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Justification as a learning practice: Its purposes in middle grades mathematics classrooms.

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JUSTIFICATION AS A LEARNING PRACTICE: ITS PURPOSES IN MIDDLE GRADES MATHEMATICS CLASSROOMS[1],[2]

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Justification is a core mathematics practice. Although its role in the mathematician community has been studied extensively (e.g., Hanna, 2000), we know relatively little about its role in K-12 classrooms. This study was conducted to clarify aspects of justification as a learning practice in middle grades mathematics classrooms. We document the views of 12 middle grades teachers who were working actively to incorporate justification into their classrooms. We further analyze differences between teachers' purposes and mathematician purposes, and how these differences may reflect the different purposes of the two communities. Implications for mathematics education and teacher development are discussed.

Purposes

The purpose of this report is to explore the purposes of justification in middle grades mathematics classrooms. Justification is a practice at the heart of the mathematics community. As a disciplinary practice, justification has many purposes: it is used to validate claims, provide insight into a result or phenomenon, and systematize knowledge (de Villiers, 1999, 2002; Hanna, 2000). We know much less about the role of justification in classrooms (when it is present). Justification may be used in classrooms for purposes similar to those of mathematicians, but it may also play a role in other classroom-relevant purposes such as assessment and pursuing content learning goals (Staples & Truxaw, 2009). In this paper, we explore the purposes of justification as a learning practice in middle grades classrooms. We reflect on the relationship between these purposes and the purposes of the mathematician community, and then analyze factors that contribute to the overlap and uniqueness of each set.

Prior literature

The purposes of proof in mathematics have been explored by de Villiers (1999, 2002) and Hanna (2000). Hanna, drawing on de Villiers and others, offered a set of purposes of proof in the mathematician community, a subset of which is shown in Figure 1.

Verification (concerned with the truth of a statement)
Explanation (providing insight into why it is true)
*Systematization (*the organization of various results into a deductive system of axioms, major concepts and theorems*)*
Discovery (the discovery or invention of the new result)

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Communication (the transmission of mathematical knowledge)
Incorporation (seeing a well-known fact into a new framework)
Figure 1. Purposes of Proof [Justification] in the Mathematician Community

De Villiers and Hanna both use the term *proof* in their work. We cast this more broadly, considering this list to reflect purposes of justification in the mathematician community and consider proof to be a specific form of justification.

One might expect a "match" between the purposes of justification in the mathematician community and the classroom community. This match might be expected as the subject of study in math classrooms is mathematics, and thus the practices in that community should reflect in some way the mathematician community. As Herbst and Balacheff (2009) note, "since a notion of proof exists in the discipline of mathematics, it might be entitled to exist in classroom activity. And if it were to exist, it would be expected to exist in a form that is accountable to, if not compatible with, how it exists in the discipline" (p. 43). On the other hand, these two communities have different overall goals. The mathematician community aims to expand the field of mathematics and produce new disciplinary knowledge. The math classroom community aims to support novices' learning of (already known) mathematical ideas and results. Thus one might expect important differences, perhaps including the purposes of justification. The finding that many middle-grades classrooms do not support justification as a practice in any form (Jacobs et al., 2006) lends support to this latter position.

Some prior studies have aimed to unpack teachers' thinking about proof and its role in their classrooms. Knuth (2002a, 2002b), in a study with 17 secondary mathematics teachers, used a subset of categories from Hanna's list to analyze interview data, which included teachers' evaluations of written proofs. Knuth found that teachers reported five roles for proof in their classrooms including developing logical thinking skills, displaying thinking, and explaining why a statement is true. Knuth (2002b) was struck by the lack of reference to the explanatory power of proofs, that is, the role of proof in "promoting insight and revealing key relationships" (p. 80). It is important to note that, because of the use of prior categories developed for the mathematician community, the study primarily uncovered the absence or presence of the mathematician purposes among this group of teachers.

Staples & Truxaw (2009) also explored 24 grades 4-9 teachers' notions of justification by asking them to identify which justification from a set of student work was the "best" justification and why. Teachers' responses highlighted valued characteristics such as: the students' work was detailed, a method was identified (or readily identifiable), and the student was able to see/use the key relationship. The researchers conclude that teachers likely preferred justifications with these characteristics because such justifications gave them insight into what students had learned as a result of their [the teachers'] instruction. The characteristics these teachers valued seemed to contrast with those a group of mathematicians might generate (e.g., demonstrates the validity of the result, builds appropriately on prior established truths).

Conceptual Framework

In approaching this research, we drew upon sociocultural frameworks and specifically the notion of *communities of practice* (Lave & Wenger, 1991; Wenger, 1998). A community of practice is a group of people who engage in a shared domain of human endeavor (joint enterprise) for a sustained period of time (Wenger, 1998). As noted above, the mathematician community and mathematics classroom community, though related, are distinct and pursue

distinct joint enterprises (purposes). This framework brings this to the fore.

The focus on *practice* also provides a useful conceptual tool. A community-of-practice lens highlights the fact that any practice is locally constituted and constantly renegotiated (sustained or modified) in interactions among the community's participants. A practice is customary within a group; it has tradition and history, and it accomplishes particular purposes. Thus, a practice such as justification, has no "absolute" form or meaning, but rather is given meaning within the context of the community. Different communities can sustain different versions of a practice. Justification, then, may have different purposes and forms across different communities.

In approaching this work, we centralized the notion of community and sought to understand, from the perspective of those members of the community (here, teachers), the role and value they saw for justification in their classroom community.

Data Methods and Analysis

Participants in the study were 12 middle school mathematics teachers (grades 7 & 8) participating in an NSF-funded project, JAGUAR. JAGUAR stands for *Justification and Argumentation: Growing Understanding of Algebraic Reasoning*. The teachers taught in five districts in two states; had 2 – 29 years of experience; and were all fully certified to teach mathematics in middle school (four held secondary credentials). Half of the teachers had previously participated in intense professional development geared towards promoting student discourse in mathematics classrooms. All but one of the other teachers had extensive exposure to related ideas (e.g., teaching for higher-order thinking) through professional development and/or prior participation in research projects. In general, this group of teachers already expected their students to participate in classroom discourse. In committing to the JAGUAR project, the teachers agreed to actively work on ideas related to justification in their practice and to collaborate with project personnel to unpack the nature of justification in middle grades mathematics classrooms. Thus, we do not expect that these teachers are representative of the larger population of middle school mathematics teachers. Rather, this was a *purposive* sample (Yin, 1994) to enhance our ability to examine the role of justification in middle grades classrooms.

A wide range of data informed the analyses reported here. Initial themes and categories related to the purposes of justification in the math classroom were developed by reviewing documents from a week-long summer course and two Saturday working sessions, videos of lessons, teacher reflective journals, and transcripts of teacher interviews. Standard qualitative methods of *open coding* and *constant comparative method* (Strauss & Corbin, 1998) were employed. As themes and categories emerged, they were recorded, and links to sources for those ideas were noted. We then conducted a systematic analysis of two data sources for confirmation and refinement. The sources were a) four discussions (~1 hour each) with subgroups of teachers during the summer course or work sessions about the role of justification in their classroom and b) teacher reflective journals (22 total) on two different lessons that were deliberately designed to engage students in justification. One of the discussions served as a *member check* (Lincoln & Guba, 1985) on the themes to enhance validity of the results. In reporting the results, we share major themes and then offer supporting evidence from the broader set of data, as appropriate.

Results and Discussion

The results of this study indicate that middle school teachers, who are actively working to incorporate justification into their practice, use justification for a wide range of purposes, some

of which overlap with the mathematician community and some of which are distinct. We report six main themes, reflect on the correspondence between these purposes and the mathematician community's purposes, and analyze the teachers' role and context to make sense of the set of purposes. We do not intend this set to be comprehensive or unique, as work with other teachers or further analyses may identify additional themes and other useful categorizations. Rather, we document the existence of purposes identified by this group of teachers, many of which are not yet found in the literature, in order to paint a picture of the potential purposes of justification in middle grades classrooms. The six purposes are *Promoting conceptual understanding, Fostering valued math skills and dispositions, Assessment (Display and monitoring), Fostering valued life-long skills and dispositions, Managing diversity, and Influencing social relationships.* We explicate this list by discussing each and offering supporting examples from the data.

Promoting conceptual understanding

One important purpose of justification in the classroom- and perhaps the most universally noted by the teachers– was to promote or deepen students' learning, particularly of concepts. Teachers regularly noted the process of justification led to deeper understanding because it required students to wrestle with the key ideas, make connections, and gain new insights. The means by which this happened were many, ranging from hearing other students explain to clarifying their own thinking as they tried to articulate their ideas.

This theme was prevalent throughout interviews (see Thanheiser et al., 2010), teacher reflective journals, and work session discussions. The following quote by one teacher offers one example of the teachers' views: "Justification pushes students beyond a procedure to a deeper understanding of the math. In order to justify their thinking, they have to justify not just the hows, but get to the whys of what they're doing." (T1, 11.14.09)

This purpose of justification is anticipated in the literature. We see this as akin to Hanna's category *Explanation* elaborated as *promotes insight into a phenomenon*. Although she positions the mathematics community as the collective recipient of this insight, it can be understood on the individual level as well. Hanna does not, however, address a dimension that seemed central for the teachers– the individual students' construction of knowledge whereby justification offered students the opportunity to figure out *for themselves* how and why something works.

Fostering valued math skills and dispositions

The teachers also identified the growth and development of other math-related skills and dispositions as an important purpose of justification. Across two work session discussions, all five process standards (NCTM, 2000) were noted. For example, teachers saw that engaging students in justification helped students develop their communication skills and representational skills (such as generating and using graphs, tables and symbols), as well as making connections across representations. Teachers also remarked that justification promoted the disposition to try lots of ways and to build vocabulary, as engaging in justification often promoted an authentic need to use mathematical terms in context.

We do not see an analogue in the mathematician community to this set of outcomes, perhaps because there is no need in the mathematician community to focus on the development of such process skills. It is assumed that they are proficient (otherwise they would not be members of that community).

Assessment (displaying and monitoring)

Justification was seen to play a critical role in creating a venue for students to display their understanding so that teachers could monitor the degree to which students had moved towards the desired learning goals. This theme encompassed formative assessment, including self-

assessment, and summative assessment.

Teachers asked students to justify as a window into students' reasoning about a particular idea. From this they gleaned information, useful for diagnostic purposes, about how the student, or class, was thinking about a problem. It helped teachers "pinpoint" where students were stuck, or where their misconceptions might lie. As one teacher noted: "It helps to guide my next instructional moves. So based on a student, or the collective class's justification, … it shows me where I need to go next." (T2, 11.14.09). Teachers also found value in justification as self-assessment, as justification prompted students to reflect on their own work and identify their own mistakes. This overlaps with the first category reported.

The emphasis teachers placed on justification as having students "show what they know" is particularly interesting to examine. Consider this teacher's comment:

> T12: Interestingly enough, our district has gone to proficiencies, … And it's now a kind of thing too, that proficient, or highly proficient, just showing an answer doesn't give me any clue as to your proficiency or understanding. So that's again, putting that responsibility back on the student to really show what they know. (11.14.09)

The focus on students demonstrating what they had learned was a theme across data sets, most notably in interviews when teachers were asked to determine whether a student response was or was not a justification. In these instances, it seemed teachers expected *more* than what might be required for a complete justification of the result. When asked if the response was a justification, teachers seemed to engage the question: "Has the student demonstrated a full understanding of the key ideas we have been working on (i.e., did the student *learn*)?" rather than the question, "Has the student demonstrated the result is true?" This distinction is captured by considering what the teachers wanted to be convinced of: that the students had learned the particular content, rather than the validity of the result.

The justifications desired by teachers from students in a summative assessment potentially then underemphasize the overall purpose of justification (to demonstrate a result is true), and emphasize a purpose of displaying what one knows. This may require the student to include non-essential details or information and not build on prior results, but rather to re-prove these results to demonstrate one's own understanding of the mathematics underlying the result. This makes sense given that teachers are responsible for both *developing* and *certifying* students' mathematical proficiency. This issue is important because it may significantly influence what students think a justification is.

Fostering valued life-long skills and dispositions

Beyond mathematics, justification was seen to play a role in promoting the development of "student goals" – goals important for students not as people who do or use mathematics, but as future adults. The teachers noted that justification fostered student perseverance, independence, critical thinking skills, and the habit of mind to support one's ideas, or request that of another.

In reflecting on this purpose, one group of teachers noted that justification played an important role in fostering these desired, long-term outcomes because it provided opportunities in their classrooms that otherwise were not generally available. That is, justification was uniquely positioned to help create these opportunities. Here are comments from two teachers:

> T7: I guess [these are] skills that are built and developed as, like through the process of justification, but may not necessarily be mathematical goals. … And I think we're teaching them how to be learners as well. (11.07.09)
>
> T4: It's like real world skills that they need for problem solving or to kind of learn. It's like learning skills that happen to come out nicely through mathematical justifications. (11.07.09)

In addition, communication as a life-long skill was also noted prominently. Note that this is different than communicating mathematically. It was seen as a more general process or outcome, applicable across fields and situations in one's adult life.

> T9: Because when you're done with a piece of work, … whatever I do, I want it to stand by itself …. I would like to have someone else pick it up, read it, understand it and say, "oh yeah, I know what she's talking about." And I want kids to be able to do that too - to be able to put something down well enough, whether it's on paper or whatever, that it can stand by itself without them having to sit there and explain it. (11.07.09)

In discussing broad goals, teachers brought in connections with other subjects, noting how each contributed to students' ability to communicate, make an argument, or become a critical thinker.

Managing diversity

Justification provided teachers other affordances for their instruction as well. They saw it as a means to make them more effective with a range of students. Justification inherently differentiated learning, as students could approach a task in any number of ways and take it as far as they could, offering more or less sophisticated justifications. It made it so not everyone had to move "lock step," but rather all could be pushed and learn even when at different levels.

> T11: This [task] will push some kids, in a heterogeneous classroom, beyond into a generalization, and this problem will push these kids here still to a justification [but not a generalization], but I can push the others beyond where the others are. (11.14.09)

Following on this idea, other teachers added that justification tasks– at least those they designed - often had multiple points of entry, which also was beneficial in a heterogeneous group.

Teachers also saw incorporating justification as a way to "reach every student." Having students share many approaches to one problem offered students multiple paths to understanding. They were bound to understand at least one approach, and teachers noted that sometimes students had better ways of helping classmates make sense of ideas than they did. For approaches that many students didn't understand, the teachers felt the exposure was still productive for student learning, laying some foundation for future work. Note that this aspect of Managing Diversity also overlaps with the first category, but is reported here for its role as a pedagogical affordance to the teacher.

Influencing social relationships

Teachers noted that justification could play influence the social relationships or impact the social dynamics in their classroom. It was felt that, if math becomes the authority, it can challenge the social status quo and shifts it from a social system organized around the popular to those who are good thinkers. One teacher stated the following:

> T7: When you add the word middle school, now I'm thinking a little more socially. I feel like justification can have a couple social functions. And one of them is sort of valuing, like, how we decide who's right. Instead of it being, you know, the teacher's right, or this kid is popular, so they're right, it has a social function of, you can prove it, so therefore, I don't care if you're the least popular kid in the class. You know, you have to be able to back you your ideas. Everybody has to justify what they're saying. … I think it takes some of those power relationships out a little bit. (07.28.09)

Similarly, teachers talked about justification "empowering" students, which potentially shifted the dynamics so that the teacher was not the only one who knew. Students might learn to trust that they can figure things out for themselves, or that they could use their peers as resources.

Verification – what needs to be verified?

We have explored six themes present in these teachers' discussions and reflections. Equally

interesting is what we did *not* find. Most notably, teachers generally did not identify a purpose of justification in their classroom communities to be establishing mathematical results or ideas as true, that is, verification. Given the centrality of this purpose to the mathematician community, we discuss some of their views. (Space precludes a discussion of other purposes not noted.)

Verification as a purpose did not appear as a theme across these data. When potentially related ideas were mentioned, such as students can check their own work, or don't have to rely on the teacher, the valued purpose was more fostering student independence and self-reliance (life-long learning goals), or to shape social relationships (as described above), and not to establish a result as true for the community to then use and build upon. The following quote from a teacher reveals his perceptions of some important differences between the classroom community and mathematician community that are relevant for understanding justification as verification:

> T4: Like verification, concerned with the truth of a statement, is really what mathematicians are concerned with because, where they are, they're forging new ground. So no one is telling them that, "yes, this is true." So they have to be *very* concerned with verification because if we take it to be true and it isn't true, then we're going to develop new ideas off of it, and those won't be true because this one isn't true. (11.07.09)

In the classroom, teachers noted that students had other resources, including the teacher, for verification. De Villiers (1999) echoes this idea that there is no *driving* need for verification in the classroom community. Furthermore, when considering whether verification could play a more prominent role, the same teacher noted that students, to some degree, are not in a position to rely on the mathematics for verification of results.

> T4: If you are using skills that you assume that a mathematician knows to be true, and only those skills to prove something more, then this *has* to be true because we know *this* … all the way back down to 1 plus 1 equals 2. … Whereas with students, they can't always use the math to justify their answers because their math is sometimes faulty. So there needs to be something else there for their verification process. (11.07.09)

In classrooms, students are (commonly) asked to make sense of new ideas when they may have gaps in their prior understanding, or some foundational ideas are still developing. One teacher noted that, as students were learning, "they need the teacher to validate because they don't "trust" the method yet" and cannot tell if they have executed it properly (T10, 02.04.10). Consequently, they are not in a position to fully rely on themselves to verify their method and/or the result. This observation connects with, and potentially thwarts in the classroom, the purpose of justification as systematization: the next result in the classroom community is not necessarily built on prior results. Teachers indeed ask students to move forwards without the requisite foundation, or to take something as true without demonstration, perhaps for pedagogical or practical reasons.

Implications and Conclusions

In this report, we explored six purposes of justification in the middle grades mathematics classroom communities of one group of teachers. These teachers' purposes for justification supported them in meeting their obligations as teachers (e.g., promoting, monitoring and assessing the learning or a diverse group of students) as well as pursuing other valued outcomes (e.g., developing important skills and dispositions for adulthood). In doing so, we outlined some important places of confluence (explanation) and some important distinctions (verification). These results broaden our understanding of the ways justification might be realized in middle grades classrooms. Prior literature has generally searched for the absence or presence of the purposes identified for the mathematician community. Such an approach identifies some

important differences, but does not uncover the valued ways teachers do use justification.

We reiterate that the meaning of a practice comes from its *use* in a community and seeing the value of that practice in that community. We know that many classrooms do not incorporate justification. This study demonstrated that, even in classrooms that do support justification, as a practice, the purposes for engaging in justification may differ in important ways from those of the mathematician community. These purposes are shaped by the overall goals of the community, which in this case is educating students. Following Chazan and Lueke (2009), we hope this work "challenges researchers interested in relationships between mathematical activity in schools and in the discipline to understand better why certain kinds of mathematical activity rarely find their way into the institutional setting of school" (p. 37). Furthermore, it is important to pursue documenting purposes that may not be singularly mathematical. By better understanding how teachers value and incorporate justification in their classrooms, we may be better positioned to work with teachers to support the development of their practice and help them see justification as something important and useful to incorporate in their own classrooms.

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