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Deterrence, Incapacitation, and Repeat Offenders

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Abstract

This paper develops an economic model of criminal enforcement that combines the goals of deterrence and incapacitation. Potential offenders commit an initial criminal act if the present value of net private gains is positive. A fraction of these offenders become habitual and commit further crimes immediately upon release from their initial prison term (if any). The optimal punishment scheme in this setting generally involves a finite prison term for first-time offenders (based on the goal of deterrence), and an infinite (life) sentence for repeat offenders (based on the goal of incapacitation).

Journal of Economic Literature Classification: K14, K42

Keywords: Deterrence, incapacitation, prison, repeat offenders
Deterrence, Incapacitation, and Repeat Offenders

1. Introduction

The economic model of crime beginning with Becker (1968) has been almost exclusively concerned with policies aimed at achieving optimal deterrence.\(^1\) While this approach has led to many important insights, it also leaves unexplained certain features of actual penalty structures. Most notably, the theory’s prediction that prison should never be imposed until an offender’s wealth is exhausted through fines seems clearly inconsistent with practice. In the case of non-violent offenders, the apparently “excessive” use of prison is likely an effort to pursue a policy of equal treatment of rich and poor defendants, but in cases involving violent offenders, prison is almost certainly imposed, at least in part, for incapacitation purposes. Indeed, this seems to be the primary justification for “three strikes” laws, which impose life sentences on offenders deemed to be habitual.

In comparison to deterrence, however, the economic literature on the incapacitation function of imprisonment is scant. A notable exception is Shavell (1987),\(^2\) who shows that if an offender is expected to impose more harm on society than the cost of imprisoning him, then he should be kept in prison for as long as that is true, possibly for the remainder of his life. This does not account, however, for the policy of imposing less than maximal sanctions for first and second offenses. This paper seeks to address that shortcoming by developing a model that combines the goals of deterrence and incapacitation. Specifically, it assumes that all offenders are deterrable on their first offense, but once they have committed an initial crime, some become “habitual” (i.e.,

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\(^1\) See, for example, the surveys by Polinsky and Shavell (2000, 2007).
\(^2\) Also see Ehrlich (1981), Levitt (1998), and Kessler and Levitt (1999).
undeterrable), while the remainder are strictly one-timers (i.e., will commit no further crimes). In this setting, repeat offenders should be imprisoned for life based on the goal of incapacitation (the “third strike”), whereas first-timers should generally receive positive but less than maximal sentences to reflect the usual trade-off between the gains from deterrence and the cost of imprisonment.

In consequence of this increasing sanction policy, the paper also contributes to the growing literature on the optimal punishment of repeat offenders. In practice, most criminal penalty structures involve increasing sanctions for repeat offenders, but standard deterrence models have had surprising difficulty in explaining this policy. The simple reason is that, if an optimal sanction achieves the first-best level of deterrence—that is, if it succeeds in deterring only those offenders who value the crime less than its cost to society—then an increasing sanction for repeat offenses would be undesirable because it would overdeter those individuals. Thus, deterrence models can only show that an increasing sanction is optimal if there is some initial underdeterrence, but the literature has an ad hoc quality to it.³ Combining incapacitation with deterrence in the manner described above, however, provides a natural explanation for a policy of increasing sanctions.⁴

Finally, because of the emphasis on incapacitation, the model explicitly incorporates time in a way that most models of crime do not. In particular, it assumes that offenders, once they choose to commit a criminal act, continue to do so until apprehended, where the time until apprehension is modeled as a random variable with

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³ See, for example, Polinsky and Rubinfeld (1991), Polinsky and Shavell (1998), Chu, Hu and Huang (2000), Emons (2003), and Miceli and Bucci (2005).
⁴ Polinsky and Shavell (2007, p. 439) note that incapacitation can justify increasing sanctions if repeat offenders have a higher propensity to commit crimes, but they do not suggest the possibility that deterrence and incapacitation together can explain the policy.
expected value equal to the inverse of the instantaneous probability of apprehension. The model is essentially a continuous version of the approach used in Shavell’s (1987) incapacitation model. The only deterrence model to use such an approach is Davis (1988), though he examines a different issue.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 then examines the pure incapacitation model, where a fraction of offenders are habitual and the remaining fraction are one-timers. In this context the optimal policy is as follows: imprison repeat offenders for life, and either release first-timers immediately or imprison them for life (in which case there obviously are no repeaters). Section 4 then develops a hybrid model in which first-time offenders can be deterred, but of those who commit an offense, a fraction become habitual. The optimal policy in this case continues to involve a life sentence for repeat offenders, but generally involves a finite term for first timers based on optimal deterrence. Finally, Section 5 draws conclusions.

2. Set up of the Model

Offenders contemplate the commission of an illegal act that yields them a private gain of $g$, and imposes a social harm of $h$, each measured per unit of time until apprehension. We treat $h$ as fixed, but assume $g$ is a random draw from the distribution function $Z(g)$ with density $z(g)$. Potential offenders begin committing crimes if, after drawing $g$, the present value of their lifetime gains exceed the present value of expected punishment costs. Potential offenders know that, once they begin committing crimes,

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Shavell (1987) allows $h$ to vary across offenders, or to decline with the offender’s age. The first possibility implies that some offenders should never be jailed for incapacitation purposes, and the second implies that those who are jailed should be released when the harm they would cause falls below the cost of imprisonment. We ignore both of these possibilities to keep the model simple.
they will be caught with (fixed) probability $p$ at each instant, and if apprehended, imprisoned for a term of $s_1$.$^6$

When released (assuming $s_1$ is finite), offenders will either be *reformed* and never commit crimes again, or they will be *habitual*. Let $\theta$ be the probability that an offender is habitual on release. Habitual offenders will begin committing crimes immediately upon release and will continue to do so in any future periods that they are not imprisoned for the remainder of their infinite lifetime. The per-period private gain from these crimes continues to be $g$ (the realized gain from the initial crime), though the offender no longer engages in a cost-benefit calculation before committing them. (These gains will, however, enter the offender’s calculation of the present value of future gains from committing the first offense.) The probability of apprehension is assumed to be the same for first-time and repeat offenders ($p$), but the prison terms, $s_2, s_3, \ldots$, can be different for repeat offenders, given that, on apprehension, an offender’s past criminal record is observable.

Let the social cost of imprisonment (not counting the offender’s disutility) be $c$ per unit of time that the offender is imprisoned, which we assume remains constant throughout the term of imprisonment. Further, we assume that

$$h > c,$$  \hspace{1cm} (1)

which says that the cost per unit of time that an offender is free, consisting of the harm he imposes, exceeds the unit cost of imprisonment. This reflects the potential social benefits of incapacitation for habitual offenders (Shavell, 1987).

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$^6$ To keep the model simple, we consider only imprisonment. One could easily imagine, however, a policy of imposing a fine on first-time offenders for deterrence purposes and then imprisoning repeat offenders for incapacitation.
The apprehension technology is modeled as follows. Time is continuous and, as noted, the instantaneous probability of apprehension is \( p \).\(^7\) Thus, the time until apprehension, \( t \), is a random variable with distribution

\[
    f(t) = p e^{-pt},
\]

and expected time to apprehension equal to \( 1/p \).\(^8\) Finally, because offenders are infinitely-lived, gains and losses are discounted at the instantaneous rate \( r \), which is the same for offenders and society. Figure 1 summarizes the structure of the model.

3. Pure Incapacitation

We begin by considering the optimal structure of penalties in a pure incapacitation model. Thus, we assume that there are a fixed number of offenders (normalized to be one), each of whom commits an initial crime with certainty. Of these offenders, a fraction \( \theta \) are habitual and resume committing crimes immediately upon release from prison, while the remainder commit no further crime upon release. In terms of Figure 1, we begin at the decision node where Nature chooses the proportion of habitual and one-time offenders. The dashed line connecting the two branches of the tree indicates that the enforcer cannot distinguish between habitual and one-time offenders following the first offense.

Consider first the optimal punishment of a repeat offender. Since a repeat offender reveals himself with certainty to be habitual, the optimal prison term is clearly infinite since, given (1), the cost of imprisonment per unit of time is less than the cost of

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\(^7\) In contrast, Shavell’s (1987) model of incapacitation treats time as discrete.

\(^8\) Mortensen (1983) and Loury (1979) use a similar technology in modeling the race for a patent. Davis (1988) is the only other model of criminal enforcement to use this apprehension technology, though his paper addresses a very different question.
harm for habitual offenders (Shavell, 1987). Thus, $s_2$ is infinite. This policy reflects the logic of three-strikes laws, which impose life sentences on repeat offenders for certain particularly harmful crimes.

Our focus in the remainder of this section will be on the optimal prison term for first-time offenders, $s_1$. To derive this we need to calculate the present value of social costs imposed by first timers. (In this section, we ignore the private gains to offenders as well as their disutility of imprisonment. These factors will be incorporated in the deterrence model below.) Consider first the cost imposed on society by a repeat (habitual) offender once he is released from prison after his first offense. Given that he will commit crimes until he is apprehended at time $t$, and then will be imprisoned for the remainder of his infinite life, we have

$$\int_{0}^{t} he^{-rt}d\tau + \int_{t}^{\infty} ce^{-rt}d\tau$$

$$= \frac{h}{r} (1-e^{-rt}) + \frac{c}{r} e^{-rt}.$$  \hspace{1cm} (3)

Now, since $t$ is a random variable with density given by (2), we obtain

$$\int_{0}^{\infty} \left[ \frac{h}{r} (1-e^{-rt}) + \frac{c}{r} e^{-rt} \right] pe^{-rt} dt$$

$$= \frac{1}{p+r} \left( h + \frac{pc}{r} \right).$$ \hspace{1cm} (4)

This expression represents the expected cost imposed by habitual offenders from their release date forward.
Now move back to the initial apprehension of a first-time offender who may or may not be habitual. The initial prison term of length $s_1$ entails a social cost, calculated as of the date of apprehension, equal to

$$\int_{0}^{s_1} ce^{-rt} d\tau = \frac{c}{r} (1 - e^{-r s_1}). \quad (5)$$

Now weight (4) by $\theta$ (the probability that the offender is habitual), discount it back to the apprehension date, and add the result to (5) to get

$$C^*(s_1) = \frac{c}{r} (1 - e^{-r s_1}) + \frac{\theta e^{-r s_1}}{p + r} \left( h + \frac{pc}{r} \right). \quad (6)$$

The final step is to account for the cost imposed by the first-time offender before apprehension. Proceeding as above, we first obtain the present value of costs imposed by the first-timer up to the date of apprehension, $t$:

$$\int_{0}^{t} he^{-rt} d\tau = \frac{h}{r} (1 - e^{-rt}). \quad (7)$$

Now discount (6) back to time zero and add it to (7) to get

$$\frac{h}{r} (1 - e^{-rt}) + e^{-rt} C^*(s_1). \quad (8)$$

Finally, integrate (8) over all $t$, weighted by the density function in (2):

$$TC(s_1) = \int_{0}^{\infty} \left[ \frac{h}{r} (1 - e^{-rt}) + e^{-rt} C^*(s_1) \right] pe^{-pt} dt$$

$$= \frac{1}{p + r} [h + pC^*(s_1)]. \quad (9)$$

This expression represents the present value of social costs imposed by a first-time offender who, with probability $\theta$, is habitual and hence, if re-apprehended, is imprisoned for life.
The social problem from the perspective of incapacitation is to choose the initial prison term, $s_1$, to minimize (9). Note that this is equivalent to minimizing $C^*(s_1)$. Thus, taking the derivative of (6) yields
\[ \frac{\partial C^*}{\partial s_1} = e^{-r_s} \left[ c - \frac{r\theta}{p+r} \left( h + \frac{pc}{r} \right) \right]. \tag{10} \]

The optimal prison term is thus a corner solution, depending on the sign of the term in square brackets.\(^9\) If this term is positive, $s_1^*=0$, meaning that the first-time offender should be released immediately upon apprehension. This will be true if the following condition holds:
\[ \theta < \frac{c(p+r)}{rh+pc} \equiv \theta^*, \tag{11} \]
where the critical value, $\theta^*$, is strictly less than one given (1). In contrast, if $\theta > \theta^*$, first-time offenders should be imprisoned for life. Intuitively, first-time offenders should be released immediately if the probability that they are habitual is less than a critical value; otherwise, they should be imprisoned for life.

The foregoing analysis provides a natural explanation, based on the goal of incapacitation, for life imprisonment for dangerous offenders. It also suggests why first-time and repeat offenders should perhaps be treated differently. It does not, however, explain positive but less than maximal punishments for first-timers. The next section seeks to address this deficiency by introducing deterrence as an additional social goal.

### 4. Introducing Deterrence: A Hybrid Model

#### 4.1. The Offender’s Problem

\(^9\) This is a consequence of the assumption that both $h$ and $c$ are constants.
We now suppose that all potential offenders make a rational decision about whether or not to commit an initial crime. As in the standard deterrence model (Polinsky and Shavell, 2000, 2007), the offender compares the gain from committing the offense to the expected cost. The current model differs from the standard model, however, in its explicit introduction of time. Thus, as noted above, the gain, $g$, is measured per unit of time. Likewise, the cost of imprisonment to the offender, $\delta$, is also measured per unit of time, and both are discounted by the rate $r$. Moreover, the offender, on making his initial commission decision, takes account of the chance that he will become habitual after the first offense, and hence may face life imprisonment for a future offense.

In calculating the potential first-time offender’s overall gain from committing a crime, we proceed as above and first consider the net gains to an offender who becomes habitual upon release from the initial prison term of $s_1$. Given a time to apprehension of $t$ for the second offense, after which he will be imprisoned for life on incapacitation grounds, we obtain

$$\int_0^t \int_0^\infty e^{-rt} d\tau \int_0^\infty e^{-rt} d\tau = \frac{g}{r} (1 - e^{-rt}) - \frac{\delta}{r} e^{-rt}. \quad (12)$$

Now integrate over all $t$, weighted by the density function in (2):

$$\int_0^\infty \int_0^\infty \left[ \frac{g}{r} (1 - e^{-rt}) - \frac{\delta}{r} e^{-rt} \right] pe^{-pt} dt$$

$$= \frac{1}{p + r} \left( g - p \delta \right). \quad (13)$$

10 The social cost of imprisonment per unit of time is therefore $c + \delta$.
11 We take the above policy prescription as given, due to the requirement of time consistency. That is, since habitual offenders cannot be deterred, only the incapacitation benefits exist and the optimal term, as argued above, is infinite. Of course, there are potential deterrence benefits from $s_2$ (and so on) in terms of its effect on the offender’s decision to commit the first offense (what Shepherd (2002) calls “full deterrence”), but any deterrence gains can be achieved by the appropriate choice of $s_1$. 
Although this expression may be positive or negative, the sign does not matter for the habitual offender’s behavior because he is, by assumption, not a rational calculator. It will, however, affect a potential offender’s decision about whether or not to commit his first crime since he knows that there is a chance he will become habitual. We turn to that decision now.

If a potential offender commits a crime for the first time, he is apprehended after $t$ periods, serves a prison term of length $s_1$, and then, with probability $\theta$, immediately begins committing further crimes, resulting in a net gain (or loss) given by (13). For a given $t$, the resulting net return is

$$
\int_0^t ge^{-\tau} d\tau - \int_0^{t+s_1} \delta e^{-\tau} d\tau + \frac{\theta e^{-r(t+s_1)}}{p+r} \left( g - \frac{p\delta}{r} \right)
$$

$$
= \frac{g}{r} (1-e^{-r}) - \frac{\delta}{r} (e^{-\tau} - e^{-r(t+s_1)}) + \frac{\theta e^{-r(t+s_1)}}{p+r} \left( g - \frac{p\delta}{r} \right).
$$

(14)

After integrating over all $t$, weighted by the density in (2), we obtain the offender’s present value of net expected gains from committing the initial crime:

$$
G(s_1) = \frac{1}{p+r} \left[ g - \frac{p\delta}{r} (1-e^{-r}) + \frac{p\theta e^{-r(t+s_1)}}{p+r} \left( g - \frac{p\delta}{r} \right) \right].
$$

(15)

A potential first-time offender will commit a crime if and only if $G(s_1)>0$. It is possible to re-arrange (15) to show that $G>0$ if and only if $g > g^*(s_1)$, where $\frac{\partial g^*}{\partial s_1}>0$. (See the Appendix for details.) Thus, an increase in the prison term for a first offense has a deterring effect.

4.2. The Social Optimum

We can now combine the present value of social costs (harm plus punishment) in (9) with the offender’s private net gains in (15) to write social welfare as
\[ W = \int_{g^*}^{\infty} [G(s_i) - TC(s_i)] z(g) dg - k(p), \tag{16} \]

where \( k(p) \) is the (fixed) cost of apprehension.\(^{12}\) We make the conventional assumption here of counting the gains of offenders as part of social welfare. Although this assumption is perhaps less justified than usual given that the current model is exclusively concerned with dangerous crimes, the formulation in (16) allows an easier comparison with the standard model of deterrence. As will be shown below, if the offender’s gains were excluded, the optimum would generally involve greater (perhaps complete) deterrence.

Taking the derivative of (16) with respect to \( s_1 \) yields

\[ \frac{\partial W}{\partial s_1} = TC(s_i) z(g^*) \left( \frac{\partial g^*}{\partial s_1} \right) + \int_{g^*}^{\infty} [G'(s_i) - TC'(s_i)] z(g) dg , \tag{17} \]

where \( G=0 \) in the first term given that \( g=g^*(s_1) \) for the marginal offender (i.e., the latter is indifferent between committing and not committing the crime). The first term in (17) is positive, reflecting the present value of savings from deterring an additional criminal act. However, the second term, which reflects the marginal effect of an increase in \( s_1 \) on net gains, is ambiguous in sign. First, consider the sign of \( G' \). Taking the derivative of (15) with respect to \( s_1 \) and rearranging yields

\[ G'(s_i) = -\frac{pe^{-r_1}}{(p + r)^2} \left[ \partial g + \delta(p(1-\theta) + r) \right] < 0 , \tag{18} \]

which reflects the decrease in lifetime benefits of offenders resulting from a longer prison term. This includes both the increasing cost of imprisonment and the foregone gains from future crimes if the offender turns out to be habitual. As for the sign of \( TC' \), the

\(^{12}\) Note that the disutility of prison for offenders is contained in \( G(s_i) \), while the cost to society is contained in \( TC(s_i) \).
analysis of the pure incapacitation model showed that it can be either positive or negative, depending on the magnitude of $\theta$, the fraction of habitual offenders, relative to the critical value, $\theta^*$. 

Consider first the special case where $\theta=0$; that is, where all offenders are one-timers. This corresponds to the pure deterrence model of crime. In this case, $TC'>0$ and (17) reduces to

$$\frac{\partial W}{\partial s_1} = \frac{1}{p + r} \left[ h + \frac{pc}{r} (1 - e^{-\tau_1}) \right] z(g^*(s_1)) p e^{-\tau_1}$$

$$- [1 - Z(g^*(s_1))] \frac{pe^{-\tau_1}}{p + r} (\delta + c). \quad (18)$$

Thus, the second term is unambiguously negative. Assuming an interior solution, this yields the following first order condition for $s_1^*$:

$$\left[ h + \frac{pc}{r} (1 - e^{-\tau_1}) \right] z(g^*(s_1)) \delta = [1 - Z(g^*(s_1))] (c + \delta), \quad (19)$$

which says that the marginal deterrence benefits from an increase in $s_1$ (the left-hand side) should be set equal to the marginal cost (the right-hand side). Note that this condition is virtually identical to the condition for the optimal non-monetary sanction derived by Polinsky and Shavell (2007) in the context of the standard deterrence model. (The only difference is that here, the expected savings in punishment costs from deterring one more crime on the left-hand side is in present value terms.)

Thus, the hybrid model contains the standard deterrence model as a special case.

Now suppose that $\theta>0$, so some offenders are habitual. If, first of all, $\theta<\theta^*$, $TC'>0$, so the right-hand side of (17) is again negative. In this case, the outcome is

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13 Also, in the version of their model in which they assume $p$ is fixed, they set $p=1$ (i.e., apprehension is certain).
qualitatively equivalent to the pure deterrence model. That is, an interior solution for \( s_1 \) will generally be optimal. The only difference is the magnitude of the marginal cost of increasing \( s_1 \).

In contrast, if \( \theta > \theta^* \), \( TC' < 0 \). Recall that this is the case where \( s_1 \) is infinite according to the pure incapacitation model. A finite \( s_1 \) is nevertheless still possible in the hybrid model if the second term in (17) is negative, meaning that there is still a positive marginal cost of increasing \( s_1 \). To illustrate, consider the extreme situation where \( \theta = 1 \) (i.e., all offenders are habitual). In this case, it is easy to show that

\[
G'(s_1) - TC'(s_1) = \frac{pre^{-e_i}}{(p + r)^2} (h - c - g). \tag{20}
\]

Generally, the sign of this expression, when summed over \( g > g^*(s_1) \), is ambiguous. The offsetting effects are as follows. First, the fact that \( h > c \) by assumption tends to make (20) positive. This works in the direction of an infinite value for \( s_1 \) according to the logic of the pure incapacitation model. That is, because all offenders are habitual, and because the harm they cause exceeds the cost of incarcerating them, they should be imprisoned for life for their first offense. In this case, the net incapacitation benefits of prison (captured by the second term in (17)) reinforces their deterrent benefits (the first term), resulting in an infinite sentence.

Offsetting this is the \(-g\) term in (20), which will make the second term in (17) negative if there is enough weight in the upper tail of the distribution of offender gains. This effect, if strong enough, limits the optimal prison term. Intuitively, if the reduction in the expected net gain to offenders from an increase in \( s_1 \) is strong enough (owing to a reduction in the amount of time they have to commit future crimes), it can outweigh the gain to society from avoiding the net harm from crime. Obviously, this effect depends
entirely on the decision to include the offenders’ gains in the social welfare function. If $G$ were not included in $W$, the second term in (17) would depend only on the sign of $TC'$, which is negative whenever $\theta > \theta^*$. In this case, an infinite $s_1$ would be optimal; that is, deterrence and incapacitation reinforce each other.

5. Conclusion

The standard economic model of crime based on optimal deterrence justifies the use of prison as a kind of last resort, to be reserved for those cases where offenders’ wealth is insufficient to provide adequate deterrence by means of a fine. Actual practice seems clearly inconsistent with this prescription. One obvious explanation is that prison is used to incapacitate offenders deemed to impose more harm on society than the cost of imprisoning them. The problem, of course, is that habitual offenders cannot generally be identified after a first offense, so a policy of punishing all first-timers as if they were habitual is deemed unduly harsh. In addition, a substantially shorter prison term is sufficient to deter many offenders from committing crimes in the first place, thereby preventing them from embarking on a life of crime. These insights suggest that a model combining both deterrence and incapacitation might be able to explain the common practice of imposing relatively short sentences for first-timers (based primarily on deterrence), and lengthy sentences for repeat offenders (based on incapacitation). The model in this paper represents a first effort to develop such a hybrid model.

In order to achieve this goal, the model explicitly introduced time in a way that most models of crime have not, but which captures some realistic aspects of the apprehension process. To keep the analysis tractable, the model also made several
simplifying assumptions. For instance, it focused exclusively on prison, and it treated the probability of apprehension as fixed. Extensions of the model would therefore relax these assumptions. Still, the model represents a useful step in the direction of explaining actual punishment policies within a coherent economic framework.
Appendix

The expression for $G(s_1)$ in (15) can be rearranged as follows:

$$G(s_1) = \frac{1}{p+r} \left( 1 + \frac{\theta e^{-rs_1}}{p+r} \right) \left[ g - \frac{p \delta}{r} \left( 1 - e^{-rs_1} \left( \frac{p(1-\theta) + r}{p+r} \right) \right) \right].$$  \hspace{1cm} (A1)

Thus, $G > 0$ if and only if $g > g^*(s_1)$, where

$$g^*(s_1) = \frac{p \delta}{r} \left( 1 - e^{-rs_1} \left( \frac{p(1-\theta) + r}{p+r} \right) \right) > 0. \hspace{1cm} (A2)$$

Note that, since the numerator of the parenthetic term is increasing in $s_1$ while the denominator is decreasing in $s_1$, $\partial g^*/\partial s_1 > 0$, as asserted in the text. In the special case where $\theta = 0$ (the pure deterrence model), (A1) reduces to

$$G(s_1) = \frac{1}{p+r} \left[ g - \frac{p \delta}{r} \left( 1 - e^{-rs_1} \right) \right], \hspace{1cm} (A3)$$

and (A2) becomes

$$g^*(s_1) = \frac{p \delta}{r} \left( 1 - e^{-rs_1} \right), \hspace{1cm} (A4)$$

where

$$\frac{\partial g^*}{\partial s_1} = p \delta e^{-rs_1} > 0. \hspace{1cm} (A5)$$
References


**Figure 1**: Structure of the model.