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Design Optimization of a Small Scale Hydropower Harvesting Device

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Design Optimization of a Small Scale Hydropower Harvesting Device

Rishav Aryal

B.S., Tribhuvan University, 2012

A Thesis

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Requirements for the Degree of

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APPROVAL PAGE

Masters of Science Thesis

**Design Optimization of a Small Scale
Hydropower Harvesting Device**

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Table of Contents

APPROVAL PAGE	iii
ACKNOWLEDGEMENTS	iv
1. Introduction.....	1
1.1 Background	1
1.2 Objectives	3
1.3 Assumptions.....	4
2. Theoretical Concepts	4
2.1 Numerical model.....	6
2.2 Genetic Algorithm	11
2.3 Fluid-Structure Interaction model.....	14
3. Methodology	15
3.1 Semi-Analytical Study	15
3.2 Optimization using the Semi-Analytical Model	16
3.3 Self-sustaining finite element model using COMSOL Multiphysics.....	21
4. Results and discussion	25
4.1 Single and multi-objective optimization	25
4.2 Cases Studied	28
4.3 Finite Element model.....	37
5. Conclusions.....	42
5.1 Summary	42
5.2 Future work.....	43
References.....	45

APPENDIX A: KINEMATIC AND KINETIC RELATIONS.....	49
APPENDIX B: SECANT METHOD	51
APPENDIX C: MATLAB CODE	53

List of Figures

Figure 1. (a) Laboratory scale model. (b) 2-D representation of the laboratory model studied herein.....	6
Figure 2. Flowchart of the semi-analytical model	10
Figure 3. Rotational velocity function for March in the Shetucket River.....	23
Figure 4. (a) Crank slider model (b) Quick return mechanism model	24
Figure 5. (a) Average distance plot between individuals through different generations. (b) Results for single optimization of time period (1 = ED, 2 = DG, 3 = CG, 4 = BC, 5 = AB, time period = 0.359 s, stroke = 1.39 cm).....	26
Figure 6. (a) Average distance plot between individuals for through different generations. (b) Results for single optimization of piston stroke (1 = ED, 2 = DG, 3 = CG, 4 = BC, 5 = AB, piston stroke = 29.67 cm, time period =5.11 sec).....	27
Figure 7. Pareto front for multi-objective optimization showing Pareto optimal solutions.....	28
Figure 8. LVDT data for piston vertical displacement for 5V and 10V from laboratory experiment reported in Malla et al. (2011)	30
Figure 9. Load and piston displacement prediction for 5 V data.	31
Figure 10. Load and piston displacement prediction for 10 V data	32
Figure 11. Location for Shetucket River data collecting station (41°41'01", 72°10'57")	33
Figure 12. Optimization results for device arm lengths for Shetucket River over 12 months.	36
Figure 13. Ratio of objective function values for optimized vs. non-optimized device lengths for Shetucket River over 12 months	36
Figure 14. Piston displacement for Crank slider type model	38
Figure 15. Piston displacement for Quick Return mechanism type model	39
Figure 16. Rotational velocity for different flow velocity	40

Figure 17. Change in rotational velocity for different number of fins	41
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List of Tables

Table 1. Monthly flow data and corresponding rotational velocity of cylinder for Shetucket River, CT ..	22
Table 2. Optimization results for reciprocating device design applicable to Shetucket River, CT	35

Abstract

This study presents a semi-analytical model that facilitates the optimum design of small-scale hydropower systems, so that maximum possible energy can be harvested under such settings. The model comprises a rotating cylinder attached to a piston producing reciprocating motion when placed in moving water. Taking this model as base, the semi-analytical model employs Genetic Algorithm based optimization and develops optimal dimensions for the system, with the objective to minimize the time period while at the same time maximizing stroke of the piston. The model is tested first with single-parameter optimization and then with multi-objective optimization. As many energy harvesting approaches are based on the reciprocating motion of the mechanical/structural system, which is greatly affected by the geometric dimensions of the system, optimization of the system geometry becomes crucial for energy harvesting. The semi-analytical model is able to reduce the arms dimension while obtaining a higher stroke of piston for lower time period. The model has limitations but is able to produce optimization results comparable to laboratory data and applicable to flow data from Shetucket River in Willimantic, Connecticut, USA.

As energy is needed to rotate the cylinder, we propose to make it self-sustainable by attaching fins to the cylinder and use the river flow to rotate the cylinder. A fluid-structure interaction type CFD model is developed to study the rotation of the cylinder and the flow measurements are used as input velocities for simulations. Two different types of self-sustaining models are studied, namely Crank-slider model and Quick-return model.

1. Introduction

1.1 Background

Hydropower, being one of the sources of clean energy, is applied vastly all over the globe. Current hydropower harvesting, at large scales, consists mostly of bulky turbines that utilize high flow discharge or high head of water or both and usually require use of dams of various sizes. Even though these hydropower plants produce reasonable amounts of energy, not all places that require energy have access to high discharge and high water heads. Linear generators have been used in generating energy by utilizing the flow and head of water. Large bodies of water, such as the sea or ocean have tidal motions that create a reciprocating action necessary for harvesting energy. However, for places with no significant waves or tidal motions, an artificial system that can generate the reciprocating motion needed to utilize the low head and low discharge in Run-of-the-River (RoR) or Run-of-the-Stream settings is more desirable. Integration of RoR hydropower with other climate related energy sources, such as solar-power and wind-power has been shown to make hydropower the highest shareholder (Francois et al. 2016). Furthermore, analysis of different possible mixes of solar-power and RoR hydropower at an hourly scale demonstrated that a high share of the RoR hydropower corresponds to higher balance between the energy load and generation (Francois et al. 2015). Remote locations that do not have high head or high discharge but still require continuous power supply would benefit more from small power generating systems that are powered by ambient and inexpensive sources and can store that energy locally (Torres et al. 2005; Kecik et al. 2013). Moreover, such systems do not suffer from the societal and environmental problems associated with hydroelectric dams that are well documented worldwide (Gleick 1992).

Malla et al. (2011) discuss a reciprocating small-scale hydropower system that aims at generating clean energy from low discharge and low head water flow that can be supplied to remote applications, such as sensors deployed on buoys or barges near river embankments. Wireless sensor systems, like the ones used for health monitoring of bridge piers, abutments, levees, etc. require continuous supply of energy for functioning, which can be provided by such a reciprocating system (Elvin et al. 2006). Furthermore, the system is environment friendly and requires no damming of the flowing water or rerouting it, does no harm to the inhabiting aquatic ecosystem, and does not prevent sunlight from penetrating the water, allowing uninhibited bottom flora growth.

The reciprocating hydropower system produces output in the form of displacement of a piston, thus requiring a generator to convert the harvested kinetic energy into electrical energy (Beeby et al. 2006). Electromagnetic, piezoelectric or electrostatic mechanisms can also be used for the conversion (Shen et al. 2006; Wang et al. 2012). Many of the energy harvesting concepts utilize mechanical/structural systems that exhibit oscillatory/reciprocating motion or vibration (Malla et al. 2011; Elvin et al. 2006; Beeby et al. 2006; Shen et al. 2006; Erturk et al. 2009; Shenck et al. 2001; Vila et al. 2012; Shen et al. 2016; Ahmed et al. 2017; Zheng et al. 2009). The amount of power generation from these systems depends on the amplitude and frequency or period of oscillation/vibration of the mechanism. These parameters in turn are greatly dependent on the geometric dimensions of various parts/members constituting the system. Therefore, optimization of these parameters is critically vital for harvesting maximum possible energy using such systems. For our reciprocating hydropower system the amount of energy generated depends on various

factors such as structural dimensions, flow velocity, lift and drag forces. The lift force created due to the Robins-Magnus effect depends on the velocity of the flow along with the dimensions of the reciprocating cylinder and the angular velocity with which the cylinder rotates (NASA 2008). Research has also been done on suppressing the vortex shedding that occurs behind the rotating cylinder by adding rotating controllers at particular positions (Goodarzi et al. 2017a, b). It is evident that with appropriate controls, one can attain optimum values for the factors influencing energy generation.

1.2 Objectives

There are two main objectives of this study. The first objective is to develop and test a model that can be used to find an optimum design in terms of physical dimensions for a typical reciprocating small-scale hydropower system similar to the laboratory-scale model discussed in Malla et al. (2011) and apply this to real field applications. This goal includes finding the lengths of the various component arms of the device as well as the radius of the rotating cylinder, given specific flow characteristics, thus achieving an optimized design as a result of maximizing the stroke of the piston and minimizing its time period. The model can also allow the incorporation of adjustments to the rotational velocity of the cylinder which is affected by the velocity of the moving water and the diameter of the cylinder, both of which greatly influence the amount of lift force generated.

The second objective of this study is to make the device self-sustainable. The hydropower harvesting system requires that a motor be used to rotate the cylinder. As the motor itself needs energy to operate, the output from the system is compromised. In order to make the device more

productive, the cylinder is modified so that it can be rotated by the flowing water, hence eliminating the need of input power.

1.3 Assumptions

Several assumptions are made in the semi-analytical model discussed in this study for the sake of simplicity. These are: (i) The various components in the model were assumed to act as rigid bodies, even though, in real cases, they will exhibit some flexibility. (ii) The entire model is supposed to be operating under water, thus experiencing not only lift but also drag and buoyancy forces due to the moving water. These forces are assumed to be negligible compared to lift and are not accounted for in the semi-analytical model. (iii) In a real field, the lift force builds up to a value over time and then stays constant. In the semi-analytical model, the maximum value of lift force is assumed to be reached instantaneously and applied all the time. (iv) Furthermore, the semi-analytical model represents a prototype in two-dimensional (2D) space while the actual physical devices are three-dimensional (3D).

2. Theoretical Concepts

The working principle of a reciprocating hydropower harvesting system is based on the Robins-Magnus effect. A cylinder rotating about its axis experiences a lift force when placed in a flowing fluid (such as running water or air) due to this effect (Bourguet et al. 2014; Sengupta et al. 2004a). When the rotating cylinder is placed in flowing water such that the flow direction is perpendicular to the cylinder's horizontal axis, the direction of rotation of the cylinder matches with the direction of flow on one side and opposes the flow direction on the other side, thus creating the lift force that

is directed from the opposing side to the matching side. This moves the cylinder in the direction perpendicular to the flow. Connecting this moving cylinder to a piston by appropriate mechanisms can result in displacement of the piston, which can translate to harvested energy. The lift force P developed due to Robins-Magnus effect is as follows (Malla et al. 2011; NASA 2008):

$$P = 0.5\rho v^2 AC_L \quad (1)$$

where ρ is the density of fluid medium, v is the velocity of flow, A is the surface area of the cylinder that is exposed to the flow, and C_L is the lift coefficient. The lift coefficient depends on the flow and the dimensions and rotation of the cylinder. Eq. (1) shows that along with fluid density and lift coefficient, the magnitude of lift force is governed by the dimensions of the cylinder and the flow velocity.

An experimental study to explore the principle discussed above for cylinders of different radii under different flow velocities and rotational speeds was presented by Malla et al. (2011). Connector arms paired the rotating cylinder with a piston that was free to move vertically in the direction perpendicular to the flow as shown in Figure 1(a). The cylinder moves up or down perpendicular to the flow as per the Robins-Magnus effect depending upon the flow and the rotation of the cylinder, thus creating an alternating motion in the piston attached. In the experiment, the flow is in a particular direction while the direction of the rotation of the cylinder is switched alternately when it reached the highest or lowest position possible in prototype assembly. Such movement of the piston can be used to harvest energy using electromagnetic induction or linear inertial

generators. The physical reciprocating hydropower system can be represented in a 2D analytical model as a see-saw frame as shown in Figure 1(b) with a hinge support at point D, the rotating cylinder at point E, free rotating hinges C and B, and the piston connection at point B.

Figure 1. (a) Laboratory scale model. (b) 2-D representation of the laboratory model studied herein.

The kinematic behavior of the 2D model can be simulated and analyzed numerically. Thus, an optimum value of the properties and factors involved can be obtained by running a set of optimization simulations using a numerical code developed for this purpose as its basis.

2.1 Numerical model

Figure 1(b) depicts the idealized 2D version of the laboratory scale model for the hydropower harvesting system adopted by Malla et al. (2011). The model discussed here aims at calculation and optimization of the vertical displacement (stroke) and the period of oscillation (time period) of the piston. Our model is adaptable to other geometric design configurations and can be started at any initial position or angle. The rotating cylinder is represented by its center at point E while B denotes the top of piston BC that moves up and down. EDGC, BC and AB are connecting arms. The model is supported at point D and A by pin supports, allowing the structure to rotate freely about those points should the geometrical constraints allow. B and C are hinge connections. Depending on the exact relative position of D and C the lengths of ED, DG, and GC can be varied.

The total lift force, P , that acts on the cylinder is applied at point E and is directed vertically upward or downward depending upon the direction in which the cylinder rotates. The magnitude of the lift force is calculated using Eq. (1). The kinematic and kinetic equations for the numerical model of the energy harvesting system are derived using fundamental analytical principles. For arm EDGC (Fig. 1), these relations are as follows:

$$\text{Angular displacement: } (\theta_{D(i+1)}) = \theta_{D(i)} + \omega_{D(i+1)}dt + 0.5 \alpha_D dt^2 \quad (2)$$

where,

$$\text{Angular acceleration: } (\alpha_{D(i)}) = (-P \cdot ED \cdot \cos\theta_{D(i)} + X_i)/Y_i \quad (3)$$

$$\text{Angular velocity: } (\omega_{D(i+1)}) = \omega_{D(i)} + \alpha_{D(i)} dt \quad (4)$$

Kinematic Equations (2) and (4) give the angular displacement (θ_D) and angular velocity (ω_D) for the arm. Equation (3) gives angular acceleration (α_D) at point D using equilibrium equations for the whole system. dt is the time step at which the simulation proceeds in time, i and $i+1$ are consecutive times.

X and Y are functions of the device member dimensions and their corresponding weights, angular velocities and displacements (Shrestha 2010) and are as follows:

$$X = 0.5(W_{ED} ED - W_{DG} DG)\cos\theta_D - W_{CG}(DG\cos\theta_D - 0.5 CG\sin\theta_D) + U(DG\cos\theta_D - CG\sin\theta_D) - S(DG\sin\theta_D + CG\cos\theta_D) \quad (5)$$

$$Y = -T(DG \cos\theta_D - CG \sin\theta_D) + R(DG \sin\theta_D + CG \cos\theta_D) + \frac{m_{ED} ED^2 + m_{DG} DG^2 + m_{CG} CG^2}{3} + m_{CG} DG^2 \quad (6)$$

Equations describing U , S , T and R from Equation (5) and (6) are presented in Appendix A.

The equations are solved numerically at different time steps for the desired number of time steps, i , to get values of angular displacement (θ_D) at point D of arm EDG as a function of time. For each time step, the angular acceleration value is considered constant. Kinematic and equilibrium equations of the system relate the angular displacement at point D with all other relevant points in the system.

Iteration is performed for obtaining the values of the angles formed by arms AB at A (θ_A), and BC at connection point B (θ_B) in relation to the rotation angle of arm EDGC at D (θ_D) produced under the action of the lift force P at the cylinder, E. This iteration is necessary to ensure that the angles conform to the geometrical constraints and employs the Secant Method, which is a root finding algorithm that uses a series of roots and secant lines to approximate the roots of a function (Diez

2003). It can be defined as the finite difference approximation of Newton's method but differs in that it does not require calculation of the derivative of the function. The derivative is replaced by an approximation, thus reducing the number of steps required. Our model couples an analytical component with a numerical iterative solution; hence, it is a semi-analytical model.

In the semi-analytical model discussed here, an educated guess for the angles is made initially and the Secant Method is applied. Iterations are conducted until the difference between two consecutive angle values is less than 10^{-4} radians. A summary of the semi-analytical model is shown in pseudo-diagram form in Figure 2.

Figure 2. Flowchart of the semi-analytical model

This 2D model was used to study different aspects of the reciprocating hydropower system. Specific details are as follows:

(1) The semi-analytical model allows one to change the values or even set a range of values to choose from for one or more lengths of the device arms simultaneously. A portion of the semi-analytical optimization model involves checks for physical incompatibility of the hydropower harvesting system and discards the combination of lengths that are not feasible (combination of lengths of connecting arms not compatible to form the system).

(2) The design of the piston in the laboratory model limited its movement in upward and downward direction, thus limiting the stroke of the piston. However, the semi-analytical model allows one to set various upper and lower boundary values for piston movement based on the geometry and lift force involved, giving it more flexibility.

(3) Along with the density of fluid and the lift coefficient, the lift force generated depends on the geometry of the cylinder, its angular velocity and the velocity of flow. The semi-analytical method allows computation of the lift force conveniently for different values of cylinder dimension, rotating at different rates for different flow velocities and the data generated can be collected and analyzed.

2.2 Genetic Algorithm

The Genetic Algorithm method was selected for optimizing the influencing factors. Genetic Algorithm optimization works on the same principle as natural selection, following survival of the fittest (David 1991; Goldberg 1989; Whitley 1994).

In Genetic Algorithm, a fitness function, which is the equivalent of an objective function, is defined first. The location of the global minimum of the fitness function is sought in parameter space. Optimization using the Genetic Algorithm can be either single or multi-objective. In single-objective optimization, the function is minimized by optimizing one particular parameter whereas in multi-objective optimization two or more parameters can be optimized simultaneously. Results can be extracted such that one has a set of optimal values to choose from based on the importance placed in each objective. In our case, the fitness function is the value of the time period (the time period of a cycle in the displacement versus time curve), or the value of the inverse of the stroke (peak to peak amplitude in the displacement versus time curve) of the piston, or a combination of both, depending on what kind of optimization (single- or multi-objective) is desired. The reciprocating system with the smallest time period or the largest stroke of the piston, or a combination of both is chosen as the optimal result in each case.

Decision variables, which describe the unknowns of the problem, are the inputs to the fitness function. The user can set initial values for these variables, or define a range as required. The fitness function, computed using specific values of the decision variables, gives the fitness value(s) as output. The initial population comprises individuals, each individual formed by choosing a random combination of the decision variables subjected to the constraints defined. Each individual in the population is a potential solution of the optimization problem. These individuals in the population are tested based on their fitness values. They are then ranked to be carried on to the next generation in parts or whole or are discarded. Using the fittest individuals from the previous generation along with some new individuals to maintain diversity, a new population is created. The process continues

until a stopping criterion is met, which could be a tolerance value, the maximum number of generations, or other as defined by the user. Details specific to the paper are discussed in the methodology section.

In Genetic Algorithm optimization different selection methods are available for creating a new population using the best individuals from the previous generation. The selection functions choose parents for next generation based on their fitness values. In stochastic uniform selection function, a line is laid and each parent is allocated a part of the line. The part of line attached to a parent corresponds to the fitness value of the parent, meaning a parent with higher fitness occupies more section of the line than a parent with lower fitness value. The algorithm then moves along the line in equal steps to select parents, thus giving more preference to the parents with better fitness values. In roulette selection function, a roulette wheel is simulated by the algorithm such that parents with better fitness values occupy larger area in the wheel. When the parents are chosen based on the roulette wheel, those parents with better fitness values have more chances of being selected. Uniform selection function selects parents for the next generation randomly from a uniform distribution that is created based on the number of parents and their probability of being chosen for next generation. In tournament selection function, parents are chosen based on the size of the tournament defined. These parents are then compared with each other based on their fitness function values and the best ones are chosen for producing off springs. Each of the selection functions discussed have their own pros and cons and a selection should be made based on which is more suitable to the optimization desired.

After the parents are selected, different parameters control how the new generation is created. Some of them are elitism count, crossover fraction and mutation. Elitism count defines how many species from one generation are carried to the next generation with certainty and helps with securing the best individuals from each generation. Crossover fraction defines what fractions of the parents will be combined with each other to form new species and ensures variation in the new generation. Mutation introduces random changes to the parents to form new population. Trial runs for optimization should be carried out to properly select the values of these parameters so that accurate results can be obtained.

2.3 Fluid-Structure Interaction model

The hydropower harvesting model can be more efficient if the input power needed to rotate the cylinder can be avoided. In order to study the rotation of cylinder purely by the flow of water, a Computational Fluid Dynamics (CFD) type model is studied in COMSOL Multiphysics 5.2a (COMSOL, Inc, Burlington, MA, USA). Fins are attached to the cylinder and it is allowed to rotate freely under the effect of the specified flow. Actual flow data are used as input values and the results obtained are compared with that of the base model. This model has advantage over the base device as it does not need input power to operate. As the cylinder's rotation direction cannot be changed, Magnus effect no longer comes into play. Instead, a new system is adopted that converts the unidirectional rotational motion of the cylinder into translational motion of the piston.

3. Methodology

3.1 Semi-Analytical Study

The 2D model shown in Figure 1(b) is the basis of the semi-analytical model that is used for optimization of the hydropower harvesting system. In the model, the rigid arms EDGC, BC and AB are connected by pin joints (hinges) at C and B, and supported by pins at A and D. Thus, the kinematic parameters (displacement, rotation, velocity and acceleration) of the arms can be related to each other such that if the kinematic parameters of one of the arms are known at a time step i , the same properties for other arms can be calculated for the next time step $i+1$. In the semi-analytical study, calculations of the kinematic parameters are started for arm EDGC (supported at D by a hinge). They are then extended to other points and arms through kinematic relations of the system. The equations move forward one time step at a time in an explicit model format. The set of values for displacement, rotation, velocity and acceleration obtained after the first run for points D, A and B are used as inputs for the next successive time step and the process continues.

Focusing on the goal of harvesting the maximum energy possible, the objective function is considered affected by two factors: the time period and the amplitude (magnitude of vertical motion) of the piston. The main idea is to perform multi-objective optimization involving the time period and the amplitude for different combinations of the potential decision variables (arm lengths, cylinder dimensions, angular velocity, and flow velocity) until the desired stopping criterion is met. The semi-analytical model gives time period and stroke of the piston as output for one run and then compares it with the output from other successive runs to get the most favorable combination of

decision variables. With the device arm lengths as decision variables, the model provides the optimized geometry to be implemented for the design of our device.

3.2 Optimization using the Semi-Analytical Model

The laboratory experiments conducted by Malla et al. (2011) used fixed values for lengths of various arms/components of the device, namely ED, DG, CG, BC and AB while changing the magnitude of the lift force produced by changing the rate of rotation of the cylinder and the water flow rate. The semi-analytical model discussed here allows one to change those arm lengths along with other factors such as rotation of the cylinder, radius of the cylinder, and flow velocity that ultimately lead to a change in the lift force. The Genetic Algorithm MATLAB optimization toolbox in combination with the semi-analytical model were used for optimizing the design of the experimental device (arm lengths).

The formal single objective optimization problem (time period or stroke of piston) is as follows:

Single Objective 1 (minimizing the time period):

$$\min \quad t \quad (7)$$

$$\text{Subject to} \quad L_i \geq 2 \quad (8)$$

$$L_i \leq 30 \quad (9)$$

where t = time period (s), L_i = length of connecting arms (cm), i = arm number ($i=1, 2, 3, 4, 5$)

Single Objective 2 (maximizing the stroke of piston):

$$\max \quad S \quad \text{or} \quad \min \quad 1/S \quad (10)$$

$$\text{Subject to} \quad L_i \geq 2 \quad (11)$$

$$L_i \leq 30 \quad (12)$$

where S =stroke of the piston (cm), L_i = length of connecting arms (cm), i = arm number ($i = 1, 2, 3, 4, 5$)

For single-objective optimization involving time period, the decision variables were varied in each generation with the objective being minimization of the time period of a cycle in displacement versus time curve while ignoring the changes in the stroke of the piston. Similarly, for single-objective optimization considering the stroke of the piston, different combinations of these decision variables were created in different generations and then the fitness values evaluated to get the combination corresponding to the maximum value of the stroke while ignoring the effects on the time period. The purpose of the single-objective optimization runs was to determine the values of other parameters germane to the Genetic Algorithm optimization such as mutation, crossover, elitism, and population size for better results (The Math Works 1993).

Single-objective optimization for minimizing the time period was performed first. Based on experiments conducted with our prototype, the value of the lift force at point E applied vertically was kept constant at 10 N while the lengths of five arms ED, DG, CG, BC and AB were chosen as the decision variables (inputs). The upper and lower limits for the lengths were chosen as 2 cm and 30 cm, based on the size of our prototypes and constructability considerations, respectively. The objective function aimed at identifying a combination of these lengths for which the time period is minimum.

Trial runs were made with different number of generations and different population sizes. Starting with a population size of 50 and the maximum number of generation as 100, optimization runs were carried out. These numbers were then gradually increased and the results compared based on convergence and accuracy. Finally, a population size of 400 for each generation with 500 as the maximum number of generations allowed was accepted based on the time of convergence and distance plots. Distance plots, which are plots of average Euclidean distance between random individuals of the population, show how different the population is in each generation. As the optimization proceeds along generations, the population becomes more and more similar, thus reducing the average Euclidean distance between them and finally becoming very close to zero (The Math Works 1993). The following four different types of selection functions were tested: stochastic uniform, uniform, roulette, and tournament. In stochastic uniform method, a line is laid where each parent corresponds to a section of the line and the algorithm moves along the line in equal steps, allocating a parent from the section it lands on. In uniform selection, parents are selected at random from a uniform distribution based on the number of parents and their probability of being chosen. In roulette selection, a roulette wheel is simulated with potential parents covering area proportional to their expectation. In tournament selection, a group of individuals of the defined tournament size is chosen at random and the best out of that set is chosen as a parent (The Math Works 1993). Tournament was chosen as it produced distance plots with more diverse individuals at the start and better convergence at the end when compared among others. The values of crossover fraction and elitism count were varied to refine the results. The results from the single-objective

optimization for time period were analyzed through distance plots that showed the variability among the generations.

After the runs for time period optimization, a second set of runs was carried out for single-objective optimization to maximize the stroke of the piston. The values of crossover fraction and elitism count found to be suitable for the time period optimization runs were used for optimization of the stroke of piston. The results from the runs were analyzed based on the distance plots as before.

For same values of lift force, inertial effects, cylinder diameter and rotational speed and flow velocity, a model of smaller dimensions and with non-deformable members will generally move faster and will thus have a smaller time period while a model of larger dimensions would provide larger stroke of the piston but will have longer time period. But an optimal model would require the minimum possible time period together with the maximum possible stroke, that is, optimized values for both time period and stroke of the piston, which are competing objectives. For this, multi-objective optimization was carried out next. These runs simultaneously minimized the time period of the piston and maximized its stroke while using different combinations of the decision variables. Pareto fronts were plotted to analyze the results, which gave a list of optimized results along with the optimization factor to choose from. Pareto front is the optimal tradeoff curve between the two objectives involved (here, minimizing time period and maximizing stroke of piston) allowing the user to choose the optimal solutions based on one's preference.

The formal optimization problem involving multi parameters (time period and stroke of piston) is as follows:

Multi objective (minimizing the time period and minimizing inverse of the stroke of piston):

$$\min \quad w_1 t + w_2 \left(\frac{1}{S} \right) \quad (13)$$

$$\text{subjected to} \quad w_1 + w_2 = 1 \quad (14)$$

$$0 \leq w_j \leq 1 \quad (j=1, 2) \quad (15)$$

$$L_i \geq 2 \quad (16)$$

$$L_i \leq 30 \quad (17)$$

where t = time period (s), S =stroke of the piston (cm), w_1 =weight for time period, and w_2 =weight for the inverse of stroke of piston, L_i = length of connecting arms, i = arm number ($i = 1, 2, 3, 4, 5$)

The values of weights w_1 and w_2 can be chosen based on the preference of the user. If time period is to be given more importance, the value of weight w_1 that corresponds to time period can be assigned a higher number and if the maximization of stroke is more preferred, a higher value for the weight w_2 corresponding to the stroke can be given a higher number. As the weights w_1 and w_2 are assigned in percentage, they add up to 1.

Taking the population size of 400, maximum number of allowed generations as 500, tournament selection function and crossover fraction of 0.8, the time period was minimized and the stroke maximized simultaneously. The results from these optimization runs were extracted in the form of distance plot, average Pareto spread plot and Pareto front plot. Pareto front plots link Pareto optimal

solutions with the optimizing function (minimizing time period and maximizing stroke). Based on which optimizing function is given how much weight, a combination of the decision variables can be obtained. Thus, one has the option to choose a combination of decision variables so that the hydropower harvesting system has the smallest calculated time period, or such that it has the largest calculated stroke, or somewhere in between depending upon the tradeoff of the time period or stroke desired.

3.3 Self-sustaining finite element model using COMSOL Multiphysics

The semi-analytical model as shown in Figure 1 has a cylinder attached to a motor that provides the cylinder a constant angular velocity. Thus, along with harvesting energy, input energy is also supplied to keep the motor running. This input energy needs to be deducted from the output energy of the hydropower harvesting device, thus decreasing the overall efficiency. To increase the efficiency of the device, CAD type fluid-structure interaction models is studied in COMSOL Multiphysics 5.2a.

In order to rotate the cylinder by means of flow, fins are attached to the cylinder. The flow velocity from Shetucket River is used as input and the cylinder is allowed to rotate until the rotational velocity remains approximately constant. Table 1 shows the 12 monthly flow data for the Shetucket River along with the corresponding rotational velocity developed in the cylinder due to the flow. Figure 3 shows the graph for rotational velocity of the cylinder over time corresponding to the 0.644 m/s stream velocity for the month of March.

Table 1. Monthly flow data and corresponding rotational velocity of cylinder for Shetucket River, CT

month	avg velocity (m/s)	Rotational velocity (rad/s)
Jan	0.533	13.7
Feb	0.490	13.6
Mar	0.644	18.0
Apr	0.625	17.7
May	0.459	11.0
Jun	0.370	10.4
Jul	0.239	6.9
Aug	0.208	4.5
Sep	0.238	5.5
Oct	0.384	10.8
Nov	0.402	11.2
Dec	0.527	13.5

Figure 3. Rotational velocity function for March in the Shetucket River

It can be observed in Figure 3 that the cylinder takes some time to come to a constant rotational velocity. Without the motor to rotate the cylinder, the cylinder is able to rotate only in the direction of flow and thus, cannot interchange between clockwise and anti-clockwise rotational direction. Reciprocating motion due to lift force is no longer possible and a new design is needed to use the rotational motion of the cylinder to induce translational motion in the piston. Two types of models, namely crank-slider model and quick return mechanism model are studied in COMSOL. Figure 4 shows a simplified prototype of the models.

Figure 4. (a) Crank slider model (b) Quick return mechanism model

Figure 4(a) shows the crank slider prototype model for self-sustaining hydropower harvesting system. The links are connected to the cylinder and to each other through pin joints. For the flow direction shown, the cylinder rotates in clockwise direction. The attached links move accordingly, causing the piston to move in reciprocating vertical motion. Figure 4(b) depicts the quick return mechanism prototype model for the system with similar connections. When the flow rotates the cylinder in clockwise direction as shown, the piston moves in reciprocating horizontal motion.

Similar to the base hydropower harvesting device, the time period and stroke of the piston for both crank slider type model and quick return mechanism type model were calculated.

The work produced per mass by the cylinder rotating due to the flow can be calculated as follows:

$$W = \frac{I\omega^2}{2m} \quad (18)$$

Where, W is the work produced per mass by the rotating cylinder in m^2/s^2 , I is the moment of inertia of the cylinder in kg.m^2 , ω is the angular velocity of the cylinder in rad/s and m is the mass of the cylinder in kg .

4. Results and discussion

4.1 Single and multi-objective optimization

Figure 5 shows the single-objective optimization results for time period using Genetic Algorithm, keeping the force at a constant value of 10 N and the upper and lower bounds for the lengths of the connecting arms set at 2 cm and 30 cm, respectively. Figure 5(a) shows the distance plot where distance between the decision values over generations is plotted. The model stops when the values of the objective function between two generations change by a number smaller than the tolerance value defined (10^{-8}). The values of the decision variables are then considered the best fit values for this run. Figure 5(b) gives the values of the optimal lengths of arms ED, DG, CG, BC, and AB equal to 5.29 cm, 2 cm, 4.07 cm, 2.67 cm, and 2.72 cm, respectively with an optimal time period of 0.359 s and stroke of 1.39 cm.

Figure 5. (a) Average distance plot between individuals through different generations. (b) Results for single optimization of time period (1 = ED, 2 = DG, 3 = CG, 4 = BC, 5 = AB, time period = 0.359 s, stroke = 1.39 cm)

Figure 6 depicts another single-objective optimization but for maximizing the stroke of the piston. As the Genetic Algorithm minimizes the function value, to obtain the maximum value of the stroke of piston, its inverse is minimized. Figure 6(a) shows the distance plot where distance between the decision values over generations is plotted. Figure 6(b) gives the values of the optimal lengths of arms ED, DG, CG, BC and AB equal to 28.09 cm, 29.98 cm, 3.24 cm, 24.98 cm, and 29.45 cm, respectively for an optimal stroke of 29.67 cm and a time period of 5.11 sec. As can be noted from Figure 5(b) and 6(b), the values of the decision variables tend to be lower when only minimization of time period is considered and are on the higher side when only the maximization of stroke is taken into account.

Figure 6. (a) Average distance plot between individuals for through different generations. (b) Results for single optimization of piston stroke (1 = ED, 2 = DG, 3 = CG, 4 = BC, 5 = AB, piston stroke = 29.67 cm, time period = 5.11 sec).

Figure 7 shows a Pareto front plot with the optimal Pareto solutions for multi-objective optimization runs so that both time period and the stroke of piston can be considered. Objective 1 is minimization of time period while objective 2 is minimization of (1/stroke) or maximization of stroke. If time period is to be given more importance than the stroke of the piston, one can select the combinations towards the tail of the curve where the values for objective 2 (1/stroke) are smaller. Similarly, if the stroke is to be given more weight, the values on the left can be chosen, where the values for objective 1 (time period) are smaller. Thus, it is possible to find a combination for the desired weights of time period and the stroke of piston. Given equal weights to both time period and the stroke, the values obtained from optimization are: time period = 0.51 sec, stroke of piston = 3.58

cm, and arm lengths $ED = 5.87$ cm, $DG = 3.17$ cm, $CG = 2.32$ cm, $BC = 4.82$ cm, and $AB = 4.82$ cm.

Figure 7. Pareto front for multi-objective optimization showing Pareto optimal solutions.

After the optimizations runs were carried, two test cases were studied using the semi-analytical model.

4.2 Cases Studied

The laboratory scale model under study has fixed dimensions of connecting arms and base plate, with a hydraulic flume as a manageable source of open channel flow. Under known values of flow velocity, cylinder dimensions and angular velocity, the piston displacement was measured and based on this power generated was estimated. The model, however, is limited by its features and is lab-specific.

The semi-analytical model discussed in this paper provides more flexibility than the laboratory scale model. As it can suggest an optimum set of values for the system, it is not site-specific. The model can be run for different streams with different flow conditions and a general dimension can be agreed upon. If it is required for a specific river or stream, even more precise results can be extracted. Flow records from that stream used as input in the semi-analytical model can help provide a reasonable estimation of the most favorable dimensions.

Two test cases were studied using the semi-analytical model. The first case serves as a check or verification for the accuracy of the semi-analytical method developed in this study while the second case optimizes the influencing factors of the hydropower harvesting system and discusses its practical applications.

Case 1: In the first case, the semi-analytical model was compared with laboratory observations. The model was used to find the amount of lift force generated, given a set of displacement data for the system. A Linear Variable Differential Transducer (LVDT) was placed on top of the piston and used to obtain displacement time history. One set of data corresponded to a supply of 5 volts to rotate the cylinder at 255 RPM while another set corresponded to a supply of 10 volts to rotate the same cylinder at 472 RPM (Figure 8).

A few adjustments were made in the semi-analytical model to be able to compare it with the laboratory model. As the data used for load prediction was from the laboratory experiments, the upper and lower limits of the piston's displacement in the semi-analytical model were applied to

match those of the laboratory scale model. Additionally, it was observed that in the laboratory experimental data, all oscillation cycles in the displacement plot were not identical due to slightly varying flow conditions during the experiment. The semi-analytical model treated every single oscillation cycle separately, calculating the time period for that specific oscillation cycle. It then used optimization to find out the force that best corresponded to that value of time period for the given lengths of connecting arms. This was done for all the oscillation cycles and the varying values of forces corresponding to the varying velocity in the laboratory experiment.

Figure 8. LVDT data for piston vertical displacement for 5V and 10V from laboratory experiment reported in Malla et al. (2011)

The experimental data for piston displacement from LVDT shown in Figure 8 were used by the semi-analytical model to predict the load and plot the displacement for that load. Figure 9 shows the semi-analytically computed lift force (load) and piston displacement for 5 V data. Even though

the wave cycles from the experiment and the ones generated by the semi-analytical model do not match perfectly (slight time lag is observed), the differences in their time periods and displacement magnitudes are minimal, thus verifying the accuracy of the load prediction by the semi-analytical model. Figure 10 represents the experimental data from LVDT for 10 V input to the cylinder along with the results from the semi-analytical model for comparison. The wave cycles are very similar in nature with negligible difference in the time periods and amplitude of displacements. Thus, one can say that the semi-analytical model can reasonably predict the varying lift force generated during the actual experiment involving the laboratory scale model. Even though the graphs are not identical we believe the model has performed reasonably well given that many factors such as drag force experienced during the experiment, rigidity, friction at hinges/connections, and three-dimensional lab prototype vs. two-dimensional simulation in the semi-analytical model have not been taken into account.

Figure 9. Load and piston displacement prediction for 5 V data.

Figure 10. Load and piston displacement prediction for 10 V data

Case 2: The second case deals with practical application of the model. For this case, the flow data of Shetucket River were chosen. Formed at Willimantic, Connecticut, United States by the junction of the Willimantic River and the Natchaug River, the Shetucket River flows southeast and south, and has an approximate average width of 150 ft. The location of the measuring station for the river is shown in Figure 11. The flow data (discharge and gage height) are available on the USGS website (USGS 2018).

Figure 11. Location for Shetucket River data collecting station ($41^{\circ}41'01''$, $72^{\circ}10'57''$)

Average monthly velocity data from 1995 to 2016 were used. With the monthly velocity of flow as the input, the semi-analytical model was run for each month. For this investigation, values of the radius of the cylinder, its angular velocity and length are taken as 2.225 cm, 985.7 rpm, and 20 cm, respectively. With these known, the only variable is the flow of the river. Multi-objective optimizations were run, thus obtaining a Pareto front for each month, with Pareto optimal solutions to choose from. These solutions have the values of decision variables (lengths of connecting arms) based on tradeoffs of the objective functions involved (here, minimizing the time period and maximizing the stroke of piston). Giving equal importance to both the objectives, the combination of decision variables for which the tradeoff is equally divided among the two optimizing functions is chosen. This is done for all 12 months and 12 sets of combination of the decision variables were obtained. These results are shown in Table 2 and were plotted and analyzed. The combination that had the lowest value for the optimizing function was chosen as the final. Thus, the dimensions of the hydropower harvesting model that is optimized as per the flow conditions of the Shetucket River are obtained.

Using the river flow velocity as input and using Eq. (1), the lift force was calculated. The decision variables were determined for each month using the multi-objective optimization analysis as described in Section 4.1. Figure 12 presents the 12 different combinations of the decision variables (lengths of various arms of the model) obtained from this analysis. The objective function value was calculated for each month, as the average of the two objective function values (objective 1: minimizing time period, objective 2: minimizing $1/\text{stroke}$). The combination with the lowest value for the objective function was chosen as the optimal one. The final comprehensive set of results obtained from this analysis is shown in Table 1. The month of March has the lowest value of the objective function, with corresponding optimal values for the length of the arms given by $ED = 5.39$ cm, $DG = 3.40$ cm, $CG = 2.20$ cm, $BC = 3.71$ cm, and $AB = 8.00$ cm. The optimal time period is 0.252 s; and the optimal stroke for the piston is 4.32 cm. The corresponding lift force is found to be the highest for this month among the 12 months.

We have elected to not include the drag force in this model. This is because the experimental results for lift coefficients versus the dimensionless velocity, given by the ratio of translational velocity of the cylinder to the velocity of the river, as obtained by Malla et al. (2011) are very consistent with the results from Tokumaru and Dimotakis (1993) and Sengupta et al. (2004a) as presented in White (2011). Moreover, for the flows shown in Table 1, the dimensionless velocity values range from 3.6 to 11 and, therefore, the drag force is less than 4% of the lift force generated according to White (2011). Thus, our assumption to not include the drag force in our model is justified.

Figure 13 shows the ratio, R, of the objective function values for optimized device lengths to the values corresponding to the device lengths (non-optimized) as reported in Malla et al. (2011) for 12 months. Equal weights were given to the time period and the stroke during these calculations. As smaller value of objective function signifies smaller time period and larger stroke of piston, a smaller value of the ratio R depicts better optimized design as compared to the lab prototype device. There exists an improvement in the objective function for all 12 months in the year; this improvement ranges from 47% to 77%, indicating that significant improvement can be achieved through optimization.

Table 2. Optimization results for reciprocating device design applicable to Shetucket River, CT

month	lengths (cm)					avg velocity (m/s)	Lift Force (N)	Time period (sec)	Stroke (cm)	Objective function
	ED	DG	CG	BC	AB					
Jan	5.1	3.2	2.1	2.9	11.4	0.533	16.91	0.29	3.9	0.273
Feb	5.2	3.0	2.1	3.5	8.9	0.490	15.53	0.30	3.6	0.285
Mar	5.4	3.4	2.2	3.7	8.0	0.644	20.45	0.25	4.3	0.242
Apr	6.1	3.5	2.1	2.6	9.4	0.625	19.83	0.39	4.5	0.306
May	4.5	2.8	2.0	2.9	4.4	0.459	14.57	0.28	3.3	0.293
Jun	5.8	3.3	2.0	2.8	7.8	0.370	11.73	0.44	4.3	0.337
Jul	4.2	2.8	2.0	2.8	5.6	0.239	7.60	0.56	3.3	0.434
Aug	4.9	2.6	2.0	2.6	4.8	0.208	6.61	0.60	2.9	0.470
Sep	4.6	3.1	2.0	2.6	6.1	0.238	7.55	0.63	3.7	0.448
Oct	4.6	3.0	2.0	2.7	6.4	0.384	12.19	0.26	3.7	0.267
Nov	5.3	3.2	2.0	2.9	6.6	0.402	12.75	0.38	4.1	0.314
Dec	4.9	3.1	2.1	3.4	10.4	0.527	16.71	0.28	3.8	0.274

Figure 12. Optimization results for device arm lengths for Shetucket River over 12 months.

Figure 13. Ratio of objective function values for optimized vs. non-optimized device lengths for Shetucket River over 12 months

Because of the short-term variability of flow conditions in a natural river, some precautions need to be taken. A cylinder with larger radius and higher angular velocity produces greater lift force. The dimensions of a hydropower harvesting system for average monthly flow velocities can be

obtained as described in Case 2 above. The river velocity, however, could vary significantly from the average. Two methods can be envisioned to account for this flow variability. One possible approach is to adjust the rotational speed of the cylinder. However, this method alone may not be effective enough as the high rotation rates of the cylinder can also cause temporal instability and vibrations that may endanger the integrity of the device (Sengupta et al. 2004b). Another method is to vary the radius of the rotating cylinder. Being able to control not only the rotational speed of the cylinder but also its diameter can give the user better control on the reciprocating action and energy production. To achieve this, the cylinder can be made up of inflatable material encased in an outer hard shell so that one can change the radius of the cylinder by a few centimeters; increase the radius by pumping air into it or decrease it by letting the air out. When the system is placed in the river, the radius can be changed accordingly to get the maximum output possible. This is done in the semi-analytical model by using the lengths as obtained in Case 2 and setting the radius of the cylinder as the variable that is allowed to vary within a suitable range of values. Optimization is then performed to find the value of cylinder diameter and rotational speed that ensure the highest power generation by the system within certain vibration/stability constraints.

4.3 Finite Element model

Finite element model developed in COMSOL Multiphysics differs from the hydropower harvesting model in terms of the input provided. By simulating the rotation due to flow in COMSOL Multiphysics 5.2a, for the velocity of 0.644 m/s that corresponds to the highest velocity for the Shetucket River, the movement of the piston was analyzed for both model types. The piston for the Crank slider type model produced a stroke of 3.75 cm and a time period of 0.35 sec as shown in

Figure 14. The quick return mechanism type model yielded a stroke of 6.76 cm and a time period of 0.37 sec as shown in Figure 15.

Figure 14. Piston displacement for Crank slider type model

Figure 15. Piston displacement for Quick Return mechanism type model

In the finite element model, fins are attached to the cylinder, allowing it to rotate purely by the flow velocity. Using the March flow velocity of the Shetucket River as the input, the work produced per mass by the cylinder when it is rotating due to the flow can be calculated as follows:

Mass of cylinder (m) = 1.662 kg

Moment of Inertia (I) = $5.593 \times 10^{-4} \text{ kg} \cdot \text{m}^2$

Rotational velocity (ω) = 18 rad/s

Thus, work produced per mass $= \frac{I\omega^2}{2m}$

$$= \frac{5.593 \times 10^{-4} \times 18^2}{2 \times 1.662} = 0.05 \frac{m^2}{s^2}$$

The rotational velocity of 18 rad/s is achieved gradually over time as shown in Figure 3 and depends on the flow of the river. Figure 16 shows the variation of the rotational velocity acquired by the cylinder for three different cases; flow velocity for the month of March in the Shetucket River, half of the flow velocity and double the flow velocity. It is observed that the rotational velocity is higher for higher values of flow velocity.

Figure 16. Rotational velocity for different flow velocity

The value of work produced per mass is greatly dependent on many influencing factors. One of the factors is the number of fins attached to the cylinder. Using the flow velocity of March as the input,

the finite element models were run for different number of fins attached to the cylinder and the results for rotational velocity were plotted as shown in Figure 17.

Figure 17. Change in rotational velocity for different number of fins

It is observed that for a constant flow velocity, the rotational velocity of the cylinder decreases as the number of fins attached to it increases. The rotational velocity acquired is highest for 4 fins and lowest for 7 fins. Besides this, the shape of the fins, radius of the fins as well as the shape of the cylinder also affect the rotational velocity that the cylinder can develop while rotating. As the connecting arms relay the circular motion of the cylinder to oscillatory motion of the piston, proper design and optimization of these connecting arms can also aid in better performance of the model. Additionally, the finite element models studied in COMSOL Multiphysics 5.2a are merely prototypes and serve the purpose of explaining a concept. More factors such as the shape of the

cylinder, shape of the fins, number of the fins attached, lengths of the connecting links, etc. that greatly affect the movement of the attached piston should be taken into account and dealt in detail in order to make the device self-sustainable.

5. Conclusions

5.1 Summary

Design optimization of a hydropower harvesting system under Run-of-the-Stream low head and low discharge conditions has been accomplished using a semi-analytical model that employs kinematic and equilibrium equations and the genetic algorithm. Compared with the non-optimized model by Malla et al. (2011), the model showed significant improvements in the objective function, ranging from 47% to 77%. Depending upon the flow conditions, the variables in the model can be altered easily and a reasonable optimum design can be agreed upon. As anticipated, the lift force generated depends highly on the combination of the flow velocity, the cylinder dimensions and its angular velocity. In reality the flow velocity in a river or a stream is never constant and fluctuates with time and seasons. Therefore, in order to make the energy harvesting device function at a near optimum level even in such situation, the angular velocity of the cylinder in the reciprocating system can be adjusted based on the forecast of the flow velocity such that the lift force is within the desirable range and does not compromise the safety of the system. Similarly, an inflatable cylinder whose radius can be varied will provide control of the dimensions. With these dynamic factors controlled properly, the system can be adjusted to different situations.

Many energy harvesting concepts depend on reciprocating motion of the structural system involved and the methodology developed in this study is found to be effective in producing optimum design of such energy harvesting devices with the stated assumptions and constraints mentioned in this paper. However, there is still room to achieve more efficient design solutions.

Finite element models are studied in COMSOL Multiphysics 5.2a to rotate the cylinder purely by the flow and thus, eliminate the need of any input to the device. Two different approaches, namely crank slider type model and quick return mechanism type model are studied. Both of these prototype models are able to produce significant oscillation of the piston with only the flow velocity as input, thus validating the significance of a self-sustaining hydropower harvesting device. The performance of the model is influenced by many factors such as the shape of the cylinder, the shape and number of fins which affect the rotational velocity developed, as well as the design of connecting arms that affect the motion of the piston. With more detail design, an efficient self-sustaining hydropower harvesting device can be developed.

5.2 Future work

The semi-analytical model is able to optimize the hydropower harvesting device under given constraints and assumptions. Nevertheless, the scope for future work exists. Only the lengths of connecting arms are considered as decision variables in our optimization but other geometric parameters such as the radius and length of cylinder also effect the motion of the piston and should be considered for more accuracy. The semi-analytical model assumes a rigid system, ignores drag force experienced by the cylinder moving through the liquid/water, and represents a small-scale

system. In reality, the system will not be a rigid body and might have longer slender, hence flexible, arms that do not follow the rigid body equations. Thus, the model should be upgraded to account for the flexible bodies. The drag force should also be considered in future work for more accuracy. Furthermore, the model should be modified and compared with real life-size prototypes to determine its accuracy and scalability.

The finite element models studied explains the self-sustaining concept of the hydropower harvesting device. However, more detail study is needed to produce feasible designs. Factors such as the shape of the cylinder, optimum radius of the cylinder, the material with which the cylinder is made, the shape and the number of fins attached, the lengths of the links that connect the cylinder to the piston, and the shape of the piston affect the performance of the model and should be analyzed separately and in more detail.

References

- Ahmed, R., Mir, F., and Banerjee, S. (2017). “A review on energy harvesting approaches for renewable energies from ambient vibrations and acoustic waves using piezoelectricity.” *Smart Materials and Structures*, 26.
- Beeby, P. S., Tudor, M. J., and White, M. N. (2006). “Energy harvesting vibration sources for micro systems applications.” *Meas. Sci. Technology*, 17, 175-195.
- Bourguet, R., and Jacono, R. D. (2014). “Flow-induced vibrations of a rotating cylinder.”, *Journal of Fluid Mechanics*, Cambridge University Press (CUP), 740, 342-380.
- Díez, P. (2003). “A Note on the Convergence of the Secant Method for Simple and Multiple Roots.” *Applied Mathematics Letters*, 16(8), 1211–1215.
- Davis, D. L. (1991). *Handbook of Genetic Algorithms*, Van Nostrand Reinhold, New York, NY.
- Elvin, G. N., Lajnef, N., and Elvin, A. A. (2006). “Feasibility of structural monitoring with vibration powered sensors.” *Smart Materials and Structures*, 15(4), 977-986.
- Erturk, A., and Inman, J. D. (2009). “An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations.” *Smart Materials and Structures*, 18(2).
- François, B., Hingray, B., Raynaud, D., Borga, M., and Creutin, J. D. (2015). “Complementarity between solar and hydro power: Sensitivity study to climate characteristics in Northern-Italy.” *Renewable Energy*, 86, 543-553.
- François, B., Hingray, B., Raynaud, D., Borga, M., and Creutin, J. D. (2016). “Increasing climate-related-energy penetration by integrating run-of the river hydropower to wind/solar mix.” *Renewable Energy*, 87, 686–696.

- Gleick, P. H. (1992). "Environmental consequences of hydroelectric development: The role of facility size and type." *Energy, Elsevier*, 17(8), 735-747.
- Goldberg, E. D. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley Longman Publishing Co., Inc. Boston, MA.
- Goodarzi, M., Dehkordi, and E. K. (2017a). "Complete Vortex Shedding Suppression Allocating Twin Rotating Controllers at a Suitable Position." *Ocean Engineering*, 137, 215–223.
- Goodarzi, M., Dehkordi, and E. K. (2017b). "Geometrical Parameter Analysis on Stabilizing the Flow Regime over a Circular Cylinder Using Two Small Rotating Controllers" *Computers & Fluids*, 145, 129–140.
- Kecik, K., and Borowiec, M. (2013). "An Autoparametric Energy Harvester." *SpringerLink, Springer*, Berlin Heidelberg, 222(7), 1597-1605.
- Malla, R. B., Shrestha, B., Bagtzoglou, A., Drasdis, J., and Johnson, P. (2011). "Hydropower Harvesting from a Small Scale Reciprocating System.", *Renewable Energy*, 36(5), 1568-1577.
- NASA, (2008). Lift of Rotating Cylinder. National Aeronautics and Space Administration, Glenn Research Center, Cleveland, OH <http://www.grc.nasa.gov/WWW/K-12/airplane/cyl.html>
- Sengupta, T. K., Kasliwal, A., De, S., and Nair, M. (2004a). "Temporal flow instability for Magnus-Robins effect at high rotation rates." *J. Fluids and Structures*, 17(7), 941-953.
- Sengupta, T. K., and Talla, S. B. (2004b). "Robins-Magnus effect: A continuing saga." *Current Science*, 86(7), 1033-1036.
- Shen, D., Choe, S. Y., and Kim, D. J. (2006). "Comparison of Piezoelectric Material for Vibration Energy Conversion Devices" *MRS Proceedings*, 966.

- Shen, W., Zhu, S., and Zhu, H. (2016). "Experimental study on using electromagnetic devices on bridge stay cables for simultaneous energy harvesting and vibration damping." *Smart Materials and Structures*, 25.
- Shenck, S. N., and Paradiso, A. J. (2001). "Energy Scavenging With Shoe-Mounted Piezoelectrics." *IEEE Micro*, 21(3).
- Shrestha, B. (2010). "Lift Force Estimation and Hydropower Harvesting Using Reciprocating System Based on Magnus Effect." M.S. Thesis, University of Connecticut, Storrs, CT. (Advisor: R. Malla).
- The MathWorks, (1993). MATLAB User's Guide, The MathWorks Inc., Natick, MA.
- Torres, E. O., and Gabriel, A. R. (2005). "Harvesting ambient energy will make embedded devices autonomous." San Francisco, CA: EE times
https://www.eetimes.com/document.asp?doc_id=1276782
- Tokumaru, P. T., and Dimotakis, P. E. (1993). "The Lift of a Cylinder Executing Rotary Motions in a Uniform Flow." *J. Fluid Mechanics*, 255, 1-10.
- USGS, Site Map for USGS, 01122500 Shetucket River near Willimantic, CT,
http://waterdata.usgs.gov/ct/nwis/nwismap/?site_no=01122500&agency_cd=USGS.
 Accessed 10 May 2018
- Vila, L., and Malla, R. B. (2012). "Investigation on Bridge Vibration Energy Harvesting Using an Electromagnetic System." *Proc., Connecticut Symposium on Microelectronics & Optoelectronics*, 79-80.

- Wang, D., Chao, C., and Chen, J. (2012). “A miniature hydro-energy generator based on pressure fluctuation in Karman Vortex Street.” *Journal of Intelligent Material Systems and Structures*, 24(5), 612–626.
- White, F. M. (2011). Fluid Mechanics, 7th edn (New York, NY) Mc-Graw-Hill Companies, Inc., 550.
- Whitley, D. (1994). A Genetic Algorithm Tutorial, *Statistics and Computing*, 4(2), 65–85.
- Zheng, B., Chang, C. J., and Gea, H. C. (2009). “Topology Optimization of energy harvesting devices using piezoelectric materials.” *Struct Multidisc Optim*, 38(17), <https://doi.org/10.1007/s00158-008-0265-0>

APPENDIX A: KINEMATIC AND KINETIC RELATIONS

Following are the kinematic equations for the semi-analytical model. The angular acceleration, angular velocity and angular displacement are represented by α , ω and θ while the subscripts A, B, C, D, E denote the points at which they are calculated. EDGC, BC and AB are the connecting arms. The equations are solved numerically at different time steps for the desired number of time steps, i , to get values of angular displacement (θ_D) at point D of arm EDGC; dt is the time step at which the simulation proceeds in time, i and $i+1$ are consecutive times. Kinematic and equilibrium equations of the system relate the angular displacement at point D with all other relevant points in the system.

$$\text{Angular displacement: } (\theta_{D(i+1)}) = \theta_{Di} + \omega_{D(i+1)}dt + 0.5 \alpha_D dt^2 \quad (\text{A.1})$$

Where,

$$\text{Angular acceleration: } (\alpha_D) = (-P.ED.\cos\theta_D + X)/Y \quad (\text{A.2})$$

$$\text{Angular velocity: } (\omega_{D(i+1)}) = \omega_{Di} + \alpha_D dt \quad (\text{A.3})$$

$$X = 0.5(W_{ED} ED - W_{DG} DG)\cos\theta_D - W_{CG}(DG\cos\theta_D - 0.5 CG\sin\theta_D) + U(DG\cos\theta_D - CG\sin\theta_D) - S(DG\sin\theta_D + CG\cos\theta_D) \quad (\text{A.4})$$

$$Y = -T(DG \cos\theta_D - CG \sin\theta_D) + R(DG \sin\theta_D + CG \cos\theta_D) + \frac{m_{ED} ED^2 + m_{DG} DG^2 + m_{CG} CG^2}{3} + m_{CG} DG^2 \quad (\text{A.5})$$

$$C = (DG\cos(\theta_B - \theta_D) + CG\sin(\theta_B - \theta_D))/(AB \cos(\theta_A - \theta_B)) \quad (\text{A.6})$$

$$D = \frac{\omega_D^2(DG \sin(\theta_B - \theta_D) - CG \cos(\theta_B - \theta_D) + AB \omega_A^2 \sin(\theta_A - \theta_B) - BC \omega_B^2)}{AB \cos(\theta_A - \theta_B)} \quad (\text{A.7})$$

$$E = \frac{-DG \sin(\theta_D - \theta_A) + CG \cos(\theta_D - \theta_A)}{BC \cos(\theta_A - \theta_B)} \quad (\text{A.8})$$

$$F = \frac{-\omega_D^2 (DG \cos(\theta_D - \theta_A) - CG \sin(\theta_D - \theta_A)) + AB \omega_A^2 + BC \omega_B^2 \sin(\theta_B - \theta_A)}{BC \cos(\theta_A - \theta_B)} \quad (\text{A.10})$$

$$G = -DG \sin \theta_D - CG \cos \theta_D - 0.5 BC . E . \cos \theta_B \quad (\text{A.11})$$

$$H = \omega_D^2 (-DG \cos \theta_D + CG \sin \theta_D) - 0.5 BC (F \cos \theta_B - \omega_B^2 \sin \theta_B) \quad (\text{A.12})$$

$$I = -DG \cos\theta_D + CG \sin\theta_D + 0.5 BC \cdot E \cdot \sin\theta_B \quad (\text{A.13})$$

$$J = -\omega_D^2 (DG \sin\theta_D - CG \cos\theta_D) - 0.5 BC \cdot F \cdot \sin\theta_B + \omega_B^2 \cos\theta_B \quad (\text{A.14})$$

$$K = m_{BC}G - 0.5 m_{AB}AB \cdot C \cdot \sin\theta_A \quad (\text{A.15})$$

$$L = m_{BC}H - 0.5 m_{AB}AB (D \sin\theta_A + \omega_A^2 \cos\theta_A) \quad (\text{A.16})$$

$$M = -m_{BC}I + 0.5 m_{BC}AB \cdot C \cos\theta_A \quad (\text{A.17})$$

$$N = m_{BC}J + W_{BC} + W_{AB} + 0.5 m_{AB}AB (D \cos\theta_A - \omega_A^2 \sin\theta_A) \quad (\text{A.18})$$

$$O = DG \sin(\theta_B - \theta_D) - CG \cos(\theta_D - \theta_B) - 0.5 BC \cdot E \quad (\text{A.19})$$

$$Q = \omega_D^2 (-DG \cos(\theta_D - \theta_B) + CG \sin(\theta_D - \theta_B)) - 0.5 BC \cdot F \quad (\text{A.20})$$

$$R = \frac{\sin\theta_B(M\cos\theta_A - K\sin\theta_A - \frac{m_{AB}AB \cdot C}{6}) - \cos\theta_A(\frac{m_{BC}BC \cdot E}{12} + 0.5 m_{BC}O)}{\cos(\theta_A - \theta_B)} \quad (\text{A.21})$$

$$S = \frac{\sin\theta_B(N\cos\theta_A - L\sin\theta_A - \frac{m_{AB}AB \cdot D}{6}) - \cos\theta_A(\frac{m_{BC}BC \cdot F}{12} + 0.5(W_{AB} + W_{BC})\sin\theta_B + 0.5 m_{BC}Q)}{\cos(\theta_A - \theta_B)} \quad (\text{A.22})$$

$$T = \frac{\cos\theta_B(-M\cos\theta_A + K\sin\theta_A + \frac{m_{AB}AB \cdot C}{6}) - \sin\theta_A(\frac{m_{BC}BC \cdot E}{12} + 0.5 m_{BC}O)}{\cos(\theta_A - \theta_B)} \quad (\text{A.23})$$

$$U = \frac{\cos\theta_B(-N\cos\theta_A + L\sin\theta_A + 0.5W_{AB} \cos\theta_A + \frac{m_{AB}AB \cdot D}{6}) - \sin\theta_A(0.5W_{BC} \sin\theta_B + \frac{m_{BC}BC \cdot F}{12} + 0.5 m_{BC}Q)}{\cos(\theta_A - \theta_B)} \quad (\text{A.24})$$

APPENDIX B: SECANT METHOD

From kinematic properties,

$$AB \sin\theta_A + BC \cos\theta_B = DG \sin\theta_D - CG (1 - \cos\theta_D) - BC \quad (B.1)$$

$$-AB \cos\theta_A - BC \sin\theta_B = -DG (1 - \cos\theta_D) - CG \sin\theta_D + AB \quad (B.2)$$

$$\text{Setting } \rho = DG \sin\theta_D - CG (1 - \cos\theta_D) - BC \quad (B.3)$$

$$\sigma = DG (1 - \cos\theta_D) + CG \sin\theta_D - AB, \quad (B.4)$$

We get

$$AB \sin\theta_A + BC \cos\theta_B - \rho = 0 \quad (B.5)$$

$$AB \cos\theta_A + BC \sin\theta_B + \sigma = 0 \quad (B.6)$$

Solving Equations B.5 and B.6,

$$AB \sin\theta_A + BC \sqrt{1 - \frac{(\sigma + AB \cos\theta_A)^2}{BC^2}} - \rho = 0 \quad (B.7)$$

Equation B.7 is used in Secant method to find θ_A .

To find θ_A , we start from an initial value θ_{A1} and assume a final value θ_{A2} that is to be found.

Then,

$$\text{Let } F11 = 1 - \frac{\sigma^2 + AB^2 \cos^2\theta_{A1} + 2.\sigma.AB.\cos\theta_{A1}}{BC^2} \quad (B.8)$$

$$F1 = AB. \sin\theta_{A1} - BC \sqrt{F11} - \rho \quad (B.9)$$

$$F22 = 1 - \frac{\sigma^2 + AB^2 \cos^2\theta_{A2} + 2.\sigma.AB.\cos\theta_{A2}}{BC^2} \quad (B.10)$$

$$F2 = AB. \sin\theta_{A2} - BC \sqrt{F22} - \rho \quad (B.11)$$

$$\theta_A = \theta_{A1} - \frac{F1 (\theta_{A1} - \theta_{A2})}{F1 - F2} \quad (B.12)$$

$$\text{Error} = \theta_{A2} - \theta_A \quad (B.13)$$

For acceptable value of error, $\theta_{A2} = \theta_A$

Else, $\theta_{A2} = \theta_{A1}$, $\theta_{A1} = \theta_A$ and the process is repeated until acceptable error is obtained.

The value of θ_A obtained is used to calculate θ_B using Equation (B.5) or (B.6).

APPENDIX C: MATLAB CODE

```
% function for multi objective optimization
function TP=MultiObjectiveOptimization(Xin) % multi objective

% function for single objective optimization for time period
% minimize the time period
%function tp=SingleObjectiveOptimizationForTimePeriod(Xin)

% function for single objective optimization for stroke
% maximize the stroke of piston
%as we can only minimize, we minimize the inverse of stroke to maximize the
%stroke
%function STRA=SingleObjectiveOptimizationForStroke(Xin)

% clc
% clear all
% close all

tp=12;
% Non-optimized Length of connecting arm, cm.
% ED = 8.255;
% DG = 8.89;
% CG = 2.223;
% BC = 8;
% AB = 10.8;

ED = Xin(1);
DG = Xin(2);
CG = Xin(3);
BC = Xin(4);
AB = Xin(5);

% to input the values of the connecting arms manually

% ED=input('ED ');
% DG=input('DG ');
% CG=input('CG ');
% BC=input('BC ');
% AB=input('AB ');
```


% Mass of the each connecting arm, kg.

```
mAB = 0.083*AB/10.8;  
mBC = 0.0628*BC/8;  
mCG = 0.0174*CG/2.223;  
mDG = 0.07*DG/8.89;  
mED = 0.065*ED/8.255;
```

% Weight of each connecting arm, N.

```
WAB=mAB*9.81;  
WBC=mBC*9.81;  
WCG=mCG*9.81;  
WDG=mDG*9.81;  
WED=mED*9.81;
```

% initial Boundary Conditions for the system

% start from rest

```
thD(1)=0;  
thB(1)=0;  
thA(1)=0;  
wD(1)=0;  
wA(1)=0;  
wB(1)=0;
```

%time step

```
dt=0.001;
```

%final time

```
time=60;
```

```
t22=0:dt:time;
```

%for time period

```
a3=1;  
a2=2;
```

% counters for checking if i1 and i2 are increasing or not

```
aa=zeros(1,10);  
bb=1;
```

% value for lift force in N

```

P(1)=-10;

%to input value of lift force
%P(1)=input('enter force in N ');

% to find the lift force created by Magnus effect in a rotating cylinder

% rh= density of fluid
% vel= velocity of fluid
% A1= surface area of cylinder exposed to impeding flow
% CL= lift coefficient

% w1= rotation/sec of cylinder in rpm
%w1=input('rotation/sec of cylinder in rpm ');

% R1= radius of cylinder % m
% R1=0.02225;
%R1=input('radius of cylinder in m ');

% vinf= far-field flow velocity
% L1= length of rotating cylinder
% L1=0.2032; % m
% rh=1000; % N/m3

% A1= surface area of cylinder exposed to impeding flow
% A1=2*pi()*R1*L1; %m2

% vel=input('enter velocity of flow in m/s ')
% vinf=0.3; % m/min

% Calculation of Lift Coefficient
% CL=2*pi()*R1*w1/vinf;

% Calculation of lift force
% P(1)=-0.5*rh*(vel^2)*A1*CL;

% for calculation of the strokes
str1=zeros(1,20);
strA1=zeros(1,20);
str2=1;

% for vertical displacement of piston at point B

```

```

dispB(1)=AB*sin(thA(1));

qw=100;
t=0;
m=10;
t1=0;
t2=0;
i1=1;
i2=1;

while t<time

if m>0
m=-1;

for i1=i2:length(t22)

% equations for when the system moves upward
y12(i1)=isnan(thD(i1)); % checking if thD becomes NAN

C(i1) = (DG*cos(thB(i1)-thD(i1))+ CG*sin(thB(i1)-thD(i1)))/AB/cos(thA(i1) -thB(i1));
D(i1) = ((wD(i1))^2 *(DG*sin(thB(i1)-thD(i1))- CG*cos(thB(i1)-thD(i1)))+ AB*(wA(i1))^2 *
sin(thA(i1) -thB(i1)) -BC*(wB(i1))^2)/AB/cos(thA(i1) -thB(i1));
E(i1) = -(DG*sin(thD(i1)-thA(i1))+ CG*cos(thD(i1)-thA(i1)))/BC/cos(thA(i1) -thB(i1));
F(i1) = (-wD(i1))^2 *(DG*cos(thD(i1)-thA(i1))- CG*sin(thD(i1)-thA(i1)))+ AB*(wA(i1))^2
+BC*(wB(i1))^2 *sin(thB(i1) -thA(i1)))/BC/cos(thA(i1) -thB(i1));
G(i1) = -DG*sin(thD(i1)) -CG*cos(thD(i1)) -BC*0.5*E(i1)*cos(thB(i1));
H(i1) = ((wD(i1))^2 *(-DG*cos(thD(i1))+CG* sin(thD(i1))) - BC*0.5*(F(i1)*cos(thB(i1)) -
((wB(i1))^2 *sin(thB(i1))));
I(i1) = -DG*cos(thD(i1)) +CG*sin(thD(i1)) +BC*0.5*E(i1)*sin(thB(i1));
J(i1) = (-((wD(i1))^2 *(DG*sin(thD(i1))-CG* cos(thD(i1)))) -BC*0.5*(F(i1)*sin(thB(i1))
+((wB(i1))^2 *cos(thB(i1))));
K(i1) = mBC*G(i1) -mAB*AB*0.5*C(i1)*sin(thA(i1));
L(i1) = H(i1)*mBC -mAB*AB*0.5*(D(i1)*sin(thA(i1))+((wA(i1))^2)*cos(thA(i1)));
M(i1) = -mBC*I(i1) + mAB*AB*0.5*C(i1)*cos(thA(i1));
N(i1) = J(i1)*mBC +WBC+WAB+ mAB*AB*0.5*(D(i1)*cos(thA(i1)) -((wA(i1))^2)
*sin(thA(i1)));
O(i1) = DG*sin(thB(i1)-thD(i1)) -CG*cos(thD(i1)-thB(i1))-BC*0.5*E(i1);
Q(i1) = (wD(i1))^2 *(-DG*cos(thD(i1)-thB(i1))+CG*sin(thD(i1)-thB(i1))) -BC*0.5*F(i1);
R(i1) = (M(i1)*cos(thA(i1))*sin(thB(i1)) -K(i1)*sin(thA(i1))*sin(thB(i1))-
((mAB*AB*C(i1)*sin(thB(i1)))/6) -((mBC*BC*cos(thA(i1))*E(i1))/12) -
mBC*cos(thA(i1))*O(i1)*0.5)/cos(thA(i1)-thB(i1));

```

```

S(i1) = (N(i1)*cos(thA(i1))*sin(thB(i1)) - L(i1)*sin(thA(i1))*sin(thB(i1))-
0.5*(WAB+WBC)*cos(thA(i1))*sin(thB(i1))-((mAB*AB*D(i1)*sin(thB(i1)))/6) -
((mBC*BC*cos(thA(i1))*F(i1))/12) -mBC*cos(thA(i1))*Q(i1)*0.5)/cos(thA(i1)-thB(i1));
T(i1) = (-M(i1)*cos(thA(i1))*cos(thB(i1))
+K(i1)*sin(thA(i1))*cos(thB(i1))+((mAB*AB*C(i1)*cos(thB(i1)))/6) -
((mBC*BC*sin(thA(i1))*E(i1))/12) -mBC*sin(thA(i1))*O(i1)*0.5)/cos(thA(i1)-thB(i1));
U(i1) = (-N(i1)*cos(thA(i1))*cos(thB(i1)) +
L(i1)*sin(thA(i1))*cos(thB(i1))+0.5*WAB*cos(thA(i1))*cos(thB(i1))-
0.5*WBC*sin(thA(i1))*sin(thB(i1))+((mAB*AB*D(i1)*cos(thB(i1)))/6) -
((mBC*BC*sin(thA(i1))*F(i1))/12) -mBC*sin(thA(i1))*Q(i1)*0.5)/cos(thA(i1)-thB(i1));

X(i1) = 0.5*(WED*ED -WDG*DG)*cos(thD(i1)) -WCG*(DG*cos(thD(i1))-
CG*0.5*sin(thD(i1))) +U(i1)*(DG*cos(thD(i1)) -CG*sin(thD(i1))) -S(i1)*(DG*sin(thD(i1)))
+CG*cos(thD(i1)));
Y(i1) = -T(i1)*(DG*cos(thD(i1))-CG*sin(thD(i1))) +R(i1)*(DG*sin(thD(i1))+CG*cos(thD(i1)))
+(((mED*(ED^2)) +(mDG*(DG^2))+(mCG*(CG^2)))/3)+mCG*(DG^2);

% calculation of acceleration, velocity, displacements at point D

% angular acceleration
alD(i1) = (-P(i1)*ED*cos(thD(i1))+X(i1))/Y(i1);
% angular velocity for next time step
wD(i1+1) = wD(i1) + alD(i1)*dt;
% angular displacement for next time step
thD(i1+1) = thD(i1) +wD(i1+1)*dt + 0.5* alD(i1) *(dt^2);

%using same value of force for next time step
P(i1+1)=P(i1);

% solving for thA and thB

% use of Secant Method to find thA and thB

% relation developed based on kinematic properties
rho(i1+1)=DG*sin(thD(i1+1))-CG*(1-cos(thD(i1+1)))-BC;
sgma(i1+1)=DG*(1-cos(thD(i1+1)))+CG*sin(thD(i1+1))-AB;

ii=1;
er1=-0.5; % initial guess for error
THA1=thA(i1); % guess for start value
THA2=thA(i1)+0.1; % guess for end value
THA=-1; % guess for final value

% secant method

```

```

while abs(er1)>0.0001
% allowable error = 0.0001

f111=abs((1-
(((sigma(i1+1))^2)+(AB^2)*((cos(THA1))^2)+2*sigma(i1+1)*AB*cos(THA1))*(1/BC^2)) );
f1=AB*sin(THA1)-BC*sqrt(f111)-rho(i1+1);

f222=abs((1-
(((sigma(i1+1))^2)+(AB^2)*((cos(THA2))^2)+2*sigma(i1+1)*AB*cos(THA2))*(1/BC^2)));
f2=AB*sin(THA2)-BC*sqrt(f222)-rho(i1+1);

THA=THA1-(f1/((f1-f2)/(THA1-THA2))); % application of Secant Method
er1=abs(THA2-THA) ; % calculation of error
ii=ii+1;

if ii>1000
break
end

%updating values of the guess based on previous step results
THA2=real(THA1);
THA1=real(THA);

end

%Final value for thA
thA(i1+1)=THA;

% for vertical displacement of point B
dispB(i1+1)=AB*sin(thA(i1+1));

%check
if ii>1000
tp=2000;
qw=10;
break
end

% using thA value to find value for thB

jj=1;
THB1=thB(i1);
THB2=thB(i1)+0.01;

```

```

er2=0.5;

while abs(er2)>0.0001
%allowable error = 0.0001

f3=BC*sin(THB1)+AB*cos(thA(i1+1))+sgma(i1+1);
f4=BC*sin(THB2)+AB*cos(thA(i1+1))+sgma(i1+1);
THB=THB1-(f3*(THB1-THB2)/(f3-f4));
er2=abs(THB-THB2);
jj=jj+1;

%check
if jj>1000
qw=10;
tp=2000;
break
end

%updating values of thB based on previous steps
THB2=THB1;
THB1=THB;

end

%final value for thB
thB(i1+1)=THB;

% calculation of angular velocity at point A (wA) and point B (wB)


$$A(i1+1) = (DG*\cos(thD(i1+1)-thB(i1+1))- CG*\sin(thD(i1+1)-thB(i1+1)))/(AB*\cos(thA(i1+1)-thB(i1+1)))$$
;

$$B(i1+1) = (- (DG*\sin(thD(i1+1)-thA(i1+1))+ CG*\cos(thD(i1+1)-thA(i1+1)))/(BC*\cos(thA(i1+1)-thB(i1+1))))$$
;

%angular velocity at A
wA(i1+1)=A(i1+1)*wD(i1+1);
%angular velocity at B
wB(i1+1)=B(i1+1)*wD(i1+1);

% check for switching the direction of force P when system is moving upward
Y1(i1)=BC;
if thD(i1)>0
X1=[AB*cos(thA(i1)) BC+CG+AB*sin(thA(i1));AB-DG+DG*cos(thD(i1))-CG*sin(thD(i1+1))
CG*cos(thD(i1+1))+DG*sin(thD(i1+1))];

```

```
Y1(i1)=sqrt(((X1(1,1)-X1(2,1))^2)+((X1(1,2)-X1(2,2))^2));
end
```

```
if Y1(i1)>(BC+0.00000000001)
```

```
%switch direction of force
```

```
P(i1)=-P(i1);
```

```
aa(bb)=i1;
```

```
bb=bb+1;
```

```
i2=i1;
```

```
wD(i2)=0;
```

```
wA(i2)=0;
```

```
wB(i2)=0;
```

```
% for time period
```

```
a1(a3)=i1;
```

```
a3=a3+2;
```

```
break
```

```
end
```

```
% checking for NAN values of thD
```

```
%check
```

```
if y12(i1)>0
```

```
qw=10;
```

```
tp=2000;
```

```
break
```

```
end
```

```
%check
```

```
if thD(i1+1)<thD(i1)
```

```
qw=10;
```

```
tp=2000;
```

```
break
```

```
end
```

```
%check
```

```
if abs(thB(i2+1)-thB(i2))>2
```

```
tp=2000;
```

```
qw=10;
```

```
break
```

```
end
```

```
end
end
```

```
% for calculation of stroke
% for point G, arm DG
max1=max(thD) ;
len1=DG*sin(abs(max1));
% for point B, arm AB
maxA1=max(thD);
lenA1=AB*sin(abs(maxA1));
t=i1*dt;
```

```
% the direction of force has been reversed
% the structure is moving downwards
```

```
%check
if i2<2
qw=10;
tp=2000;
LAL=nan;
elseif or(i2>2,i2==2)
LAL=thB(i2-1);
end
```

```
%check
if isnan(LAL)>0
qw=10;
tp=2000;
break
end
```

```
if tp<100
for i2=i1:length(t22)
```

```
y12(i2)=isnan(thD(i2));
```

```
% equations for when the system moves downward
```

```
C(i2) = (DG*cos(thB(i2)-thD(i2))+ CG*sin(thB(i2)-thD(i2)))/AB/cos(thA(i2) -thB(i2));
D(i2) = ((wD(i2))^2 *(DG*sin(thB(i2)-thD(i2))- CG*cos(thB(i2)-thD(i2)))+ AB*(wA(i2))^2 *
sin(thA(i2) -thB(i2)) -BC*(wB(i2))^2)/AB/cos(thA(i2) -thB(i2));
E(i2) = -(DG*sin(thD(i2)-thA(i2))+ CG*cos(thD(i2)-thA(i2)))/BC/cos(thA(i2) -thB(i2));
```



```

F(i2) = (- (wD(i2))^2 * (DG*cos(thD(i2)-thA(i2))- CG*sin(thD(i2)-thA(i2))))+ AB*(wA(i2))^2
+BC*(wB(i2))^2 *sin(thB(i2) -thA(i2)))/BC/cos(thA(i2) -thB(i2));
G(i2) = -DG*sin(thD(i2)) -CG*cos(thD(i2)) -BC*0.5*E(i2)*cos(thB(i2));
H(i2) = ((wD(i2))^2) *(-DG*cos(thD(i2))+CG* sin(thD(i2))) - BC*0.5*(F(i2)*cos(thB(i2)) -
((wB(i2))^2) *sin(thB(i2))));
I(i2) = -DG*cos(thD(i2)) +CG*sin(thD(i2)) +BC*0.5*E(i2)*sin(thB(i2));
J(i2) = (-((wD(i2))^2) *(DG*sin(thD(i2))-CG* cos(thD(i2)))) -BC*0.5*(F(i2)*sin(thB(i2))
+((wB(i2))^2) *cos(thB(i2))));
K(i2) = mBC*G(i2) -mAB*AB*0.5*C(i2)*sin(thA(i2));
L(i2) = H(i2)*mBC -mAB*AB*0.5*(D(i2)*sin(thA(i2))+((wA(i2))^2)*cos(thA(i2)));
M(i2) = -mBC*I(i2) + mAB*AB*0.5*C(i2)*cos(thA(i2));
N(i2) = J(i2)*mBC +WBC+WAB+ mAB*AB*0.5*(D(i2)*cos(thA(i2)) -((wA(i2))^2)
*sin(thA(i2)));
O(i2) = DG*sin(thB(i2)-thD(i2)) -CG*cos(thD(i2)-thB(i2))-BC*0.5*E(i2);
Q(i2) = (wD(i2))^2 *(-DG*cos(thD(i2)-thB(i2))+CG*sin(thD(i2)-thB(i2))) -BC*0.5*F(i2);
R(i2) = (M(i2)*cos(thA(i2))*sin(thB(i2)) -K(i2)*sin(thA(i2))*sin(thB(i2))-
((mAB*AB*C(i2)*sin(thB(i2)))/6) -((mBC*BC*cos(thA(i2))*E(i2))/12) -
mBC*cos(thA(i2))*O(i2)*0.5)/cos(thA(i2)-thB(i2));
S(i2) = (N(i2)*cos(thA(i2))*sin(thB(i2)) - L(i2)*sin(thA(i2))*sin(thB(i2))-
0.5*(WAB+WBC)*cos(thA(i2))*sin(thB(i2))-((mAB*AB*D(i2)*sin(thB(i2)))/6) -
((mBC*BC*cos(thA(i2))*F(i2))/12) -mBC*cos(thA(i2))*Q(i2)*0.5)/cos(thA(i2)-thB(i2));
T(i2) = (-M(i2)*cos(thA(i2))*cos(thB(i2))
+K(i2)*sin(thA(i2))*cos(thB(i2))+((mAB*AB*C(i2)*cos(thB(i2)))/6) -
((mBC*BC*sin(thA(i2))*E(i2))/12) -mBC*sin(thA(i2))*O(i2)*0.5)/cos(thA(i2)-thB(i2));
U(i2) = (-N(i2)*cos(thA(i2))*cos(thB(i2)) +
L(i2)*sin(thA(i2))*cos(thB(i2))+0.5*WAB*cos(thA(i2))*cos(thB(i2))-
0.5*WBC*sin(thA(i2))*sin(thB(i2))+((mAB*AB*D(i2)*cos(thB(i2)))/6) -
((mBC*BC*sin(thA(i2))*F(i2))/12) -mBC*sin(thA(i2))*Q(i2)*0.5)/cos(thA(i2)-thB(i2));

X(i2) = 0.5*(WED*ED -WDG*DG)*cos(thD(i2)) -WCG*(DG*cos(thD(i2))-
CG*0.5*sin(thD(i2))) +U(i2)*(DG*cos(thD(i2)) -CG*sin(thD(i2))) -S(i2)*(DG*sin(thD(i2))
+CG*cos(thD(i2)));
Y(i2) = -T(i2)*(DG*cos(thD(i2))-CG*sin(thD(i2))) +R(i2)*(DG*sin(thD(i2))+CG*cos(thD(i2)))
+(((mED*(ED^2)) +(mDG*(DG^2))+(mCG*(CG^2)))/3)+mCG*(DG^2);

```

% calculation of acceleration, velocity, displacements at point D

% angular acceleration

```
alD(i2) = (-P(i2)*ED*cos(thD(i2))+X(i2))/Y(i2);
```

% angular velocity at D fot the next time step

```
wD(i2+1) = wD(i2) + alD(i2)*dt;
```

% angular displacement at D for the next time step

```

thD(i2+1) = thD(i2) + wD(i2+1)*dt + 0.5* aD(i2) *(dt^2);

% same value of lift force P for the next steps
P(i2+1)=P(i2);

% solving for thA and thB

% use of Secant Method to find thA and thB

% relation developed based on kinematic properties% solving for thA and thB

rho(i2+1)=DG*sin(thD(i2+1))-CG*(1-cos(thD(i2+1)))-BC;
sgma(i2+1)=DG*(1-cos(thD(i2+1)))+CG*sin(thD(i2+1))-AB;

%iterations
ii=1;
er1=-0.5; % initial guess for error
THA1=thA(i2); % initial guess for start value
THA2=thA(i2)+0.1; % initial guess for end value
THA=-1; % initial guess for final value

%%secant method
while abs(er1)>0.0001
% allowable error = 0.0001

f111=abs((1-
(((sgma(i2+1))^2)+(AB^2)*((cos(THA1))^2)+2*sgma(i2+1)*AB*cos(THA1))*(1/BC^2)) );
f1=AB*sin(THA1)-BC*sqrt(f111)-rho(i2+1);

f222=abs((1-
(((sgma(i2+1))^2)+(AB^2)*((cos(THA2))^2)+2*sgma(i2+1)*AB*cos(THA2))*(1/BC^2))));
f2=AB*sin(THA2)-BC*sqrt(f222)-rho(i2+1);

THA=THA1-(f1/((f1-f2)/(THA1-THA2))); % application of Secant Method
er1=abs(THA2-THA); % error calculation
ii=ii+1;

if ii>1000
break
end

% updating values of guesses based on previous step
THA2=real(THA1);
THA1=real(THA);

```

```

end
ii1(i2)=ii;

% final value for thA
thA(i2+1)=THA;

% for vertical displacement of point B
dispB(i2+1)=AB*sin(thA(i2+1));

%check
if ii>1000
tp=2000;
qw=10;
break
end

% calculation of thB using thA value
jj=1;
THB1=thB(i2); % initial guess start value
THB2=thB(i2)+0.01; % initial guess for end value
er2=0.5; % initial guess for error

while abs(er2)>0.0001
%allowable error = 0.0001

f3=BC*sin(THB1)+AB*cos(thA(i2+1))+sgma(i2+1);
f4=BC*sin(THB2)+AB*cos(thA(i2+1))+sgma(i2+1);

THB=THB1-(f3*(THB1-THB2)/(f3-f4)); % application of Secant Method
er2=abs(THB-THB2); % calculation of error
jj=jj+1;

if jj>1000
break
end

% updating values of guesses based on previous step
THB2=THB1;
THB1=THB;

end
jj1(i2)=jj;

```

```

%check
if jj>1000
tp=2000;
qw=10;
break
end

% final value for thB
thB(i2+1)=THB;

%calculation of angular velocity at point A (wA) and at point B (wB)

A(i2+1) = (DG*cos(thD(i2+1)-thB(i2+1))- CG*sin(thD(i2+1)-thB(i2+1)))/(AB*cos(thA(i2+1)-thB(i2+1))) ;
B(i2+1) = (-(DG*sin(thD(i2+1)-thA(i2+1)))+ CG*cos(thD(i2+1)-thA(i2+1)))/(BC*cos(thA(i2+1)-thB(i2+1))));

wA(i2+1)=A(i2+1)*wD(i2+1);
wB(i2+1)=B(i2+1)*wD(i2+1);

Y2(i2)=BC;

if thD(i2)<0
X2=[AB*cos(thA(i2)) BC+CG+AB*sin(thA(i2));AB-DG+DG*cos(thD(i2))-CG*sin(thD(i2+1))
CG*cos(thD(i2+1))+DG*sin(thD(i2+1))];
Y2(i2)=sqrt(((X2(1,1)-X2(2,1))^2)+((X2(1,2)-X2(2,2))^2));
end

%check
if bb>300
if aa(200)==aa(300)
tp=2000;
qw=10;
break
end
end

%check
if abs(thB(i2+1)-thB(i2))>2
tp=2000;
qw=10;
break
end

```

```

%check
if (i2-i1)>4000
tp=2000;
qw=10;
break
end

%if Y2(i2)>(BC+0.000000000001)
if thD(i2)<LAL
P(i2)=-P(i2);
aa(bb)=i2;
bb=bb+1;
m=1;
i1=i2 ;
wA(i1)=0;
wB(i1)=0;
wD(i1)=0;

% for time period
a1(a2)=i2;
a2=a2+2;
break
end

% check
if y12(i2)>0
qw=10;
tp=2000;
break
end

end
else
break
end

% for calculating the stroke
min1=min(thD);
len2=-DG*sin(abs(min1)); % for point G, arm DG

minA1=min(thA);
lenA2=-AB*sin(abs(minA1)); % for point B, arm AB
t=i2*dt;
str1(str2)=abs(len1)+abs(len2);

```

```

strA1(str2)=abs(lenA1)+abs(lenA2);

str2=str2+1;
end

% for time period
if qw>10
tp2=a1(3)-a1(1);
tp3=a1(4)-a1(2);
tp1=mean(tp2,tp3);
tp=tp1*dt;
end

%check
if tp<0
tp=2000;
qw=10;
end

if tp>1000
tp=100;
end

%for single objective
%tp gives the time period

STR=max(str1);% for point G, arm DG
STR22=1/STR;

%STRA gives the stroke of piston
STRA=max(strA1);% for point B (piston), arm AB

%for multi-objective
TP=[tp 1/STRA]; % 1/STRA is the inverse of stroke

end

```