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Socially Mediated Mathematical Strategies in a Dynamic Multi-Touch Geometry Environment

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Abstract

We examine how young children work together in small groups using interactive multimodal technologies that integrate dynamic geometry software with multi-touch interfaces in order to understand a fundamental scientific idea of *covariation*. The study aims to investigate how 4th-grade children co-construct mathematical strategies for solving an Etchasketch-like activity using the Geometer' Sketchpad® and the iPad.

Introduction

Our study is situated in the investigation of how young children can explore mathematical concepts such as covariation within an interactive learning environment that integrates dynamic geometry software with multi-touch interfaces. Specifically, we examined how children working in small groups co-constructed mathematical strategies and how these strategies evolved through socially mediated metacognition because of the representational and collaborative affordances of these learning technologies.

Theoretical Framework

Multimodality reaches into education in various ways intersecting deeply with a multi-media approach to facilitate different forms of input/output interaction (Jamies & Sebe, 2007). Grounded on sociocultural theories of semiotic mediation (Mariotti, 2009) we see multimodal technologies as cultural artifacts that can mediate mathematical problem solving and understanding (Hegedus & Moreno-Armella, 2011). We implemented in mathematics classrooms the use of digital technologies through which multiple sensory representations such as dynamic-visual and haptic offered young children mutually reinforcing information that they utilized to develop an understanding of a mathematical model of co-variation.

Problem-solving settings have been historically valued because of their potential to foster strategic behavior (Schoenfeld, 1992; Siegler, 2003). *Strategies* are students' methods to solve tasks that result in various products with respect to a mathematical content (Yerulshulmy, 2000). Strategies have been conceived as a unit of higher-level cognition because they represent children's ways of thinking (Siegler, 2003). Rather than a set of steps that children use to achieve the goal of a task, a strategy is theorized as an intellectual approach to solve the problem. This means, that children use their intellectual resources at hand, their prior knowledge and information found in the exploration of the task to generate solutions, whether correct or not. Using multi-touch interfaces in addition to the dynamic geometry software allows for children collaboration to co-construct strategies to solve problems. Collaborative problem solving in small groups is critical for mathematical learning because it triggers socially mediated metacognition that promotes cognitive advancement (Goos, Galbraith & Renshaw, 2002).

Methodology

This study is part of a large research project conducted in seven 4th-grade classrooms of an elementary school from a middle-class district of Massachusetts, U.S. Nine mathematical tasks designed in Geometer's Sketchpad® for teaching various mathematical concepts were implemented with fourteen small groups of students utilizing one iPad per group.

An exemplar activity addresses the concept of *covariation*, specifically the underlying principle of using two inputs in mutually dependent ways for achieving only one output. This is an important idea related to the mathematics of change for early learners because they could develop foundations about functional relationships that will be fundamental for learning calculus.

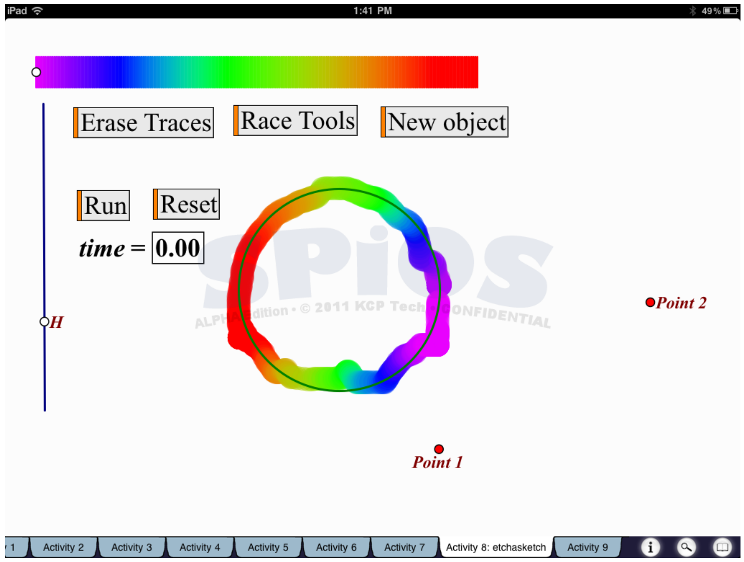


Figure 1. Etchasketch task

The goal of the activity for students is to trace a colored-blob around a fixed circle by using two points (*point 1* and *point 2*) located at the right and at the bottom sides of the circle (Figure 1). To solve the task, one student can control the lateral-moving *point 1* and another student can control the vertical-moving *point 2*, or one student can control both simultaneously. A third student (or third finger input) can adjust the color of the trace to make a rainbow around the circle (point on the spectrum), or the size of the blob (point H). Unlike a computer, the multi-touch tablet allows children to provide two inputs simultaneously to manipulate the sliders. The *point 1* is constrained in order to be moved along an invisible horizontal line segment and the *point 2* on a vertical line segment orthogonal to the horizontal one. Mathematically, in the construction of the circle we have parameterized two input functions using the software's tools.

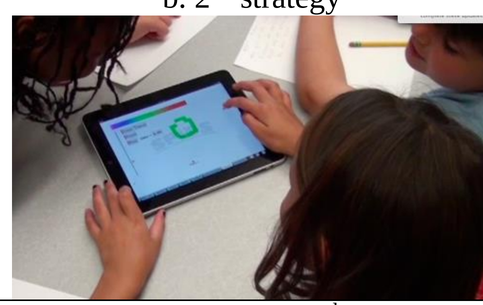
Activities were videotaped and transcribed for analysis. The analytical framework included: (a) mathematical strategies, (b) collaborative behaviors, building on Goos et al. (2003), and (c) socially mediated metacognitive processes emergent during the activity as a product of children's collaboration or technology use. Data coding consisted of a step-wise iterative process of seeking redundancy, using a First cycle-Process coding method and a Second cycle-Pattern coding method (Saldaña, 2013). In this paper, we present results of the covariation activity and show examples of the co-construction of strategies process from one small group of four children working with the technology.

Results

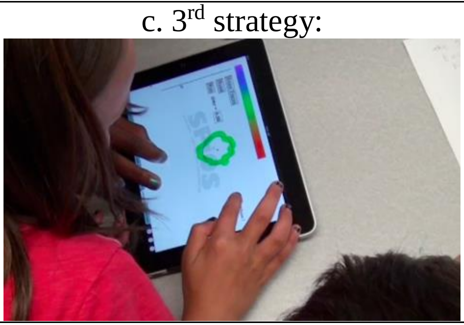
We found three collective mathematical strategies: Initially, children tried to move the points around the circle without taking into account their predetermined directions. The products were separated traces (Figure 2a). The second strategy was moving the points in the predetermined directions by taking turns. The product was a square instead of a circle (Figure 2b). Over a short time period, children co-constructed a more advanced strategy -coordinating actions between two participants to simultaneously move the points in the predetermined directions. The product was a circle (Figure 2c). They coordinated increasingly faster improving the precision of this strategy (Figure 2d).

a. 1st strategy

b. 2nd strategy



c. 3rd strategy:



d. Enhancement of 3rd strategy

Figure 2. Evolution of mathematical strategies in one small group.

The following children's collaborative behaviors were found to influence the co-construction of strategies: a) Proposing a new idea to others, b) discussing other's idea, c) posing question about one's or other's idea, d) explaining or justifying one's or other's idea, e) endorsing or criticizing other's idea, f) showing or demonstrating an idea, g) sharing one's difficulty with others, h) contributing with action to implement an idea or change a strategy and i) assigning roles/turns to others.

We also found that these collaborative behaviors triggered specific metacognitive processes such as children's awareness of their strategies, evaluation of strategy accuracy and outcomes, and regulation of activity towards reaching the goal of the task.

Conclusions

Results of the study showed a relationship between collaboration and socially mediated metacognition in the co-construction and enhancement of mathematical strategies. In order to solve the task, children used a variety of collaborative activities, which in turn, triggered metacognitive processes of the group members.

Educational implications for further research include incorporating collaborative multimodal technological environments into curriculum to support young children in understanding mathematics and sharing their work within small groups or at a whole-class level.

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Acknowledgments

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