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A Simple Formula to Calculate Azimuth without a Two-Argument Arctangent Function

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ABSTRACT: Calculating azimuths from planimetric coordinates is complicated when the available implementations of inverse trigonometric functions have ranges spanning only 180° instead of 360° as azimuth requires. A tangent half-angle formula is used to derive a formula with only one special case (due South) that computes azimuth for the whole circle.

Keywords: Azimuth, traversing

Calculating azimuths from planimetric coordinates is complicated when the available implementations of inverse trigonometric functions have ranges spanning only 180° instead of 360° as azimuth requires. Meyer (2010, p. 33) provided a table that indexes the signs of the distance increments (changes in eastings and changes in northings) to corrections applied to the angle returned by the ATAN function available on hand calculators and in programming languages, but this approach is difficult to remember and awkward to implement because of the four-case logic. A tangent half-angle formula allows an alternative, so we present a formula with only one special case (due South) that computes azimuth for the whole circle. This formula appears in standard surveying texts (Ghilani and Wolf 2011).

Given two stations with planimetric coordinates $p_0 = (e_0, n_0)$ and $p_f = (e_f, n_f)$, define the distance increments to be $\Delta e = e_f - e_0$ and $\Delta n = n_f - n_0$, and let $d = \sqrt{\Delta e^2 + \Delta n^2}$ be the length of the hypotenuse of the right triangle formed by Δe and Δn. Then, taking North to be an azimuth of 0°, the azimuth from p_0 to p_f is $\alpha = \text{atan}\ (\Delta e/\Delta n)$, and $\sin\alpha = \Delta e/d$, and $\cos\alpha = \Delta n/d$. One of the tangent half-angle formulas is:

$$\tan\frac{\alpha}{2} = \frac{\sin\ \alpha}{1 + \cos\ \alpha}$$

Substituting the above definitions for the sine, cosine, and hypotenuse yields:

$$\tan\frac{\alpha}{2} = \frac{\Delta e/d}{1 + \Delta n/d} = \frac{\Delta e}{\sqrt{\Delta e^2 + \Delta n^2} + \Delta n}$$

Now, because:

$$\alpha = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\right)$$

$$\alpha = 2\tan^{-1}\left(\frac{\Delta e}{\sqrt{\Delta e^2 + \Delta n^2} + \Delta n}\right)$$

provided $\Delta e \neq 0$ or $\Delta e = 0$ and $\Delta n > 0$. If $\Delta e = 0$ and $\Delta n < 0$, then $\alpha = 180°$. This formula's range is $-180° \leq \alpha < 180°$ Sometimes strictly positive azimuths are preferred. When the answer is negative, azimuths in the range $0° \leq \alpha < 360°$ can be obtained by adding 180°.

REFERENCES

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