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Centering Error for Range Poles

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Centering Error for Range Poles

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ABSTRACT

A land surveyor makes accurate measurements using knowledge, physical dexterity, and properly calibrated equipment. All measurements contain errors, but the professional surveyor is mindful of the various sources and consciously works to minimize them based on experience and informed decision making. One such error source is the level vial, a standard component of numerous survey instruments and accessories. This paper derives expected miscentering error from a circular sprit level with an example using a global navigation satellite system (GNSS) range pole as commonly used for real-time kinematic positioning. We demonstrate that when the length of a 2-m, 40 min:2 mm, GNSS range pole is well known (standard deviation of range pole length σ_r = ±1mm) and the observer has a decent centering ability (standard deviation of vial centering ability σ_v = ±1mm), the most probable error magnitude for a single observation is 8 mm.

INTRODUCTION

A least squares adjustment of survey observations is not complete without including centering errors as a part of the error analysis. Centering error formulas in standard texts focus on total stations to provide the variance introduced into direction observations caused by miscentering and misleveling the instrument (Brinker and Minnick 1995; Ghilani and Wolf 2015). This paper derives the formula for the variance introduced into positions caused by miscentering a global navigation satellite system (GNSS) range pole and concludes with examples.

BACKGROUND

It is usually impossible to collocate survey instrumentation with control marks or physical features of interest. The next best option is to use an offset device, either a range pole or tripod, to hold a reflective target or instrument vertically over a point, oriented level with respect to the direction of gravity. Regardless of the mounting device, a circular "bulls-eye" level vial is the primary tool that surveyors use to achieve this alignment[1]. The nickname arises from the appearance of the round vial with a transparent top etched with one or more visible concentric rings. The vial is filled with a liquid (e.g., purified alcohol) save for a single air bubble, which seeks the highest point within the vial. The inside surface of the transparent top is spherical, curving uniformly downward out from the center. Circular vials are available in a range of materials, sizes, and sensitivity grades, and might be either placed against a range pole for the duration of the observation or integrated therein by the

[1] Circular levels are most commonly used in surveying despite the fact that tube vials can be manufactured to be more sensitive. For use in fieldwork, circular vials have the advantage of plumbing in all directions simultaneously, while a tube level works only along the axis with which it is aligned, which sacrifices efficiency. The principles for both types are the same (Ghilani and Wolf 2015, p. 83).

manufacturer. The object for a rod person is to plumb the range pole on the point, as indicated by the air bubble rising to the center of the bulls-eye.

LITERATURE REVIEW

Pollard (1947) said that "when making observations with instruments, the sense organs are to be regarded as part of the instrument itself. The instrument does nothing more nor less than aid the senses". Advising engineers, Pollard warns that human vision is imperfect and subject to spherical aberrations, spectral insensitivity, and fatigue. He cites Lord Rayleigh (1893) for describing a mechanical design which balances errors in gas pressure measurements as "coincidence errors" produced by identical methods of reading. Jackson (1987, p. 13) provided a table with several examples of level vial sensitivity ratings expressed using both the angular tilt/linear unit division ratio and the vial radius of curvature reporting methods. Jackson says that the sensitivity of a vial is correlated to the sensitivity of the instrument of which it is a component by design. Wolf and Ghilani (2010, p. 102) give that a careful setup of an instrument yields a position within 0.001-ft to 0.01-ft of the true station location. They go on to discuss the potential miscentering error (σ_r) of hand-held range poles with an example where estimated centering error $\sigma_v = \pm 0.01\,ft$. Brinker and Minnick (1995, p. 370) recommend that the rod have an array of three level vials angled 120° to each other. They stipulate 10′ sensitivity for the vials which are subject to "unavoidably harsh handling" and point out that reading three vials simultaneously provides an opportunity to detect the moment at which a vial goes out of adjustment. Having one vial on a rod means that the operator does not know if the rod is out of adjustment until recalibrating and, having only two vials means that upon noticing a bubble is out of alignment the operator is not sure which one needs adjustment. For recalibrating, they specify a simple procedure for collimation of a GNSS rod in which a plumb line is established in a doorway using a standard plumb bob. The point of the rod shall be placed at the intersection of the plumb line with the floor. Then the observer shall check the vial while rotating the rod 360°. McCormac, Sarasua, and Davis (2013, p. 98) acknowledge the curiosity of those studying surveying about how to estimate the amount of error that results from miscentering. They suggest conducting an experiment to deliberately miscenter the rod and compare the readings. They also state the common division spacing was once 1/10-in. before a 2-mm interval became standard. Speaking about care of equipment, they point out that wear and tear reduces the length of a level rod over time. Kowalczyk and Rapinski (2014) pronounced error sources applicable to reflectorless EDM measurements, which are relevant to evaluating the degradation of the horizontal distance observation introduced when using a surrogate to the mark (e.g. range pole).

VIAL SENSITIVITY

The spherical surfaces inside vial tops are precisely manufactured to have a given radial length. The radial length determines the vial’s sensitivity because, as the radial length gets longer, the bubble moves farther when the vial is tilted (Ghilani and Wolf 2015, p. 58). So, radial length is one way to report sensitivity, although it is hard to visualize the arc radius inside a tiny vial. For example, a typical circular vial used in surveying has a radial length between 0.15 and 5 m. Therefore, we consider the other, more common reporting method, the relationship between angular tilt and bubble movement.

NOT ALL LEVEL VIALS ARE CREATED EQUAL

Professional surveyors take care each time they plumb a range pole, while constantly keeping an eye on the level bubble. So why does it matter how sensitive the vial is as long as the bubble winds up in the middle? To answer the question, consider that excellent physical balance is required to traverse a 500-cm-long by 10-cm-wide gymnast's balance beam. But if the beam was widened to 100 cm, the task requires no extraordinary skill. This is analogous to level sensitivity, because a vial with short-radial length is easier to center than a vial with a longer radius. Put another way, a bubble centered with a short-radius arc requires more angular tilt to put it in motion and will not reflect subtle incline. Centering the bubble on a vial with a long-radius arc is a superior indicator of level orientation.

Level-vial sensitivity can be expressed in terms of the amount of angular tilt required to cause 1 unit division of bubble movement, and a 2-mm divisional spacing is customary per standard surveying texts (Ghilani and Wolf 2015, p. 85). The level vial sensitivity rating should be found printed on the level and/or on the manufacturer's specifications document. A vial housing might be inscribed "40-min", where the term "min" serves as an intuitive abbreviation for minutes, but this is a partial expression without a specification for a unit division. To remove uncertainty, the complete ratio "40 min:2 mm" should be used. For example, Figure 1 shows a cross-section of a circular vial with multiple concentric rings inscribed on the top of the vial. If this is a 40 min: 2 mm vial and each ring is 2 mm apart, then the angle subtended by the bubble as shown is 40 min, which implies that the range pole also has an angular tilt of 40 min.

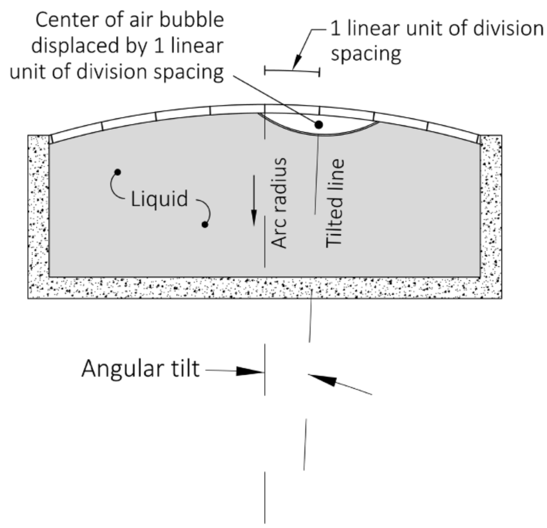

Figure 1. Cross-section of a circular vial showing the relationship between the displacement of the bubble and the angular tilt.

The definition of vial sensitivity does not reveal how miscentering translates into uncertainty of measurements. Entry-level surveyors handling a telescoping pole are taught an important lesson: the taller the range pole, the more the chance for error. For total station positioning, there is no target miscentering error when the angular and distance observations are not to a surrogate, such as a target or a prism, but rather to the point itself, as can happen when using a reflectorless electronic distance measurement (EDM). When a target is required, the least miscentering error likely occurs when the target is placed directly on top of the point. However, in practice this is rarely the case, and a target is usually placed atop a range pole or tripod and tribrach, which makes it a surrogate for the point of interest. Increasing the offset (vertical) distance between the point and a surrogate increases the uncertainty of the observations. Figure 2 depicts a scenario in which a 2-m GNSS range pole is miscentered over a point. The range pole has a built-in circular level with a sensitivity rating of

40 min: 2 mm. The fundamental formula $d = r\,\theta$ relates an angle to the arc length it subtends between two radial lines, where d is arc length, r is radius of curvature, and θ is the angle in radians[2] (Meyer 2010, p. 11). If an observation is taken with a bubble miscentered by 2 mm, we know that the pole has tilted by 40′, based on its sensitivity specification. Converting 40′ to radians gives (to four significant digits):

$$40'\left(\frac{1^\circ}{60'}\right)\frac{\pi}{180^\circ} = 0.01164\,\frac{\mathrm{m}}{\mathrm{m}}$$

Substituting 2 m = 2000 mm for the radial length, the top of the pole has moved in an arc having length:

$$0.01164\,\frac{\mathrm{m}}{\mathrm{m}}(2000\ \mathrm{mm}) = 23\ \mathrm{mm}$$

[2] The existence of the radian as a unit arises from nature because it is the ratio between a circle's radius and its circumference. These are both linear so their ratio is often taken to be unitless. However, for analytical purposes, preserving radians as arclength/radial length (m/m) can be better because that makes it easy to keep the variables ordered properly and end up with the correct units.

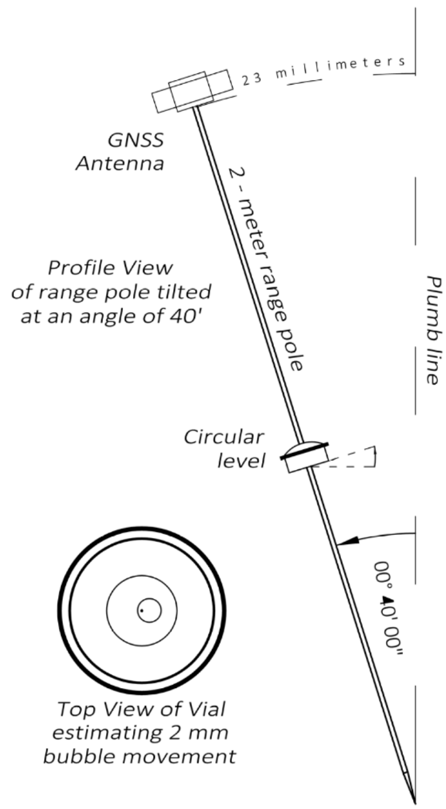

Figure 2. Profile and top views of a miscentered GNSS range pole. The graphic is not to scale because, were the graphic presented at a true scale and not exaggerated, a 40′ variation in plumbness would appear as two coinciding straight lines. In the top view of the vial, the tiny dot in the center has been added to indicate the location of the center of the vial, which is also the location of a properly positioned bubble. As is common, the radial lengths of the etched rings do not correspond to sensitivity rating division spacing.

Because professional surveyors take care when setting up the range pole, we can assume that a miscentered bubble by 2 mm would be caught and corrected. But, 2 mm is already a very small amount of movement (typically less than a bubble width), only discernible to a human eye at close range. Detecting submillimeter bubble movement can be affected by vial type and size, liquid characteristics, temperature, and whether the nature of the setup allows the observer to have a good viewpoint[3]. Probably, the question of whether or not unit divisions (i.e., 2-mm spacing) are etched on the top of the vial is a factor, because it affects ease of interpolation. The minimum amount of movement detectable by the human eye is beyond the scope of this note, but the discussion raises the question of how much variability remains after best centering effort under normal conditions.

ERROR MODEL

The miscentering distance for a range pole d_{rp} is given by $d_{rp} = r\theta$. The reader will notice that $d_{rp} = r\theta$ implies an instrument atop a range pole and not atop a tribrach on a tripod. Also, d_{rp} is a measure of arc length but is treated as straight-line horizontal displacement given the negligible difference between arc length and chord length as applied herein. Both *r* and θ are random variables: *r* is the measurement of the instrument (range pole) height above the mark, and θ comes from misleveling the range pole. Assuming *r* and θ are statistically independent, propagation of variance gives the standard deviation of d_{rp} to be

[3] The observer must look straight down at the vial to plumb properly. Inability to get into position due to site-specific challenges or failure to carefully align one's eye produces another layer of interpolation: guessing where the bubble is based on an oblique sight. The design of many GNSS range poles places the vial so close to the pole that it is impossible to look straight down at the level.

$$\sigma_{d_{rp}} = \sqrt{\left(\frac{\partial d_{rp}}{\partial r}\sigma_r\right)^2 + \left(\frac{\partial d_{rp}}{\partial \theta}\sigma_\theta\right)^2}$$

Given that $\frac{\partial d_{rp}}{\partial r} = \theta$ and $\frac{\partial d_{rp}}{\partial \theta} = r$

$$\sigma_{d_{rp}} = \sqrt{(\theta\sigma_r)^2 + (r\sigma_\theta)^2} \qquad \text{(Eq. 1)}$$

with θ and σ_θ in radians (m/m), and r and σ_r in meters (Ghilani 2010, p. 87). We assume that *r* follows a normal distribution $r \approx N(r_{\text{true}}, \sigma_r)$. We suggest 0.001 m could be a realistic value for σ_r, the standard deviation for the measurement of range-pole height. For example, under typical conditions, $r = 2$ m and $\sigma_r = \pm 0.001$ m.

For θ and σ_θ we propose that, assuming an unbiased eye and excepting for deficient equipment, miscentering errors disperse in all directions equiprobably from the center of the vial. Therefore, it is reasonable to examine this variability in terms of linear departure from the center. Further we suppose that, although the bubble can occupy any position on the surface of the vial, it is not likely that a deliberately centered bubble will stray far from the center. We modeled the bubble's departure from the center with two independently and identically distributed normal random variables $dx, dy \approx N(0, \sigma_v{}^2)$ that provide departures from the center in two orthogonal directions. The mean values of zero implies that the center of the vial is the probable place for the bubble to be in either the x or y direction independent of the other. The standard deviation σ_v depends on the vial sensitivity; supposing $\sigma_v = 20'$ implies that the bubble is typically (≈68%) within $d_v = \pm 1$ mm of the center. The *x*- and *y*-departures combine to form the linear departure from the center per Pythagoras's formula as

$$d_v = \sqrt{dx^2 + dy^2} \quad \text{(Eq. 2)}$$

Because d_v is the sum of (exactly) two identically distributed normal random variables, d_v follows a Rayleigh distribution[4]. The mean, mode, and standard deviation of the Rayleigh distribution are

$$\mu_R = \sigma\sqrt{\pi/2}$$

$$mode_R = \sigma$$

$$\sigma_R = \sigma\sqrt{(4-\pi)/2} \quad \text{(Eq. 3)}$$

where the subscript R indicates the Rayleigh distribution, σ the standard deviation of the normal distribution of the underlying variates; in this case those variates are *dx* and *dy* and σ and σ_v. Figure 3 shows a blue curve plot of the Rayleigh distribution's probability density function (PDF) for $\sigma = 1$. The PDF is nonnegative, which should be intuitive because distances cannot be negative. However, the most probable value (the mode) is not zero in spite of the fact that underlying variates both have zero means. In the same way rolling a 2 or a 12 on two six-sided dice can happen in only one way[5], which makes 2 and 12 the lowest probability outcomes, $d_{rp} = 0$ requires both *dx* and *dy* to simultaneously be zero, which has vanishing small probability (in fact, zero probability in the limit).

The histogram in Figure 3 comes from 100,000 trials of generating two random variates from a normal distribution, squaring them, summing the squares, and taking the square root. The histogram's bins fit the predicted curve extremely well.

[4] For the special case of $\sigma^2 = 1$, the Rayleigh distribution is also called a chi distribution.
[5] both ones or both sixes

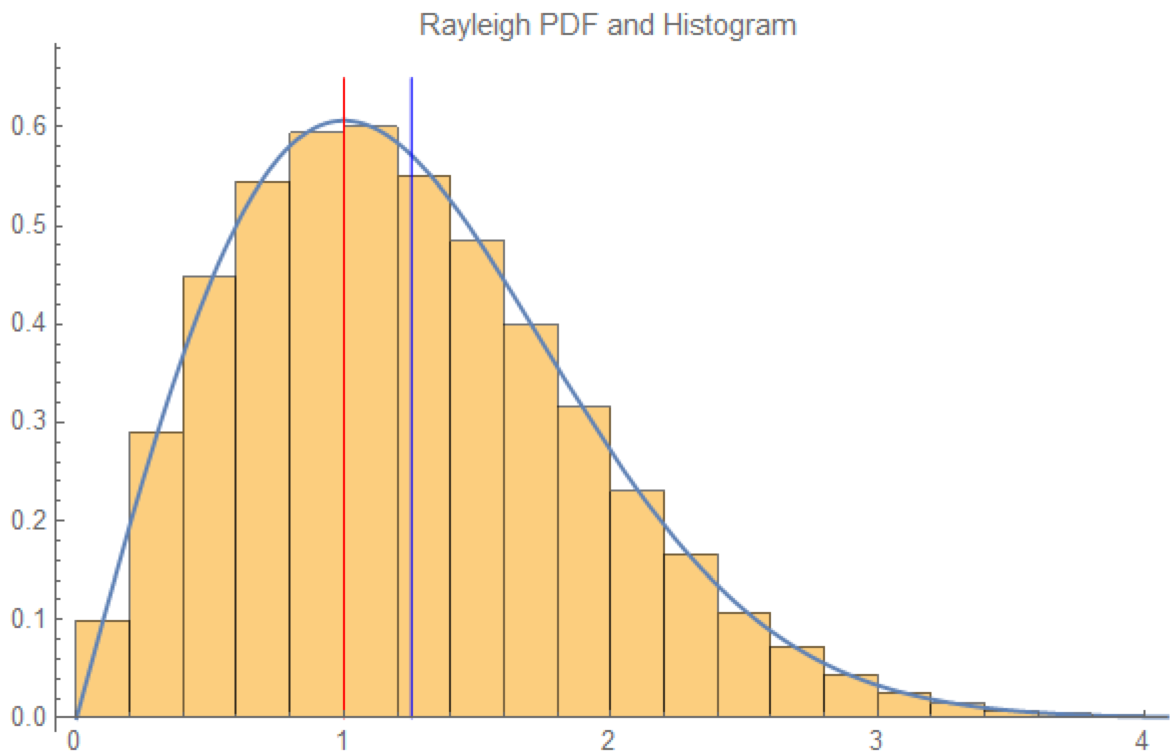

Figure 3. The blue curve is a plot of the Rayleigh distribution's PDF for $\sigma = 1$. The histogram is the outcome of a mathematical experiment that generated 100,000 random numbers using (Eq. 2). The vertical red line indicates the mode, and the vertical blue line indicates the mean.

The most probable value of any distribution is its mode, and the mode of a Rayleigh distribution is equal to σ. For this problem, the most probable value of θ is the mode of the Rayleigh distribution, which equals σ_v. So, by (Eq. 3),

$$\sigma_\theta = \sigma_v \sqrt{(4-\pi)/2}$$

Substituting the values of $\theta, \sigma_r, r,$ and σ_θ into (Eq. 1) gives our primary result:

$$\sigma_{d_{rp}} = \sqrt{(\theta\sigma_r)^2 + (r\sigma_\theta)^2} \qquad \text{(Eq. 4)}$$

EXAMPLE FOR RTK SURVEY

Utilizing the vial depicted in Figure 2, we now stipulate that the observer's ability to accurately center the bubble in the vial is $\sigma = 0.001$ m. Given the vial sensitivity (40 min:2 mm), this means that the observer is usually able to plumb the range pole within 20 min of a vertical line, and thus $\sigma_v = 0.00582$ radians to three significant digits. Then (Eq. 3) yields

$$\sigma_\theta = \sigma_v\sqrt{(4-\pi)/2} = 0.00582(0.65514) = 0.00381 \text{ radians}$$

Then, using the mode for $\theta = \sigma = 0.00582$, $r = 2$ m, and $\sigma_r = 0.001$ m,

$$\sigma_{d_{rp}} = \sqrt{(\theta\sigma_r)^2 + (r\sigma_\theta)^2} = \sqrt{(0.00582\times0.001)^2 + (2\times0.00381)^2} \cong \pm0.008 \text{ m}$$

The above calculations quantify expected miscentering for a single observation, which includes both systematic and random error(s). One contribution to systematic error is the bias from the axis of the vial not being parallel to the pole. Range poles are often subjected to torsion (bending) forces, both in transport and in use, and most surveyors do not recalibrate poles on a daily basis. There is no reason to expect misalignment in any particular direction, but the net effect of any force that causes the pole to deform creates a (mostly) horizontally dispersed observational error. Also, the exact, true center of a vial is not exactly indicated by the (usually non-existent) central dot (Figure 2), which causes error equivalent to collimation error as with a reticle in a telescope. Despite best efforts, it is possible (and in the limit, certain) that the vial has been manufactured such that the apex of the sphere will not be collinear with the center of the circular vial.

REDUNDANT OBSERVATIONS PRODUCE GREATER ACCURACY

If the satirical online news magazine *The Onion* were to feature an article on the geomatics profession, perhaps the headline would read "Check Shots Definitively Proven to be Waste of Time". But, satire is not reality, and the unwavering willingness to employ a procedure that efficiently improves accuracy is instilled in a surveyor early in his career. A large number of independent observations is best, whereas in practice, a decision-maker is confronted with a mandate to minimize crew time and maximize productivity. Therefore, we attempt to maximize the potential improvement from taking only one redundant observation.

For higher-precision work, range poles with fixed-mount circular vials are used. The cumulative-error vector produced by pole and vial defects does not change temporally. Vector algebra dictates that errors of equal magnitude and opposite direction cancel, so to optimize the marginal benefit of the redundant observation, the pole should be rotated 180° between shots. Controlling the rotation angle in the field by eye is difficult, but an experienced observer can get close. Figure 4 demonstrates how the inherent equipment bias in a single shot can be effectively canceled by taking a second shot with the pole rotated half circle and is minimized even when the rotation is marginally less than optimal.

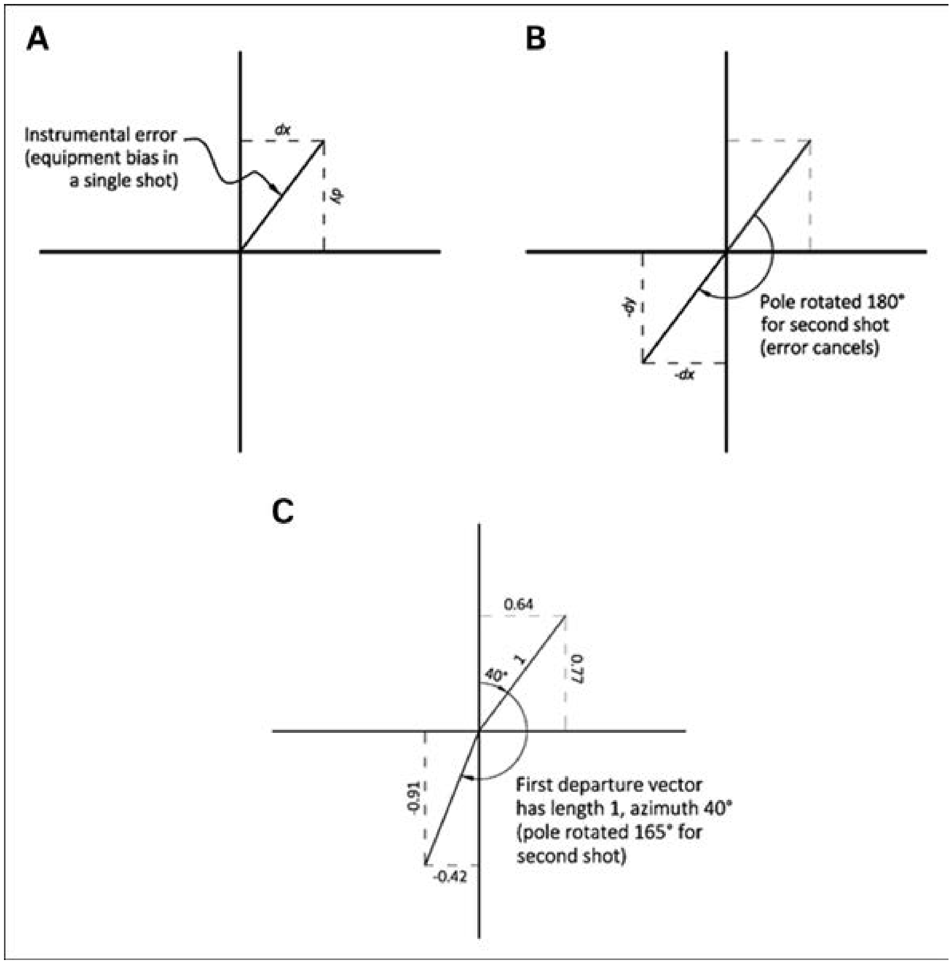

Figure 4. The two plots (A, B) compare systematic error taking instrumental bias of the range pole and circular vial as cumulative, respectively. (C) The plot depicts the worst-case error negation if the observer can rotate the range pole to within 15° of 180° for the redundant observation.

VALIDATING THE METHOD

We set three temporary magnetic nails in a flat asphalt parking lot separated by less than 20 m for the experiment. Over one, we set a round canister prism on a Seco fixed-height GNSS tripod with a 2-m center range pole. On another, we set up a Leica TCR 307 total station and used its EDM function to obtain a series of horizontal distance observations to the prism while purposefully varying the orientation and plumbness of the range pole. Figure 5 depicts the plan view of the experiment area with an aerial image in the background for reference. Table 1 holds a list of the attempted configurations and resulting distance errors. Here, “error” is the deviation (mm) from plumb, defined as the difference between the “centered” observations and the “leaned” observations.

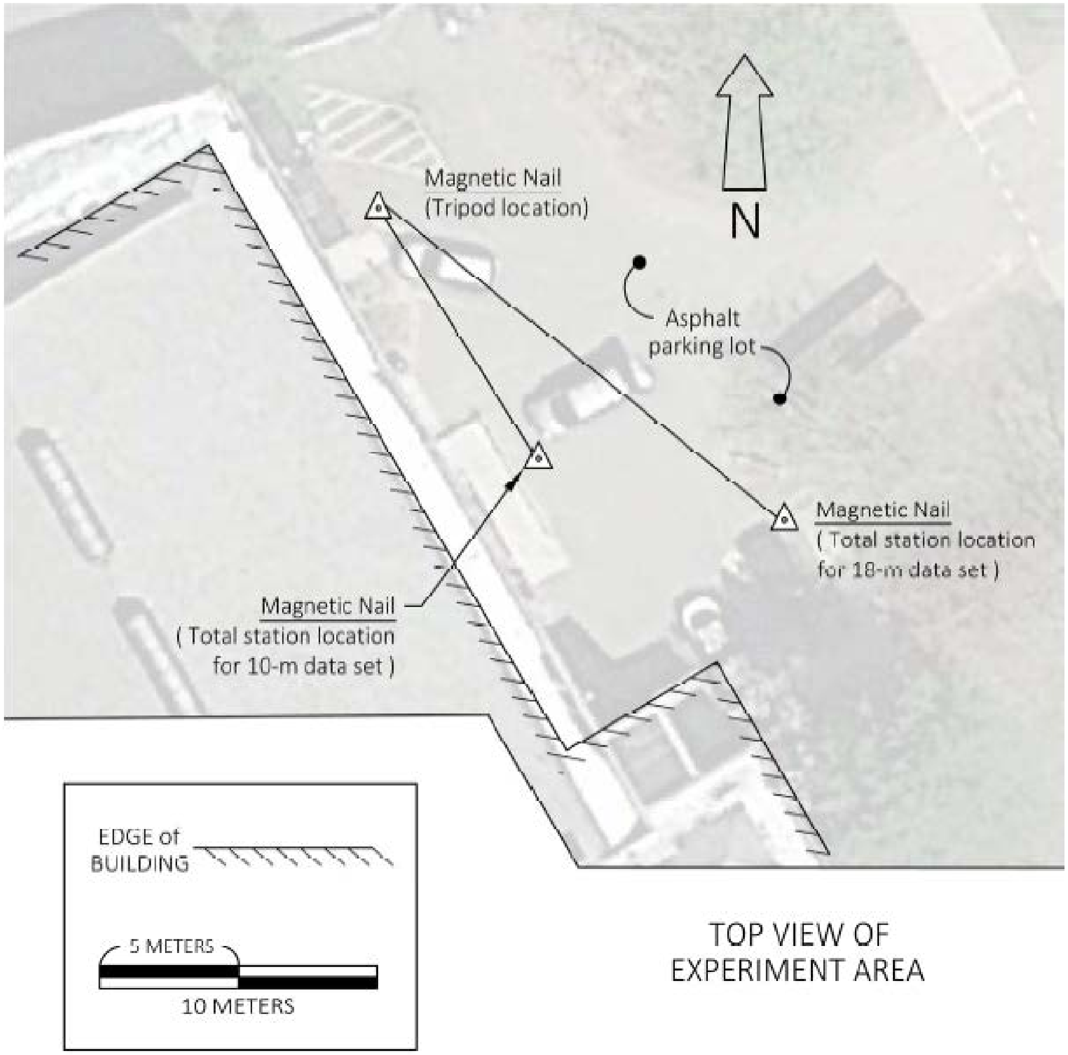

Figure 5. A diagram of the experiment area depicting the approximate layout of the three magnetic nails in a small asphalt parking lot.

Range Pole Orientation	**Bubble Position**	**HD by EDM (meters)**	**Error (millimeters)**
North	Centered	10.000	N/A
North	Leaned away	10.006	6
South	Leaned away	10.006	6
South	Centered	9.999	N/A
North	Leaned toward	9.997	2
East	Centered	10.000	N/A
East	Leaned away	10.006	6
East	Leaned toward	9.996	4
West	Leaned toward	9.992	5
West	Leaned away	10.003	6
West	Centered	9.997	N/A
North	Centered	18.015	N/A
North	Leaned away	18.022	7
North	Leaned toward	18.010	5
South	Centered	18.013	N/A
South	Leaned toward	18.008	5
South	Leaned away	18.020	7
East	Centered	18.011	N/A
East	Leaned toward	18.005	6
East	Leaned away	18.016	5
West	Centered	18.016	N/A
West	Leaned toward	18.012	4
West	Leaned away	18.024	8

Table 1. A list of horizontal distance observations taken while purposefully varying the orientation and plumbness of a range pole. The N/A value is used for the centered positions because no error is expected. The “South-Leaned toward” observation was inadvertently omitted from the 10-m data set.

In Table 1, range pole orientation comes from the built-in analog compass on the center range pole. The bubble position column is based on visual determinations, alternating between centering the bubble and leaning the range pole such that the center of the bubble is on the edge of the lone etched ring. The ring has a diameter of 1 cm, so in either the "leaned away" or "leaned toward" position, the bubble moves by 5 mm. Horizontal distance is calculated from the trigonometric relationship between an observed zenith angle and slope distance and therefore contains error from both sources. However, the fixed-height tripod kept the reflector height stable throughout the experiment so zenith angle variability was negligible by design.

The manufacturer's specifications[6] report the sensitivity of the built-in level vial on the Seco GNSS tripod is 8′ (fully expressed as 8 min: 2 mm as discussed above). Given that relationship, we expect that when the bubble is located 5 mm from the vial center, the range pole is tilted by $20' \; (20'\!: 5\ \mathrm{mm}/2\ \mathrm{mm})8' = 20'$. So, following the example above, we convert first to decimal degrees and then to radians (given to three significant digits):

$$20'\left(\frac{1^\circ}{60'}\right)\frac{\pi}{180^\circ} = 0.00582\,\frac{\mathrm{m}}{\mathrm{m}}$$

$$\sigma_\theta = \sigma_v\sqrt{(4-\pi)/2} = \pm 0.00582(0.65514) = \pm 0.00381\ \text{radians}$$

The reflector atop the fixed-height tripod was mounted on a prism stand so the actual reflector height was higher (2.12 m). When quantifying horizontal distance in this way, reflector height variability in the order of 1 – 2 decimeters does not change the result reported at the millimeter level. With mode $\theta = \sigma_v = 0.00582$ m/m, $r = 2.12$ m, and $\sigma_r = 0.001$ m,

[6] https://www.surveying.com/en/products/gnss-surveying-accesories.html?p=8 Accessed April 4, 2016

$$\sigma_{d_{rp}} = \sqrt{(\theta\sigma_r)^2 + (r\sigma_\theta)^2} = \sqrt{(0.00582\times0.001)^2 + (2.12\times0.00381\,)^2} \cong \pm0.008 \text{ m}$$

The errors in Table 1 are the differences between the "Leaned" distance observations and the respective "Centered" observation for the orientation. The sample mean error value of 0.0055 m is lower than the expected $\pm$0.008 m, even when the standard deviation is factored in. However, some of the apparent shortage can be attributed to (unintended) lateral range pole movement. Although we attempted to lean the range pole such that the motion would be entirely in the observed horizontal distance, it was impossible to eliminate this error source entirely. As a result, some portion of the bubble movement must be attributed to leaning the range pole in a lateral direction as opposed to the strictly forward-backward manipulation, which was intended. The question becomes: How much lateral movement do we anticipate? We propose that the observer was able to lean the range pole to within 15° of the line to the total station 95% of the time, which implies a standard deviation of 7.5°. Given this assumption, which is graphically depicted in Figure 6 below,

$$\varepsilon = \sigma_d \times \sin\theta = 0.008\,\times sin(7.5°) \cong \pm0.001 \text{ m}$$

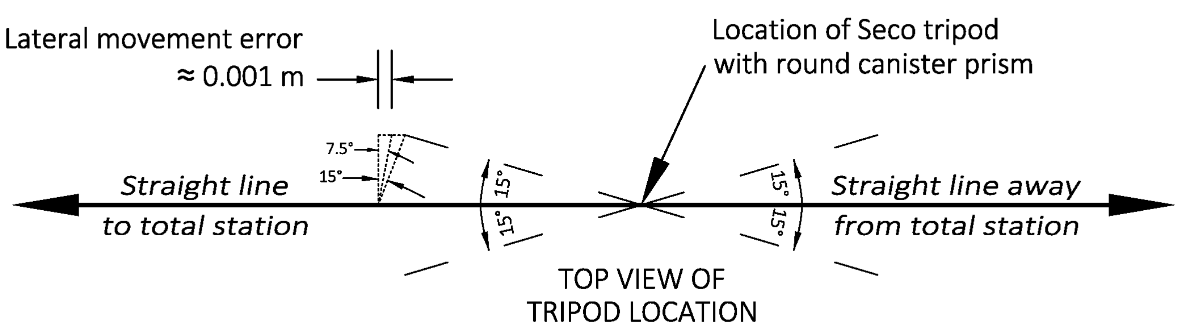

Figure 6. A top view of the tripod location (indicated by the apex of the dashed lines) referenced to the straight line to the total station. This figure depicts the lateral

movement error for an EDM observation in which the range pole was tilted by 7.5 degrees to the observer's right of the straight line to the total station.

Adding the lateral error term ε to the sample mean $\bar{x}$ gives very nearly the expected miscentering error of 8 mm.

CONCLUSIONS

Miscentering error is relevant because GNSS accuracy in the subcentimeter range is now routinely touted by equipment salesmen, specified in construction documents, and expected of surveyors. We have shown that when the length of a 2-m, 40 min: 2 mm GNSS range pole is well known ($\sigma_r = 1$mm) and the observer has a decent centering ability ($\sigma_v = 1$mm), the most probable error magnitude for a single observation is 8 mm (see RTK calculation example above). We point out that we have used 2 m for the height of the range pole, but miscentering error would need to be computed at the center of the instrument or target, which is often even 0.2 m above the top of the range pole. In practice, the variable r is not just the height of the range pole, but the height of the range pole plus the instrument/target.

In GNSS surveying, a miscentered range pole produces a solution located somewhere "else". For use in plane surveying, GNSS-derived positions are transformed from an earth-centered, earth-fixed (ECEF) system into local north, east, and up coordinates (Teunissen and Kleusberg 1998, p. 4). It is important to recognize that the impact of miscentering is complicated by the relationship between the ECEF and local systems in that vectors relative to individual axes transform in unintuitive directions.

Deriving this error's contribution in an ECEF coordinate system is beyond the scope of this paper, but the error-transformation equation can be found in Soler et al. (2011), Equation (22).

Miscentering error could be minimized through improvements to vial sensitivity, range pole height stability, and/or the observer's visual acuity. Depending on the (unexamined) potential to make meaningful progress on these fronts, a more effective way to minimize error for GNSS surveys is to take a redundant observation after rotating the pole by 180°.

The value computed using Equation (4) is dominated by range pole height r and the expected miscentering standard deviation σ_θ; in fact, the other two terms θ and σ_r are negligible for 2-m fixed height tripods and range poles. The mathematics confirm that taller rods and less accurate vials lead to greater miscentering errors.

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