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Supporting Student Justification in Middle School Mathematics Classrooms: Teachers' Work to Create a Context for Justification

Justification is an important disciplinary and learning practice. Despite a growing knowledge base regarding how teachers orchestrate mathematical discussions, few analyses have considered the orchestration of specific disciplinary practices such as justification. Using classroom video data from the JAGUAR project, we analyze two instantiations of extensive student justification in seventh-grade classrooms and document each teacher's pedagogical approach that supported students' engagement in this practice. We argue that, although there was overlap in their pedagogical repertoires, the teachers created a context for student justification in two unique ways. We document the similarities and differences in their approaches, including the nature of teachers' responses to student ideas, nature of the teachers' press prompts (for reasoning and justification), nature of the classroom culture, and priorities in task design and task implementation. Implications are discussed.

Justification activity in middle school mathematics classrooms is rare in the United States (Bieda, 2010; Jacobs et al., 2006). Despite a growing knowledge base regarding how teachers orchestrate mathematical discussions (e.g., Lampert, 2001; Smith & Stein, 2011; Staples, 2007), relatively few analyses have considered the orchestration of specific, valued disciplinary practices such as justification.

To advance this agenda, we present two cases of seventh-grade teachers who participated in the JAGUAR project (described below). Each teacher was successful at organizing student justification activity in her classroom across multiple tasks. Analyses of each teacher's *pedagogy of justification* revealed that their successes resulted from partially overlapping, yet unique pedagogical approaches.

The contribution of this analysis is twofold. First, it offers a description of two successful approaches for teaching with justification and engaging students in justification at the middle school level. Second, it prompts us to recognize that there may be different constellations of pedagogical moves and skills that support student classroom justification activity, and to argue that a drive for one model, or one set of pedagogical practices (which we see at least implicit in the literature currently), may not accomplish the desired goals of rich classroom practices across teacher strengths, grade levels, activities, etc. that support extensive student justification activity.

Justification as a Mathematics Learning and Disciplinary Practice

Justification can be thought of both as a process (practice) and as a product. As a working definition, we consider justification to be a process of developing a mathematically sound argument that uses disciplinary tools to demonstrate the truth or falsehood of a claim. Thus a class can engage in justification together, as an activity, where they aim to show a claim true or false by developing a mathematically sound argument. If the argument developed demonstrates the claim to be true or false, we call the argument a justification of the claim. In our use of the term, we also allow for modifiers of the term justification such as *incomplete* or *partial* to describe arguments that do not reach the level of a justification, e.g., *an incomplete justification.*

Justification differs from *reasoning*, which is a broader term that we take to encompass justification. A student may offer reasoning not aimed at demonstrating the truth (or falsehood) of a claim, but rather arguing the *reasonableness* of a claim (e.g., “My model is reasonable because…,” or, “the value is in the right ballpark because it should be between 100 and 1000…”.). Alternately, a student may offer reasoning that does not reference or draw on mathematical disciplinary tools (e.g., this is right because John said it was; it’s just like what we did on number 3). Our interest, however, is in mathematical reasoning.

Given that our definition of justification sounds remarkably like the definition of the term proof, one might wonder why we did not use the term *proof* (and the process of *proving*) instead. We chose not to use proof as the term tends to invoke visions of a two-column format in a geometry course, or rigorous chains of arguments published by mathematicians. We felt the choice of the term justification would allow mental space for a reader (or teacher) to see a justification as something that needs to be grade-appropriate and relate to the shared knowledge of the classroom community. It is possible that an argument considered an appropriate justification for sixth grade, may be seen as inadequate for later grades, or an argument that is a justification in one seventh grade class would not be a justification in another class because it would contain assumptions that had not yet been demonstrated true.

Theoretical Perspectives

Two perspectives informed this work: a disciplinary perspective, which considers justification as a core mathematical practice (Hanna, 2000; Lakatos, 1976); and a community-of-practice perspective (Lave & Wenger, 1991; Wenger, 1998), where justification is viewed as a practice of a specific community for a particular purpose, here, a middle school mathematics classroom that is doing and learning mathematics together. What “counts” as a justification is locally defined, and the nature of justification activity is locally constituted in the classroom through engagement of the members of the community. These two perspectives allowed us to consider how the disciplinary practice of justification is shaped by (and shapes) the community’s practices. Our analyses privileged teacher actions, as these are critical in creating opportunities and establishing criteria for justification (Bieda, 2010; Nathan & Knuth, 2003).

Sophisticated Pedagogies in Mathematics Classrooms

Over the past 30 years, a significant portion of research on mathematics teaching in the U.S. has examined the evolution of 'reform' teaching as advocated by various organizations and documents, for example, by the National Council of Teachers of Mathematics (NCTM, 1989, 2000) and more recently by the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). Although this literature is not necessarily focused on the practice of justification, we argue it is relevant for understanding pedagogies of justification and wealth of knowledge relevant for teaching with justification has developed the in of these reforms.

Researchers have studied, for example, how teachers organize classroom discussions, launch tasks, maintain high levels of cognitive demand, and position mathematics as the authority (as opposed to the teacher or high status students). Although this large body of work is not specifically focused on the practice of justification, we might hypothesize that the classroom features and pedagogical skill required to support students' participation in whole class discussions and cognitively demanding tasks is also relevant for understanding how teachers organize student justification activity. The connection can be made by considering the nature of justification. Lakatos (1976), among others, describes justification activity in a classroom community as requiring a to-and-fro of ideas, of conjectures, refutations, revision and refinements to arguments – all processes that would require those in a community to attend to each other's ideas, be willing to offer their own ideas, collectively examine ideas and errors, develop consensus in order to establish the validity of a claim, and use mathematics as the source of authority. These processes, taken collectively, are a reasonable vision of reform-aligned instruction. A goal of this paper is to attend specifically to the practice of justification that is embedded within this complex environment of reform-aligned instruction and seek to explain how it is supported.

We briefly review select articles from relevant literature to identify what the field has documented as critical for organizing classroom communities for discussion, collaboration, and inquiry – all of which we deem as relevant, though perhaps insufficient, for understanding how teachers specifically organize student justification activity in mathematics classrooms.

Selecting cognitively demanding tasks, and maintaining high level of cognitive demand during task implementation.

A body of literature focuses on the importance of selecting cognitively demanding tasks (Hiebert et al., 1997) and launching and implementing such tasks in a manner that maintains a high level of cognitive demand (Stein, Grover, & Henningsen, 1996; Stein & Smith, 1996). It was rare for a task to be written at a lower level of demand, and have that cognitive level increase during implementation.

Related to maintaining a high level of cognitive demand is teacher questioning, and specifically teacher's use of "high press" questions (Kazemi & Stipek, 2001) as well as the patterns of interaction evidence by classroom discourse. Patterns of interaction such as *focusing* tend to support high level task implementation; patterns such as *funneling* or I-R-E (Wood, McNeal & Williams, 2006) tend to reduce the cognitive demand of a task during implementation.

Eliciting student ideas and/or taking up student ideas

Although seeming almost too obvious to state, a critical aspect of organizing classrooms that align with visions of reform mathematics is having students' ideas actively elicited and taken up by the teacher. Researchers who have developed models of advancing children's thinking (e.g., Fraivillig, Murphy, & Fuson, 1999) and teacher's organization of collaborative discussions (e.g., Staples, 2007) include this component in their models.

Orchestrating discussions and positioning students for collective work on mathematical ideas

Following from eliciting student ideas, the teacher must position student ideas for collective work and have pedagogical strategies for facilitating student-to-student interactions around presenting-student ideas. The body of work focus on this component ranges from documenting the critical importance of teacher discourse moves, such as *talk moves* (Chapin & O'Connor, 2007), establishing and maintaining a *common ground* and shared space for collective work (Staples, 2007), and managing tensions between student engagement and productive math talk (Chazan, 1993; Nathan & Knuth, 2003).

Attend and respond to the details of students' mathematical thinking; attending to the "big ideas" in mathematics

A strong emphasis in the literature is the need for teachers to work with students' mathematics. As students offer contributions, teachers need to attend to the specifics of students' ideas and be able to identify the important mathematical ideas students are wrestling with or trying to express (Ball, 1993; Lampert 1990; 2001; Staples, 2007). There is also empirical evidence connecting teachers' capacity to notice students' mathematics and student learning outcomes (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Hill, Rowan, & Ball, 2005). Seymour & Lehrer (2006) have describe this as the teacher and student engaging in an *interanimated discourse* where the teacher attunes her discourse and responses to the students' thinking and the mathematics.

Establish classroom environments (classroom cultures)

Critical to supporting student participation in a community of inquiry is establishing a classroom environment (culture) governed by norms that support risk taking, position students as producers of mathematical ideas, treat errors as sites for learning, and use mathematics as the authority (as opposed to a high status student or teacher) (e.g., Cobb, Wood, Yackel, & McNeal, 1992; Hiebert et al., 1997; Hufferd-Ackles, Fuson, & Sherin, 2004; Lampert, 1990, 2001; Wood, 1999). The norms not only support student participation, but govern the types of reasoning and evidence that "count" and have weight when presenting a mathematical argument (Yackel & Cobb, 1996).

The recurrence of these themes in the literature offers a somewhat undifferentiated profile and raises the question as to whether there are different *sets of repertoires* or different *pedagogies of justification* that teachers might draw upon or enact to support student engagement in justification. As each study has contributed some subset of these features, not yet examined is the possibility that there are multiple models that accomplish the same goal of creating a context for student justification activity in mathematics classrooms. The body of literature has thus far been accumulating as if to imply that teachers must do *all* of these things well, and in each lesson, to support high level, complex student mathematical activity such as justification.

A second overlooked consideration that stems from this perhaps general depiction of reform-aligned pedagogy is that we find little guidance about pedagogical features that differentiate a pedagogy that is reform-aligned (in a general sense) from one that specifically focuses on disciplinary practices such as justification. Organizing a practice such as justification (where students must produce arguments, critique others' arguments) might place specific demands on a teacher's pedagogy. A study conducted by Nathan & Knuth (2003) supports this idea. They describe a teacher who, in Year 2 of a study, significantly enhanced student participation in class discussions, which was viewed as productive and beneficial and a step towards a pedagogy that was more aligned with reform documents. However, Nathan and Knuth also demonstrated that this shift was accompanied by classroom discussions that did not adhere to standard disciplinary criteria for determining the truth of a claim (e.g., students voted to decide what was right), and thus while prompting student participation and collaboration, may not have engaged students appropriately in justification. Thus, an important question to consider is whether there are key aspects that should be fleshed out when describing specifically a teacher's *pedagogy of justification* or one's *pedagogical repertoire with respect to justification.*

Challenges of Teaching With Justification

As noted above, justification activity in middle school mathematics classrooms is rare. Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne (2006) analyzed TIMSS video data from the United States and found no examples of students engaging in *proof* across the random sample of 50 videos of eighth-grade U.S. classrooms. Furthermore, there were no instances coded as developing reasoning, making generalization, or using counterexamples, although two videos had evidence of deductive reasoning.

We know that, even under the best circumstances, little justification happens in mathematics classrooms. Bieda (2010) studied seven middle grades teachers, all trained to use the *Connected Mathematics Project* curriculum – a curriculum designed to support students' problem solving and reasoning skills. Of the 100+ opportunities for proving that were documented in 49 lessons, only about half of the opportunities to prove became instances where students offered a justification. When students did offer a justification, rarely did a dialogue develop about the ideas. Indeed, Bieda reports only one such instance.

These studies suggest that teaching with justification is challenging, though we must leave open the possibility that the absence of justification has other causes, for example, perhaps teachers do not hold as a goal engaging students in justification due to their knowledge base, institutional pressures, or assessment requirements. Our study participants were on teachers who deliberately held justification as a goal, although we will see they had other priorities as well as they implemented lessons.

Methods and Data Sources

The case teachers were two of 12 participants in *Justification and Argumentation: Growing Understanding of Algebraic Reasoning* (*JAGUAR*[1]), an NSF-funded project focused on justification in middle grades mathematics classrooms. The two teachers were selected for this analysis as their classrooms revealed consistent justification activity among students (disciplinary practice) yet revealed different sets of learning practices (community of practice).

The JAGUAR Project: Overview

The goals of the JAGUAR project were to investigate how exemplary teachers developed specific disciplinary mathematical knowledge of justification and argumentation, transformed this knowledge to classroom learning practices, and advanced in their pedagogy to promote students' capacity for algebraic justification and argumentation. JAGUAR was a collaborative project, across multiple institutions (Portland State University, Purdue University, and University of Connecticut) working with teachers in multiple states (Oregon

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[1] Collaborators on this project include researchers from Portland State University, Purdue University and University of Connecticut: Sean Larsen (PI), Jill Newton (Co-PI), Eva Thanheiser (Co-PI), Karen Marrongelle (Co-PI), Carolyn James, Krista Strand, Joanna Bartlo, Kate Melhuish, Ann Sitomer, Briana Hennessy, Rachael Reffett, Corinne Brown and Lance Williams.

and Connecticut). We worked with teacher participants for two years, which included two summer workshops (five days each), three working sessions during each academic year, and extensive data collection on teachers' conceptions of justification and their implementation of four justification tasks in their classrooms in each year.

The JAGUAR Teachers

Participants in the study were 12 middle school mathematics teachers (grades 7 and 8). These teachers agreed to collaborate with us to better understand the nature of justification in middle grades classrooms. The teachers taught in five districts in two states; had 2 – 29 years of experience; and were all fully certified to teach mathematics in middle school (four held secondary credentials). Half of the teachers had previously participated in intense professional development on promoting student discourse in mathematics classrooms. All but one of the other teachers had extensive exposure to related ideas (e.g., teaching for higher-order thinking) through professional development and/or prior participation in research projects. In general, this group of teachers already expected their students to participate in classroom discourse. In committing to the *JAGUAR* project, the teachers agreed to actively work on ideas related to justification in their practice and to collaborate with project personnel to unpack the nature of justification in middle grades mathematics classrooms. Thus, we do not expect that these teachers are representative of the larger population of middle school mathematics teachers. Rather, this was a purposive sample (Yin, 1994) to enhance our ability to examine the role of justification in middle grades classrooms.

The case teachers . For this analysis, we focus on two teachers: Cynthia Littrell and Paige Davilla. These teachers taught in different states.

Cynthia Littrell. Cynthia had 14 years of teaching experience at the start of the project, all at the middle school level. Cynthia taught seventh grade both years of the project in a suburban school in the West. The school population had little linguistic or ethnic diversity. Approximately twenty percent of the students received free or reduced lunch. Cynthia's focal class was a heterogeneously grouped math class and they used the *Connected Mathematics Project* (*CMP*) as a curriculum, which had recently been adopted in the district. There was an average of approximately 30 students in Cynthia's focal class.

Paige Davilla. Paige was in her second year of teaching at the start of the project and held certification in secondary mathematics (grades 7-12). Paige taught seventh grade both years of the study at a school with a fairly diverse population. The school population was 70% minority, about 30% receiving free or reduced lunch, and had fewer than 5% of students classified as English language learners. Her focal class was homogeneously grouped in a pre-algebra class, a class taken by about one third of the seventh- grade students at the school based on scores on a placement test and teacher recommendation. There was an average of 25 students in Paige's focal class.

Both teachers were successful in organizing student justification activity in their classrooms. We selected these two teachers for this analysis because as the comparison is revealing and instructive. While the two teachers shared many components of their pedagogical repertoires and "fit" much of what we know about how teachers support mathematical discussions in their classrooms (e.g., elicited student ideas, positioned mathematics as the authority, pressed for justification), they offer two different approaches or models to organizing justification practices in their classrooms.

Data Sources

Data for this analysis were twelve implementations of three different justification tasks. Each teacher implemented the three justification tasks in each of two years. These are the Hexagon Task, the Number Trick task, and the Scaling task. (See Appendix A for tasks.) Each task implementation comprised 2-4 lessons. Lessons were videotaped and transcribed with a total of 600 minutes of transcribed lesson for Cynthia and 519 minutes of transcribed lessons for Paige. Off task behaviors, logistics, and unrelated warm-ups were not transcribed. Student work samples were also collected and these were used to supplement analyses when needed (e.g., to determine what a student was referencing on his or her paper).

Analyses

All lesson transcripts were important into NVivo. Three complementary analyses form the primary basis for this paper.

We conducted a Coding of Student Contributions. For this analysis, all student turns were coded as to whether the contribution was an answer/statement, explanation (steps in a procedure), reasoning (perhaps incomplete or with minor errors), or justification. See Table 1. They were also coded based on their "social function" with respect to other student ideas, specifically whether the student agreed or disagreed with a previously made statement, and whether the student offered a refutation of an answer/argument or built up (extended) an argument. We also tracked student questions. The turn level codes were used to examine the transcripts at the episode level and identify where a complete justification of the main task prompt or a task subquestion was made.

Table 1

Relevant Student Contribution Codes and Description

Student Code	Description	Example
Answer/ Statement	Used when a student made a direct statement, with no explanation of his ideas. Often used when a student answered a direct question from the teacher.	"Seven works"
Agree or Disagree	These two codes were used when students agreed or disagreed with another student without elaboration.	"I think Claire's right"
Explanation	Used when a student described a procedure that they used to find a solution.	"I added four and four."
Reasoning	Used when a student attempted to describe why his or her solution is the correct one, but the proof was incorrect or incomplete. *Reasoning together with Justification create the set we call Reasoning Contributions*	"then go back and add eight on this one, and this one's plus two fours. They both have eight."
Justification	Used when a student gave a complete justification for why their answer was correct. This was sometimes a response to a specific teacher question. This reason was mathematically valid and	Each time a hexagon is added, only four sides are added to the perimeter, but the two

	acceptable. *Justification together with Reasoning create the set we call "Reasoning Contributions"*	hexagons on the end add five sides to the perimeter, so the rule is 10 + 4(n-2)
Build/Refute	Used when a student offered something new based on what a peer had said or refutes a peers' idea. This was always double-coded with Explanation, Reasoning or Justification.	"If your variable was something different than 5 then it would not be 9."
Question	Used when a student asked a question of a teacher or peer.	"Why did you put plus two?"

Two researchers (from a team of four) independently coded each transcript and a referee compared codes and resolved discrepancies. The initial IRR for the two coders was on average above 80%.

Second, we conducted a Coding of Teacher Contributions. We focused primarily on two types of teacher contributions: *press* moves and *orchestration* moves. The first of these were teacher presses in response to student contributions – where the teacher could prompt or press for a procedure, press for additional reasoning or conceptual explanation, or press for a justification. In addition, we coded for teachers' contribution to the orchestration of student interaction around ideas. Orchestration codes were applied when the teacher's utterance prompted students to communicate about mathematics (talk to your partner) or otherwise attend to a mathematical idea of another student. See Table 2.

Table 2

Relevant Teacher Contribution Codes and Description

Code	Description	Example
Press-Justify	Teacher prompt for the student to explain how they know their solution or a part of it is correct/accurate, or why something is the way it is (mathematically). The prompt focuses	"And I'm wondering why it works, why is that your rule?" So how do you know there is 20?" "And why are we adding 10 to

	students on convincing themselves, the teacher, or their peers using mathematics. Accountable to the mathematics.	the end?
Press-Procedure	Teacher prompt, most often a follow up, that is aimed at understand what the student did to arrive at a result, and how. The prompt is often specific, but can be a follow up after a student commits to an answer. NOT an opening question about the students solution process	T: ok so tell me first what you found? S: 114 cm T: how? S: She got 90. I'm kind of on her side, but I'm- T: Okay tell me how you got the 88 and the 90. How did that happen?
Press-Other	A teacher prompt targeting a concept or relationship, reasoning, or an evaluation.	"What did you notice about the values in your table?" "Does it work for larger numbers too?" Do you agree with the expression she has on the board?
RNC (relevant no code)	RNC encompasses most other teacher utterances, where the teacher is doing the work of teaching, but these are not Press codes. RNC includes reading a task.	"That looks good. Nice job." "So that's a great connection she just made between her method and the first method."
SxS	Teacher turn that prompts students to communicate or collaborate around the mathematics. This can be direction to the listeners, speakers, or groups. All turns coded with SxS must also be coded with one of the other codes (RNC, PJ, PP, PO, NA).	Who can share a method that was different from Sarah's method? Explain to Carly what you did there by using your picture "Turn to you partner and say 'I agree because...' or I disagree because....'"

Two researchers (from a team of five) coded each transcript and one of the two researchers compared codes and resolved clear discrepancies. Other discrepancies were resolved through discussion.

Third, we reviewed the transcripts more qualitatively, based on prior literature and emerging hypotheses about critical features. We refer to this as our Themes Analysis. After becoming very familiar with the data, we conducted a "theme analysis" where each task implementation was reviewed with the goal of producing confirmed hypotheses and detailed descriptions within five categories of pedagogical work for each teacher. Table 3 lists the set of dimensions, along with a brief description of each.

Table 3

Dimensions for Theme Analysis

Dimension	Description
Classroom Community	What norms guided students' interactions around mathematical work? To what degree were students held responsible for considering the ideas of others? To what degree are students comfortable contributing to the mathematical work of the class?
Teacher Press Questions/ Prompts for justification	This is a qualitative look at how the teacher pressed for more information, specifically conceptual (reasoning) or for justification.
Patterns of Interaction	Identification of typical patterns of interaction, including funneling, exploring methods (strategy reporting), inquiry, argument, telling, teacher elaborate. See Wood, Williams and McNeal (2006).
Responsiveness to the particulars of students' ideas	To what degree do the teacher's responses to students involve attention to the specifics of that student's idea? To what degree are the teacher's responses to student's ideas "generic," transcending the specifics of the student's contribution?
Teacher Commitments	Additional "signature" commitments demonstrated by the teacher that seemed likely relevant for the justification activity in the classroom (e.g., did the teacher always push for a generalization; did the teacher push for written expression of ideas, etc.)

A team of four researchers conducted this analysis. (This team had members from both the Student Coding team and the Teacher Coding team.) Based on previous analyses and memos, a set of hypotheses was generated for each teacher. Then researchers reviewed a minimum of two of the transcripts for each task implementation for Year 1 to confirm or disconfirm the hypothesis for each lesson cycle. Researchers identified relevant passages and generated new hypotheses as appropriate. The revised set of confirmed assertions and new hypotheses were then reviewed with respect to two of the Year 2 implementations (Hexagon and Scaling) by two researchers for additional confirmation and refinement as needed. The final results of this analysis, along with supporting examples, were recorded in a memo for each teacher.

Results

In this section, we first offer evidence that both teachers supported extensive justification activity in their mathematics classrooms. We then document differences in four features of the teachers' pedagogical repertoires – features which we argue are associated with the specific contexts for justification created in each teachers' classrooms. Specifically, we focus on the nature of teachers' responses to student ideas, the nature of the teachers' press prompts (for reasoning and justification), the nature of the classroom culture, and priorities in task design and task implementation. In the final section, we argue the connection between the particulars of each teacher's pedagogy of justification and some of the observed differences in the nature of the justification activity in these two classrooms. We see the teacher's pedagogy of justification as creating different contexts for justification produced by the teachers' pedagogies, which then influence student justification activity.

Student justification activity in Cynthia and Paige's classrooms

Our analysis of task implementation and coding of student contributions provides two indicators that Cynthia and Page supported extensive justification activity in their classrooms. We analyzed the six tasks that were in common across the teachers (three tasks each year). Our "student coding" analysis categorized student contributions, which included justification, mathematical reasoning (not at the level of a justification), and explanation, among other codes (e.g., student questions). It also tracked whether there was a justification of the main task justification question(s). (See Methods section above for more detailed description of the codes and coding.)

Table 4 shows the number of student contributions per task implementation that were coded at the level of justification. Table 4 also includes the counts for other focal teachers in our cohort. Looking across task implementations of all seven teachers we see that Paige and Cynthia can also be considered successful at supporting student justifications relative to the focal cohort. Matt's students also made extensive justification contributions, and could have been used in this analysis as well. We chose Paige and Cynthia as they were more similar in how they allocated class time (whole class, small group, individual work) and we thought the parallel structure in class format would aid the analysis.

Table 4

Number of student contributions categorized as a justification by teacher and task implementation

	Year 1 task implementation			Year 2 task implementation			
	Hexagon	Number Trick	Scaling	Hexagon	Number Trick	Scaling	Total
Cynthia	**11**	**0**	**5**	**4**	**8**	**0**	**28**
Paige	**9**	**3**	**7**	**2**	**2**	**14**	**37**
Matt	9	2	3	22	5	9	50
Irene	5	0	2	1	5	1	14
Audrey	17	2	1	6	0	0	26
Bruce	0	3	0	8	3	8	22
Kelly	2	0	0	8	0	7	17
Range	0 – 17	0 – 3	0 – 7	1 – 22	0 – 8	0 - 14	14 - 50

In addition, Paige's students offered a complete justification of the main task in all six task implementations. Cynthia's students offered a complete justification of the main task in four of the six task implementations (all except Year 1 Number Trick and Year 2 Scaling).

As further evidence of student justification activity, we calculated the total number of student contributions that were categorized as either justification or mathematical reasoning, which we call *reasoning contributions,* and calculated the value for the average student reasoning contribution per minute of task implementation. (See Table 5.) We acknowledge that both absolute numbers of student reasoning contributions and rates of reasoning contributions have dubious direct interpretations, but we consider these to be general indicators of justification activity in mathematics classrooms.

Table 5

Counts and rates of student reasoning contributions

	Total time (mins) for task implementations	Number of student reasoning contributions	Average reasoning contributions per minute
Cynthia	600	138	0.23
Paige	519	147	0.28
Focal cohort: range	501- 816	72 – 180	0.10 – 0.28
Focal cohort: mean	614	117.0	0.20

These teachers' classrooms had extensive student contributions that included mathematical reasoning and justification, and not just calculations, answers or comments. On average, Cynthia and Paige's students offered some type of reasoning every four minutes, or the equivalent of 15 contributions per hour, which is quite remarkable, particularly when compared to other published reports of similar types of student responses such as that reported by Jacobs et al. (2006) in their analysis of TIMMS video data.

Comparing these values to other literature provides additional support for the claim. For example, Weaver and Dick (2008) recorded student contributions at the level of justification or generalization (level 5) and relating, conjecturing, predicting (level 4) for a large-scale Math-Science Partnership grant in a whole class format. Data from the classrooms of teachers participating in three years of the study showed that student contribution rates (normalized to per 25 students per hour) reached approximately 15-20 contributions per hour in the third year. (Please note that this value was read off from a graph and not a table of values and so is an estimate.) These values are comparable to those of Cynthia and Paige's students, whose average contributions of 15 per hour is averaged across all components of a lesson, including the launch and individual work time, and so would even have higher rates if just considering whole class discussions.

Creating a Context for Justification: Different Pedagogical Approaches

In this section, we argue that the teachers' pedagogical approaches for supporting student justification activity were different. There were many overlapping elements, but the approaches were distinct and created different contexts to for justification, where student justification activity was organized and driven in different ways. We examine four features: the nature of teachers' responses to student ideas, nature of the teachers' press prompts (for reasoning and justification), nature of the classroom culture, and priorities in task design and task implementation. We focus on whole class discussion in this part of the analysis.

Nature of teacher responsiveness to student ideas

As noted, both Cynthia and Paige gave students extensive opportunities to share their ideas with the class (Table 5). The class was expected to attend to the student's idea being presented and often the class would then be asked to work on these ideas in some manner, whether to evaluate the idea, question it, connect it, build on it, etc. Once a student idea had been elicited, Cynthia and Paige differed in how they typically took up the idea and positioned it for collective work.

Paige tended to respond to the student idea herself initially, asking follow up questions directly and having a public record made of the student work. This recording of the student was often done with Paige asking the student to restate the idea, "piece by piece" with her recording it on the board, or having the student go up to the board to "show." After this work was done, Paige would then position the idea for collective consideration.

Cynthia often responded by directing the class to work on the idea more immediately. She did not regularly ask a sequence of follow up questions herself, though she often would ask for the student idea to be recorded (on the board, or projected from a document camera) and would make sure the idea had been offered with some level of clarity. It was quite common for her to respond to a student offering an idea by focusing on facilitating the conversation or at times asking for an evaluation of the idea (e.g., do we agree? Will this work always?), which could prompt discussion (or in the case of consensus, conclusion).

The excerpt below shows a fairly typical way that Cynthia responds to a student contribution. Notice that she does not respond directly to the students' mathematics and ask a question about it. She clarifies and several pedagogical turns move student ideas into public space. As will be demonstrated, this contrasts with Paige's engagement, where she more explicitly manages the mathematical ideas.

We offer here two targeted excerpts from each teacher's implementation of the Hexagon task in Year 1 to demonstrate the differences in how they respond to student ideas and position the ideas for collective work. We present the excerpts in table form and describe the relevant features of the teacher's responses to student ideas.

In this first excerpt, Cynthia's class has noticed that based on a table, there is a "+4 pattern" as the number of hexagons in the chain increases. She is pushing them for a strategy to find the perimeter of the hexagon figures without the need to generate a table and successively add 4 to the previous values. Cynthia has asked Jackie to share her approach to thinking about the perimeter of Figure 4, a chain of four hexagons (with perimeter of 18 units). In the table, "SxS" indicates a teacher *student-student orchestration* move that either positions students to work on another student's idea or facilitates their interaction around ideas.

Table 6

Transcript CL Y1 Hexagon task, Day 2

Transcript CL Y1 Hexagon task, Day 2	Description of teacher turn
Jackie: *(at board, pointing to figure)* Okay well um every time that you have like one of these things if its n numbers, um the middle ones will always have 4 and then these are going to be 5s so I got 18, I mean, so I got 18.	
CL: So we all understand that? 18?	SxS - comprehension
S2, S3: Yeah	
CL: No, do we all understand her strategy?	SxS- comprehension
Ss: Oh no. No.	
CL: No. Then she is the presenter, you guys are the audience. Go ahead presenter, you're on.	SxS – facilitation
S3: How did you get the ones, um if the four, (inaudible)	

Jackie: Well the 4 is the (inaudible) and then another 4 right here and then you have 5 on the ends.	
CL: Ok so what if we have, (*noticing a hand raised)* okay go ahead, I'm sorry.	*Teacher starts to talk, but holds back to make space for student bid.*
S4: So what you are saying is all you did was 4 plus 4 plus 5 plus 5 (inaudible)	
…	
CL: Okay, so ask her, yeah go ahead. *(making a suggestion to a student)* Ask her how to find figure 10.	SxS – facilitation
S6: How did you find figure 10?	
Jackie: Um can I draw a picture?	
CL: Absolutely.	
CL: Alright, while she is doing that up there and drawing it why you don't at your table figure out her strategy and how to figure out figure 10. *Laughs.* Figure out figure 10. Figure out figure 10 with her strategy, use her strategy, use her strategy right now. Use her strategy right now.	SxS –press (Directly asking students to apply presenting student's strategy to a new figure number.)
Students work in small groups while Jackie works at the board. They come back together as a whole class and Jackie explains.	
Jackie: *Jackie has drawn a chain of 10 hexagons on the board and writes out her calculations as she explains.* Alright well 8 right here and then there is 4 on the bottom thing so 8 times 4 equals 32 and then the ends there is 5 and then another 5 so plus 10 equals 42.	
CL: *(directed at class)* Does it work?	SxS – evaluate
S1, S2: Yeah.	
CL: Will it work for every single one?	SxS – evaluate
S1, S2: Yes.	
CL: Do you have a question?	SxS – facilitate
S3: Um but that, like for 100 it really wouldn't work, because you would have to draw a 100 hexagons.	
CL: Okay good question, would you have to draw them? What a good question would you have to draw it? Now Jackie these guys over here did yours too can they come up and support you and show how they don't need to draw it. Alright, let's do it.	SxS – facilitate *Teacher takes up student question and positions question for collective work, and other students to support presenter.*

In this excerpt we see Cynthia's strong commitment to facilitating discussion and centralizing student ideas and questions. Her move that suggests that a student ask a particular question of the presenter is somewhat unusual, and perhaps emphasizes her commitment to student ideas. We bring to the readers attention the low number of teacher moves that directly engage the students' mathematics. At no point does Cynthia directly question Jackie about her strategy or directly question the other students about their understanding of Jackie's strategy, although she does ask them generally whether they understand it. Similarly, when Cynthia asks the class whether Jackie's strategy will always work, she does not follow up on what seems to be a consensus that it will always work. Note that a student then poses a question which challenges the value of Jackie's method, bringing up Cynthia's prior criterion that the method be able to directly find the perimeter (without drawing or using a table). With this student's bid, their inquiry is pushed deeper.

To summarize, Cynthia actively positions ideas for collective work. She does not typically question the mathematical ideas directly, but rather positions students to do this questioning themselves.

This next excerpt is also taken from the Hexagon task in year 1. Paige is working with students on determining the perimeter of Figure 25 of the hexagon pattern.

Table 7

Transcript PD Y1 Hexagon task, Day 1

Transcript PD Y1 Hexagon task, Day 1	Description of teacher turn
James: *(speaking from his seat)* Um and plus we got the 25 (inaudible) and you count the top of it you get 50 and you get 50 on the bottom, so you can just times is by 2 (inaudible)	
...	
PD: James can you actually go up and draw what you're talking about? You said something about 50, right? Okay, could you go up and draw what you're talking about? (*James goes up to the board*) ...	Elicit–making idea public Press - clarity on 50
James: It said 25 hexagons in a row	
PD: Ok	

James: and then you count the top of them and you times it by 2 and you get the same thing on top that you got on the bottom you get 50 and that's 100. And since you got 2 sides, all you do is add the 2. .	
PD: Ok so could you show us that. Let's just draw some hexagons. Ok, that's fine. Um, so here, we'll draw some hexagons so we have, so they go this way, (*draws figures on the board*) you can imagine the ones in between.	Elicit – making idea public
PD: So show me the 50 that you're talking about. …	Press – clarity on 50
James: This right here, the top's gonna be 50 and the bottom's gonna be 50.	
PD: Alright, so can you write that 50, 50. *J: writes it*	Elicit –making idea public
PD: ok, are there any questions from people in the class about where the 50 is coming from?	SxS – comprehension
There's no response from the class.	
PD: (to class) Ok, so tell me then, where is the 50 coming from?	SxS – press – clarity on the 50 *Request for a class member to demonstrate comprehension*
S: from the top and bottom	
PD: Ok, what about the top and bottom?	Press- clarity on the 50
S: Only the top part of the hexagons all 25 together is 50.	
PD: why 50?	Press- justification of the 50
S: because it's 25 hexagons and there's 2 tops for each one.	
PD: So there's 2 tops for each one and there's 25 altogether, so that's 50 and 50. Okay. And now James go ahead.	Restates
James: And this side would be 1 and this is gonna be 1 because it's the 2 ends and if you add all these up it's going to be 102. .	
PD: ok so 50 and 50 and 1 and 1. *(calling on student hand)* Ok and Robert go ahead.	Restates

In this excerpt, Paige questions the student about the mathematical ideas herself – specifically the source of the 50 in Figure 25. She works directly with James to have him produce the argument for the 50, and also represent it at the board. She then turns her attention to the class and actively checks for their comprehension of the source of the 50. There are no teacher moves that only facilitate or organize student discussion about the ideas, although Paige's moves to have James record his ideas and be explicit about the source of his numbers suggests that she is thinking about the class's access to James' ideas.

Of course both teachers engaged in a wide range of work, and their pedagogical moves overlapped. Their actions were a matter of degree with Cynthia querying the mathematics by organizing students to do so, and Paige querying the mathematics directly (both of the presenter, and of the "audience"). One implication of these patterns is more of the student ideas "go through" Paige, an idea that is returned to later.

To summarize, Paige actively addressed how James' idea is presented to the class and gave direct attention to the student's mathematics. In both Paige and Cynthia's classes, students were expected to listen to one another's ideas and understand them, but they approached this work in different ways.

The nature of teacher press prompts differed

Related to the above type of responsiveness, Paige and Cynthia differed in the verbal prompts they used to press (explicitly or implicitly) for justification. In this discussion we consider not only prompts that requested *why* something was true, but also other prompts that can be considered high-press (Kazemi & Stipek, 2001).

Cynthia frequently used press prompts that were general in nature (e.g., "What do you think?" or "What do you want to add?") and often requested some level of an evaluation of others' ideas (e.g., "Do you agree?"). Paige articulated a range of press prompts, often centralizing analysis (as opposed to evaluation) in her language (e.g., "Why does that work?" "Why is that happening?" "How does it show that it's right?"). Paige also queried much more specific aspects of the task. For example, she queries calculations – such as "why do I have to do 23 times 4?" or "Why is he taking 8 away?"

These types of prompts seem to support student contributions that would build on other's ideas. These types of prompts seem to support student contributions that can be coded *agree*, *disagree*, *build* and *refute*.

Comparing the student contributions in these two teachers' classrooms, we see that students in both Cynthia and Paige's classes participated extensively in building on each other's ideas and asking question, presumably in an effort to query ideas or advance their understanding. The values show not only activity in these areas, but the frequency of these activities relative to our focal cohort of seven teachers.

Table 8.

Student codes related to student agency in extending mathematical ideas across six task implementations

	# of student *builds*	# of student *refutation*	# of student *questions*	# of task implement-ations with *builds*	# of task implement-ations with *refutation*
Cynthia	27	30	57	5	6
Paige	19	17	36	6	2
Cohort min	7	2	9	2	1
Cohort max	27	30	57	6	6
Cohort mean	15.7	9.9	25.6	4.3	2.6
Cohort median	18	6	20	4	2

One important difference to note is that Cynthia's class engaged more extensively in *refutation,* where a student would disagree with a stated idea and offer a reason as to why he or she disagreed. Cynthia's class not only demonstrated the largest number of refutations, but she had at least one student refutation in each of the six task implementations. Although Paige's students' absolute number of refutations is higher than average for the focal cohort, her students offered refutations in two of the six task implementations.

We conjecture that this difference is likely connected to Cynthia's more extensive use of evaluative-type press prompts where she asks students whether they agree or disagree with a student's stated idea. These types of questions set up a different interaction around the mathematical ideas than ones more focused on analysis and why something works.

This difference in press prompts and refutations may also be related to a difference in how Cynthia and Paige control the student ideas that get on the table for discussion. We discuss this further below.

The language of justification

The teachers' press prompts also revealed other differences in the positioning of justification as a community practice. Cynthia often talked about wanting to know that the class was "convinced" that something was true.

Scaling task, Year 2, Day 1

> CL: Alright, is anyone willing to come up here and sell us that this works? Anyone going to sell us that that is an appropriate sentence and that that [L x n x H x n x W x n] will work?

In discussing justification, she also emphasized representations and showing connection across representation. At times, the practice of justification may have been conflated with representation.

For example, in the Hexagon task, Year 1, Day 1, Cynthia asked students how else they could represent their thinking. After a student offered one response, Cynthia remarked:

> CL: A chart, yeah. Look at that representation and try and draw a table. You guys, our goal today is to come up and I want to make a connection, a chart or a table, and our goal, is to connect all of these, make a connection, justify using all of these, alright if the pattern continues what will be a way to figure out the perimeter of any figure? How about figure 1000? 1000 hexagons!

In the Hexagon task in Year 1, Day 3, Cynthia gave the following directions to students. In doing so, she operationalized the idea of *defending,* which became effectively "showing in a model."

> CL: So I'm going to give you 3 minutes to pull out your pattern and, 1 second. And what you're going to do is you're going to then, figure out, be able to defend your pattern. And when I say defend you're gonna have a rule or pattern that tells you how to find the total of the perimeter and **when I say defend you're going to have to show it in a model**. If it is a numerical sentence you show it in the model; if it is a written sentence try and make a numerical sentence out of it.

And then as she brought the class back from pairs work, she commented:

> CL: Okay you're gonna **defend your thinking on how you got that perimeter** but you're going to **make sure we can see it in your, for figure number 4.** Are we all prepared to do it or do we have any more questions? (Pause) Alright now we have 2 minutes on our own let's do it.

Cynthia's final comment indicates that students were to use visuals and *show* connections in diagrams, or across diagrams and tables, etc., which was a component consistently emphasized in Cynthia's classroom. Note also that in this passage the phrase "defend" which came from Cynthia initially (defend your pattern), did not indicate that the students were supposed to offer a mathematical argument to show that a claim is true. She noted that they were to defend *their thinking* and noted they were defending their thinking on *how you got that perimeter.* This is different from offering a mathematical argument to show that one's derived perimeter was correct.

Paige's language emphasized to students that they needed to explain their reasoning and show why something worked. As noted above, often used phrases such as, "what's making that happen?" and "why does it work?" Her language also pointed to the idea that they were working to produce a mathematical argument to show something was true.

Scaling task, Year 2, Day 2

> PD: OK, so get started, you're talking within your groups about **why you think this is true**. Ok?

Paige also regularly asked students to provide *evidence,* particularly in the context of recording a written argument. The use of the term *evidence* was not common across our teachers. Whereas Cynthia had positioned justification and representation activity as nearly synonymous, Paige positioned representations as tools to help reveal students' reasoning and to communicate. Here is an example:

Number Trick, Year 1, Day 2

> PD: Ok well explain that to them not just to me, ok. So explain to them why you see that happening, maybe draw them a picture or show them in the table so that they know what you are talking about.

These differences in teachers' press for reasoning and justification we argue created a different context for justification. With the teachers' language focusing students' attention in different ways, students' activity will be slightly different, even though students in both teachers' classes were regularly engaged in sense making. The implications of these observed differences are not fully clear.

Nature of the classroom culture

The nature of the classroom community was to a large degree more similar than not. Students' ideas were elicited, valued, made public, and discussed. Mistakes were treated as sources of learning, and often times not positioned as mistakes but ideas-in-progress. All of these features are documented in the literature as features of classroom communities that support sense making, understanding, and mathematical inquiry (Hiebert et al., 1997).

Both teachers elicited and centralized students' ideas. They position students' ideas to be worked on by the class. Their whole class discussions were characterized by *accountable talk* (Resnick, 1995) where students were expected to listen to one another and provide reasons for their assertions. Both teachers pressed for justification of results, asking for students to make sense of ideas and demonstrate how they knew something was true.

While both classroom communities supported an *inquiry culture* and the corresponding pattern of interaction (Wood, 1999; Wood, McNeal & Williams, 2006), Cynthia's classroom supported *argument culture* as well and *pupil self-nominate* as patterns of interaction. We have already noted above that the students in Cynthia's classroom regularly offered refutations in response to arguments and other's ideas. This type of activity is associated with a culture of *argumentation* and is not commonly supported in classrooms. Cynthia's classroom also supported much more extensive student questioning. Student questioning can be seen as a reflection of the degree of agency students feel they can exercise over the course of the lesson. The number of questions in Cynthia's class far exceeded the number of questions (as well as rates of questioning) in Paige's class as well as our other focal classes.

Our analysis of transcribed lessons suggests that this difference in observed behaviors (refutations and questions) may stem from the fact that Cynthia regularly took up students' ideas. This allowed students to bid successfully for the floor and contribute new ideas, and thus both expect to be able to do this work and feel encouraged to do this work. This also allowed a wide range of ideas to be shared with the class. On occasion Cynthia would "table" an idea, asking permission to return to it later, but this was not common.

Paige by contrast had fewer student contributions that were *questions* (and *refutations*) and was more discriminating in what ideas she took up for the collective's work. At times she responded directly to a student idea herself (and did not position the idea for collective work) or tabled an idea. The fact that she was more selective in what was engaged publicly, and more directive in what ideas did receive attention, may have shaped norms where students did not feel they had as much agency in bidding for the floor and garnering attention for their particular idea or question.

The prominence of these two additional patterns of interaction in Cynthia's classroom is evidence of a different context for student justification activity – and we have speculated some about the source of this difference. That is, we have speculated about the difference in the teacher's pedagogy that created this different context for student justification activity. Students in Cynthia's classroom had extensive agency, and could get their personal question on the table for the class to consider. The flow of ideas in Paige's class was a little more controlled.

Task design and task-as-implemented.

As a final component, we comment briefly on the task design and task-as-implemented. For the JAGUAR project, teacher implemented the "same" tasks in that the main focus of the task was the same, but teachers could modify the particular task prompts as they chose in order to best align the task with their grade level, students' prior knowledge, other goals, etc.

Across task implementations, both teachers launched the task with attention to students' prior background and focusing students' attention on key concepts (e.g., scale factor). They had explicitly stated norms of listening to the speaker, etc. and visual reminders of these norms posted in their rooms. Finally, both teachers used private-think-time, small groups, and whole class format, and used these formats in roughly similar ways.

Though sharing commonalities, Cynthia and Paige developed their tasks in different ways. We highlight two differences here related to task design and implementation may be relevant for creating different contexts for justification in Paige and Cynthia's classrooms.

First, Paige and Cynthia differed in the *types of claims* for which they requested a justification. Cynthia tended to request justification only for generalized claims, and she highlighted the role of using representations in these justifications. Paige requested justifications for a wide range of claims and did not specify which (or that) representations needed to be used in producing the justification.

To elaborate, in Cynthia's written tasks, and in how it played out during lessons, there was a focus on offering justifications for generalized relationships. For example, she pursued expressing the relationship between an original volume, represented as l x w x h, and a figure scaled by a factor of *s* in her Year 2 Scaling Task and justifying that relationship. Paige tended to focus students' attention on demonstrating why a relationship held in a specific case (or small set of cases). In most cases, but not all, she then turned the class's attention to a general relationship and justifying that relationship. For example, for the Year 2 Scaling Task, Paige focused the class on justifying the relationships between the perimeters and areas of a 2 x 5 rectangle and a set of images scaled by factors of 2, 3, and 4. Although alluding to the general cases, she did not pursue this actively.

For the hexagon task in each year, Paige spent a large portion of time working on the perimeter of Figure 25. Students offered different approaches and they analyzing how these approaches did or did not account for all relevant “sides” of the perimeter. After this work, Paige then turned to the class’s attention to finding the perimeter of Figure *n.* By contrast, Cynthia’s task prompts rarely asked students to justify a result (answer) other than for the generalized relationship. (The one exception was the hexagon tasks where she asked students to find the perimeter of Figure 10 and followed up with “how do you know? This question can be seen either as a prompt for justification or as a more personal question about how one found the value that may or may not include a justification.) Her prompts related to justifying a generalized relationship often required the use of one or more representations. In addition, as indicated above in her justification prompts, she sometimes “replaced” the focus on generating an argument with a focus on creating a particular representation. For example, for the Hexagon task, Cynthia asked students to generate a rule for the perimeter of any figure and then, “Justify your thinking by showing how your way connects with the figure.”

A second, likely related, difference was an emphasis in how the task was implemented. Cynthia gave extensive attention during task implementation to recording ideas and relationships using symbolic notation and having students make sense of those expressions. For example, in the Scaling task noted above, the class had an extended discussion about whether $3*s$ and s^3 were the same. Thus Cynthia’s class engaged extensively in hashing out the meaning of symbolic notation and finding consensus as to whether an expression accurately represented the relationship of interest. Paige’s class attended to notation, but this was not the focus of their work.

Table 9 offers a summary of similarities and differences observed across Paige and Cynthia’s classroom. Text that extends across both columns reflects a feature of both of their pedagogy, their students’ activity in their classrooms, or the classroom environments. Descriptors unique to Paige or Cynthia are listed in one column under the relevant teacher heading.

Table 9

Similarities and difference across Cynthia and Paige's classes relevant for creating a context for student justification activity

Paige	Cynthia
Whole Class Discussions	
Discussions centralized student ideas. Both were skilled at eliciting student ideas, having them represented in public space, and focusing the class's attention on these ideas.	
Allowed "wrong" or incomplete answered to be presented publically for discussion	
Extensive use of clarifying prompts (e.g., what do you mean by…?)	
Expectations to attend to other students' thinking; explicitly stated norms of listening to others	
Controlled the flow of ideas in public space (monitored entrance of new ideas)	Allowed (new) student ideas at nearly any point in the discussion
Paige records students' ideas on the board at times; students do too	CL has students "pop on up" and represent ideas themselves
No formal structures or routines in place to support this (in groupwork or in whole class)	Formal structures/ routines in place to support this (e.g., familiarity with round-robin)
Press Prompts, Justification Prompts	
Focus on *why something worked* and *why something happened* as ways to press for justification; focus on "where" numbers came from and why operation used	Focus on sense making and evaluating ideas – agreeing, disagreeing Some conflation of justification with other processes such as generalizing and representing.
Criteria was that an argument showed *why* the result was true	Criteria for establishing an argument or result was having consensus on an idea (that something made sense).
Focused on justifications of specific cases, and then at times, general cases. Did not prioritize algebraic/symbolic representation and justification of such claims.	Pushed for generalization in each of the six tasks. (It is possible the teacher did not think a claim could be justified unless it was a generalized statement.
Feedback given on clarity of argument and what it showed	Feedback given on clarity and generality of visual representation
Teacher Commitments and Choices	
Strong emphasis on task completion (will push through agenda at the end of a lesson if needed)	Strong emphasis on the journey and will leave a task incomplete Strong emphasis on generalizing and representation
Task	
Extended and isolated attention to specific cases, and justification of individual cases	Work on specific cases followed immediately by work on general
Prompts regularly as part of the written task, requesting "explain your reasoning" and "explain why it works"	Prompts for justification in each task, most often with respect to the general case, and often indicating students should use a representation.
Often with linked warm-up or exit slip	Not linked with warm up or exit slip
Attention to writing out an argument in most of the tasks	Prompts to use representations or multiple representations; no final write up required

Summary

These teachers' implementations of justification tasks evinced high degrees of student participation and justification activity. Their pedagogies had many overlapping elements. The teachers, however, created a context for justification in different ways. They differed in how they orchestrated student-to-student interactions around mathematical ideas; the nature of their press prompts for justification; and the degree to which the classrooms supported a culture of *inquiry and argument* as opposed to just *inquiry,* which is linked to the degree of agency afforded students in having their ideas and questions part of the class's agenda. Finally, the teachers differed in the guidance they offered during task implementation, with Cynthia pursuing generalizations and connections across representations, and Paige more often pursuing analyses of specific cases and 'how things worked.' The relationship between this last component and its impact of the context for justification created in each classroom is not clear. It is clear that, as a result, different types of mathematical claims were being justified, and students were drawing on different resources as they engaged justification, but it is less clear how one might describe this difference as contributing to a different context that supported and drove the student justification activity.

Discussion and Conclusion

In this section we reflect on the nature of the differences seen across Cynthia and Paige's classes and consider implications for teacher learning.

Differences that make a difference?

We have positioned both of approaches to creating a context for classroom justification activity as productive. One might wonder whether one of the approaches is "better" than the other, or at least what the trade-offs are in pursuing one over the other. This cannot be decided definitively, of course, but we can offer some commentary on the observed differences.

Paige's class seemed focused and "on point" as she played a heavy role in what mathematical ideas were worked on as the lesson unfolded. Consistent with her approach where she directly queried students' mathematics, she at times addressed the student's idea and did not position it for the class for collective work. Cynthia's class, by contrast, at times could feel a bit like it was wandering or going around in circles on an idea. In the most extreme case, the class spent an entire lesson working through a question that seemed to have been generated through a misunderstanding, which was not uncovered until near the end of class. Similarly, with Paige's class, it was always quite clear what question was being addressed and the focus of the conversation. In Cynthia's class, however, there were times when one might lose a sense of orientation and it was hard to track what was being addressed at that moment. We conjecture that these differences may be responsible in part for Cynthia's students producing a task justification in four of the six task implementations, whereas Paige's students produced a task justification in all of the six task implementations. Note also, however, that Paige did not always pursue the justification of a generalization in her task design (e.g., for the Scaling task), which Cynthia seemed to do. Thus, Cynthia's pursuit of and attention to students' particular ideas may have been in tension with a goal of producing a task justification (which in some cases would be tightly linked to the lesson's objectives). Paige's class, however, at times felt a little rushed, and ideas were moved aside in an effort to develop a justification for particular results or relationships.

In terms of outcomes for students, we have noted that Cynthia's students seemed to exercise a higher degree of agency and worked more on how to represent ideas symbolically. It is reasonable to conjecture that students in Cynthia's class may develop a different sense of mathematics, their role in producing mathematical ideas, and a deeper appreciation for the power of algebraic notation (and perhaps even proficiency with it). Cynthia's students, however, may also have less of a clear sense of what a mathematical justification is, as it was often conflated with the idea of generalizing and connecting across representations. Paige's students may have a more accurate understanding of what a justification is and an appreciation for figuring out why something works, or why an observed pattern must be so. They also likely developed more skill in expressing their justifications as for all tasks, Paige required some form of written justification after the ideas had been vetted, and in nearly all, she required a more formal write up where students received feedback on their writing and had the opportunity to revise.

Teacher Resources and Implications

In considering the different contexts for justification created in each of these classrooms, it is important to consider *resources* that teachers were drawing on to create these contexts, as that has implications for what we might consider the requisite "knowledge base" for teaching with justification. Paige seemed to draw more significantly on mathematics as a resource, and her thinking about students' thinking and how ideas would be developed. Cynthia seemed to draw more significantly on the community as a resource, facilitating conversations and creating strong norms of participation and honoring student ideas.

Cynthia and Paige's use of these different resources is consistent with what we know about each teacher's strengths (based on other project data). Paige held a secondary certification and was one of the top two scores for our Mathematical Knowledge for Teaching (Hill et al., 2005) project pre- and post-assessment. Cynthia held a multiple subject (elementary) certification and was one of the three lowest scores for the pre- and post-assessments. (For our project participants, there was approximately a 30 percentage point difference between highest and lowest MKT scores.) Perhaps then it is not surprising for Paige to have developed her pedagogy in a way that centralized her mathematical work. Cynthia, by contrast, has developed a pedagogy that, while mathematical in many ways, leveraged the classroom community and not her mathematical work.

If a goal of the field of mathematics education, broadly speaking, is to support the development of teachers' pedagogies of justification, those who design professional development must be both mindful of the potential different pedagogical approaches teachers might use to successfully create a context for student justification *and* they must be mindful that teachers bring different strengths to teaching and advancement of practice, and that these should be acknowledged and built upon for each individual. Different teachers may find more or less success with different models. By documenting two instantiations of extensive student justification and connecting these instantiations to different pedagogical repertoires, we hope we have advanced this argument.

References

Ball, D. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*, 373–397.

Bieda, K. N. (2010). Enacting proof-related tasks in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education*, *41*(4), 351-382.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*, 499–531.

Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. *The learning of mathematics*, 113-139.

Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics, 24,* 359–387.

Cobb, P., Wood, T., Yackel, E., & McNeal, G. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*, 573-602.

Council of Chief State School Officers (CCSSO) & National Governors Association Center for Best Practices (NGA Center) (2010). *Common Core state Standards for Mathematics*. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in Everyday Mathematics classrooms. *Journal for Research in Mathematics Education, 30,* 148–170.

Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics, 44*, 5-23.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hill, H., Rowan, B., & Ball, D. L. (2005). Effect of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371 – 406.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2009). Describing levels and components of a math-talk learning community. *Secondary Lenses on Learning Participant Book: Team Leadership for Mathematics in Middle and High Schools*, 270.

Jacobs, J., Hiebert, J., Givvin, K.B., Hollingsworth, H., Garnier, H., & Wearne, D. (2006). Does Eighth-Grade Mathematics Teacher in the United States align with the NCTM Standards? Results from the TIMMS 1995 and 1999 Video Studies. *Journal for Research in Mathematics Education, 37*, 5-32.

Jacobs, V., Lamb, L., & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41,* 169-202.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, *102,* 59-80.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery.* Cambridge: Cambridge University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29–63.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven: Yale University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge.

Nathan, M. & Knuth, E. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and instruction, 21*(2), 175-207.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Seymour, J., & Lehrer, R. (2006). Tracing the evolution of pedagogical content knowledge as the development of interanimated discourses. *The Journal of Learning Sciences, 15*, 549-582.

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Thousand Oaks, CA: National Council of Teachers of Mathematics and Corwin Press.

Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and Instruction, 25*(2), 161-217.

Weaver, D., & Dick, T. (2008). *Oregon Math Leadership Institute: Spring 2008 Evaluation Report.*

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Wood, T., & Turner-Vorbeck, T. (2001). Extending the conception of mathematics teaching. In T. Wood, B. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school* mathematics (pp. 185-208). Mahwah, NJ: Lawrence Erlbaum Associates.

Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education, 37*, 222-252.

Voigt, J. (1985). Patterns and routines in classroom interaction. *Recherches en didactique des mathematique, 6*(1), 69-118.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for research in mathematics education, 27*(4), 458-477.

Yin, R. K. (1994). *Case study research: Design and methods.* Thousand Oaks, CA: Sage Publications.

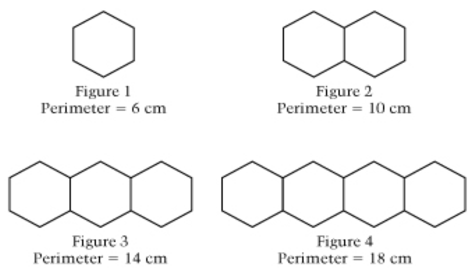
Appendix A

Justification tasks for JAGUAR

Hexagon task

One version of the Hexagon task. This is quite similar to Paige's implementation.

Each figure in the pattern below is made of hexagons that measure 1 centimeter on each side.



1. Draw and find the perimeter of Figure 5.
2. If the pattern of adding one hexagon to each figure is continued, what will be the perimeter of the 25th figure in the pattern? Justify your answer.
3. Extension: How can you find the perimeter of *any figure.* (A figure with *n* hexagons?)

Number Trick task

A version of the Number Trick task. Paige and Cynthia use this, or a very similar version, in Year 1. Both modified the task for Year 2.

Jesse discovers a number trick. She thinks of a number between one and ten, adds four to it, and doubles the result. She then writes the answer down. She goes back to the number she first thought of, she doubles it and then adds eight and writes this answer down. Will Jesse get the same answer for both methods every time?

Here is an example:

Jesse thinks of a number: 5	Jesse goes back to the first number she thought

She adds 4 to her number: 5+4 = 9 She doubles the result: 9 x 2 = 18 She writes down her answer: 18	of: 5 She doubles her number: 5 x 2 = 10 She adds 8 to the result: 10 + 8 =18 She writes down her answer: 18

1. Will Jessie's two answers always be equal to each other for any number between 1 and 10? Explain your reasoning.
2. Does your explanation show that the two answers will always be equal to each other for any number (not just numbers between 1 and 10)? Explain your answer.

How could you justify why the trick works every time?

Scaling task

Teachers had more leeway in designing the Scaling task. Some focused on the relationships of the perimeter and areas of original and scaled 2-dimensional figures. Within this group, some teachers used only one or two figures, while others used many figures or used a figure with general dimensions (e.g., l x w). Others focused on the surface area and volume of original and scaled boxes. Some focused on the surface area and volume relationships of multiple 3-D figures (e.g., cylinders, prisms).

Regardless of the set up, the core question was to articulate and then justify the relationship between the perimeter, area, and/or volume of an original figure and a scaled figure.