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James W. Boudreau
University of Connecticut

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University of Connecticut

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341 Mansfield Road, Unit 1063
Storrs, CT 06269–1063
Phone: (860) 486–3022
Fax: (860) 486–4463
http://www.econ.uconn.edu/

Abstract

This paper examines the dynamic impact of matching on economic mobility and growth. To account for complex interactions over time, experimental economies of heterogeneous agents are simulated with the match process acting as a fitness selection mechanism. Even with perfect information and substantial variety in both offspring and entrants, two-sided matching inevitably causes the population to evolve into stratified groups. Corrective measures are possible to improve mobility, but by altering the path of market evolution, a policy may have unintended negative impacts on growth and inequality.

Journal of Economic Literature Classification: C78, E24, O43

Keywords: Matching, stratification, path dependence.

1 Introduction

Bilateral matching markets - markets that exist to match two disjoint sets of economic agents - are frequent targets of policies designed to promote equal opportunity. And with good reason. Take the classic example of the labor market, in which employees match with employers; or the market for education, in which students match with schools. Participants' family backgrounds are highly influential in these markets because costly preparations beforehand can make an individual much more attractive to potential partners. Also, the match outcomes of the education and labor markets have been shown to contribute significantly to assortative mating (Mare, 1991), and this in turn may further limit intergenerational mobility (see Ermisch, Francesconi and Siedler (2006) for empirical evidence and Fernández (2002) for theoretical). Subsidization programs thus provide opportunities for deserving individuals to break out of otherwise perpetual feedback loops of socioeconomic stratification. The question remains, however: if a government makes it an objective to improve mobility by way of intervening into a matching market, might it hinder another objective, the promotion of economic growth, in the process?

Despite the fact that most developed nations have been experiencing ever widening gaps of inequality, empirical evidence of a direct link between mobility and economic growth is limited (Eriksson & Goldethorpe, 1992). In fact, even the relationship between growth and inequality itself remains unclear (Barro, 2000). Meanwhile, the theoretical work on economic mobility and growth tends to focus on the direct effects that mobility and technological development can have on each other. Works such as Bénabou (1996) and Owen and Weil (1998) consider the complementarities between skilled and unskilled workers and the resulting implications for growth, while others such as Galor and Tsiddon (1997) and Hassler and Rodríguez Mora (2000) study the roles played by technological breakthroughs and varying returns to skill. Though these relationships are certainly important, the current work approaches the link between mobility and growth from a different angle, stressing the importance of matching markets.

This paper examines the role of bilateral matching markets in driving economic stratification and growth. Also, more particularly, it looks at how redistributive policy alters incentives and outcomes in matching markets, and how this in turn can impact an economy's evolution.

Matching markets play a key part in how an economy develops due to their aforementioned relation to limited mobility, but also because of growth-enhancing human capital investments made by participants. In attempting to improve their prospects on the job market, for example, individuals often improve their productivity by going to school. This type of "pre-marital" investment - preparatory spending incurred by agents in matching markets to appear more attractive to potential partners - has recently been analyzed in work by Peters and Siow (2002) and Peters (2007). By treating pre-marital investment as a behavioral rule for agents in a dynamic environment, this paper considers how redistributive policy affects such preparatory activity. Subsidizing talented individuals so that they invest more has the potential to improve growth, but at the same time also has the potential to distort the incentives others, possibly hindering growth.

The model economy in this paper features heterogeneous agents that compete in an intergenerational match game for employment: agents known as workers wish to match with agents known as firms. Workers differ in their initial endowments of wealth and productive ability, and firms differ in the wages they are able to pay. To appeal to firms, workers can use their wealth endowments to invest in human capital, augmenting their existing ability and thereby improving their productive potential. After investments take place, the most qualified workers match with the highest paying firms. The wages they earn become the wealth endowment for their direct descendants, while firms' productivity grows as a function of their employees' ability.

A novel feature of the model is its use of the market's matching process as an evolutionary fitness selection mechanism. Workers that are unable to find a match drop out of the population and thus do not contribute to current or future productive capacity. Those that do match are able to pass on their attributes, but in a manner that is not fully deterministic. Because of the stochastic element to inheritance, results are arrived at by way of agent-based simulations. By simulating the model economy's development under alternate policy regimes, it is possible to illustrate the effects of mobility-enhancing policy on economic growth when matching influences market evolution in ways that are not fully deterministic.

Results suggest a robust tendency for agents in matching markets to swiftly move into definitive strata. Once established, deviation from the strata is unlikely, with local mobility occurring primarily among the lower classes while the upper class is more firmly entrenched. The fact that stratification is an inherent property of matching markets comes as no shock, though it does complement previous work on the causes of intergenerational mobility (for a recent example, see Anderberg and Andersson (2007)). More surprising are the consequences of correcting that stratification.

Altering market evolution via mobility-enhancing transfers can benefit long-run economic growth, but not under all circumstances and not in an egalitarian fashion. In fact, selective redistribution may exacerbate inequality, ultimately making an economy more dependent on policy. Because the most qualified workers consistently match with the economy's most preferred jobs, selective redistribution benefits that sector at the expense of less preferred sectors. This widens existing wage gaps and increases the wealth advantage of the upper class.

2 The Model

Two-sided matching markets are comprised of agents that can be grouped into two disjoint sets. Agents from one set must be matched with complementary agents in order to complete their economic objectives. [1]. The population in this model is comprised of $F = \{f^1, f^2, \ldots, f^m\}$ firms on one side of the market and $W = \{w^1, w^2, \ldots, w^n\}$ workers on the other side, $m < n$. Workers are attracted to high wages and firms require skilled labor in order to produce the economy's generic consumption good.

To narrow the focus of analysis, the job market in this model is assumed to be frictionless; the most skilled workers are always matched with the highest paying firms. Match outcomes are described by a one-to-one and invertible matching function $\mu : W \to F$, so $\mu(w^i)$ represents

[1] A vast literature has been dedicated to these markets and is expertly surveyed in Roth and Sotomayor (1990) and Roth (2008)

the firm matched with worker i. One-to-one matching and strict preferences are assumed for simplicity and tractability, but the remaining assumptions of the model allow conclusions to readily extend to cases of many-to-one and many-to-many matching, with or without strict preferences. Time is discrete, indexed as $t = 0, 1, 2, \ldots, T, \ldots$, and generations of workers are overlapping so that firms are able to fill employment in every period after time zero.

2.1 Firms

There are m infinitely lived firms that are divided into K types or "industries," with industries differing in terms of their production possibilities. The production of firm j in industry k at time t is determined by the production function $Y_{k,t}^{j} = Y(X_k; \theta_t^j(\hat{a}_t^{\mu_j}))$, where X_k is an industry-specific factor that is fixed for all firms in industry k and θ_t^j is a firm-specific technology which grows as a function of the currently employed worker's ability $\hat{a}_t^{\mu_j}$. Each firm is limited to employing only one worker at a time, and labor is assumed essential for production. Firms keep all residual output after wages are paid, so their objective is to produce as much as possible. Since the firm-specific technological factors grow with worker ability, this implies homogeneous preferences for firms: workers are ordered according to their adult ability level.

More productive industries offer higher wages. Since the highest paying industry consistently attracts the most able workers, these firms will have the highest technological growth, and thus always offer the highest wages. Worker preferences over firms therefore remain constant over time. This behavior for firms is admittedly simplified, but it is merely the presence of a wage hierarchy which drives results. A more complicated role for firms is discussed in the paper's final section.

2.2 Workers

Workers live for two periods, motivated by the utility they gain from consumption in the first period of life and the inheritance they are able to leave their offspring. Their common utility function, $U(c_t, e_{t+1})$, is strictly increasing, strictly concave, and twice differentiable in both arguments with the boundary conditions $\lim_{c_t \to 0} U_1(\cdot, e_{t+1}) = \lim_{e_{t+1} \to 0} U_2(c_t, \cdot) = \infty$. Each individual worker is born with a unique asset and ability endowment $\{e_t^i, a_t^i\}$, inherited from their parents (or simply existing in the case of the initial generation). Assets come in the form of the generic consumption good, which fully depreciates at the end of each period. Since there are no savings or credit channels in the economy, workers must base their first period consumption on their endowment and bequests on their compensation from employment.

In order to obtain the best job possible, workers can make themselves more attractive to firms by investing in their own ability. This form of human capital investment requires that the worker sacrifice a part of their asset endowment however, implying a classic intertemporal tradeoff. Formally, let $\hat{a}_t^i = g(a_t^i, I_t^i)$, where $\hat{a}_t$ is augmented ability, I_t^i is the amount invested and $g(\cdot, \cdot)$ is increasing in both arguments. The problem for worker i, born at time t, can then be stated as

$$\max_{I_t^i} \{U(c_t^i, e_{t+1}^i) | c_t^i = e_t^i - I_t^i, e_{t+1}^i = \mu_{t+1}(\hat{a}_t^i), I \geq 0\}.$$

Worker i's wage is indicated by $\mu_{t+1}(\hat{a}_t^i)$ since the firm they are matched with implies their wage, and their match is a function of their ability in the second period. The inheritance a parent can leave for their child therefore hinges upon the outcome of the job matching procedure. Without loss of generality, it is assumed that among the K industries, wages rank as $f_1^j > f_2^r > \ldots > f_K^s, \;\; \forall t = 0, 1, 2, \ldots, T, \ldots; j, r, s \in \{1, 2, \ldots, m\}$.

Typically, pre-marital investment activity of this kind is modeled as an all-pay auction. For example, see the more rigorous treatments of Peters and Siow (2002) and Peters (2007). To capture the essence of pre-marital investment competition without complicated mixed-strategies, however, workers in this model behave according to a simple heuristic.

Workers invest as if they are in a second-price auction with full information of each other's asset and ability endowments. The prizes they bid for are jobs, and the bids are the agents' post-investment ability levels. Their behavior is therefore dependent on threshold levels of investment, where the cost of investment is equal to the benefit of a higher wage:

$$U(c_t^i, e_{t+1}^i) = U[(e_t^i - I_t^{i,k}), \mu_{t+1}(g(a_t^i, I_t^{i,k}))], \forall k \in K.$$

Worker i is not willing to invest more than $I_t^{i,k}$ to achieve employment in industry k. Therefore, those agents who are willing and able to secure employment in a particular sector invest just enough to meet their closest competitor's point of indifference.

To provide an example, if the economy consists of 12 workers and 10 firms, with 4 firms in each of the higher paying industries and 2 in the lowest paying industry, investment is determined as follows:

1. Workers are ranked 1-12 according to their ability after full investment.
2. The workers ranked 1-4 invest just enough to match the 5th ranked worker at his threshold level for the highest wage, given that he invested enough to secure the 5th spot.
3. The workers ranked 5-8 invest just enough to match the 9th ranked worker at his threshold for the second highest wage, given that he has invested enough to secure his 9th position.
4. The workers ranked 9 and 10 invest enough to match the full investment ability level of the 11th ranked worker.
5. The two lowest ranked workers invest nothing.

This process then generalizes according to the number of workers and firms, and according to the size of industries. Workers invest *just enough* to outshine those who could possibly compete with them.

Here it is important to note that due to the standard properties of the utility function, those who are born both rich and talented are generally able to attain better matches. Declining marginal utility means that their threshold level of investment will exceed that of poorer workers with lower ability who would otherwise attempt to usurp their position in the rank order. Also important, however, is that individuals with relatively low ability endowments can be born with sufficient wealth to make them both willing and able to outbid others with higher ability but lower asset endowments.

2.3 Population Dynamics

The first generation of the economy begins with an exogenously given distribution of asset and ability endowments in $t = 0$. After the workers' investment decisions, the match mechanism determines the first employment scheme for their second period of life, $t = 1$. Since the number of workers exceeds the number of jobs in the economy, some workers remain unmatched. These individuals exit the job market (or "die"), leaving only the employed to reproduce and pass on their traits. The workers' asset and ability endowments thus effectively act as evolutionary strategies, determining fitness in the match environment and implicitly dictating which workers are able to reproduce.

The focus of this study is on intergenerational linkages and match behavior over time, so heritability is a key aspect of the model. To emphasize the relationship between match market success and intergenerational transmission workers mate in pairs according to job market success, meaning that the worker matched with firm 1 will mate with the worker matched with firm 2, and so on[2]. Each pair produces two new workers, and concurrent with the birth of the next generation, the remaining $n - m$ vacancies in the economy are filled by *new* agents, unrelated to any prevailing dynasties, and who enter the market fresh with randomly given asset and ability endowments. Entry and exit into the market allows new strategies to be incorporated into each generation without forcing "mutations" to occur in the worker gene pool. This is an effort to keep the model applicable to human behavior and markets, and is a response to criticisms of other models which perhaps overuse evolutionary techniques (Borgers, 1996). The market is assumed fixed in size, and thus the number of possible entries per period is restricted to the number of exits, so the total population remains constant over time.

After the initial generation, a child's asset inheritance is the same as their parents' wage. This means that by excelling in the job matching market workers better provide for their children.

[2] Assortative mating is well documented, for example see Mare (1991). All results are qualitatively the same with a more relaxed mating scheme, as long as employment status plays a significant role. The implicit restriction of m to an even number is necessary for this mating scheme, but results would not change if an odd m were used with randomized mating, as long as workers prefer those from the same industry as themselves.

Figure 1: Punnet Square Examples: Homozygous Agents on left, Heterozygous on right

	A^r	A^r
A^i	A^i, A^r	A^i, A^r
A^i	A^i, A^r	A^i, A^r

	A^r	B^r
A^i	A^i, A^r	A^i, B^r
B^i	B^i, A^r	B^i, B^r

The K different industries imply K different socioeconomic classes of workers.

Ability is more complicated, and is conveyed across generations by the genetic inheritance of two quantitative alleles[3]. All new entrants (including the first generation) begin heterozygous, with each allele having a different quantity value drawn from a distribution $\zeta(\overline{a}, \sigma_a)$. Each agent's phenotypic realization of ability is the average of their two individual alleles, plus a "luck" factor drawn from $\zeta(0, \sigma_a)$. If agent i has alleles A^i and B^i, then, their ability is

$$a_t^i = \frac{A^i + B^i}{2} + \zeta(0, \sigma_a).$$

The quantitative representation of the alleles allows for the incomplete dominance of traits; higher ability does not strictly dominate lower. Offspring of previously employed workers obtain their alleles according to the Punnet square probability.

Punnet square examples are given in Figure 1. The offspring of any two workers (w_t^i, w_t^r), have an equal chance at inheriting any of four possible allele combinations. If workers (w_t^i, w_t^r) are homozygous, with alleles $\{A^i, A^i\}$ and $\{A^r, A^r\}$, they will definitely yield heterozygous offspring, unless $A^i = A^r$. On the other hand, if the two workers are heterozygous with alleles $\{A^i, B^i\}$ and $\{A^r, B^r\}$, homozygous offspring have a 25% chance of occurring if $A^i = A^r$ or if $B^i = B^r$, and a 50% chance of occurring if both equalities hold. Of course, depending on the workers involved, it may be the case that $B^i = A^r$ and/or $B^r = A^i$ as well, with the probabilities of inheritance adjusting accordingly.

[3]See Bartels et al. (2002) for evidence of genetic influence on intelligence.

2.4 Equilibrium

A time $t \geq 0$ *equilibrium* in this economy is a family of decisions, $\{I_t^{i*}, c_t^{i*}\}, \forall i = 1, 2, \ldots, n$, and a corresponding matching, $\mu_{t+1}(\cdot)$, such that all workers act optimally given the asset and ability levels of all other workers and the wages posted by firms. Model analysis is focused on how market evolution is altered when time t equilibria are altered.

Whether or not the economy reaches a stationary state in terms of growth or mobility depends on the specification of firm technology. If the growth of technology diverges, but at different rates in different industries, then there will be very little mobility in the limit as the difference in wages will also diverge. Conversely, if technology growth converges to some productive limit, then the rate of mobility will also converge to a stationary rate, one that is determined by the limiting wage differential. Both cases of technology growth are considered below.

3 Simulation Settings

Simulation of the economy allows for the observation of its dynamic behavior in a controlled environment. For functional forms, let first period ability be augmented additively by asset investments so that $\hat{a}_{t+1}^i = a_t^i + I_t^i$. The utility function for workers takes the form

$$U(c_t^i, e_{t+1}^i) = c_t^i e_{t+1}^i = (e_t^i - I_t^i)\mu_{t+1}(a_t^i + I_t^i).$$

Workers are thus always *willing* to invest their entire endowment if necessary to secure at least some wage in the second period of life. How much each worker *actually* invests is based on the ability and asset levels of all other workers, as well as the marginal gain from a higher wage if one is possible.

For firms, two cases of technology are considered. In both cases, the production function for firm j in industry k takes the form $Y_{k,t}^j = L\theta_t^j X_k$, where L is an indicator function noting the presence of an employee. The difference in specifications is in the growth of technology, θ_t^j. In the first specification, technology grows sigmoidally according to $\theta_t^j = \exp(-\exp(\frac{\sum_t \hat{a}_t^{\mu_j}}{Q_1}))$.

With such a form, technology grows at an exponential rate initially and then tapers off so that the productivity of each firm converges to a maximum of X_k. The second specification has technology growing exponentially over time according to $\theta_t^j = \exp(\frac{\sum_t \hat{a}_t^{\mu_j}}{Q_2})$, meaning that productivity increases indefinitely. Q_1 and Q_2 are large, positive constants set to 20,000 and 100,000 respectively.

Population parameters for the number of workers, firms and industries are set to $n = 12$, $m = 10$, and $K = 3$, respectively. The three industries are separated with four firms in the first industry, four firms in the second industry, and two firms in the third, with $X_k \in \{70, 60, 50\}$. Each generation loses two of its number since they will fail to find employment and exit the market, soon to be replaced by new agents. New entrants have equal odds of receiving each asset endowment, so they may find themselves rich, poor, or middle class. Genetic alleles for the first generation, and for each new market entrant, are randomly drawn from a normal distribution with mean $\overline{a} = 60$ and a variance of σ_a =10 or 25. The random factor that is added to each worker's ability is accordingly drawn from a normal distribution with mean zero and a variance of σ_a, so they may be helped or hurt by luck.

The structure of the model economy makes it highly susceptible to stratification, irrespective of population size or number of industries. The only necessary restriction on parameter values is that industry wages are sufficiently different from one another, relative to the mean ability level. If wages are too close together, asset endowments lose their heterogeneity and the economy becomes de facto one large industry with intra-mobility driven solely by ability. Distributional specifications do impact the economy over time if the genetic variance is made to be extraordinarily large relative to the mean genetic ability level, however, even an exceptional variance can not completely eliminate stratification.

To eliminate any possible dependence on the initial conditions of the random number generator (used for stochastic processes), an economy with a particular specification is simulated 100 times. Each experimental economy lasts for 1200 generations, a suitable horizon for observable dynamic behavior given the specified parameters. That length of time allows production

Table 1: Transition Matrices from Simulated Economies

σ_a	10				25			
	0.70606	0.24915	0.03456	0.01023	0.58933	0.30235	0.07348	0.03484
	0.21727	0.48165	0.19515	0.10593	0.28607	0.40138	0.18098	0.13157
	0.03877	0.22531	0.28922	0.44670	0.12149	0.30698	0.25578	0.31574
	0.11457	0.31309	0.25137	0.32097	0.12770	0.28555	0.23531	0.35144

possibilities to converge for the first case of technology while preventing them from diverging completely in the case of exponential growth.

As a measure of stratification, economies are represented by intergenerational Markov transition matrices. This is done by keeping track of what industry each worker matches with (remaining unmatched counts as industry $K+1$), and categorizing them according to the industry of their most recent ancestor. The figures are added up and averaged over the possible number of workers in each industry. Averaging those figures over 500 generations and 100 trials yields a $(K+1) \times (K+1)$ transition matrix. Each entry (l, k) represents the fraction of workers that matched to industry k, given that their direct ancestor matched to industry l. The transition matrix thus provides a summary of socioeconomic mobility in a simulated economy. Higher values in the diagonal elements correspond with a greater tendency for descendants to have matched within the same industry as their like-numbered parent. Alternatively, smaller values of diagonal and off-diagonal elements correspond with greater mobility.

Table 1 provides the transition matrices for the first technology specification with $\sigma_a = 10$ and $\sigma_a = 25$. Most notable in both tables is that workers with successful parents (matched to the highest paying industry) generally stay successful. The remaining strata also demonstrate persistence, but persistence that declines along with income. New entrants, with randomly assigned endowments, are equally likely to gain employment in all except for the (first) highest paying industry. Such asymmetry in mobility is consistent with empirical estimates of transition probability, indicating that upward mobility from the bottom is more likely than downward mobility from the top. In fact, parameters (in particular the value of $\sigma_a = 25$) are chosen specifically so that mobility results with the first technology specification are similar both qualitatively and quantitatively to the transition matrices presented in Table 5 of Dearden, Machin and Reed (1997), as well as those presented in Table 2 of Gottschalk and Spolaore (2001), for the UK and US economies respectively. Such mobility estimates can also be obtained with various other parameter combinations (more workers, more industries, etc.), but to maintain simplicity the small population of $n = 12$, $m = 10$ and $K = 3$ is used in the next section when making comparisons to the case of a transfer scheme.

4 Correcting Stratification

Whether or not stratification is ultimately detrimental to an economy depends upon the welfare criterion used. If it is the case that a particular economic activity necessitates an egalitarian structure, limited mobility is quite undesirable. An obvious example is the market for public education, where children with varying levels of natural ability and wealth must be assigned to schools (see Balinski and Sonmez (1999) and Chen and Sonmez (2006)). Those with higher initial wealth endowments are able to engage in various preparatory activities which can improve specific attributes that factor in to schools' preferences.

The elimination of stratification also may be in an economy's best interest if production technology is positively affected by the ability of past generations. If naturally "talented" workers are able to take advantage of their potential and are given the opportunity to invest via subsidization, it increases the competition for the economy's most sought after jobs. This increased competitiveness then trickles down to the remaining sectors of the economy as more able candidates are inevitably displaced in the rank order. If human capital investment is the medium of competition, subsidizing naturally able workers therefore has a positive externality for growth. The question, however, is whether or not the positive effect is large enough to balance the distortionary effects incurred in funding the subsidization and altering future generations. The remainder of this section serves to examine that trade-off more closely.

4.1 A Transfer Scheme

To compare the development of the economy with improved mobility with that of the standard model, it is necessary to construct an experimental redistribution policy with minimal incentive distortions. The following is only one example of such a policy, but one that serves adequately as an illustration of unintended consequences.

At any time t an individual may be born with an ability endowment greater than that of all others in the time t cohort, but with an asset endowment less than that which enables them to attain employment in the first industry. Call this individual $\widehat{w}_t$. If $\widehat{w}_t$ exists, a lump sum tax is levied evenly on all workers, with the total amount of the tax just enough to guarantee $\widehat{w}_t$ employment at the highest wage. The tax is then transferred directly to $\widehat{w}_t$ in period t so that the funds are available for investment. Workers are taxed rather than firms in this scenario so that the total amount of assets available for investment remains unaltered. The tax is lump sum so that other than the change for $\widehat{w}_t$, the wealth hierarchy is unchanged.

Because of the model's highly nonlinear evolutionary process, the impacts of redistribution are not immediately clear. Although the policy in this case ensures that the worker with the highest pre-investment ability in each period is always allocated to the most productive sector, it also alters the investment incentives and mating behavior of the remaining workers in the market.

First consider the effects on strategic investment. When subsidization raises $\widehat{w}_t$ to the first industry, it forces all agents that otherwise would have been employed in that industry to invest more since one must be relegated to the second industry. The increased competition then trickles down to whichever industry $\widehat{w}_t$ would have achieved without any transfer. At the same time, however, there are less funds total for all workers except $\widehat{w}_t$ to invest. In a single period, then, redistribution can lead to either higher or lower levels technology growth in each industry.

Next consider the effects of altered matching on future generations. Just because $\widehat{w}_t$ has the highest ability in the market for a given period does not mean that their genetics are the best. It could be the case that one of $\widehat{w}_t$'s alleles is large and the other small, or that both alleles

Table 2: Transition Matrices from Simulated Economies with Redistribution, the Case of Convergent Technology Growth

σ_a	10				25			
	0.69897	0.25612	0.03462	0.01029	0.58471	0.30416	0.07549	0.03564
	0.21624	0.48359	0.19457	0.10559	0.28712	0.40021	0.18046	0.13221
	0.05369	0.21123	0.28868	0.44640	0.12409	0.30466	0.25412	0.31713
	0.11588	0.30935	0.25293	0.32184	0.13227	0.28660	0.23396	0.34717

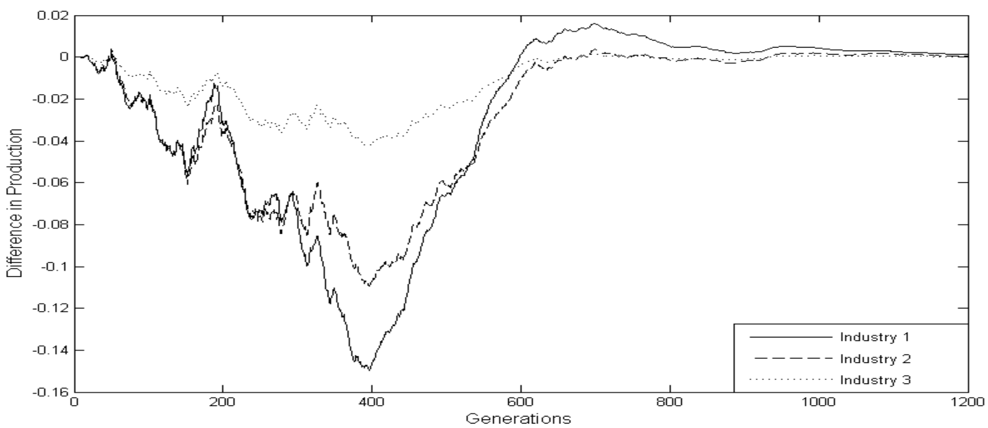
are inferior and $\widehat{w}_t$ is simply a lucky individual. Moreover, the rise of $\widehat{w}_t$ changes the mating prospects of many other agents, thus the genetic composition of the next generation. It may also mean that a different set of genes exit the market completely. Policy implementation thus has the potential for substantial echo or domino effects that may either help or hinder growth.

4.2 Comparisons of Growth and Inequality

To compare the economy's evolution with and without the policy, first consider the case of technology growth that converges to finite limits. Table 2 displays the transition matrices for simulated economies with redistribution. Compared with the numbers in Table 1, there is a small but significant increase in intergenerational mobility. The effects of those changes are illustrated in Figures 2 and 3, which depict the difference in production between the economy with the transfer scheme and the economy without as they progress through time. A positive trend indicates that the economy tends to grow faster in the presence of the transfer scheme and vice versa.

With a relatively small variance, the economy with improved mobility grows just a bit slower than the standard version until technology growth reaches its inflection point and productive capacity catches up. With higher variance in ability, transfers occur less frequently since exceptional agents are more able to succeed with ability alone. It is interesting then, that when redistribution is more selective it initially causes the economy to grow faster. Once technology progresses sufficiently, however, wealth advantages become large enough that ability alone is less likely to guarantee a good job. Transfers then become more frequent and that is when the

Figure 2: Transfer Scheme vs. Laissez Faire Economy: Convergent Technology Growth, $\sigma_a = 10$



economy falls behind. It seems that smaller, more deserving alterations in match structure have less potential for adverse effects.

Regardless of the genetic variance level, it is important to note two characteristics of the experimental economies with limiting technological growth. First, the impact of policy on productivity is quite small relative to total production, suggesting that increased intergenerational mobility comes at a fairly low cost when growth follows an s-shaped trend. Second, the differences in productivity follow qualitatively similar patterns across industries, so no one sector is benefitted at the expense of others. These traits in particular are important because neither of them holds for the case of exponential growth.

When technology growth is exponential, the impact of the transfer scheme is much more drastic. Obviously the ever-increasing difference between industries limits mobility as time goes on, making transfers both larger and more frequent, and thus making the policy more influential. Figures 4 and 5 show that influence, illustrating the difference in productivity in economies with and without redistribution for the case of exponential technology growth.

After negligible effects for the first five hundred generations, mobility-enhancing redistribution significantly alters the economy's evolution. The reallocation of skilled workers consistently

Figure 3: Transfer Scheme vs. Laissez Faire Economy: Convergent Technology Growth, $\sigma_a = 25$

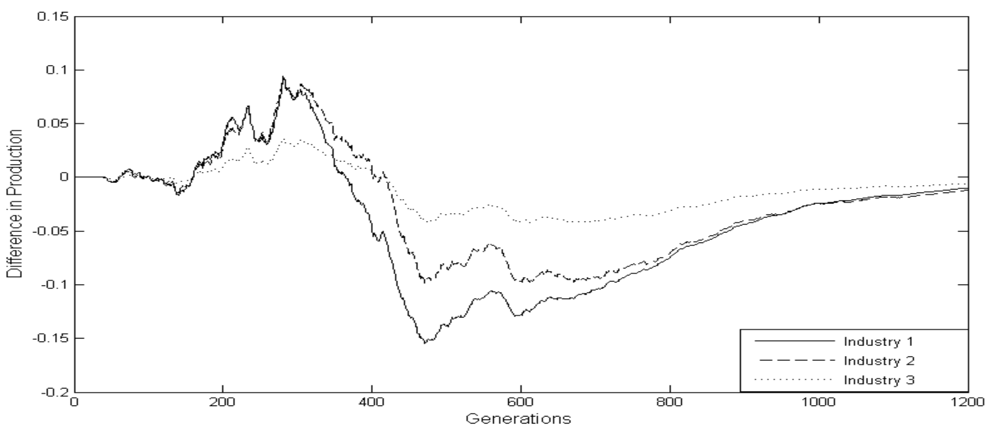


Figure 4: Transfer Scheme vs. Laissez Faire Economy: Exponential Technology Growth, $\sigma_a = 10$

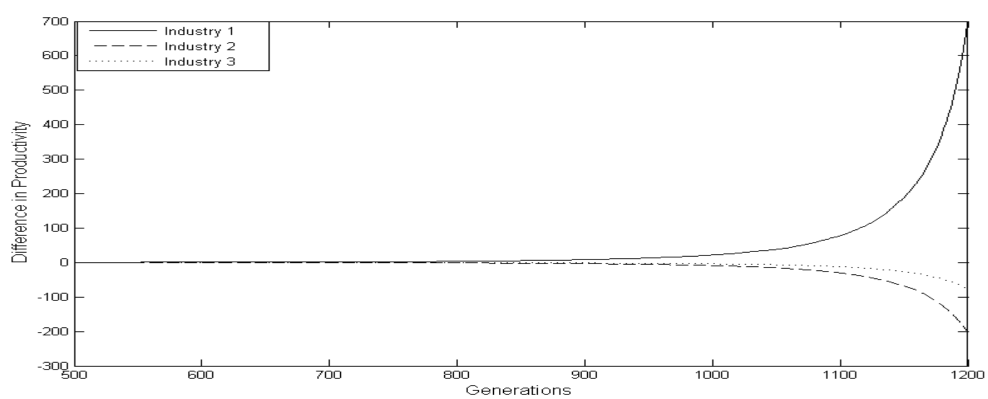
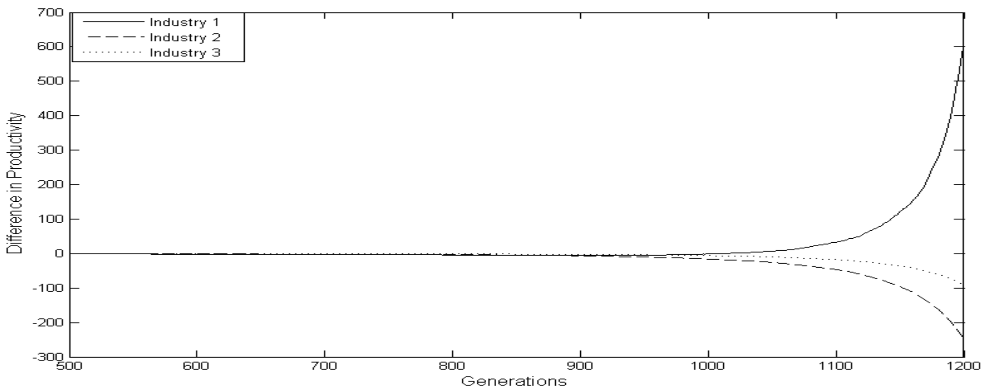


Figure 5: Transfer Scheme vs. Laissez Faire Economy: Exponential Technology Growth, $\sigma_a = 25$

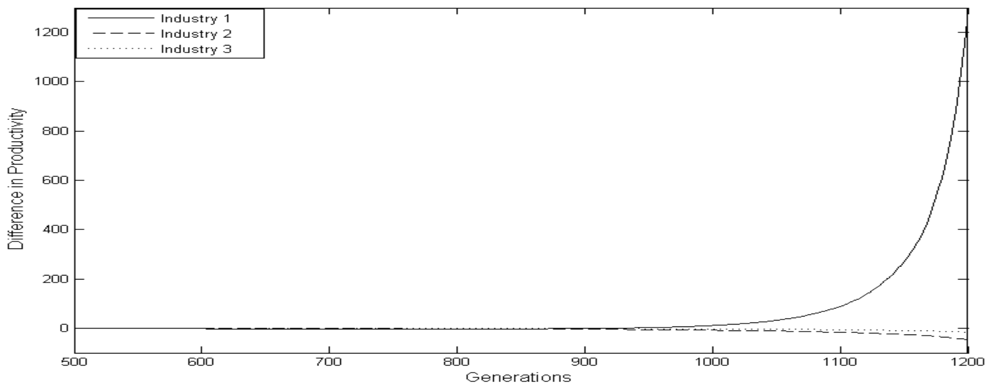


benefits the top industry, but only at the expense of the others. Redistribution increases the post-investment ability levels of the most qualified workers by making them compete with $\widehat{w}_t$, but the benefits of increased competitiveness are insufficient to improve industries other than the first. Thus, although the dramatic increase in the top sector's productivity due to the transfer scheme (about 10%) is sufficient to make total growth higher in economies with improved mobility, inequality is also dramatically elevated. Ironically, the increased wage dispersion makes the economy under mobility-enhancing policy much more susceptible to stratification should the policy ever be removed.

4.3 Manna from Heaven

A final consideration is the possibility of a mobility-enhancing policy that does not require taking away from the wealth of workers. In the preceding section, the total amount of wealth available for investment remains unchanged and is simply redistributed. Declines in productivity may therefore be due to the lower level of wealth available for investment for workers other than $\widehat{w}_t$. If, instead, subsidization is funded by a tax levied on firms, or if a benevolent government simply gives additional funds to deserving workers, the detrimental effects are in some respects reduced. They are not, however, eliminated.

Figure 6: Manna from Heaven vs. Laissez Faire Economy: Exponential Technology Growth, $\sigma_a = 25$



Working backward, consider the case of exponential technology growth. Figure 6 illustrates the deviation from the standard model caused by a policy that gives wealth to a deserving worker as defined in section 4.1, but that takes nothing away from the rest of the working population. As to be expected, the most productive industry is enormously benefitted due to the increased investment of competing workers. Surprising, however, is the fact that the remaining two industries still suffer in the presence of the policy. Their loss is much less than in the redistributive case since all workers not receiving a transfer maintain the same amount of wealth, but altering the competitive market remains detrimental to the lower income bracket.

Returning to the first case of technology, augmenting the transfer scheme so that workers are not taxed does not significantly alter the policy's impact. When technology, and thus wages and first period endowments, are limited in terms of how high they can grow, the size of the tax necessary to subsidize $\widehat{w}_t$ is relatively small. So small that less than 1% of the productivity loss returns when workers are not taxed. The overall productivity slow down in the presence of subsidization in this case is therefore not merely due to the effects of *re*distribution, but due to the more complicated dynamic effects of altering match outcomes.

This example highlights the danger of implementing policy tools in two-sided matching markets that function as dynamic systems. Any policy implemented in this environment will involve similar degrees of uncertainty regarding its effects, since altering the path of even one individual necessarily alters the paths of others, in both present and future generations. Path dependence will be a characteristic of any two-sided matching market with intergenerational influences, as matching in one period impacts the incentives and capabilities of the next.

5 Discussion

In closing, a few words may help to clarify the purpose and interpretation of the policy experiments described by this paper. First and foremost, the model is obviously a very stylized representation and is not intended as an all-encompassing account of economic activity. Rather, the intent of this model is simply to illustrate the importance of dynamic properties in two-sided matching markets, and in particular the competition in those markets that provides crucial motivation for human capital investment.

In real life, credit markets do exist and can help talented young individuals to invest in themselves despite wealth constraints. These are not explicitly modeled here, however, since preparation is in fact a lifelong process. Even if a relatively poor individual is able to receive a loan after high school so they can attend college, it still seems doubtful that they are on par with those lucky enough to be born wealthy. Those who attend private school all their lives, have access to influential social networks, and who can afford college without a loan are most likely still at a substantial advantage in readying themselves for the job market.

Subsidization in this model goes beyond cheap loans for tuition as it truly allows naturally gifted individuals to compete regardless of their initial wealth status. Intergenerational mobility is therefore increased, but depending on shape of technology growth there may also be additional effects in the long run. If technological growth is limited in its potential then mobility can be enhanced with only a slight slow-down and long-run productivity remains unaltered. When technology grows unbounded, however, changing workers' preparatory investment incentives has more drastic consequences. Increased competitive vigor only seems to occur in the battle for the economy's highest paying jobs, without trickling down to benefit the remaining industries. Though total production in the economy may increase, it does so in a lopsided manner and at the expense of those in lower income brackets.

With results so dependent on the specification of technology it is pertinent to make a final note about the specification of firms. Concepts such as entrepreneurial ability, technological innovation or serendipity, and just how technology actually does grow are intentionally omitted here. These are very complicated issues in and of themselves and are beyond the scope of this study. The intent here is simply to examine the potential impacts of altered incentives in matching markets. Certainly, it could be the case that firms all belong to one common sector and share general technology. If that were the case then subsidization would have an overwhelmingly positive impact on growth (in the case of exponential technology) without the accompanying rise in inequality. On the other hand, if firms were allowed to enter and exit sectors, or even to integrate across them, then the lopsided impact of mobility-enhancing policy could have implications for creative destruction, speeding along the exit of less desirable employers and encouraging more preferred industries. The complicated welfare implications of that sort of scenario and its dynamic effects on firm behavior, as well as the possibility of other avenues for matching markets to impact growth, are all left for future research.

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