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The Effects of Credit Risk on Dynamic Portfolio Management: A New Computational Approach

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Abstract

The study investigates the role of credit risk in a continuous time stochastic asset allocation model, since the traditional dynamic framework does not provide credit risk flexibility. The general model of the study extends the traditional dynamic efficiency framework by explicitly deriving the optimal value function for the infinite horizon stochastic control problem via a weighted volatility measure of market and credit risk. The model's optimal strategy was then compared to that obtained from a benchmark Markowitz-type dynamic optimization framework to determine which specification adequately reflects the optimal terminal investment returns and strategy under credit and market risks. The paper shows that an investor's optimal terminal return is lower than typically indicated under the traditional mean-variance framework during periods of elevated credit risk. Hence I conclude that, while the traditional dynamic mean-variance approach may indicate the ideal, in the presence of credit-risk it does not accurately reflect the observed optimal returns, terminal wealth and portfolio selection strategies.

Journal of Economic Literature Classification: G0, G10, C02, C15

Keywords: Dynamic Strategies; Credit Risk; Mean-Variance Analysis; Optimal Portfolio Selection; Viscosity Solution; Credit Default Swaps; Default Risk; Dynamic Control

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1.0 Introduction

Managers of financial assets, such as stocks and bonds, typically seek to maximize their expected returns on investment for a given level of risk. In fact the ultimate goal of modern investment theory is to construct an optimal portfolio of investments from a set of risky assets. Investment in these risky assets generally depends on the discount rate, the market return and the volatility parameters. From the classical pedagogical work of Markowitz (1952, 1959), the optimal portfolio for a given level of risk and set of constraints can be derived under the "mean-variance (MV) efficiency frontier" using known optimization algorithms such as quadratic programming (Zhou *et al* (2000), Bielecki *et al* (2005)).

Under any given mean-variance optimization framework, the portfolio with the highest expected return and the smallest minimized variance (market risk) is said to be more efficient. These properties of efficient portfolios are central to both static and dynamic optimization. However, a recognizable weakness in this methodology is the apparent exclusion of a credit risk[2] measure from the investor's operational risk frontier. As investors continuously seek higher returns on investments it becomes increasingly impractical to ignore the effects of credit risk on expected portfolio returns[3]. In fact, in any dynamic

[2] Here credit risk is defined as the risk of default or the deterioration in credit quality of a reference entity that is part of the portfolio that the investor holds. For further reading on credit risk the reader may look at Jarrow *et al* (2001), Duffie (1999), and Dunbar (2008).

[3] While complete hedging is not possible, the investor could buy credit default swaps (CDS) as a hedge against credit quality changes (risk) in the debt component of the investment portfolio opportunity set, however the cost of the CDS premium would ultimately result in a net reduction in the overall terminal hedged payoffs $\left(u^* - p_{cds} = u_h^* \;\; where\ u_h^* < u^*\right)$, because the terminal hedged payoff would be lower by the cost of the hedge.

portfolio management setting, since an infinitely lived agent may hold his portfolio $\sum_{i=1}^{n} W_i$ over some investment horizon $\tau > t$, the implied credit risk of holding the securities in the portfolio increases with the investment horizon. As such, an ideal asset in period t_0 may disappear (default) prior to the intended maturity period $\tau > t$, which poses some level of risk to the agent's expected level of return.

In section 4.0 the paper develops a baseline Markowitz-type and an extended market/credit-risk dynamic optimization set of models that were used to investigate the optimal investment strategy under credit and market risk[4]. More specifically the study derived; (a) closed-form solutions for optimal asset allocation and investment strategy in both a mean-"market" risk and a mean-"market/credit" risk framework; and (b) the reaction function of the risk-averse agent when faced with both market and credit risk. Not surprising the later analyses of the study illustrates that given added investment risk (such as credit risk) an agent will react by modifying his efficient selection strategy which could result in a lower optimal terminal return as demonstrated in figure 1.

In deriving the study's extended model we start from the basic formulation of the Markowitz (1952, 1959) framework where we denote the returns of n risky assets by a $n*1$ vector $\boldsymbol{R}$, the unobserved future return ξ_t (assumed unknown), and the instantaneous variance-covariance matrix of stock and bond fund returns;

$$\sum w_i E(\xi_t) = \mu \qquad \sum_{i=1}^{z}\sum_{j=1}^{z} w_i w_j \operatorname{cov}(i, j) = \Gamma \tag{1.1}$$

[4] Such a model develops a summation of the standard deviation of the various risk components as a proxy for the increased catastrophic impacts of total (market and credit) risk on an investor's portfolio.

where $\text{cov}(i, j)$ is the covariance between returns from investments in *i* and *j*. The mean-variance optimization solves the asset allocation w_t, which minimizes the portfolio risk σ_p^2, while achieving a certain target return μ^*. Thus our problem is to minimize

$$\sigma_p^2 \Rightarrow \left(\sigma_m^2 + \sigma_c^2\right) = \sum_{i=1}^{z}\sum_{j=1}^{z} w_i w_j \,\text{cov}(i, j) \tag{1.2}$$

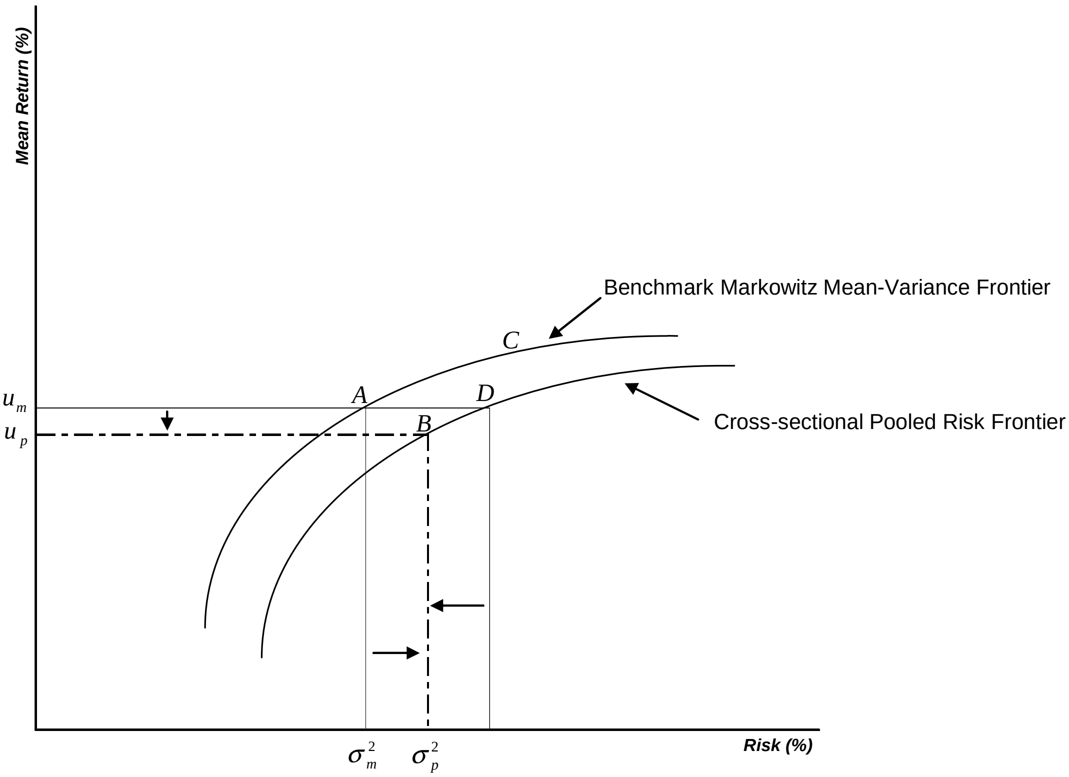
Subject to the constraints

$$\sum_{i=1}^{n} w_i E(\mu) \geq \mu^* \qquad and \qquad w_1 + w_2 + ... + w_n = 1 \tag{1.3}$$

where the portfolio strategies are defined by their weights $w_1 + ... + w_n$, the short sales constraint $\psi_i \leq w_t \leq \eta_i \quad (i = 1, 2, ..., n)$ and where we use $\sigma_p^2 \ \left(\sigma_p^2 = (\sigma_c^2 + \sigma_m^2)\right)$ to represent a pooled cross-sectional measure of credit and market risk in the extended model framework. In the traditional Markowitz/Merton framework σ_p^2 represents market risk $\left(\sigma_m^2\right)$.

Expression 1.2 demonstrates that under the extended framework, when an investor makes an investment in a firm, the investor is exposed to both market and credit risk. Economic theory and recent empirical work by Dunbar (2008) suggests that market and credit risk are intrinsically related to each other. In fact on the “investment risk continuum”, when the value of the firm’s assets unexpectedly changes, market risk is created, which increases the probability of default, subsequently generating credit risk. This underscores the need for work in portfolio optimization that investigates the dual impacts of both credit and market risk on expected returns. Moreover, as observed during the credit crisis of 2008, when financial markets experience a “flight to quality” which typically characterizes periods of high credit risk, the rational agent will receive a lower total return because of this lower portfolio efficiency curve, as depicted in figure 1.

Figure 1: Risk Return Characteristics under Pooled Market and Credit Risk



Note: Point A represents the efficient allocative market-risk adjusted returns from a risk-free investment and an investment in A. Point B represents the efficient allocative pooled-risk investment returns as credit risk increases and investors move to a lower expected return because of a flight to quality.

The remainder of the paper is organized into four sections. In section 2 we explore the existing literature on integrated market/credit risk portfolio optimization. Section 3 lays out the basic setup of the model investigated in this paper. This section introduces the dynamic framework of the model and discusses the technical background for optimal dynamic asset allocation, giving some overview of current dynamic asset allocation methodologies and the analytical procedure for including the credit risk proxy to the optimization process. Section 4 derives the optimal portfolio problem under credit risk. The model is later calibrated to U.S. interest rate, stock return, credit and market risk data. Section 5 presents the data, some representative calculations and discussions on the main empirical findings regarding the role of credit risk in the dynamic mean-variance framework. Section 6 summarizes the finding and proposes areas of future research.

2.0 Background Review of Credit Risk and Asset Allocation Models

The literature on portfolio management has focused in recent years on asset allocation using a dynamic framework, which has its roots in Markowitz's (1952, 1959) celebrated static mean-variance analyses. Since Markowitz's (1952, 1959) publication, one of the most spectacular breakthroughs to have emerged in stochastic portfolio theory was from Merton (1971, 1973), who derived optimal dynamic portfolio allocation in continuous time where security prices were allowed to follow a diffusion process. Since Merton's work there have been several extensions and applications to the classical optimization problem, particularly the derivation and solution to the optimization problem via the Hamilton-Jacobi-Bellman (HJB hereafter) equation (Zhou and Li (2000), Bielecki et al (2005)). This paper builds on Merton's vibrant strand of portfolio optimization theory, while extending the Linear Quadratic approach of Zhou and Li (2005) to incorporate credit risk into the dynamic asset allocation problem.

The effect of credit risk on the asset allocation problem has received relatively little attention, with the exception of recent work by Jobst *et al* (2001), Ramaswamy (2002) and Jobst *et al* (2006). The growing influence of credit risk in financial markets is highlighted by the 2001 proposal by the BASEL committee on banking supervision for a stricter focus on the credit risk management of banks' investment portfolio, see for example Gordy (2003), Dangl *et al* (2003) and Lopez (2003). Prior negligence of credit risk in dynamic portfolio management although surprising, maybe due in part to the relative under developed credit markets that were in existence when Merton first introduced his acclaimed paper in 1971. Since then there has been an exponential growth in financially engineered products[5] and the size of the global bond markets, (Dunbar (2008)). The British Bankers Association estimates that the credit derivative market grew from a notional $180 billion in 1997 to $5.0 trillion in 2004 and is expected to reach upwards of $8.2 trillion in 2006[6]. A review of the credit markets has shown that while overall quality of global credit has deteriorated the volume of corporate bonds (corporate credit risk) has risen dramatically over the past few years.

Although portfolio optimization models incorporating credit risk are still in a state of infancy, recent papers by Ramaswany (2002) and Jobst et al (2006) conclude that portfolio diversification of credit risk is much more difficult than for market risk. Hence credit risk appears to play a far more significant role in the outcome of an investor's terminal return than previously believed. So in assessing the effects of portfolio risks on terminal investors' returns it is imperative to derive a measure of total portfolio risk inclusive of credit and market risk, which leads to this paper; The effects of credit risk on dynamic portfolio management. However, while this paper maybe closely related to papers in the recent literature, none of these approaches by practitioners allow for a cross-sectional pooled (market-credit) risk measure that simulates the agent's total portfolio risk exposure.

The growing importance of the corporate debt component of the overall global debt market (relative to government debt) indicates a growth in global credit risk, which

[5] The creation of most of these credit derivative products were in response to the significant growth in corporate risk. These products such as Credit Default Swaps were generally created as a protection against the burgeoning global debt market.

[6] In an August 31st 2006 Wall Street Journal article "Can Anyone Police the Swaps" the current CDS market was estimated at upwards of $17 trillion.

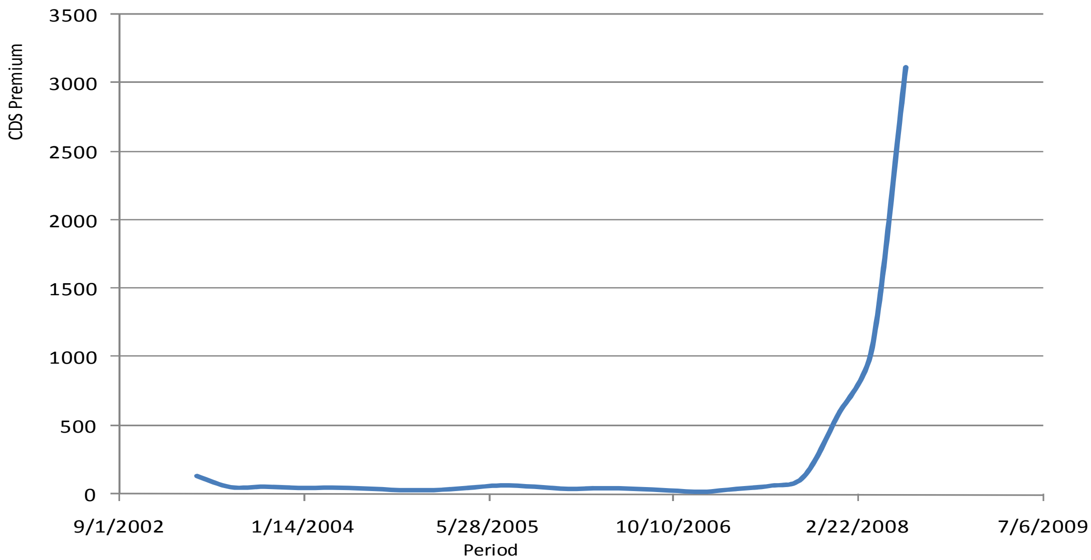
in itself partly explains the observed exponential growth in the use of credit default swaps to mitigate this growing counterparty risk. Much of this growth in counter-party risk stems from the ever increasing demand by banks, insurance companies, institutional investors and hedge funds seeking credit risk insurance to cover risky long bond exposures. As far back as November 2002, then Fed Chief Alan Greenspan appearing before the Foreign Relations Committee suggested that one positive outcome of this growth is the strengthening of the financial sector by spreading credit risk more broadly across the entire sector as against having it all concentrated among a few participants.

Observed investor attitudes suggest that investors show aversion to both market and credit risk. For example during the height of one of the most severe credit crisis of this decade, Bear Stearns (BSC) equity may have initially exhibited relatively low levels of market risk (volatility). However investors' perception of the firm's credit risk as reflected in the meteoric rise in the price of the firm's credit default swap[7] (CDS) resulted in a number of investors abandoning Bear Sterns equity and debt products prior to its near collapse.[8] See figure 2.

[7] Credit-default swaps (CDS) are financial instruments underwritten on bonds and loans that are used to speculate on a company's ability to repay its debt (credit risk). They pay the buyer face value in exchange for the underlying securities or the cash equivalent should a borrower fail to adhere to its debt agreements. A rise (decline) in the price of the CDS indicates deterioration (increase) in the perception of credit risk. Increased volatility in the price of the credit default swap provides an indication of the level of credit risk in a particular referenced entity.

[8] The 2007 credit crisis created by the elevated volatility in the U.S. credit market spread uncertainty and apprehension among market participants in many countries including some emerging markets. This resulted in an unprecedented "flight to quality" by a number of investors, where they reallocated their investment portfolios away from perceived credit risky assets into more secured lower yielding products such as U.S. Treasury and Municipal Bonds.

Figure 2: Quarterly Price Changes for Bear Sterns Credit Default Swap for the period 2003 - 2008



The graph shows that a deterioration in the credit quality of MBIA resulted in a dramatic rise in the credit risk premiums that investors pay to insure The firms debt against the risk of default. This surge in the firm's default risk premium was as a result of deteriorating U.S. and global credit markets and MBIA's esposure to corporate and derivative credit rating gaurantees

The study's makes three main contributions; firstly, it proposes a new perspective in portfolio optimization that derives a probability weighted pooled credit-market risk measure for use in deriving the agent's optimal dynamic asset allocation. Secondly, it provides a framework for the evaluation of the investor's optimal portfolio strategy under market and credit risk conditions, using quadratic utility maximization. Thirdly, it derives a convenient approach for determining an agent's risk aversion coefficient that is useful in helping to explain the portfolio choices of investors. To illustrate the analytical flexibility and potential of the extended dynamic methodology, an empirical specification was tested under a number of scenarios involving credit and market risk experiences to see how closely the results reflect actual market conditions. The study adopts changes in historical CDS bid-ask spreads as a proxy of credit risk. This is in keeping with the approach by a number of studies in the literature that have used CDS spreads as determinants of default risk, such as Longstaff *et al* (2005), Das and Hanouna (2006) and Dunbar (2008).

3.0 The Model Structure: Dynamic Framework and Technical Background

This section contains the basic setup of the extended dynamic optimization model investigated in this paper. As discussed in section 1.0, the study develops two alternative models that were used as the primary tools for investigating the investor's optimal dynamic solution given both market and credit risk. We first develop a benchmark model that allows us to determine the investor's attitude to market risk; next we creatively exposed the agent to credit risk through a more complete risk measure so as to determine any changes in investor's attitudes to credit risk. We follow the usual conditions for a dynamic portfolio optimization strategy[9] where the risky security is allowed to follow a geometric Brownian motion and a constant risk-free rate.

Assumptions: (i) Credit risk is measured by changes in the credit default swap (CDS) of each firm.
(ii) In a short sale, an investor sells borrowed shares in the hope of profiting by buying them back later at a lower price.

The study considers a pure exchange, frictionless economy with a finite horizon $[0,\tau]$ for a fixed $[\tau > 0]$. Following the usual conditions of portfolio optimization, trading can be discrete or continuous and traded are equity products as are both defaultable and default-free zero coupon bonds of all maturities. The portfolio of U.S. Treasury bonds serves as the numeraire. The underlying uncertainty in the economy is represented by a fixed filtered complete probability space $\left(\Omega, F, \mathbb{Q}, \left\{F_t^Z\right\}_{\tau\geq 0}\right)$ on which is defined a

[9] In a dynamic context we construct mean-variance efficient portfolios by optimally allocating wealth across securities as the expected returns and variance-covariance changes over time. As discussed in footnote 2 we may hedge the change in the investment opportunity set, however the hedged payoffs will be lower because of credit risk

standard $\left\{F_t^Z\right\}_{t\geq 0}$ adapted Z-dimensional Brownian motion $X(t) \equiv \left(X^1(t),...,X^Z(t)\right)'$ (Duffie (1996)). The probability space $\left(\Omega, F, \mathbb{Q}\right)$ with the filtration $\left\{F_t^Z \middle| t \leq a \leq \tau\right\}\left(-\infty \leq t < \tau \leq +\infty\right)$, Hilbert space $\mathbb{H}$ equipped with the inner product $\langle . , . \rangle$ and a Euclidean norm $\|\bullet\|_{\mathbb{H}}$, defines the Banach space. Now given the Lagrangian specification;

$$L_F^2\left(0,\tau,\mathbb{H}\right) = \left\{\varphi(\bullet)\left|\begin{array}{l}\varphi(\bullet)\, is\, an\, F_t - adapted, \mathbb{H} - valued\ measurable\ process\, on\left[\tau, t\right] \\ and\ E\int_t^\tau \left\|\varphi(\tau,w)\right\|_H^2 dt < +\infty\end{array}\right.\right. \tag{3.1}$$

With Euclidean norm;

$$\left\|\varphi(\bullet)\right\|_{F,2} = \left(E\int_t^\tau \left\|\varphi(\tau,w)\right\|_{\mathbb{H}}^2 dt\right)^{\frac{1}{2}} < +\infty \tag{3.2}$$

Where the price of the default-free bond is given by;

$$\begin{cases} d\mathbb{P}_0(t) = r(t)\mathbb{P}_0(t)dt & t \in \left[0,\tau\right] \\ \mathbb{P}_0(0) = \mathbb{P}_0 > 0 \end{cases} \tag{3.3}$$

and equity price is stochastic and risky in the economy and the price follows a $It\hat{o}'s$ process that is represented as;

$$\begin{cases} d\mathbb{P}_i(t) = \mathbb{P}_i(t)\left\{b_i(t)dt + \sum_{j=1}^{m}\left(\sigma_m^2 + \sigma_c^2\right)(t)X^j(t)\right\}, & t \in \left[0,\tau\right] \\ \mathbb{P}_i(0) = \mathbb{P}_i > 0 \end{cases} \tag{3.4}$$

where b_i is the expected return on equity per unit of time, $\left(\sigma_m^2 + \sigma_c^2\right) > 0$ is the volatility vector[10] of the real return on equity per unit of time, $r_t \geq 0$ is the instantaneous spot rate return and where σ_m^2 represents market risk.

The following assumptions are made for the study's quadratic programming models:

1. *The portfolio considered in this paper is assumed to be self financing and continuously rebalanced.*

2. *Financial Markets are dynamically complete.*

3. *For the rational investor it is assumed that the value of the expected terminal wealth "**d**" satisfies* $d \geq E\left(W_0^{e^{\int_0^\tau r(s)ds}} \right)$.

4. *It is assumed that volatility in the credit default swaps of firms is a proxy of credit risk in financial markets.*

5. *It is assumed that in the familiar Markowitz mean-variance model σ_p^2 is a probability weighted average of market risk $\left(\sigma_m^2\right)$ and credit risk $\left(\sigma_c^2\right)$*

6. *$W(t)$ is predictable with respects to the information set $F_{(0)}$ and meets the usual integrating conditions.*[11]

Now let's consider an agent (investor) with an initial wealth $W_0 > 0$ and total wealth over a fixed time interval $\tau \geq 0$ of $W(t)$. Moreover, the agent also receives a stream of investment income $E\{e_t\}$ that he can use to buy additional payoffs of *n* different given assets at prices *p*. The investor's optimization problem can be represented by expressions 1.1 – 1.3 or as;

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[10] We assume that the volatility vector $[\sigma_m, \sigma_c]$ has full rank. This assumption ensures that neither the bond nor the stock is a redundant asset in the economy.

[11] Harrison-Pliska (1981) and Duffie (1996)

$$\max_{u \in U} \in [u(t)] \ s.t. \sum_{i=1}^{n} W_i \tag{3.5}$$

where $u(t)$ = the expected terminal returns of a portfolio of stocks (*s*) and bonds (*b*), and also where $E\left(W_{(t+1)}\right) = E\left(e_t + W_t\right) = 1$.

Given the agent's investment strategy $\{u(t)\}_0^\infty$, asset allocation process $\{w(t)\}_0^\infty$ that maximizes the expected utility of his wealth and his information set $\left\{F_t^Z\right\}_{\tau \geq 0}$, the preferences of the infinitely lived agent can be represented by the time additive utility function in equation 3.6, with varying levels of risk aversion. Here we follow the Hansen-Richard (1987) and Cochrane (2008) approach which links marginal utility and the mean-variance frontier.

$$E\left[\int_0^\infty e^{-\rho t} u(t)\left[W\left(t; x, w\right)\right] dt\right] \tag{3.6}$$

We call $u_i(t), i = 0,1,2,...,m$, the total market value of the agent's wealth in the m bond $\left(B_t\right)$, and stock $\left(S_t\right)$, where $u(t) = (u_1(t),...,u_m(t))$ is a control variable or investment strategy which may change over time *t*. Since $u(t)$ is self financing it means that;

$$\frac{dW_t}{W_t} = \left[W_0(t)\frac{dS_0}{S_0}(t) + w'(t)d\underline{R}(t)\right], \tag{3.7}$$

where

$$d\underline{R} = \left(\frac{dS_1}{S_1}, ..., \frac{dS_N}{S_N}, \frac{dB_\tau}{B_\tau}\right) \tag{3.8}$$

$W(t)$ is the portfolio value at *t*, and $w'd\underline{R}$ is a scalar product. The terminal payoff d_τ has finite variance.

Definition: *The portfolio strategies are defined by the (N+1) dimensional vector of weights $w(t)$ in assets $1,...,n$ and the zero-coupon bond $B_\tau(t)$. Without loss of generality $S_n(t)$ is defined as reinvested dividends.*

From the agent's utility function in expression 3.6, the agent will try to maximize his expected optimal terminal investment returns (u^*) given his decisional wealth constraint;

$$\begin{cases} dW(t)=\left\{r(t)W(t)+\sum_{i=1}^{Z}(b_i(t)-r(t))u_j(t)\right\}dt+\sum_{j=1}^{Z}\sum_{i=1}^{Z}\left(w_{1t}\left(\sigma_{mij}(t)+\sigma_{cij}(t)\right)\right)+\left(w_{2t}\left(\sigma_{mij}(t)+\sigma_{cij}(t)\right)\right)u_i(t)dX^j(t) \\ w(0)=W_0\geq l(0),\ \ Otherwise \end{cases} \tag{3.9}$$

given that $w_1(t)$ denotes the wealth invested in risky assets (stocks), and $w_2(t)$ is the wealth invested in nominal bonds.[12] These weights sometimes have additional constraints,

$$\psi_i \leq w_i \leq \eta_i \qquad i=1,2,...,v$$

where $\psi_i \geq 0$, which represents the "short selling" constraint, is a positive constant, σ_{mij} and σ_{cij} represents the market and credit risk exposure the investor faces, $u_i(t),\ i=0,1,...,m$ denotes the total market value of the agent's wealth in the i^{th} bond or stock and where $l(0)$ is the lower bound on the decisional variable $(W_0 \geq 0)$. Note that due to the positive lower bound imposed on investments it is clear that wealth should also have a lower positive bound $\overline{W}$, since the agent needs some minimal amount of wealth for investment which cannot be negative. This condition is expressed in the assumption below;

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[12] w_1 *and* w_2 are adapted to the information structure F_t. The weights represent the agent's investment strategy.

Assumption: *The process* W_0 *describing the agent's wealth is subject to the following constraint* $W(t) \geq l(t) \qquad \forall t > 0$, *where the strictly positive function* $l(t),\ t > 0$ *represents the solvability level.*

Following Hansen and Richard (1987) the study defines $u(\bullet)$ by its Riesz representation in the Hilbert Space as an admissible portfolio strategy[13] if $u(\bullet) \in L_F^2(0,\tau;\mathbb{R}_+^Z)$. In fact the pair $\left(W(\bullet), u(\bullet)\right)$ is an admissible pair if $W(\bullet) \in L_F^2\left(0,\tau;\mathbb{R}_+^Z\right)$ is a solution of the stochastic differential equation in expression 3.9, where the control $u(\bullet) \in U[0,\tau]$.

4.0 Dynamic Portfolio Problem under Credit Risk

To add clarity to the results this section lays out the framework for the determination and addition of credit risk to the agent's investment risk exposure. Building the extended dynamic optimization model, the study uses the popular dynamic Merton framework as the benchmark, but allows the agent's investment risk frontier to be a pooled parameter of credit and market risk. Our problem is then to minimize this complete risk measure for a given level of return; expression 1.2.

Definition: *Credit risk is defined as the risk of default or the deterioration in credit quality of a reference entity that is part of the portfolio that the investor holds.*

The study uses credit default swaps (CDS) as a proxy for credit risk in financial markets. The CDS is a bellwether of increasing (eroding) investor confidence in corporate creditworthiness. A rise (decline) indicates worsening (improving) perceptions of credit quality. Credit default swaps are financial instruments that are used to speculate on a company's ability to repay debt. Unlike the extended model where we represented

[13] The set of strategies that satisfy the equality and inequality constraints are called the admissible set of the quadratic programming problem.

portfolio risk as $\sigma_p^2 = \left(\sigma_m^2 + \sigma_c^2\right)$, the benchmark model assumes that the agent's portfolio risk is only a function of market risk $\sigma_p^2 = \left(\sigma_m^2\right)$. Moreover, the study's cross-sectional pooled risk framework is assumed to nest the traditional mean-variance market risk model. In the absence of credit risk, the agent's wealth constraint depicted in expression 3.9 converges to the benchmark dynamic optimization model in equation 4.0.

$$\begin{cases} dW(t) = \left\{ r(t)W(t) + \sum_{i=1}^{Z} (b_i(t) - r(t))u_j(t) \right\} dt + \sum_{j=1}^{Z} \sum_{i=1}^{Z} (\sigma_{kij}(t))u_i(t)\,dX^j(t) \\ W(0) = W_0, \;\; Otherwise \end{cases} \tag{4.0}$$

Where the general constrained controlled linear stochastic differential notation in 4.0 can be simplified for mathematical ease without loss of generality to notation 4.1 below;

$$\begin{cases} dW(t) = \{A(t)W(t) + B(t)u(t) + f(t)\}dt + \sum_{j=1}^{Z} D_j(t)u(t)\,dX^j(t) \\ w(0) = W_0 - (d-u), \;\; Otherwise \end{cases} \tag{4.1}$$

Where;
$A(t)$ *and* $f(t)$ *are scalars*;
$u(\bullet) \in L_F^2(0,\tau;\mathbb{R}_+^Z)$;
$w(t) = W(t) - (d-u)$;
$A(t) = r(t)$;
$f(t) = (d-u)r(t)$;
$B(t) = (b_1(t) - r(t), ..., b_m(t) - r(t)$;
$D_j(t) = (\sigma_M(t) + \sigma_C(t)), ..., (\sigma_M(t) + \sigma_C(t))$;
$B(t)' \in \mathbb{R}_+^Z$ *and* $D_j(t)' \in \mathbb{R}^Z\,(j = 1, ..., z)$ *are column vectors.*
The matrix $\sum_{j=1}^{Z} D_j(t)' D_j(t)$ *is non Singular.*

Following Vasicek (1977), it is assumed that credit risk c_t (like market risk r_t) follows an Ornstein-Uhlenbeck diffusion process;[14]

$$dc_t = \kappa(\overline{c}_t - c_t)dt + \sigma_p^2 dz_c \tag{4.2}$$

where $\overline{c}$ is the long-run mean, σ_p^2 is the volatility parameter and κ is the mean reversion.

From assumption *5* in section *3* the investor's risk frontier is a combination of both market and credit risk which may be represented as;

$$\sigma_p^2 = \left(\sigma_m^2 + \sigma_c^2\right) \tag{4.3}$$

$$\Rightarrow \sigma_p^2 \approx \frac{(n-1)\sigma_m^2 + (m-1)\sigma_c^2}{n+m-2} \tag{4.4}$$

where n and m are the number of observations in both sets of risk data. From figure 2, notice that when Bear Stern's credit risk was non-existent during early 2004 through late 2006, the pooled risk measure would only reflect market risk.

4.1 The Extended Dynamic Cross-Sectional Risk Model

In this section, we derive the extended dynamic efficiency frontier (Cross-sectional Pooled Risk frontier), in the variance-expected return space $\left(\left(\sigma_{ij}^2(W_t)\right), E\left(W_T\right)\right)$.

$$\Rightarrow \left(\left(\sigma_m^2(W_t) + \sigma_c^2(W_t)\right), E\left(W_T\right)\right)$$

The CSPR frontier consists of payoffs that minimize the portfolio risk which is defined in the following way;

$$\min_{\{u(\bullet)\in U[t,\tau]\}} E\left\{\tfrac{1}{2}W(\tau)^2\right\} = \min_{\{u(\bullet)\in U[t,\tau]\}} E\left\{\left[W(\tau)-d\right]^2 + 2\mu\left[EW(\tau)-d\right]\right\} \tag{4.1.1}$$

[14] Where $\overline{c}$, σ_p^2 and κ are positive constants and dz_c is standard Brownian motion.

Subject to the constraints in "***R***" solved for different values of the expected terminal wealth "***d***"

$$(R) = \begin{cases} E\left[W_\tau\right] = d \\ u(\bullet) \in L_F^2\left(0, \tau; \mathbb{R}_+^z\right) \\ \left(W(\bullet), u(\bullet)\right) \textit{satisfies equation } 4.0 \end{cases}$$

Given expression 4.1.1 and the controlled linear representation in equation 4.0, our objective is to find a portfolio with the minimum market and credit risk for a given optimal $u(\bullet)$ that minimizes the terminal cost function

$$J(\tau, w : u(\bullet)) = E\left\{\tfrac{1}{2} W(\tau)^2\right\} \tag{4.1.2}$$

Where the associated value function of the controlled linear stochastic differential equation in 4.1 and the stochastic control problem in expression 4.1.2 (the LQ problem) is defined as;

$$V(\tau, w) = \inf_{u(.) \in u(t,\tau)} J(\tau, w : u(\bullet)) \tag{4.1.3}$$

As a necessary condition for optimality, the Hamilton-Jacobi-Bellman (HJB) equation[15] related to the family of stochastic control problems in expression 4.1.3 is given as;

$$\begin{cases} v_t(\tau, w) + \inf\limits_{u \geq 0} \left\{ v_w(\tau, w)\left[A(t) w + B(t) u + f(t)\right] + \dfrac{1}{2} v_{ww}(t, w) u' D(t)' D(t) u \right\} = 0 \\ \\ v(\tau, w) = \dfrac{1}{2} w^2, \quad \textit{Otherwise} \end{cases} \tag{4.1.4}$$

Where $D(t)' = \left(D_1(t)', \ldots, D_z(t)'\right)$

[15] The reader is referred to the optimality conditions for the HJB framework in Fleming and Rishel (1975) for additional insights.

The value function $V(\tau, w)$ defined in equation 4.1.3 is a constrained viscosity solution of the HJB equation in 4.1.4 on the interval $\left[l_t, +\infty\right]$. In fact using the uniqueness theorems of Crandall and Lions (1983, 1990, 1991) we can show that the value function is a unique smooth viscosity solution of the HJB equation.

Expressed in terms of the investment strategy and adjusted for credit risk, the HJB equation 4.1.4 may be represented as

$$\underset{\{(u,w)\in A\}}{Sup}\left\{\left\{w_1\left[\mu_S - r_R\right] + w_2\left[r_N - \pi\right] + Wr_R\right\}\frac{dd}{dw} + u(\bullet) + \frac{1}{2}\left\|w_1\left(\sigma_m^2 + \sigma_c^2\right) - w_2\left(\sigma_\pi\right)\right\|^2 \frac{d^2 d}{dw^2} = \rho d\left(w\right)\right. \tag{4.1.5}$$

Where $\rho > 0$ is a unique strict global "minimizer" or constant discount factor, and

$$H\left(x, Dv(x), D^2v(x)\right) \triangleq \sup_{\theta\in[0,1]}\left\{U(x) + \left[\left(\theta\sigma\lambda + r\right)x + \alpha w - \eta\alpha w\frac{e^{\delta\tau}-1}{\delta}\right]D_v(x) + \frac{1}{2}\theta^2\sigma^2x^2D^2v(x)\right\} \quad x\in\left[l_t, +\infty\right]$$

is the generalized Hamiltonian.

Definition: *A vector* $\rho^* \in \mathbb{R}_+^Z$ *is called a strict global minimizer of expression 4.1.6, if* $\rho^* \in F$ *and there is a neighborhood* $U\left(\rho^*\right) \in \mathbb{R}_+^Z$ *such that* $f(\rho) \geq f(\rho^*)$ *for all* $\rho \in u(\rho^*) \cap F$ *.*

To find ρ we put forward the following Lemma;

LEMMA 1. *Let Q be a continuous, strictly convex quadratic function*

$$d\left(\rho\right) \triangleq \frac{1}{2}\left\|\left(D'\right)^{-1}\rho + \left(D'\right)^{-1}B\right\|^2 \tag{4.1.6}$$

Over $\rho \in \left[0, \infty\right]^Z$ *, where* $B' \in \mathbb{R}_+^Z$ *,* $D \in R^{Z*Z}$ *and* $D'D > 0$ *. Then* ***d*** *has a unique strict global "minimizer" or constant discount factor* $\overline{\rho} \in \left[0, \infty\right)^Z$ *.*

Hence Lemma 1 implies that;

$$\left\|\left(D'\right)^{-1}\overline{\rho} + \left(D'\right)^{-1}B'\right\|^2 \leq \left\|\left(D'\right)^{-1}\rho\left(D'\right)^{-1}B'\right\|^2 \qquad \forall_\rho \in \left[0, \infty\right)^Z \tag{4.1.7}$$

The Kuhn-Tucker condition for minimization of Q in expression 4.1.6 over $\left[0,\infty\right)^{Z}$ lead to the Lagrange multiplier vector[16] $\bar{\theta} \in \left[0,\infty\right)^{Z}$ such that

$$\bar{\theta} = \nabla d\left(\bar{\rho}\right) = \left[\left(D'D\right)^{-1}\bar{\rho} + \left(D'D\right)^{-1}B'\right] \tag{4.1.8}$$

where $\bar{\theta}'\bar{\rho} = 0$ and $\bar{\xi} = \left(D'\right)^{-1}\bar{\rho} + \left(D'\right)^{-1}B'$ (The Lagrange multiplier vector)

$$\Rightarrow \bar{\theta} = \nabla d\left(\bar{\rho}\right) = D^{-1}\bar{\xi}$$

$$\Rightarrow \bar{\rho}'D^{-1}\bar{\xi} = 0$$

$$\Rightarrow d\left(\alpha\bar{\theta}\right) = d\left(\alpha D^{-1}\bar{\xi}\right) = \frac{1}{2}\alpha^{2}\left\|\bar{\xi}\right\|^{2}$$

So given this unique minimizer[17] derived in expression 4.1.6 and the Lagrange multiplier in 4.1.8, the global minimizer and the Lagrange multiplier for the extended dynamic optimization model can be expressed as;

$$\bar{\rho}\left(\tau\right) = \arg\min_{\rho_{(t)}\in\left[D,\infty\right)^{m}} \frac{1}{2}\left\|\left(\sigma_M^2 + \sigma_C^2\right)\left(t\right)^{-1}\rho + \left(\sigma_M^2 + \sigma_C^2\right)\left(t\right)^{-1}\left(b\left(t\right) - r\left(t\right)1\right)\right\|^{2} \tag{4.1.9}$$

and;

$$\bar{\theta}\left(t\right) \doteq \left(\sigma_M^2 + \sigma_C^2\right)\left(t\right)^{-1}\bar{\rho}\left(t\right) + \left(\sigma_M^2 + \sigma_C^2\right)\left(t\right)^{-1}\left(B\left(t\right) - r\left(t\right)1\right) \tag{4.1.10}$$

Hence from equations 4.1.9 and 4.1.10 the optimal portfolio selection strategy in the presence of both market and credit risk that corresponds to the expected terminal wealth $E\left[W\left(\tau\right)\right] = d$, as a function of time *t* and initial wealth W_0 can be expressed as;

$$u^{*}\left(t,W\right) \equiv \left[u_1^{*}\left(t,W\right),\ldots,u_m^{*}\left(t,W\right)\right]'$$

---

[16] The Lagrange Multiplier Vector $\left(\theta\right)$ approximates the marginal impact on the objective function $\left(\max_{u\in U} \in \left[u\left(t\right)\right]\right)$ caused by a 1 unit change in the constant of the constraint.

[17] This unique global minimizer may also be referred to elsewhere in the optimization literature as a constant discount factor.

It follows from the verification theorems of Gozzi and Russo (2006) and the results in Crandall and Lions (1983), Crandall and Newcomb (1985), Ishii (1987) and Ishii and Loreti (2002) that $u^*\left(t,w\right)$ as defined by 4.1.11 is an optimal feedback control.

$$u^*(t,W)=\begin{cases} -\left(\left(\sigma_M^2\left(t\right)+\sigma_C^2\left(t\right)\right)\left(\sigma_M^2\left(t\right)+\sigma_C^2\left(t\right)\right)'\right)^{-1}\left[\bar{\rho}\left(t\right)+\left(b\left(t\right)-r\left(t\right)1\right)\right]\left[W-\left(d-\mu^*\right)e^{-\int_t^\tau r(s)ds}\right] & if\ W-\left(d-\mu^*\right)e^{-\int_t^\tau r(s)ds}\leq 0 \\ 0 & if\ W-\left(d-\mu^*\right)e^{-\int_t^\tau r(s)ds}>0 \end{cases} \tag{4.1.11}$$

Where the expected terminal return for investor 4.1.1 can be written as

$$\mu^*=\frac{d-W_0e^{\int_0^\tau r(s)ds}}{1-e^{\int_0^\tau\left\|\bar{\theta}(s)\right\|^2ds}} \tag{4.1.12}$$

thus the agent's efficiency frontier is represented as;

$$Var\left[W\left(\tau\right)\right]=\frac{\left(d-W_0e^{\int_0^\tau r(s)ds}\right)^2}{e^{\int_0^\tau\left\|\bar{\theta}(s)\right\|^2ds}-1}\equiv\frac{\left(E\left[W\left(\tau\right)\right]-W_0e^{\int_0^\tau r(s)ds}\right)^2}{e^{\int_0^\tau\left\|\bar{\theta}(s)\right\|^2ds}-1} \tag{4.1.13}$$

LEMMA 2. *The following relation holds for* $\int_t^\tau r(s)ds$ and $\int_t^\tau\left\|\bar{\theta}\right\|^2ds$ over $\tau\in\left[0,\infty\right]$

$$\int_t^\tau r(s)ds=\gamma(t)\left(\tau-t\right)-\int_t^\tau\phi_{\tau-t}(s)dz_r(s)$$

When $t=0$

$$\Rightarrow E\int_0^\tau r(t)dt=\gamma\tau$$

hence $\int_t^\tau r(s)ds \sim N\left[\gamma(t)(\tau - t)\right]$

5.0 Data and Empirical Illustration

In this Section, we demonstrate that the extended dynamic analysis of Section 4 can easily be adapted to alternative economic environments. Section 5.1 illustrates the analytical flexibility of our methodology in relation to a benchmark baseline dynamic framework, and provides an explicit solution to the extended dynamic credit risk model.

The study selected 10 actively traded stocks, 2 corporate bonds and the U.S. 10-year Treasury bond to illustrate the approach for estimating the extended dynamic optimization model. The corporate bonds are General Electric's (GE) CUSIP# 369604AY9 and JPMorgan's (JPM) CUSIP# 014037179, while the stocks are Alcoa (AA), Procter & Gamble (PG), McDonald's (MCD), Disney (DIS), Wal-Mart (WMT), American Express (AXP), AT&T (T), Boeing (BA), Caterpillar (CAT) and International Business Machines (IBM). The study chose representative stocks from the Dow Jones industrial-30 covering a variety of industries and which had high trading volumes on the NYSE. Trade data for these stocks and bonds were taken from Bloomberg and covered 7 years, ranging from July 31st 2001 to July 31st 2008. Days with no trading activities were eliminated from the study. From this data, the return of each individual stock and corporate bond r_t, was calculated for each month so the average monthly return was given by

$\overline{r} = \frac{\sum_{t=1}^{n} r_t}{N}$, where N is the number of observations. Using this data, the covariance between each pair of stocks is then calculated by

$$\operatorname{cov}(r_a, r_b) = \frac{\sum_{t=1}^{n} (r_{a,t} - \overline{r}_a)(r_{b,t} - \overline{r}_b)}{N-1}$$

For ease of computation but without loss of generality we assume that w_t is evenly weighted across all asset classes in the portfolio.

Table 1: Summary Statistics of the study's terminal wealth portfolio's risk and returns used to obtain the optimal solutions for the two different portfolio strategies

				Terminal Wealth - *d*				
	Firm Name	Ticker	Industry	*CR*	μ	σ_m^2	σ_c^2	σ_p^2
1	Alcoa	AA	Aluminium	BBB	2.89	8.94%	21.78%	15.36%
2	American Express	AXP	Consumer Finance	A	2.36	5.81%	29.02%	17.42%
3	AT&T	T	Telecommunications	BBB	1.33	7.33%	23.25%	15.29%
4	Boeing	BA	AeroSpace & Defence	BBB	5.50	7.31%	23.54%	15.43%
5	Caterpillar	CAT	Commercial Vechicles	A	24.59	7.20%	17.37%	12.29%
6	General Electric	GE	Industrial	AAA	1.72	5.44%	24.98%	15.21%
7	International Business Machines	IBM	Computer Services	A	9.53	7.20%	20.49%	13.85%
8	JPMorgan Chase Bank	JPM	Banking	A	7.31	8.42%	24.94%	16.68%
9	McDonalds	MCD	Restaurants	BBB	16.49	6.92%	21.81%	14.37%
10	Procter & Gamble	PG	Consumer Products	A	5.49	4.03%	15.21%	9.62%
11	WalMart	WMT	Retailer	A	2.24	5.00%	21.43%	13.22%
12	Disney	DIS	Consumer Entertainment	BB	8.56	6.22%	22.22%	14.22%

Notes:
μ - Mean Returns
σ2 - Variance
CR - Credit Rating
*GE Bond - CUSIP
*JPMorgan Bond - CUSIP

5.1 Discussion

From table 1, the study's portfolio was comprised of 10 equity products and 2 corporate bonds. From table 1 we develop credit and market risk optimization models in a stochastic programming framework and found that the portfolio composition is best optimized by utilizing an asset allocation along the lines of approximately 80 percent equity and 20 percent fixed income. From the portfolio mix we let m = 12, whilst the interest rate on the 10-Year U.S. Treasury bond obtained from Bloomberg at the close of trading on July 31st 2008 was 4.02% and the appreciation rate of the *m* stocks $=(x_1, x_2 \dots x_n)'$. The resulting LaGrange Multiplier and unique minimizer in equations 4.1.9 and 4.1.10 are derived as follows;

$$\bar{\theta}(t) = (x_1, x_2 \dots x_n)'$$

while the unique minimizer over $[0,\infty)^m$ is given as

$\bar{\rho}(\tau) = [4.02,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0]'$ with a minimum value[18]

$s(\bar{\rho}) = \left\|\sigma_p^{-1}\bar{\rho} + \theta\right\|^2 = 0.0403$ for the benchmark model and 0.0092 for the extended model.

Hence from Lemma 3 the agent's efficient portfolio (allocation) strategy under the benchmark framework would be;

$$u^*(t,W) = \begin{cases} -\begin{bmatrix} 0.03 \\ 0.03 \\ 0.03 \\ 0.02 \\ 0.01 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}\left[W - \left(d - \mu^*\right)e^{4.02(\tau - t)}\right] & if\ W - \left(d - \mu^*\right)e^{4.02(\tau - t)} \le 0 \\ 0 & if\ W - \left(d - \mu^*\right)e^{4.02(\tau - t)} > 0 \end{cases} \tag{5.1.1}$$

---

[18] Suppose u and v are two orthogonal vectors in $\mathbb{R}^n$ then, $\|u+v\|^2 = \|u\|^2 + \|v\|^2$.

Proof: The proof of this theorem is fairly simple. From the proof of the triangle inequality for norms we have the following statement.

$$\|u+v\|^2 = \|u\|^2 + 2(u+v) + \|v\|^2$$

However because u and v are orthogonal we have u*v=0 and so we get; $\|u+v\|^2 = \|u\|^2 + \|v\|^2$ □

Where the optimal strategy attains its maximum value at

$$\mu^* = \frac{d - W_0 e^{\int_0^\tau r(s)ds}}{1 - e^{\int_0^\tau \|\bar{\theta}(s)\|^2 ds}} = \frac{d - W_0 e^{4.02\tau}}{1 - e^{0.0403\tau}} \tag{5.1.2}$$

And his efficient frontier can be written as;

$$Var\left[W(\tau)\right] = \frac{\left(d - W_0 e^{\int_0^\tau r(s)ds}\right)^2}{e^{\int_0^\tau \|\bar{\theta}(s)\|^2 ds} - 1} \equiv \frac{\left(E\left[W(\tau)\right] - W_0 e^{4.02\tau}\right)^2}{e^{0.0403\tau} - 1} \tag{5.1.3}$$

For the extended model we find that the expected terminal return is given by

$$u^*_{Traditional} > u^*_{Extended}$$

$$\mu^* = \frac{d - W_0 e^{\int_0^\tau r(s)ds}}{1 - e^{\int_0^\tau \|\bar{\theta}(s)\|^2 ds}} = \frac{d - W_0 e^{4.02\tau}}{1 - e^{0.0092\tau}} \tag{5.1.4}$$

Then the efficient frontier is represented as

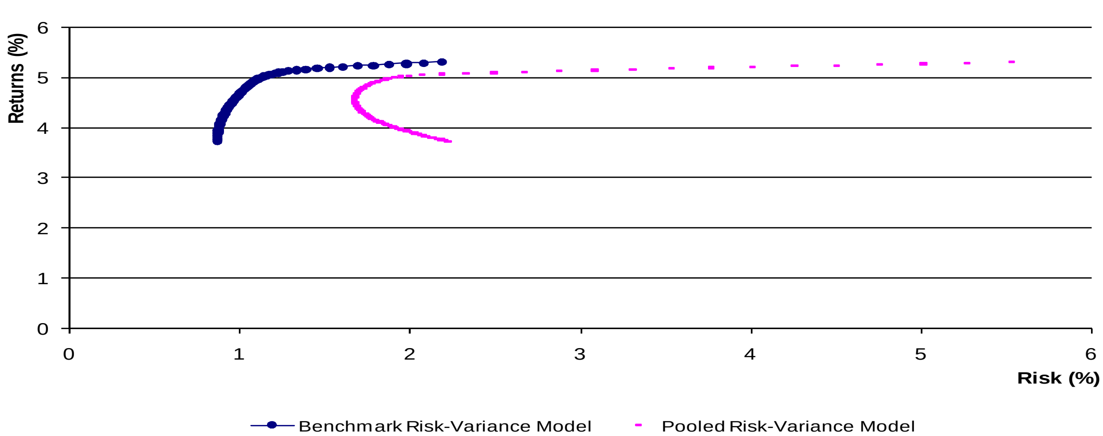
$$Var\left[W(\tau)\right]_{Traditional} > Var\left[W(\tau)\right]_{Extended}$$

$$Var\left[W(\tau)\right] = \frac{\left(d - W_0 e^{\int_0^\tau r(s)ds}\right)^2}{e^{\int_0^\tau \|\bar{\theta}(s)\|^2 ds} - 1} \equiv \frac{\left(E\left[W(\tau)\right] - W_0 e^{4.02\tau}\right)^2}{e^{0.0092\tau} - 1} \tag{5.1.5}$$

Firstly, the study demonstrates in equations 5.1.2 through 5.1.5 that given credit risk the investor's true optimal dynamic asset allocation is lower than previously indicated by the benchmark dynamic framework. In fact the investor's true terminal return may lie on or between the upper market-risk boundary and the lower credit-risk enhanced boundary as graphically illustrated using sample data in figure 3. In addition the graph shows that in the presence of credit risk, the investor's efficiency frontier moves inwards to a lower terminal return as illustrated by equations 5.1.2 and 5.1.3 respectively. However since the extended model is nested in the benchmark dynamic framework, when credit risk dissipates $c_r \leq 0$ then the agent's risk frontier and terminal return converges to that of the benchmark model.

Similarly, the analysis shows that the agent's efficient strategy of portfolio selection corresponding to the expected terminal wealth differs under both risk measurement scenarios. Expression 5.1.1 indicates that given the level of optimal return obtained from equations 5.1.2 and 5.1.4, the strategy that works for the market-risk only scenario would not work in an investment environment involving both market and credit risk. Hence as indicated in this analysis this agent will modify his investment strategy so as to better adapt to the alternative credit risky investment environment.

Figure 3: Efficiency Frontier under the Benchmark and Pooled Risk Frameworks



Secondly, the closed form solution in expression 5.1.8 demonstrates that given credit risk, investors exhibit a greater level of risk aversion as indicated by a larger risk aversion coefficient. Notice that since 5.1.5 has the smaller u^* it follows that he will have a greater risk aversion coefficient when the inverse is taken in expression 5.1.8.

$$u^* = u^f e^{\int_0^\tau r(s)ds} + \frac{1}{\varphi}\left(\hat{u} - u^f e^{\int_0^\tau r(s)ds}\right) = \frac{d - W_0 e^{\int_0^\tau r(s)ds}}{1 - e^{\int_0^\tau \|\bar{\theta}(s)\|^2 ds}} \tag{5.1.6}$$

$$\Rightarrow \frac{1}{\varphi} = \frac{u^* - u^f e^{\int_0^\tau r(s)ds}}{\bar{u} - u^f e^{\int_0^\tau r(s)ds}} \tag{5.1.7}$$

$$\Rightarrow \varphi = \left[\frac{u^* - u^f e^{\int_0^\tau r(s)ds}}{\bar{u} - u^f e^{\int_0^\tau r(s)ds}}\right]^{-1} \tag{5.1.8}$$

$$\Rightarrow \frac{1}{\varphi_E} > \frac{1}{\varphi_B}$$

Finally, the study shows that the added (credit) risk results in a reduction in the agent's terminal return on his investment because of the investor's level of risk aversion. In fact, the analysis adds support to the longstanding view that during periods of increased credit risks, investors reduce their holdings of credit risky products and move to the safety of the lower yielding risk-free products such as U.S. Treasury instruments, which results in an overall lower terminal portfolio return.

6.0 Conclusion

Bajeux and Portait (1998) concluded from their study on dynamic asset allocation that the dynamic efficient framework outperformed the standard static framework. Moving a step further, this study demonstrates that the credit risk enhance framework is fundamentally much more flexible and dynamic than the traditional dynamic framework. The paper first established a baseline dynamic optimization model which was used to determine an optimal terminal return given market risk. A more complete risk model inclusive of credit risk was later developed to investigate investors' attitude to credit risk. The empirical illustration of the extended cross-sectional pooled risk model demonstrates that the dynamic optimal portfolio return is lower than indicated by the benchmark Markowitz and Merton mean-variance framework because traditional models implicitly assumes the non-existence of credit events.

The inclusion of credit risk shows that given the variability of credit risk and risk aversion, an investor's true optimum may be below the benchmark optimum or within a given boundary region as illustrated in figure 3. In fact, the extended model exhibits the analytical flexibility whereby changes in risk reflect investors' decisions (flight-to-quality) as they move to minimize overall portfolio risks for a given level of return.

From the standpoint of policy this work will not only compliment past empirical work in dynamic asset allocation but will also provide investors a vehicle for determining a more complete measure of perceived portfolio risk in an investment environment characterized by deteriorating (improving) credit quality and rising (falling) market risk (interest rates). The flexibility of the model is highlighted in the fact that in the absence of credit risks, the model converges to the standard dynamic optimization framework.

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