Anisotropic Earth Structure Revealed by Simulated Annealing Inversion of Seismic Waves

Hao Xie
haoxie211@gmail.com

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Anisotropic Earth Structure Revealed by Simulated Annealing Inversion of Seismic Waves

Hao Xie, PhD

University of Connecticut, 2014

In a wide range of geophysical problems, the mismatch between theoretical prediction and observation may be caused by lacking the consideration of medium anisotropy. Ever increasing accuracy and scale of geophysical field observations require inclusion of anisotropy now. This PhD dissertation focuses on extracting anisotropic earth structure using simulated annealing (SA) inversion of seismic waves.

In the first part of this dissertation, the nature of seismic anisotropy, the dispersion properties of surface waves with the consideration of material anisotropy, and the stochastic inversion using SA are reviewed with certain algorithm development. In Chapter 2 the generalized Hooke’s law is presented with all information of anisotropy contained in independent elastic components of the stiffness matrix. The stiffness matrix in the medium with different symmetric systems is discussed, especially in the vertical transverse isotropy (VTI) medium. In Chapter 3 the dispersion equations of Rayleigh
wave and Love wave are derived for the VTI medium. A graphic-based method is proposed to extract dispersion curves. The impact of anisotropy on dispersion curves is discussed, showing that inclusion of multiple modes is critical to the inversion of anisotropy. Chapter 4 studies the stochastic inversion using SA. Considering the initial-model independence and different parameter sensitivities, the very fast simulated annealing (VFSA) is chosen to invert the velocity structure and the anisotropy structure simultaneously. The synthetic example shows the feasibility and effectiveness of this approach.

Four geophysical applications using VFSA inversion compose the second part of this dissertation. As the application to petroleum exploration, Chapter 5 describes the material anisotropy estimation at well location using joint-tomography inversion and VFSA inversion. As the application to the near-surface structure characterization, Chapter 6 studies the velocity and anisotropy structure at the site of Rentschler field, CT with VFSA inversion of multi-mode Love wave extracted from the active source. Chapter 7 studies the near-surface structure at Haddam Meadows, CT with VFSA inversion of multi-mode Rayleigh wave extracted from ambient noise. As the last application, Chapter 8 discusses the mechanical properties of polar firn in Greenland ice sheet from a seismic refraction survey and the inversion of Rayleigh wave dispersions.
Anisotropic Earth Structure Revealed by Simulated Annealing

Inversion of Seismic Waves

Hao Xie

B.S., Southeast University, 2007

A Dissertation

Submitted in Partial Fulfillment of the

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Hao Xie

2014
Anisotropic Earth Structure Revealed by Simulated Annealing

Inversion of Seismic Waves

Presented by

Hao Xie, B.S.

Major Advisor: 

Dr. Lanbo Liu

Associate Advisor:

Dr. Benjamin E. Barrowes

Associate Advisor:

Dr. John. W. Lane

Associate Advisor:

Dr. Amvrossios C. Bagtzoglou

Associate Advisor:

Dr. Vernon F. Cormier

University of Connecticut

2014
Dedicated to my family.
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List of Abbreviations

BA  Boltzmann Annealing
FSA  Fast Simulated Annealing
HTI  Horizontal Transverse Anisotropy
ISO  Isotropy
M0  the Fundamental Mode
M1  the First-higher Mode
M2  the Second-higher Mode
M3  the Third-higher Mode
M4  the Forth-higher Mode
M5  the Fifth-higher Mode
MASW  Multichannel Analysis Surface Wave
PDF  Probability Density Function
TI  Transverse Isotropy
TTI  Tilted Transverse Isotropy
VFSA  Very Fast Simulated Annealing
VTI  Vertical Transverse Isotropy
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Chapter 1. Introduction

Seismic anisotropy estimation has received much attention in recent years, since the neglect of anisotropy leads to the mismatch between theoretical prediction and observation (Gassmann, 1951; White and Sengbush, 1953). The increasing accuracy and scale of field observations require for consideration of anisotropy (Layet et al, 1961). Many applications benefit from the inclusion of anisotropy. For example, in the petroleum exploration, the anisotropic representation of earth parameters makes the depth imaging more accurate (Bear et al., 2005; Schleicher et al., 2010); in the earthquake hazard analysis, the consideration of anisotropy is critical to design large-scale buildings with complicated structure and high sensitivities to directions.

Several approaches have been developed to estimate the anisotropy parameters in seismic exploration. One category of those approaches estimates each anisotropy parameter separately using the information from surface seismic data (Bear et al., 2005). The other category of those approaches estimates multi-parameters simultaneously using joint tomography (Zhou, 2011). In near-surface geophysics, Liu and Xie (2014) estimate the anisotropy parameters using dispersion properties of surface waves.

Surface waves are dispersive, i.e., different frequency components of surface waves propagate with different velocities. Dispersion occurs due to depth-dependent penetration of surface waves at different wavelengths. Dispersion equations (the relationship between
frequencies and phase velocities) in multilayered isotropic media have been provided by Haskell (1953). Anderson (1961) and Harkrider (1962) extended the approach to multilayered VTI (one category of anisotropy) media. Unfortunately, the deviation in their papers and thesis contained several mistakes. As verification to their work and the correctness to mistakes, this thesis will re-derive the dispersion equation of Rayleigh wave and Love wave in a multilayered VTI medium.

Solving dispersion equations is a sophisticated task because the dispersion equation is an implicit function of two variables, frequency and phase velocity, for a given model. Previous study (Xia et al., 2003; Ke et al., 2011) solves dispersion equations using root-search algorithms. This thesis will develop an alternative approach directly from the graph. The advantages of this graphic-based approach are its simplicity, low possibility to introduce numerical errors for higher modes, and high computational efficiency. As shown in synthetic examples, anisotropy has more effect on higher modes, which implies that multi-mode surface waves are critical to estimate the anisotropy.

Simulated Annealing (SA) is a powerful stochastic global-search algorithm, especially for highly nonlinear problems with multiple local minima. Boltzmann Annealing (BA), fast SA (FSA) and very fast SA (VFSA) are three types of annealing schedules. In this thesis, VFSA is taken as the inversion algorithm since it has two more advantages over other two annealing schedules: independence to the initial model; feasibility to the case of parameters with different range and sensitivities. The synthetic
example indicates that inversion using VFSA can estimate the velocity structure and anisotropy structure simultaneously.

Four geophysical applications using VFSA inversion compose the second part of this thesis. As the application to petroleum exploration, Chapter 5 describes the material anisotropy estimation at well location using joint-tomography inversion and VFSA inversion. As the application to the near-surface structure characterization, Chapter 6 studies the velocity and anisotropy structure at the site of Rentschler field, CT with VFSA inversion of multi-mode Love wave extracted from the active source. Chapter 7 studies the near-surface structure at Haddam Meadows, CT with VFSA inversion of multi-mode Rayleigh wave extracted from ambient noise. As the last application, Chapter 8 discusses the mechanical properties of polar firn in Greenland ice sheet from a seismic refraction survey and the inversion of Rayleigh wave dispersions. The comparison with drilling logs or other methods verifies that VFSA inversion of multi-mode surface waves can characterize the near-surface structure.
Chapter 2. Seismic Anisotropy

In general cases, materials always possess certain degree of anisotropy. However, most earth materials can be considered as isotropic medium as the first degree approximation. This assumption is made for mathematical convenience. And we can obtain some reasonable approximate results from this assumption. However, in some geophysical applications, the neglect of anisotropy is somehow an oversimplification. The mismatch between the theory and the observation may be caused by medium anisotropy. Besides, ever increasing accuracy and scale of field observations require for inclusion of the consideration of anisotropy.

Gassmann (1951) theoretically proved the existence of anisotropy in granular material. The corresponding term for this type of anisotropy is intrinsic anisotropy. On the other hand, White and Sengbush (1953) observed the presence of seismic anisotropy from finely layered medium, such as shale. This type of anisotropy can be considered as extrinsic anisotropy.

This chapter begins with a discussion of the constitutive relationship for the full anisotropic elastic medium with 81 elements in the stiffness matrix. Several symmetric systems of anisotropy are discussed, and the simplified stiffness matrix is shown respectively. For example, the stiffness matrix in the generalized Hooke’s law has at most 21 independent elements for anisotropic media, but only 2 for isotropic media. Among all
anisotropy systems, a special case, transverse isotropy (TI) is widely adopted in many physical and engineering applications, since the TI approximation not only introduces mathematical simplification but also is a more accuracy approximation to the reality in many applications. Next, the measurement of anisotropy is discussed for both vertical transverse isotropy (VTI) and horizontal transverse isotropy (HTI). Several definitions for anisotropy parameters (Anderson, 1962; Thomsen, 1986) are introduced and compared in the last section.

### 2.1 The generalized Hooke’s law

The generalized Hooke’s law states that stress $\sigma$ is linearly proportional to strain $\varepsilon$. If we use Cartesian coordinates and Einstein summation convention, then the generalized Hooke’s law can be expressed by

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} \tag{2.1}$$

Each index runs from 1 to 3, so the stress and strain tensors have 9 components respectively and the stiffness tensor (the elements are $c_{ijkl}$) has 81 components.
However, not all 81 components are independent; the symmetry of stresses and strains implies that

$$c_{ijkl} = c_{jik} = c_{ijk} = c_{jlk}$$  \hspace{1cm} (2.3)$$

These symmetries lead the number of independent components to 36.

Due to the aforementioned intrinsic symmetry, the generalized Hooke’s law can be rewritten as a 6-by-6 stiffness matrix in terms of 6 stress/strain component:

$$\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{21} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
c_{1111} & c_{1112} & c_{1113} & c_{1122} & c_{1123} & c_{1133} & c_{1112} \\
& c_{1121} & c_{1212} & c_{1213} & c_{1222} & c_{1223} & c_{1212} \\
& & c_{1211} & c_{2122} & c_{2123} & c_{2112} & c_{2112} \\
& & & c_{2121} & c_{2222} & c_{2223} & c_{2212} \\
& & & & c_{2211} & c_{3322} & c_{3323} \\
& & & & & c_{3311} & c_{3312}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{23} \\
2\varepsilon_{13} \\
2\varepsilon_{12}
\end{bmatrix}$$  \hspace{1cm} (2.4)$$

Furthermore, the existence of unique strain energy potential requires that

$$c_{ijkl} = c_{klij}$$  \hspace{1cm} (2.5)$$
This relationship reduces the number of independent components further to 21. This is the maximum number of elastic constants that any arbitrarily anisotropic medium may have.

Applying the Voigt notation, the four subscripts of the stiffness tensor can be reduced to two. Each pair of indices $ij (kl)$ is replaced by one index $I(J)$.

<table>
<thead>
<tr>
<th>$ij(kl)$</th>
<th>$I(J)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>33</td>
<td>3</td>
</tr>
<tr>
<td>23,32</td>
<td>4</td>
</tr>
<tr>
<td>13,31</td>
<td>5</td>
</tr>
<tr>
<td>12,21</td>
<td>6</td>
</tr>
</tbody>
</table>

Or in the Cartesian coordinate, it is equivalent to have

$$\sigma_2 = \sigma_{yy}$$

$$\sigma_4 = \sigma_{yz}$$

$$\sigma_5 = \sigma_{xz}$$

$$\sigma_6 = \sigma_{xy}$$

Then the generalized Hooke’s law can be rewritten as

$$\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\
c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
2\varepsilon_4 \\
2\varepsilon_5 \\
2\varepsilon_6 \\
\end{bmatrix}$$

(2.7)

Only the 21 components in the upper triangular region are independent.

### 2.2 Several symmetric systems for seismic anisotropy

As mentioned above, 21 is the maximum number of elastic constants that any medium may have. This can be considered as the ultimate extreme case, with the lowest
symmetry, named as triclinic symmetry. Additional symmetry conditions will introduce further reduction of the number. The minimum is 2 for the isotropic case, with the highest symmetry. The following discussion will show several symmetric systems for anisotropy.

If the medium has one plane of symmetry, named as monoclinic system, some components are zero and only 13 independent components left. Here, if we take \( z=0 \) as the plane of symmetry, then

\[
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\
c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\
c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\
0 & 0 & 0 & c_{44} & c_{45} & 0 \\
0 & 0 & 0 & c_{54} & c_{55} & 0 \\
c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66}
\end{bmatrix}
\]

(2.8)

If the medium has three mutually perpendicular symmetry planes, named as orthotropic system, it has 9 independent components. With \( x-, y- \) and \( z- \) axis perpendicular to the symmetry plane, the elastic stiffness matrix has the form of

\[
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 \\
c_{12} & c_{22} & c_{23} & 0 & 0 \\
c_{13} & c_{23} & c_{33} & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & c_{54} & c_{55} \\
0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}
\]

(2.9)

Next, a simplification can be made when the medium has one plane of isotropy, named as hexagonal. In another word, the medium behaves in isotropic manner within that plane. This is also called transversely isotropic (TI) medium. For this case, the number of independent components reduces to 5.
For a TI medium, we can divide it into three sub-types depending on the direction of symmetry axis. Figure 2.1 shows an illustration of three types of TI media.

![Illustration of three types of TI media.](image)

**Figure 2.1.** Illustration of three types of TI media.

1) TI medium with a vertical symmetry axis (z-axis), named as vertical TI (VTI). It requires:

\[
c_{23} = c_{13}, \quad c_{55} = c_{44}, \quad c_{12} = c_{11} - 2c_{66} \tag{2.10}
\]

Then the stiffness matrix has the form with 5 independent components:

\[
C^\text{VTI} = \begin{bmatrix}
c_{11} & c_{11} - 2c_{66} & c_{13} & 0 & 0 & 0 \\
c_{11} - 2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\
c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix} \tag{2.11}
\]

This model is commonly used to describe anisotropy due to thin horizontal layers such as sediments and sedimentary rocks. VTI is a very important form of anisotropy as shale cover over 70 percent of hydrocarbon traps and most of them show VTI characteristics.
2) TI medium with a horizontal symmetry axis (x- or y-axis), named as horizontal TI (HTI). The stiffness matrix has the form with 5 independent components:

\[
C^{HTI} = \begin{bmatrix}
    c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\
    c_{12} & c_{22} & c_{22} - 2c_{44} & 0 & 0 & 0 \\
    c_{12} & c_{22} - 2c_{44} & c_{22} & 0 & 0 & 0 \\
    0 & 0 & 0 & c_{44} & 0 & 0 \\
    0 & 0 & 0 & 0 & c_{55} & 0 \\
    0 & 0 & 0 & 0 & 0 & c_{55}
\end{bmatrix}
\] (2.12)

HTI is commonly used to model parallel vertical fractures.

3) TI medium with a tilted symmetry axis, named as tilted TI (TTI). Dipping shale layers would require a TTI model. Sure, TTI model also has 5 independent elastic constants.

With more practical emphasis, this thesis emphasizes more on VTI case in the following chapters. For comparison, the stiffness matrix of the isotropic medium in terms of TI medium is show below:

\[
C^{ISO} = \begin{bmatrix}
    c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\
    c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\
    c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\
    0 & 0 & 0 & \frac{c_{11} - c_{12}}{2} & 0 & 0 \\
    0 & 0 & 0 & 0 & \frac{c_{11} - c_{12}}{2} & 0 \\
    0 & 0 & 0 & 0 & 0 & \frac{c_{11} - c_{12}}{2}
\end{bmatrix}
\] (2.13)
With the definition of the Lame’s constant $\lambda$ and $\mu$, we can express (2.13) as:

$$C^{ISO} = 
\begin{bmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}$$

(2.14)

2.3 Measurement of seismic anisotropy

As shown in (2.11) and (2.12), TI medium has five independent elastic constants in the stiffness matrix. These five components are the essential basis to measure anisotropy. We may define different anisotropy parameters for convenience in some specific applications. For example, Thomsen (1986) defined a set of parameters ($\varepsilon$, $\gamma$, and $\delta$) for VTI medium. All anisotropy parameters should be a combination of those independent elastic constants; so the most fundamental and critical measurement for anisotropy is the independent elastic constants. That is obviously true because anisotropy is a kind of medium property. At first, we will show the direct relationship between elastic constants and measurable anisotropy parameters in reality.

The elasto-dynamic equation can be expressed as

$$\rho\ddot{u}_i = \sigma_{ij,i}$$

(2.15)
where \( u \) is the particle displacement and \( \rho \) is the density. \( \sigma_{ij} \) is the derivative of the stress component in \( j \)-direction and the Einstein summation convention is understood.

Let us consider VTI medium first. Combine (2.1) and (2.11), we can rewrite the generalized Hook’s law in VTI medium as:

\[
    \sigma_1 = c_{11} \varepsilon_1 + (c_{11} - 2c_{66}) \varepsilon_2 + c_{13} \varepsilon_3 \\
    \sigma_2 = (c_{11} - 2c_{66}) \varepsilon_1 + c_{11} \varepsilon_2 + c_{13} \varepsilon_3 \\
    \sigma_3 = c_{13} \varepsilon_1 + c_{13} \varepsilon_2 + c_{33} \varepsilon_3 \\
    \sigma_4 = c_{44} \varepsilon_4 \\
    \sigma_5 = c_{44} \varepsilon_5 \\
    \sigma_6 = c_{66} \varepsilon_6
\] (2.16)

Substitute (2.16) and the definition of strain \( \varepsilon \):

\[
    \varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})
\] (2.17)

into the elasto-dynamic equation (2.15), we will get

\[
    \rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left( c_{11} \frac{\partial u}{\partial x} + c_{12} \frac{\partial v}{\partial y} + c_{13} \frac{\partial w}{\partial z} \right) + \frac{\partial}{\partial y} \left[ \frac{1}{2} (c_{11} - c_{12}) (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \right] + \frac{\partial}{\partial z} \left[ c_{44} (\frac{\partial v}{\partial x} + \frac{\partial w}{\partial y}) \right] \\
    \rho \frac{\partial^2 v}{\partial t^2} = \frac{\partial}{\partial x} \left[ \frac{1}{2} (c_{11} - c_{12}) (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \right] + \frac{\partial}{\partial y} \left[ c_{44} (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \right] + \frac{\partial}{\partial z} \left[ c_{44} (\frac{\partial w}{\partial z} + \frac{\partial v}{\partial y}) \right] \\
    \rho \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x} \left[ c_{44} (\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) \right] + \frac{\partial}{\partial y} \left[ c_{44} (\frac{\partial w}{\partial z} + \frac{\partial v}{\partial y}) \right] + \frac{\partial}{\partial z} \left[ c_{33} (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) \right] + c_{33} \frac{\partial^2 w}{\partial z^2}
\] (2.18)

where \( u, v, w \) are the displacements in \( x \)-, \( y \)-, \( z \)-directions, respectively.

For a plane wave propagated in a direction specified by direction cosines \( (l, m, n) \) we take the generalized solution for the wave equation as:

\[
    (u,v,w) = (U,V,W)e^{i\omega t}e^{-ik(x+ny+nz)}
\] (2.19)
Substitution this general solution into the wave equations (2.18) and setting the determinant of the coefficients equal to zero, we can obtain the velocity equations. We are considering three special cases now.

1) Propagation in \( z \) axis, in which the direction cosines are \( l=0, m=0, n=1 \). Then
\[ c^2 = \frac{c_{33}}{\rho} \quad \text{and} \quad c^2 = \frac{c_{44}}{\rho} \]
are solutions. They correspond to velocities of the PV and SV waves, respectively. In this case, the SV and SH waves cannot be distinguished; so we can rewrite these two velocities as follows, where the subscript \( v \) means wave propagating in vertical direction.:
\[ V_{PV}^2 = \frac{c_{33}}{\rho} \]
\[ V_{SV}^2 \ (\text{or} V_{SH}^2) = \frac{c_{44}}{\rho} \] (2.20)

2) Propagation in \( x \)- or \( y \)- or any other direction perpendicular to the \( z \) axis, in which the direction cosines are \( n=0 \). The solutions are:
\[ V_{PH}^2 = \frac{c_{11}}{\rho} \]
\[ V_{SV}^2 = \frac{c_{44}}{\rho} \]
\[ V_{SH}^2 = \frac{c_{11} - c_{12}}{2\rho} = \frac{c_{66}}{\rho} \] (2.21)

Therefore, velocity measurement along these two directions will determine four of the five elastic constants \( (c_{11}, c_{33}, c_{44}, \text{and} c_{66}) \). To determine the fifth we need another velocity measurement at some intermediate angle.
3) Propagation at $45^\circ$ to the $z$ axis, in which the direction cosines are $l = n = 1/\sqrt{2}$, $m=0$. The solutions are

$$c_{13} = -c_{44} + \sqrt{4\rho^2V_{PT}^4 - 2\rho V_{PT}^2 (c_{11} + c_{33} + 2c_{44}) + (c_{11} + c_{44})(c_{33} + c_{44})}$$

(2.22)

where $V_{PT}$ is the P-wave velocity at $45^\circ$ to the vertical direction. This is also shown by Ke et al. (2011).

In summary, for VTI medium, all five elastic constants can be measured from five velocities. Rewrite it as

$$\begin{align*}
c_{11} &= \rho V_{PH}^2 \\
c_{33} &= \rho V_{PV}^2 \\
c_{44} &= \rho V_{SV}^2 \\
c_{66} &= \rho V_{SH}^2 \\
c_{13} &= -c_{44} + \sqrt{4\rho^2V_{PT}^4 - 2\rho V_{PT}^2 (c_{11} + c_{33} + 2c_{44}) + (c_{11} + c_{44})(c_{33} + c_{44})}
\end{align*}$$

(2.23)

This is the directive relationship between elastic constants and measurable anisotropy parameters (velocities) in reality.

Thomsen (1986) suggested the following convenient notation for calculations of the weakly anisotropic medium. In combination with Anderson’s (1961) derivation, we have the following relationships:

$$\begin{align*}
\varepsilon &\equiv \frac{c_{11} - c_{33}}{2c_{33}} = \frac{V_{PH}^2 - V_{PV}^2}{2V_{PV}^2} \\
\gamma &\equiv \frac{c_{66} - c_{44}}{2c_{44}} = \frac{V_{SH}^2 - V_{SV}^2}{2V_{SV}^2} \\
\delta &\equiv \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})} \text{ (weakly anisotropy)}
\end{align*}$$

(2.24)
(Note: $\varepsilon$ and $\gamma$ are the same for both generalized anisotropic medium and weakly anisotropic medium. Formula of $\delta$ in weakly anisotropic medium is a simplification expression of that in generalized anisotropic medium.)

These three anisotropy parameters are the combinations of elastic constants. Their properties are:

a) They are dimensionless;

b) They degenerate to zero for the isotropic case;

c) Different anisotropy parameters rely on corresponding types of surface waves;

For example, the anisotropic parameter $\gamma$ relies on Love wave since it is defined by $V_{SH}$ and $V_{SV}$; and $\varepsilon$ and $\delta$ rely on Rayleigh wave since $V_P$ is included in their definitions. As the result, $\gamma$ can and only can be determined by Love wave, while the estimation of $\varepsilon$ and $\delta$ only depends on Rayleigh wave.

d) The velocities of P, SH and SV phases with directional dependence can be expressed in a simple format using these three anisotropy parameters.

For example, the velocities for these three phases with directional dependence derived by Daley and Hron (1977) are

\[ \rho V_P^2(\theta) = \frac{1}{2} \left[ C_{33} + C_{44} + (C_{11} - C_{33})\sin^2\theta + D(\theta) \right] \]
\[ \rho V_{SV}^2(\theta) = \frac{1}{2} \left[ C_{33} + C_{44} + (C_{11} - C_{33})\sin^2\theta - D(\theta) \right] \]  \hspace{1cm} (2.25)
\[ \rho V_{SH}^2(\theta) = C_{66}\sin^2\theta + C_{44}\cos^2\theta \]
where the phase angle \( \theta \) is the angle between the wave-front normal and the unique (vertical) axis. \( D(\theta) \) is the compact notation for the quadratic combination:

\[
D(\theta) = \left\{ \left( C_{33} - C_{44} \right)^2 + 2 \left[ (C_{13} + C_{44})^2 - (C_{33} - C_{44})(C_{11} + 2C_{33} - 2C_{44}) \right] \sin^2 \theta \right\}^{1/2}
\]

(2.26)

With the definition of weakly anisotropy (2.24), the three velocities with directional dependence can be simply expressed by

\[
V_p(\theta) \approx \alpha (1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta)
\]

\[
V_{sv}(\theta) \approx \beta (1 + \frac{\alpha^2}{\beta^2} (\epsilon - \delta) \sin^2 \theta \cos^2 \theta)
\]

(2.27)

\[
V_{sh}(\theta) \approx \beta (1 + \gamma \sin^2 \theta)
\]

### 2.4 Summary

In general cases, materials always have anisotropy. The neglect of anisotropy is somehow an oversimplification. The mismatch between the theory and the observation may be caused by medium anisotropy.

The information of anisotropy is totally contained in the stiffness matrix of media. The number of independent components in the stiffness matrix depends on how symmetry the anisotropic medium is. For example, 21 is the maximum number of elastic constants that the any medium may have. This can be considered as the ultimate extreme.
case, with the lowest symmetry, named as triclinic symmetry. The minimum is 2 for the isotropic case, with the highest symmetry.

Among all anisotropy systems, a special case, transverse isotropy (TI) is widely adopted in many physical and engineering applications, since the TI approximation not only introduces mathematical simplification but also is a more accuracy approximation to the reality in many applications. There are five independent elastic constants for VTI medium, which are the essential basis to measure the anisotropy.

The most direct measurement of anisotropy is the velocity. According to the corresponding relationship between elastic constants and velocities, we can obtain the anisotropy information after velocity measurement or estimation. Thomsen (1986) defined a set of anisotropy parameters for convenience. It is widely used in geophysical applications.

In recent years, several approaches have been developed to estimate the anisotropy parameters in seismic exploration. One category of those approaches estimates each anisotropy parameter separately using the information from surface seismic data (Bear, et al., 2005). The other category of those approaches estimates multi-parameters simultaneously using joint tomography (Zhou, 2011). In near-surface geophysics, Liu and Xie (2014) estimate the anisotropy parameters using dispersion properties of surface waves. Chapter 3 will derive the dispersion equation of surface waves in VTI medium and propose a graphic-based method to obtain dispersion curves.
Chapter 3. Surface Wave Dispersion

Seismic waves consist of body waves propagating in all directions within the media and surface waves travelling along the interfaces and surface. A typical seismic profile recorded by a geophone array is shown as Figure 3.1. In traditional seismic exploration, the surface waves are the most troublesome noise to get rid of during conventional seismic data processing for reflection and refraction surveys. However, surface waves contain rich information about near-surface structure characteristics since surface waves carry significant part of the wave propagation energy and provide formation structure characterization with high signal-to-noise ratio.

Surface waves are dispersive. Dispersion occurs because of depth-dependent penetration of surface waves at different wavelengths. Shorter wavelengths penetrate shallower depth of the medium, and their propagation velocities (usually slower velocities) are determined by elastic properties at shallower depth. On the other side, longer wavelengths penetrate greater depth of the medium, and their propagation velocities (usually faster velocities) are determined by elastic properties at greater depth. The generation of dispersion of surface waves is illustrated in Figure 3.2. As the result, it is feasible to characterize near-surface velocity structure using the dispersion properties of surface waves.
Figure 3.1. A typical seismic record from an active seismic survey at Rentschler, CT.

Figure 3.2. A sketch of generation of dispersion of surfaces waves (Park, 2014).
In this thesis, the dispersion properties (including higher modes) of surface waves are used to characterize near-surface anisotropic structure. There are couples of reasons to choose surface waves for investigating anisotropy. First, surface waves are an effective and non-destructive tool to estimate near-surface velocity structure. Second, dispersion curves of surface waves can be obtained from both active and passive sources (the ambient noise). Independence of active sources is the main advantage of the surface wave investigation using passive source which will be described in Chapter 7. Third, anisotropy parameter estimation has a great flexibility, since different anisotropy parameters rely on corresponding types of surface waves. For example, the anisotropic parameter $\gamma$ relies on Love wave; and $\varepsilon$ and $\delta$ rely on Rayleigh wave. In another word, $\gamma$ can and only can be determined by Love wave, while the estimation of $\varepsilon$ and $\delta$ only depends on Rayleigh wave.

Dispersion of surface waves in multilayered isotropic medium has been investigated in previous studies (e.g., Haskell, 1953; Press et al., 1961). Since the need for consideration of anisotropy as described in Chapter 2, the computing procedure has been extended by Anderson (1961) in a multilayered VTI medium, which indicates that each layer is transversely isotropic with an axis of symmetry perpendicular to horizontal layers. Then Anderson (1962) and Harkrider (1962) re-arranged the matrix elements in the dispersion equations for Love wave and Rayleigh wave separately in a manner convenient for computation. Unfortunately, the deviation in their papers and thesis
contained several mistakes. As verification to their work, in this chapter I will first re-derive the dispersion equation of Rayleigh wave and Love wave in a multilayered VTI medium.

Dispersion curves can be obtained by solving the dispersion equations. However, solving the dispersion equations is a sophisticated task because the dispersion equation is an implicit function of two variables, frequency and phase velocity, for a given model. Traditional numerical methods based on root-searching algorithms (Xia et al., 2003; Ke et al., 2011) are time-consuming and easy to introduce numerical errors for higher modes. In this chapter, I will propose a graphic-based method to acquire the dispersion curves. The advantages of this approach are its simplicity, low possibility to introduce numerical errors for higher modes, and high computational efficiency.

At the end of this chapter, the dispersion curves in isotropic medium and anisotropic medium are compared to illustrate how anisotropy affects the features of the dispersion curves.

3.1 Dispersion equation for Rayleigh wave in multilayered VTI medium

Consider plane waves of angular frequency $\omega$ propagate in the positive $x$ direction with phase velocity, $c$, in a semi-infinite medium composed of $n$ parallel homogeneous transversely isotropic (i.e., the vertically transverse isotropic, or VTI) layers. The $n^{th}$ layer
is an anisotropic half-space. The geometry of such a model is illustrated in Figure 3.3. Associated with the $m^{th}$ layer are its density, $\rho_m$, thickness, $d_m$, and five independent elastic constants as discussed in Chapter 2.

Assume the particle motion of Rayleigh wave is in $x$ and $z$ direction, no $y$-component exists ($v=0$ and $\frac{\partial}{\partial y} = 0$). Rewrite the equation of motion (2.18) for Rayleigh waves in the $m^{th}$ layer is

$$
\rho \frac{\partial^2 u_m}{\partial t^2} = \frac{\partial}{\partial x} (c_{11m} \frac{\partial u_m}{\partial x} + c_{13m} \frac{\partial w_m}{\partial z}) + \frac{\partial}{\partial z} [c_{44m} (\frac{\partial w_m}{\partial x} + \frac{\partial u_m}{\partial z})]
$$

$$
\rho \frac{\partial^2 w_m}{\partial t^2} = \frac{\partial}{\partial x} [c_{44m} (\frac{\partial w_m}{\partial x} + \frac{\partial u_m}{\partial z})] + \frac{\partial}{\partial z} (c_{13m} \frac{\partial u_m}{\partial x} + c_{33m} \frac{\partial^2 w_m}{\partial z^2})
$$

(3.1)
where \( u \) and \( w \) are displacements in the \( x \) and \( y \) direction, respectively. \( c_{11m}, c_{13m}, c_{33m}, \) and \( c_{44m} \) are the four independent elastic constants for VTI medium. The elastic constant \( c_{66m} \) will not affect Rayleigh wave so that it does not appear in Equation (3.1). That is easy to explain. Rayleigh wave is P-SV motion, but \( c_{66} \) has a direct relationship with SH motion, as shown in Equation (2.23).

For Rayleigh wave we seek the solution in the form of:

\[
\begin{align*}
  u &= U(z)e^{i(\omega t-kx)} \\
  w &= W(z)e^{i(\omega t-kx)}
\end{align*}
\]

Substitution into the equation of motion (3.1) yields,

\[
\begin{align*}
  -\rho \omega^2 U(z) &= -c_{11} k^2 U(z) - ik(c_{13} + c_{44}) W'(z) + c_{44} U''(z) \\
  -\rho \omega^2 W(z) &= c_{33} W''(z) - ik(c_{13} + c_{44}) U'(z) - k^2 c_{44} W(z)
\end{align*}
\]

where the primes denote \( \partial / \partial z \). And the solutions of these two equations are in the form of:

\[
\begin{align*}
  U(z) &= U_1 \sinh(v_1 z) + U_2 \cosh(v_1 z) + U_3 \sinh(v_2 z) + U_4 \cosh(v_2 z) \\
  W(z) &= i\gamma_1 U_1 \cosh(v_1 z) + i\gamma_1 U_2 \sinh(v_1 z) + i\gamma_2 U_3 \cosh(v_2 z) + i\gamma_2 U_4 \sinh(v_2 z)
\end{align*}
\]

Substitute (3.4) into (3.2) we can calculate the four quantities of the motion stress vector at the \( m^{th} \) layer:
\[ \tilde{u}_m = \frac{i k}{c} \sinh(v_{1m} z) U_{1m} + i k \cosh(v_{1m} z) U_{2m} \]
\[ + i k \sinh(v_{2m} z) U_{3m} + i k \cosh(v_{2m} z) U_{4m} \]
\[ \tilde{w}_m = -k \gamma_{1m} \cosh(v_{1m} z) U_{1m} - k \gamma_{1m} \sinh(v_{1m} z) U_{2m} \]
\[ - k \gamma_{2m} \cosh(v_{2m} z) U_{3m} - k \gamma_{2m} \sinh(v_{2m} z) U_{4m} \]
\[ \sigma_m = i (\gamma_{1m} v_{1m} C_m - F_m k) \sinh(v_{1m} z) U_{1m} + i (\gamma_{1m} v_{1m} C_m - F_m k) \cosh(v_{1m} z) U_{2m} \]
\[ + i (\gamma_{2m} v_{2m} C_m - F_m k) \sinh(v_{2m} z) U_{3m} + i (\gamma_{2m} v_{2m} C_m - F_m k) \cosh(v_{2m} z) U_{4m} \]
\[ \tau_m = L_m (v_{1m} + k \gamma_{1m}) \cosh(v_{1m} z) U_{1m} + L_m (v_{1m} + k \gamma_{1m}) \sinh(v_{1m} z) U_{2m} \]
\[ + L_m (v_{2m} + k \gamma_{2m}) \cosh(v_{2m} z) U_{3m} + L_m (v_{2m} + k \gamma_{2m}) \sinh(v_{2m} z) U_{4m} \]

where \( C_m = c_{i3m}, L_m = c_{44m} \) and \( F_m = c_{13m} \) are the elastic constants of the \( m^{th} \) layer.

The boundary conditions at each solid-solid interface are that these four quantities need to be continuous when crossing the interface.

As the general case, consider the \( m^{th} \) layer. First, let us place \( z=0 \) at the \((m-1)^{th}\) interface (the top interface of the \( m^{th} \) layer), the linear relationship between the motion-stress vector (contains four quantities) at the \((m-1)^{th}\) layer and the displacement coefficients can be expressed as

\[ \begin{pmatrix} \frac{\tilde{u}_{m-1}}{c}, \frac{\tilde{w}_{m-1}}{c}, \sigma_{m-1}, \tau_{m-1} \end{pmatrix} = E_m \begin{pmatrix} U_{1m}, U_{2m}, U_{3m}, U_{4m} \end{pmatrix}^T \]
\[ E_m = \begin{bmatrix} 0 & i k & 0 & i k \\ -k \gamma_{1m} & 0 & -k \gamma_{2m} & 0 \\ 0 & i (\gamma_{1m} v_{1m} C_m - F_m k) & 0 & i (\gamma_{2m} v_{2m} C_m - F_m k) \\ L_m (v_{1m} + k \gamma_{1m}) & 0 & L_m (v_{2m} + k \gamma_{2m}) & 0 \end{bmatrix} \]  

(3.6)

Setting \( z=d_m \) we can write the relationship between the motion-stress vector at the \( m^{th} \) layer and the displacement coefficients,

\[ \begin{pmatrix} \frac{\tilde{u}_m}{c}, \frac{\tilde{w}_m}{c}, \sigma_m, \tau_m \end{pmatrix} = D_m \begin{pmatrix} U_{1m}, U_{2m}, U_{3m}, U_{4m} \end{pmatrix}^T \]
\[ D_m = \begin{pmatrix} 0 & i k & 0 & i k \\ -k \gamma_{1m} & 0 & -k \gamma_{2m} & 0 \\ 0 & i (\gamma_{1m} v_{1m} C_m - F_m k) & 0 & i (\gamma_{2m} v_{2m} C_m - F_m k) \\ L_m (v_{1m} + k \gamma_{1m}) & 0 & L_m (v_{2m} + k \gamma_{2m}) & 0 \end{pmatrix} \]

(3.8)
where the matrix $D$ can be expressed as below

$$
\begin{bmatrix}
 i \sin \mathbf{h}(d_m) & i k \cos \mathbf{h}(d_m) & i k \sin \mathbf{h}(d_m) & -k \gamma_m \sinh \mathbf{h}(d_m) \\
-k \gamma_m \cosh \mathbf{h}(d_m) & i \mathbf{h}(d_m) & -k \gamma_m \sin \mathbf{h}(d_m) & -k \gamma_m \sinh \mathbf{h}(d_m) \\
 i \gamma_m v_m \cosh \mathbf{h}(d_m) & F_m \sin \mathbf{h}(d_m) & i \gamma_m \mathbf{h}(d_m) & i \gamma_m (v_m \cosh \mathbf{h}(d_m) - F_m \sinh \mathbf{h}(d_m)) \\
 L_m (v_m + k \gamma_m \cosh \mathbf{h}(d_m)) & L_m (v_m + k \gamma_m \cosh \mathbf{h}(d_m)) & L_m (v_m + k \gamma_m \cosh \mathbf{h}(d_m)) & L_m (v_m + k \gamma_m \sinh \mathbf{h}(d_m))
\end{bmatrix}
$$

Combination of (3.6) and (3.8) gives a linear relationship between the motion-stress vector at the bottom and the top of the $m^{th}$ layer,

$$
\begin{align*}
\left( \frac{\dot{u}_m}{c}, \frac{\dot{w}_m}{c}, \sigma_m, \tau_m \right)^T &= D_m E_m^{-1} \left( \frac{\dot{u}_{m-1}}{c}, \frac{\dot{w}_{m-1}}{c}, \sigma_{m-1}, \tau_{m-1} \right)^T \\
&\Delta a_m \left( \frac{\dot{u}_{m-1}}{c}, \frac{\dot{w}_{m-1}}{c}, \sigma_{m-1}, \tau_{m-1} \right)^T
\end{align*}
$$

(3.9)

where $a_m$ is the propagation matrix between two adjacent layers.

In order to make it convenient for computation, Harkrider (1962) rearranged (3.6) as:

$$
\begin{align*}
\left( \frac{\dot{u}_{m-1}}{c}, \frac{\dot{w}_{m-1}}{c}, \sigma_{m-1}, \tau_{m-1} \right)^T &= \frac{E_m}{ik} (ik)(U_{2m}, U_{1m}, U_{3m}, U_{4m})^T
\end{align*}
$$

(3.10)

Then $E_m$ becomes

$$
E_m =
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & i \gamma_{1m} & 0 & i \gamma_{2m} \\
iX_{am} & 0 & iX_{bm} & 0 \\
0 & Y_{am} & 0 & Y_{bm}
\end{bmatrix}
$$

(3.11)

and

$$
E_m^{-1} =
\begin{bmatrix}
-a_m X_{bm} & 0 & -ia_m & 0 \\
0 & -ib_m Y_{bm} & 0 & -b_m \gamma_{2m} \\
a_m X_{am} & 0 & ia_m & 0 \\
0 & ib_m Y_{am} & 0 & b_m \gamma_{1m}
\end{bmatrix}
$$

(3.12)

$D_m$ becomes

$$
D_m =
\begin{bmatrix}
\cos P_m & i \sin P_m & \cos Q_m & i \sin Q_m \\
-\gamma_{1m} \sin P_m & i \gamma_{1m} P_m & -\gamma_{2m} \sin Q_m & i \gamma_{2m} \cos Q_m \\
iX_{am} \cos P_m & -X_{am} \sin P_m & iX_{bm} \cos Q_m & -X_{bm} \sin Q_m \\
iY_{am} \sin P_m & Y_{am} \cos P_m & iY_{bm} \sin Q_m & Y_{bm} \cos Q_m
\end{bmatrix}
$$

(3.13)
Carrying out the matrix multiplication for \( a_m = D_m E_m^{-1} \), we can get each elements in the propagation matrix \( a_m \).

\[
(a_m)_{11} = a_m^* \left( X_{\beta m}^* \cos P_m - X_{\alpha m}^* \cos Q_m \right) \\
(a_m)_{12} = i \left[ -\frac{1}{g_{12m}^*} \left( \frac{\sin P_m}{r_{\alpha m}} \right) + \frac{1}{g_{12m}^*} \left( \frac{\sin Q_m}{r_{\beta m}} \right) \right] \\
(a_m)_{13} = a_m^* \left( \cos P_m - \cos Q_m \right) \\
(a_m)_{14} = i \left[ -\frac{g_{2m}}{\rho_m \beta_m^2 g_{12m}^*} \left( \frac{\sin P_m}{r_{\alpha m}} \right) + \frac{g_{1m}}{\rho_m \beta_m^2 g_{12m}^*} \left( \frac{\sin Q_m}{r_{\beta m}} \right) \right] \\
(a_m)_{21} = i a_m^* \left( -X_{\beta m}^* g_{1m}^* r_{\alpha m} \sin P_m + X_{\alpha m}^* g_{2m}^* r_{\beta m} \sin Q_m \right) \\
(a_m)_{22} = \frac{g_{1m}^* (1 + g_{2m})}{g_{12m}^*} \cos P_m - \frac{g_{2m}^* (1 + g_{1m})}{g_{12m}^*} \cos Q_m \\
(a_m)_{23} = i a_m^* \left( -g_{1m}^* r_{\alpha m} \sin P_m + g_{2m}^* r_{\beta m} \sin Q_m \right) \\
(a_m)_{24} = \frac{g_{1m}^* g_{2m}}{\rho_m \beta_m^2 g_{12m}^*} \left( \cos P_m - \cos Q_m \right) \\
(a_m)_{31} = -a_m^* X_{\alpha m}^* X_{\beta m}^* \left( \cos P_m - \cos Q_m \right) \\
(a_m)_{32} = i \left[ X_{\alpha m}^* \frac{1 + g_{2m}}{g_{12m}^*} \left( \frac{\sin P_m}{r_{\alpha m}} \right) - X_{\beta m}^* \frac{1 + g_{1m}}{g_{12m}^*} \left( \frac{\sin Q_m}{r_{\beta m}} \right) \right] \\
(a_m)_{33} = a_m^* \left( -X_{\alpha m}^* \cos P_m + X_{\beta m}^* \cos Q_m \right) \\
(a_m)_{34} = i \left[ X_{\alpha m}^* \frac{g_{2m}}{\rho_m \beta_m^2 g_{12m}^*} \left( \frac{\sin P_m}{r_{\alpha m}} \right) - X_{\beta m}^* \frac{g_{1m}}{\rho_m \beta_m^2 g_{12m}^*} \left( \frac{\sin Q_m}{r_{\beta m}} \right) \right] \\
(a_m)_{41} = i a_m^* \left[ X_{\alpha m}^* \rho_m^2 \beta_m^2 (1 + g_{1m}) r_{\alpha m} \sin P_m - X_{\beta m}^* \rho_m^2 \beta_m^2 (1 + g_{2m}) r_{\beta m} \sin Q_m \right] \\
(a_m)_{42} = -\rho_m^2 \beta_m^2 \frac{(1 + g_{1m}) (1 + g_{2m})}{g_{12m}^*} \left( \cos P_m - \cos Q_m \right) \\
(a_m)_{43} = i a_m^* \rho_m^2 \beta_m^2 \left[ (1 + g_{1m}) r_{\alpha m} \sin P_m - (1 + g_{2m}) r_{\beta m} \sin Q_m \right] \\
(a_m)_{44} = -\frac{(1 + g_{1m})}{g_{12m}^*} \frac{g_{2m}^*}{g_{12m}^*} \cos P_m + \frac{(1 + g_{2m})}{g_{12m}^*} \frac{g_{1m}^*}{g_{12m}^*} \cos Q_m
\]

All definitions of intermediate variables could be referred to Harkrider (1962).
Now, applying boundary conditions through all the layers and considering the other boundary conditions at the surface and half space, we can get the dispersion equations of Rayleigh wave in a multilayered VTI medium:

\[
\begin{bmatrix}
KN + L'M^*
\end{bmatrix} - \begin{bmatrix}
G^*N - L'H
\end{bmatrix}T^* = 0
\]

(3.15)

where

\[
K = a_n^*X_{an}|r_{an}|A_{12}^* - \frac{1 + g_{2n}^2}{g_{12n}}A_{22} + a_n^*|r_{an}|A_{32}^* - \frac{g_{2n}}{\rho_n\beta_n^2g_{12n}}A_{42}
\]

\[
L' = -a_n^*X_{an}|r_{an}|A_{11} - \frac{1 + g_{2n}^2}{g_{12n}}A_{21}^* - a_n^*|r_{an}|A_{31}^* - \frac{g_{2n}}{\rho_n\beta_n^2g_{12n}}A_{41}^*
\]

\[
M^* = -a_n^*X_{an}A_{12} - \frac{1 + g_{1n}}{g_{12n}}A_{22} + \frac{g_{1n}}{\rho_n\beta_n^2g_{12n}}A_{2n}^* - \frac{g_{2n}}{\rho_n\beta_n^2g_{12n}}A_{32}^* - A_{42}^*
\]

(3.16)

\[
G^* = -a_n^*X_{an}|r_{an}|A_{13} - \frac{1 + g_{2n}^2}{g_{12n}}A_{23}^* - a_n^*|r_{an}|A_{33}^* - \frac{g_{2n}}{\rho_n\beta_n^2g_{12n}}A_{43}^*
\]

\[
H = -a_n^*X_{an}A_{13} - \frac{1 + g_{1n}}{g_{12n}}A_{23}^* - \frac{g_{1n}}{\rho_n\beta_n^2g_{12n}}A_{33}^* - A_{43}^*
\]

\[
N = -a_n^*X_{an}A_{11} - \frac{1 + g_{1n}}{g_{12n}}A_{21}^* - \frac{1}{X_{an} - X_{an}}A_{31}^* - \frac{g_{1n}}{\rho_n\beta_n^2g_{12n}}A_{41}^*
\]

In (3.16), \(A_{ij}\) are the elements of total propagation matrix \(A = a_{n-1}a_{n-2}...a_1\) and the subscript \(n\) represents the half space.

If the medium has a free surface, then \(T^* = 0\), since the normal and tangential stresses are zero at the free surface. If the layered medium has contacted a liquid half space at the top, then \(T^*\) is a non-zero value; instead, the normal stress and the vertical displacement should be continuously extended into the liquid half space.
3.2 Dispersion equation for Love wave in multilayered VTI medium

The dispersion equation of Love wave is much simpler than Rayleigh wave, since Love wave involves horizontal motions in only one direction perpendicular to the propagation direction (a SH motion).

\[
\begin{align*}
u &= 0 \\
w &= 0
\end{align*}
\]

Follow the similar procedure as Rayleigh wave (substitute (3.17) into waves equations and apply boundary conditions), then we can get the dispersion curves of Love wave in multilayered VTI medium:

\[
A_{21} = \begin{cases} 
-(\sqrt{L_n N_n} \cdot \sqrt{\frac{c^2}{N/\rho} - 1}) A_{11}, & \text{if } \frac{c^2}{N/\rho} - 1 > 0 \\
i(\sqrt{L_n N_n} \cdot \sqrt{1 - \frac{c^2}{N/\rho}}) A_{11}, & \text{if } \frac{c^2}{N/\rho} - 1 > 0
\end{cases}
\]

where \( A_{11} \) and \( A_{21} \) are the two elements of total propagation matrix \( A = a_{n-1}a_{n-2}...a_1 \) and \( N=c_{66}, \ L=c_{44} \). The subscript \( n \) represents the half space. According to the relationship between elastic constants and velocities (2.23), \( N \) corresponds to the horizontal shear wave velocity \( V_{SH} \), while \( L \) corresponds to the vertical shear wave velocity \( V_{SV} \).
3.3 A graphic-based method to acquire dispersion curves

Solving dispersion equation is a complicate task. The traditional numerical methods are based on root-searching algorithms (Ke, et al., 2011), which are time-consuming and easy to introduce numerical errors sometimes. In this section, we present a graphic-based method to acquire dispersion curves.

For a given model, the dispersion equation is an implicit function about two variables, frequency and phase velocity. First, we divide the frequency-velocity plane into $N_f$ by $N_c$ grids, where $N_f$ is the number of frequencies and $N_c$ is the number of velocities. Then a three-dimensional plot of the dispersion function $dfun(f, c)$ versus frequency and velocity is produced. If we use two different colors represent positive values and negative values respectively, we can get a dispersion image. Figure 3.4 illustrates a dispersion image of Love wave for a 5-layer model (Hamimu et al., 2011). The left panel shows $Vs$ structure. The right panel shows corresponding dispersion image, where the red regions represent $dfun(f, c)$ with positive values and the blue regions represent negative values. Therefore, the dispersion curves should appear at locations where $dfun(f, c)$ changes signs.
The next step is extracting dispersion curves from dispersion image. I make a two-dimensional cross-section at \( dfun(f, c) = 0 \) to the three-dimensional dispersion function. As we mentioned above, dispersion curves correspond to the roots of the dispersion equation. Therefore, the two-dimensional cross-section at \( dfun(f, c) = 0 \) could provide the desired dispersion curves. Figure 3.5 shows the dispersion curves obtained from the two-dimensional cross section.
Figure 3.5. Dispersion curves obtained from the two-dimensional cross section.

Despite the simplicity of this graphic-based method, it is fast and accurate. It has low possibility to introduce numerical errors for higher modes. In order to verify this method, I will compare the dispersion curves using my method with that using other root-searching algorithms.
3.4 Comparison of dispersion curves

In this section, three cases are discussed to make a comparison between the dispersion curves obtained from my method and other algorithms. All these cases verify my method through a good match of dispersion curves.

**Case 1: Love wave in a 2-layer model.**

The 2-layer model is the same as Xia (2006). The parameters of the model are shown in Table 3.1. The dispersion curves of Love wave from Xia and my method are compared in Figure 3.6.

**Table 3.1.** A 2-layer model (Xia, 2006).

<table>
<thead>
<tr>
<th></th>
<th>Thickness (m)</th>
<th>$V_S$ (m/s)</th>
<th>$V_P$ (m/s)</th>
<th>Density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>10</td>
<td>150</td>
<td>450</td>
<td>1.5</td>
</tr>
<tr>
<td>Half space</td>
<td>Inf.</td>
<td>250</td>
<td>750</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Figure 3.6. Comparison of dispersion curves of Love wave for a 2-layer model, a) Xia’s dispersion curves and b) my dispersion curves.
Case 2: Rayleigh wave in a 2-layer model.

We use the same model as Table 3.1. The dispersion comparison is shown in Figure 3.7.

Figure 3.7. Comparison of dispersion curves of Rayleigh wave for a 2-layer model, a) Xia’s dispersion curves and b) my dispersion curves.
Case 3: Rayleigh wave in a 2-layer model

Dispersion curves of Rayleigh wave are more complicated than Love wave. They usually have an irregular shape even for a 2-layer model. In order to further verify my method, another 2-layer model used by Park (2014) chosen to make a comparison. The parameters of the model are shown in Table 3.2. The dispersion curves of Rayleigh wave from Park and my method are compared in Figure 3.8.

Table 3.2. A 2-layer model (Park, 2014).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (ft)</th>
<th>$V_S$ (ft/s)</th>
<th>$V_P$ (ft/s)</th>
<th>Density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>5</td>
<td>1000</td>
<td>1871</td>
<td>1.5</td>
</tr>
<tr>
<td>Half space</td>
<td>Inf.</td>
<td>7000</td>
<td>13096</td>
<td>2.5</td>
</tr>
</tbody>
</table>

All three cases above show a good match between dispersion curves, which verify the derivation of dispersion equation and the graphic-based method of extracting dispersion curves. This method of extracting dispersion curves from a layered model will be used as the forward modeling in the inversion. Furthermore, the capacity of obtaining multi-mode is important to invert anisotropy parameters. In the next section, we will discuss how anisotropy affects the features of the dispersion curves, especially for the higher-order modes.
Figure 3.8. Comparison of dispersion curves of Rayleigh wave for a 2-layer model, a) Park’s dispersion curves and b) my dispersion curves.
3.5 The impact of anisotropy on dispersion curves

In this section, I will discuss how anisotropy affects the features of the dispersion curves. The dispersion of Love wave is studied since only one anisotropy parameter, $\gamma$, affects the dispersion of Love wave, as discussed in Chapter 2. The dispersion curves of Love wave in isotropic medium and anisotropic medium are compared in Figure 3.9. It is a 3-layer model. The $V_{SH}$ structure is shown by blue curves and $V_{SV}$ structure is in black color on the left panel in Figure 3.9 a) ~ b). The $V_S$ structure in isotropic medium is chosen the same as $V_{SH}$. The corresponding dispersion curves are compared for different $\gamma$ on the right panel in Figure 3.9: a) $\gamma=0.1$, a) $\gamma=0.5$.

From Figure 3.7, we can find that introducing anisotropy will change the shape of dispersion curves. The most significant change is the cut-off frequency of each mode. After comparison, we can draw two conclusions. First, the curves have more changes with larger value of anisotropy parameters, which indicates the feasibility to invert anisotropy structure using dispersion curves. Second, anisotropy has more effect on higher modes than lower modes. It means the higher modes are more sensitive to anisotropy than the lower modes. As a result, in order to get an accuracy estimation of anisotropy, higher modes should be included in the inversion part.
Figure 3.9. Dispersion curves of Love wave in anisotropic medium and isotropic medium. Blue curves represent isotropic case. Black curves represent anisotropic case. a) $\gamma=0.1$ and b) $\gamma=0.5$. 
3.6 Summary

Surface waves are generated by the interferences of propagating body waves. Dispersion occurs because of the variation of seismic velocity versus depth, and the depth-dependent penetration of surface waves at different wavelengths. The dispersion properties of surface waves can be used to characterize the near-surface velocity and anisotropic structure. Dispersion curves can be extracted from surface wave generated from not only active source, but also passive source.

The dispersion equations of Rayleigh wave and Love wave in VTI medium are re-derived in this chapter. This chapter can be considered as the verification to Anderson’s work (1962). Besides, several mistakes in his paper have been corrected.

Dispersion curves can be obtained by solving the dispersion equations. However, solving the dispersion equations is a sophisticated task because the dispersion equation is an implicit function of two variables, frequency and phase velocity, for a given model. In this chapter, a graphic-based method is proposed to avoid solving equations directly. The advantages of this approach are its simplicity, low possibility to introduce numerical errors for higher modes, and high computational efficiency.

Introducing anisotropy will change the shape of dispersion curves, which indicates the feasibility to invert anisotropy structure using dispersion curves. Furthermore, anisotropy has more effect on higher modes than lower modes. It means higher modes have to be included in the inversion part to get a more accuracy estimation of anisotropy.
Chapter 4. Stochastic Inversion using Simulated Annealing

4.1 Introduction

Most geophysical inversion problems are highly-nonlinear. One primary goal of inversion is to find the global minimum of an objective function (or error function). Due to the fact that such an objective function may have several minima, classical methods for optimization, such as inversions employing gradient methods may become trapped in one of these local minima. Several global optimization algorithms are proposed to detect the global minimum (or maximum) of a given function. The common global optimization algorithms are enumerative or grid search, random search (uniform distribution of the search space sampling, such as Monte Carlo methods), and “importance sampling” (the search space is non-uniformly sampled because some function drives the search, such as Simulated Annealing (SA)).

For most geophysical problems, the model space is very large and the forward calculation is slow. Therefore, enumerative or grid search scheme is not widely used. For example, suppose there are twenty possible values for a model consisting of ten parameters; then the model space would have $10^{20}$ possible points (models), which is extremely large. However, if we use large grid increment, then the best fit model could be easily missed.
Random search overcomes this problem. A new random model can be generated by random perturbation of a specific number of model parameters in the model space. Synthetic data are then generated for the new model and compared with observations. The model is accepted based on an acceptance criterion which determines how well the synthetic data fit the observations. The generation-acceptance/rejection process is repeated until a stopping criterion is satisfied.

The random search methods have been successfully used to solve highly nonlinear inversion problems. However, they are computational expensive. Simulated Annealing, which is considered as "importance sampling", was proposed to get the trade-off between computation time and global minima.

Simulated Annealing (SA) is a powerful stochastic search algorithm applicable to a wide range of problems for which little prior knowledge is available. The annealing schedule, i.e., the temperature decreasing rate used in SA is an important factor which affects SA’s rate of convergence.

This chapter will first introduce the procedure of SA. Then different annealing schedules are investigated. They are Boltzmann Annealing (BA), fast SA (FSA) and very fast SA (VFSA). At the end, a synthetic example is given to show the feasibility to invert anisotropy structure using VFSA algorithms.
4.2 Procedure of simulated annealing

The major advantage of SA over other classic methods (gradient method) is an ability to avoid trapping in local minima. In addition, the gradient method needs to calculate the Jacobian matrix. It becomes more difficult when the number of unknowns increases.

The main idea of SA’s strategy is selecting a move randomly instead of selecting the best move among possible moves in each stage of the algorithm. Figure 4.1 illustrates how SA avoids trapping in local minima and finally reach the global minimum.

![Diagram illustrating SA searching the global minimum.](image)

**Figure 4.1.** Illustration of SA searching the global minimum.
Suppose the initial model is at Point A and our goal is to find the global minimum (Point C) of the objective function, which has many local minima. The gradient method is very likely to be trapped in local minima (e.g., Point B) since it accepts only a better model (corresponding to a down-hill move) and always reject a worse model (corresponding to an up-hill move). However, SA also accepts worse models. If the new model reduces the objective function, it is accepted as the next model and if it increases the objective function, it is accepted just with a probability of $P$. The probability $P$ is defined as

$$P = \frac{1}{1 + \exp(\Delta E / T)}$$

(4.1)

where $\Delta E = E_{\text{PRESENT}} - E_{\text{PREVIOUS}}$ represents the difference between the present and previous values of the objective function. $T$ is the “temperature” of annealing schedule. This strategy will make the search avoid trapping in local minima.

At the beginning of SA, the “temperature” $T$ is high. According to Equation (4.1), SA accepts worse models with a larger probability $P$. It makes SA easy to jump out of local minima. As the annealing schedule continues, the “temperature” $T$ decreases and SA accepts worse models with a smaller probability $P$. This strategy makes SA having high possibility to find the global minimum.

The flow chart of inversion using SA algorithm is shown in Figure 4.2. First of all, SA starts with the use of an initial model. Random search in the model space is made to
propose a new model. Then the forward modeling is applied to the new model and the objective function is calculated and the objective functions are compared at current step with the previous one. If it is better than previous one ($\Delta E<0$), then the proposed model is considered as a better model and used to update the best model at the current step. If it is a worse model ($\Delta E>0$), SA will accept it with a probability of $P$ and reject it with a probability of $(1-P)$. Generation-acceptance/rejection process is repeated until a stopping criterion is satisfied.

**Figure 4.2.** The flow chart of inversion using SA algorithm.
4.3 Three types of annealing schedules

Different annealing schedules lead to various SA algorithms. This section will investigate Boltzmann Annealing (BA), fast SA (FSA) and very fast SA (VFSA).

BA was essentially introduced as a Monte Carlo importance-sampling technique in statistical physics problems (Metropolis, 1983). Initially, the Metropolis algorithm searched the model space using a uniform PDF. Faster convergence was later achieved by using a Gaussian PDF. The searching space was narrowed as temperature decreasing (Geman, 1984). The probability density of new model generation in the model space with $M$ parameters ($x_1, x_2, … x_M$) is

$$g(x) = (2\pi T)^{-M/2} \exp(-\Delta x^2 / (2T))$$

(4.2)

where $\Delta x = x - x_0$ is the perturbation applied to the previous configuration, $x_0$. $\Delta x$ can be considered as the mean and $T$ can be considered as standard deviation of the Gaussian distribution. At high temperatures, the model space is sampled uniformly. When temperatures are low, perturbations tend to be smaller.

It has been proved (Geman, 1984) that BA can obtain a global minimum if $T$ is selected to be not faster than

$$T(k) = \frac{T_0}{\ln(1+k)}$$

(4.3)
where \( k=1,2,3,... \) and \( T_0 \) is the initial temperature. It is apparent that it takes an extremely long time for temperature \( T(k) \) to approach zero. The main shortcomings of BA are its slowness and difficult tuning of its parameters.

A lot of researchers have been trying to increase SA’s rate of convergence by faster annealing schedules. Szu (1987) proposed FSA in which the Cauchy distribution is used to generate new models:

\[
g(x) = \frac{T}{(\Delta x^2 + T^2)^{(M+1)/2}}
\]

(4.4)

It is also proved that in order to make FSA convergent, the temperature \( T \) should be not faster than

\[
T(k) = \frac{T_0}{1+k}
\]

(4.5)

As the result, FSA has an annealing schedule exponentially faster than BA.

However, in a variety of geophysical problems, different parameters have different finite ranges and different sensitivities. BA and FSA cannot satisfy these two requirements. Ingber (1989) proposed VFSA algorithm to overcome the difficulties. Consider a parameter \( x_m(k) \) in dimension \( m \) \((m=1,2,...,M)\) generated at annealing time \( k \) with the range:

\[
x_m(k) \in [A_m, B_m]
\]

(4.6)

New parameters \( x_m(k+1) \) are generated from:

\[
x_m(k+1) = x_m(k) + y_m(B_m - A_m)
\]

(4.7)
where $y_m$ is generated from a uniform distribution $u_m$:

$$u_m \in U[0, 1]$$

(4.8)

$$y_m = \text{sgn}(u_m - \frac{1}{2})T_m \left[ (1 + 1/T_m)^{|y_m| - 1} - 1 \right]$$

(4.9)

The convergence of VFSA requires temperature $T$ should be not faster than

$$T_m(k) = T_{m0} \exp(-c_m k^{1/M})$$

(4.10)

where $c_m$ is a control factor determined by the initial temperature $T_{m0}$ and the final temperature $T_{mf}$ at final time $k_{mf}$.

$$c_m = C_{1m} \exp\left(-\frac{C_{2m}}{M}\right)$$

(4.11)

$$C_{1m} = -\ln\left(\frac{T_{mf}}{T_{m0}}\right)$$

$$C_{2m} = \ln(k_{mf})$$

Different parameters can have different ranges and annealing schedules depending on their sensitivities. Furthermore, VFSA has an annealing schedule even faster than FSA.

Figure 4.3 illustrates the comparison of the annealing speed for BA, FSA and VFSA.
Figure 4.3. Comparison of various annealing schedules. The Blue curve represents annealing schedules of BA, black curve represents FSA and red curve represent VFSA.

In the next section, a synthetic example is given to estimate the anisotropy structure in a VTI medium using VFSA based on multi-mode of surface wave dispersion. We will see another advantage of VFSA is its independence to the initial model. Traditional inversion methods using multi-mode of surface dispersion reply on carefully-chosen initial model (Luo, et al., 2007). However, we can choose an arbitrary initial model in the model space for VFSA inversion due to the fact that VFSA generate proposed new models randomly.
4.4 A synthetic example using VFSA algorithm

A 3-layer VTI model is chosen as the “real” model. Multi-modes of Love wave are used to estimate the velocity structure and anisotropy structure simultaneously. Parameters of the “real” model is shown in Table 4.1 and illustrated on the left panel of Figure 4.4. The red solid and dash curves represent the real $V_{SH}$ and $V_{SV}$ respectively. The blue curves represent the initial model. The black solid and dash curves represent the inverted $V_{SH}$ and $V_{SV}$ respectively.

In a real-world survey, it is not easy to extract multi-mode of surface wave dispersion, especially for higher-modes with the order larger than four. As the result, only first three modes are used in the inversion. The VFSA algorithm begins with an arbitrary model and stops after 2000 iterations. The corresponding dispersion curves of the real model, the initial model and the inverted model are shown in the same color as the model on the right panel of Figure 4.4. As expected, the inverted $V_{SH}$ and $V_{SV}$ structure have a good match with the real model. After calculation, the comparison of shear wave velocity structure and the Thomsen parameter $\gamma$ are also shown in Table 4.1. We can find that the inversion of multi-modes dispersion of Love wave can give a good estimation of shear wave velocity and seismic anisotropy structure. The inverted parameters in the shallow depth are more reliable than that in the half space due to the smaller estimation error.
That is reasonable because surface waves propagate in the waveguide generated by the layers with large contrast, e.g., the bedrock in half space.

Figure 4.4. The synthetic example. (a) Comparison of real model and inverted model. (Blue curves represent the arbitrary initial model. Red curves represent the synthetic real model. Black curves represent the inverted model after 2000 iterations of VFSA algorithm).

Table 4.1. Parameter comparison synthetic example

<table>
<thead>
<tr>
<th>Layer</th>
<th>Real model</th>
<th>Inversed model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thick (km)</td>
<td>V_{SH} (km/s)</td>
</tr>
<tr>
<td>Layer 1</td>
<td>0.010</td>
<td>0.400</td>
</tr>
<tr>
<td>Layer 2</td>
<td>0.040</td>
<td>0.600</td>
</tr>
<tr>
<td>Half space</td>
<td>Inf</td>
<td>1.100</td>
</tr>
</tbody>
</table>
4.5 Summary

SA is a powerful stochastic global search algorithm applicable to a wide range of non-linear problems with several local minima. The strategy used in SA to avoid trapping in the local minima is that it accepts worse models with a probability, which is determined by the objective function and the temperature $T$. Three annealing schedules of SA are investigated and compared in this chapter. They are BA, FSA and VFSA. The advantages of VFSA are: 1) high capability to find global minimum; 2) independence to the initial model; 3) feasibility to the case of parameters with different range and sensitivities; 4) Less (or no) human interference is needed between iterations. The synthetic example of Love wave in a 3-layer model shows the effectiveness of this approach.

This dissertation will use VFSA algorithm to reveal the anisotropic earth structure. Three geophysical applications are discussed in the remaining chapters. Chapter 5 shows an application to petroleum exploration. Simulated annealing inversion is utilized to estimate material anisotropy at well location. Chapter 6, 7, and 8 will discuss three applications in shallow subsurface using SA inversion based on dispersion properties of surface wave. The inversion in Chapter 6 utilizes multi-mode Love wave extracted from the active source. The inversion in Chapter 7 utilizes multi-mode Rayleigh wave extracted from the ambient noise. Chapter 8 discusses the case of inversion of velocity
structures of polar firm derived from the dispersion of Rayleigh wave generated by active sources in Greenland ice sheet.
Chapter 5. Application to Petroleum Exploration: Anisotropy Estimation at Well Location

5.1 Problem statement

In recent years, estimation of anisotropy structure attracted much more attention in the petroleum industries, since they realized more accuracy results could be obtained with the consideration of anisotropy. Lots of approaches were proposed to study anisotropy. One of these approaches estimates multi-parameters simultaneously using joint tomography (Zhou, 2011). In this chapter, we applied simulated annealing inversion to petroleum exploration: anisotropy estimation at well location. Then we compared the results using joint tomography and simulated annealing. The whole procedure is conducted in the CM/MDE (Migration Development Environment) computational environment, which is provided by Petroleum Geo-Survey LLC (PGS).

Our objective is estimate anisotropy with surface seismic data and well data. In another word, we will invert anisotropy parameters \((\varepsilon, \delta)\), with the vertical velocity well known, provided by well data. In order to simplify our problem, we have assumptions for our model. First, the medium is locally one dimensional, which means anisotropy parameters only depend on depth. Second, the medium is a VTI medium.
5.2 Data description

The data used in this chapter is 2D synthetic data provided by BP Company. The full overview of velocity ($V_p$) model is illustrated in Figure 5.1. The color-bar range is from 1492 m/s (yellow) to 4554 m/s (blue), corresponding to the sea water and the salt region respectively. We select the red rectangular region as our interested region, with depth from 0 ~ 8.5 km and horizontal distance from 42 ~ 43 km. The velocity model is well known since we assume that it can be provided by well data.

![Figure 5.1. $V_p$ model of BP 2D synthetic model. The color-bar range is from 1492 m/s (yellow) to 4554 m/s (blue), corresponding to the sea water and the salt region respectively.](image)
Figure 5.2 shows the enlarged image of $\varepsilon$ and $\delta$ model in our interested region. The color-bar range is from $0 \sim 0.143$ for $\varepsilon$, and $0 \sim 0.108$ for $\delta$. The anisotropy model in our interested region varies with depth and VTI is a reasonable assumption. As a result, the two assumptions work well in this region. The anisotropy model illustrated in Figure 5.2 is considered as true model.

**Figure 5.2.** Enlarged figure of the anisotropy model. True $\varepsilon$ model is shown in a), with color-bar range from $0 \sim 0.143$. True $\delta$ model is shown in b), with color-bar range from $0 \sim 0.108$. 
5.3 Inversion using joint tomography

First, we use joint tomography method (Zhou, 2011) to invert anisotropy parameters. The work flow is shown in Figure 5.3. All modules in the work flow are in production, provided by Petroleum Geo-Survey (PGS).

![Figure 5.3. Work flow of joint tomography.](image-url)
The inversion starts from an initial model. The “aniray” module (related to ray tracing) and the “kdmig” module (related to Kirchhoff migration) are combined together to obtain angle gathers at different offsets. Then we will determine whether angle gathers are flat or not. Angle gathers will become much flatter if the model is closer to the real model. If angle gathers are not flat, we have to use joint tomography method (contains “upicker”, “jwtomoSetup2D” and “jwtomoInv” modules) to adjust the model and go to the next iteration. Quality control and events picking should be done in each iteration. And the work flow will be terminated if angle gathers are flat enough.

We set up an initial model as shown in Table 5.1 and estimate $\varepsilon$ only. The angle gathers corresponding to initial model (iteration 0) and final inverted model (iteration 4) are shown in Figure 5.4. We could observe that almost all angle gathers are flattened after 4 iterations, but it is a little bit tilt in deeper depth (below 7 km). The observation reveals that our inverted model has a good match with the true model, but a bit of mismatch below 7 km. The $\varepsilon$ comparison between inverted model and true model at one location is illustrated in Figure 5.5, which indicates more institutive conclusion similar as Figure 5.4. Both $\varepsilon$ and $\delta$ are well correlated with true values above 7 km.

Table 5.1. Initial model in the application to petroleum exploration

<table>
<thead>
<tr>
<th>Anisotropy parameters</th>
<th>Water</th>
<th>Sediment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Figure 5.4. Angle gathers at different offsets (estimate $\varepsilon$ only), corresponding to a) the initial model (iteration 0) and b) final inverted model (iteration 4). The red line shows the depth at 7 km.

Figure 5.5. The $\varepsilon$ comparison between inverted model and true model at one location. Red curve is the true model, black curve is the initial model and green curve is the final inverted model.
The inversion above is to estimate $\varepsilon$ only. Then we will consider estimating $\varepsilon$ and $\delta$ simultaneously. However, we do not get a good inversion result for BP 2D synthetic model as expected, since only $\varepsilon$ is updated in each iteration, but $\delta$ not. The $\varepsilon$ comparison and $\delta$ comparison between inverted model and true model are shown in Figure 5.6. More investigation is needed in this case.

**Figure 5.6.** a) $\varepsilon$ comparison and b) $\delta$ comparison between inverted model and true model at one location. Red curve is the true model, black curve is the initial model and green curve is the final inverted model.

### 5.4 Inversion using simulated annealing

In this section, we will reconsider to estimate both $\varepsilon$ and $\delta$ simultaneously, using simulated annealing inversion. The depth 0~9 km is divided into 30 layers; so we have 60 parameters ($\varepsilon$ and $\delta$ in each layer) to estimate. The work flow is the same as Figure 4.2.
As we mentioned above, simulated annealing inversion could start with an arbitrary model since this method is independent to the initial model. A new model is proposed by a random search in the model space, and forward modeling generates offset gathers corresponding to the proposed model. Then the scheme will calculate the objective function and determine whether the proposed model is better than last step. The objective function is defined by negative semblance, shown as Equation (5.1),

\[
obj = -\frac{\sum z \sum_{\text{offset}} f(z, \text{offset})^2}{N_{\text{offset}} \sum z \sum_{\text{offset}} f^2(z, \text{offset})}
\]  

(5.1)

where \(f(z, \text{offset})\) is the amplitude at one point \((z, \text{offset})\) in offset gathers and \(N_{\text{offset}}\) is the number of offsets in each gather. Using this definition, the objective function is bounded between \([-1, 0]\), where the value of \(-1\) corresponds to a totally flat gather. And our objective is to find the global minimum.

If the new proposed model is better, which means \(dE < 0\), then we will accept it and update current model with proposed model. If the new proposed model is worse, which means \(dE > 0\), we will accept the worse model with a probability of \(P\) and reject it with a probability of \((1-P)\). Then the scheme goes to the next iteration till a good convergence. Figure 5.7 illustrates the inversion results using fast annealing (FA). Panel a) illustrates best values of objective function at current iteration. We can see that FA inversion begin to be convergent after 10000 iterations. Panel b) shows four offset gathers at iteration 0,
800, 10000, 100000 respectively. It is clearly seen that offset gathers become flatter and flatter as iteration number increases. Panel c) and d) compare $\varepsilon$ and $\delta$ between inverted model and true model. The red curve is the true model, the Blue curve is the inverted model, and the black curve is the curve fitting result of the inverted model. After comparison, we can find that the inverted model has a good match with the real model up to the depth of 6 km, but a mismatch is also found below 6 km.

**Figure 5.7.** Estimation of $\varepsilon$ and $\delta$ simultaneously using FA inversion: a) best values of objective function at current iteration; b) four offset gathers at iteration 0, 800, 10000, 100000 respectively; c) and d) comparison of $\varepsilon$ and $\delta$ between inverted model and true model.
In order to compare FA and VFSA, figure 5.8 illustrates the inversion results using very fast simulated annealing (VFSA). We can obtain very similar results as FA inversion. But VFSA begin convergence faster (after iteration 4000) than FA (after iteration 10000).

Figure 5.8. Estimation of $\varepsilon$ and $\delta$ simultaneously using VFSA inversion; a) best values of objective function at current iteration; b) four offset gathers at iteration 0, 800, 10000, 100000 respectively; c) and d) comparison of $\varepsilon$ and $\delta$ between inverted model and true model.
5.5 Summary

Simulated annealing inversion achieves successful application to BP 2D synthetic model. It can be used to estimate anisotropy parameters ($\varepsilon$ and $\delta$) simultaneously. In this application, both $\varepsilon$ and $\delta$ are well correlated with true value above 6~7 km. There are two possible reasons for the mismatch below 6~7 km. The first reason is the VTI assumption. The true model consists of non-strictly horizontal layers. The dip is very mild in shallow but significant in deep. As a result, it may introduce accumulate error during forward modeling. The second reason is the ambiguity. The model solution corresponding to one gather may be not unique.

Compared with joint tomography inversion, simulated annealing inversion has three advantages. First, simulated annealing inversion is independent to the initial model. It is a statistical strategy; so we can use an arbitrary model in the model space as our initial model. Second, simulated annealing inversion is theoretically proved to have the ability to converge to the global minimum with enough iteration, while traditional inversion methods are easy to be trapped into local minimum if the initial models are not chosen carefully. Finally, simulated annealing inversion can be executed automatically. We do not need quality control and event picking at each iteration, as traditional inversion methods. Obviously, simulated annealing inversion also contains disadvantages. One of them is slow convergence. We need thousands of iterations more than traditional inversion methods.
Chapter 6.  Seismic Velocity Structure in Shallow Subsurface
Extracted from Active Source: Love Wave

Accurate and robust near-surface shear wave velocity information has direct impact to
earthquake engineering and seismic hazard reduction in one region. Detailed mapping of
near-surface shear-wave velocity structure generates the important parameter $Vs30$, i.e.,
the averaged shear wave velocity in the uppermost 30 meters in the ground, which is
much sought information in geotechnical engineering and foundation design (e.g.,
Morton, 2014). Inversion of seismic surface wave (Rayleigh and Love wave) dispersion
curves is commonly used to develop the $Vs30$ models. Because acquiring
compressional-wave data is easier than acquiring pure shear-wave data, Rayleigh wave
methods are used far more extensively than Love wave methods. Nevertheless, Love
wave methods offer some potentially distinct advantages over Rayleigh wave methods:
because Love waves are horizontally polarized shear waves (SH particle motion), it is
independent of Poisson’s ratio, with no contribution from the bulk modulus of the
material. Therefore, Love waves are more sensitive to shear modulus variation and layer
thickness changes than Rayleigh waves whose particle motion is P-SV type.

In this chapter we investigate the shear wave velocity structure in the Connecticut
River basin in East Hartford, Connecticut using Love wave dispersion curves for
near-surface sediments. Our approach includes numerical modeling of multi-mode Love wave generated from active (impulsive stimulation) sources, and the analysis of active Love wave field data acquired at the site of the Rentschler football field, East Hartford, CT. We compare the dispersion curves relationship obtained from the forward shot and the reverse shot; and (2) the shear wave velocity structure obtained from multi-mode Love wave and the layered structure by the horizontal to vertical spectral ratio (H/V) method in order to assess the benefit of using Love wave methods for developing high-confidence Vs30 parameters for improved seismic hazard assessments.

Multichannel analysis surface wave (MASW) is an approach that estimates the velocity structure of the subsurface in 1D or 2D profiles by collecting surface waves through a multichannel data acquisition. The overall procedure consists of three steps: 1) multichannel data acquisition, 2) dispersion analysis, and 3) inversion analysis to yield the 1D (depth) velocity structure. Then a pseudo-2D profile can be obtained by repeating 1D surveys at successively adjacent locations along a linear transect. In the same fashion, a pseudo-3D profile can be obtained by repeating 2D surveys at multiple transects parallel to each other. The last three applications (Chapter 6, 7 and 8) are based on the approach developed from MASW.
6.1 Data acquisition

The aerial view of the site of the Rentschler football field, East Hartford, CT is shown in Figure 6.1 by Google Earth. The main components in the seismic survey are multichannel recording device (Seismograph), the seismic source (sledge hammer as the active), seismic receivers (Geophones), and the seismic cable.

![Figure 6.1. The aerial view of the site of the Rentschler football field, East Hartford, CT.](image)
Figure 6.2. The active source and the horizontal geophones to record Love wave.

Twenty four 4.5 Hz geophones with the sampling interval of 0.125 ms are used as the receiver array. The spread length is 69m with the receiver spacing of 3m. In order to record Love wave, the sensing orientation of geophones is horizontal, transverse to the survey line. The SH motion of the active source is generated by hitting the wood block transversely. Figure 6.2 illustrates the hitting point and the direction of geophones.

The geophone frequency \( f_G = 4.5 \text{Hz} \) represents the “cut frequency”, which is not sharply defined but tapers down gradually. It is an important parameter to affect the maximum investigation depth \( (Z_{\text{max}}) \). Because \( f_G \) determines the lowest measurable frequency \( (f_{\text{min}} = f_G) \), which further determines the maximum investigation depth \( (Z_{\text{max}}) \) by
\[ Z_{\text{max}} \approx \frac{1}{2} \lambda_{\text{max}} \approx \frac{1}{2} \frac{V_{\text{max}}}{f_{\text{min}}} \approx \frac{1}{2} \frac{V_{\text{max}}}{f_G} \]  \hspace{1cm} (6.1)

where \( \lambda_{\text{max}} \) and \( V_{\text{max}} \) is the maximum wavelength and velocity measured. Usually, the maximum investigation depth is about 50m if geophone with \( f_G=4.5\text{Hz} \) are used.

The spread length \( L \) also affects the maximum investigation depth by considering the spatial resolution.

\[ \frac{1}{2} L \leq Z_{\text{max}} \leq L \]  \hspace{1cm} (6.2)

In the seismic survey using active source, the offset range needs to be considered carefully. If the geophone is too close to the source, surface waves cannot be developed, which is named “near-field effects”. If the geophone is too far away from the source, the energy of surface wave drops below the energy of ambient noise, which is named as “far-field effects”. Park (2014) gave the optimum offset: Equation (6.3) is to avoid both near-field effects and Equation (6.4) is to avoid far-field effects.

\[ \frac{1}{4} L \leq X_1 \leq \frac{1}{2} L \]  \hspace{1cm} (6.3)

\[ L \leq 100 \text{ m} \]  \hspace{1cm} (6.4)

where \( X_1 \) is the nearest offset from the first geophone to the source. Considering this criterion, the field data are acquired from both the forward shot \( (X_1=15\text{m}) \) and the reverse shot \( (X_2=11\text{m}) \), which are illustrated in Figure 6.3 and Figure 6.4.
Figure 6.3. Field data acquired from the forward shot. The receiver array spread is 15~84 m. The shot is at 5 m, 15 m from the first geophone.

Figure 6.4. Field data acquired from the reverse shot. The receiver array spread is 15~84 m. The shot is at 95 m, 11 m from the last geophone.
6.2 Dispersion analysis

Dispersion analysis extracts dispersion curves from the field data. Forward shot gather, Figure 6.4, is taken as an example to show the procedure of dispersion analysis.

First of all, we have to select a region in which Love waves are dominant. As we know, body waves are usually non-dispersive and the wave form is preserved in general, whereas Love waves are dispersive and become more “spread” as they propagate. As the result, we have to select a beam region to contain as many Love waves as possible. The beam region should be more “spread” at the far offset. Figure 6.5 illustrate the beam region in which the main components are dispersive Love waves.

![Diagram showing dispersion analysis](image)

**Figure 6.5.** The illustration of the beam region that contains dispersive Love waves.
In order to obtain the dispersion image, a two dimensional transformation is performed to Love waves in the beam region. In this thesis, I use the f-k transformation. The beam region should be chosen carefully to obtain a clear multi-mode dispersion image. There are two lines of guidance. First, make sure there is no dominant mode in the beam region. Otherwise, other modes will be suppressed lead to the difficulty for picking. Second, make sure to reduce the effects of computational artifact (CA) and spatial alias due to the receiver spacing as much as possible. We will discuss these two effects later.

Figure 6.6. The dispersion image of Love wave from the forward shot using f-k transform.
Figure 6.6 shows the dispersion image of Love wave using the f-k transform. We can see the three modes of Love wave very clearly. They are picked and considered as the real dispersion curves of Love wave in the inversion. In addition to these three modes, we can see some other features in the dispersion image. The rectangular region on the left (f<4.5Hz) is the computational artifact due to the geophone frequency. As we mentioned above, the cut frequency of the geophone is not sharply defined but tapers down gradually. The region (f<4.5Hz) in the dispersion image is contaminated by CA. Therefore, it is not easy to pick the fundamental mode (M0) in this case. In the next section, the inversion analysis will indicate that the three clear modes that used in the inversion correspond to the first-higher, second-higher and third-higher modes (M1, M2 and M3).

The triangle region on the bottom is another region contaminated by CA, which is due to the spatial resolution. The triangle region approximately represents the wavelength λ is smaller than the receiver spacing, dx.

\[ \lambda < dx \]  \hspace{1cm} (6.5)

The curves with strong amplitudes appearing above 60 Hz are caused by the spatial alias, which also relates to the receiver spacing, dx. The formula to determine the starting frequency of spatial alias \( f_{alias} \) from experience is expressed as

\[ f_{alias} \approx \frac{V_{min}}{dx} \]  \hspace{1cm} (6.6)

where \( V_{min} \) is the minimum phase velocity measured. In the case shown by Figure 6.6,
which is consists with the observation.

Using the same procedure, the dispersion image from the reverse shot is shown in Figure 6.7. In this figure, M0, M1 and M2 are clear and will be used for the inversion.

![Figure 6.7](image)

**Figure 6.7.** The dispersion image of Love wave from the reverse shot using f-k transform.

In the dispersion analysis, three modes are picked and considered as the real dispersion curves in the Inversion analysis.
6.3 Inversion analysis

The VFSA algorithm is used to invert the $V_s$ structure and the anisotropy structure ($\gamma$) simultaneously using the three modes of Love wave. As mentioned in Chapter 4, the inversion starts with the use of an arbitrary initial model. At the $k^{th}$ iteration, the new model is generated randomly following the special distribution, described in Equation (4.7) and Equation (4.9). The forward modeling is applied to get the corresponding dispersion curves. Then the objective function is calculated to measure how well the proposed curves fit the real curves. The objective function used here is based on the mean square error, as shown in Equation (6.8):

$$\text{obj} = \frac{1}{N_{\text{mode}}} \sum_{m=1}^{N_{\text{mode}}} \left[ \frac{1}{N_{\text{real}}} \sum_{n=1}^{N_{\text{real}}} \left( \frac{1}{N_{\text{real}}} \sum_{n=1}^{N_{\text{real}}} (\sum_{n=1}^{N_{\text{real}}} \left[ \begin{array}{c} (\Delta d)^2 \mid \text{Point } n \\ \sum_{n=1}^{N_{\text{real}}} \left[ (E_{\text{real}} - E_{\text{model}})^2 + \left( \frac{f_{\text{real}} - f_{\text{model}}}{dc} \right)^2 + \left( \frac{f_{\text{real}} - f_{\text{model}}}{df} \right)^2 \mid \text{Point } n \end{array} \right] \right) \right) \right] \right]$$

(6.8)

where $N_{\text{mode}}$ is the number of modes for comparison; $N_{\text{real}}$ is the number of real points which are picked in each mode; $dc$ and $df$ are the intervals of the phase velocity and frequency in the dispersion image; $\Delta d$ is the nearest distance in the unit of pixel from the real point to the proposed curves. It is necessary to use the distance in the unit of pixel number since the phase velocity and the frequency have different ranges and units. $dc$ or $df$ can be considered as the width of the pixels in the dispersion image; so the variables are non-dimensionalized by dividing the width of the pixels. This design of the objective function could be used for uncertain analysis in the future since the half width of the pixels corresponds to the square root of variances. In order to avoid comparing the points
from one real mode with the points from different proposed modes, named as “cross-mode comparison phenomenon”, the coherence of the points from the same mode should be introduced when calculating the nearest distance.

After obtaining the objective function at iteration $k$, compare it with previous iteration $k-1$ and apply the accept/reject rule to it. That is the procedure in one iteration. The algorithm continues until the stop criterion is satisfied. Figure 6.8 shows the inverted results of the $V_s$ structure from the forward shot and Figure 6.9 is the inversion results from the reverse shot. After the calculation using the definition of $\gamma$ (2.24), the estimated parameters are compared between the forward shot and the reverse shot in Table 6.1.

![Figure 6.8](image)

**Figure 6.8.** The estimation of $V_s$ structure from the forward shot at the Rentschler field. On the left panel, blue curves represent the initial model; black curves represent the inverted model after 1000 iterations of VFSA algorithm. The corresponding dispersion curves are shown on the right panel with the same color. The picked three modes correspond to M1, M2 and M3.
Figure 6.9. The estimation of $V_S$ structure from the reverse shot at the Rentschler field. On the left panel, blue curves represent the initial model; black curves represent the inverted model after 1000 iterations of VFSA algorithm. The corresponding dispersion curves are shown on the right panel with the same color. The picked three modes correspond to M0, M1 and M2.

The inversion results shown in Figure 6.8 and Figure 6.9 indicate that the three picked modes from the forward shot correspond to M1, M2 and M3, where the three picked modes from the reverse shot correspond to M0, M1 and M2. The best objective function at the last iteration calculated by Equation (6.8) is 2.7 grids for the forward shot and 1.9 grids for the reverse shot, which means the inverted model from the reverse shot fits the real data better and it could be considered more reliable.
Table 6.1. Comparison of estimated parameters from the forward shot and the reverse shot at the Rentschler field.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Inverted model from the forward shot</th>
<th>Inverted model from the reverse shot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thickness (km)</td>
<td>$V_{SH}$ (km/s)</td>
</tr>
<tr>
<td>Layer 1</td>
<td>0.0375</td>
<td>0.161</td>
</tr>
<tr>
<td>Layer 2</td>
<td>Inf</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Table 6.1 shows that the two cases can obtain consistent inversion results. The average depth of bedrock is about 38 m, which is close to the estimation (42.5 m) provided by Liu et al. (2010), as shown in Figure 6.10. It verifies that the VFSA inversion using multi-mode Love wave can estimate not only the $V_s$ structure, but also the anisotropy ($\gamma$) structure.
Furthermore, three conclusions can be drawn after comparison. First, lower and upper asymptotic trends of dispersion curves are approximately bounded by two shear velocities ($V_s$) of soil and bedrock. Second, the cut-off frequencies of Love wave modes appear with similar frequency-intervals and the cut-off frequency of the fundamental mode is usually larger than zero. Third, the estimated anisotropy parameters ($\gamma$) are negative values at the site of Rentschler field. According to the definition of $\gamma$ (2.24), it implies that $V_{SV}$ is larger than $V_{SH}$. From the point view of modulus, the shear modulus in the vertical direction is larger than the horizontal direction. In the other word, the earth is easier to shear in the horizontal direction at the site of Rentschler field, with the assumption of 2-layer VTI model.
As mentioned in Chapter 5, there are two possible reasons for the estimation error of anisotropy parameter. The first reason is the VTI assumption. The real earth may consist of non-strictly horizontal layers. The second reason is the ambiguity. The model solution corresponding to one set of dispersion curves may be not unique. Showing families of models that achieve acceptable minimization of the objective function will provide the estimation of the errors in the modeling.

The anisotropy parameter (γ) describes the difference between $V_{SV}$ and $V_{SH}$; so it can be determined by Love waves. Although there are no drilling logs about anisotropy at the site of Rentschler field, the estimation of bedrock depth matches well with results from H/V method. Besides, the consistence between the forward shot case and the reverse shot case verifies the method.
Chapter 7. Seismic Velocity Structure in Shallow Subsurface

Extracted from Ambient Noise: Rayleigh Wave

In this chapter we investigate the shear wave velocity structure and P-wave velocity structure in the Haddam Meadows State Park in Haddam, Connecticut using Rayleigh wave dispersion curves for near-surface sediments. Passive MASW is used to extract dispersion curves from the ambient noise. Due to the fact that the ambient noise has more low-frequency components, passive MASW can significantly increase the investigation depth. The other advantage of passive MASW is the record length can be as long as possible, which helps improving the frequency resolution or removing relevant noise.

Based on wave-interferometry principles, this chapter describes a procedure to synthesize active Rayleigh waves from ambient noise. The Rayleigh wave behaves like generating from a virtual active source. Once active Rayleigh waves are obtained, the remaining data processing procedures (dispersion analysis and inversion analysis) are identical to that of active MASW, which was discussed in Chapter 6.

7.1 Data acquisition

The aerial view of the site of the Haddam Meadows, Haddam, CT is shown in Figure 7.1 by Google Earth. The main components in the seismic survey are multichannel
recording device (Seismograph), the passive source (ambient noise), seismic receivers (Geophones), and the seismic cable.

The receiver array contains twenty four 4.5 Hz geophones with the sampling interval of 2 ms are used as the receiver array. The record length is 30 seconds with 15000 samples in each file. The spread length is 115m with the receiver spacing of 5m. In order to record Rayleigh wave, the sensing orientation of geophones is vertical. Figure 7.2 shows the field data of Line 1 acquired from ambient noise. In order to improve the frequency resolution during the dispersion analysis by increasing the record length, thirty-one files (1000.dat~1031.dat) are connected together subsequently. Therefore, the total record length is 930 seconds.

Figure 7.1. The aerial view of the site of the Haddam Meadows State Park, Haddam, CT.
Figure 7.2. Field data acquired from ambient noise. The receiver array spread is 0~115 m with 24 vertical geophones of 5m intervals.

7.2 Rayleigh wave from virtual active sources

In this section, a procedure is described to synthesize virtual active Rayleigh waves from ambient noise. The main idea is following the “wave interferometry virtual source” (WIVS) approach, which was proposed by Liu and He (2007).

Consider two receiver channels Ch1 at $x_1$, and Ch2 at $x_2$, and a source point $S$ at $x_s$, and the source is excited with a time function $e(t)$. If $T(x_1, x_s, t)$ is the wave field recorded at $x_1$, and $T(x_2, x_s, t)$ is the wave field recorded at $x_2$ due to the source at $x_s$, our goal is to
recover the response at Location x2, caused by a virtual source at location x1, denoted as
\( R(x_2, x_1, t) \), through the responses of \( T(x_1, x_o, t) \) and \( T(x_2, x_o, t) \).

First, we can express the recorded wave-field at the two locations as

\[
T(x_1, x_s, t) = e(t) * g_{1s}(t) \\
T(x_2, x_s, t) = e(t) * g_{2s}(t) 
\] (7.1)

where \( g_{1s}(t) \) and \( g_{2s}(t) \) are the impulse responses between \( Ch1 \) and \( S \), and \( Ch2 \) and \( S \), respectively; and * denotes the process of convolution. Equation (7.1) implies that the recorded responses at \( Ch1 \) and \( Ch2 \) are the convolutions of the impulse response with the source excitation function \( e(t) \), i.e., \( g_{1s}(t) \) and \( g_{2s}(t) \) are the Green’s functions for locations \( x_1 \) and \( x_2 \) when it has an excited source at \( x_s \). The cross correlation of the wave-field recorded at \( Ch1 \) and \( Ch2 \) is then

\[
R(x_2, x_1, t) = \int_{-\infty}^{\infty} T(x_1, x_s, \tau)T(x_2, x_s, t + \tau)d\tau
\]

\[
= T(x_1, x_s, t) * T(x_2, x_s, -t) \\
= e(t) * g_{1s}(t) * e(-t) * g_{2s}( -t) \\
= e(t) * e(-t) * g_{1s}(t) * g_{2s}( -t) \\
= f(t) * g_{21}(t) 
\] (7.2)

where the factor \( f(t) = e(t) * e(-t) \) depends only on the excitation function \( e(t) \) imposed at the source. Equation (7.2) indicates that 1) the impulse response between locations \( x_1 \) and \( x_2 \) is contained in \( g_{21}(t) \), so that is the Green’s function for the response at \( x_2 \) for a source at \( x_1 \); and 2) the real response \( R(x_2, x_1, t) \) and the Green’s function \( g_{21}(t) \) are proportional to each other and only differed by a factor of \( f(t) \).
As the result, the cross correlation of ambient noise received at Ch. 1 and Ch.2 can be considered as Rayleigh waves received at Ch.2 from a virtual active source at Ch.1. Applied the same procedure between Ch.1 and other channels, we can obtain Rayleigh waves received at Ch. 2 ~ Ch. 24 from the virtual source at Ch.1. Figure 7.3 demonstrates the cross correlation from Ch.1 to other channels. The red line locates at t=0. The positive correlation is on the right-hand side, while the negative correlation is on the left-hand side.

**Figure 7.3.** The cross correlation from Ch. 1 to other channels. The red line locates at t=0. The positive correlation is on the right-hand side, while the negative correlation is on the left-hand side.
Either the positive correlation or the negative correlation can be considered as time traces from the active shot at Ch.1. The remaining data processing procedures (dispersion analysis and inversion analysis) are identical to that of active MASW, as discussed in Chapter 6.

### 7.3 Dispersion analysis

Take the negative correlation as an example. First, select a beam region that contains dispersive Rayleigh waves, as shown in Figure 7.4. Second, apply the f-k transformation to this beam region to obtain dispersion image, as shown in Figure 7.5. Three clear modes can be picked from the dispersion image.

![Figure 7.4](image)

**Figure 7.4.** The illustration of the beam region that contains dispersive Rayleigh waves.
Figure 7.5. The dispersion image of Rayleigh wave from the virtual active source at Ch.1.

7.4 Inversion analysis

The VFSA algorithm is used to invert the $V_S$ structure, the $V_P$ structure and the anisotropy parameters ($\epsilon$ and $\delta$) simultaneously using three modes of Rayleigh wave picked above. The objective function is chosen as Equation (6.8), which measures how well the proposed curves fit the real curves. The results from the inversion of the
dispersion curves of the passive seismic Rayleigh wave after 2000 iterations are shown in Figure 7.6. The corresponding parameters are listed in Table 7.1.

The inversion results indicate that the three picked modes from the passive Rayleigh wave correspond to M2, M4 and M5. The estimated depth of bedrock is 38.5 m, which is very close to the depth (125 ft = 38.1 m) from drilling logs as shown in Figure 7.7. As the result, the multi-mode Rayleigh wave from ambient noise can be used to invert velocity structures. One addition step before dispersion analysis is to synthesize virtual active Rayleigh waves from ambient noise using the principle of wave interferometry.

Furthermore, three conclusions can be drawn for Rayleigh wave dispersion in the VTI medium. First, the shape of dispersion curves of Rayleigh waves is more complicated than that of Love waves. The cut-off frequency of the fundamental mode (M0) is usually zero. The lower and upper asymptotic trends of M0 are approximately bounded by two shear velocities (Vs) of soil and bedrock. Second, the effect of anisotropic $V_p$ structures can be observed directly on dispersion curves, especially on the higher-modes, as shown in the blue circle in Figure 7.6. Therefore, the phase velocities of the irregular region can be used approximately estimate the $V_p$ structure of soil. Third, anisotropy has more effect on higher modes than lower modes, which is consistent with the synthetic example in Chapter 3.

According to the definition of $\varepsilon$, (2.24), the negative value of $\varepsilon$ indicates that $V_{PV}$ is larger than $V_{PH}$, or the longitudinal modulus in the vertical direction is larger than the
horizontal direction. In the other word, the earth is easier to compress in the horizontal direction at the site of Haddam Meadows State Park, with the assumption of 2-layer VTI model.

The anisotropy parameter $\varepsilon$ describes the difference between $V_{PV}$ and $V_{PH}$, while the anisotropy parameter $\delta$ is a combination of $V_{PH}$, $V_{PT}$, $V_{PV}$ and $V_{SV}$, which is difficult to extract any physical meaning from it. Both $\varepsilon$ and $\delta$ can be determined by Rayleigh waves.

Due to the lack of drilling logs about anisotropy at the site of Haddam Meadows, we have to compare other earth parameters. The good match of bedrock depth with drilling logs verifies that the VFSA inversion of multi-mode Rayleigh waves from ambient noise can estimate the velocity structure accurately. In addition, this method also provides information about anisotropy, which will be useful in other applications in the future. Further study or lab experiments are needed to assess the estimated anisotropy structure.
Figure 7.6. The estimation of $V_S$ structure and $V_P$ structure from the Rayleigh wave generated by ambient noise at the Haddam Meadows State Park. On the left panel, solid curves represent the inverted $V_{SV}$ model; dash curves represent the inverted $V_P$ model in three directions ($V_{PH}$, $V_{PV}$ and $V_{PT}$) after 2000 iterations of VFSA algorithm. The corresponding dispersion curves are shown on the right panel. The picked three modes correspond to M2, M4 and M5.

Table 7.1. Estimated parameters from the inversion of Rayleigh wave generated by ambient noise at the Haddam Meadows State Park

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (km)</th>
<th>$V_{SV}$ (km/s)</th>
<th>$V_{PH}$ (km/s)</th>
<th>$V_{PV}$ (km/s)</th>
<th>$V_{PT}$ (km/s)</th>
<th>$\varepsilon$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>0.0385</td>
<td>0.110</td>
<td>0.430</td>
<td>0.603</td>
<td>0.515</td>
<td>-0.49</td>
<td>-0.28</td>
</tr>
<tr>
<td>Layer 2</td>
<td>Inf</td>
<td>1.393</td>
<td>2.643</td>
<td>3.241</td>
<td>2.970</td>
<td>-0.33</td>
<td>-0.16</td>
</tr>
</tbody>
</table>
Figure 7.7. a) The map of Haddam Meadows site and locations of three wells; b) Drilling logs from corresponding three wells (A) JL-1; (B) JL-2; and (C) JL-3.
Chapter 8. Seismic Velocity Structure in Greenland Firn

Seismic refraction has also been used in snow and ice investigations to determine the mechanical properties of shallow snow and ice (e.g., Bentley and Kohnen, 1976; Kirchner and Bentley, 1979; Kirchner et al., 1979). Seismic refraction measurements are faster and less expensive than core drilling to these depths, and are well suited to reconnaissance and surveying over a large number of sites. However, traditional seismic refraction method only targets to get the P-wave velocity structure and ignored the existence of the shear waves. This is because that the refraction survey sources are fraught with difficulty in producing shear waves in polar firn. Nevertheless, in most surface seismic surveys when a compressional wave source is used, more than two-thirds of total seismic energy generated is imparted into the P-SV polarized motion surface waves, i.e., the Rayleigh waves. Rayleigh waves propagate within one wavelength from the surface and their dispersive characteristics can be used to estimate shear-wave velocities (Vs) of the near-surface (Doyle, 1995).

This chapter focuses on the inversion of velocity structure in Greenland firn, using active multi-mode Rayleigh waves. First of all, the mechanic properties of firn are discussed. Then the active MASW approach is utilized to estimate the velocity structure in Greenland firn. As described in previous chapters, dispersion analysis is used to extract multi-mode Rayleigh waves generated from a seismic shotgun. According to the special
mechanic properties of polar firn, it is feasible using four parameters to invert $V_s$ and $V_p$
structure up to depth of 150 m.

### 8.1 Mechanic properties of polar firn

In polar firn a relatively simple but rather accurate empirical relations can be established to characterize the variation of firn density and velocity with respect to the increase of depth (Kohnen, 1972). Both the variations of density and velocity are of the power-law form, which can be expressed as

\[
\begin{align*}
\rho(z) &= \rho(1)(z)^b \rho \\
v_s(z) &= v_s(1)(z)^b s \\
v_p(z) &= v_p(1)(z)^b p
\end{align*}
\]

(8.1)

where $v_s(1)$, $v_p(1)$ and $\rho(1)$ are the shear wave velocity, P-wave velocity and density respectively at the depth of $z=1$ m; $bs$, $bp$ and $b\rho$ are the corresponding power indices in their relationship. Figure 8.1 illustrates the variation of the density, $V_s$ and $V_p$ as a function of depth. Due to the fact that the MASW approach is not sensitive to the density, the variation of density with respect to the depth in firn is supposed to be well known.
Figure 8.1. Variation of the density, $Vs$ and $Vp$ as a function of depth in polar firn.

8.2 Data acquisition

Seismic refraction measurements were conducted at Dye 2 to investigate the velocity structure of polar firn layers (Figure 8.2). A 24-channel engineering seismometer with 4.5-Hz vertical geophones was used to acquire the seismic refraction data. The sampling rate is 5000 per second so that the sampling interval is 0.2 ms. The geophone spacing is 3 m. Field data acquired from an active source is shown in Figure 8.3. The increasing velocity with respect to depth can be observed.
Figure 8.2. The seismic survey lines at the DYE-2 site, Greenland (Photo courtesy of Don Albert).

![Seismic survey lines at DYE-2 site](image)

Figure 8.3. Field data acquired from an active source in Greenland firn.

![Field data chart](chart)
8.3 Dispersion analysis

Select a beam region (Figure 8.4) and apply the f-k transformation to the dispersive Rayleigh waves. The dispersion image is shown in Figure 8.5. Two modes with strong amplitudes can be easily extracted and be considered as the real dispersion curves during inversion.

Figure 8.4. The illustration of the beam region that contains dispersive Rayleigh waves in Greenland firn.
Figure 8.5. The dispersion image of Rayleigh wave from the active source in Greenland firn.

8.4 Inversion analysis

As the $V_S$ and $V_P$ have a special structure in Greenland, \textit{i.e.}, the power-law form. Two parameters, velocity at $z=1$ m and the power index, can identify the variation. Therefore, four parameters ($V_S(1)$, $b_S$, $V_P(1)$ and $b_P$) are enough to characterize different $V_S$ and $V_P$ structure, no matter how many layers we are going to model the polar firn. The reduction of inversion parameters will improve the VFSA algorithm significantly. For example, suppose we use a 20-layer model to describe the polar firn and there are two unknown parameters ($V_S$ and $V_P$) in each layer. Therefore, we have to invert 40
parameters totally. All 40 parameters will be generated randomly and respectively at each iteration, which is time-consuming. However, four parameters ($V_S(1)$, $b_s$, $V_P(1)$ and $b_p$), which control the power-law form, are enough to invert the $V_S$ and $V_P$ in Greenland firn.

The inversion results from active Rayleigh waves after 260 iterations are shown in Figure 8.6. We can find that the VFSA have a good convergence after 260 iterations, which is faster than the cases in previous chapters (1000 or 2000 iterations). Besides, the inverted four parameters are listed in Table 8.1. The results verify that it is feasible using four parameters to invert $V_S$ and $V_P$ structure in Greenland firn up to depth of 150 m.

Figure 8.6. The estimation of $V_S$ structure and $V_P$ structure from the Rayleigh wave generated by the active source in Greenland firn. On the left panel, blue curves represent the initial model; black curves represent the inverted model. The corresponding dispersion curves are shown on the right panel. The picked two modes correspond to M0 and M1.
Table 8.1. Inverted four parameters in Greenland firn.

<table>
<thead>
<tr>
<th>$V_s(1)$</th>
<th>$b_s$</th>
<th>$V_p(1)$</th>
<th>$b_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8232</td>
<td>0.1696</td>
<td>1.1120</td>
<td>0.1969</td>
</tr>
</tbody>
</table>

The $V_p$ structure can be accurately determined by the classical Weichert–Herglotz–Bateman (WHB) integral (Slichter, 1932; Stoll, 2002). WHB is particularly suitable for use in the case of polar firn because the material does exhibit a continuous increase of velocity with depth. The inverted $V_p$ using WHB method is more accuracy than dispersion method since we only use first two modes in dispersion method. The features of $V_p$ in deep may be reflected on higher modes. More higher modes should be included in the inversion to obtain an accurate $V_p$ structure. However, it is very difficult to extract multi-mode (mode number larger than 5) dispersion curves in reality. They are a pair of conflicts. Further work is need to study how to extract multi-modes as many as possible.

Inversion using dispersion curves is more sensitive to $V_s$ structure than $V_p$ structure. The estimated $V_s$ structure is more reliable. As the result, we can combine $V_s$ from dispersion method and $V_p$ from WHB method to continue the study of material properties in Greenland firn. Figure 8.8 illustrates the Poisson’s ratio based on the combination of WHB method and dispersion method, where $V_p$ is from WHB method and $V_s$ is from dispersion method.
Figure 8.7. The $V_S$ structure by dispersion method (blue curves) and the $V_P$ structure by WHB method (black circles) in Greenland firn.

Figure 8.8. The Poisson’s ratio in Greenland firn. $V_S$ is obtained by dispersion method; $V_P$ is obtained by WHB method.
As the final part of inversion analysis, this thesis will take the case of Greenland firn as an example to show how picking errors affect inversion results. In the previous applications, multi-modes are picked manually from the dispersion image, at the locations with strongest amplitude, shown as white dots in Figure 8.9. However, the width of the region with the strongest amplitude contains several pixels, i.e., the mode is “fat” and it is not a single curve. As the result, it is easy to introduce errors when picking modes. Green dots are along the boundary of the strongest-amplitude region. They correspond to the two modes with maximum picking errors.

![Figure 8.9](image.png)

**Figure 8.9.** Picking errors on the dispersion image. White dots represent two “ideal” modes used in previous inversion; green dots represent two modes with maximum picking errors.
The inversion results from the modes with and without picking errors are compared in Figure 8.10. On the left panel, the inverted $V_S$ and $V_P$ from two modes without picking errors are plotted in red color, while the inverted $V_S$ and $V_P$ from two modes with picking errors are plotted in blue color. Their corresponding dispersion curves are illustrated on the right panel with the same color. Two conclusions can be drawn after comparison. First, picking errors have more effect on $V_P$ than $V_S$. Again, it verifies that the surface wave dispersion method is more sensitive to $V_S$ structure than $V_P$ structure.

![Figure 8.10](image.png)

**Figure 8.10.** The comparison of inversion results from the modes with (blue) and without (red) picking errors. On the left panel, solid curves represent inverted $V_S$ and dash curves represent inverted $V_P$. 

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Chapter 9. Summary

In general cases, materials always possess certain degree of anisotropy. The increasing accuracy and scale of field observations in many geophysical problems require for the inclusion of consideration of anisotropy.

All the information of seismic anisotropy is contained in the stiffness matrix described in the generalized Hooke’s law. There are 21 independent elastic constants in the stiffness matrix for the most general case of anisotropy. The number of independent components can be further reduced by symmetric assumption for the medium. The vertical transverse isotropic (VTI) medium is a commonly-used assumption, not only because of its simplification, but also the accuracy approximation to many geological cases in reality. The number of independent components is 5 for the VTI medium. There are many ways to measure the anisotropy. In this thesis, the anisotropy parameters (\( \varepsilon, \delta \) and \( \gamma \)) defined by Anderson (1986) are used. They are combinations of five elastic constants. Among all three parameters, \( \gamma \) can be determined by Love waves since \( \gamma \) describes the difference between \( V_{SH} \) and \( V_{SV} \); while \( \varepsilon \) and \( \delta \) can be determined by Rayleigh waves since their definitions include \( V_{PH}, V_{PT}, V_{PV} \) and \( V_{SV} \). This conclusion also indicates the feasibility to invert the anisotropy structure using surface waves.

Surface waves are dispersive. Dispersion curves illustrate the relationship between frequencies and phase velocities. Several conclusions can be drawn from synthetic
examples and real world examples. First, dispersion curves contain multi-modes, which appear in the dispersion image above their corresponding cut-off frequencies. Second, the shape of Love wave dispersion curves is simple but Rayleigh waves have complex shape. For example, cut-off frequencies of Love wave have similar frequency-intervals. Third, the lower and upper asymptotic trends of dispersion curves are approximately bounded by lowest $V_s$ and highest $V_s$, which can be used to estimate $V_s$ in the soil and $V_s$ in the bedrock. Forth, anisotropy has more effect on higher modes than lower modes. As a result, in order to get an accuracy estimation of anisotropy, higher modes should be included in the inversion part.

The inversion using VFSA shows several advantages: 1) its ability to find the global minimum; 2) Independence of the initial model; 3) good performance dealing with parameters of different ranges and sensitivities; 4) Less (or no) human interference is needed between iterations. The disadvantage is its slow convergence. Synthetic examples and real world examples shows the feasibility and effectiveness of VFSA inversion to estimate the velocity structure and anisotropy structure of the earth.

The dispersion curves of real data can be extracted from either active sources or passive sources (ambient noise). The f-k transformation is used to obtain the dispersion image. In order to extract clear modes, avoid computational artifacts and spatial alias, a beam region on the field data should be selected carefully. If the field data are ambient noise, one addition step is necessary to synthesize virtual active waves using the principle
of wave interferometry. If the field data are acquired from a special earth structure, e.g., Greenland firn with the velocity structure of the power-law form, the number of inversion parameters could be reduced significantly to improve the efficiency of VFSA inversion.

Inversion use surface wave dispersion is more sensitive to $V_S$ structure than $V_P$ structure; so the estimated $V_S$ structure is more reliable. On one side, higher modes are needed to invert accurate velocity structure and anisotropy structure. On the other side, it is very difficult to extract multi-mode (mode number larger than 5) in reality. They are a pair of conflicts. Further work is need to study how to extract multi-modes as many as possible.
Bibliography


