that bridges with lower overstrength have longer fundamental periods. The finding in this study is consistent with the current notion that the Equal Displacement method is valid for medium- to long-period structures (NCHRP, 2013).
6.5.2 Minimum Required Number of Modes

Elastic dynamic analysis (EDA) implements a modal analysis. The results from different modes are then combined together. The minimum required number of modes in modal analysis depends on the number of degrees of freedom and the structure complexity. Mass participation ratio is a good measure of required number of modes because it is a normalized index. Currently, seismic design codes require the number of modes to be considered in the analysis at least 0.90 mass participation ratio in each direction (Caltrans, 2013; AASHTO, 2011). Recent research recommends implementing modes up to a period of 0.05 sec in order to include the effect of higher modes (NIST, 2012). In this study, the minimum required mass participation ratio in order to obtain “accurate enough” displacements from EDA in transverse direction was studied as shown in Figure 6.5. The term “accurate enough” is defined as 90% displacement of any column in a bridge from EDA by capturing 0.97 mass participation ratio. It should be noted that this assessment is independent on NTH analysis.
It is seen that multi-frame systems consistently require a larger mass participation ratio (2% in average) for accurate enough displacement from EDA compared to continuous frames. Although a 2% mass participation ratio seems to be small, it could be associated with a considerable number of higher modes. This means higher modes are more critical in the response of multi-frame bridges. It is also clear that all six factors have significant effects on this parameter, regardless of the bridge system. Longer bridges require considerably more modes. Bridges with two-column bent also require considerably more modes due to a shorter period compared to single-columns. Bridges on stiff soil need more modes for the same reason. Ground motions with low intensity usually carry high frequency content that stimulate higher modes of vibration in structures. For this reason, the minimum required number of modes under low intensity motion is larger than with higher intensity. Earlier it was found that bridges with higher overstrength have a shorter fundamental period. Therefore, these bridges require a greater number of modes.

Although the average minimum required mass participation ratio is about 90%, the standard deviation is relatively large and denotes the associated uncertainties. Thus, the current provision by seismic design bridge codes on minimum number of modes is not sufficient for multi-frame bridges.
6.5.3 Maximum Ductility Demands

Figure 6.5 presents the correlation between the design ductility ratios for all of the columns and the maximum column ductility values from the NTH analysis in multi-frame bridges. The design ductility demand is not the same as the target design ductility (Sec. 5.4.5 and 5.5.3); it is defined as EDA displacements normalized by the yield displacement. The ductility from NTH analyses is obtained by dividing the maximum column displacement by the yield displacement (Sec 5.4.5). The saturation of the design ductility for the two-column bridges is because the design ductility was limited to the target design ductility in the design process. Per SDC 1.7, the values of target design ductility are 4.0 and 5.0 for the single- and two-column bents, respectively. With a few exceptions, the maximum ductility values from the NTH analyses of the single-column prototypes were less than 4.0. About 20% of the two-column prototypes, the NTH ductility exceeded the target design ductility of 5.0, while bridges are designed with the expectation that columns’ ductility do not exceed the target design ductility.
Figure 6.5  Correlation of the Transverse Design Displacement Ductility Demand with NTH Ductility Demand, Long-Span Prototypes
The statistical distribution and CDF of the maximum NTH ductility ratios for all of the columns in multi-frame bridges are shown in Figure 6.6. There is a notable difference between the distributions of the NTH ductility demands for single- and two-column prototypes. This can be attributed to smaller yield displacements in the pinned base columns (two-column bents) in comparison with the extended pile-shafts (the single-column bent). In single-column bents, the flexibility of the extended pile-shaft results in a larger yield displacement, and consequently smaller ductility values.

**Figure 6.6** Distribution and CDF of the Maximum NTH Displacement Ductility Demand of All Columns, Long-Span Prototypes
Column’s peak longitudinal drift and peak longitudinal displacement ductility demands for entire prototypes are presented in Figure 6.29 and Figure 6.8, respectively. The trend of drift with respect to factors is similar to the trend of ductility. Multi-frame systems show consistently less demand with less dispersion compared to continuous frames. This is due to the independent response of multiple frames, unlike continuous frames with shorter columns that experience larger demand. It is seen that demand on two-column bent is almost twice the single-column. Non-uniform valley has more severe effect on continuous frames. In other words, the multi-frame system is more resilient to non-uniform valley shapes. ANOVA also approves this interaction. Soft soil causes more demand on columns compared to stiff soil. Soft soils mostly carry lower frequency content (long period) of the motion and the majority of prototype bridges in this study have relatively long fundamental periods (1.2-2.5 sec). The effect of motion intensity and overstrength on a column’s demand is trivial.

![Figure 6.7](image)

**Figure 6.7** Factorial and ANOVA results of Peak Columns Drift Demand in Longitudinal Direction
6 Analysis Results and Observations

The column’s peak transverse drift and peak transverse displacement ductility demands are presented in Figure 6.9 and Figure 6.10, respectively. In the transverse direction, the trend of drift with respect to factors is different than the trend of ductility demand. Mean drift demand for different factors is almost constant except for the two last factors, while displacement ductility demand shows more variation with respect to factors. In general, drift is not a complete measure of damage. For instance, two similar columns, one with fix-fix end conditions, the other one cantilever, undergo equal lateral drift; however, damage to the fix-fix column could be more severe due to larger stiffness. Therefore, displacement ductility demand is a better index for damage assessment.

A column’s transverse demand in multi-frame bridges is almost the same as in continuous frames. Substructure systems have a significant effect on ductility demand. Demand on two-column bent is twice the single-column with higher uncertainty. Valley

Figure 6.8 Factorial and ANOVA results of Peak Columns Displacement Ductility Demand in Longitudinal Direction

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>Short Bridges</th>
<th>Long Bridges</th>
<th>Single-Column</th>
<th>Two-Column</th>
<th>Uniform Valley</th>
<th>N-Uniform Valley</th>
<th>Stiff Soil</th>
<th>Soft Soil</th>
<th>Low Over Strength</th>
<th>High Over Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{individual}$</td>
<td>0.09</td>
<td>0.00</td>
<td>0.00</td>
<td>0.85</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>$P_{interaction}$</td>
<td>0.41</td>
<td>0.37</td>
<td>0.00</td>
<td>0.97</td>
<td>0.41</td>
<td>0.43</td>
<td>0.41</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
shape and soil type have slight effects on the column’s demand in transverse direction.
The discussed reasoning for longitudinal direction is applicable for transverse direction.

**Figure 6.9** Factorial and ANOVA results of Peak Columns Drift Demand in Transverse Direction

**Figure 6.10** Factorial and ANOVA results of Peak Columns Displacement Ductility Demand in Transverse Direction
6.6 DEMAND ON ABUTMENT

6.6.1 Abutment Backwall and Backfill

Damage in abutment backwall due to longitudinal impact of superstructure is concurrent with a permanent deformation in abutment backfill. It may interrupt bridge service because of broken expansion joint and vertical offset in approach slab after an earthquake. Maximum longitudinal displacement demand on abutment backwall is presented in Figure 6.11. The measured displacement is average of displacements at two sides of the backwall or at the center line of the bridge. The backwall damage in multi-frame bridges is significantly larger than in continuous frame bridges. This is due to a smaller expansion joint (gap) in multi-frames. The gap size was 2.0 in accordance to AASHTO provisions for all multi-frame prototypes, while for continuous frames the gap size was varying from 3.5-7.5 in. depending on the length of the bridge as mentioned in Chapter 3. To reduce damage in abutment backwall, authors recommend implementing a larger gap size for multi-frame bridges in high seismicity sites. It should be noted that larger gap sizes require special expansion joint details which are usually more expensive. Although the appropriate abutment’s gap size for multi-frames was not investigated in this study, a smaller size compared to continuous frames is expected.

In both multi-frames and continuous frames, the shorter bridges cause more damage to abutment backwall due to a smaller gap size, as discussed earlier. Column type and valley shape don’t show considerable effect on this response. Soil type however has significant effect; soft soils cause twice as much damage as stiff soils. This is consistent with the column’s demand in longitudinal direction as discussed in the previous section. Ground motions with low intensity barely close the gap, thus almost no
damage is anticipated, while high intensity motions cause severe damage. Bridges with higher overstrength have more reserved capacity in columns and hence the demand on abutments is less.

![Figure 6.11](image.png)

**Figure 6.11** Factorial and ANOVA results of Displacement Demand on Abutment Backwalls

### 6.6.2 Abutment Shear Key

Shear key elements in abutments restrain the superstructure in transverse direction. They are usually designed to yield at a specific load in order to limit the damage to abutment foundation and wing walls. This will lead to relatively transverse deformation at the expansion joint and may interrupt bridge serviceability after an earthquake, unless the expansion joint has enough flexibility. The maximum transverse displacement of abutment shear keys is shown in Figure 6.12. It is seen that shear key demand in multi-frame is almost the same as continuous frames. This could be a result of transversely flexible superstructure. It is expected to observe more demand with rigid superstructure and it will probably be more severe in continuous frame compared to multi-frame bridges. Short and long bridges cause similar demand. This is due to the flexibility of
superstructure as discussed previously. Column type shows considerable effect on this response. Two-column bent causes about 70% more demands. It should be noted that superstructure associated with two-column is stiffer than in single-column and it could be the main source of this difference. However, shear keys in bridges with two-column have 60% larger capacity in this study. Other factors except intensity do not present significant effect on this response.

Figure 6.12  Factorial and ANOVA results of Displacement Demand on Abutment Shear Keys

<table>
<thead>
<tr>
<th></th>
<th>Short Bridges</th>
<th>Long Bridges</th>
<th>Single-Column</th>
<th>Two-Column</th>
<th>Uniform Valley</th>
<th>N-Uniform Valley</th>
<th>Stiff Soil</th>
<th>Soft Soil</th>
<th>Low Intensity</th>
<th>High Intensity</th>
<th>Low Over Strength</th>
<th>High Over Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Valley</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.75</td>
<td>0.40</td>
<td>0.71</td>
<td>0.05</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>ANOVA</td>
<td>P, Individual</td>
<td>0.39</td>
<td>0.04</td>
<td>0.16</td>
<td>0.02</td>
<td>0.16</td>
<td>0.08</td>
<td>0.16</td>
<td>0.08</td>
<td>0.04</td>
<td>0.00</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>P, Interaction</td>
<td>0.26</td>
<td>0.00</td>
<td>0.75</td>
<td>0.40</td>
<td>0.71</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.7  LONGITUDINAL RESPONSE OF IN-SPAN HINGES (GAP OPENING)

The unseating due to large relative longitudinal displacement at in-span hinges is the major source of seismic vulnerability in multi-frame bridges. Cable restrainers are a retrofit measure to limit relative displacements, while in new bridges the approach is maintaining larger seat width to prevent unseating. Caltrans recently installed a Pipe-Seat-Extender passing through the hinge as a backup system for unseating. Estimation of relative displacement is critical to prevent an unseating incident and collapse. The schematic of the in-span hinge is shown in Figure 6.13. The relative displacement at the
center line of the bridge ($g_{avg}$) is associated with longitudinal response while relative deformation at the edge of the superstructure ($g_{max}$) is a result of the interaction between longitudinal and transverse responses. The distribution of $g_{max}$ is shown in Figure 6.14. It has a lognormal distribution with mean 4.8 and 5.28 in. for single-column and two-column prototypes, respectively. The ANOVA result of $g_{max}$ is also presented in Figure 6.15. Since this response is particular to multi-frame bridges, two new groups based on substructure system (single- and two-column bent) are defined. It is observed that relative in-span hinge deformation in two-column bent is consistently larger than single-column. This is due to larger longitudinal displacement response (Figure 6.7 and Figure 6.8) and wider superstructure. It is also clear that all the factors have a considerable effect on this response.
Moreover, the interaction of transverse response with longitudinal relative displacement was studied through the ratio of $g_{\text{max}}$ to $g_{\text{avg}}$ as presented in Figure 6.16. This ratio (R) on average is 1.08 with the maximum of 1.6. It means the actual relative displacement at in-span hinge on average is 8% larger than those calculated at the center line but it may be as large as 60%. The bridge length has a more visible effect on this ratio compared to other factors. In short bridges, a two-frame bridge for instance, a transverse displacement causes a rotation in superstructure. The rotation will magnify the longitudinal relative displacement at the in-span hinge. This effect is expected to be more pronounced in bridges with more frames. However, it seems the interaction of transverse displacement with longitudinal relative displacement at in-span hinges in longer bridges is not significant. Surprisingly, this effect in non-uniform valley shapes is less than uniform valley shapes.
6.8 TRANSVERSE RESPONSE OF IN-SPAN HINGES

6.8.1 Shear Key Force, Displacement, and Acceleration Responses

The response histories of the in-span shear key forces, transverse displacements at the in-span hinges, and accelerations at the in-span hinges are normalized to their corresponding maximum values and plotted together in a single graph for each in-span hinge. These plots are presented in Appendix-C Part-2a and Part-2b for the single- and two-column prototype bridges, respectively. Only the large intensity portions of the responses are presented. Those multi-graphs allow for tracking the variations of these responses through time. For brevity’s sake, these results only include prototypes F2-V1, Hinge 1 of F3-V3, Hinge 2 F4-V4, and Hinge 2 of F5-V5 as samples for different valley shapes, frame numbers, and hinge locations.
6.8.2 Hysteresis Relationship of Shear Key Force with Hinge Displacement and Acceleration

The relationship between the shear key forces and the hinge displacement, as well as the shear key forces and hinge accelerations are presented in Appendix-C, Parts 3a and 3b in the form of hysteresis plots. Each figure includes three force-displacement relationships and three force-acceleration relationships (one relationship for each ground motion). Figure 6.17 is presented as a sample. The maximum forces happen at different hinge displacements or accelerations, while their values remain approximately the same. These figures show that the maximum shear key forces do not necessarily occur when the maximum displacements or accelerations of the hinge happen.

![Sample Hysteresis Plot](image)

**Figure 6.17** Sample Hysteresis Plot for a) Shear Key Force-Hinge Displacement Relationship and b) Shear Key Force-Hinge Acceleration Relationship

6.8.3 Coincidence of Peak Shear Key Forces, Displacements, and Accelerations

The statistical distribution of the ratios of displacement at the maximum shear key force to the maximum value of displacement is shown Figure 6.18 for both the single- and two-column prototype bridges. The same is plotted for accelerations in Figure 6.19. These figures suggest that, for the single-column prototypes, the values of the displacements at the maximum shear key forces tend to be closer to the maximum displacements.
From another perspective, Figure 6.20 illustrates the statistical distribution of the ratios of shear key force at the maximum displacements to the maximum value of shear key forces. Similarly, Figure 6.26 shows the same distribution, but for acceleration. The relatively uniform statistical distributions demonstrate that the maximum shear key force may happen at different hinge displacements or levels of acceleration.

**Figure 6.18** Distribution of “the Ratios of Displacement at the Instant of Maximum Shear Key Force to the Maximum Displacements”

**Figure 6.19** Distribution of “the Ratios of Accelerations at the Instant of Maximum Shear Key Force to the Maximum Value of Accelerations”
6.8.4 Correlations of Maximum Shear Key Forces with Maximum Hinge Displacements and Accelerations

The correlations of the maximum displacements and maximum accelerations with the Reference Shear Key Forces are plotted in Figure 6.22 and Figure 6.23, respectively. A large scatter is visible in these plots. The Reference Shear Key Forces and the maximum accelerations present a better correlation compared to the correlation of the Reference Shear Key Forces and the maximum displacements.
The relationship of the NTH Shear Key Forces with hinge displacements becomes nonlinear as the displacements become larger. However, the shear key force and hinge accelerations remain proportional even under larger ground motions. This is strong evidence that the accelerations that generate large shear key forces are not affected by structural yielding. This suggests that these accelerations are associated with higher modes of vibration that do not contribute to the displacement response.
Figure 6.22  Correlation of the Maximum Transverse Displacement at Hinge with NTH Shear Key Force
Figure 6.23 Correlation of the Maximum Transverse Acceleration at Hinge with NTH Shear Key Force
6.8.5 Correlation of the Maximum Shear Key Forces with Motion Characteristics

Correlations of the NTH Shear Key Forces with the peak ground acceleration (PGA) and the peak ground velocity (PGV) values of the input motion are presented in Figure 6.24 and Figure 6.25, respectively. In general, the motions with larger PGA and PGV values generate larger shear key forces. The relationship between the NTH Shear Key Force and PGV becomes nonlinear after the PGV of 20 in/sec. Increasing the PGVs after this value does not generate larger shear key forces. This shows that pulse type motions do not necessarily generate larger shear key forces. The Reference Shear Key Force remains relatively proportional to PGA. This confirms the contribution of the higher dynamic modes that are not typically suppressed by the yielding of the structure.
Figure 6.24  Correlation of Peak Ground Acceleration (PGA) with the NTH Shear Key Forces
Figure 6.25 Correlation of Peak Ground Velocity (PGV) with the NTH Shear Key Forces
6.9 SYSTEM STATE AT THE PEAK SHEAR KEY FORCE

The state of the systems at the instant of peak shear key force in one of the in-span hinges is presented in Appendix-C Parts 4a and 4b. Two sample figures are shown in Figure 6.26 and Figure 6.27.

Each figure includes two graphs. The top graph shows the profiles of the displacements and acceleration, using solid black and dashed lines, respectively. The gray line shows the profile of the design displacements per SDC 1.7. The left and right vertical axes show the scale for displacements and accelerations, respectively. The hinges with the maximum forces are indicated with a red dot, and the other hinges are shown with blue dots. The horizontal axis shows the node numbers. The bottom graphs show the base reactions of the column in the transverse direction with a black stem, and their corresponding over strength shear demand with a red stem.

![Figure 6.26](image)

**Figure 6.26** System State of Prototype F4-V3, at the Instant of Maximum Force in the 3rd Shear Key, Ground Motion 25, Single-Column
The shape of the displacement and acceleration profiles, at the instant of the maximum shear key force resembles the response of the bridge in its higher modes. The large flexural deformations in the superstructure suggest that assuming rigid movement of the individual frames in multi-frame bridges is not correct. In addition, the base reactions of the columns at the instant of maximum shear key force are typically smaller than their yield forces.

### 6.10 MAXIMUM IN-SPAN SHEAR KEY FORCES

The maximum shear key forces for every in-span hinge shear key for all of the prototype bridges under thirty-three ground motions are shown in the Part 1 of Appendix-C. These graphs show the data from the Main Analyses. They are presented in the form of bar charts where horizontal axis indicating the ground motion numbers, ranging from 1 to 33 (refer to Table 3.2) and vertical axis showing the Reference Shear
Key Forces. The name of the prototype and the hinge numbers are indicated in the graph titles. Hinge numbers start from the left end. These charts present the complete picture of the in-span shear key force responses.

6.10.1 Definition of “NTH Shear Key Forces” and “Reference Shear Key Force”

Each ARS group includes three ground motions as indicated in Chapter 3. The maximum shear key force values, from the “Main Analysis” series, for individual ground motions are called the “NTH Shear Key Forces”. The value of the geometric mean of the three shear key forces, for each ARS, is named as “Reference Shear Key Force” (Eq. 6.3). This terminology will be used throughout this chapter. The geometric averaging of the values from the three motions is expected to eliminate some of the uncertainties attributed to the non-stationary response characteristics.

Reference Shear Key Force = \[ (F_1 \times F_2 \times F_3)^{1/3} \]

To investigate the variation of the NTH Shear Key Forces within each set of three ground motions two methods are utilized. In the first method, the difference between the maximum and minimum NTH Shear Key Forces within each set of three motions is normalized by the geometric mean of the three values. These values are presented in Figure 6.28. The coefficient of variation (Eq. 6.4), defined as the ratio of the standard deviation to the mean (Montgomery, et al., 2010), is used as the second measure of variation of the forces within each set of three motions. Figure 6.29 shows the values of the coefficient of variation. The relatively small values of these two measures of variation within the set of three motions confirm that NTH Shear Key Forces obtained from the three motions are comparable.
\[ CV = \frac{\sigma}{\mu} \]  
\text{Eq. 6.4}

where \( \sigma \) and \( \mu \) are the standard deviation and the mean, respectively.

Because each set of motions is matched to one of the design ARSs, the geometric mean of the responses from the three motions may be related to the response obtained from EDA when the associated ARS is used. In addition, demands will be proportional to the capacity of the prototype bridge that leads to viable and consistent results.

Figure 6.28 Normalized Variation of the NTH Shear Key Forces from Three Motions

Figure 6.29 Coefficients of Variation (CV) of the NTH Shear Key Forces within the Three Motions of each ARS
6.10.2 Statistical Distribution of “NTH Shear Key Forces”

The statistical distribution and CDF of NTH Shear Key Forces is shown in Figure 6.30. The shear key forces are between 100 kip and 1,000 kip, for single-column prototypes, and 200 kip and 2,000 kip, for the two-column prototypes. The mean and median values for these prototypes are 378 kip and 763 kip; and 358 kip and 723 kip, respectively. This implies that the shear key forces in two-column prototypes were twice that of the single-column ones.

Figure 6.30  Distribution and CDF of “NTH Shear Key Forces” from the Main Analyses
6.10.3 Factorial Comparison of Reference Shear Key Force

The “Reference Shear Key Force” of the critical in-span hinge in a bridge for the factors mentioned in Table 6.2 is shown in Figure 6.31 and Figure 6.32 for single-column and two-column, respectively. Number of frames shows some effects on shear key force especially in two-column prototypes. However this result cannot be generalized because numbers of prototypes with non-uniform valley shapes are not the same within each prototype set. Referring to Fig. 12 and considering only uniform valley shape (V1), it can be concluded that in general number of frames has not significant effect on the shear key force.

![Figure 6.31](image1.png)  
**Figure 6.31** Effects of Different Factors on Shear Key Force in Single-Column Prototypes

![Figure 6.32](image2.png)  
**Figure 6.32** Effects of Different Factors on Shear Key Force in Two-Column Prototypes
Single-column system is not sensitive to CVCS while in two-column, the shear key force in bridges with more variation in columns stiffness is larger. This is consistent with findings in the previous section. At the same time, it should be noted that location of the in-span hinge (close to stiffer or softer column) is important. Soil type is also effective on shear key force in two-column. Soil type E shows similar results to soil type D. However this could be as a result of excluding the sever hazard level for soil type E in this study. In single-column the effective height of columns is increasing with softer soils which may compromise the soil effect for this system. Effect of PGA on shear key force is significant as expected. The column over strength also has a significant inverse effect. Bridges with smaller over strength, show a larger shear key force response. The reason for this pattern could be linked to motion intensity (PGA). Usually for small PGAs, the design is governed by code minimum requirements which results in larger over strength. Finally, period does not show significant effect.

6.10.4 Effect of Valley Shapes on the Reference Shear Key Forces

This section presents the variation of the shear key forces in each prototype for different valley shapes. In Figure 6.33 to Figure 6.36, each bar shows the mean of NTH Shear Key Forces for all the in-span hinges. A table is presented below the bar charts, which illustrates the shape of the valleys and the columns' clearance height in each frame. The in-span hinge with the maximum NTH Shear Key Force is indicated by a circle in the tables, except in the case of the two-frame and symmetric prototypes. The presence of two circles on a prototype means that both hinges have almost equal demands. Results for all prototypes show that “balanced” condition is achieved when the stiffness of the left column divided by the distance of the hinge from the left column
is comparable with that of the right column. In-span hinges with balanced adjacent frames have smaller forces in comparison to the hinges with unbalanced frames. This implies that the closer the hinge is to the stiffer columns, the larger the shear key force demand. For instance, valley shapes V3 and V2 of the two-frame prototype have significantly different shear key demands because in V2, the hinge is located to the softer column, but in V3 it is closer to stiffer. This is more pronounced in the two-column prototypes, which is probably due to their stiffer superstructure in the transverse direction. From another perspective, the non-uniform valley shapes do not necessarily increase the shear force. In the two-frame prototypes, valley shapes V2 and V4 have less demand compared to valley shape V1. However, for the three-, four- and five-frame prototypes, the uniform valley shape leads to the least shear key forces. Therefore, where possible in design practice, it is suggested that the in-span hinge be placed close to the softer of the adjacent columns.

![Figure 6.33](image.png) **Figure 6.33** Effect of Valley Shape on Shear Key Force, two-Frame Prototypes
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Figure 6.34 Effect of Valley Shape on Shear Key Forces, Three-Frame Prototypes

Figure 6.35 Effect of Valley Shape on Shear Key Forces, Four-Frame Prototypes
In the four- and five-frame prototypes, the effect of the non-uniform valley shape on the shear key force is less significant. The reason is that, in long bridges with a large number of frames, there are often one or two hinges close to a stiff column, which governs the force as the critical hinge. For instance, for the five-frame prototype, valley shape V8 has a very different shape when compared to V3. However, the end hinge is critical in both of them, of almost equal forces.

6.10.5 Effect of Adjacent Frame Properties on Reference Shear Key Force

The Sec. 7.1.2 of SDC 1.7 defines the balance condition as Eq. 6.5

\[
\frac{T_i}{T_j} \geq 0.7
\]  

Eq. 6.5
where \( T_i \) is the natural period of the less flexible stand-alone frame and \( T_j \) is that of the more flexible frame. SDC states that there is a greater likelihood for the out-of-phase responses of adjacent frames when the stand-alone periods are significantly different. To examine this criterion with respect to the magnitude of the shear key forces, the correlation of the mean NTH Shear Key Forces for each hinge with the value of \( T1/T2 \) is shown in Figure 6.37, where \( T1 \) is the fundamental transverse period of the stand-alone frame with a shorter cantilever (closer to the hinge), and \( T2 \) is that of the frame with a longer cantilever (further from the hinge). This definition is slightly different than the SDC’s. Thus, Eq. 6.5 is always less than 1.0, while \( T1/T2 \) can be larger than 1.0. The reason for the updated definition is that it was found that the closer bent has more effect on the hinge response. Although Figure 6.37 shows a poor correlation, it is seen that shear key forces decreases with an increase in \( T1/T2 \). This means that, the stiffer the closer column, the larger the shear key force.

![Figure 6.37](image-url)  
**Figure 6.37** Correlation of Stand-Alone Frames Period Ratio (\( T1/T2 \)) with the Mean NTH Shear Key Forces
6.10.6 Effect of the Yielding of Abutments Shear Keys on the In-Span Hinges
Reference Shear Key Force

Two cases are studied to understand the effect of yielding of the abutment shear keys on the in-span shear key forces. A strong and weak abutment shear key with a capacity equal to 2.0 and 0.5 times the abutment dead load reaction respectively are studied. These analyses were performed only for the prototypes with uniform valley shape. The results are shown in Figure 6.38 and Figure 6.39. They show the statistical distribution of the ratio of the in-span shear key force to the Reference Shear Key Force for weak and strong abutment shear keys, respectively.

The change in the capacity of the abutment shear keys may increase or decrease the in-span shear key forces due to the changing of the boundary conditions. The over strength of the abutment shear keys slightly increase the in-span shear key forces in the two-column models while it has almost no effect on the single-column models. A weaker abutment shear key may lead to smaller in-span shear key forces in both systems, which is more pronounced in the two-column models.

![Figure 6.38](image-url)

**Figure 6.38** Effect of Stronger Abutment Shear Key (Capacity of 2.0 Dead) on In-Span Shear Key Force
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6.10.7 Effect of the Impact between Frames

As mentioned earlier, the Main Analysis is performed with closed transverse gaps, extended longitudinal gaps, and only transverse motion is applied. To investigate the full dynamic response and the dynamic amplification on shear key force due to gap closure, all prototypes are analyzed with a 1.0-in transverse gap and a 2.0-in longitudinal gap under biaxial ground motion. This set of analyses is called the “Impact Analysis.” The results of this analysis are shown in Appendix-C Part 5a (single-column) and 5b (two-column). The shear key force response histories from the Main Analysis and the Impact Analysis are plotted together. Only results of prototypes with uniform valley shape are presented in here.

Figure 6.40 shows the impact factor with respect to the Reference Shear Key Force. The impact factor is the ratio of shear key force from the impact analysis to that of the Main Analyses (no gap). It can be seen that the impact factor decreases as the shear key force increases. The reason for this is the increase in damping at the contact surface due to an increase in penetration. The distribution and CDF of the impact factor
is shown in Figure 6.41 and Figure 6.42 for long-span and short-span prototypes, respectively. The mean and median impact factor in all three systems is approximately 2.5.

The impact phenomenon that results from the transverse gap in multi-frame bridges is a complex problem which is affected by many parameters, including: 1) torsional displacement of the frames about the vertical axis upon the gap closure and non-uniform change in momentum; 2) longitudinal gap closure and friction in the transverse direction at the hinge contact surface; 3) shear wave velocity along the frame deck; 4) contact surface conditions and the coefficient of restitution; 5) ground motion frequency and pulse content; and 6) non-uniform base excitation. In addition, the dynamics of a discrete system with gaps is completely different from the dynamics of a continuous system, such that, before gap closure, the vibration modes are related to the stand-alone frames.

The individual correlation between the impact factor and some parameters, including PGA, PGV; the maximum acceleration at the hinge, the maximum velocity, spectral acceleration, spectral velocity, and columns’ ductility are studied. However, no traceable relationship was found that addresses the complexity of the problem. Therefore, a straightforward analytical solution for the impact problem in the transverse direction of multi-frame systems cannot be developed.
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Figure 6.40 Impact Factor with Respect to Reference Shear Key Force

Figure 6.41 Distribution and CDF of the Impact Factor, Long-Span Prototypes
6.10.8 Effect of Non-Uniform Base Excitation

Long bridges, including multi-frames, are prone to be excited non-uniformly at different supports due to the traveling speed of seismic waves. To study this phenomenon on the shear key force response, all of the main analyses are repeated with non-uniform base excitations (NUBE). For this purpose, each support is excited with a time lag proportioned to the ratio of the distance from the left abutment to the shear wave velocity of the soil. The velocity of the shear wave is assumed as an average of the velocity range for soil types B, C, D, and E. According to recommendations found in the OpenSees users’ forum, the displacement history of ground motion is applied instead of acceleration. The displacement history is obtained from the double integration of the acceleration history. To ensure that the method of analysis is working properly, it is checked for a uniform base excitation (UBE).

The results in Figure 6.43 show the distribution of the shear key amplification ratio. The amplification ratio is the ratio of shear key force with non-uniform base excitation to
the Reference Shear Key Force. Each graph is related to one of the following soil types: B, C, D, and E.

It can be seen that, for all cases, the effect of NUBE may increase the shear key force by 200%-300% or decrease it by 50%. This effect for soil type B is negligible because of the high velocity of the shear wave. However, for the other soils, the effect of NUBE is more visible. Figure 6.44 shows the median ratio of NUBE/Reference Shear Key Force for different soil types and compares the single-column with the two-column bent system. Almost no change is seen in the shear key force of soil type B while the increase in the shear key force for other soils is 20%-30%. This happened consistently, except in the single-column prototype for soil type D, which showed, on average, a 75% increase in force. The reason for this sudden change is not known.
Analysis Results and Observations

Figure 6.43 Effect of Non-Uniform Base Excitation on Shear Key Force
Figure 6.44  Effect of Non-Uniform Base Excitation on Shear Key Force for Different Soil Types
6.11 POSSIBLE METHODS FOR ESTIMATION OF SHEAR KEY FORCES

A correlational study is performed to determine the relationship between the Reference Shear Key Force, and those calculated from several other analysis methods. The correlational studies enable the identification of methods with a consistent performance for the entire set of prototype models.

The general analysis methods that are investigated in the correlation studies are as follows: 1) the nonlinear static analysis or pushover analysis, 2) the modal analysis or spectral analysis, and 3) the rigid modal analysis. For some of these analysis methods, a few variations are also investigated.

With the insight gained on the strengths and weaknesses of each analysis method, a set of modifications is proposed and the degree of improvement of the modified analysis method is investigated. To this end, the correlations of the shear key forces predicted by the modified analysis method with the Reference Shear Key Forces are evaluated. Two major criteria are considered in the evaluation of the suitability of each method: reliability and accuracy. Reliability refers to the consistency of the results when the method is utilized for different prototype models and hazard levels. The accuracy refers to the degree of the correlation of the predicted force with Reference Shear Key Forces.

The modified analysis methods that are studied are the following: 1) the pushover method combined with inertial effects, 2) the rigid dynamic method, 3) the combined pushover and rigid dynamic method, 4) the modal analysis method with inelastic spectrum, 5) the modal analysis method modified with displacement ductility, and 6) the modal analysis method with modification of multiple modes. Some of these methods are examined with different assumptions and refinements. The 1st, 2nd, 4th, and 5th
methods are well established for the seismic analysis of structures, while the 3\textsuperscript{rd} and 6\textsuperscript{th} methods are developed in this study.

6.11.1 Pushover Methods

Conventional force-based pushover analyses use a uniform load pattern or load patterns obtained from the combination of multiple modes. Refined pushover analysis methods, like adaptive pushover or spectral pushovers, implement a varying load pattern during the analysis or a sophisticated combination of multiple pushovers (Wilson, 2010).

The shear key forces obtained from a force- or displacement-based pushover analysis, where all the nodes along the length of the bridge are pushed, is not realistic because of the equilibrium of the column forces and the push forces. Therefore, in accordance with the concept of capacity design, only the nodes of the in-span hinges are pushed to their corresponding design displacements, as shown in Figure 6.45. The pushover shear key forces are obtained using two methods: 1) the difference of the absolutes of the difference of the shear forces in the left and right neighboring frame elements, i.e. $|V_1|-|V_2|$; and 2) the quarter of the absolutes of these two values, which is equal to the push force divided by four, i.e. $P/4= (|V_1|+|V_2|)/4$.

![Figure 6.45 Defining Pushover Forces](image-url)
The correlations of $|V1|-|V2|$ and the Reference Shear Key Forces are shown in Figure 6.46. Similar scatter plots with the $P/4$ are presented in Figure 6.47. It is evident that the method based on $|V1|-|V2|$ lacks reliability, as it fails in the case of the two-column prototypes. It is because $|V1|-|V2|$ leads to small values when the left and right shear forces are comparable.

The second method, based on $P/4$, is more reliable but is not always accurate. The observation that the Reference Shear Key Forces are comparable with $P/4$ is supported by the data shown in Sec. 6.13, which demonstrates that the shear key force is correlated to one-half of the plastic shear demand of its adjacent column. The value of $P$ approximates the summation of the plastic shear demands of the two adjacent columns.
Figure 6.46  Correlation of $|V1|/|V2|$ from Pushover Analyses with the Reference Shear Key Force
Figure 6.47 Correlation of \((P/4)\) from Pushover Analyses with the Reference Shear Key Force
6.11.2 EDA Method

The correlation between the shear key force from the elastic dynamic analyses (EDA) and the Reference Shear Key Forces are presented in Figure 6.48. The ground motion specific spectrums are used to find the values of spectral accelerations instead of ARSs. The complete quadratic combination (CQC) method is used to combine the modal responses. The EDA method offers the advantage of including the effects of higher modes of vibration. It is established in Chapter 2 that these modes, most of which are related to the in plane deformations, of the superstructure, significantly contribute to the in-span shear force response. However, one should be aware that all methods utilized to combine spectral responses of individual modes only provide an approximation of response values, which are not exact when compared to a time history method.

For low amplitude motions, there is a relatively good correlation between EDA and NTH results. In this range, the models remain elastic or undergo minimal plastic deformations. On the other hand, under large amplitude motions, the shear key forces derived from NTH analyses are significantly smaller than those values derived from the EDA because NTH suppresses the forces and accelerations due to yielding. The figures show that the EDA method may over predict the maximum shear key forces by 300%.
Figure 6.48  Correlation of Force from EDA with the Reference Shear Key Force
The statistical distribution of the cumulative mass participation ratio needed to achieve more than 90% of the total transverse displacement at in-span hinge is shown in Figure 6.50. The total displacement is obtained by combining a large number of modes of vibration (over 100 modes corresponding to 97% mass participation ratio). It is seen that in some cases 90% mass participation ratio is not adequate for accurate displacement response. This is consistent with the findings in section 6.5.2. A similar statistical distribution is plotted in Figure 6.49 for the cumulative mass participation ratio needed to mobilize more than 90% of the total shear key force. This figure shows the accumulation of bars toward 90% and more. On average, the contribution of the natural modes, which make up 95% of the total mass, should be included in the EDA for mobilizing a relatively precise estimation of the shear key forces. Two general observations can be made based on Figure 6.50 and Figure 6.49: 1) A dynamic mode that does not participate in the displacement response may significantly participate in the total shear key force, and 2) the effect of the modes mobilizing 90% of the mass participation ratio may not be adequate for an accurate estimation of the shear key forces.

![Distribution of the Mass Participation Ratios for Mobilizing 90% of the Total Displacement at Hinges](image)

**Figure 6.49** Distribution of the Mass Participation Ratios for Mobilizing 90% of the Total Displacement at Hinges
The increase in the mobilized shear key forces achieved by including higher modes is plotted in Figure 6.51. This figure demonstrates that modes with frequencies as high as 10 Hz and 15 Hz should be considered in single- and two-column bridges to mobilize a large percentage of the shear key force.

6.11.3 Rigid Dynamic Method

It is generally accepted that the responses in modes having frequencies greater than the rigid frequency (modes in the constant acceleration region of the spectrum at high
frequencies) are almost perfectly correlated with the input acceleration history and, therefore, are mutually and perfectly correlated (Gupta, 1984). Higher modes can be assumed to respond in phase with the PGA and with each other. Hence, these modes are combined algebraically. This is equivalent to a pseudo static response to the inertial forces from these higher modes excited by PGA. The pseudo static shear key forces associated with inertial forces at the two sides of a hinge are equal to \((M_1 - M_2) \times \text{PGA}\). The values of \(M_1\) and \(M_2\) are dependent on superstructure flexibility. Figure 6.52 and Figure 6.53 show how these masses are defined. \(M_1\) is the mass of 0.75 and 0.625 of the length of the longer cantilevers for single- and two-column bridges, respectively. Similarly, \(M_2\) is the mass of the 0.5 length of the shorter cantilever in both bridge systems. These values are found by trial and error to get the best correlation in results. Therefore, the values of the differential masses at in-span hinges (i.e. \(M_1 - M_2\)) are 3.5 kip.sec\(^2\)/in and 4.48 kip.sec\(^2\)/in for single- and two-column bridges, respectively. The inertial effects are subtracted because the masses experience the same acceleration. Thus, the shear key force is equal to the difference of these inertial forces. This is illustrated in Figure 6.54.

![Diagram](image.png)

**Figure 6.52** Participating Mass of Superstructure, Single-Column Prototypes
6 Analysis Results and Observations

Figure 6.53 Participating Mass of Superstructure, Two-Column Prototypes

Figure 6.54 The Inertial Forces in Rigid Dynamic Method

Figure 6.55 shows the correlation of the forces obtained by this method with the Reference Shear Key Forces. This method presents a reliable approach, but, in this simple form, it lacks the accuracy. It should be emphasized that this approach only considers the inertial effects, which means that no stiffness forces are accounted for.
Figure 6.55  Correlation of Force from Rigid Mode Method with the Reference Shear Key Force
6.11.4 Combination of Pushover and Inertial Effects

The major deficiency of the static nonlinear analysis (pushover) method is that it ignores the higher modes’ inertial effects. To address this, a simple approach is developed during this study to combine pushover and inertial forces. It is done by combining the shear key forces, from the pushover analysis, i.e. \(0.25P\), with the inertial forces from the rigid dynamic method, i.e. \((M_1-M_2) \times PGA\). This combination is called the “M_V” analysis method where “M” is related to inertial force and “V” represents shear key force from the pushover analysis. These values are combined using the root mean square (RMS) method presented in Eq. 6.6. Some other combination methods including the arithmetic mean (Eq. 6.7), the geometric mean (Eq. 6.8), and SRSS (Eq.6.9) were examined; however, the RMS equation was found to be the best:

\[
V_{RMS}^{Sk} = 2\sqrt{0.5 \times [(0.25P)^2 + (M_1 - M_2)^2 \times PGA^2]}
\]  
Eq. 6.6

\[
V_{AM}^{Sk} = 0.5 \times [(0.25P) + (M_1 - M_2 \times PGA]
\]  
Eq. 6.7

\[
V_{GM}^{Sk} = 2\sqrt{(0.25P) \times (M_1 - M_2)PGA}
\]  
Eq. 6.8

\[
V_{SRSS}^{Sk} = 2\sqrt{(0.25P)^2 + (M_1 - M_2)^2 \times PGA^2}
\]  
Eq. 6.9

The correlation between the shear key forces obtained from the "M_V" method and the Reference Shear Key Forces are shown in Figure 6.56. This method has a decent reliability but large scatters are still visible in the data.
Figure 6.56  Correlation of Force from M_V Method with the Reference Shear Key Force
6.11.5 EDA Method with Inelastic Spectrum

To address the over prediction of the shear key forces by the EDA method under large ground motions, the elastic spectrum is replaced with a constant-ductility inelastic spectrum. The inelastic spectrums are constructed for each ground specific spectrum by applying the modification factor $R_y$ presented as Eqs. 6.7 and 6.8. (Krawinkler & Nassar, 1992). The elastic response spectrums of the ground motions are divided by this reduction factor to obtain the inelastic spectrums.

$$ R_y = [c(\mu - 1) + 1]^{1/c} $$  

Eq. 6.10

where

$$ c(T, a) = T^a / (1 + T^a) + b / T $$  

Eq. 6.11

The numerical coefficients $a$ and $b$ depend on the post-elastic stiffness ratio (hardening slope) of the bilinear force-displacement relationship. In this study these parameters are assumed as $a=1.0$ and $b=0.37$, corresponding to a hardening slope of 2%. The parameter $T$ is the fundamental period of the structure and $\mu$ is the displacement ductility ratio, which was taken as the design displacement ductility demand, $\mu_D$.

The correlation of the results from this method and the Reference Shear Key Forces are given in Figure 6.57. While the inelastic spectrum method corrects the overprediction of the shear key force by EDA, the correlation of the data is poor. This method may significantly under predict the shear key forces. The reason is that the higher modes of vibration that mainly contribute to the shear key forces are mostly elastic modes, and using a constant-ductility inelastic spectrum suppresses the effects of these modes.
Figure 6.57  Correlation of Force from EDA with Inelastic Spectrum with Reference Shear Key Force
6.11.6 EDA Method Modified by Ductility

The concept of equal displacements is based on the assumption that forces are linearly proportional with the displacements for ductility ratios smaller than approximately 4.0. This concept is used in the displacement-based design method adopted by SDC 1.7 (see Figure 5.1).

In an effort to account for the nonlinear response of the model under strong motions, the total elastic shear key forces obtained from the EDA method (Sec. 6.11.2) are divided by the maximum column displacement ductility ratio, $\mu_D$. The ductility ratios that were smaller than 1.0 were set to be 1.0. The displacement ductility is the maximum of column ductility values. Since the EDA forces are found using the spectrum of ground motions, to preserve the consistency, the ductility values are also obtained from NTH analysis. The correlation of the results from this method with the Reference Shear Key Forces is shown in Figure 6.58. This method lacks reliability and accuracy.
Figure 6.58  Correlation of Force from EDA Modified by Ductility with Reference Shear Key Forces
6.12 EDA SHEAR KEY FORCE WITH MODIFICATION OF MULTIPLE MODES

Results of the two previous methods showed that the uniform modification of modal shear key forces leads to significant errors. A refined method is developed in this study to target the reducing effects of the nonlinearities at certain modes.

6.12.1 Modal Displacement Ductility

Recent research studies for building types of structures have shown that they typically become inelastic at the first mode of vibration and remain elastic at higher modes. Therefore, applying a reduction factor to the effects of the higher mode can lead to the underprediction of responses. Thus, it has been proposed that the reduction factor, $R$, only be applied to the effects of the first mode of building structures (NIST, 2012).

For this study, reducing the effects of the first mode does not yield accurate results. This is due to the unique dynamic characteristics of multi-frame bridges that are explained in Chapter 2. In multi-frame bridges, the higher modes can generate inelastic displacements that are even with or larger than the inelastic displacements due to the first mode. To address this difference, the concept of modal displacement ductility is developed in this study as a new concept for bridges. The displacement ductility of the system is defined by, $\mu_{D,n}$, where $n$ is defined as the equivalent mode to the largest displacement ductility in the columns under the $n^{th}$ modal displacement.

The ductility values calculated for each mode are plotted in Figure 6.59. In the single-column models, the 1\textsuperscript{st} to 4\textsuperscript{th} modes show inelastic deformations, while, in the two-column prototypes, modes as high as 22\textsuperscript{nd} become inelastic. It should be
mentioned that these mode numbers include the lateral, longitudinal, and vertical modes of vibration.

![Figure 6.59](image1) Transverse Modal Ductility Ratios for Different Modes

Figure 6.60 shows the modal ductility values versus the modal periods. For a single-column system, the inelastic modes have periods between 1.0 sec. and 2.8 sec., while, in a two-column system, they happen between 0.5 sec. to 2.2 sec. with some exceptions.

![Figure 6.60](image2) Transverse Modal Ductility Ratios versus Modal Periods
6.12.2 EDA with Modification of Multiple Modes (MEDA)

This data demonstrates the importance of treating each mode separately for the effects of nonlinearities. The approach of the proposed EDA method with modification of multiple modes is to modify the participation of each mode in shear force by the ductility of the same mode. The modified modal forces are then combined using CQC.

Figure 6.61 compares the results of the EDA method and the proposed modified EDA method, named MEDA with respect to the Reference Shear Key Force. The modification of the individual modes has a significant effect in reducing the elastic forces to realistic levels. This is more pronounced in the two-column systems. Figure 6.63 presents the performance of the MEDA method for three hazard levels. Acceptable values of correlation coefficients demonstrate the accuracy of this method. The MEDA is found to be a reliable and accurate method.
Figure 6.61 Comparing the Results of EDA and EDA with Modification of Multiple Modes (MEDA)
Figure 6.62 Comparing the Results of EDA with EDA with Modification of Multiple Modes (MEDA) for Short-Span Prototypes
Figure 6.63 Performance of MEDA Method
The results of the MEDA method for all long-span prototypes are condensed in Figure 6.64. The same results for all short-span bridges are shown in Figure 6.65.

Figure 6.64  Performance of MEDA for All Long-Span Prototypes

Figure 6.65  Performance of MEDA for All Short-Span Prototypes

The proposed MEDA method is adopted as the preferred analysis method for the estimation of the shear key force demand. This method is easy to implement once the values of the force and ductility values are known for each mode.
6.12.3 Sources of Error in MEDA

The differences between the results from the NTH analysis and MEDA may be associated with the following factors. 1) The spectral analysis method is formed around theories of random vibration and is expected to present an approximate estimation for maximum response values. 2) In a seismic event, a multi-column bridge will be subjected to multi-support excitation. Even for a uniform support excitation, the non-uniform yielding of the columns generates changes in the relative stiffness of the frames. This effect may not be captured by the EDA method. 3) Non-transient dynamic effects of the base excitation in the form of wave propagation in the superstructure cannot be seen when implicit methods like spectral analysis are used.

6.13 UPPER BOUND SHEAR KEY FORCE

As a simple design approach, in-span hinge shear keys are currently designed for a portion of the maximum expected transverse capacity (also known as the column overstrength shear, $V_{o}^{col}$ in SDC, typically defined as $M_{o}^{col} / L$, SDC Sec. 2.3.2) of the two adjacent bents to the hinge. For the two-column bents, the bent capacity is the summation of the individual columns’ capacities in the bent. The column overstrength shear is $1.2M_{p}^{col}$ divided by the length of the column, where $M_{p}^{col}$ is the plastic moment of the column obtained from the bilinear moment-curvature relationship. For the single-column bents, the length of columns is the distance between the superstructure centroid and the mid-point of the plastic hinge.

The statistical distribution of the ratios of the NTH Shear Key Forces to the maximum overstrength shear of the two adjacent bents is presented in Figure 6.66. The
medians of the data sets are approximately 0.9 and 0.8 for the single- and two-column bent prototypes with 200-ft spans, respectively. For these prototypes, the maximum shear key forces, including the impact effect, do not exceed two times the overstrength shear of the bent. However, for the 110-ft span prototypes the median ratio is 1.5 and the maximum ratio may be as high as 3.5. Thus, the capacity of the bent may not be used to define the upper bound design force.

Figure 6.67 shows the distribution and CDF of the ratio of the maximum shear key force to the transverse overstrength shear of the weaker of the two adjacent frames. The distributions are fairly similar for all the prototypes. The median ratios are approximately 0.33 and the maximum ratios are approximately 0.8. The CFD graphs show that with a confidence higher than 95%, the ratio of the maximum shear key force to the transverse overstrength shear of the weaker frame is less than 0.75. In other words with a high level of confidence, it may be assumed that the maximum shear key force does not exceed 75% of the transverse overstrength shear of the weaker frame. Therefore, this matric may be suggested as an upper bound value for shear key force demands.
Figure 6.66  Distribution and CDF of the Ratio of the Reference Shear Key Force with Impact Effect to Capacity of the Bent Adjacent to the Hinge
Figure 6.67  Distribution and CDF of the Ratio of Reference Shear Key Force with Impact Effect to the Transverse Capacity of the Weaker of two Adjacent Frames
6.14 DUCTILE SHEAR KEY DESIGN FOR IN-SPAN HINGE

In contrast with the elastic shear key design that ensures the shear key remains elastic in the event of the design earthquake, a ductile shear key design allows for yielding of the shear keys. For ductile shear keys, the performance measure may be in terms of the maximum relative displacement of the adjacent frames and the residual relative displacements. Currently two approaches are implemented for the design of multi-frame bridges. 1) Frames are designed assuming a full transverse integrity of the frames. For that, the shear key needs to be designed to remain intact. 2) Frames can be designed as individual structures ignoring the shear key and transverse integrity. This approach is suggested by SDC when T1/T2 < 0.7 sec. For the latter case, a nominal shear key may be provided to preserve the integrity only under service level seismic events. In this study, only the first approach has been considered.

The concept of the ductile shear key is briefly studied through the analysis of all of the prototypes with yielding shear keys, open gaps, and biaxial ground motions. The yielding capacity of the shear key was taken to be equal to the maximum of the adjacent bents’ capacities (overstrength shear). The modeling of the ductile shear key was explained in Chapter 4. In Sec. 6.10.7, it was demonstrated that the median ratio of the maximum shear key force to the plastic shear demand of the adjacent is approximately 0.9 and 0.8 for the single-column and two-column systems, respectively. Therefore, it can be expected that more than 50% of shear keys remained elastic in the recent analysis. Figure 6.68 shows a representative sample of the response of the ductile shear keys for single-column prototype F1-V1 under ground motion #27 (D33). To assess the bridge performance with ductile shear keys, three matrices are studies: 1)
the maximum relative displacement at the hinges; 2) the residual relative displacement at the hinges; and 3) the displacement ductility demands of the columns.

![Force-Displacement Response of Ductile Shear Key](image)

**Figure 6.68** Force-Displacement Response of Ductile Shear Key

The distribution of the maximum relative transverse displacement at the hinges is shown in Figure 6.69. The median relative displacement is approximately 1.35 in, where 1.0 in. of this displacement is the initial gap opening, and the remaining 0.35 in. is related to the maximum plastic deformation of the ductile shear key. This is a reasonable and small deformation.
Figure 6.69 Distribution and CDF of the Maximum Hinge Relative Displacement with Ductile Shear Key

The distribution of the residual relative displacement at the hinges is shown in Figure 6.70. To find this value, the analysis was continued 2.0 sec. after the end of the ground motion duration. The residual transverse deformations in the hinges are acceptable in terms of post-earthquake repair. Only a few cases have residual displacements larger than 1.0 in.

Finally, the maximum displacement ductility demand of the columns, in models with ductile shear keys, is compared with the results of the impact analysis. The change in the ductility demand was smaller than 10%. This can be attributed to the small plastic deformation of the shear keys and the correspondingly small change in the displacement of the columns. It can be concluded that a ductile shear key with a capacity equal to the adjacent bent’s overstrength shear is able to preserve the integrity of the frames without any considerable change in bridge performance during and after an earthquake.
Figure 6.70 Distribution and CDF of Residual Hinge Transverse Relative Displacement with Ductile Shear Key
A rational analysis method is proposed for the calculation of the seismic design forces of in-span shear keys in multi-frame bridges. This method is developed based on the concept of EDA. A methodology is advanced to modify elastic forces to account for nonlinear behavior in the transverse. The components of this method are presented in this chapter. The modification factors that are proposed to account for the effects of impact and non-uniform base excitation are discussed. An upper bound force is suggested. The final form of the proposed analysis method is presented in a format that is consistent with SDC V1.7. The bridge modeling considerations for the proposed method are presented. This chapter includes two comprehensive design examples.
7.1 GENERAL FORM OF THE PROPOSED ANALYSIS METHOD

The higher modes of vibration may participate significantly in shear key force demand. Thus, an analysis method that does not include effect of higher modes may yield inaccurate shear key force demands. Several possible analysis methods are studied through this research project. This study proposes a reliable method for modifying the EDA forces. This method is named as “MEDA” or Modified EDA. Compared to other analysis approaches, MEDA yields accurate shear key forces. It is proposed that the ultimate seismic force demands on in-span shear keys ($V_{u,sk}$) are calculated using the relationship given in Eq. 7.1.

$$V_{u,sk} = \text{Factor 1} \times \text{Factor 2} \times V_{MEDA}^{sk}$$

Eq. 7.1

where:

Factor 1: Impact Factor
Factor 2: Non-uniform Base Excitation Factor

$V_{MEDA}^{sk}$: Shear Key Force from MEDA Method

7.2 THE MEDA ANALYSIS METHOD

In Chapter 6, it was discussed that the modification of EDA force with a single scalar or using a nonlinear design spectrum leads to an unreliable and non-conservative estimation of the shear key force demands. Therefore, the concept of MEDA proposes modification of modal forces individually. In this method, the modal shear key forces are modified by the displacement ductility ratio of the corresponding mode. Modified modal shear key forces are then combined using conventional modal combination methods.
7.2.1 Modal Ductility

The concept of modal displacement ductility ($\mu_{D,n}$) is developed in this study to enable the estimation of forces in a nonlinear system. It considers the nonlinear responses at higher modes. Eq. 7.2 is proposed for the values of modal ductility.

\[ \mu_{D,n} = \max \left| \frac{\Delta_{(i)D,n}}{\Delta_{(i)Y}^{col}} \right| \geq 1.0 \]

Eq. 7.2

where $\Delta_{(i)D,n}$ is the contribution of the $n^{th}$ mode of vibration to the global displacement of the $i^{th}$ column, in the transverse direction. $\Delta_{(i)Y}^{col}$ is the transverse yield displacement of the $i^{th}$ column (SDC Sec. 2.2.4). These parameters are illustrated in Figure 7.1. Where $\Delta_{(i)D,n}$ is smaller than $\Delta_{(i)Y}^{col}$, modal displacement ductility shall be taken as 1.0.

![Figure 7.1 Obtaining the Value of Modal Displacement Ductility](image)

7.2.2 Modified Modal Shear Key Force

Elastic modal shear key force is divided by the corresponding modal displacement ductility to obtain the modified modal shear key force. The concept of reducing elastic forces with displacement ductility ratio is consistent with the concept of the equivalent displacement method (EDM). For mid-range period structures, the force reduction
factor is assumed proportional to the inverse of the displacement ductility ratio. The equivalent displacement method is valid for bridges with a fundamental period of vibration between 0.7 sec. and 3.0 sec. (Caltrans, 2010).

7.2.3 Modal Combination of Modified Modal Forces

Modified modal shear forces shall be combined to obtain the MEDA shear key force, \( V_{MEDA}^{sk} \). Both SRSS and CQC methods are studied. The CQC method is more accurate. The SRSS method is still acceptable as an easy to implement method. It is found that modes of vibration with frequencies as high as 10Hz to15 Hz may contribute to the shear key force. Thus, the minimum number of considered modes, \( m \), is suggested to be the number of the modes with a period smaller than 0.05 sec.

7.2.4 Validation of the Results with the NTH Method

Figure 7.2 and Figure 7.3 show a comparison of the shear key forces estimated using the MEDA method, \( V_{MEDA}^{sk} \), and the Reference Shear Key Forces obtained from the nonlinear time history analyses of all the prototype bridges. \( V_{MEDA}^{sk} \) is obtained using the design ARS spectrum. The correlation coefficient of approximately 0.9, is comparable with the values obtained for the correlation of EDA displacements and NTH displacements, as discussed in Sec 6.5.1. This confirms that the accuracy of the proposed method in estimating the shear key force demands is comparable with the accuracy of the EDA method in estimating the global displacement demands.
Figure 7.2  Distribution and Correlation of MEDA Forces using Design ARS with Reference Shear Key Forces for All Single-Column Prototypes
Figure 7.3 Distribution and Correlation of MEDA Forces using Design ARS with Reference Shear Key Forces for All Two-Column Prototypes
7.3 MODIFICATION FACTORS

Two modification factors are proposed in the analysis formulation. Factor 1 accounts for the increase in shear key forces due to the transverse and longitudinal pounding. Factor 2 includes the amplifying effects of non-uniform base excitation.

7.3.1 Effect of Pounding

The discussion on the impact forces caused by transverse gap closure is presented in Sec. 6.10.7. Because of the complexity of the impact phenomenon, developing an exact formulation to estimate the impact forces is impractical. Based on the results from the NTH analysis, an impact factor equal to 2.5 is proposed for the single pile-shaft and the two-column long-span bridges, as well as for the short-span bridges. Figure 7.4 shows the correlation of the amplified $V_{MEDA}^{sk}$ with the Reference Shear Key Forces from impact analysis. The design ARS's are used in the MEDA method.

![Figure 7.4 Correlation of Design Force with NTH Shear Key Force Including Impact Effect](image-url)
7.3.2 Non-Uniform Base Excitation

The effect of non-uniform base excitation on shear key forces is discussed in Sec. 6.10.8. Amplification factors of 1.0 for soil Type A and B and 1.25 for soil Type C, D, and E are suggested to be applied to $V_{MEDA}^{sk}$.

7.4 UPPER BOUND FORCE

According to SDC 1.7, the upper bound force for in-span shear keys is equal to the smaller of the transverse capacities of the adjacent frames. This study presented that the maximum shear key forces, including the impact effect, do not exceed 75% of the transverse capacity of the weaker of the adjacent frames. The 75% of the capacity of the weaker proposed as an upper bound for the shear key design force.

7.5 PROPOSED ANALYSIS METHOD

The ultimate seismic shear demand for in-span hinge shear keys shall be calculated using Eq. 7.3.

\[
V_u^{sk} = \text{Factor 1} \times \text{Factor 2} \times V_{MEDA}^{sk} \leq V_{up}^{sk}
\]

Eq. 7.3

\[
V_{MEDA}^{sk} = \left[\sum_{n=1}^{m} \left(\frac{V_n^{sk}}{\mu_{D,n}}\right)^2\right]^{1/2}
\]  \hspace{1cm} (SRSS Method)

Eq. 7.4a

or

\[
V_{MEDA}^{sk} = \left[\sum_{n=1}^{m} \sum_{l=1}^{m} C_{nl} \left(\frac{V_n^{sk}}{\mu_{D,n}}\right) \times \left(\frac{V_l^{sk}}{\mu_{D,l}}\right)\right]^{1/2}
\]  \hspace{1cm} (CQC Method)

Eq. 7.4b

where $V_n^{sk}$ is the modal shear key force obtained from the $n^{th}$ mode of vibration using the design ARS, $m$ is the mode number associated with the period of 0.05 sec, and $C_{nl}$ is the modal response correlation coefficient between modes $n$ and $l$, in the CQC method.
\[ \mu_{D,n} = \max \left| \frac{\Delta_{(i)D,n}}{\Delta_{(i)Y}^{col}} \right| \geq 1.0 \quad \text{Eq. 7.5} \]

where \( \Delta_{(i)D,n} \) is the global system displacement of the \( i \)th column in the transverse direction due to the \( n \)th mode of vibration, \( \Delta_{(i)Y}^{col} \) is the idealized yield displacement of the \( i \)th column (SDC Sec. 2.2.4).

Factor 1 = 2.5 \quad \text{Eq. 7.6}

Factor 2 = \begin{cases} 1.0 & \text{Soil Type A, and B} \\ 1.25 & \text{Soil Type C, D, and E} \end{cases} \quad \text{Eq. 7.7}

\[ V_{upsk} = 0.75 \times \min \left( V_{o}^{Left \ Frame}, V_{o}^{Right \ Frame} \right) \quad \text{Eq. 7.8} \]

where \( V_{o}^{frame} \) is the summation of the transverse over strength shear capacities (SDC Sec. 2.3.2.1) of the columns in each frame.

### 7.6 Modeling and Analysis Considerations

To calculate shear key force demands based on the proposed design method, the following structural modeling and analysis requirement should be considered. The modeling and analysis method is consistent with the SDC provisions; however, some minor modifications are needed.

**Modeling of Bridge Structure:** An elastic model similar to what is used for estimating design displacements may be used for the MEDA method with the following considerations:

1) The superstructure shall be divided into a minimum of five segments in each span in order to capture higher mode effects.
2) To read the shear key forces, a short-length “dummy element” needs to be modeled connecting adjacent frames. A flexural and axial hinge shall be assigned to one end of this element, while torsion and shear stiffnesses remain engaged. The modal shear force outputs from this element are used in the proposed method. **The shear force values read from the adjacent superstructure segments cannot be used because of the inertial forces on diaphragms.**

3) Masses shall be assigned to the nodes for the six degrees of freedom. Ignoring the rotational and torsional masses of the superstructure leads to incorrect estimation of shapes and frequencies of higher modes. The diaphragms and bent caps are massive members. They largely affect the shear key force response. The mass of the diaphragms should be assigned to the nodes at each end of the dummy element.

4) For bridges with more than five frames, sub models with boundary frames may be used in accordance with SDC Sec.5.3.

5) Effective section properties according to SDC Sec.5.6 shall be assigned to column and superstructure elements.

6) The abutments shall be modeled as transversely restrained. Modeling the abutments as springs (per SDC 1.7 Sec. 7.8.2) may lead to non-conservative in-span shear key forces.

**Analysis of Bridge Structure Model:** Elastic Dynamic Analysis (EDA), in accordance with SDC1.7 Sec. 5.2.2, shall be performed in the transverse direction with the following considerations:
1) All the modes of vibration with periods larger than 0.05 sec. shall be considered for modal analysis. The target mass participation of 90% is not adequate for estimating shear key force demands.

2) Both the shear key forces and displacements associated with all the modes shall be extracted from the model to enable implementation of the method.

7.7 DESIGN EXAMPLES

Two examples are presented in this section to demonstrate the proposed analysis method. The configurations of the example bridges are different from the prototypes used in the analytical studies for cross-referencing validation.

7.7.1 Example Bridges Configurations

Both examples are three-frame bridges, shown in Figure 7.5. Column heights are different for each frame. Example 1 is a single-column and Example 2 is a two-column bent bridge, as shown in Figure 7.6. Columns heights are given in Table 7.1 for both example bridges. All other specifications, including the superstructure dimensions, the location of in-span hinges, hinge and abutment details, bent caps, as well as dead loads are assumed the same as those of the prototype bridges explained in Chapter 3.

Figure 7.5 Elevation for Example Bridges
Figure 7.6  Sections a) Example 1and b) Example 2

Table 7.1  Example Bridges Columns Height (feet)

<table>
<thead>
<tr>
<th>#</th>
<th>Frame 1</th>
<th>Frame 2</th>
<th>Frame 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>30* (66.6)**</td>
<td>40 (78.75)</td>
<td>20 (55.1)</td>
</tr>
<tr>
<td>Example 2</td>
<td>30 (39)</td>
<td>40 (49)</td>
<td>20 (32)</td>
</tr>
</tbody>
</table>

* Clearance height
** Total Height from the mid-height of the superstructure to the base or point of fixity

7.7.2 Hazard Level and Design Spectrum

The design spectrum is ARS-C33, which represents a severe hazard level. This spectrum represents magnitude of 7.75-8.25 (Level 3) and a PGA of 0.6g on soil Type C. The design spectrum and the spectrums of the three matched ground motions are shown in Figure 7.7. The bridges are designed for the specified ARS following SDC. These motions are implemented for the NTH analyses of the example bridges. The ARS spectrum is used in the MEDA methods. The shear key force obtained using the proposed analyses method is compared with the average of the values from the three NTH analyses. The specifications of the ground motions are presented in Table 3.2.
7.7.3 Example 1

The bridge is designed according to SDC V1.7 with the same method discussed in Chapter 5. The design results are summarized in Table 7.2.

<table>
<thead>
<tr>
<th>Description</th>
<th>Frame 1</th>
<th>Frame 2</th>
<th>Frame 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (in)</td>
<td>102</td>
<td>102</td>
<td>102</td>
</tr>
<tr>
<td>Longitudinal Reinforcement Ratio (%)</td>
<td>1.4</td>
<td>2.8</td>
<td>1.0</td>
</tr>
<tr>
<td>Transverse Reinforcement Ratio (%)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\frac{I_{eff}}{I_g}$</td>
<td>0.364</td>
<td>0.540</td>
<td>0.311</td>
</tr>
<tr>
<td>$D_{\gamma^{col}}$ (in)</td>
<td>15.71</td>
<td>22.83</td>
<td>10.32</td>
</tr>
<tr>
<td>Design Ductility Demand</td>
<td>2.3</td>
<td>1.65</td>
<td>1.80</td>
</tr>
<tr>
<td>$V_{o,frame}$ (kip)</td>
<td>1972</td>
<td>5084</td>
<td>2928</td>
</tr>
</tbody>
</table>

SAP2000 15.1.0 (CSI, 2011) is used for modeling and conducting the EDA analysis of the example bridges (Figure 7.8). The modeling and analysis considerations of Sec. 7.6 are incorporated. The three analysis cases are defined in the following ways:
1) Dead, 2) Modal, and 3) Response Spectrum, which are named as “DEAD”, “MODAL” and “EDA-ARS-D33”, respectively.

![Analytical Model of Example1 using SAP2000](image)

**Figure 7.8** Analytical Model of Example1 using SAP2000

After running the analyses, the first step is finding the modal ductility values. For this purpose, the nine column nodes are selected. Using the “Show Tables...” tab from the “Display” menu, two result tables of “Joint Displacements” and “Response Spectrum Modal Information” are recalled, as illustrated in Figure 7.9.

![Extracting Modal Displacements from SAP2000 “Show Tables” Tab](image)

**Figure 7.9** Extracting Modal Displacements from SAP2000 “Show Tables” Tab
From the “Joint Displacements” table, normalized mode shapes in the transverse direction (U2) are exported to a spreadsheet and organized as Table 7.3. This value is indicated by “φD” in the table. Then, from the “Response Spectrum Modal Information” table, illustrated in Figure 7.9, the “U2Amp” for all modes is extracted and inserted into Table 7.3. This parameter is the same for all the nodes in the transverse direction. The global modal displacement demands of the i\textsuperscript{th} column are found using Eq. 7.9. This equation is based on the definition of the parameters in SAP2000 (CSI, 2013). The mode shape values, φD, are unit-less and shall be multiplied by “U2Amp” scalar to obtain the modal displacements. Then modal ductility is calculated using Eq. 7.5, given the yield displacement for each column.

\[ \Delta_{(i)D,n} = U2Amp \times \phi_D \]  
Eq. 7.9

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period (sec)</th>
<th>U2Amp</th>
<th>Col. 1</th>
<th>Col. 2</th>
<th>Col. 3</th>
<th>Col. 4</th>
<th>Col. 5</th>
<th>Col. 6</th>
<th>Col. 7</th>
<th>Col. 8</th>
<th>Col. 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.49</td>
<td>211.6</td>
<td>0.05</td>
<td>0.64</td>
<td>-0.10</td>
<td>1.37</td>
<td>-0.15</td>
<td>1.40</td>
<td>-0.14</td>
<td>1.33</td>
<td>-0.13</td>
</tr>
<tr>
<td>2</td>
<td>2.09</td>
<td>16.72</td>
<td>0.06</td>
<td>0.07</td>
<td>0.12</td>
<td>0.13</td>
<td>0.15</td>
<td>0.16</td>
<td>0.04</td>
<td>0.03</td>
<td>-0.07</td>
</tr>
<tr>
<td>4</td>
<td>1.48</td>
<td>53.65</td>
<td>0.07</td>
<td>0.24</td>
<td>0.09</td>
<td>0.31</td>
<td>0.04</td>
<td>0.12</td>
<td>-0.11</td>
<td>0.26</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

... Other Modes are Elastic

In this example, only the first mode is inelastic with a modal ductility of 2.04, and the other modes remain elastic. The next step is finding the modal shear key forces. For this purpose, the two shear key elements in the model are selected. This piece of data is obtained from the “Element Forces - Frames” table in SAP2000, as shown in Figure 7.10.
Then the “V3” response (transverse shear) for each shear key is exported to another spreadsheet and indicated by “$\phi_V$” in Table 7.4. Again, one should note that $\phi_V$ is unitless. Modal shear key forces, $V_n^{sk}$, are calculated using Eq. 7.10, in which “U2Amp” is the same value that is obtained in the previous step (this value is applicable to both displacement and force responses in transverse direction (CSI, 2013)).

$$V_n^{sk} = U2Amp \times \phi_V$$  \hspace{0.5cm} \text{Eq. 7.10}

The value of $V_{MEDA}^{sk}$ is calculated using SRSS method to combine the modified spectral shear key forces. Finally, the ultimate design force of the shear keys, $V_u^{sk}$, is calculated in Table 7.5 for in-span Hinges 1 and 2.
Table 7.4 Modal Shear Key Forces, Example 1

<table>
<thead>
<tr>
<th>Mode</th>
<th>U2Amp</th>
<th>$\mu_{D,n}$</th>
<th>$V_{n,sk}^{*}$ (kip)</th>
<th>$V_{n,sk}^{*}/\mu_{D,n}$</th>
<th>$V_{n,sk}^{*}$ (kip)</th>
<th>$V_{n,sk}^{*}/\mu_{D,n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\varphi \nu$ (U2Amp$\times\varphi \nu$)</td>
<td>$\varphi \nu$ (U2Amp$\times\varphi \nu$)</td>
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<td></td>
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<td>1</td>
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<td>-353.1</td>
<td>-173.2</td>
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</tr>
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<td>-63.7</td>
<td>-63.7</td>
<td>-2.0</td>
</tr>
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<td>-407.6</td>
<td>-407.6</td>
<td>-6.4</td>
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<td>11.8</td>
<td>61.9</td>
<td>61.9</td>
<td>-6.0</td>
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<td>37.5</td>
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<tr>
<td>11</td>
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<td>85.6</td>
<td>85.6</td>
<td>39.5</td>
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<td>-64.8</td>
<td>228.3</td>
<td>228.3</td>
<td>-42.0</td>
</tr>
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<td>87.9</td>
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<td>-212.4</td>
<td>-35.6</td>
</tr>
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<td>1.0</td>
<td>-11.6</td>
<td>17.8</td>
<td>17.8</td>
<td>108.4</td>
</tr>
<tr>
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<td>1.0</td>
<td>20.4</td>
<td>93.4</td>
<td>93.4</td>
<td>-39.6</td>
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<tr>
<td>24</td>
<td>-0.5</td>
<td>1.0</td>
<td>-32.2</td>
<td>17.3</td>
<td>17.3</td>
<td>-15.8</td>
</tr>
<tr>
<td>27</td>
<td>0.5</td>
<td>1.0</td>
<td>-14.0</td>
<td>-6.9</td>
<td>-6.9</td>
<td>-14.6</td>
</tr>
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<td>28</td>
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<td>1.0</td>
<td>177.6</td>
<td>12.7</td>
<td>12.7</td>
<td>-178.2</td>
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<td>1.0</td>
<td>-36.6</td>
<td>2.3</td>
<td>2.3</td>
<td>-0.9</td>
</tr>
<tr>
<td>30</td>
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<td>4.4</td>
<td>1.3</td>
</tr>
<tr>
<td>31</td>
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<td>1.0</td>
<td>155.8</td>
<td>-91.9</td>
<td>-91.9</td>
<td>246.7</td>
</tr>
<tr>
<td>32</td>
<td>0.1</td>
<td>1.0</td>
<td>-16.2</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-18.4</td>
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<tr>
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<td>-288.5</td>
<td>-87.3</td>
<td>-87.3</td>
<td>-24.7</td>
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<td>37</td>
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<td>1.0</td>
<td>-119.8</td>
<td>15.6</td>
<td>15.6</td>
<td>-171.5</td>
</tr>
<tr>
<td>39</td>
<td>0.1</td>
<td>1.0</td>
<td>-228.4</td>
<td>-32.7</td>
<td>-32.7</td>
<td>-295.5</td>
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<tr>
<td>46</td>
<td>0.0</td>
<td>1.0</td>
<td>16.2</td>
<td>-0.4</td>
<td>-0.4</td>
<td>75.2</td>
</tr>
<tr>
<td>48</td>
<td>-0.1</td>
<td>1.0</td>
<td>130.0</td>
<td>-17.1</td>
<td>-17.1</td>
<td>-392.9</td>
</tr>
<tr>
<td>50</td>
<td>-0.1</td>
<td>1.0</td>
<td>-745.8</td>
<td>82.6</td>
<td>82.6</td>
<td>235.3</td>
</tr>
<tr>
<td>55</td>
<td>0.0</td>
<td>1.0</td>
<td>-161.9</td>
<td>-1.8</td>
<td>-1.8</td>
<td>353.0</td>
</tr>
<tr>
<td>56</td>
<td>0.0</td>
<td>1.0</td>
<td>-198.5</td>
<td>-3.4</td>
<td>-3.4</td>
<td>510.7</td>
</tr>
<tr>
<td>58</td>
<td>0.1</td>
<td>1.0</td>
<td>-174.1</td>
<td>-9.5</td>
<td>-9.5</td>
<td>-531.0</td>
</tr>
</tbody>
</table>

Note: Modes with zero participation are excluded from table.
Table 7.5  Design Shear Keys Forces from MEDA Method, Example 1

<table>
<thead>
<tr>
<th>Description</th>
<th>Shear Key at Hinge 1 (kip)</th>
<th>Shear Key at Hinge 2 (kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{sk}^{MEDA}$ (Eq. 7.4a)</td>
<td>586</td>
<td>599</td>
</tr>
<tr>
<td>Factor 1 (Eq. 7.6)</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Factor 2 (Eq. 7.7)</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>Factor 1 x $V_{sk}^{MEDA}$</td>
<td>1465</td>
<td>1498</td>
</tr>
<tr>
<td>Ultimate Design Force (Eq. 7.3)</td>
<td>1831</td>
<td>1872</td>
</tr>
<tr>
<td>Upper Bound Force (Eq. 7.8)</td>
<td>1480 (governs)</td>
<td>2196</td>
</tr>
</tbody>
</table>

For the NTH analysis and the cross checking of the EDA results for Example 1, OpenSees 2.4.4 is used. The modeling assumptions are the same as they were in Chapter 4. The mean shear key forces from the NTH analysis are indicated in Table 7.6. The shear key force without impact is comparable with $V_{sk}^{MEDA}$ in the above table. The NTH analysis is performed with a uniform base excitation; hence, the design forces without Factor 2 are compared to the NTH forces.

Table 7.6  Comparing NTH and the Method Results, Example 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Hinge 1 Shear Key Force (kip)</th>
<th>Hinge 2 Shear Key Force (kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NTH</td>
<td>Method</td>
</tr>
<tr>
<td>without Impact</td>
<td>527 (err=11%)</td>
<td>586 (err=11%)</td>
</tr>
<tr>
<td>with Impact</td>
<td>1155 (err=26%)</td>
<td>1465 (err=26%)</td>
</tr>
</tbody>
</table>

Note: Values do not include effect of NUBE (Factor 2)
7.7.4 Example – 2

A procedure similar to the one used for Example 1 is repeated for Example 2. Thus, only the calculation tables are presented.

**Table 7.7** Column Design Results, Example 2

<table>
<thead>
<tr>
<th>Description</th>
<th>Frame 1</th>
<th>Frame 2</th>
<th>Frame 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (in)</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Longitudinal Reinforcement Ratio (%)</td>
<td>1.2</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Transverse Reinforcement Ratio (%)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \frac{I_{eff}}{I_g} )</td>
<td>0.327</td>
<td>0.430</td>
<td>0.301</td>
</tr>
<tr>
<td>( D Y_{col} ) (in)</td>
<td>4.54</td>
<td>7.72</td>
<td>2.44</td>
</tr>
<tr>
<td>Design Ductility Demand</td>
<td>4.28</td>
<td>3.72</td>
<td>3.34</td>
</tr>
<tr>
<td>( V_0^{frame} ) (kip)</td>
<td>3232</td>
<td>7108</td>
<td>5877</td>
</tr>
</tbody>
</table>

**Figure 7.11** Analytical Model of Example-2 Using SAP2000

**Table 7.8** Finding Modal Ductility, Example 2

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period (sec)</th>
<th>U2Amp</th>
<th>( \Phi_D )</th>
<th>( \mu_D )</th>
<th>( \Phi_D )</th>
<th>( \mu_D )</th>
<th>( \Phi_D )</th>
<th>( \mu_D )</th>
<th>( \Phi_D )</th>
<th>( \mu_D )</th>
<th>( \Phi_D )</th>
<th>( \mu_D )</th>
<th>( \Phi_D )</th>
<th>( \mu_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.745</td>
<td>152.31</td>
<td>-0.12</td>
<td>3.96</td>
<td>-0.13</td>
<td>2.61</td>
<td>-0.08</td>
<td>1.57</td>
<td>-0.02</td>
<td>0.43</td>
<td>0.00</td>
<td>0.27</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>1.529</td>
<td>-71.33</td>
<td>0.03</td>
<td>0.42</td>
<td>-0.13</td>
<td>0.73</td>
<td>0.10</td>
<td>0.56</td>
<td>-0.09</td>
<td>0.52</td>
<td>-0.10</td>
<td>1.68</td>
<td>-0.05</td>
<td>0.91</td>
</tr>
<tr>
<td>6</td>
<td>1.007</td>
<td>42.32</td>
<td>-0.09</td>
<td>0.94</td>
<td>-0.11</td>
<td>1.04</td>
<td>0.13</td>
<td>0.73</td>
<td>0.10</td>
<td>0.56</td>
<td>-0.09</td>
<td>0.52</td>
<td>-0.10</td>
<td>1.68</td>
</tr>
<tr>
<td>7</td>
<td>0.871</td>
<td>38.27</td>
<td>0.06</td>
<td>0.51</td>
<td>0.04</td>
<td>0.37</td>
<td>-0.02</td>
<td>0.12</td>
<td>-0.03</td>
<td>0.16</td>
<td>0.00</td>
<td>0.01</td>
<td>0.05</td>
<td>0.27</td>
</tr>
<tr>
<td>9</td>
<td>0.773</td>
<td>-23.58</td>
<td>0.18</td>
<td>0.93</td>
<td>0.08</td>
<td>0.39</td>
<td>-0.13</td>
<td>0.41</td>
<td>0.04</td>
<td>0.13</td>
<td>0.08</td>
<td>0.25</td>
<td>-0.05</td>
<td>0.16</td>
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</table>

... Other Modes are Elastic
Table 7.9 Finding Modal Shear Key Force, Example 2

<table>
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<tr>
<th>Mode</th>
<th>U2Amp</th>
<th>$\mu_{D,n}$</th>
<th>$\phi v$</th>
<th>$V_{n,k}^{sk}$ (kip)</th>
<th>$V_{n,k}^{sk}/\mu_{D,n}$</th>
<th>Hinge 1</th>
<th>Hinge 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>152.31</td>
<td>3.96</td>
<td>-3.13</td>
<td>-476.72</td>
<td>-120.39</td>
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<td></td>
</tr>
<tr>
<td>4</td>
<td>-71.33</td>
<td>2.56</td>
<td>-4.94</td>
<td>352.52</td>
<td>137.50</td>
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</tr>
<tr>
<td>6</td>
<td>42.32</td>
<td>1.68</td>
<td>-39.37</td>
<td>-1666.44</td>
<td>-989.40</td>
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<tr>
<td>7</td>
<td>38.27</td>
<td>3.01</td>
<td>18.12</td>
<td>693.32</td>
<td>230.40</td>
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<td></td>
</tr>
<tr>
<td>9</td>
<td>-23.58</td>
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<td>21.56</td>
<td>-508.37</td>
<td>-508.37</td>
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</tr>
<tr>
<td>16</td>
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<td>1.00</td>
<td>42.13</td>
<td>-143.70</td>
<td>-143.70</td>
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<td></td>
</tr>
<tr>
<td>20</td>
<td>-2.00</td>
<td>1.00</td>
<td>-195.12</td>
<td>390.51</td>
<td>390.51</td>
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<td></td>
</tr>
<tr>
<td>24</td>
<td>1.28</td>
<td>1.00</td>
<td>269.88</td>
<td>344.88</td>
<td>344.88</td>
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<td></td>
</tr>
<tr>
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<td>0.03</td>
<td>1.00</td>
<td>3.87</td>
<td>0.12</td>
<td>0.12</td>
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</tr>
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<td>1.00</td>
<td>0.47</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>-1.03</td>
<td>1.00</td>
<td>42.24</td>
<td>-43.40</td>
<td>-43.40</td>
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<td></td>
</tr>
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<td>-106.97</td>
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<td></td>
</tr>
<tr>
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<td>0.01</td>
<td>1.00</td>
<td>-16.63</td>
<td>-0.17</td>
<td>-0.17</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>14.57</td>
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<td>-0.08</td>
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<td>773.51</td>
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<td>-124.15</td>
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<td>716.06</td>
<td>52.38</td>
<td>52.38</td>
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<td>-292.38</td>
<td>-19.28</td>
<td>-19.28</td>
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<td></td>
</tr>
</tbody>
</table>

Note: Modes with Zero Participation are Excluded from Table

Table 7.10 Finding Shear Keys Design Forces, Example 2

<table>
<thead>
<tr>
<th>Description</th>
<th>Shear Key at Hinge 1 (kip)</th>
<th>Shear Key at Hinge 2 (kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{n,k}^{sk_{MEDA}}$ (Eq. 7.4a)</td>
<td>1287</td>
<td>1037</td>
</tr>
<tr>
<td>Factor 1 (Eq. 7.6)</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Factor 2 (Eq. 7.7)</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>Factor 1 x $V_{n,k}^{sk_{MEDA}}$</td>
<td>3217</td>
<td>2592</td>
</tr>
<tr>
<td>Ultimate Design Force (Eq. 7.3)</td>
<td>4021</td>
<td>3240</td>
</tr>
<tr>
<td>Upper Bound Force (Eq. 7.8)</td>
<td>2424 (governs)</td>
<td>4407</td>
</tr>
</tbody>
</table>
For Hinge 1, the design force is exceeding the upper bound, thus the upper bound is considered as a design force.

\[\text{Table 7.11} \quad \text{Comparing NTH and the Method Results, Example 2}\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Hinge 1 Shear Key Force (kip)</th>
<th>Hinge 2 Shear Key Force (kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NTH Method</td>
<td>NTH Method</td>
</tr>
<tr>
<td>without Impact</td>
<td>930 1287</td>
<td>1197 1037</td>
</tr>
<tr>
<td>with Impact</td>
<td>1920 2424</td>
<td>2063 2592</td>
</tr>
</tbody>
</table>

Note: Values do not include effect of NUBE (Factor 2)
8 Summary and Conclusions

8.1 SUMMARY

This study was conducted to: 1) demonstrate the transverse dynamic response characteristics of a multi-frame bridges, to examine the effects of geometric, ground motion and design parameters on the seismic response of multi-frame bridges and force demands on in-span shear keys, 2) investigate the significance of in-span shear keys to the seismic performance of multi-frame bridges, and 3) develop a rational method for the reliable estimation of in-span shear key force demands as well as a realistic upper bound design force. The findings of this study are anticipated to expand the current understanding of seismic behavior of multi-frame bridges and the proposed method is
intended to be used for determining seismic force demands for elastic design of in-span shear keys. An analytical approach was taken to achieve these objectives.

Approximately 9400 nonlinear time history (NTH) analyses were conducted on a set of 56 box-girder multi-frame prototype and 52 equivalent box-girder continuous frame bridges that were designed in accordance with the seismic design practices of Caltrans. Prototype bridges were comprised of single-columns of extended pile shaft, and pinned-base two-column piers with 200-ft long spans. To study the effect of the number of frames, sets of two-, three-, four-, and five-frame bridges were considered. The frame lengths vary from 600 ft to 760 ft as a practical length for post-tensioned superstructures. Prototype models with different realistic valley shapes were included in each set. Four 110-ft long span multi-frame bridge models with a uniform valley shapes were also studied as short-span models. A suite of thirty-three spectrum matched ground motions were used for the nonlinear time history analyses.

The dynamic response characteristics, unique to the multi-frame bridge system, were first investigated by performing steady state and modal analyses on two simple bridge models. A limited number of time history analyses were performed to quantify the isolated effects of different system properties, such as stiffness and strength ratios of adjacent frames, on the seismic response of this bridge system.

For the main body of the analytical studies, three-dimensional spine models of the prototype bridges were developed using OpenSees 2.4.4 through a robust code (with about 10,000 lines). These analytical models are used to design the prototype bridges and conduct the response history analyses. The elastic frame elements were used for
the superstructure, diaphragms, and bent caps. The columns were modeled using the inelastic beam-column elements with fiber sections. For the single column bridges, soil parameters were considered to locate the depth of fixity and define the depth and length of the plastic hinge. For the two-column bents, the plastic hinges were assigned to the top of the column elements. The abutments and in-span hinges were explicitly modeled. Their assembly was composed of elements representing diaphragms, bearings, shear keys, gap and impact, backwall, and backfill soil. A series of sensitivity analyses were performed to define the modeling parameters of shear keys and transverse impact elements. The inelastic response of an individual column model was verified using the data from shake table experiments. The system level response of a trial OpenSees model was compared to that of the same model obtained from SAP2000 v15.1.

A suite of eleven acceleration response spectrums (ARS) was selected for the seismic design of each prototype model according to Caltrans SDC v1.7. Three seismic hazard levels of moderate, large, and severe were defined for this project. The corresponding ARS’s were obtained from Appendix B of SDC 1.7 for four soil types of B, C, D, and E for the three hazard levels. Five design criteria were controlled when designing each prototype: 1) the minimum local displacement ductility capacity of the bents, 2) the maximum displacement ductility demands of the bents, 3) the global displacement capacity of bridge, 4) the minimum lateral load capacity of the bents, and 5) the maximum permissible P-Delta effects.
The Pacific Earthquake Engineering Research Center (PEER) NGA ground motion database was used to select and scale eleven sets of three biaxial ground acceleration histories. Each set of motions was selected to have a soil type, moment magnitude, and intensity level compatible with the target ARS. SeismoMatch software was used to match the scaled ground motions to the target ARS. Each prototype model was subjected to the three compatible ground acceleration histories that corresponded to its design ARS. Because of the compatibility of the design demands with the input motions, the nonlinear response history analyses are expected to be proportional to the design, and present a realistic estimation of the force demands in in-span shear keys.

In addition to the “Main Analyses”, a subset of analyses were conducted to investigate effects of the following parameters on shear keys forces: 1) gap closure and pounding of adjacent frames in transverse and longitudinal directions, 2) yielding of abutment shear keys, 3) variation of the base excitation along the length of the bridge, and 4) effects of limited yielding of ductile in-span shear keys on seismic performance of multi-frame bridge systems. Moreover, an equivalent set of continuous frame bridges were analyzed as a benchmark for comparison of responses with multi-frame bridges.

MATLAB was used for post processing and statistical analysis of 260 GB raw data. Some critical seismic responses of multi-frame bridges including columns ductility and drift demand, abutment backfill and shear key displacement demand, and in-span hinge deformation demand were studied. Responses of equivalent continuous frame bridges were also compared with multi-frame bridges. Statistical distributions of responses were presented. A factorial design was used to investigate the effects of six independent
factors on the responses. The independent factors include: 1) number of frames, 2) substructure system 3) valley shape 4) soil type 5) PGA and, 6) overstrength. Analysis of variance (ANOVA) was utilized in order to test hypothesizes including significance of the independent factors on the responses.

The correlations between shear key forces and other response parameters, including the maximum displacement and acceleration at hinges, base shears, displacement ductility of column, ratios of the periods of the standalone frames, elastic design forces, and pushover forces were extracted from NTH analyses results. The maximum shear key forces, obtained from the NTH analyses, were considered as the reference force. Several possible analysis methods were investigated including pushover method, rigid dynamic method, and spectral analysis method (i.e. EDA) to predict shear key force demands.

Finally, a refined EDA method was developed, where the effect of the nonlinear behavior is accounted for by modification of multiple modal forces. Factors are proposed to adjust the elastic shear key force for the effects of transverse impact of frames and non-uniform excitation at the base of columns along the length of the bridge. In addition, a rational upper bound force is defined to cap the shear key design force demands. At the end, two examples are presented to facilitate the implementation of the proposed method by bridge designers.

8.2 OBSERVATIONS AND CONCLUSIONS

The following present the observations and the conclusions that are made based on the comprehensive analytical simulations:
1. If bridges are designed in accordance with Caltrans SDC or AASHTO seismic provisions, the multi-frame design in general does not lead to larger column diameter and reinforcement compared to continuous frame bridges.

2. The Equal Displacement method for multi-frame bridges is equally reliable for continuous frame bridges. In both systems, the two-column bent results in under prediction to some extent. Thus, it is recommended for the seismic design of bridges with a two-column pinned base (possibly multiple-column pinned base) take a more conservative approach compared to single-column bent. Also, the Equal Displacement method is more reliable for bridges with lower overstrength and lower ground motion intensity.

3. The minimum required number of modes to implement in the elastic dynamic analysis (EDA) is larger in multi-frame bridges compared to continuous frame. The 90% mass participation is not sufficient, especially in either of these situations: long bridges, two-column bent, stiff soils, or bridges with high overstrength.

4. Columns in multi-frame bridges are more resilient than continuous frames in non-uniform valley shapes in longitudinal direction. In transverse direction, multi-frame and continuous frame impose similar demands on columns. Displacement ductility demand in two-column bent is twice as much as in single column bent. Soft soil causes more demand on columns. Drift is not a robust measure of damage in columns, so the displacement ductility ratio is preferred. In cases of non-uniform valley shapes it is recommended to position in-span hinges close to the softer adjacent column.
5. Displacement demand on abutment backwall in multi-frame bridges is significantly larger than continuous frames due to smaller expansion joints in multi-frames. Gap size in accordance with AASHTO provisions for service loads is not adequate for seismic displacement demand in multi-frame abutments. In transverse direction, abutments shear key displacement demand is 2.5 in. on average. Two-column bent causes more demand on abutment shear keys compared to single-column bent.

6. In-span hinge relative longitudinal displacement in bridges with two-column bent is about 75% larger than single-column. Soft soil and low over strength also increases this response. The interaction of transverse response with longitudinal increases the probability of unseating at in-span hinges by 8% on average; however, it can be as high as 60%. Thus, it is recommended this effect to be considered in design codes. Further research is needed on this matter.

7. The in-plane vibration of the superstructure significantly contributes to the shear key forces; thus, assuming in-plane rigid superstructure yields inaccurate estimation of shear key force demands.

8. In general, the maximum shear key force does not coincide with the maximum transverse displacement or acceleration at the corresponding in-span hinge, and/or the maximum base shear.

9. At the instance of the maximum shear key force, the profile of accelerations along the length of superstructure - and consequently inertial forces - is a high-order polynomial. Thus, the equation of equilibrium of column forces, inertial forces, and shear key forces may not be easily solved as closed form.
10. Shear key force demands increase approximately linearly, with the increase in the peak ground acceleration (PGA). In contrast, increase in the peak ground velocity (PGV) beyond 20 in/sec. does not generally increase shear key force demands. This confirms that pulse frequencies in pulse-type motions do not excite the local/higher modes of superstructure. Therefore, pulse-type motions do not generate shear key forces larger than that of non-pulse motions with the same amplitude.

11. The average of the shear key force demands obtained from the analyses of the two-column bent bridges is approximately two times that of the single-column bent bridges. This ratio is comparable to the ratio of the average plastic shear of two-column bents to that of single column bents.

12. As a general observation, the scatter of the seismic responses, including the maximum shear forces, is larger for the two-column bent bridges. This may be associated with the larger transverse stiffness and mass of the superstructure in two-column bent bridges.

13. The shear key force demands in bridges with a non-uniform valley shape are not always larger than the shear key force demands in bridges with a uniform valley shape. The distance of the hinge from the stiffer frame plays the main role.

14. Only a weak correlation exists between the maximum shear key forces and the ratio of the periods of the adjacent frames.

15. Standard pushover method fails to estimate shear key force demands. However, when only in-span hinges are pushed to their design displacement demands,
individually or together, a quarter of the pushover force at each hinge is comparable to the shear key force demand in the hinge.

16. Elastic dynamic analysis (EDA) significantly over predict the shear key forces as it ignores the de-amplification of elastic forces due to nonlinear response. Under severe base excitations, the shear key force may be over predicted approximately three and four times for single- and two-column bent bridges, respectively. EDA shear key forces shall not be directly used for designing and sizing shear keys.

17. In EDA, the mass participation ratio necessary for accurate estimation of elastic shear key forces is larger than that needed for estimating displacements. Modes with frequencies as high as 15 Hz (period of 0.06 sec.) should be included in spectral analysis for mobilizing 90% of the total shear key force. For that, approximately 10 transverse modes should be considered. If the number of longitudinal and vertical modes is included, the total number of modes will be approximately 30. However, these numbers may be different for different prototypes; hence, the modal frequency is a more reliable measure for defining the number of required modes. It should be noted that including modes with a total mass participation of 90% of the total mass is not adequate for estimating shear key forces.

18. Using inelastic response spectrum or modifying EDA forces with one response factor to account for the nonlinear behavior leads to significant under prediction of shear key forces. This is because nonlinear behavior does not affect shear key forces generated by the higher modes.
19. It is found that the effect of yielding should only be applied to modal forces that generate large displacement ductility ratios. A modification of multiple modes present the most reliable and consistent results. In order to modify individual elastic modal forces, they should be divided by the ductility ratio of the corresponding mode (modal ductility ratio).

20. The displacement ductility ratios of higher modes (up to 15th mode) may be as large as five for the two-column bent bridges with a large number of frames. For single-column bent bridges, large modal ductility ratios are limited to the first few modes (up to 5th mode).

21. Abutment shear keys, designed for forces equal to the dead load reactions at the abutments (SDC v1.7, Eq. 7.8.4-4), may extensively yield under large motions. The transverse displacements at the abutments may be as large as 5.0 in. in two-column bent bridges with relatively stiff superstructure. However, if the abutment shear keys are designed per SDC 1.7, Eq. 7.8.4-1, smaller displacements are expected. For instance for the shear key capacity of twice the dead load the maximum displacement does not exceed 2.0 in.

22. The impact forces due to the closure of the transverse gap between the shear key and insert/sleeve may increase the forces by a factor of four under moderate levels of seismic excitation. Under large earthquakes, the impact factor falls under 2.0. The median value of the impact factor across the range is determined as 2.5. Further research is required for more accurate prediction of impact factor.
23. Effect of non-uniform base excitations may increase the shear key forces. For soil Types C, D, and E, the mean increase factor was approximately 1.25. For soil Type B, the increase in shear key forces due to non-uniform excitation was negligible. Further research is required on this topic.

24. The median value of the ratios of the shear key force demands to the overstrength shear of the closer bent is approximately 0.9 and 0.8 for the single- and two-column bent prototypes with 200-ft spans, respectively. For these prototypes, the maximum shear key forces, including the impact effect, do not exceed two times the overstrength shear of the bent. However, for the 110-ft span prototypes the median ratio is 1.5 and the maximum ratio may be as large as 3.5.

25. The median value of the ratios of the shear key force demands to the transverse overstrength shear of the weaker of the two adjacent frames is fairly similar for all the prototype bridges. This median ratio is approximately 0.33. The maximum ratio is approximately 0.8.

26. With 95% confidence interval, the ratio of the maximum shear key force to the transverse overstrength shear of the weaker frame is less than 0.75. In other words, 75% of the transverse overstrength shear of the weaker frame is an upper bound value for shear key force demands.

27. When the shear key was modeled as a ductile element (ductile shear key) with a transverse yield strength equal to the overstrength shear of the closer bent, the maximum inelastic deformation of the shear key was approximately 1.5 in.
28. In models where ductile shear key elements with a yield strength equal to the overstrength shear of the adjacent bent are implemented, the residual relative transverse displacement at in-span hinges were found to be smaller than 1.0 in. The residual relative displacements in bridges with ductile shear keys with a yield strength that is notably smaller than what was assumed in this study might be larger. In order to study the seismic response of bridges when large plastic deformations are expected in shear keys, a comprehensive understanding of the cyclic load-deformation relationship of the ductile shear key elements is necessary.

29. The median of relative longitudinal displacements at hinges at the instance of peak shear key force was approximately 2.0 in (the initial longitudinal gap size was 2.0 in). The median of maximum relative longitudinal displacements of adjacent frames was 5.0 in.
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Figure C.76  Response Histories of Prototype F2-V1, Hinge 1, Motions B11

Figure C.77  Response Histories of Prototype F2-V1, Hinge 1, Motions B22
Figure C.78  Response Histories of Prototype F2-V1, Hinge 1, Motions B33

Figure C.79  Response Histories of Prototype F2-V1, Hinge 1, Motions C11
Figure C.80  Response Histories of Prototype F2-V1, Hinge 1, Motions C22

Figure C.81  Response Histories of Prototype F2-V1, Hinge 1, Motions C33
Figure C.82  Response Histories of Prototype F2-V1, Hinge 1, Motions D11

Figure C.83  Response Histories of Prototype F2-V1, Hinge 1, Motions D22
Figure C.84  Response Histories of Prototype F2-V1, Hinge 1, Motions D33

Figure C.85  Response Histories of Prototype F2-V1, Hinge 1, Motions E11
### Appendix C Selected Analysis Results

#### Figure C.86  Response Histories of Prototype F2-V1, Hinge 1, Motions E22

- **E22 Motion 1**
  - Force: 585 (kip)
  - Disp.: 31 (in)
  - Accl.: 254 (in/sec²)

- **E22 Motion 2**
  - Force: 668 (kip)
  - Disp.: 24 (in)
  - Accl.: 277 (in/sec²)

- **E22 Motion 3**
  - Force: 667 (kip)
  - Disp.: 25 (in)
  - Accl.: 260 (in/sec²)

#### Figure C.87  Response Histories of Prototype F3-V3, Hinge 1, Motions B11

- **B11 Motion 1**
  - Force: 141 (kip)
  - Disp.: 2 (in)
  - Accl.: 42 (in/sec²)

- **B11 Motion 2**
  - Force: 169 (kip)
  - Disp.: 3 (in)
  - Accl.: 55 (in/sec²)

- **B11 Motion 3**
  - Force: 149 (kip)
  - Disp.: 2 (in)
  - Accl.: 42 (in/sec²)
Figure C.88  Response Histories of Prototype F3-V3, Hinge 1, Motions B22

Figure C.89  Response Histories of Prototype F3-V3, Hinge 1, Motions B33
Figure C.90  Response Histories of Prototype F3-V3, Hinge 1, Motions C11

Figure C.91  Response Histories of Prototype F3-V3, Hinge 1, Motions C22
Figure C.92  Response Histories of Prototype F3-V3, Hinge 1, Motions C33

Figure C.93  Response Histories of Prototype F3-V3, Hinge 1, Motions D11
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Figure C.94  Response Histories of Prototype F3-V3, Hinge 1, Motions D22

Figure C.95  Response Histories of Prototype F3-V3, Hinge 1, Motions D33
**Figure C.96** Response Histories of Prototype F3-V3, Hinge 1, Motions E11

**Figure C.97** Response Histories of Prototype F3-V3, Hinge 1, Motions E22
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Figure C.98  Response Histories of Prototype F4-V4, Hinge 2, Motions B11

Figure C.99  Response Histories of Prototype F4-V4, Hinge 2, Motions B22
Figure C.100  Response Histories of Prototype F4-V4, Hinge 2, Motions B33

Figure C.101  Response Histories of Prototype F4-V4, Hinge 2, Motions C11
Figure C.102  Response Histories of Prototype F4-V4, Hinge 2, Motions C22

Figure C.103  Response Histories of Prototype F4-V4, Hinge 2, Motions C33
Figure C.104  Response Histories of Prototype F4-V4, Hinge 2, Motions D11

Figure C.105  Response Histories of Prototype F4-V4, Hinge 2, Motions D11
Figure C.106  Response Histories of Prototype F4-V4, Hinge 2, Motions D33

Figure C.107  Response Histories of Prototype F4-V4, Hinge 2, Motions E11
Figure C.108  Response Histories of Prototype F4-V4, Hinge 2, Motions E22

Figure C.109  Response Histories of Prototype F5-V5, Hinge 2, Motions B11
Figure C.110  Response Histories of Prototype F5-V5, Hinge 2, Motions B22

Figure C.111  Response Histories of Prototype F5-V5, Hinge 2, Motions B33
Figure C.112  Response Histories of Prototype F5-V5, Hinge 2, Motions C11

Figure C.113  Response Histories of Prototype F5-V5, Hinge 2, Motions C22
**Figure C.114** Response Histories of Prototype F5-V5, Hinge 2, Motions C33

**Figure C.115** Response Histories of Prototype F5-V5, Hinge 2, Motions D11
Figure C.116  Response Histories of Prototype F5-V5, Hinge 2, Motions D22

Figure C.117  Response Histories of Prototype F5-V5, Hinge 2, Motions D33
Figure C.118  Response Histories of Prototype F5-V5, Hinge 2, Motions E11

Figure C.119  Response Histories of Prototype F5-V5, Hinge 2, Motions E22
PART-2b: Time History of Shear Key Forces, Transverse Displacement at Hinges, and Transverse Acceleration at hinges. Two-Column Bent Prototypes.
**Figure C.120**  Response Histories of Prototype F2-V1, Hinge 1, Motions B11

**Figure C.121**  Response Histories of Prototype F2-V1, Hinge 1, Motions B22
Figure C.122  Response Histories of Prototype F2-V1, Hinge 1, Motions B33

Figure C.123  Response Histories of Prototype F2-V1, Hinge 1, Motions C11
Figure C.124  Response Histories of Prototype F2-V1, Hinge 1, Motions C22

Figure C.125  Response Histories of Prototype F2-V1, Hinge 1, Motions C33
Figure C.126  Response Histories of Prototype F2-V1, Hinge 1, Motions D11

Figure C.127  Response Histories of Prototype F2-V1, Hinge 1, Motions D22
Figure C.128  Response Histories of Prototype F2-V1, Hinge 1, Motions D33

Figure C.129  Response Histories of Prototype F2-V1, Hinge 1, Motions E11
Figure C.130  Response Histories of Prototype F2-V1, Hinge 1, Motions E22

Figure C.131  Response Histories of Prototype F3-V3, Hinge 1, Motions B11
Figure C.132  Response Histories of Prototype F3-V3, Hinge 1, Motions B22

Figure C.133  Response Histories of Prototype F3-V3, Hinge 1, Motions B33
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Figure C.134  Response Histories of Prototype F3-V3, Hinge 1, Motions C11

Figure C.135  Response Histories of Prototype F3-V3, Hinge 1, Motions C22
Figure C.136  Response Histories of Prototype F3-V3, Hinge 1, Motions C33

Figure C.137  Response Histories of Prototype F3-V3, Hinge 1, Motions D11
Figure C.138  Response Histories of Prototype F3-V3, Hinge 1, Motions D22

Figure C.139  Response Histories of Prototype F3-V3, Hinge 1, Motions D33
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**Figure C.140**  Response Histories of Prototype F3-V3, Hinge 1, Motions E11

**Figure C.141**  Response Histories of Prototype F3-V3, Hinge 1, Motions E22
Figure C.142  Response Histories of Prototype F4-V4, Hinge 2, Motions B11

Figure C.143  Response Histories of Prototype F4-V4, Hinge 2, Motions B22
Figure C.144  Response Histories of Prototype F4-V4, Hinge 2, Motions B33

Figure C.145  Response Histories of Prototype F4-V4, Hinge 2, Motions C11
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Figure C.146  Response Histories of Prototype F4-V4, Hinge 2, Motions C22

Figure C.147  Response Histories of Prototype F4-V4, Hinge 2, Motions C33
Figure C.148  Response Histories of Prototype F4-V4, Hinge 2, Motions D11

Figure C.149  Response Histories of Prototype F4-V4, Hinge 2, Motions D22
Figure C.150  Response Histories of Prototype F4-V4, Hinge 2, Motions D33

Figure C.151  Response Histories of Prototype F4-V4, Hinge 2, Motions E11
Figure C.152  Response Histories of Prototype F4-V4, Hinge 2, Motions E22

Figure C.153  Response Histories of Prototype F5-V5, Hinge 2, Motions B11
Figure C.154  Response Histories of Prototype F5-V5, Hinge 2, Motions B22

Figure C.155  Response Histories of Prototype F5-V5, Hinge 2, Motions B33
Figure C.156  Response Histories of Prototype F5-V5, Hinge 2, Motions C11

Figure C.157  Response Histories of Prototype F5-V5, Hinge 2, Motions C22
Figure C.158  Response Histories of Prototype F5-V5, Hinge 2, Motions C33

Figure C.159  Response Histories of Prototype F5-V5, Hinge 2, Motions D11
**Figure C.160** Response Histories of Prototype F5-V5, Hinge 2, Motions D22

**Figure C.161** Response Histories of Prototype F5-V5, Hinge 2, Motions D33
Figure C.162  Response Histories of Prototype F5-V5, Hinge 2, Motions E11

Figure C.163  Response Histories of Prototype F5-V5, Hinge 2, Motions E22
PART-3a: Hysteresis Curves for: Shear Key Force vs. Transverse Displacement and Shear Key Force vs. Transverse Acceleration, Single-Column Bent Prototypes.
Figure C.164  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions B11

Figure C.165  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions B22
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Figure C.166  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions B33

Figure C.167  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions C11
Appendix C Selected Analysis Results

Figure C.168  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions C22

Figure C.169  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions C33
Figure C.170  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions D11

Figure C.171  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions D22
Figure C.172  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions D33

Figure C.173  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions E11
Figure C.174  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions E22

Figure C.175  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions B11
Figure C.176  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions B22

Figure C.177  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions B33
Figure C.178  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions C11

Figure C.179  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions C22
Figure C.180  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions B11

Figure C.181  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions D11
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Figure C.182  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions D22

Figure C.183  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions D33
Figure C.184  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions E11

Figure C.185  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions E22
Figure C.186  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions B11

Figure C.187  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions B22
Figure C.188  Hysteresis Curves for  Prototype F4-V4, Hinge 2, Motions B33

Figure C.189  Hysteresis Curves for  Prototype F4-V4, Hinge 2, Motions C11
Figure C.190  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions C22

Figure C.191  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions C33
Figure C.192  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions D11

Figure C.193  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions D22
Figure C.194  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions D33

Figure C.195  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions E11
Figure C.196  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions E22

Figure C.197  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions B11
Figure C.198  Hysteresis Curves for  Prototype F5-V5, Hinge 2, Motions B22

Figure C.199  Hysteresis Curves for  Prototype F5-V5, Hinge 2, Motions B33
Figure C.200 Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions C11

Figure C.201 Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions C22
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Figure C.202  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions C33

Figure C.203  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions D11
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Figure C.204 Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions D22

Figure C.205 Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions D33
**Figure C.206** Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions E11

**Figure C.207** Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions E22
PART-3b: Hysteresis Curves for: Shear Key Force vs. Transverse Displacement and Shear Key Force vs. Transverse Acceleration, Two-Column Bent Prototypes.
Figure C.208  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions B11

Figure C.209  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions B22
Figure C.210  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions B33

Figure C.211  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions C11
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Figure C.212  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions C22

Figure C.213  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions C33
Figure C.214  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions D11

Figure C.215  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions D22
**Figure C.216** Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions D33

**Figure C.217** Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions E11
Figure C.218  Hysteresis Curves for Prototype F2-V1, Hinge 1, Motions E22

Figure C.219  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions B11
Figure C.220  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions B22

Figure C.221  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions B33
Figure C.222  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions C11

Figure C.223  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions C22
Figure C.224  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions C33

Figure C.225  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions D11
Figure C.226  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions D22

Figure C.227  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions D33
Figure C.228  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions E11

Figure C.229  Hysteresis Curves for Prototype F3-V3, Hinge 1, Motions E22
**Figure C.230** Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions B11

**Figure C.231** Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions B22
Figure C.232  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions B33

Figure C.233  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions C11
Figure C.234  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions C22

Figure C.235  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions C33
Figure C.236  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions D11

Figure C.237  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions D22
Figure C.238  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions D33

Figure C.239  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions E11
Figure C.240  Hysteresis Curves for Prototype F4-V4, Hinge 2, Motions E22

Figure C.241  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions B11
**Figure C.242** Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions B22

**Figure C.243** Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions B33
**Figure C.244** Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions C11

**Figure C.245** Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions C22
Figure C.246  Hysteresis Curves for  Prototype F5-V5, Hinge 2, Motions C33

Figure C.247  Hysteresis Curves for  Prototype F5-V5, Hinge 2, Motions D11
Figure C.248  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions D22

Figure C.249  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions D33
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Figure C.250  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions E11

Figure C.251  Hysteresis Curves for Prototype F5-V5, Hinge 2, Motions E22
PART-4a: System State at the time of the Maximum Shear Key Forces, Single-Column Bent Prototypes

**Legend:**

- Solid Black = Displacements
- Dash line = Accelerations
- Gray = Design Displacement Demands
- Red Stem = Bent capacities
- Black Stem = Base reactions
Figure C.252  System State at Maximum Force, Prototype F2-V1, motion-3

Figure C.253  System State at Maximum Force, Prototype F2-V1, motion-9
Figure C.254  System State at Maximum Force, Prototype F2-V1, motion-17

Figure C.255  System State at Maximum Force, Prototype F2-V2, motion-9
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Figure C.256  System State at Maximum Force, Prototype F2-V2, motion-27

Figure C.257  System State at Maximum Force, Prototype F3-V1, motion-13
Figure C.258  System State at Maximum Force, Prototype F3-V1, motion-13

Figure C.259  System State at Maximum Force, Prototype F3-V1, motion-27
Figure C.260  System State at Maximum Force, Prototype F3-V1, motion-27

Figure C.261  System State at Maximum Force, Prototype F3-V2, motion-11
Figure C.262  System State at Maximum Force, Prototype F3-V2, motion-11

Figure C.263  System State at Maximum Force, Prototype F3-V4, motion-27
Figure C.264  System State at Maximum Force, Prototype F3-V4, motion-27

Figure C.265  System State at Maximum Force, Prototype F4-V1, motion-27
Figure C.266  System State at Maximum Force, Prototype F4-V1, motion-27

Figure C.267  System State at Maximum Force, Prototype F4-V1, motion-27
Figure C.268  System State at Maximum Force, Prototype F4-V3, motion-25

Figure C.269  System State at Maximum Force, Prototype F4-V3, motion-25
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Figure C.270  System State at Maximum Force, Prototype F4-V3, motion-25

Figure C.271  System State at Maximum Force, Prototype F4-V6, motion-6
**Figure C.272** System State at Maximum Force, Prototype F4-V6, motion-6

**Figure C.273** System State at Maximum Force, Prototype F4-V6, motion-6
Figure C.274  System State at Maximum Force, Prototype F5-V1, motion-18

Figure C.275  System State at Maximum Force, Prototype F5-V1, motion-18
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Figure C.276  System State at Maximum Force, Prototype F5-V1, motion-18

Figure C.277  System State at Maximum Force, Prototype F5-V1, motion-18
Figure C.278  System State at Maximum Force, Prototype F5-V2, motion-9

Figure C.279  System State at Maximum Force, Prototype F5-V2, motion-9
Figure C.280  System State at Maximum Force, Prototype F5-V2, motion-9

Figure C.281  System State at Maximum Force, Prototype F5-V2, motion-9
Figure C.282  System State at Maximum Force, Prototype F5-V7, motion-11

Figure C.283  System State at Maximum Force, Prototype F5-V7, motion-11
Figure C.284  System State at Maximum Force, Prototype F5-V7, motion-11

Figure C.285  System State at Maximum Force, Prototype F5-V7, motion-11
PART-4b: System State at the time of the Maximum Shear Key Forces, Two-Column Bent Prototypes

**Legend:**
- Solid Black = Displacement
- Dash line = Acceleration
- Gray = Design Displacement Demand
- Red Stem = bent capacity
- Black Stem = base reaction
Figure C.286  System State at Maximum Force, Prototype F2-V1, motion-3

Figure C.287  System State at Maximum Force, Prototype F2-V1, motion-9
Figure C.288  System State at Maximum Force, Prototype F2-V1, motion-17

Figure C.289  System State at Maximum Force, Prototype F2-V2, motion-9
Appendix C Selected Analysis Results

Figure C.290  System State at Maximum Force, Prototype F2-V2, motion-27

Figure C.291  System State at Maximum Force, Prototype F3-V1, motion-13
Figure C.292  System State at Maximum Force, Prototype F3-V1, motion-13

Figure C.293  System State at Maximum Force, Prototype F3-V1, motion-27
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Figure C.294  System State at Maximum Force, Prototype F3-V1, motion-27

Figure C.295  System State at Maximum Force, Prototype F3-V2, motion-11
Figure C.296  System State at Maximum Force, Prototype F3-V2, motion-11

Figure C.297  System State at Maximum Force, Prototype F3-V4, motion-27
Figure C.298  System State at Maximum Force, Prototype F3-V4, motion-27

Figure C.299  System State at Maximum Force, Prototype F4-V1, motion-27
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Figure C.300  System State at Maximum Force, Prototype F4-V1, motion-27

Figure C.301  System State at Maximum Force, Prototype F4-V1, motion-27
Figure C.302 System State at Maximum Force, Prototype F4-V3, motion-25

Figure C.303 System State at Maximum Force, Prototype F4-V3, motion-25
Appendix C Selected Analysis Results

Figure C.304  System State at Maximum Force, Prototype F4-V3, motion-25

Figure C.305  System State at Maximum Force, Prototype F4-V6, motion-6
Figure C.306  System State at Maximum Force, Prototype F4-V6, motion-6

Figure C.307  System State at Maximum Force, Prototype F4-V6, motion-6
Figure C.308  System State at Maximum Force, Prototype F5-V1, motion-18

Figure C.309  System State at Maximum Force, Prototype F5-V1, motion-18
Figure C.310  System State at Maximum Force, Prototype F5-V1, motion-18

Figure C.311  System State at Maximum Force, Prototype F5-V1, motion-18
Figure C.312  System State at Maximum Force, Prototype F5-V2, motion-9

Figure C.313  System State at Maximum Force, Prototype F5-V2, motion-9
Figure C.314  System State at Maximum Force, Prototype F5-V2, motion-9

Figure C.315  System State at Maximum Force, Prototype F5-V2, motion-9
PART-5a: Shear Key Force Responses with and without Transverse Gaps, Single-Column Bent Prototypes
Figure C.316  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions B11

Figure C.317  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions B22
Figure C.318  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions B33

Figure C.319  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions C11
Figure C.320  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions C22

Figure C.321  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions C33
Appendix C Selected Analysis Results

Figure C.322  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions D11

Figure C.323  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions D22
Figure C.324  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions D33

Figure C.325  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions E11
Figure C.326  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions E22

Figure C.327  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions B11
Figure C.328  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions B22

Figure C.329  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions B33
Figure C.330  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions C11

Figure C.331  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions C22
Figure C.332  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions C33

Figure C.333  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions D11
Figure C.334  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions D22

Figure C.335  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions D33
Figure C.336  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions E11

Figure C.337  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions E22
Figure C.338  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions B11

Figure C.339  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions B22
Figure C.340  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions B33

Figure C.341  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions C11
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**Figure C.342**  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions C22

**Figure C.343**  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions C33
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Figure C.344 Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions D11

Figure C.345 Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions D22
Figure C.346  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions D33

Figure C.347  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions E11
Figure C.348 Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions E22

Figure C.349 Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions B11
Figure C.350  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions B22

Figure C.351  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions B33
Figure C.352  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions C11

Figure C.353  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions C22
Figure C.354  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions C33

Figure C.355  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions D11
Figure C.356  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions D22

Figure C.357  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions D33
Figure C.358  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions E11

Figure C.359  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions E22
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Figure C.361  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions B22
Figure C.362  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions B33

Figure C.363  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions C11
Figure C.364  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions C22

Figure C.365  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions C33
Figure C.366  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions D11

Figure C.367  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions D22
Figure C.368  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions D33

Figure C.369  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions E11
Figure C.370  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions B11

Figure C.371  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions B22
Figure C.372  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions B33

Figure C.373  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions C11
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Figure C.377  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions D22
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Figure C.378  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions D33

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Figure C.384  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions C11
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Figure C.390  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions E11
Figure C.391  Shear Key Force Response w/ and w/o Gap, Prototype F2-V1, Hinge-1, Motions E22

Figure C.392  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions B11
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Figure C.396  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions C22
Figure C.397  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions C33

Figure C.398  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions D11
Figure C.399  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions D22

Figure C.400  Shear Key Force Response w/ and w/o Gap, Prototype F3-V1, Hinge-1, Motions D33
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Figure C.404  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions B22
Figure C.405 Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions B33

Figure C.406 Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions C11
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Figure C.410  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions D22
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Figure C.412  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions E11
Figure C.413  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-1, Motions E22

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Figure C.416  Shear Key Force Response w/ and w/o Gap, Prototype F4-V1, Hinge-2, Motions B33
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Figure C.430  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions C33
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Figure C.435  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-1, Motions E22

Figure C.436  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions B11
Figure C.437  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions B22

Figure C.438  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions B33
Figure C.439  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions C11

Figure C.440  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions C22
**Figure C.441**  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions C33

**Figure C.442**  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions D11
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Figure C.444  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions D33
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Figure C.446  Shear Key Force Response w/ and w/o Gap, Prototype F5-V1, Hinge-2, Motions E22
Data Loading and Processing Algorithm

1) Open and read the txt files generated by OpenSees.
2) Process the data; extract the target information such as maximum displacement, maximum force and etc.
3) Place them in a pre-allocated 2D matrix. Each matrix row is related to one prototype due to one ground motion, and matrix columns are corresponding target parameters.
4) For modal responses a 3D matrix is formed: matrix rows are same as before, matrix columns are corresponding modal response for mode 1 to n, and the third dimension is related to different responses (force, period, participation factors, etc.)
5) Close the txt file, repeat steps 1-5 for all prototypes and motions.
6) Multiple other scripts are developed to try different methods for prediction of shear key force and for parametric study.
7) A few functions and scripts are developed for plotting the processed results. Three of them are presented as following.

Script-1: plot actual and log-normal distribution bar chart and CDF.

```
function PDFCDF(Input, BinNo, AxisLimitX, AxisLimitY, XLabel, bentType)
axisLabelSize =12.5;
axisFontWeight= 'bold'
```

% Input Variables:
Input is the designated variable or array.
BinNo is desired number of bins for bar chart.
AxisLimitX is horizontal axis limit.
AxisLimitY is vertical axis limit.
XLabel is desired label string for x axis.
bentType is substructure type; 1.0 for Single-Column and 2.0 for Two-Column bents.

Example: PDFCDF(NTH_ShearKeyForce, 50, 1000 ,300 ,'Maximum Shear Key Forces (kip)',1.0)
lableFontWeight = 'bold'
h1FaceColor = [0.25 0.50 0.70];
lineWidth = 1.45;
figure('Position',[100 500 1100 450]);
subplot(1,2,1)
parmhat = lognfit(Input);
[H,Bin]=hist(Input,BinNo);
ht=bar(Bin,H); hold on
xt=0:max(Input)/50:max(Input);
scale=sum((Bin(2)-Bin(1)).*H);
plot(xt,scale*lognpdf(xt,parmhat(1),parmhat(2)), 'LineWidth', 2.5, 'color', 'k');
str=['Median= ' num2str(median(Input))];
set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'FontWeight', lableFontWeight);
str=['Mean= ' num2str(mean(Input))];
set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'FontWeight', lableFontWeight);
str=['Total= ' num2str(length(Input))];
set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'FontWeight', lableFontWeight);
set(h1, 'facecolor', h1FaceColor, 'LineWidth', lineWidth, 'BarWidth', 0.6) % use color name
set(gca, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'XScale', 'linear', 'YScale', 'linear', 'FontWeight', axisFontWeight);
xlabel(XLable , 'FontName', 'Arial', 'FontSize', 1.3*axisLableSize, 'FontWeight', lableFontWeight);
ylabel('Number', 'FontName', 'Arial', 'FontSize', 1.3*axisLableSize, 'FontWeight', lableFontWeight); % axis([0 AxisLimitx 0 AxisLimity]);
str=['Theoretical' ; 'Lognormal' ; 'Distribution']);
set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.0*axisLableSize, 'FontWeight', lableFontWeight);
if bentType== 1; str='Single-Column Prototypes'; else str='Two-Column Prototypes'; end
ht =text(AxisLimitx*.75,AxisLimity*1.06, str); set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.3*axisLableSize, 'FontWeight', lableFontWeight)
subplot(1,2,2)
PDF=H./length(Input); CDF_R=cumsum(PDF);
CDF = cdf('Lognormal',xt,parmhat(1),parmhat(2));
plot(Bin, CDF_R, 'LineStyle', '--', 'LineWidth', 2.5, 'color', h1FaceColor); hold on
plot(xt, CDF, 'LineStyle', '-', 'LineWidth', 2.5, 'color', 'k'); hold on
set(gca, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'XScale', 'linear', 'YScale', 'linear', 'FontWeight', axisFontWeight);
xlabel(XLable , 'FontName', 'Arial', 'FontSize', 1.3*axisLableSize, 'FontWeight', lableFontWeight);
ylabel('CDF', 'FontName', 'Arial', 'FontSize', 1.3*axisLableSize, 'FontWeight', lableFontWeight); % axis([0 AxisLimitx 0 1]);
grid on;  end
Script-2: Scatter Plots with Coefficient of Correlation and Number of Data Points

```matlab
figure('Position',[10 50 950 950]);
margin1=0.12; margin2=0.0195; W=(1-margin1-4*margin2)/2;
positionVector(1,1:4)=[margin1, margin1+margin2+W, W, W];
positionVector(2,1:4)=[margin1+margin2+W, margin1+margin2+W, W, W];
positionVector(3,1:4)=[margin1, margin1, W, W];
positionVector(4,1:4)=[margin1+margin2+W, margin1, W, W];
markerShape = [ 's' 'd' '^' 'o' ];
axisFontWeight= 'bold';
lableFontWeight= 'bold';
axisLableSize =12.5;
for pro = 2 : 5 ;

% Input Variables:
X_moderate , Y_moderate , X_large, Y_large, X_severe, Y_severe are desired processes variables for three seismic hazard levels.
bentType is substructure type; 1.0 for Single-Column and 2.0 for Two-Column bents.

plot(X_moderate, Y_moderate, markerShape(4) , 'LineWidth',2.2,'MarkerEdgeColor', [.2 .8 1] , 'MarkerSize',4.5);hold on; %
plot(X_large, Y_large, markerShape(4) , 'LineWidth',2.2,'MarkerEdgeColor', [1 .1 0] , 'MarkerSize',4.5);hold on; %
plot(X_severe, Y_severe, markerShape(4) , 'LineWidth',2.2,'MarkerEdgeColor', [0 0 0], 'MarkerSize',4.5);hold on;

CC = corrcoef(X_moderate, Y_moderate); cc = CC(1,2);
str_cc_moderate = [ 'CC = ' num2str(cc ) ];
CC = corrcoef(X_large, Y_large); cc = CC(1,2);
```

**Appendix D Data Processing MATLAB Scripts**
Appendix D Data Processing MATLAB Scripts

str_cc_moderate = ['CC =', num2str(cc)];
CC = corrcoef(X_severe, Y_severe); cc = CC(1,2);
str_cc_moderate = ['CC =', num2str(cc)];

AxisLimity = 1000 * bentType;
AxisLimitx = AxisLimity;

ht = text(AxisLimitx*.05, AxisLimity*.925, str_cc_moderate(1:21)); set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'FontWeight', lableFontWeight);
ht = text(AxisLimitx*.05, AxisLimity*.925, str_cc_large(1:21)); set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'FontWeight', lableFontWeight);
ht = text(AxisLimitx*.05, AxisLimity*.925, str_cc_severe(1:21)); set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'FontWeight', lableFontWeight);

str_datapoint = num2str(length(X));
ht = text(AxisLimitx*.7, AxisLimity*.175, str_datapoint); set(ht, 'rotation', 0, 'FontName', 'Arial', 'FontSize', 1.2*axisLableSize, 'FontWeight', lableFontWeight);

axis equal;
axis([0 AxisLimitx 0 AxisLimity]);

if pro==2; set(gca,'xtick',[0 AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'xticklabel', [], 'ytick',[0 AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'yticklabel', []);
else pro==3; set(gca,'xtick',[0 AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'xticklabel', [], 'ytick',[0 AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'yticklabel', []);
elseif pro==4; set(gca,'xtick',[0 AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'xticklabel', [], 'ytick',[0 AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'yticklabel', []);
elseif pro==5; set(gca,'xtick',[AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'xticklabel', [], 'ytick',[0 AxisLimitx/4 AxisLimitx/2 3*AxisLimitx/4 AxisLimitx], 'yticklabel', []);
end

set(gca, 'FontName', 'Arial', 'FontSize', 1.3*axisLableSize, 'XScale', 'linear', 'YScale', 'linear', 'FontSize', 1.3*axisLableSize);
str_f = [num2str(pro) '-Frame' ];
TxBxPosVec = positionVector(pro-1,1)+.65*W, positionVector(pro-1,2)+.015, .12, .04];
annotation('textbox', TxBxPosVec, 'String', str_f, 'FontName', 'Arial', 'FontSize', 1.5*axisLableSize, 'FontWeight', lableFontWeight);
end; % pro

legend('Moderate', 'Large', 'Severe');
ax=axes('Units', 'Normal', 'Position', [.6*margin1 .8*margin1 0.95 0.85], 'Visible', 'off', 'tag', 'sublabel');

set(get(ax,'Title'), 'Visible', 'on');
if bentType==1;
title('Single-Column Prototypes', 'FontName', 'Arial', 'FontSize', 1.75*axisLableSize, 'FontWeight', lableFontWeight);
else
title('Double-Column Prototypes', 'FontName', 'Arial', 'FontSize', 1.75*axisLableSize, 'FontWeight', lableFontWeight);
end

set(get(ax, 'XLabel'), 'Visible', 'on');
xlabel('Shear Key Force'; 'from EDA with Modification of Multiple Modes(MEDA) (kip)'); 'FontName', 'Arial', 'FontSize', 1.75*axisLableSize, 'FontWeight', lableFontWeight);
```matlab
set(get(ax,'YLabel'),'Visible','on')
ylabel('Reference Shear Key Force (kip)', 'FontName','Arial', 'FontSize', 1.75*axisLableSize, 'FontWeight', lableFontWeight);

Script-3: Bar-Charts and Analysis of Variance (ANOVA)

![Bar-Chart Image]

Note: The table below the chart is manual.

```function` BARANOVA( InputVector,G, Split , YLable,YaxisRange)
axisLableSize =20;
axisFontWeight= 'bold';%
lableFontWeight= 'bold';
h1FaceColor =[0.25 0.50 0.70];
lineWidth = 1.0;
barWidth = 1.0;

a= (size(G,2)-Split) ;
b=2; % Levels
MEAN=zeros(a,b);
STD=zeros(a,b);
MEDIAN=zeros(a,b);
MEAN1=zeros(a,b);
MEAN2=zeros(a,b);
i=1;
for A=1:a
    for B=1:b
        if Split==0;
            MEAN(A,B)= mean(InputVector(G(A)==B) ) ; STD(A,B)=std(InputVector(G(A)==B) ) ;
            MEDIAN(A,B)=median(InputVector(G(A)==B) ) ;
            MEAN1(A,B)= mean(InputVector(G(1)==1 & G(A)==B )) ;
            MEAN2(A,B)= mean(InputVector(G(1)==2 & G(A)==B )) ;
        elseif Split==1;
            MEAN(2*A-1,B)= mean(InputVector(G(1)==B & G(A+1)==1)) ; STD(2*A-1,B)=std(InputVector(G(1)==B & G(A+1)==1)) ; MEDIAN(2*A-1,B)=median(InputVector(G(1)==B & G(A+1)==1)) ;
        end
    end
end

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<thead>
<tr>
<th>ANOVA</th>
<th>Short Bridges</th>
<th>Long Bridges</th>
<th>Single-Column</th>
<th>Two-Column</th>
<th>Uniform Valley</th>
<th>N-Uniform Valley</th>
<th>Stiff Soil</th>
<th>Soft Soil</th>
<th>Low Intensity</th>
<th>High Intensity</th>
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</tr>
</tbody>
</table>
```

```
Appendix D Data Processing MATLAB Scripts

```
MEAN(2*A, B) = mean(InputVector(G{1}==B & G{A+1}==2)) ;
STD(2*A, B)=std(InputVector(G{1}==B & G{A+1}==2)) ;
MEDIAN(2*A, B)=median(InputVector(G{1}==B & G{A+1}==2)) ;
p(i) = anovan(InputVector(G{A+1}==B),G{1}(G{A+1}==B),
    'alpha',0.25, 'model', 'interaction', 'display', 'off') ;
i=i+1;
end

if Split==0;  Diff(A) = abs((MEAN(A,1) - MEAN(A,2)));
elseif Split==1; Diff(2*A-1) = abs((MEAN(2*A-1,1) - MEAN(2*A-1,2)));  Diff(2*A) =
    abs((MEAN(2*A,1) - MEAN(2*A,2)));
end
if Split == 0; X=1:a;s=1; Range= 1;
else
  Range= b; d=1.0; s=0.4*d; X=[S S+s 
    S+d S+2*d S+3*d S+4*d S+5*d] ;
end
figure('Position',[100 50 b/2*a/6*(Split+1)*950 550]);
h = bar(X(1:Range*a),MEAN, 'grouped' ) ; hold on;
set(h(1), 'facecolor', [255/255 69/255 0], 'LineWidth', lineWidth, 'EdgeColor', [1 1 1],
    'BarWidth',barWidth) % use color name
set(h(2), 'facecolor', [139/255  0 0], 'LineWidth', lineWidth, 'EdgeColor', [1 1 1],
    'BarWidth',barWidth) % use color name
set(gca,'FontName','Arial', 'FontSize',axisLableSize, 'FontWeight', axisFontWeight,
    'XTick', X, 'XTickLabel',[], 'YTick', YaxisRange );

ytick = get(gca,'YTick');
xlim = get(gca,'XLim');
for ss=1:length(YaxisRange)
    string{ss} = num2str(ytick(ss), ' %.2f');
end
set(gca,'Yticklabel',string)
Yg = repmat(ytick,2,1); Yg(:,1)=[]; Yg(:,end)=[];
Xg = repmat(xlim',1,size(ytick,2)); Xg(:,1)=[]; Xg(:,end)=[]; Xg(1,:)=Xg(1,:)+.1;
Xg(2,:)=Xg(2,:)-.1;
% Plot line data
plot(Xg,Yg,'w', 'LineWidth', 2.0)

numgroups = (Split+1)*a;
numbars = b;
groupwidth = .7*s ; %min( barWidth, numbars/(numbars+1.5));
for i = 1:numbars
    x = (1:numgroups) - groupwidth/2 + (2*i-1) * groupwidth / (2*numbars);  %
    % Aligning error bar with individual bar
    x=X(1:Range) - groupwidth/2+ (2*i-1) * groupwidth / (2*numbars);
    errorbar(x, MEAN(:,i), STD(:,i), 'ks', 'linestyle', 'none', 'lineWidth', 2.5 ,
    'markersize',8, 'color', 'k');

plot(x,MEDIAN(:,i),'d', 'LineWidth',2.5,'MarkerEdgeColor','k','MarkerFaceColor','w','MarkerSize',10)
```

520
str  =  { {'2' ; '<0.3' ; 'B' ; '<0.25' ; '<1.5' ; '<1.23'} ;  {'3' ; '<0.3' ; '<0.4'; 'C' ; '<0.25 < 0.4' ; '<1.5 < 1.30' ; '<0.4 < 0.5' ; '<1.65 < 1.7'; '<1.30 < 1.75'} ;  {'5' ; '>0.6' ; '>0.5' ; '>1.7' ; '1.75 < 2.1'} );

y=repmat(YaxisRange(1)+YaxisRange(2)*.02, 1 , length(x));

end
if b==2;
   for i=1:(Split+1)*a
      str  = [ 'diff.' num2str( Diff(i), '%.2f' ) ];
      ht =text(X(i)-.66*groupwidth,YaxisRange(1)+YaxisRange(2)*.05,str); set(ht, 'rotation', 90 , 'FontName', 'Arial', 'FontSize',0.9*axisLableSize, 'FontWeight','bold')
   end
end

% Interaction Plots
if Split==0;
   anovan(InputVector,G,'alpha',0.25, 'model','interaction');
   figure('Position',[100 100 750 600]); subplot1(2, 3, 'Min', [ .1 .3]);
   for A=2:a
      subplot1(A-1);
      plot([1 2], [MEAN1(A,1) MEAN2(A,1)], '-o', 'LineWidth',3.3,'color' ,[255/255 69/255 0], 'MarkerEdgeColor','k', 'MarkerSize',6.5); hold on;
      plot([1 2], [MEAN1(A,2) MEAN2(A,2)], '-^', 'LineWidth',3.3,'color' ,[139/255  0 0 ], 'MarkerEdgeColor','k', 'MarkerSize',6.5); hold on;
      axis ([0.5 2.5 YaxisRange(1) YaxisRange(end)])
      legend( str, 'Location','north', 'Orientation', 'Horizontal', 'FontSize',0.75 *axisLableSize,'FontWeight','bold');
      set(gca,'FontName','Arial','FontSize',0.9*axisLableSize,'FontWeight','axisFontWeight, 'XTick', [1 2], 'XTickLabel', {'Multi-F.' ; 'Cont. F.' },'YTick', YaxisRange);
      rotateXLabels( gca(), 90 )
      if A==2 || A==5;
         ytick = get(gca,'YTick');
         for ss=1:length(ytick)-1
            string(ss) = num2str(ytick(ss), '%.2f');
         end
         if A==5; string{ss+1}='';end
         set(gca,'Yticklabel',string)
      end
   end
end

end
Appendix D Data Processing MATLAB Scripts

Script-3: Plotting System State of the Bridge at the Instance of Peak Shear Key Force

% Input Variables:
DispProfile is displacement time history of all nodes along the length of the bridge, loaded from txt file.
AcclProfile is acceleration time history of all nodes along the length of the bridge loaded from txt file.
AllKEY_Th is shear key force time history Response Histories of all shear keys within the bridge loaded from txt file.
DemandDispProfile is design displacement demand of all nodes along the length of the bridge loaded from txt file.
BaseReactions is columns reaction time history in 3 directions: longitudinal, transverse and vertical loaded from txt file.
BentCapacity_All is overstrength shear capacity for all columns loaded from txt file.
Key is designated hinge number in multi-frame bridge.
bentType is substructure type; 1.0 for Single-Column and 2.0 for Two-Column bents.

valleyTag = [ 4 5 7 10 ]; % Number of valley shapes for two-, three-, four- and five-frame prototypes;
[ TH_KeyForce,TH_KeyForce_index] = max( abs( AllKEY_Th(:,key+3) ) );
deformedShape= DispProfile(TH_KeyForce_index,2:end);
Acceleration= AcclProfile(TH_KeyForce_index,2:end) / g ;

DemandDisp = DemandDispProfile(end,2:end);
if bentType == 1;
    BaseReactions= BaseReactions(TH_KeyForce_index,3:3:end);
signSet = BaseReactions ./ abs(BaseReactions);
else
    BaseReactions_double= BaseReactions(TH_KeyForce_index,3:3:end);
clear BaseReactions;
    BaseReactions(1)=BaseReactions_double(1);
    for i=2:length(BaseReactions_double)/2
        BaseReactions(i) = BaseReactions_double(2*i-2)+BaseReactions_double(2*i-1);
    end
    BaseReactions(i+1)=BaseReactions_double(end);
signSet = BaseReactions ./ abs(BaseReactions);
BentCapacity_frame = BentCapacity_All(3:4:end); % only for Single-Column
AllKeysForce= round( AllKEY_Th(TH_KeyForce_index,4+pro-2));

clear BentCapacity;
if valley ~= valleyTag(end);
j=1;
for i=1:3*pro
if i > 3*j ; j=j+1 ; end;
BentCapacity(i) = BentCapacity_frame(j);
end
BentCapacity =  signSet .* [0 BentCapacity 0 ];
else
BentCapacity =  signSet .* [0 BentCapacity_frame 0 ];
end

figure;
subplot(2,1,1);
X= 1:length(deformedShape);
[ax, h1, h2] =  plotyy(X,deformedShape, X, Acceleration, 'plot' , 'plot');hold on

set(h1, 'linestyle', '--', 'LineWidth', 1.8, 'color', 'k') % use color name
set(h2, 'linestyle', '--', 'LineWidth', 1.9, 'color', 'k') % use color name
set(ax(1), 'xlim', [ 1 X(end)] , 'ylim', [-50 50] , 'YTick',[ -50 -25 0.0 25 50] , 'FontName', 'Arial', 'FontSize',0.7*axisLableSize, 'FontWeight',axisFontWeight ,'YColor', 'k') % use color name

set(ax(2), 'xlim', [ 1 X(end)] , 'ylim', [-0.75 0.75] , 'YTick',[-0.75 0.0 0.75] , 'FontName', 'Arial', 'FontSize',0.7*axisLableSize, 'FontWeight',axisFontWeight ,'XTickLabel',[] , 'YColor', 'k') %

line([0 X(end)], [ 0 0 ], 'LineWidth', 1.25, 'color', 'b');%ylim([-1 1]); ;
xlim([ 1 X(end)]);

x_demand=[];
for node=1: length(X)
if ismember(node, HingeIndex) ==1 ;
plot(X(node), deformedShape(node), 'o', 'MarkerSize',7,'MarkerEdgeColor','b','MarkerFaceColor','w','LineWidth',1.5 )
x_demand=[x_demand node];
end
if node == HingeIndex(key) ;
plot(X(node), deformedShape(node), 'o', 'MarkerSize',7,'MarkerEdgeColor','r','MarkerFaceColor','r','LineWidth',1.5 )
end
if fix((node-1)/5) == (node-1)/5 ;
plot(X(node), deformedShape(node), 'o', 'MarkerSize',4,'MarkerEdgeColor','k','MarkerFaceColor','k','LineWidth',1.5 )
x_demand=[x_demand node];
end
end

plot(x_demand,DemandDisp, 'linestyle', ' linestyle', 'LineWidth', 1.5 , 'color' , [ 0.4 0.4 0.4] );hold on
plot(x_demand,-DemandDisp, 'linestyle', 'linestyle', 'LineWidth', 1.5 , 'color' , [ 0.4 0.4 0.4] );hold on
title([ num2str(bentType) 'Col ' num2str(pro) ' -V' num2str(valley) ' Key' num2str(key) ' motion' num2str(e) ] , 'FontName','Arial', 'FontSize',0.8*titleSize, 'FontWeight',lableFontWeight ) % 'V'
set(get(ax(1), 'Ylabel'), 'String', {'Displacement (in)'}, 'FontName','Arial', 'FontSize',axisLableSize, 'FontWeight',axisFontWeight);
set(get(ax(2), 'Ylabel'), 'String', 'Acceleration (g)' , 'FontName','Arial', 'FontSize',axisLableSize , 'FontWeight',axisFontWeight);

xlabel('DOF', 'FontName','Arial', 'FontSize',axisLableSize, 'FontWeight',lableFontWeight);

subplot(2,1,2);
x=0:length(BentCapacity)-1;
stem(x,BentCapacity, 'LineWidth', 1.8 , 'Color', 'r', 'MarkerSize', 2.75, 'MarkerFaceColor', 'k');hold on
stem(x,BaseReactions, 'LineWidth', 1.8 , 'Color', 'k', 'MarkerSize', 2.75, 'MarkerFaceColor', 'k');
set(gca, 'FontName','Arial', 'FontSize',0.7*axisLableSize, 'FontWeight',axisFontWeight, 'YColor', 'k')

str=[ 'Key Forces= ' num2str(AllKeysForce) 'kip'];
if bentType == 1;
  ylim([-2000 2000]); xlim([ 0 x(end)]);
  h =text(0.2,-1700,str); set(h, 'rotation', 0 , 'FontName','Arial', 'FontSize',0.75*axisLableSize,'FontWeight',lableFontWeight)
else
  ylim([-2500 2500]); xlim([ 0 x(end)]);
  h =text(0.2,-2200,str); set(h, 'rotation', 0 , 'FontName','Arial', 'FontSize',0.75*axisLableSize,'FontWeight',lableFontWeight)
end

ylabel('Base Reaction(kip)', 'FontName','Arial', 'FontSize',axisLableSize, 'FontWeight',lableFontWeight);

xlabel('Bent Number', 'FontName','Arial', 'FontSize',axisLableSize, 'FontWeight',lableFontWeight);