Faculty Articles and Papers   School of Law

1975

On What a Rule Is

Robert Birmingham
University of Connecticut School of Law

ON WHAT A RULE IS

by Robert Birmingham*

> As Augustine with Time, I knew what a rule was until asked. I
> asked myself and proceeded to become quite perplexed.
>
> <div align="right">Wilfrid Sellars†</div>

0. Introduction

I want to talk about rules; it is best that I find a sort of paradigm
of a rule to start with. Friedman in his very good paper "Legal Rules
and the Process of Social Change" gives as an example of a rule the
rule of *Hadley v. Baxendale*:[1]

> It is very clear that some of the propositions enunciated
> in appellate cases are (or purport to be) rules. Thus, the
> famous case of *Hadley v. Baxendale* asserts as a rule:
>
> > Where two parties have made a contract which one
> > of them has broken, the damages . . . should be
> > such as may fairly and reasonably be considered
> > either arising naturally . . . from such breach of
> > contract itself, or such as may reasonably be sup-
> > posed to have been in the contemplation [of both]
> > parties, at the time they made the contract, as the
> > probable result of the breach of it.
>
> To a certain set of facts (a broken contract), this rule appends
> certain consequences (a particular measure of damages). Stat-
> utory phrases or sentences are also rules.[2]

His example will do as my paradigm. There are many problems with
it; but the problems are part of the point.

---

* Professor of Law, University of Connecticut. A.B., J.D., Ph.D., University of Pitts-
burgh; LL.M., Harvard Law School. Nuel Belnap helped me a lot with this; mistakes
that are left are my own.

† Sellars, *Language, Rules and Behavior*, in JOHN DEWEY: PHILOSOPHER OF SCIENCE
AND FREEDOM 289, 296-97 (S. Hook ed. 1950).

1. 9 Ex. 341, 156 Eng. Rep. 145 (Ex. 1854).

2. Friedman, *Legal Rules and the Process of Social Change*, 19 STAN. L. REV. 786,
787 (1967) (footnote omitted).

1. *Sentences and Propositions*

A first problem has to do with the sort of thing a rule is. Fried-
man in what I have so far quoted from him speaks of rules as sen-
tences and as propositions. And he says this:

> In general, the word "rule" is used in law to describe a
> proposition containing two parts: first, a statement of fact
> (often in conditional form) and, second, a statement of the
> consequences that will or may follow upon the existence of
> that fact, within some normative order or system of govern-
> mental control. Or, as Roscoe Pound has put it, a rule is a
> "legal precept attaching a definite detailed legal conse-
> quence to a definite detailed state of fact." Pound's defini-
> tion is accurate enough for present purposes. It is broad
> enough to include statements of common-law doctrine as
> well as statutory provisions, administrative regulations, ordi-
> nances, decrees of dictators, and other general propositions
> promulgated by legitimate authorities which are intended to
> govern or guide some aspect of social or individual conduct.
> All of these propositions may be called legal "rules" in that
> they all append legal consequences to given facts.[3]

Sentences and propositions are not things of the same sort. It is
usual to distinguish them in this way:

(1) A sentence is a sequence of words and so on. It is a
    linguistic object.
(2) A proposition may be the meaning of a sentence. It is an
    abstract object.

What a statement comes to is not so clear. It can be a sentence or a
proposition, or a sentence said at some time. But most often it is the
last of these.[4]

I should say what a proposition is; a rule may be one. I take it
that I have to do with a set of possible worlds. I think of each subset
p of the set of possible worlds which I have to do with as a proposi-
tion. I think of the possible worlds which are members of p as the
possible worlds in which p is true. There are other ways to look at
propositions. So one can take a proposition to be a set of sentences
which mean the same thing. But what I do gives me a good sense of

---

3. *Id.* at 786-87 (footnote omitted).
4. Cresswell, *Hamblin on Time,* 9 Noûs 193 (1975).

what a proposition is. Moreover it is philosophically more or less usual.[5]

One wants to say that a rule as Friedman looks at it is a sentence or a proposition; not both. Friedman fails to choose; and he is in good company: most notoriously, that of Russell.[6] So far there is some reason to say that a rule is a proposition and not a sentence. One does not want necessarily to have to do with rules that are not the same when one has to do with sentences that are not the same—for instance, a sentence and the conjunction of it with itself, or some translation of it.

2. *Cases*

Holmes says this: "It seems to me well to remember that men begin with no theory at all . . . . They begin with particular cases . . . ."[7] As Holmes does I come to rules by way of cases. Since I do this I have to get clear on what a case is. I think of a case as made up from two parts: a situation and a result. By this a case is somewhat spare: no doctrine. But this rightly reflects practice.[8] And I think of both of the parts of a case as propositions.

The first of them is naturally so. A judge starts with the situation in a case. It is usual to think of it as a set of facts. But a set of facts is a proposition. Or if it is not then it is usual to associate a proposition with it; which way one relates facts and propositions does not matter much.[9]

The second of them is less naturally so. A judge ends with the result in a case. It is perhaps usual to think that it is a command, or that it has a deontic aspect. At least a command is not propositional. But a normative analysis of some sort is not needed here for two reasons:

> (1) Anderson suggests that 'it is obligatory that p' reduces to 'if not p then V' where V is that some bad thing happens. Thus whether obligations are propositional or not

5. Belnap, *Restricted Quantification and Conditional Assertion*, in TRUTH, SYNTAX AND MODALITY 48, 53 (Studies in Logic and the Foundations of Mathematics Vol. 68, H. Leblanc ed. 1973). As to what a possible world is see A. PLANTINGA, THE NATURE OF NECESSITY 44-45 (1974).

6. Meyer, *Entailment Is Not Strict Implication*, 52 AUSTRALASIAN J. OF PHIL. 212 (1974).

7. Holmes, *Law in Science and Science in Law*, 12 HARV. L. REV. 443, 448 (1899).

8. W. TWINING, KARL LLEWELLYN AND THE REALIST MOVEMENT 132 (1973); Gilmore, *The Storrs Lectures: The Age of Anxiety*, 84 YALE L.J. 1022, 1035 (1975).

9. Fitch, *Propositions as the Only Realities*, 8 AM. PHIL. Q. 99 (1971).

perhaps one can get rid of them. And a move of the same sort may be made as to commands.[10]

(2) A result may from the start be not normative. Equity acts on the person. But at law a court does not command a defendant to do something. It says that something is to be so; if the defendant does not make it so then the sheriff does, or tries to.[11]

I give an example of a case as I understand it. Let this ordered pair of propositions be a case:

$$\langle\, p,\, q\,\rangle$$

The first member of this pair is the proposition which is the situation in this case. The second member of this pair is the proposition which is the result in this case. It might be hard to know from say the record of this case just what these propositions are. But lawyers do well enough at this.

Where this case is say *Hadley v. Baxendale* this summary by Gilmore perhaps expresses p and q:

Situation: Plaintiffs were owners of a mill which had been shut down because of a broken crankshaft. Defendants were common carriers who had been engaged to transport the broken crankshaft to Greenwich where it was to be used by the manufacturer as a "pattern" for the new crankshaft. Delivery of the old crankshaft to the manufacturer was delayed by the carrier's negligence; wherefore the mill was shut down for several days longer than it would otherwise have been.

Result: There was a jury verdict for the plaintiffs in the amount of £ 25, which was reversed in the Court of Exchequer.[12]

For now I take it to do so; that there are problems with it becomes clear as I go on.

There is one more point, that I put but crudely for now. It is that cases as I understand them are possible cases with respect to a

---

10. Anderson, *Some Nasty Problems in the Formal Logic of Ethics*, 1 Noûs 345 (1967).

11. L. FULLER & M. EISENBERG, BASIC CONTRACT LAW 47 (3d ed. 1972).

12. G. GILMORE, THE DEATH OF CONTRACT 49 (1974).

jurisdiction. The cases which I consider as to this jurisdiction need not have been decided in it, though they should be such that they might be or might have been decided in it in the weak sense that a judge who does decide them acts as he is authorized to act. Once more: there are possible cases which are not actual as well as actual cases.

3. The Standard Damage Measure

Let me look now at something more simple than the rule of *Hadley v. Baxendale*. It is more or less usual to put the standard measure of damages for breach of contract much as Williston does:

> [T]he general purpose of the law is . . . to put the plaintiff in as good a position as he would have been in had the defendant kept his contract.[13]

I look at this measure as a rule; I get back to the rule of *Hadley v. Baxendale* by way of it.

I want to look at what a rule ought to be so long as cases are as I have them. In what I quote from Friedman there is this claim by Pound: a rule is a legal precept attaching a definite detailed legal consequence to a definite detailed state of fact. And Friedman says that a result is an application of a rule.[14] So this much is clear: a rule takes a judge from situations in cases to results in cases.

What are situations and results for purposes of the standard damage measure? Well, I can apply this measure so long as I know these numbers:

P = Gain to plaintiff from contract as breached
D = Gain to defendant from contract as breached
C = Gain to plaintiff from contract if not breached

And if I do apply it I get these numbers:

C = Gain to plaintiff from contract as adjusted at law
$P + D - C$ = Gain to defendant from contract as adjusted at law

I take it that C is at least as large as P; most of the time a plaintiff would not be in court if this were not so. And I take it that gain is in cents instead of say in satisfaction.

---

13. 11 S. WILLISTON, CONTRACTS § 1338 (3d ed. W. Jaeger 1968).
14. Friedman, *supra* note 2, at 768.

So I take the situation and the result in a case to correspond to vectors, and represent them in a usual mathematical notation in this way:

$$\begin{bmatrix} P \\ D \\ C \end{bmatrix}$$

$$\begin{bmatrix} C \\ P + D - C \end{bmatrix}$$

I want to get from one to the other of them. At the level of the representations of them this is easy; I do it with this matrix, by multiplication:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

So I say that the standard damage measure applied to the situation in some case gives the result it does like this:[15]

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} P \\ D \\ C \end{bmatrix} = \begin{bmatrix} C \\ P + D - C \end{bmatrix}$$

I have now to find out what it is that the matrix that I take to represent the standard damage measure represents; that is, what the standard damage measure as a rule is. To start with this matrix represents a mathematical function; but there is more.

4. *Rules as Functions*

Situations in cases are propositions and results in cases are propositions. Therefore a rule is perhaps best thought of as a function from propositions to propositions. Then the standard damage measure is a function from propositions such as this:

> That gain to plaintiff from contract as breached is P & gain to defendant from contract as breached is D & gain to defendant from contract if not breached is C

And it is a function to propositions such as this:

---

15. R. ALLEN, BASIC MATHEMATICS 364-402 (1962).

> That gain to plaintiff from contract as adjusted is C & gain to defendant from contract as adjusted is $P + D - C$

I have so far associated—'identified' is too strong—these propositions with vectors which characterize them. This correspondence is significant; I should consider it with care. The relation of my mathematical formalism to the way I conceptualize a rule is this: the mathematical entities correspond to and stand for the entities of my conceptualization; the mathematical system is isomorphic to the conceptualized system. This should be enough; anything more is more than I need.[16]

Let the function f be a rule which takes a judge from the proposition p to the proposition q. I get this:

$$f(p) = q$$

In contexts outside the law it is not unusual casually to identify rules with functions.[17] Likewise let the function f be a set of ordered pairs of this sort:

$$\langle\, p,\ q \,\rangle$$

In contexts outside the law one does this too.[18]

I look at $\langle\, p,\ q \,\rangle$ as a case. So a rule is no more than the set of cases that are made up from the situations to which this rule applies together with the results which it directs. For Holmes men begin with particular cases. Also for the legal realists "cases past and potential are the essential substance of the field of law."[19] The way that I look at a rule reflects their view; but it contrasts sharply with what Simpson has said is the "predominant conception today," that law "consists of a system of rules."[20]

---

16. MacKinnon, *Ontic Commitments of Quantum Mechanics*, in LOGICAL AND EPISTEMOLOGICAL STUDIES IN CONTEMPORARY PHYSICS 255 (Boston Studies in the Philosophy of Science Vol. 13, R. Cohen & M. Wartofsky eds. 1974). On the difficulty of distinguishing between identification and correspondence see Sklar, *The Evolution of the Problem of the Unity of Science*, in PHILOSOPHICAL FOUNDATIONS OF SCIENCE: PROCEEDINGS OF SECTION L, 1969, AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE 535, 541 (Boston Studies in the Philosophy of Science Vol. 11, R. Seeger & R. Cohen eds. 1974).

17. Thomason, *Introduction* to FORMAL PHILOSOPHY: SELECTED PAPERS OF RICHARD MONTAGUE 1, 38 (R. Thomason ed. 1974).

18. R. MONTAGUE, *English as a Formal Language*, in *id.* at 188, 206.

19. Bingham, *What Is the Law?*, 11 MICH. L. REV. 1, 18 n.17 (1912).

20. Simpson, *The Common Law and Legal Theory*, in OXFORD ESSAYS IN JURISPRUDENCE 77, 79 (2d ser. A. Simpson ed. 1973).

Russell states: "The supreme maxim in scientific philosophising is this: Wherever possible, logical constructions are to be substituted for inferred entities."[21] I do not infer rules from cases but construct rules out of cases.

5. *Modification*

It may be that the standard damage measure will do for the rule of *Hadley v. Baxendale*. If not, I can put the rule of *Hadley v. Baxendale* in place of it, so that C is determined by it. Or I can consider the rule of *Hadley v. Baxendale* to be a modification of it, so that one has to do with this number as well:

L = That part of loss by plaintiff not natural or contemplated

I take it that C less L is at least as large as P; I say that the standard damage measure as so modified applied to the situation in some case gives the result it does like this:

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} P \\ D \\ C \\ L \end{bmatrix} = \begin{bmatrix} C - L \\ P + D - C + L \end{bmatrix}$$

This is of course too simple. I have taken the rule of *Hadley v. Baxendale* to be a restriction of the standard damage measure. As Gilmore points out it was when announced more plausibly an extension of it, or of the several rules then used in place of it: the reaction to it was "distinctly hostile" in that it was thought that the author of it—Baron Alderson—"had gone much too far in the direction of allowing recovery."[22] Gilmore adds that it "has meant all things to all men."[23] He is right, I guess.

6. *Computability*

In the end the world is not such a simple place; but let me say more of what it would be like if it were. The standard damage measure in it would correspond to a function from ordered triples of integers to ordered pairs of integers; the rule of *Hadley v. Baxendale* in it would correspond to a function from ordered quadruples of integers

---

21. B. RUSSELL, MYSTICISM AND LOGIC 155 (1918).
22. G. GILMORE, *supra* note 12, at 50.
23. *Id.*

to ordered pairs of integers. The point is this: I can find the value of either function at any argument of it which is given to me. This is so even though there are infinitely many arguments of these functions most of which I have not thought of. And I can find this value by a mechanical procedure in a finite number of steps.

What I say wants to be made more precise. I take computation to be manipulation of pieces of language—most often numerals, or words which express numbers—according to rules; and an algorithm to be a recipe by which one computes. I adopt a usual definition, which is to be generalized in the obvious way from integers to ordered n-tuples of them:

> A function f from the integers to the integers is computable
> if and only if there is an algorithm A such that, given a
> numeral $\bar{n}$, A applied to $\bar{n}$ yields a numeral $\bar{m}$ if and only if
> $f(n) = m$.[24]

A function is recursive just in case it can be built up from the zero function, the successor function, and various identity functions by three sorts of operations: composition, primitive recursion, and minimization. This is difficult; I do not do more with it now. A function which is recursive is computable, and by Church's thesis a function which is computable is recursive. I take Church's thesis to be true, so that I do not distinguish between functions which are computable and functions which are recursive.[25]

Now the point is this: the standard damage measure and the rule of *Hadley v. Baxendale* as a modification of it are computable functions or recursive functions. If a judge is given a description of one of these rules and a description of a situation to which it applies—both descriptions of the right sort, such as I have set out—he can just compute the result, or a description of it. This is law as Leibniz loved it; he said that he had prepared

> a table, comparable in size to a map, which uses a unique
> arrangement and method to present the entire *common pri-*
> *vate law* of the Empire today, with all of its fundamental
> rules and propositions, and reduces them to *first principles*
> so that any one who understands this table, or has it lying
> before him, can decide any fact or case of private law, and at

24. Ross, *Church's Thesis: What Its Difficulties Are and Are Not*, 71 J. PHIL. 515, 516 (1974).
25. G. BOOLOS & R. JEFFREY, COMPUTABILITY AND LOGIC 19-20, 70-88, 98 (1974).

once put his finger on the basis for the decision in the table itself . . . .[26]

7. *Cases, Again*

I must say more about what a case is, or the way that one is to individuate cases. I have said that to apply the standard damage measure one might look at a case like this:

$$\langle \langle P, D, C \rangle, \langle C, P + D - C \rangle \rangle$$

It is pretty clear that when one does not have the standard damage measure in mind this way to look at a case will not do; if the situation in a case is to be an argument of a function which is a rule one has to be able to pick it out.

But then it is hard for me to say just what a case is. I start with what is required for a definite description of it. I want to be able to say something like this:[27]

$$(\iota x)A$$

Read: 'The case such that so and so.' To come out right this can refer to just one case.

But a great many cases might correspond to the same sequences of numbers; nor will the summary by Gilmore of *Hadley v. Baxendale* do, since a second case like it so far as this summary shows can come up. There are possible cases other than actual cases, and I must take them into account. The prospect is parodied by Quine:

> Take, for instance, the possible fat man in that doorway; and, again, the possible bald man in that doorway. Are they the same possible man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one?[28]

There is not a natural standard by which to individuate possible fat men in doorways; but perhaps there is such a standard by which to

---

26. Letter from Gottfried Leibniz to John Philip, March 27, 1669, in Loemker, *Introduction: Leibniz as Philosopher*, in GOTTFRIED WILHELM LEIBNIZ: PHILOSOPHICAL PAPERS AND LETTERS 1, 6 (2d ed. L. Loemker transl. & ed. 1969) (emphasis in original).
27. 1 A. CHURCH, INTRODUCTION TO MATHEMATICAL LOGIC 41 (1956).
28. W. QUINE, FROM A LOGICAL POINT OF VIEW: 9 LOGICO-PHILOSOPHICAL ESSAYS 4 (2d ed. rev. 1961).

individuate possible cases. It has to do with the doctrine of res judicata: by it a judge has to decide whether a case has been decided; if it has, he is not to decide it again. Then possible cases are different so long as they might both be actual. This does in part; one must distinguish results as well. By it descriptions of cases may be a good bit more complicated than I have yet made them. But as before cases are just ordered pairs of propositions, and so ordered pairs of sets.

I do not pursue this. Nor do I go on from situations to results. But I make this point: since cases can be distinguished only by descriptions of them there can be only denumerably many of them. So a function such as the standard damage measure defined not as I have taken it to be but on situations in cases as properly individuated—or on mathematical entities that correspond to such situations—can still be computable or recursive.[29] Exception: one may have to do with demonstrative elements, thus, 'this much'.

8. Contracts as Inferred Entities

But this is not enough because situations in cases are described wrong. Situations—results as well—ought in the first instance to be described not as so far they have been but in terms that have to do with the world as it is apart from law. Law takes one from situations to results, both described in these terms. As it does so it often redescribes situations at least; but it does not start from its redescriptions. The notion of a rule that I have to do with is most of the time not adequate when this is seen to be so, in so far as it is tied to computability or recursiveness.

Both the standard damage measure and the rule of *Hadley v. Baxendale* presuppose this:

There is a contract.

This is clear: a contract is not a thing that is in the world as it is apart from law. A judge must find it, or make it up; he must do so in situations not described with regard to it. The trouble comes from how he does it.

A contract is not a construction from aspects of the world as it is apart from law: it is not say the consequences of it. It is instead inferred—as a rule is not from cases—from aspects of this world. So

---

29. Gabbay, *A General Theory of the Conditional in Terms of a Ternary Operator*, 38 THEORIA 97, 99-100 (1972).

that a person has said 'I promise' and so on is evidence that there is a contract; but it is not more than this.

But then there can be no mechanical procedure by which to go from the world as it is apart from law to contracts; that is, there can be no computable or recursive function from situations in cases as law comes on them to situations in cases as the standard damage measure or the rule of *Hadley v. Baxendale* applies to them, in the sense that as I have it these last can be such functions. Thus, more importantly, there can be no such functions from situations as law comes on them to what one would want for results. It is not so much that whether there is a contract in a certain situation cannot most of the time be found out; but this may require wit and insight, not a mere ability to compute.[30]

9. *Conclusion*

I have started with a simple notion of a rule. I have gone on to make this notion more precise; then I have made plain the limitations of it. They are more substantial than they might be: thus there might be a contract just in case the parties to it have said certain things, or have used seals. But the rule that a judge is to do justice is not recursive; something of this sort would betray it.

A last note. I construct rules but I infer contracts. In part it just works out this way. But in part too I do this because I take it that contracts explain law in a way that rules do not. There are just too many rules. As Wittgenstein puts it: "Can't anything be derived from anything by means of *some* rule . . . ?"[31]

---

30. M. STEINER, MATHEMATICAL KNOWLEDGE 102-07, 136 (1975).

31. L. WITTGENSTEIN, REMARKS ON THE FOUNDATIONS OF MATHEMATICS 5e (G. von Wright, R. Rhees & G. Anscombe eds. G. Anscombe transl. 1967) (emphasis in original).