

Spring 5-1-2020

Evaluating the Cost of a Lapse in Life Insurance and its Implications on Developing a Policyholder Retention Strategy for a Company

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**Evaluating the Cost of a Lapse in Life Insurance
and its Implications on Developing a Policyholder
Retention Strategy for a Company**

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Spring 2020

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Abstract

In the insurance industry, many companies focus on policyholder retention as one of their key tools to retain premiums collected from customers. Customers may lapse after their purchases. Insurance companies will not receive premiums and have to pay out surrender benefits which makes it costly for an insurer. Indeed, understanding the cost of a lapse is important to retain policyholders. This paper focuses on the cost of a lapse in life insurance, and its implications on developing policyholder retention strategies. The first part of the paper summarizes the general background of life insurance, lapses, and conservation strategies. The second part introduces the modeling and simulations of three types of life insurance policies. The economic gain of a life insurance policy is defined as the accumulated value (AV) of past premiums plus the present value (PV) of future premiums until death/lapse less the PV of future benefits and less the AV of acquisition costs at issue. Next, the paper proceeds to analyze the cost of a lapse of a policy and quantify it as the difference between the economic gain of a policy at 0% lapse rate and that at 10% lapse rate. Based on the cost of a lapse, which is the same as the gain from conservation strategies, the insurance company will be able to rank its policies and prioritize which policies to focus on. Recommendations on developing conservation strategies to retain policyholders are discussed in the following section from both an actuarial perspective and a business perspective.

1. Introduction

1.1 Life Insurance

Individuals, companies, and organizations face different risks every day. Most of the time, risks arise due to the uncertainties of multiple outcomes that may happen. For example, when a person falls sick, he may recover, be critically ill, or even die. As one of the main purposes of insurance is to provide protection to its beneficiaries or policyholders, there is an important relationship between risk and insurance. According to the 2019 Insurance Fact Book, 52% of the Net Premiums written in the U.S insurance industry in 2017 comes from Life and Health Insurance policies (The III 5), which indicates the importance of life insurance to people's lives. Moreover, the total life insurance premiums amounted to USD 2.6 trillion, which is 3.5% of the global GDP in 2016 (KPMG 6). From the insurer's perspective, it is also valuable to investigate how life insurance can bring in more benefits to the company. Therefore, the main focus of this paper is going to be life insurance.

Insurance is an agreement between two parties - one is the insurer or the insurance company, and the other is the policyholder or his designated beneficiary. The policyholder pays a stipulated payment called premium to the insurance company, in exchange for a defined amount of money upon the occurrence of any specific loss (Anderson & Brown 3). Specifically in life insurance, the defined amount of money is usually a surrender charge when someone lapses, or a death benefit to the beneficiary of a policyholder when death occurs.

Two of the most common types of life insurance policies are used for the modeling in the paper. One of them is term life insurance, with ranges of term length between 1 year and 30 years. The premiums of term insurance are usually the lowest amongst all types of life insurance, but it will increase with the age of the policyholder if the term policy is renewed (Smith & Hayhoe 1).

During the term length, the policyholder may voluntarily choose to lapse by stopping the payment of premiums, and the insurer has to pay out a surrender charge. At the end of the term, if the policyholder survives and the policy matures, he has the option to end the policy or to extend/renew it. If the policyholder dies, the insurer will pay out death benefits to the beneficiaries of the policyholder.

Whole life insurance is the other one used for the modeling. It works similarly to the Term Life, but whole life policies will pay out the death benefit regardless of when the policyholder dies as the term length of whole life policies is lifelong. The premiums of whole life insurance are much higher than term life insurance as it is a lifetime protection.

1.2 Lapses

With the tumultuous changes in the economy and technology, some people may disagree in regards to investing in life insurance, because of its expensive premiums and the existence of other financial priorities with faster rewards. Policyholders may also assume that they are healthy and life insurance is not necessary, after continuously paying premiums for a few years. Lapses happen when policyholders choose to discontinue their policies with insurance companies, impacting the economic gains of insurers.

When a lapse occurs, the policyholder drops out of the insurance policy and stops paying regular premiums. The insurance company needs to pay out a surrender charge, which is close to the reserve at this point. A higher lapse rate means fewer people renew policies, which may adversely affect the profitability of the insurance company. People who usually think that they are healthy choose to lapse their policies, leaving the group of policyholders who are not confident in their health situation with insurance companies. The mortality rate of the remaining group could

go up compared to the original group, and there is a higher probability that insurers will pay out death benefits. Nevertheless, if the policyholder is holding a term life insurance policy and he is at the later part of the term length, then it might be better for this policyholder to lapse. There is a higher chance that policyholders may accidentally pass away. Paying out death benefits at such a late stage is detrimental to the insurer, and lapses would be preferred in this particular case. Hence, retaining policyholders and managing the lapse rate has to be strategically managed, and is one of the most crucial goals for insurance companies to earn profits in the long run.

In theory, there is no clear definition or formula about what the cost of a lapse is. It is also hard to quantify the cost of a lapse, because it depends on when the lapse happens during the term length, the type of insurance policy and how this policy is priced. In this paper, the cost of a lapse is viewed the same as gain from conservation strategies to retain policyholders, and it is defined as the difference between the economic gain at 0% lapse rate and the economic gain at the policy's underlying lapse rate assumed to be 10% in the modeling of this paper.

1.3 Terminologies

1) Duration

The duration of a life insurance policy is the maximum number of years over which the policy coverage will continue, provided that the policyholder is alive and the policy is active.

2) Premiums

Premiums are the payments made by the policyholders to their insurance companies regularly. In general, the three most common types of premiums are level premiums, yearly renewable term premiums, and decreasing term premiums.

3) Acquisition Costs

The acquisition costs are the direct costs an insurer incurs when acquiring a new policy with incoming premiums.

4) Economic Gain

The economic gain of a life insurance policy is defined as the accumulated value of past premiums plus the present value of future premiums until death/lapse less the present value of future benefits and less the accumulated value of acquisition costs at issue.

5) Cost of a Lapse

In this paper, the cost of a lapse is essentially the same as the gain from implementing conservation strategies to retain policyholders, and it is defined as the difference between the economic gain at 0% lapse rate and the economic gain at an assumed underlying 10% lapse rate.

6) Conservation Strategies

In this paper, the conservation strategies for life insurance is the same as the policyholder retention strategies that the insurance company develops to control its cost of lapses.

2. Conservation Strategies

2.1 History of Conservation Activities in Insurance

The cost is paid for at first by the insurance company, and the company makes a profit because of the fact that the savings from conservation are larger than the cost. These savings through rate adjustment are passed on to the insured, and at the same time, the expense is gradually passed on to the insured. Eventually, there comes a time through the operation of the law of diminishing returns when further conservational activity will not pay for itself and further activity can therefore not be undertaken except for reasons other than economics (Whitney 237).

2.2 Conservation Strategies in Life Insurance

As a protection to human lives or physical properties, insurance usually reduces the negative impacts of unfortunate events. However, insurance does not prevent misfortunes from happening, and in many cases, the costs of these misfortunes can be detrimental to a company.

For a life insurance policy, the insurance company will conduct examinations and assess risks associated with the policyholder to decide if it will accept this policyholder. After that, the policyholder pays premiums regularly, and the only uncertainty left in this policy is how long he can live. If the policyholder survives beyond the term length of the insurance, the insurance company will make money due to the premiums collected and the interest earned on the premiums over time. However, if the person did not make it to the end of the term, the insurance company has to pay out benefits due to death or lapses. While it is hard to foresee how many people are going to drop their policies in the future, there are conservation strategies that insurance companies can implement to help them better estimate the lapse rate levels so as to achieve cost efficiency and better profits.

In this paper, the focus of the modeling is to quantify the cost of a lapse and to demonstrate the importance for insurance companies to manage the lapses in different situations. Conservation strategies will be provided in a later section with both an actuarial perspective and a business perspective, bringing a more comprehensive business acumen to the insurance company.

3. Modeling

The ultimate goal of this model is to evaluate the cost of lapses and how it can help companies implement conservation strategies to retain their policyholders. There are three situations that can occur to a policyholder:

1. The policyholder survives throughout the whole term, and the policy matures. All premiums collected are the gains to the insurance company.
2. The policyholder decides to stop paying premiums to the insurance company and lapses the term insurance policy.
3. The policyholder dies during the term period. The insurance company pays out the Death Benefit (Face Amount) to the beneficiary of the policyholder.

In total, 20,000 scenarios are simulated by using Microsoft Excel. Each scenario is represented by an index number from 1 to 20,000, and it will be assigned with one of the status among “alive”, “lapse” or “death”. The status is assigned based on the testing lapse rate and mortality rate from the VBT table. For example, with a 10% lapse rate and a mortality rate of 0.03% for the first year, there should be 2,000 scenarios that lapse, 6 scenarios that die, and the rest 17,994 scenarios stay alive in the first year. If the index number is smaller or equal to 17,994, the policy will be an “alive”. If the index number falls between 17,994 and 19,994 (inclusive), the policy will be a “lapse”. Otherwise, a “death” will be assigned to the policy. If the policy is not “lapse” or “death”, it will move on to the next year and use a 10% lapse rate and next year’s mortality rate.

Assumptions

1. Use the select-and-ultimate mortality table - 2018 Valuation Basic Table (VBT).
2. Use 20,000 scenarios to stabilize policy status.

3. Interest rate is 3%.
4. Expense is assumed to be 0%.
5. The gross premium is 1.2 times the net premium.
6. The acquisition cost is 2 times the gross premium in the first year.
7. The surrender charge is assumed to be the reserve calculated at the time when lapse occurs.
8. The death benefit is \$100,000.

Tested Life Insurance Products

There are three types of life insurance policies tested, and they are listed as following:

Total Term Length & Products	Duration
25-year Term Life Insurance	0 (New Issue)
	5
	10
	15
15-year Term Life Insurance	0 (New Issue)
	5
	10
	15
Whole Life Insurance	0 (New Issue)
	5
	10
	15

Calculation of the Economic Gain

All cash flows are evaluated at the beginning of the term length for each policy. The overall economic gain is the average of economic gains of 20,000 scenarios. All premiums are treated as cash inflows. The death benefit paid at death, surrender charge paid at lapse, and the acquisition costs at the beginning are treated as cash outflows. The overall economic gain equals to:

PV (Future Premiums) + AV (Past Premiums)

- AV (One-Time Acquisition Costs at the beginning)

- PV (Surrender Charge, assumed as the reserve at that specific time when lapses)

- PV (Death Benefit when death happens)

Methodology

1. Set the lapse rate as 10%.
2. Compare the average present value of economic gain through all scenarios.
3. Set the lapse rate to 0% which means no lapse. Calculate the average present value of economic gain through all scenarios.
4. Calculated the gain from conservation strategies as the difference between the economic gains in (2) and (3).

Quantify the Cost of a Lapse

This paper analyzes the cost of a lapse, i.e. the gain from conservation, as:

Gain from conservation (cost of a lapse) = Economic gain at 0% lapse rate

– Economic gain at 10% lapse rate

Furthermore, how the cost of a lapse/gain from conservation strategies changes with variations in policyholder duration, term length, and lapse rate will be analyzed. These analyses will help to demonstrate the relationship between the cost of a lapse and those changing factors, and contribute to how conservation strategies will be developed in the best interest of the company.

4. Analysis

Economic Gain by Duration

The economic gain of a policy increases as the duration increases for all three types of life insurance policies tested. During earlier durations, the economic gain at 0% lapse rate is always higher than the economic gain with 10% lapse rate. However, after a certain duration, the economic gain with no lapse will become smaller than the economic gain at 10% lapse rate until the end of duration. The trend of economic gains at 0% and 10% lapse rate by duration explains why there is a positive gain from conservation at earlier durations, and decreases to negative values later on.

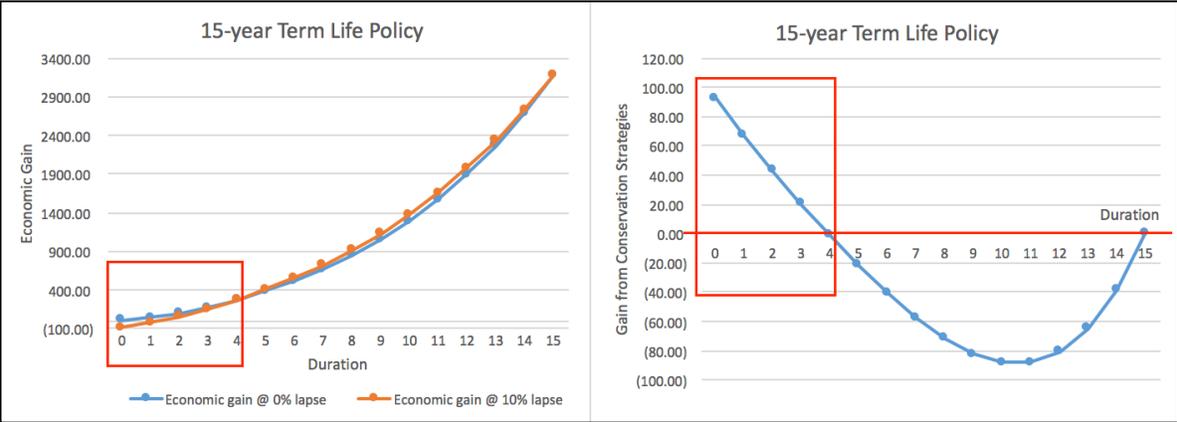


Table 1: Economic Gain at 0% and 10% Lapse Rates by Duration for a 15-Year Term Life Policy

For example, the economic gain at 0% lapse is larger than the economic gain at 10% lapse rate from duration 0 to duration 4. After duration 5, the economic gain at 0% lapse rate becomes less than that at 10% lapse rate, which leads to a negative gain from conservation strategies. This is also observed in both the 25-year term life policy and the whole life policy.

Gain from Conservation Strategies by Duration

1) 15-Year & 25-Year Term Life Policy

The gain from conservation strategies of the policy at different durations reflects how much costs a lapse is to an insurer. The graphs of 15-Year and 25-Year Term Life

Policy exhibit a similar trend. The gain from conservation strategies decreases gradually from positive to negative values, and increases back to 0 at the end.

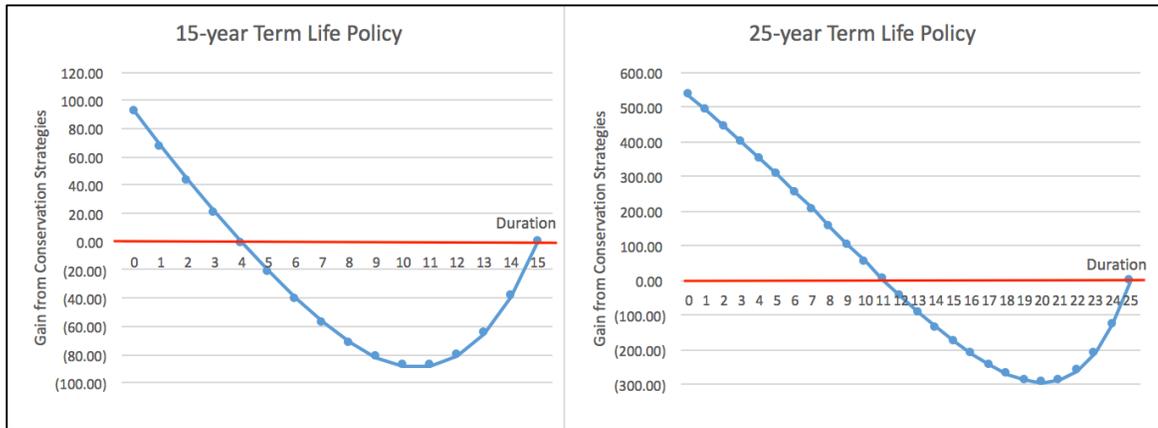


Table 2: Gain from Conservation Strategies by Duration for 15-Year & 25-Year Term Life Policies

This shows that the longer the duration, the less the gain is obtained from conservation strategies and controlling the lapse rate. Moreover, there are only positive gains from conservation strategies at the early durations. Evidently, it is more beneficial to control lapse rates at earlier durations.

The gain from conservation strategies increases after reaching its minimum value. Even though the economic gain at 10% lapse rate is larger when the gain from conservation strategies becomes negative, the increasing rate of the economic gain at 0% lapse rate accelerates towards the end. Eventually, the economic gains at 0% and 10% lapse rate become the same. Hence, controlling the lapse rate at a later stage is not necessary, because economic gains do not differ much whether lapses are present or not.

2) Whole Life Policy

In general, the whole life policy exhibits a similar trend as what is observed in the 15-Year and 25-Year Term Life Policy. The 33-year period of positive gain from conservation strategies is much longer than term insurance policies. This means that

controlling the lapse rate during these 33 years will be more profitable to the company. Between the duration 40 and duration 60, the gain from conservation strategies remains almost steady. A sharp decrease is observed after the 60-year duration. Since the issue age of the modeling policy is at age 45, and the mortality table becomes artificial after 60 years, i.e. at age 105, it is possible that there is a sudden drastic change.

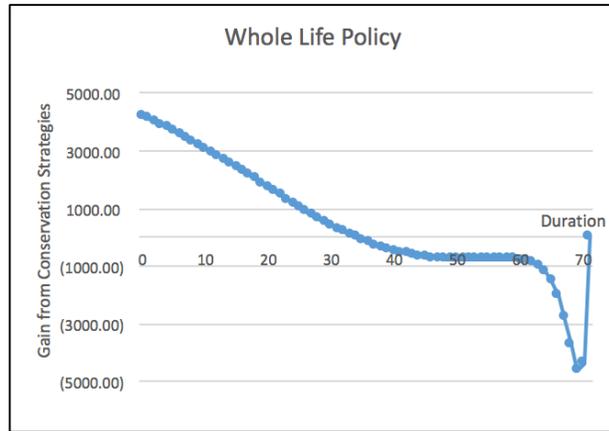


Table 3: Gain from Conservation Strategies by Duration for a Whole Life Policy

Gain from Conservation Strategies Per Thousand for Whole Life Policies

It is worthwhile to notice the gain from conservation strategies for a large number of whole life policies, because whole life policies are commonly purchased by policyholders. For a newly issued (at duration 0) whole life policy, developing conservation strategies to retain policyholders can generate a gain per 1,000 as \$4.2. For a group of similar policies, this gain will exceed what is needed to implement conservation strategies, illustrating the benefits of conservation strategies.

Whole Life Policy			
Face Amount	100,000		
Net Premium	1,424		
Gross Premium	1,709		
Economic gain			
Duration	0% Lapse Rate	10% Lapse Rate	Gain from Conservation Strategies
0	3149.98	(1073.05)	4,223

Table 4: Gain from Conservation Strategies Per Thousand for a Whole Life Policy

Gain from Conservation Strategies by Term Length

With the same duration and all other constant inputs, the gains from conservation strategies will increase if the term length increases.

Gain from Conservation Strategies	15-Year Term	25-Year Term	Whole Life
New Issue 0-year duration	92.44	534.50	4,223.03
5-year duration	(21.53)	303.36	3,685.76
10-year duration	(88.05)	52.82	3,089.23

Table 5: Relationship between Term Length and Gain from Conservation Strategies of Life Insurance Policies

For instance, for new-issued policies, i.e. 0-year duration, the gain from implementing conservation strategies to bring the 10% lapse rate down to 0 for a \$100,000 policy is \$92.44 for a 15-Year Term Life policy, and 46 times more (\$4223.03) for a Whole Life policy. The table below illustrates the trend of gain from conservation strategies for three types of life policies discussed in this paper at duration 0, 5, and 10.

This trend shows that it is more beneficial to introduce conservation strategies to decrease the lapse rate for longer term length policies. When a policy has longer term length, it also has more time at the beginning to generate a positive gain from lowering the lapse rate.

5. Proposed Policyholder Retention Strategies

5.1 Actuarial Perspective

The actuarial perspective of policyholder retention strategies will help the insurance company to manage their financial risks, since the actuarial analysis done in previous sections provides the results of how much gain the company will earn at different durations across a range of products. It is commonly known that managing financial risks is one of the top priorities of companies. Insurance companies will be more prepared if they can model and estimate the right time to implement conservation strategies on particular types of insurance products. Three strategies can be applied from an actuarial perspective:

- 1) Implementing conservation strategies at the early stage is essential

Conservation strategies to retain policyholders are beneficial in increasing its overall economic gain, especially at early stages. At the beginning of each policy, when a policyholder just purchased it, the insurance company has to pay out acquisition cost and other compulsory expenses, and consistent premiums paid by policyholders in the future are necessary for insurance companies to earn profits in a long-term.

- 2) Companies should focus more on policies with longer term length

For policies with longer term length, insurance companies will have more time to collect premiums. From the analysis above, the longer the term length of a life insurance policy, the larger the gain generated from conservation strategies are generated.

- 3) The effect of conservation strategies diminishes at a later stage

At later durations, it is not necessary to control the lapses. From a study conducted by the SOA, the highest lapse rate occurs in the first year and decreases most significantly in the first three policy years (Shaughnessy & Tewksbury 19).

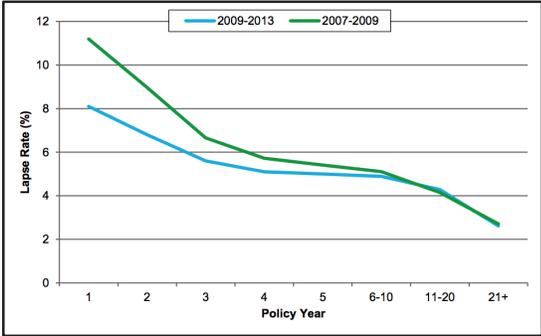


Table 6: Current Study (2009-2013) versus Prior Study (2007-2009) of Policy Lapse Rate by Policy Year

Besides, when the gain from conservation strategies becomes negative, lapses are no longer a cost. Therefore, insurance companies will no longer benefit from implementing conservation strategies to control lapses.

5.2 Business Perspective

The actuarial perspective provides a foundation to develop policyholder retention strategies with financial modeling. Due to the current economic situation with low interest rate and investment yield, insurance companies should know the importance of managing their non-financial risks to achieve cost efficiency. According to an article published by McKinsey & Company, non-financial risks (operational risks) are a second critically important risk type that might be largely overlooked (Buehler, Carpineti, Michel-Kerjan, Nauck & Serino). Operational risks, such as critical misconduct and cyber-attacks, have an impact on daily business operations. Hence, building policyholder retention strategies from a business perspective together with actuarial analysis will help companies to obtain better economic gains and reduce both financial and non-financial risks to a greater extent.

1) Risk Management Team

The risk management team conducts risk assessment on all policyholders when they purchase a policy. It is important to closely monitor and update the process of risk classification, because individuals who are expected to have the same cost should be grouped together. The more accurate the risk classification is, the easier it is for the management team to implement conservation strategies to certain targeted groups.

2) Information Technology (IT) Department

The IT department processes the demographics and data of policyholders. To develop conservation strategies, the IT department should automate its system of processing data and delivering results. The performance of policyholders should also be captured in the system, so it will be easier to track and follow-up with certain groups that need conservation strategies to control the lapse rate.

3) Customer Call Service Center

When there are customers complaining about insurance agents or policies, call-center representatives should indicate these policyholders in the system for follow-up purposes. Complaint management is also useful in identifying policyholders with high economic gain who are likely to have issues with their policies and may potentially drop out.

Since it is important to manage the lapse rate at early durations, the call center can generate a list of new policyholders or policyholders who are at their early durations. It will be helpful for call center representatives to perform conservation strategies to retain policyholders, as they can reach out to those policyholders, particularly those with high economic gain, to check if they have any questions or show their concerns in general.

6. Conclusion

In conclusion, in this paper, the cost of a lapse is evaluated as the gain from developing conservation strategies to retain policyholders, and it is mathematically quantified as the difference in economic gain at 0% lapse rate and 10% lapse rate. The implications of policyholder retention strategies are analyzed from both an actuarial perspective and a business perspective. As there are many other ways to quantify the cost of a lapse and how it can affect an insurance company's profitability, it is undeniable that controlling the lapse rate and policyholders is important to insurance companies. Conservation strategies are used to help achieve this goal in different ways. There are further improvements that can be made to the modeling used in this paper. The modeling can be more dynamic if a feature of risk classification is added as an input. R may also be used to run 20,000 scenarios for multiple policies at the same time.

Overall, from calculations in the modeling, the cost of a lapse can be costly, especially in early durations. Insurance companies should invest in developing policyholder retention strategies, because these strategies will be beneficial to their overall economic gain, especially from policyholders who are at early durations. Although people commonly know that retaining policyholders is important for insurance companies, it is hard to quantify the retention value due to varying durations and different types of policies. From the modeling and analyses in this paper, insurance companies can calculate their cost of lapses/gain from conservation strategies, and prioritize their conservation strategies by ranking each inforce policy by highest to lowest economic gain. Efforts from different teams and departments in the company are also important to support and boost the gain calculated from the actuarial modeling.

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Appendix

1 Assumptions and Notations

1. Lapse rate: l
2. Discount rate: v
3. Net Premium: P
4. Gross Premium: G
5. Death Benefit: B
6. Reserve at time k : ${}_kV_x$
7. n years term product with t duration years
8. Evaluate at the begin of year $t + 1$
9. Calculate future cash flows, prospective evaluation.
10. Net premium reserve is calculated traditionally.

In one year,

- Probability of lapse: α
- Probability of alive: $(1 - \alpha)p_x = \beta p_x$
- Probability of death: βq_x

2 Calculation of the Economic Gain

Everything below is standing at the beginning of year $t + 1$ to see a n years term product with t years of duration.

1. Death Benefit is paid when the policyholder dies. The expected payment to death benefit is:

$$\begin{aligned} E[DB] &= B * (v\beta q_{x+t} + v^2\beta p_{x+t}\beta q_{x+t+1} + \dots) \\ &= \sum_{k=0}^{n-t-1} B(v\beta)^{k+1} {}_k|q_{x+t} \end{aligned}$$

2. Expected gross premiums collected is :

$$\begin{aligned} E[GP] &= G * (1 + v\beta p_{x+t} + v^2\beta^2 p_{x+t} + \dots) \\ &= \sum_{k=0}^{n-t-1} G(v\beta)^k {}_k|p_{x+t} \end{aligned}$$

3. Net premium reserve is paid if lapse. The reserve is calculated prospectively. Expected payment for Lapse Benefit is:

$$\begin{aligned}
 E[LB] &= {}_{t+1}V_x v \alpha + {}_{t+2}V_x v^2 (1 - \alpha) p_{x+t} \alpha + {}_{t+3}V_x v^3 (1 - \alpha) p_{x+t} (1 - \alpha) p_{x+t+1} \alpha + \dots \\
 &= {}_{t+1}V_x v \alpha + {}_{t+2}V_x v^2 \beta p_{x+t} \alpha + {}_{t+3}V_x v^3 \beta^2 p_{x+t} \alpha + \dots \\
 &= \frac{\alpha}{\beta} [{}_{t+1}V_x (v\beta) + {}_{t+2}V_x (v\beta)^2 p_{x+t} + \dots] \\
 &= \frac{\alpha}{\beta} \sum_{k=1}^{n-t} {}_{t+k}V_x (v\beta)^{k-1} p_{x+t}
 \end{aligned}$$

4. Reserves are calculated using recursive formula:

$${}_{k+1}V_x = ({}_kV_x + P)(1 + i) - (B_{k+1} - {}_{k+1}V_x)q_{x+k}$$

5. Economic Gain: (Accumulated Value = AV, Present Value = PV)

$$\begin{aligned}
 \text{Economic Gain} &= AV[\text{Past Premiums}] + PV[\text{Future Premiums}] \\
 &\quad - AV[\text{Acquisition Costs}] \\
 &\quad - PV[\text{Surrender Charge}] \\
 &\quad - PV[\text{Death Benefits}]
 \end{aligned}$$

where surrender charge is assumed to be the net premium reserve when it lapses.