The Wave Boundary Layer Over the Open Ocean and the Implications to Air-Sea Interaction

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Wave field and atmospheric observations during the Southern Ocean Gas Exchange experiment 2008 were used to explore air-sea boundary layer dynamics. The closure of a momentum budget at the air-sea interface allows the selection and tuning of a wave growth parameter consistent with the observed conditions. An energy balance between the atmospheric energy input and the observed wind-wave spectral energy is posed based on the turbulent kinetic energy budget. The energy input is defined as the rate of work done by the wave-induced stress over the wind velocity profile. Wave induced perturbations on the airflow are modeled by an exponential decay function with a variable dimensional decay rate ($A$ m$^{-1}$). Wave-induced perturbations are incorporated into the atmospheric input term to account for the wind-wave coupling. The decay rate is tuned iteratively by minimizing the difference between the input and the wind-wave spectral energy. Under weaker forcing the model works within 40-45%. It is hypothesized, that this is due to long-wave modulation and an upward ocean–atmosphere momentum flux. Under stronger forcing ($i.e.$ $0.4 < u^* < 0.9$ m s$^{-1}$) results are within 10-20% predicting progressively slower decay rates ($A \sim 0.5 \pm 0.4$ m$^{-1}$). This suggests that longer waves support the wave-induced momentum flux, extending the depth of the wave
boundary layer to an average height of 2 m inducing stronger perturbations on the airflow. Under weaker forcing the model suggests that wind and waves become uncoupled exhibiting a shallower wave boundary layer.
The Wave Boundary Layer over the Open Ocean and the implications to Air-Sea Interaction

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Doctor of Philosophy Dissertation

The Wave Boundary Layer over the Open Ocean and the implications to Air-Sea Interaction

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1. Introduction to Air-Sea interaction

The lower marine atmosphere and upper ocean share a common interface and studies of air-sea interaction require investigation of processes that act across the coupled boundary layers. Wind related forcing drives waves and currents and heat exchange between the atmosphere and ocean. Large-scale atmospheric and oceanic circulation is what ultimately impacts the shorter local scales. Fluxes of energy, momentum, moisture, and trace gases are critical components of the Earth’s climate system, regulating geophysical cycles on Earth. Therefore, an improved understanding the atmosphere-ocean system is of critical importance to successful modeling and prediction efforts of the scientific community.

A great deal of attention has been placed on the role of anthropogenic greenhouse gases in climate change. Carbon dioxide (i.e. CO$_2$) is a prominent greenhouse gas and the consensus of the scientific community is that the oceans serve as a global sink of anthropogenic CO$_2$. However, this global sink is the result of a small difference between much larger source and sink terms and numerous factors could tip the balance the other way. Therefore, accurate parameterization of CO$_2$ transfer across the air-sea interface is a key component to climate models. Other climate relevant gases that are exchanged between the atmosphere and the ocean, include methane, ozone and DMS. All of them impact the climate system with further implications in global ecosystems, and planetary dynamics (e.g., zonal winds and cloud formation). As the overall physics of the gas transfer and air-sea interaction become clearer under a wider range of physical conditions, the overall predictive capabilities of global climate model forecasts, and
ultimately predictions of climate change will improve. However, the wide range of physical conditions makes global parameterizations hard to achieve and limited observations make validation an ongoing challenge.

1.1 Air-Sea Interaction and Surface Waves

Of particular interest are the often-neglected wave effects on heat, mass and momentum transfer. Wave generation and evolution has been called the “gear box” of the air-sea interaction phenomenon. For example, as the wave field evolves (i.e. through wave grow and decay), the waves have the potential to interact and modify the local wind flow (e.g., through surface roughness, flow separation and sheltering). Waves perturb the near surface atmospheric flow and transfer momentum and energy to the water column by wave breaking along with wind generated shear and Langmuir circulation.

The evolution of the wave field is explored and parameterized with the different terms in the wave action equation:

\[
\frac{\partial \Phi}{\partial t} + c_g \nabla \Phi = S_{in} + S_{diss} + S_{nl} \tag{1}
\]

where \( \Phi \) is the wave spectrum, \( c_g \) is the group phase speed, \( S_{nl} \) is the non-linear wave-wave interaction term (e.g., Hasselmann et al. 1973), and \( S_{diss} \) corresponds to the dissipation term due to large and small-scale wave breaking (e.g., Melville, 1994) and \( S_{in} \) is the wind input term (e.g., Donelan et al. 2006). The wind input \( (S_{in}) \) at any time corresponds to a fraction of the total energy of the wave field, where the energy flux to the waves \( (E_{aw}) \) and the wind input are related by:
\[ E_{aw} = \rho_w g \int S_{in}(\omega) d\omega = \rho_w g \int \omega \beta(\omega) \Phi(\omega) d\omega \]  \hspace{1cm} (2)

where \( \rho_w \) is the water density, \( g \) is the gravitational acceleration and \( \omega \) is the angular frequency. The fractional energy increase associated only with the wind input is captured via the wave growth parameter:

\[ \beta(\omega) = \frac{1}{\omega \Phi(\omega)} \frac{\partial \Phi(\omega)}{\partial t} \]  \hspace{1cm} (3)

The air-sea interaction and connection between the two fluids is captured via the input term and the wave growth parameter. One of the first wave generation theories was the sheltering hypothesis by Jeffrey (1925). Jeffrey’s sheltering theory suggested the existence of a pressure difference along the waves. This pressure difference is caused by a pressure drop on the lee side of the wave due to the wave blocking the air-flow (i.e. the sheltering effect). Later laboratory studies suggested that the pressure drop due to sheltering was too small to fully account for the energy into the wave field and explain wave growth.

Miles (1957) and Phillips (1957) complementary resonance theories are considered the foundation of the wave generation theories. Phillips (1957) considered the turbulent pressure fluctuations over the waves. The Miles (1957) approach focused on the interaction of the wave induced perturbations and the free surface, leaving turbulence only to support the wind velocity profile. Combination of the theoretical approach of the Miles critical layer theory and field campaigns during the 1970’s resulted in parameterizations of the wind-input source function (i.e. \( S_{in} \)) that provide good results in operational wave models (Cavaleri et al., 2007). The Miles theory is mainly criticized because it neglects turbulent perturbations in the transfer mechanism. Experimental studies (e.g., Hasselmann and Bösenberg, 1991; Snyder et al. 1981) show the Miles
theory predicting lower values of energy transfer than the observations. Nonetheless
observations and theory agree in the order of magnitude, suggesting the validity of the
approach.

Fully turbulent theories were developed to improve Miles approach. These fully
turbulent theories define an inner and outer layer. The division into layers is
characteristic in fluid dynamics in an effort to define boundary conditions and simplify
the transport equations \(i.e.\) Navier-Stokes equations). The inner layer is defined as
being adjacent to the interface where turbulent transport dominates the exchange and the
outer layer is assumed inviscid. Belcher and Hunt (1993) identify a non-separated
sheltering effect associated to the thickening of the boundary layer on the lee side of the
wave. It is the thickening of the boundary layer that is associated with a pressure
asymmetry and ultimately wave growth.

Wave growth parameterizations and input functions are key elements in the ongoing
exploration of energy, momentum and mass transport between the atmosphere and
ocean. Full theories incorporating air-sea interaction and wave dynamics are explored in
wave boundary layer (WBL) models. The more common models rely on bulk formulas
and parameterizations. These correspond to empirical formulations (wind and wave state
dependent, \(e.g.,\) Fairall \textit{et al.} 1996, 2003; Taylor and Yelland, 2000; Edson \textit{et al.} 2013).
Bulk formulations have the advantage of being observation driven providing a robust
relation between the observed fluxes and their associated means through key variables
such as the drag coefficient and gas transfer velocities (section 1.3.1). The disadvantage
lies in the lack of detailed information about the structure of the WBL and not
accounting for a wind-wave feedback. In an effort to add information to the WBL,
theoretical approaches have been developed. These are numerically and analytically
driven (e.g., Chalikov and Rainchik, 2011; Janssen, 1991; Kudryavtsev et al. 1999;
Makin and Kudryavtsev, 1999) with some of them including previous observations (e.g.,

These WBL models attempt to incorporate the wind-wave/wave-wind feedback
present at the air-sea interface. The feedback is related to the partition between wave
induced and turbulent momentum fluxes, which ultimately depends on the state of the
wave field driven by the wind input (i.e. wave generation) and evolution (i.e. wave-wave
interaction and wave breaking). The work presented here develops a simple WBL model
relying on the conservation of momentum and energy. A theoretical set of assumptions
is used to account for the wave induced perturbations on the airflow. These assumptions
are ultimately validated with direct observations under the open ocean.

1.2 Boundary Layers

At a Mathematical congress in 1904, Ludwig Prandtl first introduced the concept of
a boundary layer (Prandtl, 1904), where he showed the flow past a rigid body could be
divided into regions or layers. A very thin layer was defined to be in direct contact with
the object. This immediate layer above the object contains all of the viscous effects of
the fluid interacting with the surface. The outer region above the viscous boundary layer
was defined to be inviscid (i.e. no viscous effects). Classic fluid dynamics deals with
rigid surfaces and fluids of relatively low viscosity (e.g., air and water), which follow
the Newtonian behavior. These flows are often characterized by the Reynolds number,
\( Re \), which is defined as the ratio of the inertial to viscous forces. At sufficiently high
Reynolds numbers the flow becomes fully turbulent. Therefore, flows over a rigid surface exhibit three distinct regions: an outer flow (assumed inviscid and turbulence free), a turbulent inner flow characterized by a high $Re$, and a very thin region directly at the surface known as the **viscous sublayer**, where $Re$ is low and viscous effects are again relevant. Within the inner layer the turbulent effects generate an **apparent turbulent friction** (i.e., as opposed to viscous friction), which overcomes any viscous influence and dominates the stress. These apparent frictional forces result from turbulent perturbations on the flow. For example, the vertical transport of horizontal momentum is given by the stress vector:

$$\vec{\tau} = -\rho \left[ i \ u' \ w' + j \ v' \ w' \right]$$  \hspace{1cm} (4)

where $\rho$ is the density of the fluid, and $u'$, $v'$, and $w'$ are the turbulent perturbations (hereinafter all turbulent perturbations are denoted by primes) of the velocity vector in the along-wind (defined as the $i$ direction), cross-wind (the $j$ direction) and vertical direction, respectively. Equation (4) represents the Reynolds stresses (named after O. Reynolds who introduced the concept). The Reynolds stresses arise from the decomposition of the velocity vector into mean and turbulent perturbations, for example the along-wind component of the velocity vector ($u$):

$$u(t) = \bar{u} + u'(t)$$  \hspace{1cm} (5)

where the overbar ($\bar{u}$) denotes the mean velocity of the flow.
1.2.1 The Rigid Boundary Layer and the Atmospheric Velocity Profile

A well-known example of turbulent flows relevant to all boundary layer flows adjacent to a wall is known as the Couette Flow. The Couette flow corresponds to a simple two-dimensional pure shear flow with a uniform stress distribution. The problem is set for a fluid between two plates, where the lower plate is at rest and the upper plate moves at constant velocity. This generates a uniform shear throughout the fluid once the flow is fully developed. In a two-dimensional coordinate system with the vertical axis defined by $z$ and the horizontal by $x$, the objective becomes the definition of velocity profile $u(z)$.

For a two-dimensional flow over a rigid plate, the conservation of mass for an incompressible fluid (a more rigorous description can be found in Schlichting et al. 2000, 1968) is given by the following form of the continuity equation:

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial z} = 0$$

The Couette flow is driven by a constant stress applied at the top of the incompressible fluid. This is the only driving force with no pressure gradients. The mean momentum equation in the $z$ direction then reads as:

$$\bar{w} \frac{\partial \bar{u}}{\partial z} = \frac{\partial}{\partial z} (\tau^v + \tau^t)$$

where $\tau^v$ and $\tau^t$ are the viscous and turbulent components of the shear stress for a Newtonian fluid the viscous stress is:

$$\tau^v = \rho \nu \frac{\partial \bar{u}}{\partial z}$$

and the Reynolds stress is:

$$\tau^t = -\rho \bar{u}' \bar{w}'$$
where $\nu$ is the kinematic viscosity of the fluid. For a fully developed and horizontally homogeneous flow, $\partial u / \partial x = 0$ and from the continuity equation all inertial terms vanish. The shear stress applied at the upper moving plate ($\tau^p$) keeps the plate moving at a constant velocity leading to the following force balance (or from equations 6 and 7 under given conditions):

$$\tau^p = \tau^t + \tau^r$$  \hspace{1cm} (10)

The following dimensionless variables can be defined as:

$$\eta = \frac{z}{H} ; \quad u^* = \frac{\bar{u}}{u_c} ; \quad Re = \frac{\bar{u} H}{\nu} ; \quad \tau^t_r = \frac{\tau^t}{\rho u_c^2}$$

where $H$ is the height at which the moving plate is set, $Re$ is the Reynolds number for this flow, and $u^*$ is the friction velocity; equation (9) can be written as:

$$\frac{1}{Re} \frac{du^*}{d\eta} + \tau^t_r = 1$$  \hspace{1cm} (11)

Following from dimensional analysis alone, the friction velocity is defined as:

$$u_c = \sqrt{\frac{\tau^p}{\rho}}$$  \hspace{1cm} (12)

and represents the velocity scale of the turbulence. The purely shear driven flow is defined as the core layer in the Couette flow. This classic result extends to planetary boundary layers and it is used to define the depth of the atmospheric boundary layer. In the near surface layer (analogous to the core layer) the magnitude of the stress is constant \textit{i.e.} varies by less than 20\% (Lumley and Panofsky (1964); implying a fully shear driven flow with a constant turbulent momentum flux. The velocity profile can be determined by acknowledging that equation (10) does not satisfy the condition of $\tau^t_r = 0$ at the surface requiring the definition of another layer, which is called a wall layer. A
matching condition is introduced by requiring that the velocity profile be continuous across these layers:

\[
\frac{du^*}{d\eta} = \frac{1}{\kappa \eta}
\]

(13)

where \(\kappa\) is the Karman constant (after v. Karman 1930). From the wall layer perspective:

\[
\frac{du^*}{dz^*} = \frac{1}{\kappa z^*}
\]

(14)

where \(z^*\) is the dimensionless height of the wall layer \((z^* = z/\delta_v)\) and \(\delta_v\) is the depth of the purely viscous layer. Integration of (13) or (14) leads to a logarithmic velocity profile, characteristic of all turbulent flows with finite shear. This is called the universal law of the wall.

From equation (14) it follows:

\[
u^+(z) = \frac{1}{\kappa} \ln(z^*) + C
\]

(15)

where \(C\) is a constant of integration that is defined by the roughness of the wall.
The classic logarithmic velocity profile applies to the *wall layer* of geophysical fluids. The planetary boundary layer velocity profile can also be derived under similarity considerations. Similarity considerations lead to the assumption that close to the *wall* the velocity profile can only depend on the height above the surface and the friction velocity. It then follows that:

\[
\frac{d\bar{u}}{dz} = \frac{\bar{u}_*}{\kappa(z + z_o)}
\]  

(16)

where \(z_o\) corresponds to the roughness length (or the *aerodynamic* roughness length). The roughness length is a theoretical height at which the logarithmic velocity profile goes to zero. The roughness length physically accounts for the length scale of individual roughness elements on the surface. The order of magnitude of the roughness length over the ocean is within millimeters (i.e. \(O(10^{-3} \text{ to } 10^{-4})\) m). The friction velocity is the atmospheric friction velocity and its definition is given by equation (12), where the total
shear is wind driven. Equation (16) is derived solely on similarity considerations and corresponds to a classic approach to the logarithmic velocity profile found on many geophysical fluid texts (e.g., Lumley and Panofsky 1964; Krauss and Bussinger 1994).

Integration of (16) leads to a logarithmic velocity profile analogous to (15):

$$\bar{u}(z) - \bar{u}(0) = \frac{u_*}{\kappa} \ln\left(\frac{z + z_0}{z_o}\right)$$

Equation (17) is the logarithmic velocity profile at the planetary wall layer under neutral conditions (no buoyancy effects). The planetary wall layer is commonly referred to as the surface layer or “constant” flux layer.
1.2.2 Rigid Boundary Layer and the Turbulent Kinetic Energy (TKE) Budget

Large Reynolds numbers and turbulence intensities often characterize geophysical fluid flows. This generates turbulent perturbations on the fluid that are of particular interest. Investigation and quantification of the turbulent flow is often undertaken using the turbulent kinetic energy (TKE) budget. For steady state, horizontally homogeneous and neutral conditions (no buoyancy) the TKE budget above a rigid surface is given by:

\[
\begin{align*}
- \overline{u'w'} \frac{\partial \bar{u}}{\partial z} - \frac{\partial}{\partial z} \left( \frac{w'p'}{\rho} + \overline{w'w'} \right) - \varepsilon &= 0 \\
(18)
\end{align*}
\]

The term \( p \) corresponds to pressure fluctuations and \( \varepsilon \) is the turbulent kinetic energy (\( \varepsilon = 0.5 [(u')^2 + (w')^2], \) with \( v' = 0 \)). The first term on the left hand side of (26) corresponds to the mechanical generation of turbulence (shear driven). The second term neither produces nor consumes TKE and only acts to redistribute it. The third term represents the viscous dissipation of TKE.

A very important result is obtained in a neutrally stable wall layer, where it was shown by Wyngaard and Cote (1971) that the TKE budget can be reduced to:

\[
\begin{align*}
- \frac{\partial \bar{u}}{\partial z} \overline{u'w'} &= \varepsilon \\
(19)
\end{align*}
\]

implying a balance between turbulent production and dissipation. In a turbulent flow away from the pure viscous layer: \( \tau_{\text{total}} / \rho_a = \overline{u'w'} = u_*^3 \) leading to:

\[
\varepsilon = \frac{u_*^3}{\kappa (z + z_o)} \\
(20)
\]

for a wall layer type of behavior, where the velocity profile is logarithmic.
1.2.3 The Effect of Stratification

Buoyancy effects are incorporated by the addition of the buoyancy flux ($w'b$) term to the TKE budget introduced in Equation (18):

$$\frac{-\partial u}{\partial z} w'w' + w'b - \frac{\partial}{\partial z} \left[ \frac{1}{\rho_a} w'p' + \epsilon w' \right] - \varepsilon = 0$$ (21)

where $b$ corresponds to the buoyancy and $w'b$ is the buoyancy flux which will suppress or enhance the TKE depending on its direction (given by the sign). A positive term will enhance the TKE, whereas a negative term will suppress it. In the atmospheric surface layer, the buoyant term is approximately constant (Kraus and Businger, 1994), whereas the shear production decreases with height as shown in Figure 1.

The Obukov length ($L$) was defined (Obukhov, 1946) to quantify the buoyancy effects on turbulent flows. Obukov equated the pure shear production to the buoyant production leading to:

$$L = -\frac{u_3^3}{kw'b}$$ (22)

The Obukov length is used to describe the structure of the turbulence in the surface layer. For example, in unstable flows, the height at which $z = -L$ defines height where shear production equals buoyant production of TKE. Shear production dominates below this height and buoyant production becomes dominant above. As such, $z/L$ is very similar to the flux Richardson number commonly used in oceanography.

Monin-Obukov similarity theory (MOS) states that turbulence statistics are universal functions of $z/L$ implying that thermal and or mechanical sources of turbulent fluxes across the boundary layer can be stated purely in terms of $z/L$. MOS has been validated over land by several independent studies (e.g., Wyngaard and Cote 1971) among many
others. It is important to note that over the ocean, the MOS can only be applied above the influence of waves. This is because similarity arguments do not include wave-induced pressure and velocity perturbations (e.g., Edson and Fairall, 1998).

1.3 The Boundary Layer over the Ocean

Over the ocean surface, waves make the air-sea interface strongly deviate from the rigid wall previously presented in the boundary layer introduction. Waves have the potential to influence the adjacent flow adding wave-induced perturbations (hereinafter denoted by a tilde) to the already existing turbulent perturbations:

\[ u(t) = \bar{u} + u'(t) + \tilde{u}(t) \]  

(23)

where \( \tilde{u} \) is the wave-induced along-wind component of the horizontal velocity. Analogous to the turbulent stress defined by the turbulent velocity perturbations in equation (7), the wave-induced stress follows from wave induced perturbations on the velocity components:

\[ \tau^w = -\rho_a \bar{u} \tilde{w} \]  

(24)

where the \( \tilde{w} \) is the wave induced perturbation on the vertical velocity. This term modifies the overall momentum balance in the surface layer above the wavy interface (i.e. in the core layer), where the total stress is now:

\[ \tau_{total} = \rho_a u'^2 = \tau'(z) + \tau^v + \tau^w(z) \]  

(25)

The total stress (\( \tau_{total} \)) is still constant throughout the atmospheric boundary layer, but now it is partitioned into a turbulent and wave induced component. The viscous term becomes relevant only in the purely viscous layer (\( \delta_v \)) for heights \( z < z_v \).
The KE budget for a homogeneous neutral flow over the ocean has then to be modified to include the presence of the wave induced pressure and velocity perturbations on the KE budget:

\[
- \frac{\partial u'}{\partial z} \left[ u' w' + \bar{u} \bar{w}' \right] + \frac{\partial}{\partial z} \left[ \frac{1}{\rho_a} w' p' + \bar{e} w' \right] - \frac{1}{\rho_a} \frac{\partial \bar{w} p}{\partial z} - \varepsilon = 0
\]  

(26)

The third term corresponds to the wave induced energy flux that supports wave growth at the surface (i.e. wave induced energy flux divergence). In the presence of waves at the interface the shear production deviates from the dissipation-production balance. The imbalance leads to a TKE dissipation rate ($\varepsilon$) relative to a wall layer that will depend on the sign of the wave induced flux divergence.

The wave induced velocities and pressure perturbations ultimately define what is referred to as the wind-wave coupling, which becomes a key element to the air-sea interaction phenomenon. How far up into the atmosphere these perturbations protrude is what ultimately defines the wave boundary layer (WBL). This layer is then a region adjacent to the ocean surface overlapping a portion of the surface layer of the purely shear-driven wind flow, where the total momentum flux remains approximately constant (within 20%). In this layer wave-induced perturbations modify the momentum (total stress) and energy equations.

In order to explore potential wave-induced deviations on the log-profile the local friction velocity is defined. The local friction velocity ($u'^*$) reflects the reduction of the turbulent momentum flux in the WBL due to wave effects. For a constant momentum flux (equation 25) the local turbulent plus viscous stress is given by:

\[
\tau^l(z) = \tau'(z) + \tau^v(z) = \rho_a u_*^2 - \tau^v(z)
\]  

(27)
and the local friction velocity follows as:

\[ u'_l(z) = \sqrt{\tau'_l(z)/\rho_a} \]  

(28)

which equals the total stress and friction velocity above the WBL. The viscous component is included because the local friction velocity is used to investigate the wind profile near the interface in section 4.22. This definition maintains a continuous velocity profile across the atmospheric surface layer that takes the form of the traditional semi-logarithmic profile above the WBL.

The local effect of the waves on the logarithmic velocity profile inside the WBL can be determined following the approach of Makin and Kudryavtsev (1999). They assume that the local turbulent plus viscous momentum flux is given by:

\[ \tau'_l(z) = \rho_a u'_l(z)^2 = \rho_a K \frac{\partial u'_l}{\partial z} \]  

(29)

where \( K \) is the eddy diffusivity or eddy viscosity characteristic of turbulent flows. The velocity profile can be written as:

\[ \frac{\partial u'_l(z)}{\partial z} = \frac{(u'_l)^2}{K} = \frac{u'_l^2}{K} \left[ 1 - \frac{\tau''/\rho}{u'_l^2} \right] \]  

(30)

Notice that the shear and the friction velocity \( (u'_l(z)) \) are referred to as local variables and are used to investigate the effect of the waves on the air-flow in the WBL. In order to solve for the velocity profile in Equation 30 and define the shear, the vertical behavior of the wave induced momentum flux \( (\tau''(z)) \) and eddy diffusivity needs to be defined. These represent an effort to capture the wave-induced perturbations on the air-flow above the wave field. The determination of these values is the focus of section 4.2.1 and 4.2.2.
1.4 Implications to Air-Sea Interaction

This investigation focuses on characterization of the wind-wave interaction by accounting for the presence of the waves at the interface. From a kinematic perspective, the velocity profile is modified by a reduced turbulent shear close to the interface. The total momentum flux is now partitioned between the turbulent and wave induced components. From an energetic perspective, investigations have shown that the sign of the energy flux divergence term is related to the state of development of the wave field or wave-age. The wave age is often defined as the ratio between the phase speed of the waves (generally at the spectral peak) and the wind speed at a reference height (generally defined at 10 m). For example, conditions where the 10-m wind speed is blowing faster than the peak phase speed \( \text{\(U_{10} > c_p\)} \) or \( \frac{c_p}{U_{10}} < 1 \) are characterized by young or developing seas. Under these conditions, profiles of TKE dissipation from the Air-Sea Interaction Tower (ASIT) during the CBLAST-LOW program (Edson et al. 2007) and R/P FLIP during the Marine Boundary Layer (MBL) Experiment show how the MOS-predicted values overestimate the observed dissipation rates over growing seas. Simply put, less TKE dissipation is required on the atmospheric side because some of the energy is going into the waves and transmitted to currents by wave breaking. Observations and theory have shown that the departure to MOS is linked to the flux divergence of the wave-induced pressure transport term and the non-zero flux of energy into the growing wave field (Edson et al. 1997; Janssen, 1998). Conversely, a dissipation surplus on the atmospheric side is expected in the presence of decaying seas where swell transfers a fraction of its energy to the atmosphere. This implies that the
inertial dissipation technique \((e.g.,\ \text{Edson\ et\ al.\ 1991})\) over the ocean has the potential to underestimate or overestimate the real stress above the wave field. This is consistent with the observations of fast waves \((i.e.,\ \text{non-locally\ generated\ swell})\) aligned with slower overlying winds \((i.e.,\ \text{cp} > \text{U}_{10}\ \text{or} \ \text{cp}/\text{U}_{10} > 1)\), having the capacity to reduce and even reverse the energy and momentum flux. This effect generates what has been called a wave-driven wind \((e.g.,\ \text{Hanley\ and\ Belcher\ 2008})\) due to an upward wave-induced flux with local effects on the wind flow directly above the waves \((e.g.,\ \text{Shaikh\ and Siddiqui,\ 2011})\). This is also predicted by large eddy simulation (LES), where in-line wind and swell leads to a low-level jet and transfer of energy from waves to wind \((\text{Sullivan\ and\ McWilliams,\ 2010;\ Sullivan\ et\ al.,\ 2008})\).

Wave breaking, due to wave slope instabilities, can also impact the aerodynamics of the flow. The wind flow above the wave field undergoes flow separation and the sea surface becomes rougher under stronger forcing \((e.g.,\ \text{Donelan\ et\ al.,\ 2004;\ Kudryavtsev\ and\ Makin,\ 2001;\ Maat\ and\ Makin,\ 1992})\). Nonetheless under severe forcing a \textit{full separation} condition has been observed \((\text{Donelan\ et\ al.\ 2006})\), where the energy and momentum transfer between wind and waves becomes weaker; the wind and wave field becomes less coupled.

The impact of breaking waves extends below the surface where breaking has the potential to contribute to a TKE enhancement and dissipation surplus on the water side, leading to observed deviations of a \textit{wall layer} type of behavior \((e.g.,\ \text{Melville,\ 1994;\ Terray\ et\ al.,\ 1996})\). The consequences of an enhanced TKE dissipation rate on the waterside vary and are considered extremely relevant to processes such as gas exchange \((e.g.,\ \text{Weber\ 2008})\). The wave generated turbulence structure can be explored by
tracking the TKE dissipation rate profiles in the water column. The depth of the wave-affected boundary layer is usually defined by the depth at which the dissipation rate follows wall layer type of behavior and a production-dissipation balance results. Previous work on this topic has shown sensitivity to scaling arguments and has predicted different decay rates. For example, Craig and Banner (1994) predicted a decay rate following $z^{-3.4}$, which conformed to early observations, whereas data from lakes (e.g., Donelan et al. 1996) predict a $z^{-4}$ depth dependence. A $z^{-2}$ decay rate was supported by the work presented by Gerbi et al. (2009). It is clear that at both sides of the air-sea interface waves have the capacity to alter and modify the flow and overall dynamics resulting in a clear departure from a classic wall-layer-law.

1.4.1 Air-Sea Interaction Parameterizations: Bulk Formulations

Improvement in our understanding of the WBL and overall air-sea interaction dynamics will further elucidate turbulence exchange of momentum, heat and mass that affect wave field evolution and larger scale atmosphere and ocean dynamics. For example, estimates of the momentum exchange between the wind and sea used in global atmosphere-ocean models rely on parameterizations that relate the total surface momentum flux (i.e. the surface stress) to the square of the wind speed through a of the drag coefficient

$$\tau_{\text{total}} = \rho_a C_{DN} U_r^2$$

where $\rho_a$ is the density of air, $U_r$ is the wind speed relative to the ocean surface and $C_D$ is the drag coefficient. Equation 31 corresponds to a bulk formulation of the momentum flux. The drag coefficient has been found to depend on the state of the wave field
through the surface roughness length \((z_o)\). For the logarithmic velocity profile the drag coefficient under neutral conditions \((C_{DN})\) is given by:

\[
C_{DN} = \left( \frac{\kappa}{\ln\left(\frac{z + z_o}{z_o}\right)} \right)^2
\]

(32)

where \(\kappa\) is the von Karman constant and \(z\) is the height above the mean sea level.

Charnock (1955) used scaling arguments to estimate the sea surface roughness as:

\[
z_o = \alpha \frac{u^2}{g}
\]

(33)

where \(g\) corresponds to the gravitational acceleration, and variable \(\alpha\) was originally known as the Charnock constant. This parameterization was intended to effectively model the form drag due to the wind waves. An additional roughness length is often added to model the viscous component (e.g., Fairall et al., 1996; Smith, 1988) such that:

\[
z_o = a_1 \frac{\nu}{u_s} + \alpha \frac{u^2}{g}
\]

(34)

where \(a_1\) is a numerical constant. Equation 34 suggests that the roughness elements of the sea surface are a combination of the purely viscous ones (first term on the right hand side) plus the roughness elements arising from the presence of the wave field (second term on the right hand side).

Experimental observations have shown variability in \(\alpha\) leading to investigations of the potential dependence of the roughness length on the state of the wave field (e.g., Donelan et al. 1993; Johnson et al. 1998)). Early work based on scaling arguments such
as the wave steepness (e.g., Hsu, 1974) and wave age (e.g., Stewart 1974) explored the functionality of the roughness length to the waves. These early parameterizations were given in terms of significant wave height ($H_s$), spectral peak frequency ($f_p$), and peak phase speed ($c_p$), where the peak represents the dominant wave scale for a given wave spectrum. The peak characterization of the wave field works rather well for purely wind seas on lakes, where the swell counterpart is not present. However, agreement between the modeled and measured drag coefficients in the presence of swell under open ocean conditions remains elusive (e.g., Drennan et al., 2003). Although parameterizations of the roughness length through wave field scaling arguments sounds ideal and intuitive, determination of the appropriate scaling is not straightforward. For example, work done by Maat (1991) comments on the importance of and difficulty in capturing the wind-wave peak in order to parameterize the surface roughness elements. However, difficulties in identifying the wind–wave peak make the spectral peak a common and widely use reference for scaling parameters, which sometimes fails to capture the true wind-wave dynamics.

Under open ocean conditions, it is hypothesized that the scales associated with the higher frequency wind-waves are most appropriate when investigating scaling arguments for the WBL dynamics, as opposed to those associated with the spectral peak. For example, work done by Donelan (1982) proposed a roughness length proportional to a wave scale captured by twice the peak frequency ($f = 2f_p$). It is these shorter waves that support most of the momentum transfer from wind to waves, with estimations suggesting that only 10% of the total form drag is supported by waves longer than 10 meters (Makin and Kudryavtsev 1999).
The work presented here focuses on developing an observationally driven WBL model that captures the structure of the wave-induced perturbations above the wave field \((i.e.\) wave induced momentum flux) and the stress partition at the interface. A theoretical approach is complemented with observations made during the Southern Ocean Gas Exchange experiment in 2008. Momentum and energy conservation across the WBL are used as the main constrains. The investigation attempts to define the depth of the WBL and the vertical behavior of the wave induced momentum flux throughout this layer. An effort is made to identify the strongly coupled wind-waves to improve air-sea interaction parameterizations \((i.e.\) classic bulk formulations).

2. Experiment Description

The Southern Ocean Gas Exchange Experiment (SO GasEx) was designed to investigate the processes governing gas exchange at high winds by implementing the dual tracer (Ho \textit{et al.} 2011) and the direct covariance technique (Edson \textit{et al.} 2011). The experiment took place in the southern portion of the Atlantic Ocean between the months of March and April 2008, where a sufficiently large CO\textsubscript{2} concentration difference between atmosphere and ocean was expected \((\Delta \text{CO}_2 > 50\mu\text{atm})\) in order to implement the direct covariance method. Figure 2 presents a seasonal average global circulation pattern based on the Wave Watch 3 (WW3) model. The global circulation is presented in order to interpret the local wave and meteorological data collected during the SO GasEx. Figure 2 includes the predicted significant wave heights (top panel) based on the wind measurements and predictions (bottom panel). From Figure 2 is easy to appreciate the severity of the predicted conditions that dominate the Southern Ocean and adjacent
southern Pacific and Atlantic regions. The wind exhibits a relative homogeneous 
direction with a measured average of $\theta_{\text{wind}} = 278 \pm 68^\circ$. The average dominant wave 
direction was measured to be $\theta_{\text{wave}} = 256 \pm 46^\circ$. Spectrally, the wave field exhibits a 
weakly bimodal spectral distribution, with a wind-wave peak usually buried under the 
swell component of the wave field. This behavior follows from the general circulation 
picture of the system (Figure 2), which predicts, on average, a Westerly wind between 
the sub-tropical high pressure and the sub-polar low-pressure systems. This prevalent 
Westerly wind drives the Antarctic Circumpolar Current through the Drake Passage, 
connecting the Pacific and the Atlantic Ocean as shown in Figure 2. The Westerly 
winds averaged between 10 – 13 m s$^{-1}$ and exert a total stress ranging between 0.12 -0.2 
N m$^{-2}$, with an almost constant eastward direction. With no fetch limitations, the waves, 
winds and currents wrap around this landless stretch of ocean, with ample space and 
time to evolve and develop. It is this relatively constant wind direction that forces a 
wave field with a weakly bimodal spectral distribution. The scale of the dominant 
waves show a frequency range of: $0.08 < f < 0.12 \text{ Hz}$, which is associated with the 
background swell component of the wave field. During storm passages and strong 
forcing the spectra eventually develops a more clearly defined secondary peak in the 
frequency range $0.13 < f < 0.2 \text{ Hz}$. The relative directional homogeneity of the wind 
forcing leads this secondary peak to develop in the same direction ($\pm40^\circ$) as exhibited by 
the swell component, with the wind-waves riding on top of the swell component.
Figure 2: Seasonal circulation patterns (March-April-May) of significant wave height (top panel) and wind speed (bottom panel) based on the model Wave Watch III (WW3) with 31 years of wind forcing. The figure is obtained from Stopa et al. (2012). The red rectangle shows the cruise location during March-April 2008. Wind speed ranging from $5 < U_{10} < 20 \text{ ms}^{-1}$ and significant wave heights ranging $2 < H_s < 8 \text{ m}$.

The atmospheric instrumentation package shown in Figure 3 consisted of three Gill R-3 sonic anemometers for fast response velocity and temperature measurements; two Vaisala PTU200 sensors to measure pressure, temperature and humidity; and three open path Licor 7500 to measure fast response atmospheric water vapor and CO$_2$ concentrations.

The sonic anemometers were paired with inertial measurement units (IMU;
Systron-Donner MotionPaks) to remove platform motion before calculation of the fluxes (Edson et al. 1998). The experiment was carried out with consistently moderate to high winds \((5 \leq U_{10} \leq 18 \text{ m s}^{-1})\) with a severe wind event \((U_{10} > 18 \text{ m s}^{-1})\) encountered at the end of the field campaign (Figure 4, top panel). More details about the meteorological measurements are given below. Wave field measurements were collected to explore gas transfer as a function of the state of the wave field \((i.e.\) wave age, wave breaking, and wave steepness). The wave measurements were also collected in order to assist the investigation of the boundary layer dynamics and the role of the wave boundary layer in gas transfer across the air-sea interface.

The principal instrument system for collecting wave field statistics was the Wave Monitoring System II (hereinafter WaMoS), mounted on the flying bridge of the NOAA ship Ronald H. Brown. The WaMoS system uses an X-Band radar (microwave region of the electromagnetic spectrum) to measure the sea surface anomalies and obtain a three dimensional image of the wave field \((e.g.,\) Borge et al., 1999). The 3D image is transformed into frequency space using the Fast Fourier Transform (FFT), ultimately yielding a directional wave number and frequency spectra. The deployment of the WaMoS system was complemented with a laser altimeter (RIEGL LD90-3800 VHS) and a TSK SWHM nadir-looking Doppler radar in order to capture the higher frequency range of the wave field. The WaMoS frequency spectrum for the entire cruise is presented in the lower panel of Figure 4. The ship motion was captured by IMU sensors deployed on the jack-staff of the R/V Brown. These set of measurements were used in the motion correction algorithm and in the calculation of the heave of the ship at its center of mass (ship-CM) by a projection of the accelerations at the bow to the ship’s
center of mass.

The final frequency spectrum is generated by merging the data from the instrumentation systems. The WaMoS was used for the low frequency portion of the spectra $0.035 \leq f \leq 0.35$ Hz (Figure 4, bottom plot), The data obtained from the laser altimeter RIEGL and the microwave sensor TSK, both mounted on the bow of the ship, were used for the higher frequency range. The frequency range was extended to 2 Hz with a slope of $f^{-4}$ after merging the WaMoS, TSK and RIEGL. For further detail in the signal processing of wave spectra obtained from a moving vessel refer to Cifuentes-Lorenzen *et al.* (2013).
Figure 3: Experimental Setup: picture of the instrumentation on the jack-staff of the R/V Brown. Top left shows the position of the radar based WaMoS system. Right panel shows the sonic anemometers and the respective IMU sensors. Bottom left shows the three sonic anemometers.
3. Measurement Details

The relative direction between the ship’s heading and the wind and wave fields was a primarily consideration in the signal processing and data analysis presented here. Aside from the radar based WaMoS, all other instruments were deployed on the bow of the ship and are sensitive to their relative orientation with the wind and waves. The direction of the wind relevant to the ship’s bow was of primary importance to the atmospheric measurements (and ultimately flux measurements), particularly wind speed and friction velocity. These measurements are best behaved when flow distortion due to
wind blockage by the ship is minimized. Given the instrument setup, this is achieved by having the bow of the ship headed into the wind, which minimizes the flow distortion at the sensor location on the mast. Directional constraints also apply to the wave field measurements, particularly to the point measurements of the RIEGL and TSK, where the best behavior is expected for waves coming towards the ship’s bow. This avoids blockage/reflection from the ship’s bow and keeps the measurements out of the ship’s wake. Therefore the focus was set on observations that satisfied the following conditions:

\[
0.8 \leq \frac{\theta_{wind}}{\Theta_{heading}} \leq 1.2
\]

\[
0.8 \leq \frac{\theta_{wind}}{\theta_{wave}} \leq 1.2
\]

where \(\theta_i\) corresponds to the direction of wind and waves in meteorological convention (i.e. direction from), \(\Theta_{heading}\) is the ship’s heading. For example, \(\theta_{wind}/\Theta_{heading} = 1\) when the winds are from the west and the ship is heading west. This minimizes blockage of the wave field by the ship for the instrumentation mounted on the bow.

3.1 Wave Field Observations: Wave Spectra

Retrieving accurate measurements of significant wave height and directional wave spectra from research vessels remains a challenge due to platform motion. Corrections include, but are not limited to, the removal of heave for point measurements and effects of Doppler shifting while underway. Open ocean wave measurements were collected by a RIEGL LD90-3800VHS laser altimeter, a TSK SWHM microwave sensor by the Tsurumi-Seiki Co., Ltd. the Wave Monitoring System (WaMoS II) developed by OceanWaves GmbH and Inertial Measurement Units (IMU; Systron-Donner MotionPak
6-variable 2-axis linear acceleration and 3-axis angular velocity) proving the ship’s motion at its center mass (ship-CM). Figure 5 shows an example of the frequency spectrum obtained from the different instrumentation. The WaMoS and motion packs provide the low frequency range (bottom left and right Figure 5) whereas the high frequency end is provided by the RIEGL and TSK (top left and right Figure 5).

![Figure 5](image)

**Figure 5**: A representative spectra from the RIEGL, TSK, WaMoS and the ship-CM is presented for different winds: $U_{10} = 5.1$ m s$^{-1}$ (solid line) and $U_{10} = 14.0$ m s$^{-1}$ (dash line). The Donelan *et al.* (1985) parameterization is added as reference: black dashed line ($U_{10} = 14.0$ m s$^{-1}$) and solid black line ($U_{10} = 5.1$ m s$^{-1}$) in all plots. No added correction or restriction aside the motion correction algorithm (Edson *et al.* 1998) has been applied so far.

The Donelan (1985) parameterization was developed for a purely wind driven wave field. For the high wind conditions in Figure 5 the Donelan *et al.* (1985) does capture the frequency range of the peak with an overestimation of the observed spectral energy. At
low winds the Donelan (1985) fails to capture the frequency range and spectral energy of the wave field as it does not account for the presence of the swell. Nonetheless, this parameterization is used as a reference in the study presented here as it shows a $f^{-4}$ Hz spectral tail rolloff, which coincides with the measured spectrum from the wave instrumentation. A full frequency spectrum follows from merging the measurements and applying Doppler and motion corrections (Cifuentes-Lorenzen et al. 2013). Figure 6 shows an example of the final product while the ship is station keeping, and Figure 8 shows the significant wave height comparison for all data when the ship’s speed over ground (SOG) was less than 3 ms$^{-1}$.

3.1.1 On Station Spectral Measurements

In order to isolate potential issues arising from making wave field observations from a moving vessel the dataset was divided in two main categories: Station Keeping and Underway observations. The motion correction algorithm applied by Edson et al. (1998) is required in both scenarios for the point measurements take on board of the R/V Brown. While on station, instrument tilt and Doppler shifting and aliasing effects are negligible and the instrumentation provides a consistent and coherent ensemble of measurements. Measurements of wave height ($H_s$ estimated from the spectral variance) and spectral peak frequency ($f_p$) are found to be in good agreement among the instrumentation (within 10% of each other). They capture the overall spectral shape of the wave field providing a frequency spectra in the 0.035 to 1.5 Hz range (Figure 6). Contamination from wave breaking and wave refraction at the bow of the ship is also negligible and the ship provides a good platform for measurements under a wide range
of physical conditions.

Figure 6: Wave Spectra: Final spectral product for SOG < 1 m/s with a mean wind speed $U_{10}$ of 12.5 m/s. The solid black line corresponds to the WaMoS frequency spectra, solid blue corresponds to the TSK-SWHM and the solid red is the RIEGL laser altimeter. The green dashed line corresponds to the Donelan et al. 1985 spectral parameterization. The frequency slope corresponds to a $f^{-4}$ type of behavior.

The behavior of the RIEGL at frequencies $f > 1.5$ Hz is associated with white noise with power ranging between -20/-30 dB. The faster spectral rolloff of the TSK (blue line Figure 6) at frequencies $f > 0.4$ Hz is explained by the lower spectral resolution offered by the instrument. The TSK resolution is based on its footprint. The Doppler radar based measurement has a 1.8 m (diameter) footprint corresponding to a frequency resolution of 0.7 Hz based on the half angle antenna radiation beam of 6.5°. The green dashed line corresponds to the Donelan (1985) spectral parameterization, which is used as a reference. The dashed black line shows the $f^{-4}$ tail rolloff and the dashed red lines mark
the variance of the RIEGL under different wind and sea conditions. The secondary peak observed at \( \sim 0.2 \) Hz can be associated to wind-waves.

3.1.2 Underway Spectral Measurements

The WaMoS processing algorithm accounts for Doppler shifting while calculating the surface currents, whereas the point measurements (i.e. RIEGL, TSK and ship based estimates of surface displacement) need Doppler correction. The RIEGL and TSK spectral measurements are contrasted against the WaMoS in Figure (7), where the ship-CM is also plotted as a reference for the ship’s motion (dashed magenta in Figure 7).

Following Hanson et al. (1997) the maximum Doppler shifted frequency is given by:

\[
f = \frac{g}{4\pi U_s} \left( 1 + \frac{8\pi U_s f}{g} - 1 \right)
\]

(35)

where \( U_s \) is the ship speed relative to the mean surface current, here the frequency \( (f) \) is given in hertz. Conservation of energy under the spectral density function is guaranteed by:

\[
S(f) = \hat{S}(f) \frac{\delta \hat{f}}{\delta f}
\]

(36)

where \( S(f) \) is the corrected spectral density function and \( \delta f \) corresponds to the frequency resolution. The Doppler correction works well and is able to account for the perceived frequency enhancement of the point measurements up to SOG < 3 m s\(^{-1}\). At larger ship speeds the shape of the RIEGL and TSK frequency spectra looks distorted, failing to capture the overall energy distribution, relative to the more stable WaMoS. While the ship cruises into the wave field (given the directional constraints) there is an aliasing
issue to consider. It is found that while underway (3 < SOG ≤ 6 m s⁻¹) the TSK spectral measurements suffers from aliasing and the spectral resolution is greatly diminished. The signal then exhibits a faster spectral rolloff (Figure 7) closer to $f^5 - f^6$ Hz rather than the $f^4$ observed in the RIEGL and WaMoS.

![Figure 7](image.png)

**Figure 7:** Underway Spectral Measurements (3 < SOG ≤ 6 m s⁻¹) as a function of frequency. Left Panel: 135 spectral realizations of 30 min each are plotted for the RIEGL (grey). The red solid line is the total spectral average for the RIEGL. Right Panel: 135 spectral realizations of 30 min each are plotted for the TSK (grey). Both Panels: The WaMoS spectral average is plotted in solid blue. The dashed magenta line corresponds to the average ship-CM spectrum. The "ghost wave field" from added pitch and roll is plotted in black plus signs. The green solid line corresponds to the Donelan et al. 1985 spectral parameterization for a mean U₁₀ = 9.2 m s⁻¹. The dash black line corresponds to a frequency slope of $f^4$ Hz

The faster rolloff is again associated with the reduced capacity of the TSK to resolve wave scales of frequencies larger than 0.35 Hz. The tail of the RIEGL appears to be
better behaved after corrections are applied. The instrument issues are also found at the spectral peak. At larger SOG the RIEGL underestimates the peak frequency ($f_p = 0.074 \pm 0.04$ Hz), which is estimated by the WaMoS system to be $f_p = 0.096 \pm 0.016$ Hz (Cifuentes-Lorenzen et al. 2013). The TSK exhibits the same behavior ($f_p = 0.075 \pm 0.02$ Hz). The main issues of the TSK are the evident overestimation of the energy (Figure 7 and Figure 8), which confirms the behavior presented by the significant wave height estimates, where the TSK appears to overestimate the variance (equation 37). In Figure 8 spectral observations are plotted under different underway SOG ranges. From figure 8 it is observed an overall peak misalignment (based on the WaMoS estimates as a reference) and the deterioration of the spectral shape. The lowest SOG considered was $1 < \text{SOG} < 3 \text{ m s}^{-1}$ (left hand side Figure 8) and the largest was for SOG $> 6 \text{ m s}^{-1}$. Figure 8 shows the spectral deterioration as the SOG increases. The green dashed line marks the spectral peak based on WaMoS and is used as a reference to show the progressive spectral peak misalignment of the RIEGL and TSK.
Figure 8: Wave Spectra: Final spectral product for different SOG m/s. The solid black line corresponds to the WaMoS frequency spectra, solid blue corresponds to the TSK-SWHM and the solid red is the RIEGL laser altimeter. The green dashed line corresponds to the Donelan et al. 1985 spectral parameterization. The vertical dashed line marks the spectral peak.

Analyzing the spectral energy (i.e. the variance of the signal $\sigma^2$) via the significant wave height ($H_s$) definition makes a final comparison of the agreement between instrumentation (Figure 9):

$$H_s = 4\sqrt{\sigma^2}$$  \hspace{1cm} (37)$$

where $\sigma^2$ follows directly from:

$$\sigma^2 = \int S(f)df$$  \hspace{1cm} (38)$$

For the RIEGL and TSK point measurements, the frequency spectrum (i.e. $S(f)$) follows from a Fourier transform on the time series of the sea surface displacement point observations ($\eta(t)$).
Figure 9: Significant Wave Height Comparison ($H_s$). Integration of the spectra is carried on for $[0.05 : 0.35]$ Hz. The comparison is presented for SOG < 3 m s$^{-1}$. Bottom plot shows the bin-averaged data with the corresponding variance.

While on station the instrumentation provides a frequency spectra in the range 0.035 -1.5 Hz once appropriate motion corrections and relative ship speed constrains are applied (e.g., see Figure 6). While underway Doppler-shifting corrections are shown to be necessary for SOG > 1 m s$^{-1}$, with a limit at SOG ~3 m s$^{-1}$, due to spectral shape degradation related to wave breaking and refraction at the bow of the ship. Instrument frequency resolution also becomes an issue due to aliasing. The max frequency resolution for the RIEGL for an SOG = 3 m s$^{-1}$ is ~1.3 Hz and the TSK falls to ~0.35 Hz. There is good agreement in the spectral energy captured by the instrumentation, which is reflected in the estimates of $H_s$ (m) shown in Figure 9.
The tilt of the instruments (pitch and roll) and added motion contamination was also evaluated by projecting the tilt angle into a vertical displacement $\delta h(t)\ m$ where spectral analysis shows little energy at relevant frequencies making it negligible once the motion correction algorithm is applied. For more details on the subject refer to Cifuentes-Lorenzen et al. (2013).

3.2 Meteorological Observations

As shown in Figure 3, the meteorological instrumentation was installed in the jack-staff of the R/V Brown containing the air-sea flux package and instrumentation to measure wind speed, wind direction, air temperature, humidity, pressure, and downwelling solar and IR radiation. A Direct Covariance Flux System (DCFS) was used to measure the momentum, energy and buoyancy fluxes using sonic anemometer/thermometers. The velocity measurements were corrected for platform motion using linear accelerometers and angular rate sensors (IMU; Systron-Donner MotionPak) to remove platform motion to estimate the turbulent perturbations ($u'$, $v'$, $w'$) as described by Edson et al. (1998). The measurements are used to estimate the mean neutral wind speed at a reference height of 10 meters and the corresponding friction velocity (Figure 10). The momentum flux computed at 18-m above the mean sea level with the DCFS is shown in Figure 10. Additional estimates of the momentum flux were made using the COARE 3.5 algorithm (Edson et al. 2013).
Figure 10: Neutral wind speed ($U_{10}$) (top) and friction velocity ($u^*$), (bottom) as a function of yearday for the entire cruise duration.

Figure 11 shows the direct estimates of the momentum flux based on the direct covariance method (solid red line), which relies on the turbulent perturbations of the wind velocity measured by the sonic anemometers. The direct covariance is compared to the COARE 3.5 bulk algorithm (solid black line Figure 11) showing very good agreement. These set of measurements define the total stress at a height of 18 meters where the momentum flux is fully turbulent.
Figure 11: Momentum flux comparison: Direct Covariance \((u^'w^')\) technique (red) compared to the bulk COARE algorithm (black) plotted for the entire duration of the cruise. The negative indicates a momentum flux into the ocean. At a height of 18 m this corresponds to the total momentum flux \((\tau_{tot})\).

4. Boundary Layer Analysis

In the following sections the theoretical introduction on boundary layers of sections 1.1 and 1.2 is applied to the open ocean under more realistic \((i.e.,\ \text{less idealized})\) conditions.

4.1 Momentum Flux across the Wave Boundary Layer

The atmospheric surface layer is often defined as a constant momentum flux layer, \(i.e., \frac{\partial(\tau(z))}{\partial z} = 0\) as in a pure shear driven flow. However, there is always some flux divergence in the atmospheric boundary layer. A conditional definition of the height of the surface layer is then needed. The momentum flux will be assumed constant as long as it varies by less than 20\% from their surface value. This condition is expected to be satisfied in the lower 10\% of the atmospheric boundary layer. Within the surface layer
the total momentum flux is divided into three different components at the air-sea interface: the turbulent, wave induced, and viscous stresses, where the total stress is supported by different components throughout the atmospheric boundary layer (Figure 12). As in (25), the three stress components aligned with the surface wind above the wave field are given by:

\[
\rho_s u_c^2 = \tau^t(z) + \tau^w(z) + \tau^v
\]  

(39)

Although the total stress is approximately constant in the surface layer, the turbulent and wave induced components exhibit a strong functionality with height. For example, at the interface, the turbulent flux becomes negligible and most of the flux is supported by waves and viscous stress (lower layer in Figure 12). Above the interfacial layer, wave induced perturbations and therefore the wave induced momentum flux is expected to decay with height. As the wave induced momentum flux decays, the turbulent component increases until all of the flux is supported by atmospheric turbulence (upper layer in Figure 12).
Figure 12: Schematic of the triple-layered system: The interface, a wave boundary layer and a fully turbulent layer. The lower undulations (solid black) represent a random wave field. The red dotted line represent an extremely thin layer, with purely viscous effects, which can disappear entirely where the small-scale roughness elements protrude through the layer.

The layering concept in Figure 12 defines different sections where different assumptions can be made. At the interface the turbulent stress vanishes; the eddies are too small and do not support stress as they dissipate into heat. The interfacial stress is supported by viscous stress and form drag over all wave scales. In the WBL, the nearly constant total stress is portioned between the turbulent and wave induced fluxes with negligible viscous stress effects. A fully turbulent flow exists above the WBL (to be defined). All other stress components vanish in is this layer.
4.1.1 The Viscous Layer and the Viscous Stress at the Interface

Classic boundary layer theory for flow over a rigid surface assumes the existence of purely viscous layer at the interface. In the purely viscous layer the only frictional effects are generated at a molecular level and turbulence plays no role in it (e.g., Schlichting and Gersten, 2004; Schilichting, 1968). Viscous considerations are not limited to a rigid layer and do need to be considered over the ocean surface as well. Viscous effects at the air-sea interface are of particular importance at low wind speeds ($U_{10} < 5 \text{ m s}^{-1}$). The viscous stress is often thought of as the tangential stress where the momentum is directly transferred to the ocean via friction. At higher wind speeds ($U_{10} > 8 \text{ m s}^{-1}$), a fully rough flow is observed (Donelan 1990), with negligible viscous stress. Under these conditions, the wave induced form drag dominates the interfacial stress.

The form drag supported by the wave field is commonly referred to as the normal stress as arises from pressure differences across the waves. However, part of the wave-induced stress is supported by small-scale waves that quickly exchange their momentum with the ocean through micro-breaking. Therefore, the wave induced stress associated with this rapid transfer is often considered part of the tangential stress, i.e., the tangential stress can be thought of as the viscous stress plus form drag due to microbreaking waves.

The relative importance of the purely viscous and normal stress is required to investigate the momentum balance at the air-sea interface. It is useful for quantification of the aerodynamic roughness length and the drag coefficient. The drag coefficient ($C_d$) corresponds to an air-sea interaction parameterization used to capture the total momentum transfer between atmosphere and ocean.
This investigation attempts to quantify the partition by defining the total stress at the interface as

\[
\tau_{\text{total}} = \tau^v(z_0) + \mu \frac{du^v}{dz}
\]  

(40)

where the wave induced stress is evaluated at the surface and the viscous stress is given by:

\[
\tau^v = \mu \frac{du^v}{dz}
\]

(41)

where \(\frac{du^v}{dz}\) is the velocity profile inside the purely viscous region and \(\mu=\rho_a \nu\) is the dynamic viscosity of air. The wind profile is assumed to be linear in the viscous sublayer such that (41) can be written

\[
\frac{\bar{u}(z)}{u^*_v} = \frac{u^*_v}{\nu} z
\]

(42)

where \(u^*_v = \sqrt{\tau^v/\rho_a}\) is the viscous friction velocity. The height of the viscous sublayer is given by

\[
\delta_v = d \frac{\nu}{u^*_v}
\]

(43)

where \(u^*_v\) represents the total friction velocity and \(d\) is a numerical constant. Inside the interfacial layer the total flux has to be conserved, therefore the question is how much of the total stress is supported by viscosity (i.e. viscous stress) and how much by the wave induced flux (i.e. form drag). Parameterizations of the viscous stress have been developed by patching the linear velocity profile in the viscous layer to the logarithmic
velocity profile expected above the waves (e.g., Kudryavtsev et al., 1999; Makin and Kudryavtsev, 2002; Mueller and Veron, 2009). However, this approach gives a viscous stress that is higher than anticipated at high winds (i.e. the transition from smooth to fully rough flow is observed at approximately 12 m s\(^{-1}\) instead of the 8 m s\(^{-1}\) threshold previously defined). Better agreement of the transition threshold is found by instead matching \(\bar{u}(\delta_v)/u_\ast\) with the normalized wind speed from the logarithmic profile at the height of the viscous sublayer, i.e.,

\[
\frac{\bar{u}(\delta_v)}{u_\ast} = \frac{u_\ast}{\nu} \frac{\delta_v}{\nu} = \frac{1}{\kappa} \ln \left( \frac{\delta_v + z_o}{z_o} \right)
\]

which assumes that \(u_\ast = u_\nu\) at the top of the viscous sublayer, and the form of the logarithmic profile allows the wind speed to go to 0 at \(z = 0\) in agreement with (16) and (42). This approach is also consistent with the approach commonly used in fluid dynamics where the dimensionless velocity \(u^+\) is used (described in section 1.2.1). This provides an expression for the viscous stress given by:

\[
\tau^+ = \rho a \left[ \frac{u_\ast}{\kappa d} \ln \left( \frac{\delta_v + z_o}{z_o} \right) \right] \]

which is constant in the viscous sublayer, but becomes negligible a few centimeters away from the surface. The numerical constant \(d\) in Equation 45 has been reported to be of \(O(10^0)\)-\(O(1^1)\). Large values for \(d\) have been previously set as \(d \sim 12\) by Schilchting (1968), Wu (1975), Kudryavtsev and Makin (2002). This value is consistent with the value found for smooth flow where \(u_\ast = u_\ast'\) and \(z_o = a_i \frac{\nu}{u_\ast}\) as given by the smooth flow
regime limit of Equation 34. The smooth flow assumption with the commonly used value of $a_1 = 1/9$ in (44) requires that $d = 11.7$. Lower values of $d$ are also found in the literature (e.g., $d \sim 5$; Monin and Yaglom 1971).

Equation 45 provides a means to separate the viscous and wave-induced stress at the surface. This partition predicts a transition to fully rough flow (roughness Reynolds number, $Re_o = u_* z_o / \nu \sim 2$) at a wind speed of approximately 8 m s$^{-1}$, as shown in Figure 12. In this figure, the total friction velocity and roughness length are computed using the COARE 3.5 algorithm (Edson et al. 2013) with $d = 11.7$. At low wind speeds, the stress is purely viscous (blue line in Figure 13), whereas the flow is fully rough above 10 m s$^{-1}$ (red line in Figure 13) where the roughness elements (i.e. waves) supporting the stress protrude above the viscous sublayer.
Figure 13: Stress partition as a function of wind speed. The stress components follow from the COARE 3.5 algorithm by Edson et al. (2013). Alternatively, the average viscous stress inside the viscous layer can be defined by assuming that velocity at the top of the viscous sublayer is proportional to the wind-induced surface current.

In an effort to capture the viscous stress based on observations the following relation is introduced, where from (41) it follows that:

$$\tau^* = \mu \frac{\overline{u(\delta_v)}}{\delta_v} = \mu \frac{c_1 U_s}{\delta_v} = \rho_a u_* c_1 U_s$$

(46)

where $U_s$ which is the wind induced surface current and $c_1$ is a constant assumed to be of order 1. Early experimental work by Wu (1975) and Wu (1984) defined the surface current relative to the atmospheric friction velocity:

$$U_s = 0.53 u_*$$

(47)
This approximation is contrasted to direct observations based on the WaMoS system mounted on the R/V Brown (Figure 14). A direct comparison of the Wu (1975) surface current parameterization (red line) and the WaMoS surface currents ($U_s$ m s$^{-1}$) estimates is presented in Figure 14 (black line). The WaMoS system estimates $U_s$ as part of the signal processing algorithm made on the radar image in order to provide Doppler correction to the spectral density.

**Figure 14:** Wind driven surface velocity ($U_s$) as a function of yearday. The red line corresponds to the Wu (1975) parameterization (Equation 47) and the solid black line corresponds to the observations provided by the WaMoS instrumentation. Figure 14 is based on the measurements taken during days 74.5-77.

There is good agreement (order of magnitude) between the Wu (1975) parameterization and the WaMoS estimates. Notice that the Wu (1975) parameterization is based solely on the atmospheric measurement of the friction velocity, whereas WaMoS follows from wave field observations.

In an effort to capture the viscous stress at the interface an approximation to the viscous stress is presented based on Equation 46 and compared to Equation 44 (Figure
16), which follows from patching the viscous velocity profile with the log-profile (Equation 42). Figure 15 shows the respective wind velocities during both periods.

![Graph](image)

**Figure 15:** Neutral wind speeds ($U_{10N}$) as a function of yearday 2008 for the periods under study.

Figure 16 shows the behavior of the viscous parameterizations for both time periods analyzed. Both parameterizations are based on the selection of a viscous layer depth defined by $d = 11.7$ and equation (46) uses the dimensionless constant of $c_1 = 0.12$. In Figure 16 both periods are analyzed, where for the less energetic time (yearday 74.5-77) the agreement is better (within the order of magnitude) and deviations are larger for the period between yearday 98-100. A larger discrepancy is linked to the very different nature of the measurements.
Figure 16: Viscous stress to total stress ratio as a function of yearday. The black dots correspond to equation (45). The red dots correspond to equation (46) based on the surface velocities reported by WaMoS. Maximum viscous stress based on (45) corresponds to the lowest winds during both periods Figure 15

For example, the low value of $c_1$ and the disagreement between the formulations are likely due to the inclusion of the tangential stress supported by short gravity-capillary waves when Equation 46 is used. In other words $U_s$ m s$^{-1}$ from the WaMoS system is not purely viscous, but includes the contribution of breaking-waves to the surface currents. Further investigation of this topic is left for the future.
4.1.2 Momentum Balance and the selection of a Wave Growth Parameter

Once the viscous stress at the interface is defined, the momentum balance at the interface can be stated as:

\[ \tau^w(z_o) = \rho_o u^2 - \tau^\nu \]  \hspace{1cm} (48)

where the viscous stress is computed from Equation 45 using the COARE 3.5 algorithm and the total stress is found from direct covariance estimates measured by the DCFS. The wave induced momentum flux at the interface can also be determined from the wave spectra using (2), where the energy input is divided by the phase speed to yield momentum:

\[ \tau^w(z_o) = M_w = \int \frac{\Phi(\omega)}{\omega} \frac{\beta(\omega)}{c(\omega)} \Phi(\omega) d\omega \]  \hspace{1cm} (49)

where \( \Phi(\omega) \) is the one-dimensional wave spectrum, \( \omega \) is the angular frequency \( (\omega = 2\pi f) \), and the relation between momentum and energy at a given frequency is:

\[ M(\omega) = \frac{E(\omega)}{c(\omega)} \]

Equation 49 corresponds to the stress directly supported by the wave field (surface gravity waves), where the wave growth parameter, \( \beta(\omega) \), is defined as the fractional spectral energy increase due to wind input given by (3). The wave growth parameter captures much of the wind-wave interaction and accounts for the momentum and energy of the wave field by direct wind input.
The momentum partition at the interface given by (48) can be used to select a wave growth parameter that satisfies momentum conservation at the interface. Previously accepted parameterizations are tested in order to find a wave growth parameter that satisfies (48). The wave growth parameterizations tested include those of Snyder (1981), Plant (1982), Hisao and Shemdin (1983), Donelan (1995), Janssen (1991) and Burgers and Makin (1993). The frequency dependence of these parameterizations is shown in Figure 17 for a 10-m wind speed of 10 m s\(^{-1}\). The models developed by Plant (1982), Janssen (1991) and Burgers and Makin (1993) parameterize the wave-growth parameter as a function of inverse wave age using the friction velocity, \(i.e., \frac{u_*}{c}\) where the relation between the friction velocity and the wind speed is approximated by \(U_{10} = a u_*\) is used in Figure 17 to allow comparison, where \(a\) is set to 27. The model developed by Donelan (1995) models the wave-growth parameter as a function of inverse wave age given by \(U(z_{ref})/c\), where \(z_{ref} = \frac{\lambda_p}{2}\) and \(\lambda_p\) is the wavelength at the spectral peak. The Snyder (1981) parameterization is based on observations for seas that are assumed to be purely wind driven.
Figure 17: Wave Growth Parameters. Dimensionless Wave Growth Parameter as a function of spectral frequency. Different parameterizations are plotted as a function of frequency for a mean wind speed of 10 m s\(^{-1}\) at a 10 m height with the corresponding friction velocity \((U_{10} \sim 27 u^*)\). The characteristic swell frequency is located at the frequency range: 0.08 - 0.1 Hz.

The Wave Modeling (WAM) third generation ocean prediction model (WAM, 1988) uses explicit parameterizations for the dissipation, input and non-linear wave-wave interactions. The WAM uses the input term \((S_w)\) based on the Janssen 1991 (Figure 17) for the integration of the wave action equation.

The WAM-Janssen (1991), Burgers and Makin (1993) and Donelan (1995) parameterizations allow for the upward exchange of energy and momentum from waves to wind at the spectral peak. The upward flux is modeled through a negative growth rate when \(c_p > U(z) \approx 30u^*\). This is expected to be the case in swell dominated regimes where the peak phase speed is faster than the wind speed. The wind speed \(U(z)\) is usually
evaluated at 10 m, except in the Donelan (1995) formulation where \( z \) is set has half the peak wave length \( z = \lambda_p/2 \) in an effort to account for the variability of the depth of the WBL.

These parameterizations are also used to evaluate the behavior of the normalized cumulative wave induced momentum at the surface, given by:

\[
\Gamma(f) = \frac{1}{\tau^w(z_o)} \sum_{i=1} M(f_i) \Delta f
\]  

(50)

where \( M(f) \) is the momentum of the wave field at a given frequency \( 2\pi f = \omega \) directly obtained from equation (49), where it is relevant to note that \( M(f) = 2\pi M(\omega) \) such that \( M(f) df = M(\omega) d\omega \). Equation (50) can be used to assess what fraction of the momentum flux is supported by a particular wave scale (Figure 18). For example, under fully developed conditions \( (c_p/U_{10} = 1.2 \text{ or } c_p/u_* = 32) \), Makin et al. (1995) shows a cumulative wave induced momentum flux reaching 65% of the total wave induced flux for wavelengths shorter than 7 m corresponding to the frequency range between 0 and 0.5 Hz (or 3 rad s\(^{-1}\)). However, different wave growth parameterizations have a different impact on the cumulative momentum. Plant (1982), Hsiao and Shemdin (1983) and Donelan (1995) heavily weight the higher frequency (i.e. spectral tail). For example, the Plant 1982 parameterization suggests that 65% of the momentum is supported by waves in the range 0.8 to 2 Hz (Figure 18), which represents wavelengths between 2.4 and 0.4 m. This heavy weighting of shorter wave scales is due to the origin of the Plant 1982 parameterization, which mainly follows from laboratory wave tank experiments. Under open ocean conditions, a heavier weight on intermediate to long wave scales is assumed based on the work by Makin et al. (1995).
Figure 18: Normalized Cumulative Wave Induced Momentum Flux, $\Gamma(f)$ (Equation 52) as a function of frequency ($f$). A comparison between different wave growth parameters with default coefficients is shown. The lower plot on the right hand side is a blow-up of the cumulative wave induced momentum at lower frequencies. Only the Donelan 1995 and the Janssen 1991 allow a negative contribution at the spectral peak.

The wave induced cumulative momentum flux at the surface ($\Gamma(f)$) for the WAM-Janssen (1991) and the Burgers and Makin (1993) parameterizations show that 50% of the total flux is supported by waves in the frequency range 0 to 0.5 Hz and 80% of the wave induced momentum flux is supported by waves in the frequency range given by 0-1.2 Hz. This behavior is assumed to capture open ocean conditions. At the same time there is a reduction of the wave-induced momentum flux due to the upward momentum flux (ocean to atmosphere) in the frequency range 0.05-0.13 Hz is shown in Figure 16. The reduction of the momentum flux due to wave-driven winds has been observed over the open ocean (e.g., Drennan et al. 1999; Grachev and Fairall 2001; Grachev et al.)
2003 and Hanley and Belcher 2008; Edson et al. 2013). The upward momentum flux is also evident in the $u'w'$ cospectra measured during SO GasEX, which is seen as a reduction in the cospectral estimates around the swell dominated frequencies.

The wave spectrum is given by the combination of the WaMoS and the RIEGL measurements in the frequency range of 0.035 to 1.5 Hz. The frequency spectrum was extended to 2 Hz by adding a tail that followed an $f^4$ Hz roll off. The shortest waves the measurements will resolve are ~0.7 m in length with phase speeds 1 m s$^{-1}$. The selection of the wave growth parameter provides an estimate of the momentum flux at the interface based on spectral estimate based on (49), which can be compared with the atmospheric measurements and viscous parameterization given by (48). The Burgers and Makin (1995) and Janssen (1991) parameterizations used in the WAM model are the best behaved when closing the momentum balance at the interface (Figure 19), where the implications of the swell is accounted for. The data analyzed is presented separately in Figure 19, where the top plot shows the momentum closure for yearday 74.5-77. The bottom plot in Figure 19 shows the momentum closure under more severe seas yearday 98-100. A good momentum closure follows a 1:1 correlation between estimates where the spectral estimates of the wave induced stress are plotted in the y-axis. The difference between the total stress and the viscous estimates are plotted on the x-axis.
**Figure 19**: Wave induced stress from spectral estimates (Equation 49) as a function of the total momentum flux from equation minus the viscous parameterization (Equation 48) Black dots in both plots mark the 1:1 line. (a) Momentum closure at the interface for yearday 74.5-77. The grey squares are the entire data for the period, the blue squares are the bin-averaged data with and the standard deviation of the bin. The $R^2$ for the 1:1 regression is reported to be 0.81 with a mean bias high of 6% for the spectral estimates versus the total momentum flux minus the viscous parameterization (A2). (b) Momentum closure at the interface for yearday 98-100. The $R^2$ for the 1:1 regression is reported to be 0.49. Characterization of the viscous stress follows from (45) which accounts for approximately 5-8% of the total stress.

The less energetic period 74.5-77 closes the balance at the interface with an average wind stress overestimation of 6%. Figure 17 shows good agreement in the mean for the range: $0.4 < \tau_{\text{tot}} < 0.8$ [Pa] for both periods. However, the momentum closure is better for the less energetic period shown in the upper panel ($R^2 = 0.81$) than for the more
energetic period shown in the lower panel. At larger stresses $\tau_{tot} > 1$ [Pa] the normal stress estimated from the spectrum using the wave growth parameterization shows a bias low. The bottom plot also exhibits an overestimation (in the mean) of the spectral estimates at $\tau_{tot} < 0.3$ [Pa].

Therefore, it is possible that the wave growth parameterization needs to be fine-tuned to fully capture the behavior of the wave induced stress at higher forcing under more severe seas. The observed deviations and variance is explored in Figures 20 and 21. Figure 20 shows a time series for the period yearday 98-100, where the total stress (atmospheric observations) is plotted in solid black against the spectral estimates plotted in red dots. Figure 20 shows a clear picture of periods where the spectral estimates are biased low and high, also showing periods of good agreement in the closure scheme.

**Figure 20:** (a) Momentum closure at the interface for yearday 98-100. Solid black line corresponds to the total stress minus the viscous parameterization. The red dots show the spectral estimates based on a.1.0.
Figure 21 explores the deviation by looking at the stress ratio defined as the wave induced to total stress minus the viscous component, $\tau'/(\tau_{\text{tot}} - \tau')$, where the numerator follows from spectral estimates and the denominator from atmospheric measurements and the viscous parameterization given by (45). For $u^*/c_p < 0.03 – 0.04$, the ratio deviates from the 1:1 line exhibiting a bias high of the spectral estimates, suggesting a stronger reversal at the spectral peak that is not accounted for in (51). Figure 21 indicates that the bias in the stress ratio is dependent on the age of the wave field. This further supports the hypothesis of a long wave modulation on the system, which at this point is not fully accounted for. The bias low suggests that the spectral weighting by the wave-growth parameter needs some adjustment to properly estimate the energy and momentum flux to the waves.
Figure 21: (a) stress ratio as a function of the atmospheric forcing \( (u^*/c_p) \) for yearday 74.5-77. (b) stress ratio as a function of the atmospheric forcing \( (u^*/c_p) \) for yearday 98-100. The red and blue correspond circles correspond to the binned averaged data shown in grey. Values of the ratio above the line indicate an overestimation of the wave induced stress. Values below suggest an underestimation of the wave supported momentum flux at the interface.

The sensitivity to the wave-growth parameterization is briefly explored in Figure 22, where the momentum reversal (negative peak at \( f \sim 0.1 \text{ Hz} \)) has been enhanced by simply adjusting the wave growth parameterization given by:

\[
\beta(\omega) = A_1 \frac{\rho_u}{\rho_w} \left[ B_1 \frac{u_*}{c} - X \right] 
\]

where the constant \( X \) represents the value at which the transition to developing seas is observed. The frequency spectrum of the wave stress (form drag) follows from
where $S_{in}(f)$ is the spectral wind input based on the wave growth parameter. Examples of this spectrum for several value of X are shown in Figure (22). Integration of (52) yields the wave induced stress evaluated at the interface (i.e. $\tau^w(\zeta_\omega)$). Note that the high frequency portion ($f > 0.6$ [Hz]) remains unmodified. An enhancement of the reversal will reduced the deviation of the stress ratio observed in Figure 21 under weaker forcing. Potentially, the spectrum will need to be extended to higher frequencies to fully capture the input at shorter wave scales.

$$M_w(f) = \rho_w g \frac{S_{in}(f)}{c(f)}$$ (52)

Further investigation of the magnitude and frequency range of the momentum reversal the spectral shape of the momentum flux needs to be complemented with the wave action equation given by (1). At the same time the frequency resolution needs to

Figure 22: Example of the Frequency distribution of the wave stress at the interface. The green dashed line corresponds to the behavior of the wave growth parameterization with $X = 1$. The black dashed corresponds to $X = 1.2$. Larger X enhances the momentum reversal at lower frequencies.
be expanded in order to account for smaller waves, which ultimately support the largest fraction of the wave induced momentum flux. The brief analysis presented here shows how the wave dynamics and feedback could to be incorporated in the future analysis.

4.2 The Wave Boundary Layer Structure: Kinematic Considerations

The behavior of the momentum flux at the interface is defined using viscous and spectral observations. The viscous component vanishes quickly away from the interface and the transition to a fully turbulent momentum flux will then depend on the vertical structure and behavior of the wave induced perturbations. Ideally, direct measurements of the wave induced momentum flux would be performed to quantify the behavior of the momentum flux components \( \tau''(z) \) and \( \tau'(z) \). Such measurements would allow a direct means to define the wave boundary layer and scale the wave-induced perturbations as a function of wave and turbulence related scaling parameters. Unfortunately, direct measurements of theses fluxes across the marine boundary layer are extremely difficult to make. Estimates of the surface flux and flux divergence away from the surface require static pressure and wave perturbed velocities to be measured as close to the surface as possible. This complicates the instrument deployment due to the presence of the waves and analysis due to any motion of the measurement platform and sensors. At the same time, separating the turbulent from purely wave induced signal is hard to achieve (e.g., Reider, 1998; Hristov et al. 2003). The most direct measurements of the surface stress involve atmospheric pressure observations and subsequent extrapolation from a given height down to the surface from wave followers, towers and stabilized platforms (e.g., Snyder 1981; Hasselman and Bosenberg 1991; Donelan et al. 2006). The extrapolated pressure measurements are then correlated with the wave slope
to estimate the wave stress supported by the longer waves. However, this approach
unavoidably misses the contribution of the shorter waves due to the rapid decay of the
wave-induced signature of these waves. Phase averaging of the velocity field has also
been attempted (Wetzel 1996). However, this approach is difficult to implement unless
the waves field has a very narrow spectral peak.

The aforementioned measurement issues are mitigated by introducing a model of the
deck of the wave-induced momentum away from the surface based on previous
observational and modeling studies (e.g., Snyder 1981; Hare 1997; Chalikov and
Rainchik 2011; Makin and Kudryavtsev 1999). The wave-affected layer is then
analyzed from kinematic and dynamic considerations by combining a full range of
observations from the SO GasEx cruise 2008.

4.2.1 The Decay of the Wave Induced Momentum Flux

When exploring the structure of the WBL, the potential flow solution to the
equations of motion is commonly used as a guideline. A potential flow solution is based
on an irrotational and incompressible fluid. This leads to an exponential decay of the
wave-induced perturbations of the velocity components (i.e., $u$, $v$, $w$). The exponential
function has the general form, $g(kz) = e^{-\alpha kz}$, where $k$ is the wavenumber of a given wave
component and the constant $\alpha$ equals one for the velocity perturbations. The
exponential behavior has proven to be consistent with limited observations (e.g., Snyder,
1981; Hare et al. 1997), where the difficulty is in defining the actual decay rate. Snyder
(1981) found that the exponential decay rate of the wave-induced pressure on the
atmospheric side was slower than the predictions based on potential theory. This is also
consistent with numerical efforts by Chalikov and Rainchik (2011), where the wave
induced pressure decays exponentially, but at a slower rate than potential predictions. Hare et al. (1997) shows that the wave induced pressure perturbations associated with form drag depart from the exponential behavior for older seas, particularly for dimensionless heights $kz > 2$ (*i.e.*, well above the waves). Nonetheless, under developing to mature sea conditions in the lower portion of the boundary layer where $kz < 2$ (*i.e.*, close to the waves), Hare et al. (1997) shows good agreement with an exponential behavior and appears to be consistent with Snyder (1981). The wave age dependence in Hare et al. (1997) is also explored in Donelan et al. (2006), where the pressure decay rate is found to be $\alpha = 1.09$ and $\alpha = 1.88$, for light and strong wind forcing conditions (based on inverse wave age), respectively.

The wave-induced momentum flux as a function of height is defined here as:

$$\tau^w(kz) = \tau^w(z_o)g(kz)$$

where $g(kz)$ is a function exhibiting exponential decay from the surface. Alternatives to the potential type of decay functions have been explored analytically (*e.g.* Chalikov 1995) and numerically (*e.g.* Makin and Kudryavtsev 1999). The numerical efforts of Makin and Kudryavtsev (1999) propose the following decay function:

$$g(kz) = e^{-10 \times k \times z} \cos(\pi k z)$$

The oscillating multiplier (*i.e.*, $\cos(x)$) makes the physical interpretation of this function rather interesting, indicating a potential flux reversal above a certain height. A slight modification to the more classic potential decay is introduced:

$$g(z) = e^{-Az}$$
where the decay rate \( A \) has units of inverse height and is defined as:

\[ A = \alpha k \]

which combines the wavenumber of a given wave component and the dimensionless decay parameter, \( \alpha \). Introducing the same modification into Equation (54):

\[ g(z) = e^{-A z} \cos(A z) \tag{56} \]

This simplification allows an investigation of the overall decay of the wave-induced momentum flux within the WBL. The decay rate is expected to provide information about the height of the WBL and length scale of the waves that support the momentum flux. This simplification avoids the \textit{a priori} selection of the decay parameter \( \alpha \) and implicitly captures the physical conditions on which the decay rate \( A \text{ m}^{-1} \) is assumed to depend.

4.2.2 The Wind Velocity profile in the wave Boundary Layer

The decay rate of the wave induced momentum flux given by (53) provides an expression for the local (turbulent plus viscous) friction velocity \( u^* \) given by:

\[ u^*(z) = u_r \sqrt{1 - \alpha_c g(z)} \tag{57} \]

where \( \alpha_c \) is the wind-wave coupling ratio given by

\[ \alpha_c = \frac{\tau^w(z_0)}{\rho_a u_r^2} \tag{58} \]
which captures the fraction of the surface stress directly supported by the wave field. Therefore, \( \alpha_c \) is expected to approach 1 as the sea becomes fully rough. From Equation (29) it follows:

\[
\frac{\partial u'}{\partial z} = \frac{u^2}{K} [1 - \alpha_c g(z)]
\]  

(59)

The following eddy viscosity parameterization is used as a first approach (e.g., Chalikov, 1995; Janssen, 1989):

\[
K = u'_h \kappa (z + z_o)
\]  

(60)

where the local friction velocity is used as a scaling argument instead of the total friction velocity in an effort to characterize the immediate wave effect. The basic assumption used to validate this approach is that the eddy viscosity time the shear provides the turbulent component of the momentum flux.

4.3 The Wave Boundary Layer Structure: Dynamic Considerations

In this section, the model of the velocity profile given in the previous section is combined with the momentum balance at the air-sea interface to investigate the structure of the WBL. An assessment of the \( g(z) \) functions is presented based on an energy balance across the WBL. The KE budget for neutral stability conditions introduced in Section 1.2.1 provides the starting point:

\[
-\frac{\partial \bar{u}}{\partial z} [\bar{u}' \bar{w}' + \bar{u} \bar{w}'] + \frac{\partial}{\partial z} \left[ \frac{1}{\rho_a} \bar{w}' \bar{p}' + \bar{w} \bar{p} \right] - \frac{1}{\rho_a} \frac{\partial \bar{w} \bar{p}}{\partial z} - \varepsilon = 0
\]  

(61)
where the wave induced perturbations are identified with tildes and turbulent perturbations with primes. It is customary to neglect the turbulent transport terms in equation (61), as they have been shown to cancel each other in the atmospheric surface layer (e.g., McBean and Elliott, 1975; Wyngaard and Coté, 1971). Equation (61) reduces to:

\[-\frac{\partial \tilde{u}}{\partial z} \left[ \bar{u}' \bar{w} + \bar{u} \bar{w}' \right] = \frac{1}{\rho_a} \frac{\partial \bar{w}\bar{p}}{\partial z} + \epsilon \]  

(62)

The turbulent KE (TKE) production (i.e. the shear times the turbulent component of the flux) is assumed to be locally balanced by the TKE dissipation term (last term on the right-hand-side). This local production-balance reduces Equation 62 to the purely wave induced terms:

\[-\bar{u}\bar{w} \frac{\partial \tilde{u}}{\partial z} = \frac{1}{\rho_a} \frac{\partial \bar{w}\bar{p}}{\partial z} \]  

(63)

which is analogous to the steady-state equation presented in Mastenbroeck (1996), Hara and Belcher (2004), and Chalikov and Makin (1996). Integration of (63) over height leads to:

\[-\rho_a \int \bar{u}\bar{w} \frac{\partial \tilde{u}}{\partial z} dz = \bar{w}\bar{p}_z - \bar{w}\bar{p}_o \]  

(64)

For \( z \rightarrow \infty \) the first term on the right-hand-side goes to zero, as the wave induced perturbations vanish away from the interface. The second term on the right-hand-side is the energy going into the wave field given by (2) and it can be spectrally defined as:
where \( \rho_w \) is the water density. As such, Equation 64 represents the extraction of energy from the velocity profile by the wave-induced flux. The here defined extraction mechanism supplies the energy flux to the wave field (e.g., Lighthill, 1962; Makin and Mastenbroek, 1996).

4.3.1 A Constant Momentum Flux

Integration of Equation 64 requires knowledge of the way in which the wave induced perturbations and velocity profile change with height. Miles (1957) assumed a constant wave induced momentum flux throughout the WBL and a logarithmic profile based on a constant value of \( u* \) such that \( \partial u / \partial z = u* / kz \). This assumption leads to a step function type of behavior, where the wave induced momentum flux is constant and then sharply jumps to zero at a given height. This height was defined as the critical height and lead to the concept of a critical layer where the wave-induced momentum flux is dominant. The Miles (1957) theory predicted that the critical height is found where the wind speed matches the phase speed of the dominant waves (originally developed for a monochromatic wave field).

The left hand side of Equation 64 is easily evaluated using the step-function for the wave-induced momentum flux, where the flux is zero above and constant within the critical layer. Integration is carried up to the critical height yielding:

\[
E_{aw} = \overline{\rho w} \overline{\beta(\omega)} \Phi(\omega) d\omega = \rho_w g \int \omega \beta(\omega) \Phi(\omega) d\omega
\]  

(65)

where \( \rho_w \) is the water density. As such, Equation 64 represents the extraction of energy from the velocity profile by the wave-induced flux. The here defined extraction mechanism supplies the energy flux to the wave field (e.g., Lighthill, 1962; Makin and Mastenbroek, 1996).

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The left hand side of Equation 64 is easily evaluated using the step-function for the wave-induced momentum flux, where the flux is zero above and constant within the critical layer. Integration is carried up to the critical height yielding:

\[
E_{aw} = \rho_p \overline{u \beta}[U(z_o) - U(z_o)] = \rho_p \overline{u \beta}[c_p - U(z_o)]
\]  

(66)
where $c_p = U(z_c)$ is the phase speed of the dominant waves, $\overline{uw_o}$ the wave induced flux evaluated directly at the interface (i.e. $z \sim z_o$) and $E_{aw} = \overline{wp_o}$ is used hereafter to represents the energy input from the atmosphere to the wave field.

In the Miles shear flow theory (MSF), turbulence is only included as a means to support the wind velocity profile, leading to a simplified statement of the air-sea interaction and wave generation. The MSF has proven useful, but has never been validated beyond reasonable doubt due to its simplified approach to the matter. Nonetheless, observations by Hristov et al. (2003) show air flow patterns exhibiting the characteristic cat-eye pattern of the closed streamlines at the critical height, where a maximum vortex force would be observed (Lighthill, 1962). The physical explanation of the mathematical MSF provided by Lighthill (1962) shows how this vortex maximum at the critical height leads to the step-function type of behavior for the wave induced momentum flux. The momentum transfer is limited to a very shallow region above the wave field for the short gravity waves that support most of the momentum exchange.

MSF theory initially assumed a monochromatic wave field. Obviously, the real ocean surface is comprised of a continuous spectrum of waves. Therefore, it is possible to think of an effective critical layer in which the sum of the wave contribution to the wave induced momentum flux dominates the exchange. For example, evaluating (66) at an effective critical height leads to the Terray et al. (1996) approach, where the effective phase speed is related the total stress to the energy input:

$$E_{aw} = \tau_{total} \overline{c}$$  \hspace{1cm} (67)
The height at which this effective phase speed matches the wind speed therefore defines the effective critical height. In Terray et al. (1996), the energy transfer is related to the total momentum flux. As such, the effective phase speed accounts for the difference between the total and wave-induced flux due to viscous stress over a realistic and evolving wave field. As such, the normalized effective phase speed, $\bar{c}/c_p$, was shown to be a function of inverse wave-age, $u_*/c_p$, to account for the evolving waves and energy input.

4.3.2 Introducing the Height Dependence

The integrand in (64) corresponds to the rate of work done by the wave induced momentum flux on the wind shear (Lighthill, 1962). Integration over height leads to the atmospheric energy input into the wave field (i.e., the energy extraction mechanism). In this section, the structure of the WBL is explored assuming an exponential decay of the wave-induced momentum flux and corresponding modification of the shear, rather than the unrealistic assumption of a step-like behavior in the flux and a traditionally-defined logarithmic profile. Using the concept introduced in Section 4.2, the height dependence of the wave-induced momentum flux and wind shear is given by:

$$-\rho_a \int_z u^w \frac{\partial \bar{u}}{\partial z} dz = -\tau^w(z_o) \int_z g(z) \frac{\partial u^l}{\partial z} dz$$

Equation (68) includes the decay function and the modified shear based on Sections 4.2.1 and 4.2.2. This leads to the energy balance between the atmospheric input (i.e.
extraction) and the wind-generated energy of the wave field (\textit{i.e.} energy that the wave field supports by direct wind input):

\begin{equation}
-\rho_a \tau^v(z_o) \int_z \left( g(z) \frac{\partial u}{\partial z} \right) \, dz = \rho_w g \int_\omega \beta(\omega) \Phi(\omega) \, d\omega \tag{69}
\end{equation}

The right hand side of (69) can be directly evaluated using the wave field spectral measurements (Section 3.1) and the already selected wave growth parameter. The right hand side of (69) corresponds to the most direct estimates of the wind inputted energy to the wave field based on the observations made during the SO GasEx 08. The left-hand-side is evaluated using estimates of the wave-induced momentum flux at the interface (from Section 4.1.2) with the decay functions and modified velocity profiles (from Section 4.2.2).

5. Results

Figure 23 shows the behavior of the local friction velocity with an exponential decay under different decay rates (A m\(^{-1}\)). The local friction velocity converges to the total friction velocity \((u_*)\) at a height dependent on the value of the decay rate, \(A\).
Figure 23. Local friction velocity \(u^*(z)\) as a function of height (Equation 57). Different decay rates, \(A\) are explored for an exponential decay. The local friction velocity converges to the total friction velocity \(u^*\), which is constant throughout the atmospheric boundary layer.

The variable of integration in (68), \(g(z)\frac{\partial u^l}{\partial z}\) provides the vertical structure of the extraction mechanism, where the wind shear is modified using the local friction velocity as given by (59). These variables are then normalized by those for a traditional logarithmic profile in order to provide a dimensionless function:

\[
r(z) = g(z)\frac{\partial u^l}{\partial z} \frac{\kappa z}{u_*}
\]

(70)

where \(u_*\) is the total atmospheric friction velocity and \(u^*/\kappa z\) is the shear profile from similarity theory. Figure 21 shows the vertical behavior of the dimensionless extraction
mechanism for the two proposed height dependencies, the exponential and the cosine-exponential function. These functions are plotted in Figure 24 for several combinations of model coefficients to provide examples of the perturbed velocity profile driven by changes in the local friction velocity.

Figure 24: Dimensionless $r(z)$ as a function of height. Equation 70 is plotted based on the vertical behavior assumed by Equations 63 and 64 with different decay rates ($A$, exponential behavior). Decay rates $A_2$ and $A_3$ follow the $\cos(x)\exp(x)$ type of behavior.

Oscillations of the wave induced momentum and pressure perturbation were observed in Wetzel (1996) and Hare et al. (1997), respectively, and in the modeling studies of Zou et al. (1995) and Mastenbroek et al. (1996). Physically, the oscillations
have been interpreted as the decay of the wave field, where the momentum is being transferred from waves to winds (Hare et al. 1997). The difference between these studies is the height at which the oscillations are modeled and observed respectively. For example, Hare et al. (1997) shows oscillations at dimensionless heights $kz > 2$ for mature seas, with the lower region being well characterized by a potential decay. Numerical studies set the oscillation at much lower heights $kz < 0.5$. The oscillating decay function given by (59) is explored later in a TKE budget across the WBL.

The Makin and Kudryavtsev (1999) function is in reasonable agreement with the exponential decay of the dimensional gradient function (i.e. $r(z)$), and reaches a maximum at approximately the same height $z \sim 0.25-0.5$ m (Figure 24). Despite this agreement, the negative oscillation - appearing for $A_2 A_2 < 2$ - makes it difficult to physically interpret the function as well as the behavior of the wave induced momentum flux. Therefore, the exponential decay is preferred over the more convoluted behavior of the Makin and Kudryavtsev (1999) for this investigation.

The vertical integral of $r(z)$ is proportional to the effective phase speed, i.e. the multiplier of the wave-induced momentum flux that quantifies the energy flux to the waves. For example, the faster decay rates shown in Figure 24 will extract less energy from the perturbed velocity profile because the wave-induced perturbations will protrude up to a shallower height. Therefore, the decay rate can be adjusted to improve the comparison between the wave spectral estimates of the energy flux given by (65) and the wind-wave extraction model given by (69). The spectral estimates follow directly from the wave growth parameter that gives the fraction of the wave field energy directly transferred from the wind to the waves (denoted as $E_{aw}$: atmosphere to wave field). The
spectral estimates for both periods in the present study are plotted in Figure 25. These are spectral estimates of the input in watts per unit surface area. Both periods are shown in the same plot. The more energetic seas are shown in blue corresponding to the yearday 98-100 and the more moderate conditions are shown in red (yearday 74.5-77).

Figure 25: Energy input from the atmosphere to the waves from spectral measurements ($w_{po}$). Estimates ($w_{po}$) were retrieved from 15-minute spectral averages using the parameterization provided in Equation 52 modified using coefficients $A_1$ and $B$. The blue time series corresponds to observations made during the yearday 98-100, under severe seas (mean $H_s = 5$ m) with mean wind speeds ($U_{10}$) of 14.8 m s$^{-1}$. The red time series correspond to observations made during yeardays 74.5 through 77 under milder conditions ($H_s = 3.4$ m) with mean wind speeds ($U_{10}$) of 12.3 m s$^{-1}$. During both times, the SOG < 3 m s$^{-1}$.

The spectral estimates of the sea surface displacement followed directly from merging the RIEGL laser altimeter at high frequencies and the WaMoS radar based system at lower frequencies. This procedure provides spectra in the frequency range 0.035-2 Hz. The time series of the wave-input using these wave spectra and the
modified Janssen input parameter shown in Figure 25 are considered the most direct observations of the wind-wave energy of the wave field.

A direct comparison between the spectral observations and the extraction model presented so far is given in Figure 26. The modeled extraction mechanism lies on average within a 20-25% difference between its estimates and the observed spectral measurements. The 1:1 line in Figure 23 corresponds to the dotted black line. Strong deviations are observed for yearday 98-100 with severe seas with predicted values reaching 7 W m\(^{-2}\) (Figure 23). In the range < 2.5 W m\(^{-2}\), the modeled extraction mechanism and the spectral observations show good agreement with an average lying on top of the 1:1 line (Figure 23). The algorithm based on the energy balance across the WBL exhibits low sensitivity to variations in \(A\) for \(0.75 \leq A \leq 3\) m\(^{-1}\), with an error near ~20% for wind speeds between 9.3 and 20.6 m s\(^{-1}\).
Fig. 26: Extraction Mechanism ($E_{aw}$) as a function of Spectral Energy Estimates ($E_{aw}(\omega)$). Both periods are plotted in Fig. 23. Decay rates from $1 \leq A \leq 3 \text{ m}^{-1}$ are presented. The black dashed line represents the 1:1 relation.

Deviations from the 1:1 line observed in Fig. 26 are investigated by exploring the decay rate under different forcing ranges (i.e. different ranges of the friction velocity). The goal is to capture the sensitivity of A to different physical conditions. Different forcing based on the turbulent velocity scale is the most simple and straightforward step at this point. Further analysis considering the wave field (e.g. wave slope) is left for future research.

5.1 The WBL Depth and the Scale of the Wind Coupled Waves

The depth of the WBL follows directly from the decay rate where the e-folding depth is defined as $z_e = 1/A$, where approximately 30% of the total momentum flux is
supported by the wave-induced momentum flux. However, the results shown in Figure 26 suggest that a constant decay rate across physical conditions may not be the best option. Therefore, the dependence of $A$ on the forcing is explored by minimizing the difference between the two estimates of the wave-induced energy exchange given by (69) under different ranges of the turbulent velocity scale $u_*$.

For every forcing range, modifying the decay rate minimizes the difference between the model and the spectral observations of the energy input into the wave field. This minimization is based on (69) and the values of the decay rate for various ranges of $u_*$ as shown in Table 1 and Figure 27. Under weaker forcing conditions (i.e., $u_* < 0.35$), the modeled extraction mechanism does not capture the energy input with an overall overestimation of $\sim 50\%$. Even for larger decay rates ($A > 25 \text{ m}^{-1}$), the improved agreement is marginal.

**Table 1.** Average decay rates ($A$, m$^{-1}$), and the associated standard deviation, for different forcing ranges (i.e. $u_*$, m s$^{-1}$) during yeardays 74.5-77 (Period 1) and yeardays 98-100 (Period 2). The binned-averaged results are plotted in different colors in Figure 27.

<table>
<thead>
<tr>
<th>Range of $u_*$ (m s$^{-1}$)</th>
<th>0.35$&lt; u_*$&lt; 0.4</th>
<th>0.4$&lt; u_*$&lt; 0.5</th>
<th>0.5$&lt; u_*$&lt; 0.6</th>
<th>0.6$&lt; u_*$&lt; 0.7</th>
<th>0.7$&lt; u_*$&lt; 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$, Period 1 (m$^{-1}$)</td>
<td>20.86 ± 10</td>
<td>5.8 ± 2</td>
<td>1.64 ± 0.7</td>
<td>0.53 ± 0.4</td>
<td>N/A</td>
</tr>
<tr>
<td>$A$, Period 2 (m$^{-1}$)</td>
<td>7.0 ± 3.0</td>
<td>4.1 ± 3.0</td>
<td>1.73 ± 0.6</td>
<td>0.98 ± 0.4</td>
<td>0.9 ± 0.4</td>
</tr>
</tbody>
</table>

Figure 27 shows the individual and bin-averaged estimates of the energy input from the wave spectra (i.e. the right-hand-side of (69)) and model (i.e. the left-hand-side of
(69)) as a function of the atmospheric friction velocity. The grey squares and black triangles are the individual bine-average values from the spectral estimates, respectively. The colored squares represent the bin-averaged values found by minimizing the difference between model and observations over a range of $u_*$. The decay rates used to generate these bin-averaged values in Figure 27 are presented in Tables 1 and 2. The minimization of (68) is done within 20\% for $u_* > 0.5$ m s$^{-1}$ with maximum agreement (i.e. within 5\% in the mean) at $u_* > 0.65$ m s$^{-1}$.

**Figure 27:** Modeled energy input ($E_{aw}$) and spectral observations of the atmospheric energy input (grey squares) as a function of the friction velocity ($u_*$). The black triangles are averaged values of the spectral observations. Colored squares correspond to averaged values of the modeled input under different decay rates (see Tables 1 and 2).
For $u^* > 0.9 \text{ m s}^{-1}$ (only observed during yeardays 98-100), a large decay rate ($A \sim 4 \text{ m}^{-1}$) is needed to minimize (69). This behavior sheds light on the observed trend in Figure 27, where an overestimation of the modeled extraction mechanism is observed for constant decay rates ranging from 0.75 to 3 $\text{m}^{-1}$. However, due to the lack of data points that reach those conditions (~11% of the observations), no conclusive analyses can be presented. Nonetheless, the modeled energy input shows good agreement with spectral observations using an inversely proportional decay rate with the forcing ($u^*$) in the range 0.4 to 0.85 $\text{m s}^{-1}$. The decay rates (Table 1) are used to define the depth of the WBL and make the link to the actively wind coupled wave scale (i.e. the effective wave scale).

The height scale (i.e. $A \text{ m}^{-1}$) of the actively wind-coupled waves was assumed to be inversely proportional to the effective wavenumber ($z \sim 1/k$, where $k$ is the peak wavenumber for purely wind seas, $k_p$) as discussed in Chalikov (1995). Analogously, the effective wave scale can be determined from the effective phase speed using the Terray et al. (1996) approach modified to include the viscous stress to be consistent with the analysis used in this investigation:

$$E_{aw} = \bar{c} \tau^w \quad (71)$$

Equation 71 follows directly from Equation 69, where the effective phase speed is related to our energy extraction model defined using:

$$\bar{c} = \int_g \frac{\partial u^l}{\partial z} g(z) dz \quad (72)$$

where $c$ in (67) differs from $\bar{c}$ in (72) because the wave induced rather than the total momentum flux is used. This definition is expected to be consistent with Equation 66.
for the much younger seas observed in Terray et al. (1996) as \( \tau_{\text{tot}} \sim \tau^w(\zeta_o) \) under those conditions. However, the viscous stress is expected to be non-negligible for the low wind, older sea conditions commonly found over the open ocean. Therefore, it is more appropriate to use (71) for the energy flux to the waves.

An interesting feature displayed in Figure 28 corresponds to the constant value reached for the phase speed ratio \((c_i/c_p = 0.5)\) for very young seas, which implies a saturated wave field. Under the open ocean conditions analyzed here is not possible to confirm or deny the saturation level because the maximum observed inverse wave age is approximately 0.07. Instead, the open ocean data shows a steady transition to smaller normalized values, which can be parameterized by:

\[
\frac{c}{c_p} = \Gamma \frac{U_*}{c_p}
\]  

(73)

This is consistent with Terray et al. (1996), except that these observations given a smaller slope (Figure 28). This smaller slope \(i.e.\) less steep) suggests a slower transition to a potential saturation level, meaning that under the conditions presented here the wave field responds slower to the wind input.

This result, combined with Terray et al. (1996), suggest that the effective phase speed is a function of the friction velocity for wave-ages older than approximately 15 and equal to one-half the peak phase speed for younger seas. Figure 28 shows the linear fit (red dashed line in Figure 28) of the relation based on Equation (73), where \(\Gamma = 5.9\) (slope) with an intercept of zero.
Figure 28: Ratio $c_i/c_p$ as a function of the wind forcing ($u*/c_p$). $c_i$ follows directly from Equation 72. The light blue squares correspond to the period during yeardays 74.5-77 and the grey squares correspond to the period including yeardays 98-100. The bin-averaged data (red squares) correspond to both time periods together. The black dashed line represents the behavior of the ratio $c_i/c_p$ presented in Terray et al. (1996), where $c_i$ corresponds to the effective phase speed from different studies.

This expression can be used to compute decay rates for comparison with our energy extraction model. This is accomplished by determining the effective wavenumber from the phase speed using the dispersion relationship

$$A = \alpha \frac{\bar{k}}{c} = \alpha \frac{g}{c^2} = \frac{\alpha}{\Gamma} \frac{g}{u^*} \quad (74)$$

where is $\alpha$ is varied between 1 and 3. This parameterization of $A$ is plotted as a function of the friction velocity in Figure 29 as a family of curves for different values of $\alpha$, using the experimentally defined value of $\Gamma=5.9$. 
Figure 29 also shows the values of $A$ determined from iteration (Table 1) plotted as a function of the friction velocity (red and black squares). The black squares follow from period yearday 74.5-77 with a narrow friction velocity range. The red squares correspond to the more energetic period with rougher conditions (yearday 98-100). The dashed and solid lines from (74) are in reasonably good agreement with the model results. The higher the $\alpha$ value, the larger the dimensional decay rate $A$ is for a given friction velocity.
Figure 29: Decay rate \( (A, \text{m}^{-1}) \) as a function of the friction velocity. Equation (74) is evaluated with \( \Gamma = 5.9 \) (Figure 24) and \( \alpha \) values ranging from 1–3. The red squares correspond to the period during yeardays 98-100. The black squares correspond to the period during yeardays 74.5-77, reported in Table 1. Each bin averaged value is plotted with its corresponding standard deviation.

With classic values of \( \alpha \) reported \((i.e. \text{defined a priori})\) between 0.75–2. The range of decay rates for the wind-coupled waves are consistent with previous values used in the literature \((e.g. \text{Makin and Kudryavtsev}, 1999)\). The dimensionless decay rate \( (a) \) usually defined as a constant has been explored as a function of forcing \((\text{Donelan et al.}, 2006)\). Donelan et al. (2006) found that \( a \) increases under stronger forcing, making the longer waves decay faster. In terms of the decay rate \( (A, \text{m}^{-1}) \), this would suggest that \( A \) should reach a constant value for young seas. Exploration of \( A \) for younger seas is left open at this point.
Under the assumptions presented here, weaker forcing means a faster decay (i.e. larger $A$), leading to minimal wave induced perturbation on the airflow and therefore little energy input to the wave field. This translates into a shallow WBL. Figure 30 shows the behavior of the WBL depth as a function of forcing. The depth of the WBL ($z_e$) follows directly from $1/A$. The red and black squares in Figure 30 follow from yearday 74.5-77 and 98-100 respectively. The solid and dashed lines correspond to the parameterization defined in (74).

**Figure 30:** WBL height as a function of forcing. The depth of the WBL defined as $1/A$. The red dashed line corresponds to the linear trend of the model based on Table 1. The WBL depth is also defined based on (74) with $\alpha = 1$, $\alpha = 1.5$ and $\alpha = 3$. 
In Figure 30, the best agreement between (74) and the model predicted height of the WBL is found for $\alpha = 1.6 \pm 0.1$. An increase in the forcing implies a deeper WBL in the range $u^*[0.4 \ 0.9]$. Under the assumptions so far presented, this implies that the effective wave scale (wind-coupled wave) becomes longer and faster with increasing wind stress, as shown in Table 2.

Therefore, the scale of the wind-coupled waves and WBL depth under open ocean conditions strongly depends on the physical forcing. This observationally supported model predicts a decrease of $A$ with increasing surface stress. This implies a deeper WBL ($i.e.$, proportional to increasing $1/A$) under stronger forcing with a shift towards longer waves ($i.e.$, proportional to increasing $1/k$ and therefore increasing wavelength) supporting the WBL depth (Table 2). However, no conclusive comment can be made about the saturation value $c/c_p = 0.5$ and its implications on the effective wave scale. The behavior of $A$ under younger seas and the potential limit under strong forcing is a question for future research.

**Table 2:** Mean effective wavelength ($\lambda$) for different forcing ranges ($i.e.$ $u^* \text{ m s}^{-1}$) for yeardays 74.5-77 (Period 1) and yeardays 98-100 (Period 2). For both periods, $\alpha = 2$.

<table>
<thead>
<tr>
<th>Range of $u^*$ (m s$^{-1}$)</th>
<th>0.35-0.4</th>
<th>0.4-0.5</th>
<th>0.5-0.6</th>
<th>0.6-0.7</th>
<th>0.7-0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$, Period 1 (m)</td>
<td>0.6</td>
<td>2.2</td>
<td>7.7</td>
<td>24.2</td>
<td>N/A</td>
</tr>
<tr>
<td>$\lambda$, Period 2 (m)</td>
<td>1.8</td>
<td>3.0</td>
<td>7.3</td>
<td>12.8</td>
<td>14.0</td>
</tr>
</tbody>
</table>

The model is less successful under weaker forcing where large decay rates are required showing larger variability. Large decay rates imply a shallow WBL and that the wind-wave coupling happens at shorter scales. Research has shown that long wave-
modulation of the shorter waves becomes increasingly more important under these conditions. Additionally, these longer waves are thought to impart a significant upward component to the wave-induced stress that depends on direction of the wind versus dominant waves. The model may not adequately capture these effects, i.e. it does not incorporate long to short wave modulation and may not properly account for an energy and momentum reversal between the atmosphere and the ocean.

6. Summary

During the SO GasEx 08 field campaign, a set of wave field spectral observations were made onboard of a moving vessel. It was reported that Doppler shifting corrections combined with the motion correction algorithm of Edson et al. (1998) provides a reliable frequency spectrum for SOG < 3 ms\(^{-1}\) (Cifuentes-Lorenzen et al. 2013). Under these conditions the RIEGL and the radar based WaMoS measurements were combined to provide a frequency spectrum in the 0.035–1.5 Hz range. The frequency spectrum was extended to 2 Hz following the observed \(f^{-4}\) Hz tail roll off. These spectral observations correspond to the wave field statistics for open ocean conditions that were complemented with the meteorological data set collected during the same period.

The data set was used to conduct an air-sea boundary layer study that included a momentum closure at the air-sea interface and an energy balance across the WBL. At the wavy air-sea interface, the turbulent stress becomes negligible and the total stress is supported by form drag (normal stress) and viscous effects (tangential stress). An estimate of the magnitude of the form drag on the atmospheric side was made after
defining the magnitude of the viscous component at the interface. The purely viscous stress followed from the Makin and Kudryavtsev (2000) parameterization. This parameterization was modified by matching $\bar{u}(\delta_v)/u^*$ with the normalized wind speed from the logarithmic profile at the high of the viscous sublayer ($z = \delta_v$). The transition to fully rough flow was estimated at a wind speed of 8 m s$^{-1}$.

The spectral observations were then equated to the atmospheric estimate of the wave-induced stress (i.e. total stress minus the viscous fraction) to select a wave growth parameter that satisfied momentum conservation at the interface. Several wave growth parameters were evaluated and the Janssen (1991) parameterization used in the WAM model with the following tuned coefficients: $B = 32$ and $A_l = 0.2283 \pm 0.0884$ for high wing conditions (yeardays 98-100) and $A_l = 0.2083 \pm 0.0504$ for lower wind conditions (yeardays 74.5-77).

The definition of the wave induced momentum flux at the interface was then coupled with a decay function in order to explore the vertical structure of the WBL. An exponential decay function was preferred for simplicity, where the decay rate $A = \alpha k$ was introduced in order to avoid the a priori definition of a decay rate ($\alpha$) and implicitly capture the physical variability with the existing physical conditions. The decay function (i.e. $g(z)$) was then used to estimate the vertical behavior of the wave perturbations in the WBL. These perturbations are included in the velocity profile, where a local atmospheric friction velocity and the eddy viscosity were defined to capture the wave induced effects on the airflow.
The spectral observations combined with the wave growth parameter are used to estimate the observed wind driven energy of the wave field. These observations are then equated to the modeled atmospheric extraction mechanism. This leads to an energy balance across the WBL. The atmospheric extraction mechanism is modified to account for wave-induced perturbations on the airflow. This allows the selection of a decay rate that satisfies the energy balance across the WBL. The selection of the decay rate was done iteratively. This formulation closes the energy balance within 10-20% depending on the forcing. Under weaker forcing the model only allows closure within 40-45%. It is hypothesized that this is due to long wave modulation and an upward ocean–atmosphere momentum flux that is not fully accounted for. For stronger forcing (i.e., $u^* > 0.4$ m s$^{-1}$), the model works well and predicts progressively slower decay rates. The slow decay rate ($A \sim 0.5 \pm 0.4$ m$^{-1}$) suggests longer and faster waves supporting the wave-induced momentum flux. This shift in wave scales makes the wave-induced perturbations protrude higher into the atmosphere as shown in Figure 27 and summarized in Table 2, resulting in a higher WBL. Under open ocean conditions, a WBL height of 1-2 m is predicted at moderate to high wind speeds.

The more classic dimensionless decay rate $\alpha$ is found to vary between 1-3 with best agreement given by a constant $\alpha = 1.6 \pm 0.1$ for the $0.5 < u^* < 0.9$ m s$^{-1}$ range. At weaker forcing, this study suggests that $\alpha$ is not constant and exhibits a dependency on the wind forcing and/or wave field conditions.

The effective wave scale follows from the decay rate ($A$ m$^{-1}$) and defines the scale of the actively wind-coupled waves. The approach presented here is consistent with Terray
et al. (1996) and sheds light on the physical meaning of the Terray et al. (1996) characteristic phase speed. The phase speed is closely related to the work done by the wave-induced momentum flux on the wind speed that is responsible for the energy going into the wave field.

The scales supporting the WBL depth reflect the necessary shift from the spectral peak (i.e. $k_p$ and $c_p$) needed to capture air-sea interaction dynamics under the open ocean. For example, the results given by Terray et al. (1996) suggest that the characteristic phase speed is approximately half the peak phase speed for very young waves (i.e. $\bar{c} = 0.5c_p$). However, the characteristic phase speed and wavelength decrease with decreasing stress for older seas, suggesting that shorter and shorter wind waves are supporting the wave-induced energy and momentum flux as the wind speed decreases until viscous stress is primarily responsible for the exchange. Finally, the defined depth of the WBL under open ocean ultimately shows that the MOS theory remains valid as long as measurements are made above the effective critical height, which is generally satisfied as long as the measurements are made above 3 m.
7. Conclusions

Wave and atmospheric measurements were successfully collected during the SO GasEx 08 field campaign under open ocean conditions. The combination of wave and meteorological instruments are proven to work well on a moving vessel provided the appropriate motion correction algorithm (i.e. Edson et al. 1998) and constraints on the data collection (i.e. SOG < 3 and the respective directional considerations). Based on these measurements a study of the kinematics and dynamics of the WBL was conducted, focusing on momentum and energy transfer between wind and waves.

The momentum closure scheme at the interface was successful within the uncertainty of the measurements (i.e. wave and atmospheric measurements from a moving vessel under the open ocean conditions) and the assumptions (i.e. constant momentum flux layer and the viscous parameterization) made at the interface.

The viscous component at the interface was parameterized in order to satisfy the transition to a fully rough flow at a wind speed of 8 m s$^{-1}$. The viscous parameterization suggests that 5-8% of the total momentum is purely viscous directly at the interface. The wave field corresponding to frequencies in the 0.035-2 Hz range of the wave spectrum directly supports the rest of the momentum (~90%). The Janssen (1991) parameterization used in the WAM model is selected as it is in good agreement with the directly measured momentum flux and captures the expected behavior at the spectral peak. Based on the wave growth parameterization, 80-90% of the wave momentum is supported by waves ranging between 7 – 15 m in wavelength.

The momentum closure at the interface is the cornerstone for the rest of the modeling efforts at the WBL. An exponential decay function is defined to capture the
vertical behavior of the wave induced perturbations on the airflow. The dimensional decay rate $A \text{ m}^{-1}$ is found to vary between 1 – 6. The iterative selection of $A$ works best when the data is divided into ranges of atmospheric forcing as summarized in Table 1. This strongly suggests a wave induced decay rate that depends on the physical conditions of the system. The depth of the WBL is defined as $1/A$, corresponding to the e-folding depth of the decay function. At this height, approximately 30% of the stress is supported by waves with scales defined by the characteristic wave scales (Table 2). It is found that the actively wind coupled waves under open ocean conditions are at significantly higher frequencies than the spectral peak. When the characteristic phase speed (i.e. $\bar{c}$) is plotted following Terray et al. (1996) approach, it is found that the ratio $\bar{c} / c_p$ is a function of inverse wave-age and does not reach an equilibrium value as a function of the forcing. The slope of the curve from the $\bar{c} / c_p$ vs. $u_\ast / c_p$ relation is defined as the parameter $\Gamma$ which is used to parameterize $A$ as a function of the atmospheric friction velocity and the more common dimensionless decay rate $\alpha$. This proposed parameterization can be used interchangeably to define the scale of the wind coupled waves and the depth of the WBL (i.e. $1/A$).

It is found that the decay rates for the vertical behavior of the wave induced perturbations ($A$ and $\alpha$) are dependent on the physical conditions of the system. In this case an inverse dependence was found between $A$ and the forcing (i.e. $u_\ast$). It is found that the dimensionless decay rate $\alpha$ that best captures the conditions under open ocean is approximately $\alpha = 1.6 \pm 0.1$ for the $0.5 < u_\ast < 0.9$ range. This value is in agreement with a priori used values in the literature.
8. Future Work

Continuation of the work presented here will be pursued in order to further expand the understanding of the physics at the WBL by incorporating the wave-induced perturbations on the atmospheric side. The proposed line of work will focus on answering the following questions:

1. What fraction of the stress at the interface does the wave field support? How does it depend on the wave field conditions?
2. What fraction of the tangential stress is purely viscous and what fraction is supported by the small gravity-capillary waves associated with microbreaking?
3. Can the turbulent kinetic energy (TKE) imbalance on the atmospheric side be linked to the observed enhancement of subsurface TKE under breaking waves?
4. Can the misalignment of the surface waves and winds be responsible for some of the disagreement between the wave-spectral estimates of the energy input and the energy extraction model?
5. Can the removal of the energy flux that goes into the gravity-capillary waves that are quickly transferred to ocean currents improve the agreement between the spectral estimates and the model?
6. Can the wave-growth parameter be modified to provide better agreement between the direct covariance and spectral estimates of the wave-induced momentum flux as discussed in Section 4.1.2?

Questions 1 and 2 touch on a very relevant aspect of the physics at the air-sea interface; i.e., the contributions of *microbreaking* to the tangential stress at the interface.
which drives surface currents. The viscous ($\tau^v$) stress and this small-scale wave driven stress together define the tangential stress. The momentum at the interface is then supported by the larger-scale wave induced stress retained by the waves leading to growing waves (and ultimately large-scale breaking) and the tangential stress immediately driving surface currents.

Exploration of this will ideally lead to a definition of a tangential stress parameterization based on surface currents:

$$\tau_{\text{tan}} = f(U_s, u^l, \Phi(\omega))$$

(75)

where $U_s$ is the surface current (from radar observations), $u^l$ is the local friction velocity (reduced turbulent scale) at the interface and $\Phi(\omega)$ is the frequency spectrum of the wave field.

Expected outcomes include a definition of the appropriate spectral cutoffs for the momentum transfer from the wind to waves (i.e. wind-wave spectral region), from the waves to wind (i.e. momentum reversal at the spectral peak $u^l(\omega) < c$) and wind to ocean (i.e. surface current generation). The ultimate goal is to define a spectral drag coefficient at the air-sea interface and to provide a parameterization to be used under open ocean conditions based on both the wind forcing and on wave scaling arguments.

Question 3 focuses the subsurface dynamics by building a turbulent kinetic energy budget at the air-sea interface. The objective is to close the turbulent kinetic energy (TKE) budget by studying the difference between the observed and predicted TKE dissipation rates ($\Delta \varepsilon = \varepsilon_{\text{MOS}} - \varepsilon_{\text{obs}}$) on both sides of the interface. On the atmospheric
side, the delta TKE dissipation rate can be linked to the wave-induced contribution to
the pressure transport term:
\[
\rho_a \Delta \varepsilon \approx -\frac{\partial \tilde{w} \tilde{p}}{\partial z}
\]  
(76)

The tildes in (76) indicate wave-induced perturbations. The energy flux divergence can
be approximated using the local velocity profile and the wave-induced perturbations on
the wind velocity components:
\[
-\frac{\partial \tilde{w} \tilde{p}}{\partial z} = \rho_a \tilde{u} \tilde{w} \frac{\partial \tilde{u}}{\partial z}(z)
\]  
(77)

Equation (77) provides an estimate of the KE energy imbalance on the atmospheric side
due to the energy transfer between the atmosphere and the waves. Local variables are
used in the right-hand side of (77), which are inherent to the wave-affected layer.

Focusing on the TKE imbalance across the surface can provide better parameterizations
of subsurface dissipation rates and improve the accuracy of operational global wave
forecasting methods applied to satellite data (e.g., Anguelova and Hwang, n.d.; Ardhuin
et al., 2009).

Question 4 involves the misalignment of the surface waves and wind stress that
drives them. The current investigation assumes that the winds and waves are perfectly
aligned such that \(\cos(\theta_{\text{wind}} - \theta_{\text{wave}}) = 1\) in (52). However, the relative mean wind–
wave angle is 26° for period 1 (yeaday 74.5 -77) and 28° for period 2 (yeaday 98-100). This is based on the wave direction provided by the WaMoS instrumentation and
corresponds to the dominant wave reported in the directional wave spectrum. Although
the smaller-scale waves are expected to be more closely aligned with the wind, there is clearly a possibility that there is some misalignment that is not accounted for in the analysis. Additionally, sudden changes in wind direction are not included in the momentum closure at the interface. Long wave to short wave modulation is also not considered in the momentum closure at the interface. Therefore, a more detailed investigation that includes the misalignment is a good topic for further research.

Question 5 involves the removal of the viscous stress from the total stress to provide an estimate of the wave-induced stress supported by the surface waves. The viscous parameterization given by (45) accounts only for the \textit{purely} viscous stress associated to molecular friction at the viscous layer. This parameterization does not include the potential effects of microbreaking at high frequencies. Small wave scales that microbreak drive surface currents rather than wave growth and removing this fraction from the momentum balance at the interface could account for the spectral underestimation of the stress (Figure 18) under stronger forcing. Therefore, the phenomenon of microbreaking could add to the total tangential stress at the interface, but this remains as a hypothesis as the spectral resolution during the SOGasEx does not allow for a direct evaluation of this. However, Deviations in the energy balance across the WBL were attributed to the state of the wave field relative to the wind (\textit{i.e.} wind forcing). It was found that under weaker forcing the model only allows closure within 40-45%. Therefore, seems reasonable to explore the stress partitioning at the air-sea interface and its dependence on the wind forcing (\textit{i.e.} $u^*/c_p$).
References

Anguelova, M.D., Hwang, P.A., n.d. 6.7 WHITECAP FRACTION OF ACTIVELY BREAKING WAVES: TOWARD A DATABASE APPLICABLE FOR DYNAMIC PROCESSES IN THE UPPER OCEAN.


Lighthill, M.J., 1962. Physical interpretation of the mathematical theory of wave
Lumley, J.L., Panofsky, H.A., 1964. The structure of atmospheric turbulence,
Interscience Publishers.
Meteorol. 54, 89–103.
Boundary Layer Meteorol. 60, 77–93.
Boundary Layer Meteorol. 73, 159–182.
Makin, V.K., Mastenbroek, C., 1996. Impact of waves on air-sea exchange of sensible
evidence of the rapid distortion of turbulence in the air flow over water waves.
2041–2049.
185–204.
breaking waves: Feedback mechanisms from air-flow separation. Boundary


