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The SureMath Approach to Success in Freshman Chemistry

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Introduction

This material is being presented as an alternative to methods of solving chemistry problems which are hoary with age; specifically, the factor label method, which has been in use for at least 50 years, fails for some of us. And it is for those of us who need an alternative to the traditional schemes shown in textbooks, that this manuscript is offered.

The scheme, a paper and pencil variant on the "SureMath" scheme [1] works for some of us who find that traditional word problems don't "click".

What you will see is a method of solution in which you are asked to write down the thing that you want as an answer, on the left hand side of an equal sign, and then, on the right hand side of the equal sign, you will be asked to write down the definition of what's on the other (left hand) side. That definition, written in terms of physical/chemical quantities, suggests what we need to know to solve for the "answer", and so we "drill-down" and look at each term of the right hand side and make each the left hand side of a new line which, in essence, is a new problem whose new right hand side needs interpreting in the same manner. We just repeat the procedure, until we end up with quantities on the right hand side which are all numbers, ready for our calculators. Then, we zip up all those numbers, moving up the ladder, until we end up with the answer we sought.

If this seems complicated, just start doing the problems in this text and you will see what I mean.

In the following, you should cover the answers, and just reveal them one line at a time as you practice thinking in this reverse scheme.

Good luck!

[1] H. C. McAllister, http://www.hawaii.edu/suremath/home1.html

1 Introductory Materials

Before we start, there are some classic conversions which need to be addressed.

1.1 Conversions & Things Numerical (some preliminary problems)

1.1.1 Conversions of the non-religious kind

How fast are you travelling in miles per hour when you are travelling at 100 kilometers per hour (in Europe)?

There are two ways that are commonly used to solve a problem like this (and all the others we will be doing). The first method starts with what's given, and attempts to create what's desired. We have:

1.1.2 Solution 1

$$100\frac{kilometers}{hour} \times factor\frac{miles}{kilometers}$$

where factor is a conversion factor which converts kilometers to miles. We may not be able to look this up, so we need to calculate it:

$$\frac{miles}{kilometers} = \frac{miles}{\cancel{feet}} \times \frac{\cancel{feet}}{in} \times \frac{in}{cm} \times \frac{cm}{km}$$

and

$$\frac{miles}{kilometers} = \frac{miles}{\cancel{feet}} \times \frac{\cancel{feet}}{\cancel{in}} \times \frac{\cancel{in}}{cm} \times \frac{cm}{km}$$

and

$$\frac{miles}{kilometers} = \frac{miles}{\cancel{feet}} \times \frac{\cancel{feet}}{\cancel{in}} \times \frac{\cancel{in}}{\cancel{cm}} \times \frac{\cancel{cm}}{km}$$

Now, putting numbers into the fractions we have

$$\frac{miles}{kilometers} = \frac{1\ miles}{\cancel{5280\ feet}} \times \frac{1\ \cancel{feet}}{12\ \cancel{in}} \times \frac{1\ \cancel{in}}{2.54\ \cancel{cm}} \times \frac{100\ \cancel{cm}}{1\ \cancel{m}} \times \frac{1000\ \cancel{m}}{1\ km}$$

Therefore

$$100\frac{\cancel{kilometers}}{hour} \times \frac{1\ miles}{5280} \times \frac{1}{12} \times \frac{1}{2.54} \times \frac{10^5}{1\ \cancel{kilometer}} = 62mph$$

Notice that the kilometers cancel in this last expression, giving us 100 km/hour as its new value with resultant units miles/hour.

Next time you look at your car's speedometer, notice that the 65 miles hour mark is approximately over the 100 kilometers per hour mark on the alternate (European) dial.

1.1.3 Conversions of the non-religious kind, revisited

How fast are you travelling in miles per hour when you are travelling at 100 kilometers per hour (in Europe)?

There are two ways that are commonly used to solve a problem like this (and all the others we will be doing). The second method starts with what's given, and attempts to create what's desired. We have:

1.1.4 Solution 2

(α) velocity in miles/hour = velocity in $\frac{feet}{hour} \times \frac{1\ miles}{5280\ feet}$

(β) velocity in feet/hour = velocity in $\frac{inches}{hour} \times \frac{1\ feet}{12\ inches}$

(γ) velocity in inches/hour = velocity in $\frac{cm}{hour} \times \frac{1\ inch}{2.54\ cm}$

(δ) velocity in cm/hour = velocity in $\frac{meters}{hour} \times \frac{100\ cm}{1\ meter}$

(ϵ) velocity in meter/hour = velocity in $\frac{kilometers}{hour} \times \frac{1000\ m}{1\ kilometer}$

(ϵ) velocity in meter/hour = 100 kph $\times \frac{1000\ m}{1\ kilometer}$

(ϵ) velocity in meter/hour = $10^5 \frac{meter}{hour}$

(δ) velocity in cm/hour = velocity in $\frac{meters}{hour} \times \frac{100\ cm}{1\ m}$

(δ) velocity in cm/hour $= 10^5 \frac{meter}{hour} \times \frac{100\ cm}{1\ meter}$

(δ) velocity in cm/hour $= 10^7 \frac{cm}{hour}$

(γ) velocity in inches/hour = velocity in $\frac{cm}{hour} \times \frac{1\ inch}{2.54\ cm}$

(γ) velocity in inches/hour = $10^7 \frac{cm}{hour} \times \frac{1\ inch}{2.54\ cm}$

(γ) velocity in inches/hour = $\frac{10^7}{2.54} \frac{inches}{hour}$

(β) velocity in feet/hour = velocity in $\frac{inches}{hour} \times \frac{1\ feet}{12\ inches}$

(β) velocity in feet/hour $= \frac{10^7}{2.54} \times \frac{1\ feet}{12\ inches}$

(α) velocity in miles/hour = velocity in $\frac{feet}{hour} \times \frac{1\ miles}{5280\ feet}$

(α) velocity in miles/hour = $\frac{10^7}{2.54} \times \frac{1\ feet}{12\ inches} \times \frac{1\ miles}{5280\ feet}$

1.1.5 Comments

Notive that this method is the “backwards” of the method we used before. In the first method, we started with a given and worked towards an answer, while in the second, we started with what was being asked for, and worked backwards towards what was given.

1.1.6 Distance from the Earth to the Moon

The moon is approximately 384,000 kilometers from the earth. How far is that in miles?

1.1.7

(α) distance in miles = distance in feet $\times\frac{1\ mile}{5280\ feet}$

(β) distance in feet = distance in inches $\times\frac{1\ feet}{12\ inches}$

(γ) distance in inches= distance in cm $\times\frac{1\ inch}{2.54\ cm}$

(δ) distance in cm = distance in meters $\times\frac{100\ cm}{1\ meter}$

(ϵ) distance in meter = distance in kilometers $\times\frac{1000\ m}{1\ kilometer}$

(ζ) distance in kilometers = 384,000 km

(ϵ) distance in meter = 384,000 km $\times\frac{1000\ m}{1\ kilometer}$

(δ) distance in cm = 384,000 km$\times\frac{1000\ m}{1\ kilometer}\times\frac{100\ cm}{1\ meter}$

(γ) distance in inches= 384,000 km$\times\frac{1000\ m}{1\ kilometer}\times\frac{100\ cm}{1\ meter}\times\frac{1\ inch}{2.54\ cm}$

(β) distance in feet = 384,000 km$\times\frac{1000\ m}{1\ kilometer}\times\frac{100\ cm}{1\ meter}\times\frac{1\ inch}{2.54\ cm}\times\frac{1\ feet}{12\ inches}$

(α) distance in miles = 384,000 km$\times\frac{1000\ m}{1\ kilometer}\times\frac{100\ cm}{1\ meter}\times\frac{1\ inch}{2.54\ cm}\times\frac{1\ feet}{12\ inches}\times\frac{1\ mile}{5280\ feet}$ = 238,000 miles

1.1.8 Comments

1.1.9 Size of an Atom

An atom is about 1 Å in diameter. 1 Å is 10^{-8} cm. What is the diameter of an atom in inches?

1.1.10

(α) diameter in inches = diameter in cm $\times \frac{1\ inches}{2.54\ cm}$

(β) diameter in cm = diameter in Å$\times \frac{1\ cm}{10^8\ \mathring{A}}$

(γ) diameter in Å= 1 Å

(β) diameter in cm = 1 Å$\times \frac{1\ cm}{10^8\ \mathring{A}}$

(α) diameter in inches = 1 Å$\times \frac{1\ cm}{10^8\ \mathring{A}} \times \frac{1\ inches}{2.54\ cm} = 3.9 \times 10^{-9}$ inches

1.1.11 Comments

1.2 A problem concerning mixtures

When 215 grams of potassium nitrate, 0.35 grams of sodium chloride and 500 grams of water are mixed at about 90^oC, the volume of the resultant solution is found to be 585.3 mℓ. What is the density of the solution?

1.2.1 Solution

(α) density $= \frac{\text{mass}}{\text{volume}}$

- (β) mass = mass of potassium nitrate + mass of sodium chloride + mass of water
  - (γ) mass of potassium nitrate = 215 grams
  - (γ) mass of sodium chloride = 0.35 grams
  - (γ) mass of water= 500 grams
- (β) mass = 215 + 0.35 + 500 grams
- (β) volume = 585.3 mℓ

(α) density $= \frac{215+0.35+500 \text{ grams}}{585.3 \text{ m}\ell}$

(α) density $= \frac{715.4}{585.3} = 1.22\frac{\text{grams}}{\text{m}\ell}$

1.2.2 Comments

The SureMath scheme for doing word problems such as these consists of starting an outline, using the α level first, and writing down what's asked for in the question. In this case, it's the density.

On the other side of the equal sign, we write down the definition (or meaning) of the thing on the left hand side, i.e., mass divided by volume.

Next, at the indented first level, β, we write two lines, one for the mass and the other for the volume.

If we know these values, we place them on the right hand side of the equal sign.If we don't know one or more of these values, we search the problem for a statement about them, or employ a definition. In this case, we find statements in the problem about the mass, and write that out on the right hand side of the mass equation.

Then, we indent one level, to the γ level, and write out each of the terms in the preceding β level, with an equal sign, and the value from the problem text. Of course, we might have had to use formula instead, it depends on what we know at the indentation level we're working at.

Anyway, once all the right hand sides are numbers, we can un-indent, and calculate back to the β level, and then when these β level items are all numbers, we can back-indent to the α level and finish the problem.

1.3 A problem concerning the density of a person

Assume that a person is a cylinder. Perhaps this person is 5.3 feet tall and has a waistline of 25.94 inches. Perhaps this person weighs (has a mass of) 160 pounds. What is the density of this person (in grams/cc)?

1.3.1 Solution

- (α) density $=\frac{\text{mass}}{\text{volume}}$
  - (β) mass (in grams) = mass (in ~~pounds~~)*453.6 $\frac{\text{grams}}{\cancel{\text{pound}}}$
    - (γ) mass (in pounds) = 160 pounds
  - (β) mass (in grams) = (160 ~~pounds~~)*453.6 $\frac{\text{grams}}{\cancel{\text{pound}}}$
  - (β) mass (in grams) = 72576 grams
  - (β) volume = $\pi r^2 h$; r = radius of person, h = height
    - (γ) $h = 5.3\ \cancel{\text{feet}} \times 12\ \frac{\cancel{\text{in}}}{\cancel{\text{ft}}} \times 2.54\ \frac{\text{cm}}{\cancel{\text{in}}}$
    - (γ) h = 161.5 cm
    - (γ) $r = \frac{\text{waist}}{2\pi}$; waist = circumference
      - (δ) waist = 25.94 ~~inches~~ $\times$ 2.54 $\frac{\text{cm}}{\cancel{\text{inch}}}$
      - (δ) waist = 65.89 cm
    - (γ) $r = \frac{65.89}{2\pi}$
    - (γ) r = 10.49 cm
  - (β) volume $= \pi \times (10.49)^2 \times 161.5$; in cubic centimeters
  - (β) volume = 55830 cc
- (α) density $= \frac{\text{mass}}{\text{volume}} = \frac{72576 \text{ grams}}{55830 \text{ cc}}$
- (α) density = 1.3 $\frac{\text{grams}}{\text{cc}}$

1.3.2 Comments

Notice that we use "cc" (for cubic centimeters) and "mℓ" (for milliliters) interchangeably. This is common.

It might be a good idea to approximate the answer, i.e.,

$$\frac{70,000}{50,000} > 1$$

which is a fine check of your work.

1.4 A problem concerning the density of a cow

Assume that a cow is a sphere. Perhaps this cow is 5 feet 4 inches in diameter. Perhaps this cow weighs (has a mass of) 0.21 tons. What is the density of this cow (in grams/cc)?

1.4.1 Solution

(α) density $=\frac{\text{mass}}{\text{volume}}$

- (β) mass (in grams) = mass (in $\cancel{\text{tons}}$) $* \frac{2000 \cancel{\text{pounds}}}{1 \cancel{\text{ton}}} * 453.6 \frac{\text{grams}}{\cancel{\text{pound}}}$
  - (γ) mass (in pounds) = 0.21 tons
  - (β) mass (in grams) = 0.21 (in $\cancel{\text{tons}}$) $* \frac{2000 \cancel{\text{pounds}}}{1 \cancel{\text{ton}}} * 453.6 \frac{\text{grams}}{\cancel{\text{pound}}}$
- (β) mass (in grams) = 190,512 grams
- (β) volume $= \frac{4}{3}\pi r^3$; r = radius of cow
  - (γ) $r = \frac{5\times 12+4}{2}$ inches
  - (γ) $r = \frac{64}{2}$ inches
  - (γ) $r = 32 \cancel{\text{ inches}} \times 2.54 \frac{\text{cm}}{\cancel{\text{inch}}}$
  - (γ) $r = 32 \times 2.54$ cm
  - (γ) $r = 81.28$ cm
- (β) volume $= \frac{4}{3}\pi \times (81.28)^3$; in cubic centimeters
- (β) volume = 715,962 cc

(α) density $= \frac{\text{mass}}{\text{volume}} = \frac{190000 \text{ grams}}{716000 \text{ cc}}$

(α) density $= 0.02 \frac{\text{grams}}{\text{cc}}$

1.4.2 Comments

5 foot 4 inches has to be converted to inches by first converting the feet, i.e., 5 foot is the same as 60 inches. Then, one adds the 4 inches to get 64 inches. Clearly, this is a cow that floats!

1.5 A solubility problem at two temperatures

The solubility of a certain compound is 130.9 g/100g water at 100^{o}C and 5.9 g/100 g water at 25^{o}C. When 124 grams of solvent which is saturated (with solute) at 100^{o}C is cooled to 25^{o}C, calculate the amount of solute which precipitates out, in grams.

1.5.1 Solution

(α) grams precipitated =grams solute beginning−grams solute still in solution

- (β) grams solute at beginning $= 124\, \cancel{\text{grams solvent at } 100^o\text{C}} \times \frac{130.9}{100} \frac{\text{grams solute}}{\cancel{\text{grams solvent}}}$
- (β) grams solute at beginning = 162.3 grams
- (β) grams solute at end $= 124\, \cancel{\text{grams solvent at } 25^o\text{C}} \times \frac{5.9}{100} \frac{\text{grams solute}}{\cancel{\text{grams solvent}}}$
- (β) grams solute at end = 7.316 grams

(α) grams precipitated = 162.3 grams - 7.3 grams = 155.0 grams

1.5.2 Comments

The beginning is the high temperature situation, 100^{o}C. The end is the low temperature situation, 25^{o}C

We start with a saturated hot solution, and as we cool it, crystals of the solute appear and fall to the bottom of the container. When we've cooled to 26^{o}C, a goodly portion of the solute shows up at the bottom of the container as a powder (solid). What is left in the supernatant solution (the one above the solid precipitate) contains some of the original solute, the rest of the solute is now solid in the bottom of the container.

1.6 A solubility problem; two temperatures and two species

The solubility of a certain compound (X) is 226 g/100 g water at 80^{o}C and 172.4 g/100 g water at 25^{o}C. Another substance (Y) has a solubility of 125.5 g/100 g solvent at 80^{o}C and 65.4 grams/100 g solvent at 25^{o}C. 401.6 grams of solvent (water) is used to prepare a solution saturated in both X and Y at 80^{o}C. The solution is cooled to 25^{o}C. What is the ration of the amount of X to the amount of Y in the solid precipitate?

1.6.1 Strategy

First, we need to know how much of each substance has been dissolved Then

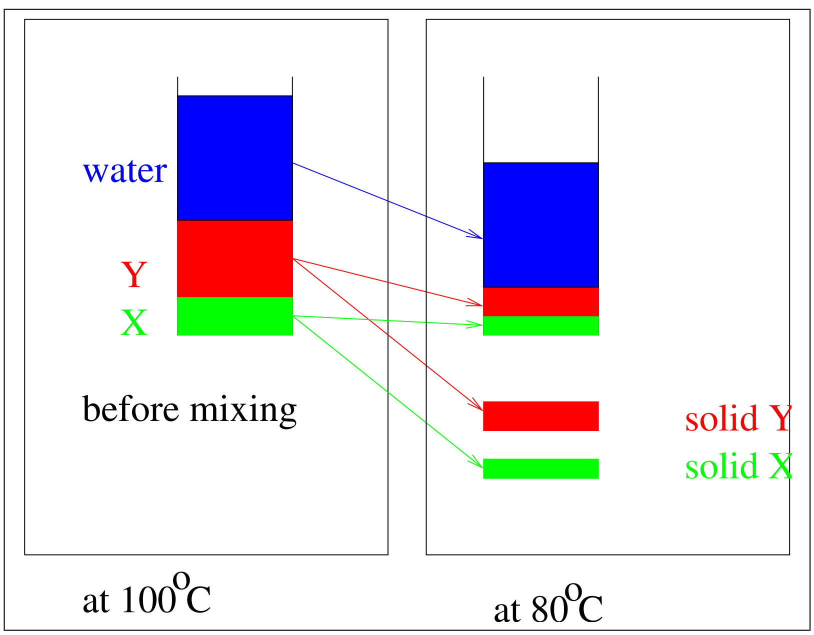

Figure 1: Schematic showing why solid X and solid Y appear (problem 1.5).

we have at 80^{o}C. Then we will compute how much is dissolved at 25^{o}C, and subtract these values from the original amounts, to get the masses of X and Y which, not being in solution, must be in the solid precipitate.

Then, we will get the ratio of the amounts of the two substances in the precipitate.

1.6.2 Solution

(α) grams X precipitated =grams X beginning – grams X still in solution

- (β) grams X at beginning = $\cancel{401.6 \text{ grams solvent at } 80^oC} \times \frac{226}{100} \frac{\text{grams X}}{\cancel{\text{grams solvent}}}$
- (β) grams solute at beginning = 401.6×2.26 grams = 908 grams
- (β) grams solute at end = $401.6\ \cancel{\text{grams solvent at } 25^oC} \times \frac{172.4}{100} \frac{\text{grams X}}{\cancel{\text{grams solvent}}}$
- (β) grams solute at end = 401.6×1.724 grams =692 grams

(α) grams X precipitated =908 grams X beginning−692 grams X still in solution = 216 grams of X

(α) grams Y precipitated =grams Y beginning – grams Y still in solution

- (β) grams Y at beginning = $401.6\ \cancel{\text{grams solvent at } 80^oC} \times \frac{125.5}{100} \frac{\text{grams Y}}{\cancel{\text{grams solvent}}}$
- (β) grams Y at beginning = 401.6×1.255 grams = 504.0 grams
- (β) grams Y at end = $401.6\ \cancel{\text{grams Y at } 25^oC} \times \frac{65.4}{100} \frac{\text{grams Y}}{\cancel{\text{grams solvent}}}$
- (β) grams Y at end = 401.6×0.654 grams = 262.6 grams

(α) grams Y precipitated = 504 grams - 263 grams = 241 grams

Ratio = $\frac{692}{241} = 2.87$

1.6.3 Comments

1.7 Conversion Problem, gallons to cubic centimeters

A car's gas tank contains about 16 gallons of fuel. How many ℓiters of fuel does it contain?

1.7.1 Solution

(α) ℓiters of fuel = gallons of fuel $\times \frac{\ell iters}{gallon}$

(α) ℓiters of fuel = 16 gallons $\times$ 3.785 $\frac{\ell iters}{gallon}$

(α) ℓiters of fuel = 16 $\times$ 3.785 ℓ iters

1.7.2 Comments

2 Atoms & Molecules

2.1 Percent by Mass (%)

A certain organic compound is found by analysis to conform to the formula $C_6H_1C\ell_1$. What is the percent by mass of chlorine in the compound?

2.1.1 Solution

We assume one mole of compound, yielding, therefore, 6 moles of C, one mole of H, and 1 mole of chlorine.

(α) $\%\ C\ell = \frac{\text{mass } C\ell}{\text{mass of compound}} \times 100$

(β) mass of $C\ell$ = 35.45 grams

(β) mass of compound = mass of $C\ell$ + mass of C+ mass of H

(γ) mass of C $= 6\ \cancel{\text{moles}} \times 12.01\ \frac{\text{grams}}{\cancel{\text{mole}}}$

(γ) mass of H $= 1\ \cancel{\text{moles}} \times 1.008\ \frac{\text{grams}}{\cancel{\text{mole}}}$

(γ) mass of $C\ell = 1\ \cancel{\text{moles}} \times 35.45\ \frac{\text{grams}}{\cancel{\text{mole}}}$

(β) mass of compound = $6 \times 12 + 1.008 + 35.45$ grams

(β) mass of compound = 109.5 grams

(α) $\%\ C\ell = \frac{35.45}{109.5} \times 100$

2.1.2 Comments

Some problems really aren't amenable to treatment using suremath, and this is one of them.

2.2 Empirical Formula from mass %

A certain organic compound is found by analysis to contain 31.79% carbon (by mass), 5.59% hydrogen (by mass). Since it is known that only C, H, and $C\ell$ are present in this compound, what is the empirical formula for this compoud?

2.2.1 Solution

We are asked for the values of x, y, and z in the following formula: $C_xH_yC\ell_z$. That means, we need to known the number of moles of C, H and $C\ell$ in an arbitrary sample. Choose a 100 gram sample, so there are 31.79 grams of C, 5.59 grams of H, and 100-31.79-5.59 grams of $C\ell$.

(α) moles of C$= \dfrac{\text{mass C}}{\text{atomic mass of C}}$

- (β) mass of C$=$ 31.79 grams
- (β) atomic mass of C $= 12\ \dfrac{\text{grams}}{\text{mole}}$
- (β) moles of C$= \dfrac{31.79\ \cancel{\text{grams}}}{12\ \cancel{\text{grams}}/\text{mole}}$

(α) moles of C$=$ 2.65 moles

(α) moles of H$= \dfrac{\text{mass H}}{\text{atomic mass of H}}$

- (β) mass of C$=$ 5.59 grams
- (β) atomic mass of C $= 1.008\ \dfrac{\text{grams}}{\text{mole}}$
- (β) moles of H$= \dfrac{5.59\ \cancel{\text{grams}}}{1.008\ \cancel{\text{grams}}/\text{mole}}$

(α) moles of H$=$ 5.55 moles

(α) grams of $C\ell$ = 100 - 31.79 - 5.59 grams

(α) moles of $C\ell = \dfrac{\text{mass}\ \ C\ell}{\text{atomic mass of }C\ell}$

(β) mass of $C\ell = 62.62$ grams

(β) atomic mass of $C\ell = 35.45 \ \frac{\text{grams}}{\text{mole}}$

(β) moles of $C\ell = \frac{62.62 \ \cancel{\text{grams}}}{35.45 \ \cancel{\text{grams}}/\text{mole}}$

(α) moles of $C\ell = 1.766$ moles

The implication is that the formula of the compound is $C_{2.65}H_{5.55}C\ell_{1.766}$ which we could reduce to integer subscripts by dividing through by the smallest, i.e.,

$$C_{\frac{2.65}{1.766}}H_{\frac{5.55}{1.766}}C\ell_{\frac{1.766}{1.766}}$$

which would be $C_{1.5}H_{3.14}C\ell_1$ which might be $C_3H_6C\ell_2$

2.2.2 Comments

If the "answer" $C_3H_6C\ell_2$ were correct, then we would have

- C accounts for 3×12 grams in a mole
- H accounts for 6×1 grams in a mole, and
- $C\ell$ accounts for 2×35.45 grams in a mole

which means that

$$3 \times 12 + 6 \times 1 + 2 \times 35.45 = 93.9\frac{\text{grams}}{\text{mole}}$$

so the % of each composition should have been

1. C is $\frac{36}{113.9}\%$ $= 31.6\%$compared to 31.8%
2. H is $\frac{6}{113.9}\%$ $= 5.3\%$compared to 5.59%
3. $C\ell$ is $\frac{71}{113.9}\%$ $= 62.3\%$compared to 62.6%

2.3 Number of electrons in a sample %

Assume that we have 10×10^{10} molecules (!) of ammonium cation, $NH_4{}^+$. How many electrons are in the sample?

2.3.1 Solution

(α) Number of e^- = molecules of ammonium $\times$ # of electrons per molecule

- (β) molecules of ammonium $= 10 \times 10^{10}$
- (β) $\frac{\text{number of e}^-}{\text{molecule}} = \frac{4\ \text{e}^-}{\text{H-atom}} + \frac{7\ \text{e}^-}{\text{N-atom}} - 1$
  ('-1' for $NH_4 \rightarrow NH_4{}^+ + e^{-1}$)

(α) Number of e^- $= 10 \times 10^{10} \times 10$

(α) Number of e^- $= 10 \times 10^{11} = 10^{12}$

2.3.2 Comments

2.4 Calcium Nitrate's oxygen atoms

How many grams of calcium nitrate, $Ca(NO_3)_2$, contain 8.65 grams of oxygen *atoms*?

2.4.1 Solution

(α) grams of $Ca(NO_3)_2$ = grams O atoms $\times \frac{\text{grams of } Ca(NO_3)_2}{\text{grams of O atoms}}$

(β) grams of O atoms = 8.65 grams.

(β) $\frac{\text{grams of } Ca(NO_3)_2}{\text{grams of O atoms}} = \frac{\text{grams of Ca} + \text{ grams of N} + \text{ grams of O}}{\text{grams of O atoms}}$

(γ) $\frac{\text{grams of Ca}}{\text{grams of O atoms}} = \frac{8 \text{ grams of Ca}}{6 \times 16 \text{ grams of O atoms}}$

(γ) $\frac{\text{grams of N}}{\text{grams of O atoms}} = \frac{2 \times 14 \text{ grams of N}}{6 \times 16 \text{ grams of O atoms}}$

(γ) $\frac{\text{grams of O}}{\text{grams of O atoms}} = \frac{6 \times 16 \text{ grams of O}}{6 \times 16 \text{ grams of O atoms}}$

(β) $\frac{\text{grams of } Ca(NO_3)_2}{\text{grams of O atoms}} = \frac{40.08 \text{ grams of Ca} + 2 \times 14 \text{ grams of N} + 6 \times 16 \text{ grams of O}}{6 \times 16 \text{ grams of O atoms}}$

(α) grams of $Ca(NO_3)_2 = 8.65 \cancel{\text{grams O atoms}} \times \frac{(40.08+28+96) \text{ grams of } Ca(NO_3)_2}{96 \cancel{\text{grams of O atoms}}}$
$= 14.8$ grams

2.4.2 Comments

2.5 Mixture of NO and O_2

A certain amount of nitrogen oxide and of oxygen are contained in a sealed flask. The mass of the gases in the flask is 146 grams and the total number of moles of gas is 4.61. What is the number of moles of NO(g) in the mixture?

2.5.1 Solution

(α) moles of NO $= \dfrac{\text{mass of NO}}{(16{+}14)\frac{\text{grams}}{\text{mole}}}$

(β) mass of NO = mass of sample - mass of oxygen

(γ) mass of oxygen = moles of O_2 $\times 32\frac{\text{grams}}{\text{mole}}$

(δ) moles of O_2 = total moles - moles of NO

(δ) moles of O_2 = 4.61 - moles of NO

(γ) mass of oxygen =(4.61 - moles of NO) $\times$ 32 $\frac{\text{grams}}{\text{mole}}$

(β) mass of NO = mass of sample - (4.61 - moles of NO) $\times$ 32 $\frac{\text{grams}}{\text{mole}}$

(α) moles of NO $= \dfrac{\text{146 grams} - (\text{ 4.61 - moles of NO}) \times \frac{\text{32 grams}}{\text{mole}}}{(16{+}14)\frac{\text{grams}}{\text{mole}}}$

2.5.2 Special Comments

Stop. What have we here? We have the text "moles of NO on both sides of an equal sign, i.e., we have an equation for this quantity. Letting this quantity be symbolize by a letter, say z, we have

$$z = \frac{146 \text{ grams} - (4.61 - z)\text{moles} \times 32\frac{\text{grams}}{\text{mole}}}{(16+14)\frac{\text{grams}}{\text{mole}}}$$

which can be solved for "z"!

2.5.3 Comments

This is a hard problem. The SureMath method is not necessarily optimum here.

Consider that we have x moles of NO and y moles of O_2. Then x+y = 4.61. Further, the mass of this gas must be

$$30 \text{ x} + 32 \text{ y} = 136 \text{ grams}$$

based on the defintions of x and y.

We have two equations in two unknowns, which need to be solved simultaneously.

$$y = 4.61 - x$$

so

$$30x + 32(4.61 - x) = 136$$

so

$$-2x = 136 - 4.61 \times 32$$

which allows us to solve for x

3 Elementary Stoichiometry

3.1 Isotopic Distributions

An element has two isotopes. Isotope A has an atomic mass of 23.00 amu. Isotope B has an atomic mass of 24.00 amu. The average atomic mass of the element is 23.80 amu. What is the % (abundance) of isotope A in the naturally occuring mixture?

3.1.1 Solution

Let x = the abundance of isotope A

$$x \times 23.00 + (1 - x) \times 24.00 = 23.80$$

Solve for x

3.1.2 Comments

3.2 Mass of CO_2 from burning carbon

When 130 grams of carbon are completely burned in an excess of oxygen, carbon dioxide (CO_2) is formed. What is the mass of this carbon dioxide?

3.2.1 Solution

$C(s) + O_2(g) \rightarrow CO_2(g)$

(α) mass CO_2 = moles of CO_2 × Molecular Mass (CO_2)

- (β) Molecular Mass (CO_2) $= 12 + 2 \times 16 = 44\ \frac{\text{grams}}{\text{mole}}$
- (β) moles of $CO_2 = \cancel{\text{moles of C}} \times \frac{1 \text{ mole } CO_2}{\cancel{1 \text{ mole C}}}$
  - (γ) moles of C$=$ grams C$\times \frac{1 \text{ mole C}}{12 \text{ grams C}}$
    - (δ) grams C$=$ 130 grams
  - (γ) moles of C$=$ 130 $\cancel{\text{grams C}} \times \frac{1 \text{ moles C}}{12\ \cancel{\text{grams C}}}$
- (β) moles of $CO_2 = \frac{130}{12}\ \cancel{\text{moles of C}} \times \frac{1 \text{ mole } CO_2}{1\ \cancel{\text{mole C}}}$
- (β) moles of CO_2 = 10.8 moles

(α) mass $CO_2 = 10.8\ \cancel{\text{moles}} \times 44\ \frac{\text{grams}}{\cancel{\text{mole}}}$

(α) mass CO_2 = 475 grams

3.2.2 Comments

3.3 Mass of CO_2 from burning carbon in (possibly) limiting oxygen

120 grams of carbon are mixed with 122 grams of oxygen and burned to carbon dioxide (CO_2). What is the mass of the carbon dioxide formed?

3.3.1 Solution

$C(s) + O_2(g) \rightarrow CO_2(g)$

(α) mass CO_2 = moles of CO_2 $\times$ Molecular Mass (CO_2)

(β) Molecular Mass (CO_2) $= 12 + 2 \times 16 = 44\ \frac{\text{grams}}{\text{mole}}$

(β) moles of $CO_2 = \cancel{\text{moles of C}} \times \frac{1\text{ mole } CO_2}{\cancel{1\text{ mole C}}}$

(γ) moles of C$= \cancel{\text{grams C}} \times \frac{\text{moles C}}{\cancel{\text{grams C}}}$

(δ) grams C$=$ 120 grams

(γ) moles of C$= 120\ \cancel{\text{grams}}\text{ C} \times \frac{1\text{ moles C}}{12\ \cancel{\text{grams C}}}$

(β) moles of $CO_2 = \frac{120}{12}\ \cancel{\text{moles of C}} \times \frac{1\text{ mole } CO_2}{\cancel{1\text{ mole C}}}$

(β) moles of CO_2 = 10.0 moles

(α) mass $CO_2 = 10.0\ \cancel{\text{moles}} \times 44\ \frac{\text{grams}}{\cancel{\text{mole}}}$

(α) mass CO_2 = 440 grams

(β) moles of CO_2 = moles of $O_2 \times \frac{1\text{ mole } CO_2}{1\text{ mole } O_2}$

(γ) moles of O_2 = grams $O_2 \times \frac{\text{moles } O_2}{\text{grams } O_2}$

(δ) grams O_2 = 122 grams

(γ) moles of $O_2 = 122\ \cancel{\text{grams } O_2} \times \frac{1\text{ moles } O_2}{32\ \cancel{\text{grams } O_2}}$

(β) moles of $CO_2 = \frac{122}{32}\ \cancel{\text{moles of } O_2} \times \frac{1\text{ mole } CO_2}{\cancel{1\text{ mole } O_2}}$

(β) moles of CO_2 = 3.81 moles

(α) mass CO_2 = 3.81 moles$\times$ 44 $\frac{\text{grams}}{\text{mole}}$

(α) mass CO_2 = 168 grams

Choose the lesser amount, 168 grams.

3.3.2 Comments

3.4 Mass of carbon dioxide from burning carbon in (possibly) limiting oxygen

132 grams of carbon are mixed with 268 grams of oxygen and burned to carbon dioxide (CO_2). What is the mass of the carbon dioxide formed?

3.4.1 Solution

$C(s) + O_2(g) \rightarrow CO_2(g)$

Molecular Mass (CO_2) $= 12 + 2 \times 16 = 44 \ \frac{\text{grams}}{\text{mole}}$

(α) mass CO_2 = moles of CO_2 $\times$ Molecular Mass (CO_2)

(β) moles of CO_2 = moles of C$\times$ $\frac{1 \text{ mole } CO_2}{1 \text{ mole C}}$

(γ) moles of C= grams C$\times\frac{\text{moles C}}{\text{grams C}}$

(δ) grams C= 132 grams

(γ) moles of C= 132 grams C$\times\frac{1 \text{ moles C}}{12 \text{ grams C}}$

(β) moles of CO_2 = $\frac{132}{12}$ moles of C$\times$ $\frac{1 \text{ mole } CO_2}{1 \text{ mole C}}$

(β) moles of CO_2 = 11.0 moles

(α) mass CO_2 = 11.0 grams$\times$ 44 $\frac{\text{grams}}{\text{mole}}$

(α) mass CO_2 = 444 grams

(β) moles of CO_2 = moles of O_2 $\times$ $\frac{1 \text{ mole } CO_2}{1 \text{ mole } O_2}$

(γ) moles of O_2 = grams O_2 $\times\frac{\text{moles } O_2}{\text{grams } O_2}$

(δ) grams O_2 = 268 grams

(γ) moles of O_2 = 268 grams O_2 $\times\frac{1 \text{ moles } O_2}{32 \text{ grams } O_2}$

(β) moles of $CO_2 = \frac{268}{32}$ moles of $O_2 \times \frac{1 \text{ mole } CO_2}{1 \text{ mole } O_2}$

(β) moles of $CO_2 = 8.375$ moles

(α) mass $CO_2 = 8.375$ moles$\times$ 44 $\frac{\text{grams}}{\text{mole}}$

(α) mass $CO_2 = 369$ grams

Choose the lesser amount, 369 grams.

3.4.2 Comments

Too much information usually means that one should suspect a limiting reagent problem.

3.5 Feldspar Weathering

Feldspar, $KA\ell Si_3O_8$ weathers in carbon dioxide and water to form potassium carbonate and a clay (aluminosilicate) according to the reaction

$$2KA\ell Si_3O_8\,(s) + 2H_2O(\ell) + CO_2(g) \rightarrow K_2CO_3(aq) + A\ell_2Si_2O_5(OH)_4(s) + 4SiO_2(s)$$

10.9 grams of feldspar, 16.7 moles of water and 21.39 moles of carbon dioxide are mixed and allowed to react. How many moles of silica (SiO_2) are formed?

3.5.1 Solution

molecular mass (Feldspar) = 39.1 + 26.98 + 3 × 28.09 + 8 × 16 grams = 278 grams per mole

(α) moles of SiO_2 =

$$\frac{10.9 \text{ grams feldspar}}{278 \text{ grams/mole feldspar}} \times \frac{4 \text{ moles } SiO_2}{2 \text{ moles of } KA\ell Si_3O_8}$$

= 0.0784 moles

(α) moles of SiO_2 = 16.7 moles $H_2O \times \frac{4 \text{ moles } SiO_2}{2 \text{ moles of } H_2O}$ = 33.4 moles

(α) moles of SiO_2 = 21.4 moles $CO_2 \times \frac{4 \text{ moles } SiO_2}{1 \text{ mole of } CO_2}$ = 85.6 moles

Choose the smaller.

3.5.2 Comments

3.6 Feldspar Weathering

Feldspar, $KA\ell Si_3O_8$ weathers in carbon dioxide and water to form potassium carbonate and a clay (aluminosilicate) according to the reaction

$$2KA\ell Si_3O_8\,(s)+2H_2O(\ell)+CO_2(g) \rightarrow K_2CO_3(aq)+A\ell_2Si_2O_5(OH)_4(s)+4SiO_2(s)$$

835 grams of feldspar, 198 grams of water and 510.4 grams of carbon dioxide are mixed and allowed to react. How many grams of silica (SiO_2) are formed?

3.6.1 Solution

molecular mass (Feldspar) = 39.1 + 26.98 + 3 × 28.09 + 8 × 16 grams = 278 grams per mole

$$(\alpha)\ \text{moles of } SiO_2 = \frac{835 \text{ grams feldspar}}{287 \text{ grams per mole}} \times \frac{4 \text{ moles } SiO_2}{2 \text{ moles of } KA\ell Si_3O_8} = 6$$

$$(\alpha)\ \text{moles of } SiO_2 = \frac{198 \text{ grams } H_2O}{18 \text{ grams per mole}} \times \frac{4 \text{ moles } SiO_2}{2 \text{ moles of } H_2O} = 22$$

$$(\alpha)\ \text{moles of } SiO_2 = \frac{510.4 \text{ grams } CO_2}{44 \text{ grams per mole}} \times \frac{4 \text{ moles } SiO_2}{1 \text{ mole of } CO_2} = 46$$

Choose the smaller (6 moles SiO_2). Then

$$6\ \cancel{\text{moles } SiO_2} \times \frac{60 \text{ grams } SiO_2}{\cancel{\text{mole } SiO_2}}$$

is the number of grams of silicon dioxide produced.

3.6.2 Comments

3.7 Feldspar Weathering

Feldspar, $KA\ell Si_3O_8$ weathers in carbon dioxide and water to form potassium carbonate and a clay (aluminosilicate) according to the reaction

$$2KA\ell Si_3O_8\,(s) + 2H_2O(\ell) + CO_2(g) \rightarrow K_2CO_3(aq) + A\ell_2Si_2O_5(OH)_4(s) + 4SiO_2(s)$$

835 grams of feldspar, 198 grams of water and 510.4 grams of carbon dioxide are mixed and allowed to react. How many grams of water are left over?

3.7.1 Solution

molecular mass (Feldspar) = 39.1 + 26.98 + 3 × 28.09 + 8 × 16 grams = 297 grams per mole

---

$$(\alpha)\ \text{moles of } SiO_2 = \frac{835 \text{ grams feldspar}}{287 \text{ grams per mole}} \times \frac{4 \text{ moles } SiO_2}{2 \text{ moles of } KA\ell Si_3O_8} = 6$$

$$(\alpha)\ \text{moles of } SiO_2 = \frac{198 \text{ grams } H_2O}{18 \text{ grams per mole}} \times \frac{4 \text{ moles } SiO_2}{2 \text{ moles of } H_2O} = 22$$

$$(\alpha)\ \text{moles of } SiO_2 = \frac{510.4 \text{ grams } CO_2}{44 \text{ grams per mole}} \times \frac{4 \text{ moles } SiO_2}{1 \text{ mole of } CO_2} = 46$$

Choose the smaller (6 moles SiO_2). Then

$$6\ \cancel{\text{moles } SiO_2} \times \frac{2 \text{ moles } H_2O}{4\ \cancel{\text{mole } SiO_2}}$$

is the number of moles of water consumed. Then $\frac{198}{18} - 3$ is the number of moles of water left over, the original amount minus the amount consumed.

$$18 \times \left(\frac{198}{18} - 3\right) = 18 \times (11 - 3) = 18 \times 8 = 146 \text{ grams}$$

is then the number of grams of water left over.

3.7.2 Comments

3.8 Limiting Reagent (I)

For the reaction
$N_2(g) + 3H_2(g) \rightarrow 2\ NH_3(g)$
if 4.31 *moles* of N_2 and the exactly correct number of *moles* of H_2 are reacted, how many *moles* of NH_3 are produced?

3.8.1 Solution

$$(\alpha)\ \text{moles of } NH_3 = 4.31 \text{ moles of } N_2 \times \frac{2 \text{ moles of } NH_3}{1 \text{ moles of } N_2}$$

3.8.2 Comments

The number of moles of H_2 simultaneously needed is

$$4.31 \text{ moles } N_2 \times \frac{3 \text{ moles } H_2}{1 \text{ mole } N_2} = 12.9 \text{ moles } N_2$$

3.9 Limiting Reagent (I)

For the reaction
$N_2(g) + 3H_2(g) \rightarrow 2\ NH_3(g)$
if 4.31 *moles* of N_2 and 14.53 *moles* of H_2 are reacted, how many *moles* of NH_3 are produced?

3.9.1 Solution

The fact that initial amounts of both reactants is known tells us that this is most likely a limiting reagent problem. Therefore, we will do the calculation twice, once based on N_2 and once based on H_2 and choose the case which produces *the least* NH_3.

$$(\alpha)\ \text{moles of}\ NH_3 = 4.31\ \text{moles of}\ N_2 \times \frac{2\ \text{moles of}\ NH_3}{1\ \text{moles of}\ N_2} = 8.62\ \text{moles of}\ NH_3\ \text{possibly produced}$$

$$(\alpha)\ \text{moles of}\ NH_3 = 14.53\ \text{moles of}\ H_2 \times \frac{2\ \text{moles of}\ NH_3}{3\ \text{moles of}\ H_2} = 9.53\ \text{moles of}\ NH_3\ \text{possibly produced}$$

Choose the smaller.

3.9.2 Comments

The number of moles of ammonia produced = 8.62 moles NH_3.
The number of moles of N_2 left over = 4.31 - $\frac{8.62}{2}$
The number of moles of H_2 left over is 14.53 - $\frac{8.62 \times 3}{2}$

3.10 Atoms in Molecules

Calculate the number of oxygen atoms in a sample of 126 grams of ethyl alcohol, C_2H_6OH.

3.10.1 Solution

(α) number of O atoms $= \dfrac{\text{atoms O}}{\text{molecules } C_2H_5OH} \times$ molecules of C_2H_5OH

(β) molecules of $C_2H_5OH =$ number of moles of $C_2H_5OH \times \dfrac{6.022 \times 10^{23}\text{molecules}}{\text{mole}}$

(β) moles of $C_2H_5OH = \dfrac{126 \text{ grams } C_2H_5OH}{46 \ \frac{\text{grams}}{\text{mole}}}$

(β) moles of $C_2H_5OH =$ 3.74 moles

(β) molecules of $C_2H_5OH =$

$$3.74 \text{ moles of } C_2H_5OH \times \frac{6.022 \times 10^{23}\text{molecules}}{\text{mole}}$$

(β) molecules of $C_2H_5OH = 22.5 \times 10^{23}$ molecules

(α) number of O atoms $= \dfrac{2 \text{ atoms O}}{\cancel{\text{molecule } C_2H_5OH}} \times 22.5 \times 10^{23} \ \cancel{\text{molecules } C_2H_5OH}$

(α) number of O atoms $= 4.5 \times 10^{22}$ atoms of O

3.10.2 Comments

3.11 Advanced Problem on $\mathbf{X}C\ell_2$ versus $\mathbf{X}Cl_4$

A 10.5 gram sample of $XC\ell_2$, where X is some unknown element, reacts with $C\ell_2$ to form 12.6 grams of $XC\ell_4$. What is the atomic mass of X?

3.11.1 Solution

Fundamentally, we have to compute the number of moles of $XC\ell_2$ (symbolically, and using the balanced equation, equate that to the number of moles of $XC\ell_4$, which we can compute in another way, thereby creating an equation. The general rule of thumb is that saying the same thing two different ways is the path to creating an equation!

---

1. the number of moles of $XC\ell_2$ is just

$$\frac{10.5}{\mathrm{M}_X + 2 \times 35.45}$$

   and since the balanced equation is just

$$X\mathrm{C}\ell_2 + \mathrm{C}\ell_2 \rightarrow X\mathrm{C}\ell_4$$

   the number of moles of $XC\ell_4$ is the same, which can be expressed as

$$\frac{10.5}{\mathrm{M}_X + 2 \times\ 35.45}$$

2. Since the number of moles of $XC\ell_4$ is *also* just

$$\frac{12.6}{M_X + 4 \times 35.45}$$

   so we have two ways of saying the same thing:

$$\frac{10.5}{M_X + 2 \times 35.45} = \frac{12.6}{M_X + 4 \times 35.45}$$

   which are expressions for the number of moles of $XC\ell_4$, and is an equation for M_X.

3.11.2 Comments

The suremath scheme really doesn't work here.

3.12 A Very Difficult Question

A mixture of oxygen and hydrogen is analyzed by passing it over hot copper oxide and through a drying tube. Hydrogen reduces the CuO according to the equation

$$CuO(s) + H_2(g) \rightarrow Cu(s) + H_2O(g)$$

Oxygen then reoxides the copper formed:

$$Cu(s) + \frac{1}{2}O_2(g) \rightarrow CuO(s)$$

321.2 grams of an unknown mixture is passed over the copper oxide, and 10.5 grams of dry oxygen are found in the effluent gas. What is the original composition of the mixture?

3.12.1 Solution

This is a hard problem, perhaps harder than any you've seen so far. It requires understanding the chemistry fully.

Let x = the number of moles of H_2 in the original mixture, and y = the number of moles of O_2 in the same mixture.

Certainly

$$2^*x + 32^*y = \text{grams of mixture}$$

If x moles of H_2 are oxidized to water, then $\frac{x}{2}$ moles of oxygen are used up in that oxidation, since

$$\cancel{CuO(s)} + H_2(g) \rightarrow \cancel{Cu(s)} + H_2O(g)$$
$$+$$
$$\cancel{Cu(s)} + \frac{1}{2}O_2(g) \rightarrow \cancel{CuO(s)}$$

results in (after adding the equations and appropriately canceling)

$$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g)$$

Then $y - \frac{x}{2}$ is the number of moles of O_2 in the effluent. Thus

$$32\left(y - \frac{x}{2}\right) = 10.5$$

and

$$2 * x + 32 * y = 321.2$$

This constitutes two equations in two unknowns, which can be solved for both x and y.

3.12.2 Comments

As one way of solving this set of equations, we offer the following:

$$x = \frac{321.2 - 32 * y}{2}$$

so

$$\left(y - \frac{321.2 - 32 * y}{4}\right) - \frac{10.5}{32}$$

which would mean that

$$y - 80.3 + 8 * y = 0.328$$

and

$$9 * y = 80.3 + 0.328 = 80.628$$

ending up with y = 8.96 moles

That means that

$$x = \frac{321.2 - 32 * 8.96}{2} = 17.3$$

Since the problem didn't specify how to express the composition, we are free to leave it this way, with $8.96 + 17.3$ moles of mixture, and

$$\text{mole fraction } H_2 = \frac{17.3}{8.96 + 17.3}$$

4 Acid Base, Precipitation and Oxidation Reduction

4.1 A problem concerning mixtures and molarity

108.4 mℓ of 1.1 M KCℓ and 101.3 m 2.36 M NaCℓ solutions are mixed together. What is the molarity of Cℓ^-(aq) in the resultant solution? Assume the volumes are additive.

4.1.1 Solution

(α)

$$\text{molarity of C}\ell^- = \frac{\text{moles of C}\ell^-}{\text{total volume}}$$

- (β) moles of Cℓ^- = moles of KCℓ+ moles of NaCℓ
  - (γ) moles of KCℓ = $0.1084\ \ell \times 1.1\frac{\text{moles}}{\ell\text{iter}}$
  - (γ) moles of NaCℓ = $0.1013\ \ell \times 2.36\frac{\text{moles}}{\ell\text{iter}}$
- (β) volume = $0.1084 + 0.1013\ \ell$

(α)

$$\text{molarity of C}\ell^- = \frac{\text{moles of C}\ell^-}{\text{total volume}} \tag{1}$$

(α) density = $\frac{715.4}{585.3} = 1.22\frac{\text{grams}}{\text{m}\ell}$

4.1.2 Comments

4.2 A problem concerning solutions and molarity

102.4 mℓ of 2.62 M NaCℓ is to be diluted by adding 109.2 mℓ if water. What is the molarity of Cℓ^-(aq) in the resultant solution? Assume the volumes are additive.

4.2.1 Solution

(α)

$$\text{molarity of C}\ell^- = \frac{\text{moles of C}\ell^-}{\text{total volume}}$$

(β) moles of Cℓ^- = moles of NaCℓ

(γ) moles of NaCℓ = $0.1024\ \ell \times 2.62 \frac{\text{moles}}{\ell\text{iter}}$

(β) volume = $0.1024 + 0.1092\ \ell$

(α) molarity of Cℓ^- = $\frac{0.1024 \text{ x } 2.62}{0.1024+0.1092}$ moles/ℓiter = $\frac{0.268\ moles}{0.212\ \ell iters} = 0.126M$

4.2.2 Comments

4.3 Another problem concerning solutions and molarity

108 mℓ of 2.92 M NaCℓ is to be diluted by adding 101.4 mℓ of a 1.4 M NaOH solution. What is the molarity of Cℓ^-(aq) in the resultant solution? Assume the volumes are additive.

4.3.1 Solution

(α)

$$\text{molarity of C}\ell^- = \frac{\text{moles of C}\ell^-}{\text{total volume}}$$

(β) moles of Cℓ^- = moles of NaCℓ

(γ)

$$\text{moles of NaC}\ell = 0.108\ell \times 2.92\frac{\text{moles}}{\ell\text{iter}}$$

(β) volume = 0.108 + 0.1014 ℓ

(α)

$$\text{molarity of C}\ell^- = \frac{0.108 \text{ x } 2.92}{0.108 + 0.1014}\frac{\text{moles}}{\ell\text{iter}} = \frac{0.315 \; moles}{0.209 \; \ell iters} = 1.51M$$

4.3.2 Comments

4.4 Yet another problem concerning solutions and molarity

100.6 mℓ of 2.96 M NaCℓ is to be diluted by adding 109.3 mℓ of a 1.15 M NaOH solution. What is the molarity of Na+(aq) in the resultant solution?

4.4.1 Solution

(α) molarity of $\mathrm{Na}^+ = \frac{\text{moles of Na}^+}{\text{total volume}}$

(β) moles of Na^+ = moles of $\mathrm{NaC\ell}$ + moles of NaOH

(γ)

$$\text{moles of NaC}\ell = 0.1006\ell \times 2.96\frac{\text{moles}}{\ell\text{iter}} + 0.1093\ell \times 1.15\frac{\text{moles}}{\ell\text{iter}}$$

(γ)

$$\text{moles of NaC}\ell = 0.298\frac{\text{moles}}{\ell\text{iter}} + 0.126\frac{\text{moles}}{\ell\text{iter}} = 0.424\frac{\text{moles}}{\ell\text{iter}}$$

(β) volume = 0.1006 + 0.1093 ℓ

(α) molarity of
$\mathrm{Na}^+ = \frac{\text{moles of Na}^+}{\text{total volume}}$

(α) molarity of Na^+ =

$$\frac{0.424 \text{ moles of Na}^+}{0.2099\ \ell} = 2.02\frac{\text{moles of Na}^+}{\ell}$$

4.4.2 Comments

4.5 A precipitation problem

112.6 mℓ of 12.7 M $NaC\ell$ is mixed with 105.3 mℓ of a 1.76 M $AgNO_3$ solution. What is the mass (in grams) of the resultant precipitate?

4.5.1 Solution

(α) moles of precipitate based on $NaC\ell$ = moles of $NaC\ell$

- (β) moles of $NaC\ell$ = molarity of $NaC\ell$ $\times$ volume of $NaC\ell$ solution
  - (γ) molarity of $NaC\ell$ = 12.7 moles/ℓiter
  - (γ) volume of $NaC\ell$ solution = 0.1126 ℓiters
- (β) moles of $NaC\ell$ = 12.7 $\times$ 0.1126 moles

(α) moles of precipitate based on $NaC\ell$ = 12.7 $\times$ 0.1126 moles = 1.43

(α) moles of precipitate based on $AgNO_3$ = moles of $AgNO_3$

- (β) moles of $AgNO_3$ = molarity of $AgNO_3$ $\times$ volume of $AgNO_3$ solution
  - (γ) molarity of $AgNO_3$ = 1.76 moles/ℓiter
  - (γ) volume of $AgNO_3$ solution = 0.1053 ℓiters
- (β) moles of $AgNO_3$ = 1.76 $\times$ 0.1053 moles

(α) moles of precipitate based on $AgNO_3$ = 1.76 $\times$ 0.1053 moles= 0.185

Choose the smaller! 0.185 moles ppt means

$$0.185\ moles \times \frac{108 + 35.5\ grams}{1\ mole}$$

4.5.2 Comments

Once you've chosen the appropriate number of moles of precipitate expected multiply that number by the molecular mass to obtain the mass of precipitate expected.

4.6 A precipitation problem

A human patient suffering from a duodenal ulcer may show a hydrochloric acid concentration of 0.080 mol/liter in his gastric juice. It is possible to neutralize this acid with aluminum hydroxide, $A\ell(OH)_3$, which reacts with $HC\ell$. If the patient's stomach receives 3.0 liters of gastric juice per day, how much aluminum hydroxide must he consume (in gram) per day to counteract the acid?

4.6.1 Solution

0.08M $\times$ 3 ℓ/day = 0.24 moles H^+/day

0.24 moles H^+/day $\times \frac{1 \text{ mole } Al^{3-}}{3 \text{ moles } H^+}$ = 0.08 moles Al^{3+}/day

MW of $Al(OH)_3$ = 27 + 3*(16+1) = 78 grams/mole

0.08 $\times$ 78 = grams of $Al(OH)_3$ to be consumed per day

4.6.2 Comments

4.7 A Difficult Oxidation Reduction Balancing Problem (ch4q10)

Balance the following equation

$$Hg_2I_2(s) + HC\ell(aq) + HNO_3 \rightarrow HgC\ell_2(s) + HIO_3(s) + NO(g) + H_2O(\ell)$$

4.7.1 Solution

Treating the oxidation first, we have

$$Hg_2I_2(s) \rightarrow 2HgC\ell_2(s) + HIO_3(s) \tag{2}$$

which is balanced for mercury, and then

$$Hg_2I_2(s) \rightarrow 2HgC\ell_2(s) + 2HIO_3(s) \tag{3}$$

which is balanced for iodine. Notice that the mercury goes from +1 to +2, i.e., is oxidized, and the iodine goes from -1 to +5, i.e., it is also oxidized. That's what makes this problem unique!

Continuing with the chemical balance of Equation 2 (above), we have when balancing the chlorines

$$Hg_2I_2(s) + 4C\ell^-(aq) \rightarrow 2HgC\ell_2(s) + 2HIO_3(s)$$

and, when balancing the oxygens we have

$$Hg_2I_2(s) + 4C\ell^-(aq) + 3H_2O(\ell) \rightarrow 2HgC\ell_2(s) + 2HIO_3(s)$$

and finally, balancing the protons,

$$Hg_2I_2(s) + 4C\ell^-(aq) + 6H_2O(\ell) \rightarrow 2HgC\ell_2(s) + 2HIO_3(s) + 10H^+(aq)$$

Next, we balance the electrons:

$$Hg_2I_2(s) + 4C\ell^-(aq) + 6H_2O(\ell) \rightarrow 2HgC\ell_2(s) + 2HIO_3(s) + 10H^+(aq) + 14e^-$$

For the reduction we have

$$NO_3^-(aq) \rightarrow NO(g)$$

to begin, which is balanced for nitrogen. To balance for oxygen, we have

$$NO_3^-(aq) \rightarrow NO(g) + 2H_2O(\ell)$$

and penultimately, we have for protons

$$NO_3^-(aq) + 4H^+(aq) \rightarrow NO(g) + 2H_2O(\ell)$$

All we have to do now is get the charges correct. We have +3 on the left and nothing on the right, so we need 3 electrons on the left, i.e.,

$$3e^- + NO_3^-(aq) + 4H^+(aq) \rightarrow NO(g) + 2H_2O(\ell)$$

$$Hg_2I_2(s) + 4C\ell^-(aq) + 6H_2O(\ell) \rightarrow 2HgC\ell_2(s) + 2HIO_3(s) + 10H^+(aq) + 14e^-$$
$$3e^- + NO_3^-(aq) + 4H^+(aq) \rightarrow NO(g) + 2H_2O(\ell)$$

Cross multiplying to get equal numbers of electrons in the oxidation and the reduction, we have:

$$14\left\{3e^- + NO_3^-(aq) + 4H^+(aq) \rightarrow NO(g) + 2H_2O(\ell)\right\}$$

$$3\left\{Hg_2I_2(s) + 4C\ell^-(aq) + 6H_2O(\ell) \rightarrow 2HgC\ell_2(s) + 2HIO_3(s) + 10H^+(aq) + 14e^-\right\}$$

which, after addition yields

$$\cancel{42e^-} + 14NO_3^-(aq) + 56H^+(aq) + 3Hg_2I_2(s) + 12C\ell^-(aq) + 18H_2O(\ell) \rightarrow$$
$$14NO(g) + 28H_2O(\ell) + 6HgC\ell_2(s) + 6HIO_3(s) + 30H^+(aq) + \cancel{42e^-}$$

The electrons have cancelled out, as they must!

All we have left is some pseudo-algebra to clean up the equation for components which appear both left and right of the "→", i.e.,

$$14NO_3^-(aq) + 26H^+(aq) + 3Hg_2I_2(s) + 12C\ell^-(aq) \rightarrow$$
$$14NO(g) + 6HgC\ell_2(s) + 6HIO_3(s) + 10H_2O(\ell)$$

4.7.2 Comments

For the H^+ we have
56 $H^+ \rightarrow$ 30 H^+
or
56-30 $H^+ \rightarrow$ 30 - 30H^+
or
26 $H^+ \rightarrow$ 0 H^+

5 Gases

5.1 Gas problem concerning contents of a balloon of Helium

Calculate the mass of Helium required to fill a 12400 meter^3 balloon at 25^{o}C and 1.72 atm.

5.1.1 Solution

(α)

$$\text{mass of He} = \text{moles of He} \times 4\frac{\text{grams}}{\text{mole}}$$

(β) moles of He$= \frac{\text{p}\times\text{V}}{\text{R}\times\text{T}}$

(γ) p = 1.72 atm

(γ) V = 12,400 meter^3 $\times\frac{\text{liters}}{\text{meter}^3}$

(γ) T = 25^{o}C + 273^{o}C = 298^{o}K

(β) moles of He=

$$\frac{1.72 atm \times 1.24 \times 10^7 \ell}{0.082\frac{\text{liter- atm}}{\text{mole}^o\text{K}} \times 298^o K}$$

(β) moles of He=

$$\frac{1.72 \times 1.24 \times 10^7}{0.082 \times 298} = 8.7 \times 10^9 \; moles$$

(α)

$$\text{mass of He} = \text{moles of He} \times 4\frac{\text{grams}}{\text{mole}} = 3.48 \times 10^{10} \; grams$$

5.1.2 Comments

5.2 Density of a balloon of Helium

Calculate the density of a Helium balloon filled to 15,900 meter^3 at 25^{o}C and 1.56 atm.

5.2.1 Solution

(α)

$$\text{density of He} = \frac{\text{grams He}}{\text{volume}}$$

(β) grams of He = moles of He $\times$ 4 $\frac{\text{grams}}{\text{mole}}$

(γ) moles of He=

$$\frac{p \times V}{R \times T}$$

(δ) p = 1.72 atm

(δ) V = 15,900 meter^3 $\times \frac{\text{liters}}{\text{meter}^3}$

(δ) T = 25^{o}C + 273^{o}C = 298^{o}K

(β) moles of He=

$$\frac{1.72 atm \times 1.59 \times 10^7 \ell}{0.082 \frac{liter-atm}{mole^o K} 298^o K}$$

(β) moles of He=

$$\frac{1.72 \times 1.59 \times 10^7}{0.082 \times 298}$$

(α)

$$\text{density of He} = \frac{\text{grams He}}{\text{volume}} = \frac{3.49 \times 10^6 \ grams}{15,900 \ m^3} = 219.5 \frac{grams}{m^3}$$

5.2.2 Comments

This seems high, so let's convert it to grams per cc:

$$219.5 \frac{grams}{m^3} \times \left(\frac{1 \ meter}{100 \ cm} \right)^3 = 219.5 \times 10^{-6} \frac{grams}{cc}$$

5.3 Density of a balloon of Helium

Calculate the lifting power (in grams) of a Helium balloon filled to 10000 meter^3 at 25^{o}C and 1.01 atm. To do this, compute the mass in excess of the Helium mass which would make the density just equal to that of ambient air at 298^{o}K and 1.01 atm. Take the molecular mass of air to be 28.8 grams/"mole", i.e., (1/5)*32 + (4/5)*28.

The lifting power is here defined as the mass of material that the balloon can lift over and above its own mass (assumed to be negligible i.e., zero) and the mass of the Helium gas.

5.3.1 Solution

(α) lifting power = Mass of load in basket of balloon

(α) Density of balloon =

$$\frac{\text{mass of balloon + mass of load}}{\text{volume of balloon}}$$

- (β) mass of balloon = grams of He = moles of He $\times$ 4 $\frac{\text{grams}}{\text{mole}}$
  - (γ) moles of He=
    $$\frac{p \times V}{R \times T}$$
    - (δ) p = 1.01 atm
    - (δ) V = 10,000 meter^3 $\times \frac{\text{liters}}{\text{meter}^3}$
    - (δ) T = 25^{o}C + 273^{o}C = 298^{o}K
  - (γ) moles of He=
    $$\frac{1.01 atm \times 1.00 \times 10^7 \ell}{0.082 \frac{liter-atm}{mole^o K} 298^o K}$$
  - (γ) moles of He=
    $$\frac{1.01 \times 1.00 \times 10^7}{0.082 \times 298} = 4.13 \times 10^5$$
- (β) mass of balloon = grams of He = 4.13 $\times$ 10^5 moles of He $\times$ 4 $\frac{\text{grams}}{\text{mole}}$

(α) Density of balloon =

$$\frac{\text{mass of balloon + mass of load}}{\text{volume of balloon}} = 1.91 grams/\ell iter$$

(α) Density of balloon =

$$\frac{4 \times 6.6 \times 10^5 + mass\ of\ load}{10^7} = 1.91 grams/\ell iter$$

5.3.2 Comments

Solving the above equation for "the mass of the load", we have

$$mass\ of\ load = \frac{1.91 \times 10^7}{4 \times 6.6 \times 10^5} = 7.3\ grams$$

Not very big, is it? Perhaps its wrong?

5.4 Partial Pressure in a Wet Gas

A sample of wet gas whose volume is 104.5 liters at 1.14 atm pressure is in contact with liquid water at 25^{o}C. The vapor pressure of the water is 23.76 mm Hg at this temperature. What is the mole fraction of water in this gaseous mixture?

5.4.1 Solution

The gas is exerting its pressure, and the water is exerting its pressure, so the sum of the two pressures must be 1.14 atm (1.14*760 mm Hg). Therefore, the pressure of the gas sample itself, not counting the water vapor, must be 1.14*760-23.8 mm Hg.

(α) mole fraction of water =

$$\frac{moles\ water}{total\ gaseous\ moles}$$

(β) moles of water =

$$\frac{pV}{RT} = \frac{\left(\frac{23.8}{760}\right) 104.5}{0.082 \times 298^o K} = 0.134\ moles$$

(β) moles of sample gas =

$$\frac{pV}{RT} = \frac{\left(\frac{1.14 \times 760 - 23.8}{760}\right) 104.5}{0.082 \times 298^o K} = 4.74\ moles$$

(α) mole fraction of water =

$$\frac{moles\ water}{total\ gaseous\ moles} = \frac{0.134}{4.74 + 0.134} = 0.027$$

5.4.2 Comments

5.5 Stoichiometry I

Consider the reaction of nitrogen and hydrogen to prepare ammonia:
$N_2(g) + 3\ H_2(g) \rightarrow 2\ NH_3(g)$
How many liters of nitrogen at 1.62 atm, and 283.8^{o}C are required to make 19.3 moles of ammonia?

5.5.1 Solution

(α) liters of nitrogen = $\frac{nRT}{p}$

- (β) n = moles of ammonia $\times \frac{moles\ nitrogen}{moles\ ammonia}$
  - (β) n = 19.3 $\times \frac{1\ moles\ nitrogen}{2\ moles\ ammonia}$
- (β) R = 0.082 $\frac{\ell - atm}{mole\ ^oK}$
- (β) T = (283.8^{o}C + 273.16^{o}C)^{o}K
  - (β) T = 557^{o}K
- (β) p = 1.62 atm.

(α) liters of nitrogen = $\frac{\left(\frac{19.3}{2}\right)\times 0.082\times 557}{1.62} = 272\ell iters$

5.5.2 Comments

5.6 Stoichiometry II

Consider the reaction of nitrogen and hydrogen to prepare ammonia:
$N_2(g) + 3\ H_2(g) \rightarrow 2\ NH_3(g)$
How many liters of nitrogen at 1.62 atm, and 283.8^{o}C are required to make 10.5 ℓiters of ammonia at STP?

5.6.1 Solution

To do this problem, we need to remember what STP is, not the car oil additive, but the Standard Temperature and Presure, i.e., 0^{o}C and 1 atm.

Then one need to compute required number of moles of ammonia from the problem's data, ie., $n_{ammonia} = \frac{pV}{RT}$.

$$n_{ammonia} = \frac{1 \times 10.5}{0.082 \times 273} = 0.469$$

(α) liters of nitrogen = $\frac{nRT}{p}$

- (β) n = moles of ammonia $\times \frac{moles\ nitrogen}{moles\ ammonia}$
  - (γ) n = 0.469 $\times \frac{1\ moles\ nitrogen}{2\ moles\ ammonia}$
- (β) R = 0.082 $\frac{\ell - atm}{mole\ ^oK}$
- (β) T = (283.8^{o}C + 273.16^{o}C)^{o}K
  - (γ) T = 557^{o}K
- (β) p = 1.62 atm.

(α) liters of nitrogen = $\frac{\left(\frac{0.469}{2}\right)\times 0.082 \times 557}{1.62} = 6.61 \ell iters$

5.6.2 Comments

6 Thermochemistry with Stoichiometry

6.1 Stoichiometry with Thermochemistry

When one mole of methane ($CH_4(g)$) and 2 moles of oxygen ($O_2(g)$) are burned at constant pressure 890 kJoules of energy are released. What amount of heat (in kJ) is observed when 19.9 moles of methane react with an excess of oxygen?

6.1.1 Solution

Always, always, write the chemical equation out first, i.e.,
$CH_4(g) + 2\ O_2(g) \rightarrow CO_2(g) + 2\ H_2O(g)$; $\Delta\ H^o$ = -890 kJ

(α) heat emitted = moles of methane reacted × 890 kJ/mole

  (β) moles of methane = 19.9

(α) heat emitted = 19.9 × 890 kJ/mole

6.1.2 Comments