Reachability Analysis of Power Electronic Converters

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Reachability Analysis of Power Electronic Converters

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Abstract

This work presents a reachability analysis approach for dual active bridge (DAB) converters in the presence of heterogeneous uncertainties induced by manufacturing tolerance, temperature, humidity, etc. Reachability analysis can overapproximate the set of all possible reachable states from the set of initial states and parameter values based on the reachability algorithm. The novelty of this work includes: 1) it gives a comprehensive introduction of how to build a hybrid automata model for a DAB converter, and use the hybrid automata model for the purpose of reachability analysis through SpaceEx; 2) it develops the procedures of building the SpaceEx model for an arbitrary model by using Atom Text Editor; 3) different non-determined input values are incorporated in the reachability analysis to further improve the performance; and 4) multiple non-determined input values are the first time proposed to discover the influence of the system with diverse uncertain. Test results validate the effectiveness and excellent performance of the presented method.
Chapter 1

Introduction

1.1 Problem Background

The design and verification of new types of power electronic devices such as DC-DC converters require numerical simulations [1]. There are many types of simulation software tools that can offer the environment for users to model and test their designs, including Labview, Plexim PLECS, Simulink, and many more, their introductions are shown in Fig 1.1. However, only using simulations to analyze the models is intrinsically incomplete, because a simulation is running one time only has a single execution of the system. Due to the infinite number of the initial conditions in the presence of heterogeneous uncertainties, just using simulations to test the designs of electronics devices can detrimentally increase the safety risk. For instance, Toyota Motor Corporation recalled nearly 1.9 million Prius cars in 2014 because of a mismatch between a boost converter and its software design [2]. It is critical to developing a formal verification tool to consider all possible executions of the system with various uncertainties.
1.2 Previous Work

The formal verification community has developed many reachability analysis tools including UPPAAL, HyTech, PHAVer, SpaceEx, CORA, and HyLaa [3, 4, 5, 6, 7, 8, 9, 10], as shown in Fig 1.2. Among these tools, the SpaceEx is a widely used verification platform for hybrid automata. It combines polyhedra and supports function representations of continuous sets to compute an over-approximation of the reachable states [11, 12, 13]. For a reachability analysis using SpaceEx, a critical point is to build a SpaceEx model. A common method to build the SpaceEx model is using the Hybrid Source Transformer (HYST) to obtain this model directly. However, for those
complicated models such as the dual active bridge (DAB) converters, using HYST cannot accurately obtain the SpaceEx models [14, 15]. To solve this problem, we need to find a general method to build the SpaceEx model for an arbitrary system with deterministic and non-deterministic inputs.

**Reachability Analysis Tools**

<table>
<thead>
<tr>
<th>Tool</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SpaceEx</td>
<td>Combines polyhedra and supports function representations of continuous sets to compute an over-approximation of the reachable states.</td>
</tr>
<tr>
<td>CORA</td>
<td>MATLAB toolbox for prototypical design of algorithms for reachability analysis, and for various kinds of systems with purely continuous dynamics and hybrid dynamics.</td>
</tr>
<tr>
<td>UPPAAL</td>
<td>An integrated tool environment for modeling, validation and verification of real-time systems modeled as networks of timed automata.</td>
</tr>
<tr>
<td>HyTech</td>
<td>A symbolic model-checking procedure by representing and manipulating regions geometrically, which consists a collection of C++.</td>
</tr>
<tr>
<td>HyLaa</td>
<td>A verification tool for system models with linear ODEs, time-varying inputs, and possibly hybrid dynamics.</td>
</tr>
<tr>
<td>PHAver</td>
<td>A tool for verifying safety properties of hybrid systems.</td>
</tr>
</tbody>
</table>

**Fig. 1.2:** The list of exist reachability analysis tools.
1.3 Purpose

This thesis gives a comprehensive introduction of how to build a hybrid automata model for an electronic converter, and use the hybrid automata model for the purpose of reachability analysis through SpaceEx. It also develops the procedures of building the SpaceEx model for an arbitrary model including Buck and DAB converters by using Atom Text Editor. Moreover, the non-deterministic input values are incorporated in the reachability analysis to improve the performance further. Case studies are provided to validate the effectiveness and performance of the presented method.

1.4 Structure of Thesis

The thesis is organized as follows:

Chapter 2 introduce two different dc-dc converters used in this thesis. Buck converter is presented first as the simple converter circuit, then more complicated circuit design as dual active bridge (DAB) converter is explained. The steady state equation with each operation modes will be calculated based on the converter circuit model. For DAB model, we also give three different types of simulation method (Dynamic, Simscape, Stateflow) to check our model before we do reachability analysis.

Chapter 3 presents a comprehensive introduction of reachability analysis based on the model from chapter 2. There are four sections in this chapter. Firstly, the reachability algorithm of SpaceEx will be discussed. Secondly, the details of the reachability analysis tool SpaceEx is illustrated. Then the hybrid automaton model of DAB converter
is presented. Finally, the SpaceEx model of DAB is built by Atom Text Editor (ATE) based on the hybrid automaton model.

**Chapter 4** discuss the results of reachability analysis using the method in chapter 3. The reachability results with buck converter and DAB converter are presented individually. For DAB converter, five different situations are discussed to investigate the influence with different non-deterministic values.

**Chapter 5** summarizes the contributions and concludes this work. Chapter 5 also presents the direction for future research of reachability analysis on power electronic device.

1.5 **List of publications**

In chapter 2, two types of converter models will be introduced with their circuit design and working modes. Firstly, the typical buck converter is discussed, then a more complex design of dual active bridge (DAB) dc-dc converter is illustrated to prove the versatility of our methods.

2.1 Buck Converter Models

The buck converter is a well-known switched-mode converter that is capable of producing a dc output voltage lower in magnitude than the dc input voltage.

2.1.1 System Description of Buck Converters

The buck converter has the simplest converter structure, which is good to start with. The circuit model of a typical buck converter is given in Fig 2.1. $V_{in}$ is the input DC voltage. $L$ is the inductor. $C$ is the capacitor. $R_0$ is the load resistance. The system is controlled by close and open of the switches $S$, where the duty cycle is $D$ and the period is $T$. 
According to the status of the switch, the system operates in two modes, which are open and closed described as follows:

Mode closed: In this mode, the switch is closed. The state variables are defined by the voltage across the capacitor $v_C$, and the current through the magnetizing inductor $i_L$. According to the Kirchoff’s voltage law (KVL) and Kirchoff’s current law (KCL), the ordinary differential equations (ODEs) of $i_L$ and $v_C$ are obtained as

Inductor ODEs:
\[
\frac{di_L}{dt} = -\frac{1}{L} V_C + \frac{1}{L} V_{in}
\] (2.1)

Capacitor ODEs:
\[
\frac{dv_C}{dt} = \frac{1}{C} i_L - \frac{1}{RC} V_C
\] (2.2)

Mode open: In this mode, the switch is open. According to the Kirchoff’s voltage law (KVL) and Kirchoff’s current law (KCL), the ordinary differential equations (ODEs) can be shown as

Inductor ODEs:
\[
\frac{di_L}{dt} = -\frac{1}{L} V_C
\] (2.3)

Capacitor ODEs:
\[
\frac{dv_C}{dt} = \frac{1}{C} i_L - \frac{1}{RC} V_C
\] (2.4)

2.2 Dual Active Bridge Converter Models

Dual active bridge dc-dc converters have been widely used in distributed power systems and energy storage equipment, because the topology fits applications of battery power
Fig. 2.1: The circuit model of a Buck Converter.

Table 2.1: Parameters of A Buck Converter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{in}$</td>
<td>15 V</td>
<td>$R_0$</td>
<td>5 Ω</td>
</tr>
<tr>
<td>$L$</td>
<td>3 mH</td>
<td>$T$</td>
<td>10 ms</td>
</tr>
<tr>
<td>$C$</td>
<td>200 $\mu$Ω</td>
<td>$D$</td>
<td>0.6</td>
</tr>
</tbody>
</table>

interfaces and solid-state transformers due to its capability for bidirectional power flow, its high power density, the controllability of the power flow, the inherent soft switching, and the flexibility of voltage conversion ration. DAB converter also has shortcoming as it is much expensive than usually converters and it needs complex control design for the system. There are two types of DAB converters, the single-phase and the three-phase. In our work, the single-phase DAB converter is used. Usually the converter contains two parts: Inverter Bridge which converts DC to AC and Rectifier Bridge which converts AC to DC.
2.2.1 Literature Review of DAB Converters

There are many different circuit topologies of DAB converter about the DAB converter in following paper. The stability analysis of a controlled DAB converter is discussed in paper [16]. In paper [17], different dynamic control theory is illustrated. An improved steady state model is developed in paper [18] to correctly predict DAB dc characteristics. In paper [19], the stability of a DAB converter with closed loop controller is presented. Paper [20] presents an averaged and small-signal model of DAB DC-DC converter. The hybrid phase-shift control is used in paper [21] for a wide input voltage range. In paper [22], a improved state-space averaged model of a DAB converter based on causal system is detailed explained. The paper [23] addresses the controller design issue for a dc-dc DAB converter. The paper [24] derives a function to build large and small signal state-space model of a single-phase shift DAB converter.

2.2.2 System Description of DAB Converters

The circuit model of a typical DAB converter is given in Fig. 2.2. It consists of a primary bridge (PB), a secondary bridge (SB), and a transformer. Each bridge consists of four switches \((S_1-S_4 \text{ or } S_5-S_8)\). \(R_i\) represents the sum of switch on-resistances, line resistance, and transformer winding resistance. \(L\) is the transformer leakage inductance. \(C_0\) and \(R_c\) are the shunt capacitor and its equivalent series resistance, respectively. \(V_{in}\) is the input DC voltage, and \(V_{out}\) is the output DC voltage on the final load of \(R_0\). \(V_p\) and \(V_s\) are the voltages at the two sides of the inductor \(L\), and are used to control the
switches. Using the parameters in Table 4.21, Fig. 2.3 gives the waveforms of \( V_p \) and \( V_s \) under the single phase-shift (SPS) control, where the duty cycle \( D \) is set as 0.5, the period \( T \) is set as 20 \( \mu s \), and the phase shift \( \phi \) between \( V_p \) and \( V_s \) is \( \pi/2 \).

**Fig. 2.2:** The circuit model of a typical DAB converter.

According to the status of the switches, the system operates in four modes, which are described as follows:

**Fig. 2.3:** The waveforms of \( V_p \) and \( V_s \) under the single phase-shift (SPS) control.

Mode 1: In this mode, the switches of the primary bridge \( S_1 \) and \( S_4 \), and the switches of the secondary bridge \( S_6 \) and \( S_7 \) are closed while the other switches are
Table 2.2: Parameters of A DAB Converter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{in}}$</td>
<td>30 V</td>
<td>$R_0$</td>
<td>12.5 Ω</td>
</tr>
<tr>
<td>$L$</td>
<td>35.49 μH</td>
<td>$T$</td>
<td>20 μs</td>
</tr>
<tr>
<td>$R_i$</td>
<td>0.38 Ω</td>
<td>$D$</td>
<td>0.5</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.45 Ω</td>
<td>$C_0$</td>
<td>455 μF</td>
</tr>
</tbody>
</table>

open during the switching cycle $[0, DT)$. The state variables are defined by the voltage across the capacitor $v_C$, and the current through the magnetizing inductor $i_L$. According to the Kirchhoff’s voltage law (KVL) and Kirchhoff’s current law (KCL), the ordinary differential equations (ODEs) of $i_L$ and $v_C$ are obtained as

Inductor ODEs:

$$\frac{di_L}{dt} = -\frac{R_t + R_0R_C/(R_0 + R_C)}{L}i_L + \frac{R_0}{L(R_0 + R_C)}v_C + \frac{1}{L}V_{\text{in}} \quad (2.5)$$

Capacitor ODEs:

$$\frac{dv_C}{dt} = -\frac{R_0}{C_0(R_0 + R_C)}i_L - \frac{1}{C_0R_0 + R_C}v_C \quad (2.6)$$

Mode 2: In this mode, the switches of the primary bridge $S_1$ and $S_4$, and the switches of the secondary bridge $S_6$ and $S_7$ are closed while the other switches are open during the switching cycle $[DT, T)$. Similarly,

Inductor ODEs:

$$\frac{di_L}{dt} = -\frac{R_t + R_0R_C/(R_0 + R_C)}{L}i_L - \frac{R_0}{L(R_0 + R_C)}V_C + \frac{1}{L}V_{\text{in}} \quad (2.7)$$
Capacitor ODEs:

\[
\frac{dv_C}{dt} = \frac{R_0}{C_0(R_0 + R_C)} i_L - \frac{1}{C_0R_0 + R_C} V_C
\]  

(2.8)

Mode 3: In this mode, the switches of the primary bridge \(S_2\) and \(S_3\), and the switches of the secondary bridge \(S_5\) and \(S_8\) are closed while the other switches are open during the switching cycle \([T, (1 + D)T]\). Similarly,

Inductor ODEs:

\[
\frac{di_L}{dt} = -\left(\frac{R_t + R_0R_C/(R_0 + R_C)}{L}\right) i_L - \frac{R_0}{L(R_0 + R_C)} V_C - \frac{1}{L} V_{in}
\]  

(2.9)

Capacitor ODEs:

\[
\frac{dv_C}{dt} = \frac{R_0}{C_0(R_0 + R_C)} i_L - \frac{1}{C_0R_0 + R_C} V_C
\]  

(2.10)

Mode 4: In this mode, the switches of the primary bridge \(S_2\) and \(S_3\), and the switches of the secondary bridge \(S_6\) and \(S_7\) are closed while the other switches are open during the switching cycle \([(1 + D)T, 2T]\). Similarly,

Inductor ODEs:

\[
\frac{di_L}{dt} = -\left(\frac{R_t + R_0R_C/(R_0 + R_C)}{L}\right) i_L + \frac{R_0}{L(R_0 + R_C)} V_C - \frac{1}{L} V_{in}
\]  

(2.11)

Capacitor ODEs:

\[
\frac{dv_C}{dt} = -\frac{R_0}{C_0(R_0 + R_C)} i_L - \frac{1}{C_0R_0 + R_C} V_C
\]  

(2.12)

After building the model of the buck converter and Dual Active Bridge converter with each mode. Simulations will be done before the reachability analysis. In this paper, three different kinds of simulation methods will be given in Appendix part B.
Chapter 3

Reachability Analysis Procedure

In this section, the reachability analysis of a DAB DC-DC converter is presented, including 1) reachability algorithm, 2) polyhedra and support functions, 3) SpaceEx description, 4) hybrid automation model of a DAB converter, and 5) DAB converter modeling in SpaceEx.

3.1 Reachability Algorithm

The reachability algorithm used in this paper is a classical fixed-point computation which is operated on symbolic states. A symbolic state is a pair $R = (l, \Omega)$, where $l$ is a location and $\Omega$ is a convex continuous set. The discrete post-operator $post_d(R)$ is defined as the set of states reachable by a discrete transition from $R$, and the continuous post-operator $post_c(R)$ is defined as the set of states reachable from $R$ by letting an arbitrary amount of time elapse. The set of reachable states is the fixed-point of the sequence: $R_0 = post_c(Init)$ and $R_{k+1} := R_k \cup post_c(post_d(R_k))$. The algorithm uses a passed list of states found so far and a waiting list of states whose successors are
yet to be computed. The flowchart of the algorithm is given in Fig. 3.1.

3.2 Polyhedra and Support Functions

Because it is hard to compute the image of the post-operators \( post_c(R) \) for continuous sets. The SpaceEx uses an efficient algorithm to represent convex continuous sets with support functions. The support function defined a closed and bounded continuous set \( S \subseteq \mathbb{R}^n \) and a direction vector \( l \in \mathbb{R}^n \) with equation \( \rho(l, S) = \max_{x \in S} l^* x \). Here the convex set \( S \) is an exact representation of the set, and defined as \( S = \bigcap_{l \in \mathbb{R}^n} \{ x | l^* x \leq \rho(l, S) \} \).

Which means a convex set can be efficient applied to set operations by the support function in SpaceEx algorithm. Here we have definitions of three sets operations as
Fig. 3.2: Reachable sets of discrete and continue modes

following:

- **Linear map**: defined as a map $M \in \mathbb{R}^n \times \mathbb{R}^n$, $\rho(l, MS) = \rho(M^Tl, S)$.

- **Minkowski sum**: defined as sets $S_1, S_2$, where $\rho(l, S_1 \oplus S_2) = \rho(l, S_1) + \rho(l, S_2)$.

- **Convex hull**: defined as sets $S_1, S_2$, where $\rho(l, CH(S_1, S_2)) = \max(\rho(l, S_1), \rho(l, S_2))$

Besides that, the SpaceEx algorithm needs two more operations which uses another set representation **template polyhedra** rather than the **support function**. **Template polyhedra** is polyhedra useing priori normal vectors as faces. A set $D = \{l_1, l_2, ..., l_m\}$ is defined as **template directions**, then a template polyhedron $P_D \subseteq \mathbb{R}^n$ is a polyhedron which satisfies $P_D = \left\{ x \in \mathbb{R}^n \mid \bigwedge_{l_i \in D} l_i \cdot x \leq b_i \right\}$ with exist coefficients $b_1, ..., b_m \in \mathbb{R}$. When working with sets of template polyhedra, the template polyhedron can be substituted by its coefficients $b_i$. In order to connect the support function to a template polyhedron with a set $S$, **template hull** is defined here as $TH_D(S)$, where the template polyhedron is defined by coefficients $b_i = \rho(l_i, S)$. There are three different kind of directions $l$ in
SpaceEx algorithm.

- $2n$ box directions: $x_i = \pm 1$, $x_k = 0$ for $k \neq i$.

- $2n^2$ octagonal directions: $x_i = \pm 1$, $x_j = \pm 1$, $x_k = 0$ for $k \neq i$ and $k \neq j$.

- $m$ uniform directions: directions distributed over the unit sphere evenly.

In the SpaceEx algorithm, support functions and template hulls work efficiently for different operations. Support function can represent convex sets entirely and precisely as function objects in any direction while template hulls can correct the directions $D$.

3.3 SpaceEx Description

SpaceEx is a robust and user-friendly verification platform for hybrid systems. Its modeling language, called SX, is to allow the exchange of models with a graphical user interface and model editor, as well as the exchange with other tools and modeling languages via automatic translation. The SpaceEx consists of three components, namely, SpaceEx analysis core, web interface, and system model as shown in Fig. 3.4.

- The analysis core is a program which takes a model file in SX and a configuration file in CFG. The configuration file specifies the initial states, the scenarios and other options and then analyzes the system and produces a series of output files.

- The web interface as shown in Fig. 3.3 is a graphical user interface where users can use virtual machines to access the analysis core on the web server. It can be
Fig. 3.3: Web Interface of SpaceEx.

used to specify the initial states and other parameters, and can also visualize the output graphically.

- The model editor is a graphical editor for creating models of complex hybrid systems. Users can directly edit the SX file to build the model.

- The Atom Text Editor is a text editor where users can directly edit and modify the SX file and the CFG file without using other hybrid system translation tools such as HYST [13].
Fig. 3.4: Software architecture of the SpaceEx platform.

3.4 Hybrid Automaton Model of A DAB Converter

Hybrid automaton uses a mathematical model to describe the systems in which digital computational processes interact with analog physical processes. A hybrid automaton is a finite state machine with a finite set of continuous variables whose values are described by a set of ordinary differential equations. More details about hybrid automaton can be see in [27, 28, 29]. A hybrid automaton $H$ can be defined by a tuple $(L, S, P, T, G, F,)$, which are described as follows:

1. $L$ is the infinite set of the topology that represents the control model of a hybrid system: $L = \{l_1, l_2, ..., l_n\}$. For a DAB converter in this paper, there are four modes which are therefore denoted as $L = \{l_1, l_2, l_3, l_4\}$.

2. $S$ is a group of the continuous state variables: $S = \{i_L, V_C, t\}$, where $t$ is the real
3. \( P \) is the set of the inputs to the system for each mode: \( P = \{ p_1, p_2, ..., p_n \} \). For a DAB converter in this paper, \( P = \{ [V_{in}, 0, 0]', [V_{in}, 0, 0]', [-V_{in}, 0, 0]', [-V_{in}, 0, 0]' \} \).

4. \( T \) is the set of feasible discrete transitions occurring among the modes. Each element is defined as \( t_{ij} = (l_1, l_2) \in T \) which means there is a discrete transition from the \( i^{th} \) mode to the \( j^{th} \) mode. For a DAB converter in this paper, \( T = \{(l_1, l_2), (l_2, l_3), (l_3, l_4), (l_4, l_1)\} \).

5. \( G \) is the guard set, which refers to the discrete transition from a given topology to another pre-defined topology. For a DAB converter in this paper, \( G = \{ (t \geq DT), (t \geq T), (t \geq (1 + D)T), (t \geq 2T) \} \).

6. \( F \) is the set of ODEs that are defined for each topology \( l \in L \) with the continuous state variable \( s \in S \).

### 3.5 DAB Converter Modeling in SpaceEx

This paper develops an approach which uses the Atom Text Editor (see Fig. 3.4) to build the SpaceEx model for an arbitrary model including DAB converters. The flowchart of the presented method is given in Fig. 3.6. For each element we defined as follow:

- Parameters: Define the characteristics of a variable. (controlled, dynamics, name, type).
• Locations: Each location is a mode in the system.

• Invariants: The constraints of remaining in a location.

• Flows: A set of differential equations that define the time-driven evolution of the continuous variables.

• Transitions: Edges of a topology, allow the system to jump between locations.

• Guards: The requirement of the transition to happen.

• Assignments: New value of continuous variables after the jump.

• Binds: Connection between component A and network B. (formal parameter of B, numeric value).

For an arbitrary model, its variables are defined step by step in the Atom Text Editor, and the SpaceEx model can thus be obtained. The code of DAB with deterministic and non-deterministic inputs by using ATE method are shown in Appendix Part A. The work panel of Atom Text Editor is shown as Fig. 3.5. An example of the SpaceEx model for a DAB converter obtained via the presented method is illustrated in Fig. 3.7. The SX file of the SpaceEx model and the CFG file are then uploaded to the web server through the web interface as shown in Fig. 3.4. The reachability analysis is then implemented in the SpaceEx analysis core, and the results are sent back to the web interface.
Fig. 3.5: Work panel of Atom Text Editor.

Fig. 3.6: Flowchart of the presented method.
Fig. 3.7: The illustration of the SpaceEx model for DAB converters obtained via the presented method. (Model System)

Fig. 3.8: The illustration of the SpaceEx model for DAB converters obtained via the presented method. (Model Network)
Chapter 4

Result and Analysis

4.1 Buck Converter Analysis

This section presents the formal verification results of a buck converter via verifying the states. By using reachability analysis tools SpaceEx, we got the comparison of reachability results of buck converter between deterministic and non-deterministic value. More details about non-deterministic inputs will be described with the DAB converter model.

Figs. 4.1-4.3 give the comparison results of deterministic and single non-deterministic where $v_{in}$ is 15V as deterministic input and is within [14.95, 15.05]V during non-deterministic inputs. The blue curves represent the reachability results using the deterministic input for a buck converter model, and the black curves represent the reachability results using the non-deterministic input $v_{in}$ for the system.

According to the Tab. 4.1-4.2, the reachable range with deterministic input is between [0.0001, 0.0002]. While the reachable range with non-deterministic input is between [0.01, 0.22]. Then for the Tab. 4.3-4.4, the reachable range with deterministic in-
Fig. 4.1: Buck Converter with deterministic and non-deterministic inputs on $i_L$.

Table 4.1: Selected points of SpaceEx in Buck: Deterministic (Blue) ($i_L$-t)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Current (A)</th>
<th>Time (s)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>1.43527</td>
<td>0.002</td>
<td>2.72953</td>
</tr>
<tr>
<td>0.0005</td>
<td>1.43533</td>
<td>0.003</td>
<td>1.99359</td>
</tr>
<tr>
<td>0.001</td>
<td>2.38472</td>
<td>0.003</td>
<td>2.00137</td>
</tr>
<tr>
<td>0.001</td>
<td>2.38491</td>
<td>0.004</td>
<td>1.5426</td>
</tr>
<tr>
<td>0.002</td>
<td>2.72902</td>
<td>0.004</td>
<td>1.54299</td>
</tr>
</tbody>
</table>
Table 4.2: Selected points of SpaceEx in Buck: Non-deterministic (Black) $(i_L-t)$

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Current (A)</th>
<th>Time (s)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>1.40023</td>
<td>0.002</td>
<td>2.75441</td>
</tr>
<tr>
<td>0.0005</td>
<td>1.41665</td>
<td>0.003</td>
<td>1.922</td>
</tr>
<tr>
<td>0.001</td>
<td>2.36809</td>
<td>0.003</td>
<td>2.06598</td>
</tr>
<tr>
<td>0.001</td>
<td>2.40154</td>
<td>0.004</td>
<td>1.44358</td>
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<td>0.002</td>
<td>2.67624</td>
<td>0.004</td>
<td>1.66547</td>
</tr>
</tbody>
</table>

Fig. 4.2: Buck Converter with deterministic and non-deterministic inputs on $V_c$. 
Table 4.3: Selected points of SpaceEx in Buck: Deterministic (Blue) \( v_C-t \)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Voltage (V)</th>
<th>Time (s)</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>1.58928</td>
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<td>10.464</td>
</tr>
<tr>
<td>0.0005</td>
<td>1.61171</td>
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<td>11.1665</td>
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<tr>
<td>0.001</td>
<td>4.85725</td>
<td>0.003</td>
<td>11.176</td>
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<tr>
<td>0.001</td>
<td>4.92871</td>
<td>0.004</td>
<td>9.4575</td>
</tr>
<tr>
<td>0.002</td>
<td>10.4438</td>
<td>0.004</td>
<td>9.44638</td>
</tr>
</tbody>
</table>

Table 4.4: Selected points of SpaceEx in Buck: Non-deterministic (Black) \( v_C-t \)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Voltage (V)</th>
<th>Time (s)</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
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<td>10.3595</td>
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<td>0.0005</td>
<td>1.56483</td>
<td>0.003</td>
<td>10.9908</td>
</tr>
<tr>
<td>0.001</td>
<td>4.8003</td>
<td>0.003</td>
<td>11.3587</td>
</tr>
<tr>
<td>0.001</td>
<td>4.82979</td>
<td>0.004</td>
<td>9.12352</td>
</tr>
<tr>
<td>0.002</td>
<td>10.3414</td>
<td>0.004</td>
<td>9.77368</td>
</tr>
</tbody>
</table>
Fig. 4.3: Buck Converter with deterministic and non-deterministic inputs on $i_L$ and $V_c$.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.199505</td>
<td>1.5H</td>
<td>9.48086</td>
</tr>
<tr>
<td>0.5</td>
<td>0.204178</td>
<td>2.5L</td>
<td>5.49377</td>
</tr>
<tr>
<td>1.5L</td>
<td>1.75903</td>
<td>2.5L</td>
<td>5.53609</td>
</tr>
<tr>
<td>1.5L</td>
<td>1.78203</td>
<td>2.5H</td>
<td>11.2251</td>
</tr>
<tr>
<td>1.5H</td>
<td>9.44761</td>
<td>2.5H</td>
<td>11.2442</td>
</tr>
</tbody>
</table>
Table 4.6: Selected points of SpaceEx in Buck: Non-deterministic (Black) \( i_L-u_C \)

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Voltage (V)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.198423</td>
<td>1.5H</td>
<td>9.31884</td>
</tr>
<tr>
<td>0.5</td>
<td>0.200587</td>
<td>2.5L</td>
<td>5.39068</td>
</tr>
<tr>
<td>1.5L</td>
<td>1.7721</td>
<td>2.5L</td>
<td>5.45972</td>
</tr>
<tr>
<td>1.5L</td>
<td>1.78319</td>
<td>2.5H</td>
<td>11.02</td>
</tr>
<tr>
<td>1.5H</td>
<td>9.29935</td>
<td>2.5H</td>
<td>11.21</td>
</tr>
</tbody>
</table>

put is between [0.0002, 0.011]. While the reachable range with non-deterministic input is between [0.02, 0.65]. Finally for the Tab. 4.5-4.6, the reachable range with deterministic input is between [0.01, 0.02]. While the reachable range with non-deterministic input is between [0.002, 0.2]. The above data depicts the range is small at the beginning and rising rapidly when the system comes to a steady state. So the non-deterministic input \( u_{in} \) have a larger impact on the system when it comes to steady states.

4.2 DAB Converter Analysis

This section presents the formal verification results of a DAB converter via verifying the states, e.g., \( i_L \) and \( v_C \), which include: 1) comparison of Stateflow and SpaceEx; 2) reachability with different uncertainty levels of \( u_{in} \); 3) reachability with non-deterministic \( v_C \) and \( i_L \) initial values; 4) reachability with deterministic and non-deterministic \( R_0 \); and 5) reachability with multiple and single non-deterministic inputs.
4.2.1 Comparison of Stateflow and SpaceEx

The traditional method to verify the performance of a DAB converter is to perform simulations using tools such as a combination of Stateflow and Simulink. Figs. 4.4-4.6 give the comparison results of Stateflow and SpaceEx, where $v_{in}$ is deterministic as 30V in Stateflow and is within [29.95V, 30.05V] in SpaceEx. The other parameters of the DAB converter are the same as shown in Table 4.21. It can be seen that the Stateflow results (see the blue dotted lines) are contained in the reachability set calculated by SpaceEx (see the black solid lines), which verifies that SpaceEx computes an over-approximation of the set of reachable states of the system, and in turn can be used to ensure that the system satisfies all desired safety properties for all possible executions.

Fig. 4.4: Comparison results of Stateflow and SpaceEx on $i_L$. 
Table 4.7: Selected points of SpaceEx in DAB (Part1: $i_L$-t)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Current (A)</th>
<th>Time (s)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>12.3435</td>
<td>0.002</td>
<td>2.53056</td>
</tr>
<tr>
<td>0.0005</td>
<td>12.5052</td>
<td>0.003</td>
<td>2.23946</td>
</tr>
<tr>
<td>0.001</td>
<td>4.34569</td>
<td>0.003</td>
<td>2.23902</td>
</tr>
<tr>
<td>0.001</td>
<td>4.53397</td>
<td>0.004</td>
<td>2.23534</td>
</tr>
<tr>
<td>0.002</td>
<td>2.33559</td>
<td>0.004</td>
<td>2.43066</td>
</tr>
</tbody>
</table>

Fig. 4.5: Comparison results of Stateflow and SpaceEx on $v_C$. 
Table 4.8: Selected points of SpaceEx in DAB (Part1: $v_C$-t)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Voltage (V)</th>
<th>Time (s)</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>21.7531</td>
<td>0.002</td>
<td>29.0869</td>
</tr>
<tr>
<td>0.0005</td>
<td>21.8257</td>
<td>0.003</td>
<td>29.0596</td>
</tr>
<tr>
<td>0.001</td>
<td>27.4959</td>
<td>0.003</td>
<td>29.1571</td>
</tr>
<tr>
<td>0.001</td>
<td>27.6182</td>
<td>0.004</td>
<td>29.0631</td>
</tr>
<tr>
<td>0.002</td>
<td>28.9901</td>
<td>0.004</td>
<td>29.1601</td>
</tr>
</tbody>
</table>

Fig. 4.6: Comparison results of Stateflow and SpaceEx on $v_C$ and $i_L$. 
Table 4.9: Selected points of SpaceEx in DAB (Part1: $i_L$-$v_C$)

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Voltage (V)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>30.3694</td>
<td>20</td>
<td>14.8072</td>
</tr>
<tr>
<td>5</td>
<td>30.4708</td>
<td>25</td>
<td>7.77692</td>
</tr>
<tr>
<td>10</td>
<td>27.3869</td>
<td>25</td>
<td>8.0187</td>
</tr>
<tr>
<td>10</td>
<td>27.7539</td>
<td>29</td>
<td>2.23532</td>
</tr>
<tr>
<td>20</td>
<td>14.6532</td>
<td>29</td>
<td>2.43066</td>
</tr>
</tbody>
</table>

4.2.2 Reachability with Different Uncertainty Levels of $v_{in}$

In reality, the different impacts of uncertainties (i.e., manufacturing tolerance, temperature, humidity, etc.) will result in different initial input values to the SpaceEx model. Considering the non-deterministic characteristics of the components’ values, the non-deterministic initial input values are added for the reachability analysis. According to ODE, $\dot{x}(t) = Ax(t) + Bu$, where $x(t) = [i_L, v_C]'$. The non-deterministic value $v_{in}$ is parameter $u$ in the ODE. Where $v_{in}$ is within [29.5V, 30.5V] for the high-level uncertainties and [29.95V, 30.05V] for the low-level uncertainties. The other parameters of the DAB converter are the same as shown in Table 4.21. The reachability results with high-level input and low-level input are given in Figs. 4.7-4.9. It can be seen that compared with the low-level uncertainties input, the high-level uncertainties input gives a larger set of reachable states of the system, making the system satisfies desired safety properties for more possible executions.
Fig. 4.7: Reachability results with high-level and low-level uncertainties input $v_{in}$ on $i_L$.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Current (A)</th>
<th>Time (s)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>12.3435</td>
<td>0.0005</td>
<td>11.6419</td>
</tr>
<tr>
<td>0.0005</td>
<td>12.5052</td>
<td>0.0005</td>
<td>13.2575</td>
</tr>
<tr>
<td>0.001</td>
<td>4.34569</td>
<td>0.001</td>
<td>3.55926</td>
</tr>
<tr>
<td>0.001</td>
<td>4.53397</td>
<td>0.001</td>
<td>5.43987</td>
</tr>
<tr>
<td>0.002</td>
<td>2.33559</td>
<td>0.002</td>
<td>1.45339</td>
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<td>0.002</td>
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<tr>
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<td>2.23946</td>
<td>0.003</td>
<td>1.35997</td>
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<td>2.23902</td>
<td>0.003</td>
<td>3.31268</td>
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<td>2.23534</td>
<td>0.004</td>
<td>1.35655</td>
</tr>
<tr>
<td>0.004</td>
<td>2.43066</td>
<td>0.004</td>
<td>3.30937</td>
</tr>
</tbody>
</table>
Fig. 4.8: Reachability results with high-level and low-level uncertainties input $v_{in}$ on $v_C$.

Fig. 4.9: Reachability results with high-level and low-level uncertainties input $v_{in}$ on $v_C$ and $i_L$. 
4.2.3 Reachability with Non-deterministic $v_C$ and $i_L$ Initial Values

As shown in (2.6)-(2.12), there are three types of parameters that can affect $v_C$ and $i_L$, namely, $v_{in}$, the initial values of $v_C$ and $i_L$, and the weights such as $R_o$. Figs. 4.10-4.12 give the reachability results with different non-deterministic $v_C$ and $i_L$ initial values. According to ODE, $\dot{x}(t) = Ax(t) + Bu$, where $x(t) = [i_L, v_C]'$. The non-deterministic $v_C$ and $i_L$ is the parameter initial $x(t)$ here. In the one-side case, $v_C$ is within [0V, 1.456V] and $i_L$ is within [0A, 0.117A], while in the two-sides case, $v_C$ is within [-1.456A, 1.456A] and $i_L$ is within [-0.117A, 0.117A]. The other parameters of the DAB converter are the same as shown in Table 4.21. It can be seen that the one-side results
Table 4.12: Selected points of SpaceEx in DAB (Part2: $i_L$-$v_C$)

<table>
<thead>
<tr>
<th>Low-Level</th>
<th>High Level</th>
</tr>
</thead>
<tbody>
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<td>Voltage (V)</td>
<td>Current (A)</td>
</tr>
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<td>30.3694</td>
</tr>
<tr>
<td>5</td>
<td>30.4708</td>
</tr>
<tr>
<td>10</td>
<td>27.3869</td>
</tr>
<tr>
<td>10</td>
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<tr>
<td>20</td>
<td>14.6532</td>
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<td>14.8072</td>
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<td>29</td>
<td>2.23532</td>
</tr>
<tr>
<td>29</td>
<td>2.43066</td>
</tr>
</tbody>
</table>

(see the blue dotted lines) are contained in the reachability set calculated by the two-sides results (see the black solid lines).

4.2.4 Reachability with Deterministic and Non-deterministic $R_0$

Figs. 4.13-4.15 give the reachability results with deterministic and non-deterministic $R_0$. According to ODE, $\dot{x}(t) = Ax(t) + Bu$, where $x(t) = [i_L, v_C]'$. The non-deterministic $R_0$ is the parameter $A$ in ODE, where the deterministic $R_0$ is set as $12.5 \Omega$ and the non-deterministic $R_0$ is within $[11.25 \Omega, 13.75 \Omega]$. The other parameters of the DAB converter are the same as shown in Table 4.21. It can be seen that compared with the deterministic $R_0$, the non-deterministic $R_0$ results in a larger set of reachable states of
Fig. 4.10: Reachability results with non-deterministic $v_C$ and $i_L$ initial values on $i_L$.

Table 4.13: Selected points of SpaceEx in DAB (Part3: $i_L$-t)

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**Fig. 4.11:** Reachability results with non-deterministic $v_C$ and $i_L$ initial values on $v_C$.

**Table 4.14:** Selected points of SpaceEx in DAB (Part 3: $v_C$-t)

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<th>Voltage (V)</th>
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Fig. 4.12: Reachability results with non-deterministic $v_C$ and $i_L$ initial values on $v_C$ and $i_L$.

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<tr>
<th>Voltage (V)</th>
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<th>Voltage (V)</th>
<th>Current (A)</th>
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</thead>
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<td>15.0168</td>
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</tr>
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<tr>
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<tr>
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</tbody>
</table>
the system.

**Fig. 4.13**: Reachability results with deterministic and non-deterministic $R_0$ on $i_L$.

### 4.2.5 Reachability with Multiple Non-deterministic Inputs

To further evaluate the impacts of different parameters on reachability results, Figs. 4.16-4.18 give the comparison of reachability analysis using multiple non-deterministic inputs and a single non-deterministic input. The red, green and blue curves represent the reachability results using the non-deterministic $v_C$ and $i_L$, $v_{in}$, and $R_0$, respectively, and the black curves represent the reachability results using all the three types of parameters. It can be seen that 1) different parameters have different impacts; and 2) compared with using a single non-deterministic input, using multiple non-deterministic inputs gives a larger set of reachable states of the system.

The time consumption of the reachability analysis of DAB converter is variations
### Table 4.16: Selected points of SpaceEx in DAB (Part4: $i_L$-t)

<table>
<thead>
<tr>
<th>Low-border</th>
<th>Non-deterministic</th>
<th>Up-border</th>
<th>Non-deterministic</th>
<th>Deterministic $R_0$</th>
</tr>
</thead>
<tbody>
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<td>Time (s)</td>
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<td>Time (s)</td>
<td>Current (A)</td>
<td>Time (s)</td>
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**Fig. 4.14:** Reachability results with deterministic and non-deterministic $R_0$ on $v_C$. 
Table 4.17: Selected points of SpaceEx in DAB (Part 4: $v_C-t$)

<table>
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<tr>
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<th>Up-border</th>
<th>Non-deterministic</th>
<th>Deterministic $R_0$</th>
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</thead>
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<td>Time (s)</td>
<td>Voltage (V)</td>
<td>Time (s)</td>
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</table>

between each other. In part 1 with deterministic input, it usually takes 2 to 3 minutes to get the results. While in part 2 with non-deterministic input $v_{in}$, it takes almost 15 minutes to get the reachable sets. When there are multiple non-deterministic inputs as in part 5, the SpaceEX spends near 30 minutes to get the reachability results. So how to reduce the time consumption is also a significant problem need to solve.

The time consumption of the reachability analysis will be longer for a complicated system, but the results of reachability analysis is much more reliable and convincing than repeated simulation results.
Fig. 4.15: Reachability results with deterministic and non-deterministic $R0$ on $v_C$ and $i_L$.

Fig. 4.16: Reachability results of multiple non-deterministic inputs and a single non-deterministic input on $i_L$. 
Table 4.18: Selected points of SpaceEx in DAB (Part4: $i_L$-$v_C$)

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<th>Low-border Non-deterministic Voltage (V)</th>
<th>Current (A)</th>
<th>Up-border Non-deterministic Voltage (V)</th>
<th>Current (A)</th>
<th>Deterministic $R_0$ Voltage (V)</th>
<th>Current (A)</th>
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</thead>
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Table 4.19: Selected points of SpaceEx in DAB (Part5: $i_L$-$t$)

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Fig. 4.17: Reachability results of multiple non-deterministic inputs and a single non-deterministic input on $v_C$.

Table 4.20: Selected points of SpaceEx in DAB (Part5: $v_C$-t)

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**Fig. 4.18:** Reachability results of multiple non-deterministic inputs and a single non-deterministic input on $v_C$ and $i_L$.

**Table 4.21:** Selected points of SpaceEx in DAB (Part5: $i_L$-$v_C$)

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<th>Voltage (V)</th>
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<th>Voltage (V)</th>
<th>Current (A)</th>
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</thead>
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Chapter 5

Conclusions

This paper contributes a reachability analysis approach using SpaceEx for the arbitrary converter. The reachability results with five different situations consider all the exist non-deterministic parameters in ODE. The multiple non-deterministic inputs give us a new way to find the influence level of each non-deterministic inputs to a system. This work not only presents the procedures of reachability analysis for buck and DAB converters but also provides insightful situational awareness of power electronic devices in the presence of heterogeneous uncertainties.

There are many research direction arising from this thesis which can be used for future work. Firstly, the ATE method will be evaluated based on more complicated designs. For example, the closed-loop (PID control) DAB converter will be built and tested by using ATE method. Secondly, more realistic uncertainties such as switch diode parameters will be considered in our future research. Last but not least, the way to reduce the calculating time of reachability analysis with multiple non-deterministic inputs also deserves further investigation.
Bibliography


Appendix A

ATE Source Code

A.1 DAB-deterministic.xml (Model)

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    namespaces/spaceex" math="SpaceEx" version="0.2">
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  <param controlled="true" d1="1" d2="1" dynamics="const"
      local="false" name="a21" type="real" />
  <param controlled="true" d1="1" d2="1" dynamics="const"
      local="false" name="a22" type="real" />
  <param controlled="true" d1="1" d2="1" dynamics="const"
      local="false" name="b1" type="real" />
  <param controlled="true" d1="1" d2="1" dynamics="const"
      local="false" name="bounds" type="real" />
  <param controlled="true" d1="1" d2="1" dynamics="const"
      local="false" name="Ts" type="real" />
```
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="vi" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="tmax" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="D" type="real" />
<location id="1" name="model" x="421.0" y="196.0" width="410.0" height="344.0">
  <invariant>t \&gt;= 0 &amp; amp;
  t \&lt;= D * Ts &amp; amp;
  gt \&gt;= 0 &amp; amp;
  gt \&lt;= tmax &amp; amp;
  mode == 1</invariant>
  <flow>il' == a11 * il + a12 * vc + b1 * vi &amp; amp;
  vc' == a22 * vc a21 * il &amp; amp;
  t' == 1 &amp; amp;
  gt' == 1 &amp; amp;
  mode' == 0</flow>
</location>
<location id="2" name="mode2" x="1157.0" y="190.0" width="332.0" height="322.0">
  <invariant>t \&gt;= D * Ts &amp; amp;
  t \&lt;= Ts &amp; amp;
  gt \&gt;= 0 &amp; amp;
  gt \&lt;= tmax &amp; amp;
  mode == 2</invariant>
  <flow>il' == a11 * il a12 * vc + b1 * vi &amp; amp;
  vc' == a21 * il + a22 * vc &amp; amp;
  t' == 1 &amp; amp;
  gt' == 1 &amp; amp;
  mode' == 0</flow>
</location>
<location id="3" name="mode3" x="1135.0" y="620.0" width="410.0" height="358.0">
  <invariant>t \&gt;= Ts &amp; amp;
  t \&lt;= (1 + D) * Ts &amp; amp;
  gt \&gt;= 0 &amp; amp;
  gt \&lt;= tmax &amp; amp;
  mode == 3</invariant>
  <flow>il' == a11 * il a12 * vc b1 * vi &amp; amp;
  vc' == a21 * il + a22 * vc &amp; amp;
  t' == 1 &amp; amp;
\begin{verbatim}
gt’ == 1 &amp;
mode’ == 0</flow>
</location>
<location id="4" name="mode4" x="433.0" y="604.0"
        width="490.0" height="350.0">
        <Invariant>t &gt;=(1 + D) * Ts &amp;
        t’ &lt;= 2 * Ts &amp;
t &lt;= tmax &amp;
mode == 4</Invariant>
<flow>il’ == a11 * il + a12 * vc   b1 * vi &amp;
vc’ == a22 * vc   a21 * il &amp;
t’ == 1 &amp;
gt’ == 1 &amp;
mode’ == 0</flow>
</location>
<transition asap="false" bezier="false" source="1"
        target="2" timedriven="false">
        <guard>t &gt;= D * Ts</guard>
        <assignment>il’ == il &amp;
vc’ == vc   &amp;
t’ == 0 &amp;
gt’ == gt   &amp;
mode’ == 2</assignment>
<labelposition x="29.0" y="135.0" width="144.0"
        height="136.0" />
<middlepoint x="743.0" y="170.0" />
</transition>
<transition asap="false" bezier="false" source="2"
        target="3" timedriven="false">
        <guard>t &gt;= Ts</guard>
        <assignment>il’ == il &amp;
vc’ == vc   &amp;
t’ == 0 &amp;
gt’ == gt   &amp;
mode’ == 3</assignment>
<labelposition x="295.0" y="125.0" width="138.0"
        height="130.0" />
</transition>
<transition asap="false" bezier="false" source="3"
        target="4" timedriven="false">
        <guard>t &gt;= (1 + D) * Ts</guard>
\end{verbatim}
<assignment>il' == il &amp;  
vc' == vc &amp; 
t' == 0 &amp;  
gt' == gt &amp;  
mode' == 4</assignment>

<labelposition x="55.0" y="51.0" width="164.0"
    height="170.0" />
</transition>
<transition asap="false" bezier="false" source="4"
    target="1" timedriven="false">
<guard>t &gt;= 2 * Ts</guard>
<assignment>il' == il &amp;  
vc' == vc &amp; 
t' == 0 &amp;  
gt' == gt &amp;  
mode' == 1</assignment>
<labelposition x="261.0" y="67.0" width="162.0"
    height="148.0" />
</transition>
</component>

.createComponent(id="$DAB_conv_net$">
<param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="il" type="real" x="999.0" y
    ="59.0" />
<param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="vc" type="real" x="1011.0" y
    ="91.0" />
<param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="t" type="real" x="1051.0" y
    ="143.0" />
<param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="gt" type="real" x="1071.0" y
    ="165.0" />
<param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="mode" type="real" x="961.0" y
    ="218.0" />
<param controlled="true" d1="1" d2="1"
    dynamics="cons"
    local="false" name="a11" type="real" />
<param controlled="true" d1="1" d2="1"
    dynamics="cons"
    local="false" name="a12" type="real" />
<param controlled="true" d1="1" d2="1"
    dynamics="cons"
    local="false" name="a21" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="a22" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="b1" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="bounds" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="Ts" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="vi" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="tmax" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="const" local="false" name="D" type="real" />
<bind as="$DAB_conv$" component="$DAB_conv_sys$" x="706.0" y="124.0" width="130.0" height="160.0">
  <map key="i1">i1</map>
  <map key="vc">vc</map>
  <map key="t">t</map>
  <map key="gt">gt</map>
  <map key="mode">mode</map>
  <map key="a11">22946</map>
  <map key="a12">27198</map>
  <map key="a21">2121.4</map>
  <map key="a22">170</map>
  <map key="b1">28177</map>
  <map key="bounds">500.0</map>
  <map key="Ts">0.1</map>
  <map key="vi">30</map>
  <map key="tmax">10</map>
  <map key="D">0.5</map>
</bind>
</component>
</spaceex>
A.2 DAB-deterministic.cfg (Parameters and Initials)

```
system = DAB_conv_net
initially = "loc(DAB_conv) == model & il == 0 & vc == 0
       & t == 0 & gt == 0 & mode == 1"
forbidden = ""
scenario = stc
directions = box
set aggregation = "chull"
sampling time = 1E 8
flowpipe tolerance = 0.0001
time horizon = 0.02
clustering = 100
iter max = 1
output variables = "gt, il, vc"
output format = GEN
verbosity = m
rel err = 1.0e 12
abs err = 1.0e 13
```
A.3 DAB-multiple-non-deterministic.xml (Model)

```xml
<?xml version="1.0" encoding="iso 8859 1"?>
<spaceex xmlns="http://www.verimag.imag.fr/xml
    namespaces" math="SpaceEx" version="0.2">
    <component id="DAB_conv_sys">
        <param controlled="true" d1="1" d2="1" dynamics="any"
            local="false" name="il" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            local="false" name="vc" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            local="false" name="t" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            local="false" name="gt" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            local="false" name="mode" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="all" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="a12" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="a21" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="a22" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="b1" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="bounds" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="Ts" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="vi" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="tmax" type="real" />
        <param controlled="true" d1="1" d2="1" dynamics="any"
            const local="false" name="D" type="real" />
        <param name="r1" type="real" local="false" d1="1" d2="1" dynamics="any"
            controlled="false" />
        <location id="1" name="model" x="412.0" y="194.0"
            width="410.0" height="344.0">
            <invariant>t &gt;= 0 &amp;
```
t &lt;= D * Ts &amp; 
gt &gt;= 0 &amp; 
gt &lt;= tmax &amp; 
mode == 1 &amp; 
rl &gt;= 14090 &amp; 
rl &lt;= 14090 &lt;/invariant>

&lt;flow&gt;
il' == a11 * il + a12 * vc + b1 * vi + r1 &amp;
vc' == a22 * vc + a21 * il &amp; 
t' == 1 &amp; 
gt' == 1 &amp; 
mode' == 0 &lt;/flow&gt;
&lt;/location&gt;
&lt;location id="2" name="mode2" x="1157.0" y="190.0"
    width="332.0" height="322.0"&gt;
&lt;invariant&gt;t &gt;= D * Ts &amp; 
t &lt;= Ts &amp; 
gt &gt;= 0 &amp; 
gt &lt;= tmax &amp; 
mode == 2 &amp; 
rl &gt;= 14090 &amp; 
rl &lt;= 14090 &lt;/invariant&gt;
&lt;flow&gt;
il' == a11 * il + a12 * vc + b1 * vi + r1 &amp;
vc' == a22 * vc + a21 * il &amp; 
t' == 1 &amp; 
gt' == 1 &amp; 
mode' == 0 &lt;/flow&gt;
&lt;/location&gt;
&lt;location id="3" name="mode3" x="1135.0" y="620.0"
    width="410.0" height="358.0"&gt;
&lt;invariant&gt;t &gt;= Ts &amp; 
t &lt;= (1 + D) * Ts &amp; 
gt &gt;= 0 &amp; 
gt &lt;= tmax &amp; 
mode == 3 &amp; 
rl &gt;= 14090 &amp; 
rl &lt;= 14090 &lt;/invariant&gt;
&lt;flow&gt;
il' == a11 * il + a12 * vc + b1 * vi + r1 &amp;
vc' == a21 * il + a22 * vc &amp; 
t' == 1 &amp; ;
<transition asap="false" bezier="false" source="3"
  target="4" timedriven="false">
  <guard>t &gt;= (1 + D) * Ts</guard>
  <assignment>il' == il &amp;&amp;
    vc' == vc &amp;&amp;
    t' == 0 &amp;&amp;
    gt' == gt &amp;&amp;
    mode' == 4</assignment>
  <labelposition x="55.0" y="51.0" width="164.0"
    height="170.0"/>
</transition>

<transition asap="false" bezier="false" source="4"
  target="1" timedriven="false">
  <guard>t &gt;= 2 * Ts</guard>
  <assignment>il' == il &amp;&amp;
    vc' == vc &amp;&amp;
    t' == 0 &amp;&amp;
    gt' == gt &amp;&amp;
    mode' == 1</assignment>
  <labelposition x="261.0" y="67.0" width="162.0"
    height="148.0"/>
</transition>
</component>

<component id="DAB_conv_net">
  <param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="il" type="real" x="999.0" y ="59.0"/>
  <param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="vc" type="real" x="1011.0" y ="91.0"/>
  <param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="t" type="real" x="1051.0" y ="143.0"/>
  <param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="gt" type="real" x="1071.0" y ="165.0"/>
  <param controlled="true" d1="1" d2="1"
    dynamics="any"
    local="false" name="mode" type="real" x="961.0" y ="218.0"/>
  <param controlled="true" d1="1" d2="1"
    dynamics="const"
    local="false" name="all" type="real"/>
</component>
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="a12" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="a21" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="a22" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="b1" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="bounds" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="Ts" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="vi" type="real" />
<param controlled="true" d1="1" d2="1" dynamics="
  const" local="false" name="tmax" type="real" />
<param name="r1" type="real" local="false" d1="1" d2 ="1" dynamics="any" controlled="false" x="519.0" y 
 ="121.0" />
<bind as="DAB_conv" component="DAB_conv_sys" x 
 ="706.0" y="124.0" width="130.0" height="160.0">
  <map key="i1">i1</map>
  <map key="vc">vc</map>
  <map key="t">t</map>
  <map key="gt">gt</map>
  <map key="mode">mode</map>
  <map key="a11">22899</map>
  <map key="a12">27093</map>
  <map key="a21">2113.3</map>
  <map key="a22">188</map>
  <map key="b1">28177</map>
  <map key="bounds">500.0</map>
  <map key="Ts">0.1</map>
  <map key="vi">30</map>
  <map key="tmax">10</map>
  <map key="D">0.5</map>
  <map key="r1">r1</map>
</bind>
</component>
</sspaceex>
A.4 DAB-multiple-non-deterministic.cfg (Parameters and Initials)

```plaintext
system = DAB_conv_net
initially = "loc(DAB_conv) == model & il >= 0.116665 & il <=0.11665 & vc >= 1.4555668 & vc <=1.455668 & t == 0 & gt == 0 & mode == 1"
forbidden = ""
scenario = stc
directions = box
set aggregation = "chull"
sampling time = 1E 8
flowpipe tolerance = 0.0001
time horizon = 0.02
clustering = 100
iter max = 1
output variables = "gt, il, vc"
output format = GEN
verbosity = m
rel err = 1.0e 12
abs err = 1.0e 13
```
Appendix B

Dual Active Bridge Simulations

B.1 DAB Dynamics Simulation Model and Results

The dynamics simulation was used here to simulate the DAB converter before the reachability analysis. The dynamic model of DAB converter is shown as Fig B.1. And for the DAB subsystem part, it is shown as Fig B.2. The results of the dynamic model is given as Fig B.3 and the simulation coordinate of scope 1 to scope 4 are defined as following: 1) The voltage of the primary bridge (PB) side $V_p$. 2) The voltage of the secondary bridge (SB) side $V_s$. 3) The current through inductor $i_L$. 4) The output power $P_0$.

![Fig. B.1: The DAB Converter Model Simulation](image)

B.2 DAB Simscape Model and Results

The Simscape toolbox was used to simulate the DAB converter. The model of DAB is shown as Fig B.4. The results of the Simscape model is given as Fig B.5 and the simulation coordinate of scope 1 to scope 4 are defined as following: 1) The voltage of the primary bridge (PB) side $V_p$. 2) The voltage of the secondary bridge (SB) side to the primary side $V_s$. 3) The voltage of the secondary bridge (SB) side $V_2$. 4) The current through inductor $i_L$. 

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Fig. B.2: The DAB Converter Model Simulation (DAB subsystem)

Fig. B.3: The Dynamics Dual Active Bridge Converter Model Simulation Results
B.3 DAB Stateflow Model and Results

The Stateflow is a control logic tool used to model reactive systems via state machines and flow charts within a Simulink model. Stateflow is helpful for our SpaceEx mode design, so we also show the Stateflow model of DAB in Fig. B.6. The results of the Stateflow model is given as Fig. B.7 and the simulation coordinate of scope 1 to scope 3 are defined as following: 1) The current through inductor $i_L$. 2) The voltage across capacitor $v_C$. 3) The operation modes of the system.
Fig. B.5: The DAB Simscape Simulation Results
Fig. B.6: The DAB Stateflow Simulation
Fig. B.7: The DAB Stateflow Simulation Results