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The Great Moderation and the Relationship between Output Growth and Its Volatility

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Abstract

This study examines the effect of the Great Moderation on the relationship between U.S. output growth and its volatility over the period 1947 to 2006. First, we consider the possible effects of structural change in the volatility process. In so doing, we employ GARCH-M and ARCH-M specifications of the process describing output growth rate and its volatility with and without a one-time structural break in volatility. Second, our data analyses and empirical results suggest no significant relationship between the output growth rate and its volatility, favoring the traditional wisdom of dichotomy in macroeconomics. Moreover, the evidence shows that the time-varying variance falls sharply or even disappears once we incorporate a one-time structural break in the unconditional variance of output starting 1982 or 1984. That is, the integrated GARCH effect proves spurious. Finally, a joint test of a trend change and a one-time shift in the volatility process finds that the one-time shift dominates.

Journal of Economic Literature Classification: C32; E32; O40

Keywords: Great Moderation, economic growth and volatility, structural change in variance, IGARCH

1. Introduction

Macroeconomic volatility declined substantially during the past 20 years. Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), Stock and Watson (2003), and Ahmed, Levin, and Wilson (2004), among others, document this Great Moderation in the volatility of U.S. GDP growth. Moreover, the current Federal Reserve Board Chairman Bernanke (2004) also addressed this issue. Most research focuses on the causes of the Great Moderation such as good policies, structural change, good luck, or output composition shifts.[1] This paper empirically investigates the effect of the Great Moderation on the relationship between the output growth rate and its volatility.[2]

Macroeconomists have long focused on business cycles and economic growth. Recently, increasing attention considers the relationship between business cycle volatility and the long-run trend in growth. Alternative models give rise to negative, positive, or independent relationships between the output growth rate and its volatility. For example, the misperceptions theory, proposed originally by Friedman (1968), Phelps (1969), and Lucas (1972), argues that output fluctuations around its natural rate reflect price misperceptions due to monetary shocks, whilst the long-run

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[1] Good policies refer to better management of the economy by monetary policy makers. Structural change refers to better inventory management. Good luck refers to the reduction in economic shocks (e.g., oil price shocks). Output composition shifts refer to the fall in the volatility of output components, such as consumption and investment. Bernanke (2004) uses a lower-bound frontier on inflation and output volatilities to organize his thinking. Inefficient monetary policy or inventory management leaves the economy above the frontier, whereas changes in the volatility of random shocks will shift the lower-bound frontier. Stock and Watson (2003) attribute the Great Moderation to good luck, implying that the frontier shifted toward the origin. Bernanke (2004) argues that a substantial portion of the Great Moderation reflects better monetary policy, implying a movement toward the frontier. The distinction proves important, because if Stock and Watson (2003) are correct, then good luck can turn into bad luck and the frontier can shift back to a more unfavorable trade-off. If Bernanke (2004) is correct, then maintaining good policy can continue the benefits of the Great Moderation. Finally, Blanchard and Simon (2001) and Eggers and Ioannides (2006) find that most of the decline reflects a decline in the volatility of consumption, investment, and/or manufacturing output.

[2] At least two interpretations can explain the decline in output growth-rate volatility. For example, McConnell and Perez-Quiros (2000) invoke a step decrease sometime in the mid-1980s. Blanchard and Simon (2001) detect a trend decline, temporarily interrupted in the 1970s and early 1980s. This study generally follows the approach of a step decrease and then investigates its effect on the relationship between the output growth rate and its volatility. Section 4, however, does consider the relative effect of a trend decline against a one-time shift in volatility.

growth rate of potential output reflects technology and other real factors. The standard dichotomy in macroeconomics implies no relationship between the output growth rate and its volatility.

Bernanke (1983) and Pindyck (1991) demonstrate that irreversibility makes investment especially sensitive to various forms of risk. Output volatility generates risk about future demand that impedes investment, leading to a negative relationship between output volatility and growth. Martin and Rogers (1997) argue that learning-by-doing generates growth whereby production complements productivity-improving activities and stabilization policy can positively affect human capital accumulation and growth. One natural conclusion, therefore, implies that short-run economic instability can prove detrimental to human capital accumulation and growth (Martin and Rogers, 2000).

In contrast, Black (1987) argues that technology comes with varying levels of risk and expected returns that associate with the degree of specialization. More specialization means more output volatility. Investment occurs in specialized technologies only if expected returns sufficiently compensate for associated risk. Thus, when high expected return technologies emerge, high output volatility and high growth coexist. Mirman (1971) argues that higher output volatility leads to higher precautionary saving, implying a positive relationship between output volatility and growth. Bean (1990) and Saint-Paul (1993) show that the opportunity cost of productivity-improving activities falls in recessions, implying that higher output volatility may positively affect growth. According to Blackburn (1999), a relative increase in the volatility of shocks increases the pace of knowledge accumulation and, hence, growth, implying a positive relation between output variability and long-term growth.

In a simple stochastic growth model, Blackburn and Galindev (2003) illustrate that different mechanisms of endogenous technological change can lead to different implications for the relationship between output variability and growth. Generally, the relationship more likely exhibits a positive correlation, if internal learning drives technological change through deliberate actions that substitute for production activities. The relationship exhibits a negative correlation, if external learning drives technological change through non-deliberate actions that complement production activity. Blackburn and Pelloni (2004) predict that real shocks generate a positive correlation between output variability and growth and nominal shocks produce a negative relationship.

The statistical evidence also exhibits ambiguity. The empirical literature employs two approaches. Using cross-country data, Kormendi and Meguire (1985) and Grier and Tullock (1989) find a positive relationship between growth and its standard deviation, but Ramey and Ramey (1995), Miller (1996), Martin and Rogers (2000), and Kneller and Young (2001) report a negative relationship. More recently, Rafferty (2005) discovers that unexpected volatility reduces growth and expected volatility increases it, while the combined effect of expected and unexpected volatility reduces it.

Applying generalized autoregressive conditional heteroskedasticity in mean (GARCH-M) models, Caporale and McKiernan (1996, 1998) find a positive relationship between output volatility and growth for the U.K. and the U.S., whereas Fountas and Karanasos (2006) find a positive relationship for Germany and Japan. Speight (1999), Grier and Perry (2000), and Fountas and Karanasos (2006), however, conclude that no relationship exists in the U.K and the U.S. In contrast, Macri and Sinha (2000) and Henry and Olekalns (2002) discover a negative link between volatility and growth for Australia and the U.S.

The lack of robust evidence on the relationship between the output growth rate and its volatility motivates our analysis. While many empirical studies employ post-war data, no one explicitly considers the effect of the Great Moderation on this relationship.[3] The volatility of U.S. GDP growth fell by more than half since the early to mid-1980s. Although no agreement exists on the causes of the Great Moderation, the reduced volatility implies that empirical models for output growth over periods that span the break may experience model misspecification.

In addition to considering the relationship between the output growth rate and its volatility, we first consider the possibility that structural change affects the process generating the volatility of output growth. Deibold (1986) first raised the concern that structural changes may confound persistence estimation in GARCH models. He notes that Engle and Bollerslev's (1986) integrated GARCH (IGARCH) may result from instability of the constant term of the conditional variance, that is, nonstationarity of the unconditional variance. Neglecting such changes can lead to spuriously measured persistence; the sum of the estimated autoregressive parameters of the conditional variance is heavily biased towards one. Lamoreux and Lastrapes (1990) explore Diebold's conjecture and provide confirming evidence that not accounting for discrete shifts in unconditional variance, the misspecification of the GARCH model, can bias upward GARCH estimates of persistence in variance and, thus, vitiates the usefulness of GARCH when the degree of persistence proves important. The longer the sample period is, the higher the probability is that such changes will occur. Including dummy variables to account for such shifts diminishes the degree of GARCH persistence. More recently, Mikosch and Stărică (2004) argue theoretically that the IGARCH model makes sense when non-stationary data reflect changes in the unconditional variance. Hillebrand (2005) shows that in the presence of neglected parameter change-points, even

[3] Grier and Tullock (1989), using pooled cross-section data on 113 countries between 1950 and 1980, investigate empirical regularities in post-war economic growth. They find significant time-period dummy variables or a trend variable in their mean growth models for OECD countries and conclude the average growth rate, holding other variables constant, rises in the post-1961 period. Similarly, Caporale and McKiernan (1998) include two dummies for periods of the Great Depression and World War II in their mean equation over long sample period from 1870 to 1993. This paper focuses on structural changes in the variance equation.

a single deterministic change-point, GARCH inappropriately measures volatility persistence. Before carrying out GARCH estimations, we perform a thorough change-point study of the data to avoid the spurious effect of almost-integration.

The identification of change points will occur endogenously in the data generating process. We employ Inclán and Tiao's (1994) iterated cumulative sums of squares (ICSS) algorithm to detect sudden changes in the variance of output growth, as well as the time point and magnitude of each detected change in the variance.[4] The algorithm finds one change point at 1982:1, two-year earlier than that of 1984:1 in McConnell and Perez-Quiros (2000). Most analysts argue that the break date occurs some time in the early to mid-1980s, but the exact timing of the decline remains controversial. For example, Blanchard and Simon (2001) analyze the large decline in U.S. output volatility starting in 1982:1.

This paper employs GARCH-M and ARCH-M models to examine the effect of the Great Moderation on the volatility-growth relationship over the period 1947:1 to 2006:4 with the break date of 1982:1. Our empirical results show strong evidence of IGARCH effects and no evidence of significant links between volatility and growth for the U.S. Moreover, the time-varying variance falls sharply or even disappears once we allow for the structural break in the unconditional variance of output growth. That is, the IGARCH effect proves spurious due to the Great Moderation. These results prove robust to the alternative break 1984:1. Section 2 discusses the data and the Great Moderation in output volatility. Section 3 presents the methodology and the empirical results. Section 4 considers additional evidence. Finally, Section 5 concludes.

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[4] Aggarwal, Inclán, and Leal (1999) apply this algorithm to identify the points of sudden changes in the variance of returns in ten emerging stock markets, in addition to Hong Kong, Singapore, Germany, Japan, the U.K., and the U.S.

2. Data and the Great Moderation

Output growth rates (y_t) equal the percentage change in the logarithm of seasonally adjusted quarterly real GDP (Y_t), measured in billions of chained 2000 dollars, that come from the U.S. Bureau of Economic Analysis (BEA) over the period 1947:1 to 2006:2. A rather dramatic reduction in output volatility in the most recent two decades relative to the previous four produces the most striking observation. McConnell and Perez-Quiros (2000), applying tests of Andrews (1993) and Andrews and Ploberger (1994), detect a unique break in the variance of the growth rate in 1984:1 for the sample 1953:2 to 1999:2 with no break in the mean growth rate. This paper extends the data from 1947:1 to 2006:2.

As discussed earlier, the methodology used in this study to detect structural changes in the variance employs the ICSS algorithm described by Inclán and Tiao (1994). The analysis assumes that the time series of output growth displays a stationary variance over an initial period, and then a sudden change in variance occurs. The variance then exhibits stationarity again for a time, until the next sudden change. The process repeats through time, yielding a time series of observations with an unknown number of changes in the variance.

Let $\{\varepsilon_t\}$ denote a series of independent observations from a normal distribution with mean zero and unconditional variance σ_t^2. When N variance changes occur in T observations, $1 < k_1 < k_2 < ... < k_N < T$ equal the set of change points. Let C_k equal the cumulative sum of the squared observations from the start of the series to the k^th point in time (i.e., $C_k = \sum_{t=1}^{k} \varepsilon_t^2$, k = 1,…,T). Then, define D_k as : $D_k = (C_k / C_T) - k/T$, $k = 1,...,T$ with $D_0 = D_T = 0$. If no changes in variance occur over the sample period, the D_k statistic oscillates around zero. If one or more sudden variance changes exist in the series, then the D_k values drift either up or down from zero.

Critical values based on the distribution on D_k under the null hypothesis of homogeneous variance provide upper and lower boundaries to detect a significant change in variance with a known level of probability. When the maximum of the absolute value of D_k exceeds the critical value, we reject the null hypothesis of no changes. Let k^* equal the value of k for which $\max_k |D_k|$ occurs. If $\max_k \sqrt{T/2}|D_k|$ exceeds the predetermined boundary, then k provides an estimate of the change point. The factor $\sqrt{T/2}$ standardizes the distribution. Under the null, D_k asymptotically behaves as a Brownian bridge. The critical value of 1.36 defines the 95$^{\text{th}}$ percentile of the asymptotic distribution of $\max_k \sqrt{T/2}|D_k|$. Therefore, upper and lower boundaries occur at ±1.36 in the D_k plot. Exceeding these boundaries marks a significant change in variance of output. To examine multiple change points, the ICSS algorithm successively evaluates D_k at different parts of the series, dividing consecutively after finding a possible change point.

The procedure identifies one, and only one, change point at 1982:1. That is, the shift lasts to the end of our sample period with no breaks in other periods. Figure 1 plots the series of real GDP and its growth rate and marks the break with a gray area. We further conduct structural stability tests for the unconditional mean and variance of the growth rate by splitting the sample into two sub-periods: 1947:1 to 1981:4 and 1982:1 to 2006:2. For the unconditional mean, a Wald statistic tests for the equality of means for two different samples, while a variance-ratio statistic tests for the equality of the unconditional variances.

Table 1 reports descriptive statistics for the data and the results of the structural stability tests. The mean growth rate in each sub-sample nearly equals the 0.8358 percent growth rate average for the full 60-year sample. The Wald statistics, distributed as $\chi^2(1)$, that test for structural change in the mean between the samples cannot reject the null hypothesis of equality of means. In contrast, a clear decline in the standard deviation of the growth rate occurs, equaling 1.1844 percent a quarter in the pre-1982 period and 0.6094 percent in the post-1982 period, a decline of 49 percent. The p-values for the variance-ratio F-test significantly reject the null of variance equality between the samples. Economists call the substantial drop in the variance of output in the post-1982 period as the Great Moderation. Skewness statistics support symmetric distributions for the full and pre-1982 sample periods, but not the post-1982 period. Kurtosis statistics suggest that the full and post-1982 sample series exhibit leptokurticity with fat tails. Consequently, Jarque-Bera tests reject normality for these two samples, but cannot reject normal distributions in the pre-1982 sub-sample. The Ljung-Box Q-statistics (LB Q), testing for autocorrelation of up to 6 lags, indicate serial correlation in growth for all three periods. The Lagrange Multiplier statistics (LM) test for ARCH effects (Engle, 1982) up to 6 lags, suggesting a time-varying variance in output growth for the full and post-1982 sample periods and no heteroskedasticity in the pre-1982 period. The ADF unit-root test implies that the growth rate exhibits stationarity for each of the three samples.

Useful information emerges that can assist in the model building and verification stages. First, the evidence of autocorrelation suggests an autoregressive moving-average (ARMA) model for the mean growth equation to capture temporal dependence and to generate white-noise residuals for all the three periods. Second, the emergence of a time-varying variance argues for GARCH-M and ARCH-M models to examine the effect of volatility on growth. The robustness of this conclusion to the Great Moderation, however, requires the allowance for a one-time shift in the unconditional variance. Third, no hetroskedasticity in the pre-1982 sub-period suggests the use of other volatility measures such as a moving-sample standard deviation to examine the effect for the sub-sample. Fourth, and most importantly, the significant decline in the variance combined with no change of the mean growth rate in the post-1982 sub-period may imply a weak relationship between volatility and growth.

3. Methodology and Empirical Results

We first construct an ARMA model for the growth series to remove any linear dependence in the data, and then add output volatility as an explanatory variable in the mean equation. Based on Schwarz Criterion (SC), an AR(2) process proves adequate to capture growth dynamics and produces white-noise residuals for all the three-sample periods. The mean growth equation equals the following:

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t , \tag{1}$$

where the growth rate $y_t \equiv 100 \times (\ln Y_t - \ln Y_{t-1})$, $\ln Y_t$ equals the natural logarithm of real GDP, ε_t equals the white-noise random error; and σ_t equals a measure of output volatility. The estimate of λ may exceed or fall below zero and prove significant or insignificant.

We, now, derive an operational measure for output volatility. The descriptive statistics show that ARCH effects emerge in the full and post-1982 sample periods. The GARCH(1,1) specification proves adequate to represent most financial and economic time series (Bollerslev et al., 1994). For example, Caporale and McKiernan (1996), Speight (1999), Grier and Perry (2000), Henry and Olekalns (2002), and Fountas and Karanasos (2006) apply this process to parameterize the time varying conditional variance of output growth. Caporale and McKiernan (1998) and Macri and Sinha (2000), however, use ARCH(1) to examine the time-dependence of the conditional variance. To provide more evidence, we employ both GARCH(1,1) and ARCH(2) models to account for periods of high- and low-output volatility for our samples as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \text{, and} \tag{2}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 , \tag{3}$$

where equations (2) and (3) equal the GARCH(1,1) and ARCH(2) specifications, and σ_t^2 equals the conditional variance of the growth rate, given information available at time t-1. The presence of the square root of σ_t^2, σ_t, in the mean equation of the growth rate makes equations (1) and (2) or (3) a GARCH-M or an ARCH-M model (Engle *et al*., 1987). The conditions that $\alpha_i \geq 0$, $\beta_i \geq 0$, and $\alpha_1 + \beta_1 < 1$ in the GARCH model or $\alpha_1 + \alpha_2 < 1$ in the ARCH model ensure positive and stable conditional variances of ε_t. The sums, $\alpha_1 + \beta_1$ or $\alpha_1 + \alpha_2$, measure the persistence of shocks to the conditional variances. Evidence of an integrated GARCH (IGARCH), or in general, evidence of high persistence proves analogous to a unit root in the mean of a stochastic process. This persistence may result from occasional level shifts in volatility. If β_1 (or α_2) equals zero, the process reduces to an ARCH(1). When α_1 and β_1 (or α_2) both equal zero, the variance equals a constant. We estimate each of the models using maximum likelihood under normality and using the Berndt et al. (1974) (BHHH) algorithm.

For the pre-1982 sub-period, since we do not find time-varying variance, we also construct a moving-sample standard deviation to measure output volatility for 1947:1 to 1981:4 as follows:

$$\sigma_{t+m} = [(\frac{1}{m})\sum_{i=1}^{m}(\ln Y_{t+i-1} - \ln Y_{t+i-2})^2]^{0.5}, \qquad (4)$$

where we select m = 4, 8, and 12 as the orders of the moving average. The choice of the moving-average order does not affect the results, however.

Table 2 reports the estimation results of our GARCH-M and ARCH-M models for the full sample with standard errors in parentheses, p-values in brackets, and statistics for the standardized residuals. In the mean equation, AR(2) estimates verify significance at the 5-percent level, lending support to the autoregressive specification. The coefficient of the conditional standard deviation (λ) possesses no statistical significance. Each estimate in the variance equation exceeds zero. The volatility persistence of 0.9961 in the GARCH and 0.6837 in the ARCH process, however, proves high. The likelihood ratio (LR) tests for $\alpha_1 + \beta_1 = 1$ in the GARCH and $\alpha_1 + \alpha_2 = 1$ in the ARCH process, respectively, do not reject the null hypothesis of an IGARCH effect at the 5-percent level. It does reject the IGARCH effect for the ARCH process, however, at nearly 10-percent level. The fitted models adequately capture the time-series properties of the data in that the Ljung-Box Q-statistics for standardized residuals ($LB\ Q$) and standardized squared residuals ($LB\ Q^2$), up to 6 lags, do not detect remaining autocorrelation and conditional heteroskedasticity. The standardized residuals exhibit symmetric distributions, but with significant excess kurtosis. Thus, they do not exhibit the characteristics of a normal distribution, as observed in Table 1.

The insignificant estimate of the conditional standard deviation (λ) in the mean equation implies no relationship between output growth and its volatility. This result conforms to Friedman's (1968), Phelp's (1969), and Lucas's (1972) misperceptions hypothesis and the previous empirical findings, using GARCH-M models, of Grier and Perry (2000) and Fountas and Karanasos (2006) for the U.S. and Speight (1999) for the U.K. This finding, however, proves inconsistent with the discovery of a positive relationship by Caporale and McKiernan (1996, 1998) for the U.K. the U.S., and by Fountas and Karanasos (2006) for Germany and Japan, as well as the discovery by Macri and Sinha (2000) and Henry and Olekaln (2002) of a negative relationship for Australia and the U.S.

Although different countries, sample periods, data frequencies, or econometric models may lead to different findings, two issues emerge from the empirical results. First, existing research efforts do not limit the phenomenon of the Great Moderation to U.S. output only. Mills and Wang (2003) and Summers (2005) find structural breaks in the volatility of the output growth rate for the G7 countries and Australia, although the break occurs at different times. The well-documented moderation in the volatility of GDP growth in the U.S. and other developed nations suggests that the finding of (G)ARCH-M effects and volatility persistence may prove spurious, since researchers fail to account for the structural change in the variance.[5] Lastrapes (1989) shows that changes in the unconditional variance should receive consideration when specifying ARCH models. In his study, for instance, the persistence of volatility in exchange rates decreases after accounting for three U.S. monetary policy regime shifts between 1976 and 1986, diminishing the likelihood of integration-in-variance. Tzavalis and Wickens (1995) find strong evidence of a high degree of persistence in the volatility of the term premium of bonds. Once they allow for the monetary regime shift between 1979 and 1982, the high persistence in the GARCH(1,1)-M model disappears. Poterba and Summers (1986) note that the degree of persistence in variance of a variable importantly affects the relationship between the variable and its volatility, for example, stock returns and their volatility.

Second, the significant statistical property of excess kurtosis provides a cautionary note. Kurtosis for the standardized residuals (i.e., ε_t / σ_t) generally falls below that for the unconditional standard deviation (see Tables 1 and 2). According to the distributional assumptions in the (G)ARCH specification, the standardized residuals should reflect a normal distribution, if the (G)ARCH model totally accounts for the leptokurtic unconditional distribution. The sample kurtosis in Table 2 for the standardized residuals indicates that (G)ARCH accounts for some, but

[5] Employing the GARCH-M modeling approach, Caporale and McKiernan (1998) and Fountas and Karanasos (2006) use annual real GNP or IP (industrial production) data from the mid-1800s to the 1990s for the U.S. Japan, and Germany; Macri and Sinha (2000) use Australian quarterly GDP index and IP from the late-1950s to the end of 1999; Henry and Olekalns (2002) use U.S. quarterly real GNP from 1947 to 1998; Caporale and McKiernan (1996), Speight (1999), and Grier and Perry (2000) use monthly IP from 1948 to the mid-1990s for the U.K. and the U.S. to examine the relationship between output growth and its volatility. The longer the sample period is, the more likely structural changes in variance occur. Only Caporale and McKiernan (1996) create a dummy variable equaling several periods of high volatility in their GARCH process. None of other studies consider possible structural changes in variance.

not all, of the leptokurtosis for the output growth rate.[6] Blanchard and Simon (2001) note that the distribution of output growth exhibits excess kurtosis (or skewness), if large and infrequent shocks occur. This suggests that the evidence of excess kurtosis may also reflect the Great Moderation.

Thus, we expect to resolve the two puzzles by modeling the non-stationarity variance arising from the Great Moderation. First, the high persistence of output volatility decreases after accounting for the Great Moderation, diminishing the likelihood of biasing the sum of the estimated autoregressive parameters toward one. Second, leptokurtosis in the unconditional distribution of output growth vanishes after adjustment for (G)ARCH with conditional normality.

To consider the effect of the Great Moderation on the variance of output in the GARCH-M specification, we include a dummy variable in the conditional variance equation, which equals unity after 1982:1; zero otherwise. To provide more evidence regarding the effect of the Great Moderation, we also estimate the variance process with the break date 1984:1.

Table 3 reports the new estimates, showing that the structural dummy proves highly significant in the variance equation in all four cases. The substantial and significant increase of the value of the maximum log-likelihood indicates that including the dummy in the (G)ARCH equation provides a superior specification. The log-likelihood ratio test statistics (i.e., $\chi^2(1)$ = 24.77 for the GARCH-M or $\chi^2(1)$ = 41.64 for the ARCH-M) prove significant at the 1-percent level. Based on the log-likelihood values, the two models perform almost equally well. The Ljung-Box Q-statistics of the standardized residuals and the squared standardized residuals show

[6] As a general rule, empirical studies report the first- and second-order serial correlation in the standardized residuals of the GARCH estimation based on Ljung-Box diagnostic statistics, but lack skewness, excess kurtosis, and normality tests in most research. We argue that the higher moments of the standardized residuals provide important diagnostic information regarding accurate model specification and the true data generating process. Speight (1999) reports excess kurtosis and significant Jarque-Bera statistic after GARCH adjustment. Other studies pay no attention to the behavior of kurtosis before and after GARCH estimation.

no evidence of autocorrelation and heteroskedasticity, providing further support for these specifications. The coefficients of skewness and excess kurtosis prove insignificant at the 5-percent level.[7] And, thus, the standardized residuals conform to a normal distribution. All results prove robust to the choice of the alternative break at 1984:1, as shown in Table 3.[8]

The estimate of λ remains insignificant. That is, no relationship exists between growth and volatility measured by the two GARCH-M models with two different break dates. Two important consequences emerge by allowing for a structural change in the conditional variance. First, a large decline occurs in the estimated degree of persistence in the conditional variance. Each estimate in the variance equation in Table 3 falls below that in the model without the dummy in Table 2. The highly significant LR statistic in Table 3 proves no IGARCH effect. In addition, the estimates of α_1 and β_1 or α_1 and α_2 not only fall in size but also become insignificant in the specification that includes the post-1984 dummy variable. The dummy variable replaces the (G)ARCH effects. Second, a strong interaction emerges between the dummy variable and the excess kurtosis, which previously proved significant (see Tables 1 and 2). This interaction now proves insignificant. These results suggest that the statistical evidence for time-varying variance and for excess kurtosis in the growth rate may reflect a shift in the unconditional variance caused by the Great Moderation. Figures 2-5 plot the conditional variances with and without a dummy for the four models, respectively. The solid line includes the dummy variable while the dashed line

[7] Blanchard and Simon (2001) calculate skewness and excess kurtosis statistics for the error term from their estimated rolling first-order autoregressive process, finding significant skewness and excess kurtosis only around the early 1980s recession. These results match our findings. That is, by incorporating a one-time shift in either 1982:1 or 1984:1 in the GARCH and ARCH variance equations, we observe insignificant skewness and excess kurtosis.

[8] We also examine descriptive statistics for the data in the pre- and post-1984 subsamples. The same conclusions emerge as when the break equals 1982:1 (see Table 1). The Wald statistic cannot reject the null of equality of means between samples and the variance ratio rejects the null of variance equality between the samples. One minor difference does occur, however. No ARCH effects exist in either the pre- and the post-1984 periods. To save space, we do not report detailed statistics.

excludes the dummy variable. One common characteristic appears in the four Figures -- a clear shift in the variance. The high volatility appears in the period before 1982 or 1984.

The structural break for the Great Moderation in 1982:1 (or 1984:1) suggests that we divide the sample into pre- and post-1982 (or post-1984) groups to estimate the relationship between the growth rate and its volatility separately for each period. Since the descriptive statistics indicate no time-varying variance in the growth series for three sub-samples (pre- and post-1984 and pre-1982), we construct a moving-sample standard deviation to proxy for output volatility and use OLS to estimate the relationship. For the post-1982 period, higher-order ARCH tests yield insignificant results in Table 1, suggesting the appropriateness of a simple ARCH(1) model.[9] Table 4 presents the results. For the two periods 1947:1 to 1981:4 and 1947:1 to 1983:4, the coefficient of the autoregressive term at lag two proves insignificant. Thus, we report the estimation results for an autoregressive model with only one lag. The Ljung-Box diagnostic statistics show no evidence of first- and second-order autocorrelation in the residuals for the four sub-samples and the residuals reflect a normal distribution. The insignificant estimate of λ again verifies our earlier finding that no relationship exists between the growth rate and its volatility in the U.S.[10]

4. Further Evidence

This section considers two additional tests. First, we examine the possibility that the output growth

[9] We also experimented with a GARCH(1,1) model, but a negative and insignificant estimate appears in the variance equation, violating the non-negativity assumption.

[10] One referee notes that some inherent limitations may exist in GARCH-M models for examining the relationship between growth and volatility. Particularly, the GARCH-M models usually employ high-frequency data. We use quarterly data. But, the GDP growth in Figure 1 exhibits volatility clusters. That is, certain time periods experience high volatility while other periods experience low volatility. This basic characteristic of the data suggests applying the methodology to measure volatility and its effect on growth. Our approach focuses mainly on modeling the non-stationarity variance arising from the Great Moderation. In Table 4, the full sample splits and neither the moving-sample standard deviation nor the ARCH process produces a significant effect on growth.

rate affects its volatility, exploring whether an endogeneity bias exists in the GARCH and ARCH processes. Second, we study whether a trend decrease in the volatility of output growth provides a better specification than the one-time shift considered above. The first test follows the analysis of Fountas and Karanasos (2006) while the second test addresses the conclusion of Blanchard and Simon (2001).

Fountas and Karanasos (2006) recently find, using annual industrial production data from 1860 to 1999, that the output growth rate volatility exhibits no effect on the growth rate, but the output growth rate affects its volatility negatively in the U.S. and a bidirectional causality between output growth and its volatility in Germany. The causal relationship between the output growth rate and its volatility suggests that the GARCH-M approach suffers from an endogeneity bias. Fountas and Karanasos (2006) include lagged growth in the conditional variance equation (the level effect) to test for the effect of growth on volatility in their GARCH(1,1)-M model. Following this specification, Table 5 reports the GARCH(1,1)-M estimation results, where we consider this level effect. The insignificance of λ continues, even while the lagged growth estimate (δ) proves significant in the variance equation for either break date, 1982:1 or 1984:1. All other estimates and diagnostic statistics close mirror those in the models without this level effect. The findings that the output growth rate does not depend on changes in its volatility and that the output growth rate does affect its volatility negatively prove consistent with evidence in Fountas and Karanasos (2006), although they employ the long series of annual output data and we use quarterly data.

Blanchard and Simon (2001) argue that a trend decline in the volatility of output growth provides a better explanation of output growth volatility than does the one-time shift. Tables 6 and 7 present the evidence. Table 6 introduces a time trend in specifications of the GARCH and ARCH processes, but without a one-time shift dummy variable. The coefficient of the time trend proves negative for both specifications, although only significantly negative at the five-percent level in the ARCH process. All other coefficients and diagnostic statistics closely mirror those in the models estimated without the time trend or the one-time shift dummy variable (see Table 2). One exception exists; the LR statistics now prove significant at the five-percent level when the time trend appears, which rejects the null hypothesis of an IGARCH. Furthermore, although the volatility persistence falls substantially with the time trend, excess kurtosis remains. Thus, the time trend captures some, but not all, of the time-varying property of the variance. Finally, the volatility measure remains insignificant in the growth rate equation, matching the results of Tables 2 and 3.

Table 7 includes the time trend and the one-time shift dummy variable together in the GARCH and ARCH processes. In all four models, the coefficient of the time trend proves insignificant. Moreover, the coefficient of the one-time shift dummy variable proves significantly negative in each specification. All remaining coefficients and diagnostic statistics nearly match those in Table 3, including the insignificant coefficient of the variance measure in the growth rate equation.

In summary, the one-time shift dummy variable dominates the time trend across our various tests. That is, based on the log-likelihood value, the corresponding specifications in Tables 3 and 7 do not exhibit significant differences, whereas the corresponding specifications in Tables 2 and 6 do exhibit significant differences with those in Tables 3 and 7.

5. Conclusion

This paper examines the effect of the Great Moderation on the relationship between quarterly real GDP growth rate and its volatility in the U.S. over the period 1947:1 to 2006:2. We begin by considering the possible effects, if any, of structural change on the volatility process. Our initial results, based on either a GARCH-M or an ARCH-M model of the conditional variance of the residuals, find strong evidence of volatility persistence and excess kurtosis in the growth rate. Subsequent analysis reveals that this conclusion does not remain robust to a one-time shift in output variability due to the Great Moderation. First, the findings of a time-varying variance measured by the GARCH-M or ARCH-M model disappear in the specifications that include the post-1984 dummy variable. That is, the GARCH effect proves spurious. In any case, no GARCH-M effect emerges. Second, excess kurtosis vanishes in the specifications that include either the 1982 or the 1984 dummy variable in either the GARCH or the ARCH process. Both the data analysis and the OLS estimates generally suggest no relationship between U.S. output volatility and growth, favoring macroeconomic models that dichotomize the determination of output volatility and growth. In sum, our results add to the conclusion that the relationship between the output growth rate and its volatility in the U.S. proves weak, at best.

The independence between the output growth and its volatility needs careful interpretation. Endogenous growth theory, for example, does not imply any importance for the second moment. Blackburn and Galindev (2003) and Blackburn and Pelloni (2004) model the link between the mean and variance of the output growth rate explicitly. Different mechanisms of endogenous technological change and nominal or real shocks can lead to positive or negative relationship between growth and volatility. In his model, Blackburn (1999) shows for a linear endogenous learning function, the effect of the output growth-rate volatility on the output growth rate equals zero. A concave (convex) learning function generates a negative (positive) effect. That is, an independent relationship may exist with or without the Great Moderation. The discrepancy of our findings from those in previous studies highlights the sensitivity of the results to the country considered, the time period examined, the frequency of the data, and the methodology employed. This apparent inconclusiveness warrants further investigation of the relationship between growth and its volatility. Moreover, since studies generally focus on developed countries, additional analysis from developing countries may prove illuminating. For example, the Asian newly industrializing countries may provide totally different scenarios because of their high growth rates.

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Table 1: Descriptive Statistics for Quarterly Growth of Real GDP

	1947:1 - 2006:2	1947:1 - 1981:4	1982:1 - 2006:2
Sample size	237	139	98
Mean	0.8358	0.8700	0.7872
Standard deviation	0.9872	1.1844	0.6094
Skewness	-0.0773	-0.0858	-0.5931*
	(0.1591)	(0.2077)	(0.2474)
Excess kurtosis	1.3124*	0.3723	2.2179*
	(0.3184)	(0.4155)	(0.4948)
Jarque-Bera	17.246*	0.9739	25.8337*
	[0.0001]	[0.6144]	[0.0000]
***LB Q*(1)**	26.0163*	13.0015*	13.9116*
***LB Q*(3)**	34.154*	16.8830*	34.0089*
***LB Q*(6)**	45.754*	25.1725*	38.7083*
LM (1)	3.3967**	0.0439	5.7063*
	[0.0653]	[0.8338]	[0.0169]
LM (3)	8.1946*	1.7582	6.2496
	[0.0421]	[0.6240]	[0.1000]
LM (6)	12.407*	4.1906	4.2203
	[0.0534]	[0.6508]	[0.6468]
ADF(*n*)	-10.888(0)*	-8.4126(0)*	-4.4736(1)*
Structural stability test for unconditional mean			
	H_0 **: full-sample = pre-1982**	H_0 **: full-sample = post-1982**	H_0 **: pre-1982 = post-1982**
	H_1 **: full-sample ≠ pre-1982**	H_1 **: full-sample ≠ post-1982**	H_1 **: pre-1982 ≠ post-1982**
***Wald* test**	-0.2871	0.5460	0.7024
Structural stability test for unconditional variance			
	H_0 **: full-sample = pre-1982**	H_0 **: full-sample = post-1982**	H_0 **: pre-1982 = post-1982**
	H_1 **: full-sample < pre-1982**	H_1 **: full-sample > post-1982**	H_1 **: pre-1982 > post-1982**
***F* test**	0.6947	2.6242	3.7772
	[0.0072]	[0.0000]	[0.0000]

Note: Standard errors appear in parentheses; p-values appear in brackets. The measures of skewness and kurtosis are normally distributed as $N(0,6/T)$ and $N(0,24/T)$, respectively, where T equals the number of observations. $LB\ \ Q(k)$ equals a Ljung-Box statistic testing for autocorrelations in growth up to *k* lags. *LM* (*k*) equals the Lagrange multiplier test for conditional heteroskedasticity, distributed asymptotically as $\chi^2(k)$. ADF(*n*) equals the augmented Dickey-Fuller unit-root test with lags *n* selected by the SC. The Wald statistic $(\hat{\mu}_i - \hat{\mu}_j)^2 / (SD_i^2 + SD_j^2)$ tests for structural change in the mean between the samples *i* and *j*, distributed as $\chi^2(1)$. The *F* statistic equals the variance-ratio test between the samples *i* and *j*, asymptotically distributed as $F(df_i, df_j)$ and *df* denotes degrees of freedom.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 2: GARCH-M Results without Structural Break

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

a_0	a_1	a_2	λ	α_0	α_1	β_1	
0.4267*	0.2371*	0.2002*	0.0911	0.0106	0.1606*	0.8355*	
(0.1648)	(0.0692)	(0.0725)	(0.1960)	(0.0123)	(0.0492)	(0.0434)	
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
1.6828	4.2863	0.7517	6.7173	0.1388	0.8898*	8.5442*	0.0213
[0.6407]	[0.6379]	[0.8609]	[0.3477]	[0.3870]	[0.0059]	[0.0139]	[0.8839]

Maximum log-likelihood function value: -76.5102

ARCH-M Results without Structural Break

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2$

a_0	a_1	a_2	λ	α_0	α_1	α_2	
0.3891**	0.3223*	0.1611*	0.1130	0.3661*	0.3107*	0.3730*	
(0.2247)	(0.0755)	(0.0651)	(0.2497)	(0.0564)	(0.0883)	(0.1190)	
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
2.6000	5.8491	1.0033	8.7188	-0.0293	1.2352*	15.0378*	2.6977
[0.4575]	[0.4403]	[0.8004]	[0.1904]	[0.8551]	[0.0001]	[0.0005]	[0.1005]

Maximum log-likelihood function value: -85.6076

Note: Standard errors appear in parentheses; p-values appear in brackets; $LB\,Q(k)$ and $LB\,Q^2(k)$ equal Ljung-Box Q-statistics testing for standardized residuals and squared standardized residuals for autocorrelations up to k lags. LR equals the likelihood ratio statistic following a χ^2 distribution with one degree of freedom that tests for $\alpha_1 + \beta_1 = 1$ or $\alpha_1 + \alpha_2 = 1$.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 3: GARCH-M Results with Structural Break, 1982:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1982:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	β_1	γ
0.4427*	0.2634*	0.1757*	0.0519	0.3948*	0.0801	0.6271*	-0.3322*
(0.1361)	(0.0694)	(0.0726)	(0.1633)	(0.1958)	(0.0692)	(0.1767)	(0.1651)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
2.4247	5.4342	0.8279	4.9614	0.0944	0.1846	0.6862	17.5179*
[0.4890]	[0.4894]	[0.8427]	[0.5487]	[0.5560]	[0.5684]	[0.7095]	[0.0000]

Maximum log-likelihood function value: -64.1225

ARCM-M Results with Structural Break, 1982:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1982:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	α_2	γ
0.4644*	0.2880*	0.1559*	0.0254	0.9886*	0.1891*	0.0959	-0.8191*
(0.1366)	(0.0751)	(0.0714)	(0.1619)	(0.1843)	(0.0844)	(0.1125)	(0.1754)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
2.3699	5.3571	0.7853	4.2135	0.1307	0.1385	0.8607	18.1591*
[0.4992]	[0.4988]	[0.8529]	[0.6478]	[0.6967]	[0.4153]	[0.6502]	[0.0000]

Maximum log-likelihood function value: -64.7860

GARCH-M Results with Structural Break, 1984:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1984:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	β_1	γ
0.4396*	0.2365*	0.1888*	0.0516	1.0989*	0.1025	0.0737	-0.9326*
(0.1296)	(0.0728)	(0.0671)	(0.1621)	(0.5406)	(0.0893)	(0.4186)	(0.4615)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
2.0877	4.0421	1.2643	6.3881	0.0652	0.2646	0.8633	26.3497*
[0.5543]	[0.6709]	[0.7376]	[0.3811]	[0.6830]	[0.4115]	[0.6494]	[0.0000]

Maximum log-likelihood function value: -62.2568

ARCH-M Results with Structural Break, 1984:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1984:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	α_2	γ
0.4360*	0.2592*	0.1806*	0.0533	1.0628*	0.0913	0.0823	-0.8914*
(0.1369)	(0.0706)	(0.0746)	(0.1618)	(0.1946)	(0.0821)	(0.1073)	(0.1828)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
2.6621	5.0311	0.4254	5.1444	0.0759	0.1963	0.6057	22.1019*
[0.4467]	[0.5398]	[0.9349]	[0.5254]	[0.6360]	[0.5441]	[0.7386]	[0.0000]

Maximum log-likelihood function value: -62.7772

Note: Standard errors appear in parentheses; p-values appear in brackets; $LB\,Q(k)$ and $LB\,Q^2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to *k* lags. LR equals the likelihood ratio statistic, following a χ^2 distribution with one degree of freedom that tests for $\alpha_1 + \beta_1 = 1$ or $\alpha_1 + \alpha_2 = 1$.

* denotes 5-percent significance level.

Table 4: Sub-Sample Results with Structural Break, 1982:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_{t+m} = [(m^{-1})\sum(\ln Y_{t+i-1} - \ln Y_{t+i-2})^2]^{0.5}$ **, where *m*=8.**

AR(1): 1947:1−1981:4

a_0	a_1	λ				
0.8160*	0.3089*	-0.2937				
(0.4157)	(0.0900)	(0.6735)				
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**
2.3929	4.4391	4.0181	5.2361	0.0609	0.7565**	3.1319
[0.4949]	[0.6174]	[0.2595]	[0.5139]	[0.7806]	[0.0891]	[0.2088]

AR(2)-ARCH-M estimation: 1982:1−2006:2

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2$

a_0	a_1	a_2	λ	α_0	α_1	
0.6398*	0.1717	0.2881*	-0.4121	0.1578*	0.3386**	
(0.2474)	(0.1113)	(0.0973)	(0.5470)	(0.0373)	(0.1821)	
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**
1.8017	2.4284	0.8315	2.6803	0.2668	-0.1686	1.2793
[0.6145]	[0.8763]	[0.8419]	[0.8477]	[0.2882]	[0.7423]	[0.5274]

Sub-Sample Results with Structural Break, 1984:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_{t+m} = [(m^{-1})\sum(\ln Y_{t+i-1} - \ln Y_{t+i-2})^2]^{0.5}$ **, where *m*=8.**

AR(1): 1947:1−1983:4

a_0	a_1	λ				
0.8244*	0.3401*	-0.3740				
(0.4033)	(0.0839)	(0.6578)				
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**
1.9239	4.6216	2.1934	2.6126	0.0358	0.6570	2.2728
[0.5883]	[0.6417]	[0.5333]	[0.8566]	[0.8655]	[0.1260]	[0.3209]

AR(2): 1984:1−2006:2

a_0	a_1	a_2	λ			
0.4893*	0.1907**	0.3931*	-0.4156			
(0.1825)	(0.1039)	(0.1066)	(0.5190)			
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**
2.2561	2.5491	2.6447	6.0366	-0.0174	0.0461	0.0121
[0.5218]	[0.8636]	[0.4503]	[0.4193]	[0.9479]	[0.9327]	[0.9939]

Note: Standard errors appear in parentheses; p-values appear in brackets; $LB\,Q(k)$ and $LB\,Q^2(k)$ equal Ljung-Box Q-statistics, testing for residuals (standardized residuals) and squared residuals (squared standardized residuals) for autocorrelations up to *k* lags.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 5: GARCH-M Results with the Level Effect and Structural Break, 1982:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \delta\ y_{t-1} + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1982:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	β_1	γ	δ
0.3948*	0.2325*	0.1978*	0.1066	0.4156*	0.0805	0.6532*	-0.2784*	-0.0907**
(0.1544)	(0.0726)	(0.0727)	(0.1633)	(0.1685)	(0.0690)	(0.1477)	(0.1255)	(0.0478)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**	
2.2408	4.7920	0.7720	4.7959	0.0759	0.0103	0.2281	18.5340*	
[0.5239]	[0.5707]	[0.8561]	[0.5702]	[0.6358]	[0.9746]	[0.8921]	[0.0000]	
Maximum log-likelihood function value: -61.5242								

GARCH-M Results with the Level Effect and Structural Break, 1984:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \delta\ y_{t-1} + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1984:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	β_1	γ	δ
0.3737*	0.2112*	0.1914*	0.1583	0.6903*	0.0953	0.4373	-0.4708**	-0.1391*
(0.1517)	(0.0741)	(0.0721)	(0.1600)	(0.3448)	(0.0796)	(0.2850)	(0.2672)	(0.0643)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**	
2.4636	4.6720	0.5555	4.3049	0.0395	0.0011	0.0616	17.7487*	
[0.4819]	[0.5865]	[0.9065]	[0.6354]	[0.8051]	[0.9972]	[0.9696]	[0.0000]	
Maximum log-likelihood function value: -60.5848								

Note: Standard errors appear in parentheses; p-values appear in brackets; $LB\,Q(k)$ and $LB\,Q^2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to *k* lags. LR equals the likelihood ratio statistic following a χ^2 distribution with one degree of freedom that tests for $\alpha_1 + \beta_1 = 1$ or $\alpha_1 + \alpha_2 = 1$.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 6: GARCH-M Results with Time Trend

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \varpi\ t$

a_0	a_1	a_2	λ	α_0	α_1	β_1	ϖ
0.4497*	0.2448*	0.1716*	0.0765	0.2833*	0.1170**	0.7077*	-0.0011
(0.1469)	(0.0738)	(0.0725)	(0.1686)	(0.1917)	(0.0633)	(0.1331)	(0.0007)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
1.6637	4.3703	1.2280	5.1846	0.1258	0.8619*	7.9283*	6.1271*
[0.6450]	[0.6266]	[0.7462]	[0.5203]	[0.4328]	[0.0077]	[0.0189]	[0.0133]

Maximum log-likelihood function value: -71.6664

ARCH-M Results with Time Trend

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \varpi\ t$

a_0	a_1	a_2	λ	α_0	α_1	α_2	ϖ
0.4202*	0.2885*	0.1595*	0.0828	1.1111*	0.1144	0.2047*	-0.0043*
(0.1484)	(0.0706)	(0.0799)	(0.1659)	(0.1944)	(0.0753)	(0.0979)	(0.0008)
LB Q **(3)**	***LB*** Q **(6)**	***LB*** Q^2 **(3)**	***LB*** Q^2 **(6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**
2.0563	4.7750	0.4943	4.1656	0.0708	0.7629*	5.9219*	13.6673*
[0.5608]	[0.5729]	[0.9201]	[0.6542]	[0.6588]	[0.0184]	[0.0517]	[0.0002]

Maximum log-likelihood function value: -70.4836

Note: Standard errors appear in parentheses; p-values appear in brackets; $LB\,Q(k)$ and $LB\,Q^2(k)$ equal Ljung-Box Q-statistics testing for standardized residuals and squared standardized residuals for autocorrelations up to *k* lags. LR equals the likelihood ratio statistic following a χ^2 distribution with one degree of freedom that tests for $\alpha_1 + \beta_1 = 1$ or $\alpha_1 + \alpha_2 = 1$.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 7: GARCH-M Results with Time Trend and Structural Break, 1982:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \varpi\ t + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1982:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	β_1	ϖ	γ
0.4427*	0.2634*	0.1756*	0.0519	0.3963*	0.0802	0.6263*	-0.0000	-0.3318*
(0.1371)	(0.0696)	(0.0729)	(0.1638)	(0.2106)	(0.0697)	(0.1795)	(0.0004)	(0.1674)
***LB* Q (3)**	***LB* Q (6)**	***LB* Q^2 (3)**	***LB* Q^2 (6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**	
2.4190	5.4276	0.8264	4.9524	0.0945	0.1848	0.6878	17.4906*	
[0.4901]	[0.4902]	[0.8431]	[0.5499]	[0.5555]	[0.5679]	[0.7089]	[0.0000]	

Maximum log-likelihood function value: -64.1224

ARCH-M Results with Time Trend and Structural Break, 1982:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \varpi\ t + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1982:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	α_2	ϖ	γ
0.4582*	0.2878*	0.1562*	0.0327	1.0336*	0.1757*	0.0982	-0.0005	-0.7598*
(0.1379)	(0.0745)	(0.0725)	(0.1626)	(0.2189)	(0.0885)	(0.1110)	(0.0011)	(0.2104)
***LB* Q (3)**	***LB* Q (6)**	***LB* Q^2 (3)**	***LB* Q^2 (6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**	
2.2990	5.2517	0.7697	4.1941	0.1232	0.1582	0.8441	18.3301*	
[0.5127]	[0.5119]	[0.8566]	[0.6504]	[0.4422]	[0.6249]	[0.6556]	[0.0000]	

Maximum log-likelihood function value: -64.6902

GARCH-M Results with Time Trend and Structural Break, 1984:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \varpi\ t + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1984:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	β_1	ϖ	γ
0.4432*	0.2361*	0.1889*	0.0476	1.0756*	0.1058	0.0747	0.0002	-0.9623*
(0.1301)	(0.0731)	(0.0673)	(0.1618)	(0.5134)	(0.0910)	(0.4091)	(0.0011)	(0.4950)
***LB* Q (3)**	***LB* Q (6)**	***LB* Q^2 (3)**	***LB* Q^2 (6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**	
2.1568	4.0880	1.2042	6.4179	0.0653	0.2568	0.8232	26.1636*	
[0.5404]	[0.6647]	[0.7519]	[0.3780]	[0.6827]	[0.4254]	[0.6625]	[0.0000]	

Maximum log-likelihood function value: -62.2309

ARCH-M Results with Time Trend and Structural Break, 1984:1

$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \lambda\sigma_t + \varepsilon_t$ **; and**

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \varpi\ t + \gamma\ Dummy$ **, where *Dummy* = 1 for *t* ≧ 1984:1; 0 otherwise.**

a_0	a_1	a_2	λ	α_0	α_1	α_2	ϖ	γ
0.4374*	0.2589*	0.1808*	0.0515	1.0512*	0.0935	0.0809	0.0001	-0.9106*
(0.1377)	(0.0708)	(0.0750)	(0.1619)	(0.2188)	(0.0838)	(0.1075)	(0.0011)	(0.2268)
***LB* Q (3)**	***LB* Q (6)**	***LB* Q^2 (3)**	***LB* Q^2 (6)**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**LR**	
2.7064	5.0671	0.4301	5.1829	0.0756	0.1906	0.5823	22.0962*	
[0.4391]	[0.5352]	[0.9339]	[0.5205]	[0.6374]	[0.5558]	[0.7473]	[0.0000]	

Maximum log-likelihood function value: -62.7692

Note: Standard errors appear in parentheses; p-values appear in brackets; $LB\,Q(k)$ and $LB\,Q^2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to k lags. LR equals the likelihood ratio statistic, following a χ^2 distribution with one degree of freedom that tests for $\alpha_1 + \beta_1 = 1$ or $\alpha_1 + \alpha_2 = 1$.

* denotes 5-percent significance level.

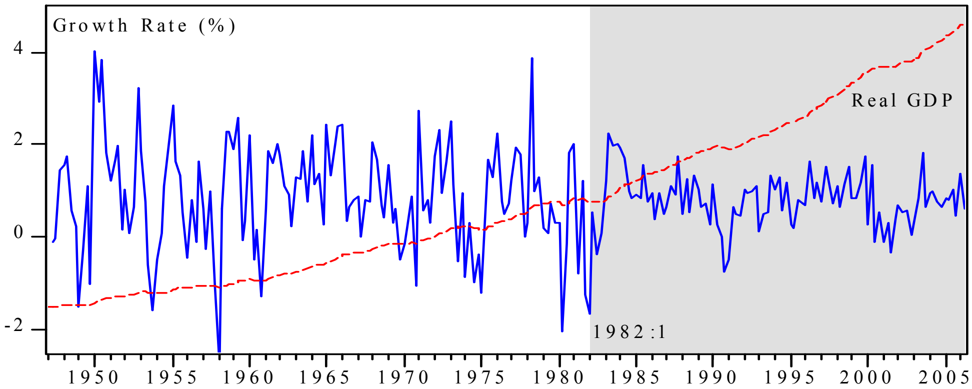

Figure 1. Real GDP and GDP Growth Rate

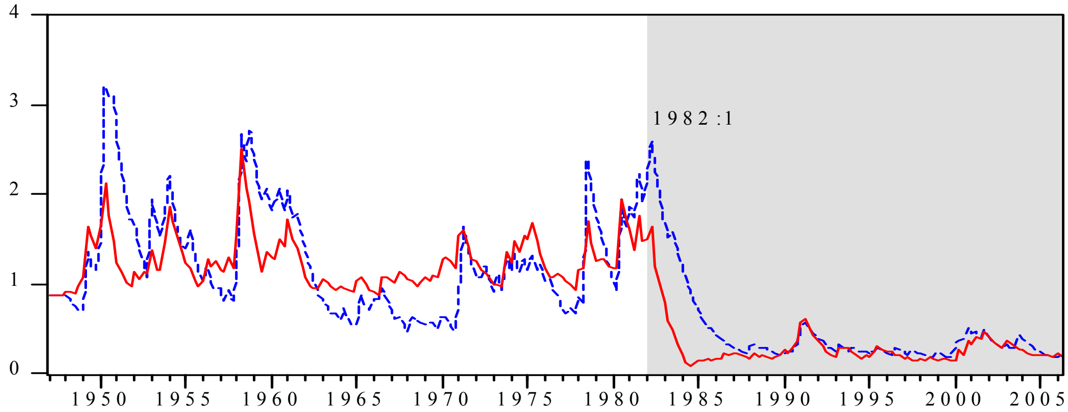

Figure 2. GARCH Variance with (Solid Line) and without (Dashed Line) Dummy.

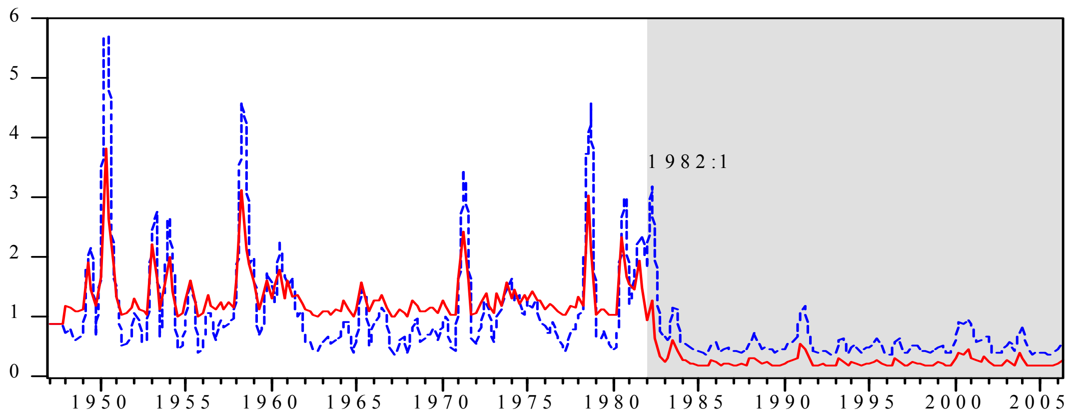

Figure 3. ARCH Variance with (Solid Line) and without (Dashed Line) Dummy.

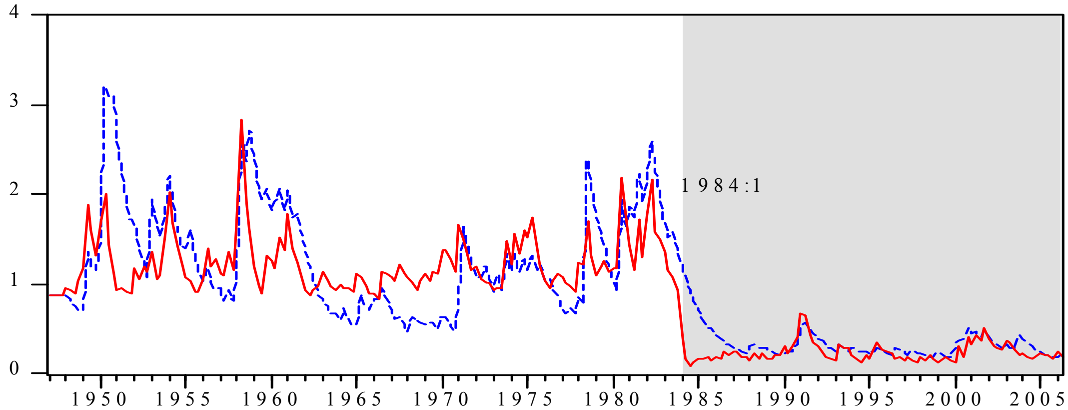

Figure 4. GARCH Variance with (Solid Line) and without (Dashed Line) Dummy.

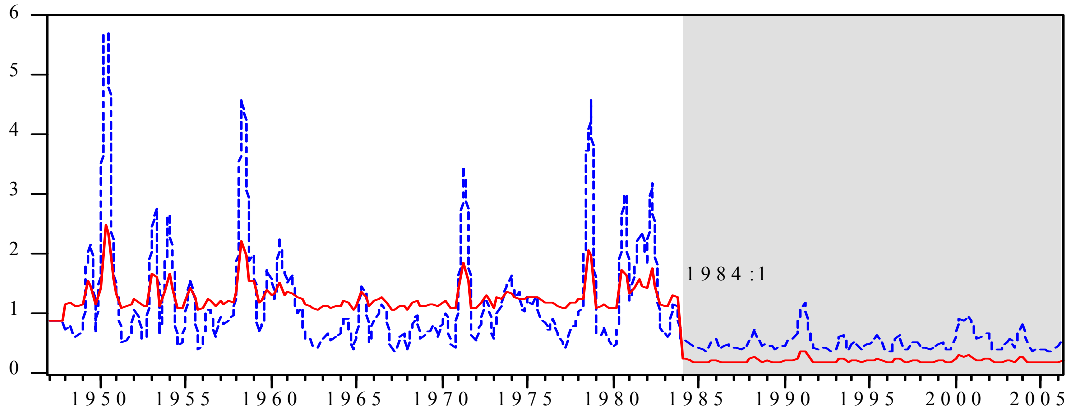

Figure 5. ARCH Variance with (Solid Line) and without (Dashed Line) Dummy.