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# Supporting Whole-Class Collaborative Inquiry in a Secondary Mathematics Classroom

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SUPPORTING WHOLE-CLASS COLLABORATIVE INQUIRY IN A SECONDARY  
MATHEMATICS CLASSROOM

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## ABSTRACT

Recent mathematics education reform efforts call for the instantiation of mathematics classroom environments where students have opportunities to reason and construct their understandings as part of a community of learners. Despite some successes, traditional models of instruction still dominate the educational landscape. This limited success can be attributed, in part, to an underdeveloped understanding of the roles teachers must enact to successfully organize and participate in collaborative classroom practices. Towards this end, an in-depth longitudinal case study of a collaborative high school mathematics classroom was undertaken guided by the following two questions: What roles do these collaborative practices require of teacher and students? How does the community's capacity to engage in collaborative practices develop over time? The analyses produced two conceptual models: one of the teacher's role, along with specific instructional strategies the teacher used to organize a collaborative learning environment, and the second of the process by which the class's capacity to participate in collaborative inquiry practices developed over time.

In recent decades, there have been a multitude of reform efforts in mathematics education in the United States. One major focus of these efforts is to effect classroom-level changes. Specifically, efforts have aimed to create more collaborative and student-centered environments, where students have opportunities to reason and construct their understandings as part of a community of learners (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000). The value of these reform practices has been demonstrated with respect to student learning particularly students' conceptual understanding (Boaler, 1997; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997; Silver & Stein, 1996; Wood & Sellers, 1997), and in the production of potentially more equitable learning environments (Boaler, 1997; Cohen & Lotan, 1997; Silver, Smith & Nelson, 1995). Furthermore, students participating in these kinds of practices tend to develop positive identities with respect to mathematics (Boaler & Greeno, 2000; Cobb & Hodge, 2002) and their beliefs about mathematics more accurately reflect the nature of the discipline than those of their traditionally schooled peers (Schoenfeld, 1988, 1992).

Despite these compelling successes, traditional models of instruction, particularly at the secondary level, still dominate the educational landscape (Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006; Kawanaka & Stigler, 1999; Stigler & Hiebert, 1999). Teachers find it very challenging to organize and support student participation in these discourse-intensive practices that centralize students' ways of thinking. There are myriad factors contributing to the staying-power of traditional models and the lack of implementation of more collaborative, inquiry-oriented environments. An assumption of this study is that this limited success, in part, can be attributed to an underdeveloped understanding of the nature of the practices of these communities and the roles required of teachers and students to enact these practices successfully (Boaler, 2003).

This article offers two conceptual models of a teacher's instructional strategies that supported students' participation in whole-class discussions, a central event in inquiry-oriented environments. Whole-class discussions are used by teachers to promote and extend students' mathematical understandings and develop their proficiency with a range of mathematical practices such as conjecturing, justifying and reconciling (Ball & Bass, 2000). The conceptual models are based on data from a longitudinal case study of a teacher and class that regularly engaged in collaborative inquiry practices. The first model represents the role of the teacher during collaborative whole-class discussions. It aims to capture the critical work that the teacher did as she supported students' participation in collaborative inquiry practices. The second model represents the process by which the class's capacity to enact collaborative practices developed over time. In their prior mathematics courses, the students had

experienced primarily traditional modes of instruction. Thus an important part of the teacher's work was establishing new norms and expanding the class's repertoire of learning practices (Boaler, 2002; Cohen & Ball, 2001).

Although whole-class discussion is the identified participation structure, this study focuses on *collaborative inquiry mathematics practices* as a specific instantiation of reform ideals. Rochelle and Teasley (1995, as cited in Dillenbourg, 1999, p. 17) define collaboration as "a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conceptualization of a problem" (p. 70). The term *collaborative* then implies a joint production of ideas, where students offer their thoughts, attend and respond to each other's ideas, and generate shared meaning or understanding through their joint efforts. Collaboration is distinct from cooperation which only implies sharing. Students can cooperate to accomplish a task, by sharing answers or creating a poster or other product together, but may not engage in collaboration with respect to the joint production of mathematical ideas. Collaboration is also distinct from practices involving extensive participation or talk from students. Too often "surface features" associated with reform practices, such as high levels of interaction among students or groupwork, are taken as evidence of successful realization of reform ideals. Yet, classrooms supporting these surface features may not give sufficient attention to the quality of students' engagement with mathematical ideas (Nathan & Knuth, 2003; NCTM, 2000).

*Inquiry* is a practice or stance, and indicates a particular way of engaging with and making sense of the world. In this article, I focus on inquiry as it relates to students' mathematical activities and I take the term to mean both inquiry *into* mathematics and inquiry *with* mathematics. Inquiry into mathematics involves delving into mathematical ideas and concepts and trying to understand the structure, power, and limitations of mathematics. Inquiry with mathematics involves using mathematics as a tool to make sense of problem situations and come to some reasonable resolution. This type of work involves problem solving, modeling, and applications to business, physics or other "real world" phenomena.

The focus of this research on practices is driven by developments in learning theories, particularly the advent of sociocultural and situative perspectives. These perspectives understand learning as a fundamentally participatory and social process. From a situative perspective, "mathematics is a set of practices of inquiry and sense-making that include communication, questioning, understanding, explaining, and reasoning" and "learning mathematics is marked by increasing participation in an expanding range of such practices" (Greeno & MMAP, 1997, p. 104). Learning results from, and is evidenced by, student participation in both standard disciplinary practices (e.g., justifying, representing algebraically) and an array

of other practices of mathematical communities (e.g., questioning, communicating, informal reasoning). Thus, understanding the social context and practices of a classroom, and how the teacher organizes and supports these practices, is essential to understanding the learning opportunities afforded students.

This study builds on prior research on the successful enactment of collaborative inquiry practices in mathematics classrooms. Several studies offer insights into the pedagogical demands of these socially and intellectually intensive learning communities with analyses of teaching practices or particular aspects of pedagogy that support student participation in reform-oriented mathematics classrooms (e.g., Ball, 1993; Chazan, 2000; Fraivillig, Murphy, & Fuson, 1999; Lampert, 1990, 2001; Lobato, Clarke, & Ellis, 2005; Rasmussen & Marongelle, 2006). For example, Fraivillig et al. (1999) developed a framework, *Advancing Children's Thinking (ACT)*, to describe teaching at the elementary level that supported the advancement of students' mathematical thinking in an *Everyday Mathematics* classroom. They identified three overlapping but distinct "patterns of practice": eliciting children's solution methods; supporting children's conceptual understanding; and extending children's mathematical thinking along with corresponding instructional strategies for each pattern. Such a framework offers a conceptualization of the many facets of the teacher's role in a reform-aligned classroom and the specific actions teachers may employ to support student participation and mathematics learning. Other studies have focused on analyzing the role of the teacher in organizing student participation in practices authentic to communities of mathematicians (e.g., Ball, 1993; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Lampert, 1990, 2001) such as argumentation, justifying, and revising.

Another line of research has analyzed the critical work of teaching that supports students' participation in a new set of learning practices (e.g., Boaler, 2002; Chazan, 2000; Goos, 2004; Heaton, 2000; Hufferd-Ackles, Fuson, & Sherin, 2004; Lampert, 1990, 2001; Wood, 1999). These classroom cultures and communities represent a radical departure from more conventional classrooms, which requires students to participate in new ways. For example, Goos (2004) studied an expert secondary teacher of an advanced mathematics course. Drawing on a Vygotskian perspective, she identified teacher actions and assumptions that helped to create a culture of inquiry in this classroom and the corresponding learning practices that students developed in response to these teacher actions. Although the students in Goos's study seemed receptive to these new learning practices, the development of new norms and students' proficiency with new practices may not proceed so fluidly (Chazan, 2000; Heaton, 2000).

Each of these studies offers a vision of the nature of reform-aligned classroom communities as well as a representation of the corresponding teaching to enact this vision. Continued efforts are needed to

expand our collective knowledge regarding how to productively conceptualize and understand teaching that supports the development of students' mathematical proficiencies through collaborative inquiry (Fraivillig et al., 1999). As Chazan and Ball (1999) argue, one of the most prominent images of reform is that of a facilitator. In this role, teachers believe they must "not tell," which leaves open the question of what teachers can do to support collaborative inquiry practices and guide the development of mathematical ideas. To address this issue, Lobato, Clarke & Ellis (2005) have offered a reformulation of telling that aligns with reform practices and specifies some conditions under which telling productively supports inquiry mathematics. Such efforts begin to specify aspects of the teacher's role in inquiry classrooms and the practices that appropriately support this new role.

To better support teachers in instantiating collaborative inquiry learning environments, the mathematics education communities needs an increasingly sophisticated understanding of the nature of the teacher's work and ways to communicate this to teachers. In addition, research has generally focused on collaborative classrooms *only once the practices have been established* (Fraivillig et al., 1999; Kazemi & Stipek, 2001). We need detailed accounts of the development of these communities for teachers who are faced with the challenge of transforming their students' participation, and potentially their own as well. The findings reported here aim to extend our current understandings of the teaching and learning practices of collaborative classrooms, their organization, and how they develop over time.

#### RESEARCH DESIGN AND METHODS

The exploratory nature of my research questions and their demand for longitudinal involvement in a classroom made this study well suited to a case design (Yin, 1994). The focal case was a ninth-grade pre-algebra class (Math A) taught by a highly experienced and accomplished teacher, Ms. Nelson, during 2001-2002. Ms. Nelson was selected because she consistently organized collaborative inquiry activities in her classes, regardless of course level or student composition. This was validated by a research team from the Stanford Mathematics Teaching and Learning Study (Principal Investigator, Dr. Jo Boaler; see Boaler & Staples, 2003, in press). Two other teachers also participated in this study. Ms. Nelson became the focal case, however, as her class supported the highest levels of collaborative interaction in a whole-class format. The Math A class also comprised a lower-attaining group of students, which provided a particularly challenging teaching context, and thus I anticipated would yield results that might be more revealing or speak to a broader audience than those from a class of high-attaining students. Thus, the sampling strategy was *purposive* (Yin, 1994).

### *Participants and Context*

*Ms. Nelson.* At the time of the study, Ms. Nelson was in her twenty-second year of teaching. She was Nationally Board certified and had won awards for her teaching. Ms. Nelson had high expectations for her students. She emphasized the importance of developing students' abilities to analyze problem situations, generate solution methods and develop conceptual and procedural fluency. Ms. Nelson viewed mathematics as the product of human thinking; held strong beliefs that all students could be successful, regardless of past levels of achievement; and adapted curricular materials and instructional strategies to meet students' needs. Ms. Nelson, then, is perhaps somewhat unusual, as most high school mathematics teachers see mathematics as a rigid and fixed body of knowledge; take as their responsibility transmitting this knowledge to their students; and believe students' mathematical ability is a stable attribute (Stodolsky & Grossman, 1995).

*The Math A students.* On average, there were 20 students enrolled in the Math A course, with roughly a 2:1 ratio of boys to girls<sup>1</sup>. A core group of 14 students were in the course the full year. The students were primarily Caucasian, reflecting the demographics of the school (90% Caucasian) and area. Eight of the students enrolled were also supported by the Resource Department for mathematics, and Ms. Fabrique, a resource specialist, worked with Ms. Nelson as part of this arrangement.

The Math A students had very weak computational and basic skills. On a 75-question incoming basic competency test including operations on integers, decimals and fractions, the class's average was 25.3 (34%). The test comprised 5 problems on each of 15 different math topics including addition, subtraction, multiplication, and division of whole numbers; decimal values and fractions; conversion of fractions to decimals; percentages; and rounding and estimation. It was untimed and calculators were not permitted.

On the addition section, 9 of 19 students for whom tests were available answered the following problem correctly:

$$(BC 4) \$595 + \$73.28 + \$1,479 + \$.93 =$$

answer: \$2148.21

For the five problems in this section, the average number correct was 2.75 of 5. This was highest average number correct of the 15 sections.

On the multiplication section, only 3 of 19 students answered this problem correct:

$$(BC 6) 45 \times 70.9 =$$

answer: 3190.5

For all problems in this section, at least one multiplicand was a decimal value. The average score on the section was 0.9 correct out of 5. Similar results across the fifteen topics—ranging from 0.11 (computing with percents) to 2.75 (addition)—indicated extremely low levels of procedural fluency.

In prior courses, the Math A students had experienced primarily traditional modes of instruction where the teacher explained mathematics and they practiced the demonstrated procedures. These students' prior experiences are fairly typical for students in the U.S. (Kawanaka & Stigler, 1999; Schoenfeld, 1992; Stigler & Hiebert, 1999). These histories comprised a significant part of the context of Ms. Nelson's work, as they shaped students' beliefs and perceptions about the nature of mathematics and their selves as learners and doers of mathematics.

*The course and curriculum.* The Math A course was the lowest track math class at this high school. It was designed to support students who were not yet prepared to take algebra while also remediating students' basic mathematics skills. This was one of three sections of Math A. The course drew on a range of curricular materials. The two focal units for this study (Patterns and Overland Trail) were from the Year 1 *Interactive Mathematics Program* (IMP) designed by Fendel, Resek, Alper, and Fraser (1997). Relative to more conventional materials, these are more exploratory and conceptually focused. Concurrent with these units was a continuing emphasis on basic numeracy skills and middle school content often in the form of individualized basic competency worksheets (self-paced). Table 1 shows the primary instructional foci across the year.

Table 1

Primary instructional foci of the Math A course, 2001-2002

<b>Time of year</b>	<b>Unit</b>	<b>Math Topics in Unit</b>	<b>Other significant activities or topics</b>
September – early January	Patterns Unit	Patterns In-out tables and functions Positive and negative integers Use of variables to represent generalizations Summation notation	Order of operations Basic geometry (polygons and angles) Basic competency work

Late January – early March	Visualization (not an IMP unit)	Perspective and isometric drawings  3-d projections into the plane	Basic competency work  Warm-up activities to review middle school math topics (for state testing)
Late March – June	Overland Trail	Using variables to write meaningful expressions and interpret expressions  Constructing, interpreting and solving equations with variables  Graphing and graphical analysis  Multiple representations (correspondence among a table, graph, and equation)	Warm-up activities to review middle school math topics

### *Data Collection*

The research was conducted from an interpretivist paradigm (Eisenhart, 1988), which assumes that “human behavior and human learning are responsive to a context that is interpreted by participants” (p. 101) from which it followed that I employed primarily ethnographic methods of data collection. Such an approach is congruous with situative perspectives and socio-cultural theories of learning. I became intimately familiar with the class, its students, and the school, although my role in the classroom was *observer as participant* (Merriam, 1988). I was an outside observer in that it is not typical to have a researcher in the classroom, and this was my only formal role and reason to be present. At the same time, I spent many hours in this class and became a regular participant in the classroom community. I developed relationships with the Ms. Nelson, Ms. Fabrique, and students. During classtime, I responded to students’ questions and requests for assistance as they worked in small groups or individually, always making efforts to do so in a manner that was consistent with Ms. Nelson’s practice. These relationships and my

participation in this classroom community enhanced the quality of my observational data and provided me with unique access to students' stances towards and engagement with the classroom activities.

Data sources used to document lessons including videotapes, written field notes, class materials (overheads, worksheets, a record of board work) and frequent conversations with the teacher. Overall, 66% of class time (105 hours) was observed and 65% was videotaped (103 hours) across the school year. Observations and videotaping were particularly intensive during the first twelve weeks of school (Patterns Unit, 32 of 33 lessons), and again during March and April (Overland Trail unit, 11 of 19 lessons) to provide data for comparison. The school followed a modified block system where classes met 3 times per week, once for 54 minutes and twice for 110 minutes. After each lesson, I wrote a reflective memo summarizing the lesson and documenting various aspects of the class with a focus on the practices students' participated in that day, the teacher's work to organize and support their participation, and the mathematics they encountered.

Lesson observations and my interactions with students during lessons also provided data on students' perceptions and interpretations. Further information about their views were elicited using questionnaires and interviews. I gave students questionnaires three times during the year (1 fall, 1 mid-year, 1 spring) and conducted interviews in the fall (18) and spring (13). Interviews were generally conducted in friendship pairs. Questions focused on students' perceptions and beliefs about math class (e.g., What does it take to do well in this math class?), their views of their roles and participation in the class (e.g., How do you feel about going to the board? What do you do when you don't understand someone else's explanation?), and their prior experiences with math (e.g., Is this class similar to or different from your last year's math class? In what ways?). By interviewing students at the beginning and end of the year I was able to track changes in their perceptions and beliefs over time. Of the 13 students interviewed in the spring, 12 were class members for the entire year. I also conducted 2 video viewing sessions with groups of 4-5 students where I showed lesson segments I thought were revealing of particular practices or norms and elicited feedback from students on their perceptions of these events. These were conducted in May.

I also actively elicited Ms. Nelson's perceptions of lessons and regularly engaged with her in dialogue about the class, its progress as a collective, and what we were both noticing about their mathematical thinking and participation. These discussions were documented in the reflective notes. I conducted 4 formal interviews with Ms. Nelson, and 1 video viewing session with her, along with Ms. Fabrique (the resource specialist working with the Math A group) and another teacher participating in the

study. In the months following the completion of the class, Ms. Nelson and I met 5 times to review segments of video, discuss my emerging understandings of the class, and elicit her responses to my interpretations. I also shared with Ms. Nelson some of the final written analyses for this project. These latter activities served as a form of respondent validation.

### *Data Analysis Procedures*

Data analysis followed principles of grounded theory (Glaser & Strauss, 1967; Strauss & Corbin, 1998) and drew upon other analytic techniques such as conversation analysis (Clark, 1996). To identify episodes of collaborative inquiry activity, I used as criteria Dillenbourg's (1999) three characteristics of collaborative interactions: *interactivity*, *synchronicity*, and *negotiability*. Interactivity requires that participants exert reciprocal influence on one another. Synchronicity indicates that interactions must be coordinated. Negotiability captures the ability of the participants to influence the process in which they are engaged. Classrooms, however, are systems comprising inter-related elements (Schoenfeld, 1992) and inquiry practices must be supported by other elements and practices of the classroom system. Thus I cast my net widely, analyzing a broad range of classroom interactions (not just collaborative) and examining students' and teacher's understandings of the ever-evolving set of classroom practices and participation structures.

*Phase 1:* In the first phase of analysis, which was concurrent with my fieldwork, fieldnotes, reflective notes and video content logs<sup>2</sup> were coded employing processes of *open coding* and *constant comparison* to identify initial patterns and themes related to the teacher's and students' participation in mathematical activities, and specifically collaborative inquiry activities. I periodically wrote analytic memos to document my emerging understandings of the classroom regarding the teacher's role, the students' role, the mathematics, and the nature of their collaborative activities together.

*Phase 2.* In the next phase of analysis, I used a set of 35 codes and NuDIST™ software to code two sets of video content logs—four lessons each from the fall and the spring (approx 800 minutes of video). These codes were developed from the first phase. There were two main categories of codes: teacher moves and student moves. For example, one teacher move was “student-student,” which was used to identify instances where the teacher promoted one student's uptake of another student's idea (either responding to it, building on it, etc.). An example of a student move was “abort,” where a student made an initial effort to share an idea, but chose to terminate his or her participation in the particular exchange. Coding reports (which comprised all instances of a particular code) were then subjected to the constant comparative method to refine the codes and develop a summary report for each code. I then used these

reports to query each code's relationship to other codes and to refine my evolving understanding and hypotheses of the class's collaborative inquiry practices.

Drawing on this analysis and my earlier coding and analytic memos, I synthesized the findings by generating a conceptual map that related the demands of collaborative inquiry practices and the teacher's role in supporting students in meeting those demands. This also revealed the set of collaborative practices engaged by this Math A community. From this work, I inductively created preliminary categories for the teacher's role. Finally, I transcribed whole-class collaborative episodes from the coded lessons and two additional lessons that also evinced high levels of whole class collaborative inquiry to further refine and extend the analyses of student-teacher interactions and the teacher's role in supporting the class's collaboration.

*Phase 3.* To understand the development of practices over time, I relied more heavily on my fieldnotes and the video content logs, as these allowed me to access more readily a range of data across a larger expanse of time. Early in the school year, it became clear that the negotiation of learning practices and the meanings of various practices might play a central role in the development of this community's capacity to participate in collaborative inquiry mathematics. Student comments and participation indicated that they held interpretations and understandings of what it meant to do mathematics that were antithetical, in some cases, to the meanings the teacher held and hoped to promote among her students. I carefully attended to and documented these points of negotiation.

Looking across the two sets of lessons from the fall and spring also provided a comparison between the class early and late in the school year. These lessons were among the beginning lessons in the Patterns (fall) and Overland Trail (spring) units, which both focused on developing algebraic thinking and proficiency with symbolic representations. This choice of lessons provided some continuity of content supporting the comparative process. In addition, I conducted an intensive review of the content logs and videos of the first 6 weeks of lessons, and content logs and reflective notes for the first 12 weeks. My initial analysis focused on identifying efforts Ms. Nelson made to organize new forms of participation, including instances where Ms. Nelson made explicit her expectations, and students' responses to these efforts. These videos and content logs of sequential lessons were also analyzed to trace the development of mathematical ideas over time.

Concurrent with these activities, I analyzed students' interpretation of classroom events using student questionnaires, student interviews, fieldnotes and records of the joint video viewing sessions with students. The interviews had probed students' perceptions of mathematics, their participation in classroom

activities, and their views of normative practices. I wrote case reports on each student who was in the class for the majority of the school year. This process helped to document changes in students' perceptions and stances between the fall and the spring. I then worked comparatively across the interviews and case reports and developed themes that captured trends and key issues across the class members. These analyses of students' perceptions, practices, and moments of negotiation informed the development of the second model.

Throughout the analysis, I worked iteratively, moving among the data, developed codes, themes, and tentative hypotheses. I used two forms of triangulation to enhance validity. Data on the same phenomena were collected from multiple sources (e.g. teachers, students and myself, as participant observer) and using multiple methods (e.g. surveys, informal conversations, interviews, observations of practices). For example, in this research, the practices of the class were very important. The data I gathered on practices included direct observations of the practices; students' perspectives through interviews and questionnaires; and the teachers' perspectives through conversations after class, interviews, and reviewing videotapes. In conducting my analysis, then, I had the opportunity to check one data source against another, as well as perceptions from multiple parties involved. In addition, analyses were refined and validated by reviewing videotapes and sharing my analyses with Ms. Nelson. She confirmed that the results presented an accurate portrayal of her practice, both the description and how it functioned to support students' participation and learning in the classroom. Thus, the findings presented here are supported by a variety of data, which converged to produce a coherent understanding of the research setting.

Naturally, there are limitations to a case study design. The depth of analysis afforded by a single case necessarily means that resources were not devoted to analyzing other cases of teaching. Data collection in two other collaborative classrooms helped highlight important aspects of the teacher's practice and trained my eye on previously unnoticed features of classroom life. However, these other cases were not fully analyzed, which may have led to more refined analyses and augmented the validity of the results. Despite these limitations, the use of multiple data sources, processes of triangulation, respondent validation, and the depth of my involvement ensured ecological validity and produced robust results.

## RESULTS

In Ms. Nelson's class, students participated in a wide range of collaborative mathematics *learning practices* (Cohen & Ball, 2001) which provided them opportunities to develop their mathematical

understanding through inquiry and to participate in an array of mathematical practices (see Table 2). Several of these collaborative practices will be recognizable in later excerpts.

Table 2

Collaborative mathematical learning practices in Ms. Nelson's class

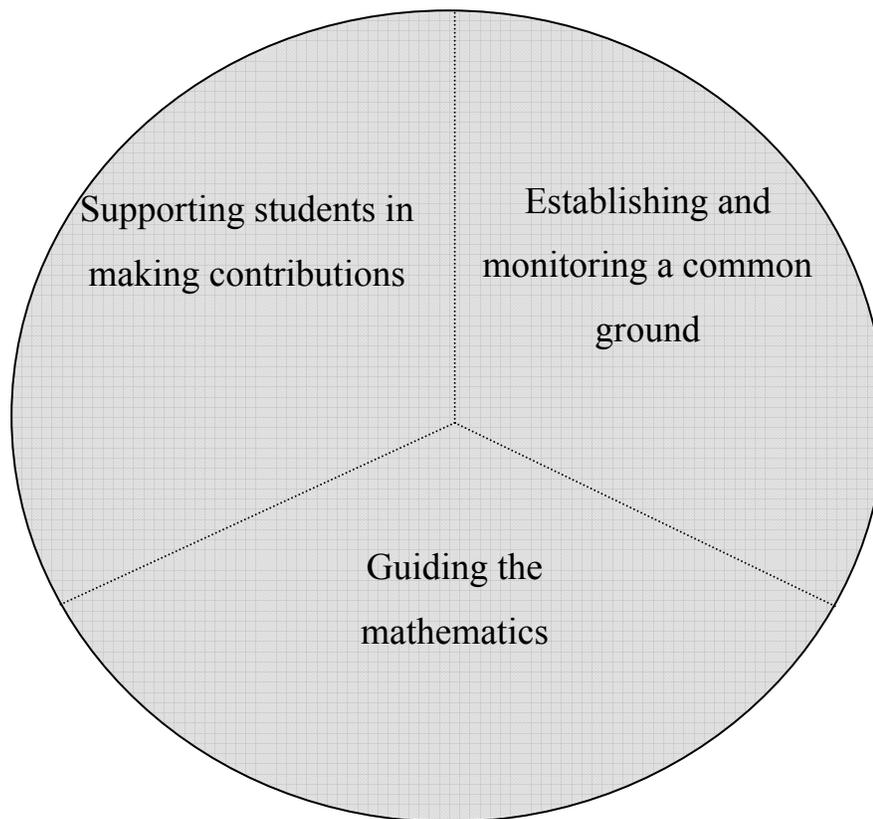
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- (Co)-constructing a line of reasoning or method of problem solving
- (Co)-constructing a representation
- Diagnosing and analyzing errors
- Explaining one's approach (including reflecting on decisions made during problem solving)
- Interpreting a problem; examining another's interpretation of a problem
- Justifying or proving results for others
- Juxtaposing or comparing strategies, approaches, and ways of understanding
- Reflecting on the nature of the problem; juxtaposing two or more problems
- Representing one's ideas for others
- Understanding and evaluating another's argument

This set is not exhaustive but represents regular patterns of practice organized in the Math A class. Learning communities evincing such practices are uncommon and these mathematics learning practices provide a beginning source. Other collaborative classrooms may evidence other practices. Ms. Nelson played a central role in developing and sustaining these practices. First, I present findings in relation to Ms. Nelson's role in supporting the community as they engaged in whole class collaborative inquiry. Next, I present a conceptual model that represents the process by which the development of the community's practices took place over time.

#### The Teacher's Role in Supporting Whole-class Collaborative Inquiry

From analysis and synthesis of the data sources, the teacher's role in supporting collaborative inquiry is conceptualized as a tripartite model comprising the following three components: *Supporting Students in Making Contributions*; *Establishing and Monitoring a Common Ground*; and *Guiding the Mathematics* (see Figure 1). The dotted lines indicate that these are analytic distinctions but the boundaries between these components are actually more blurred in practice.



*Figure 1:* The role of the teacher in supporting whole class collaborative inquiry

Ms. Nelson played a critical role in Supporting Students in Making Contributions, the first component of the model. Collaborative inquiry fundamentally relies upon students making their thinking public - their conjectures, their proposed next steps to problems, their ideas and justifications. Ms. Nelson had a range of instructional strategies by which she elicited from students their initial thinking and supported them as they represented their ideas to the class. This second component – Establishing and Monitoring a Common Ground - provided the basis for the students' and teacher's collaborative work. Ms. Nelson focused many of her efforts on setting up the exchange of ideas among students. She positioned students to work with one another, focused their collective attention, and made sure that they had access to the conversation by ensuring some shared conception of the problem they were working on. Without common ground, students would not have the opportunity to consider and respond to others' ideas in meaningful ways. The third component of the model - Guiding the Mathematics – is a shortened version of “fostering the development of the mathematical ideas during collaborative work.” This is not a surprising

aspect of Ms. Nelson's role, but the particular ways she attended to students' thinking and supported their mathematical inquiry were unusual. She centralized students' thinking while simultaneously organizing their work around core mathematical ideas and practices.

Detailed explication of these three components, along with specific instructional strategies corresponding to each component, are discussed in the remainder of this section (see Staples, 2004 for further analysis). Table 3 provides an overview. The three components of the teacher's role are listed at the top of the columns. Within each component are various *categories of work* (e.g., Eliciting student ideas, Creating a shared context). Within each of these are specific instructional strategies the teacher used to accomplish the category of work and enact the corresponding component of her role.

### *Supporting Students in Making Contributions*

Collaborative inquiry activities are built upon the exchange of ideas, as revealed by both the definition of the term and the set of collaborative inquiry practices engaged by this Math A class. Although norms can support student sharing of ideas to some degree, the teacher plays an ongoing role and has considerable influence over whether or not a student's idea is elicited and done so in a manner so that it can be taken up by the collective. To share and articulate their ideas, students need both socio-emotional and intellectual supports, or social and analytic scaffolds (Lampert, 2001; Nathan & Knuth, 2003; Staples & Hand, in preparation).

Three categories of work were identified in relation to this aspect of Ms. Nelson's role. These are *Eliciting student ideas*, *Scaffolding the production of student ideas*, and *Creating contributions*. My argument is that eliciting, scaffolding, and creating contributions are categories of work Ms. Nelson undertook to support students in making contributions which are necessary to foster the class's inquiry.

*Eliciting student ideas.* Ms. Nelson constantly made space for students' thinking by seeking their input, ideas, thoughts, comments and questions. The extent to which student thinking was incorporated into lessons was substantial and thought to be unusual. Many teachers use generic questions such as "Why?", "How do you know?", or "Who else has an idea?" These were also part of Ms. Nelson's repertoire. These *requests* open space for student ideas (both initial responses and explanations), but merely requesting information may be insufficient for students to share their thinking in a manner that supports collaborative inquiry (Kazemi & Stipek, 2001). Requests assume that a student has an idea that is reasonably well formulated, and, given the opportunity, can and would be willing to share her idea with others. However, the novice, who is simultaneously constructing her mathematical understanding, may need support articulating and publicly sharing her thinking.

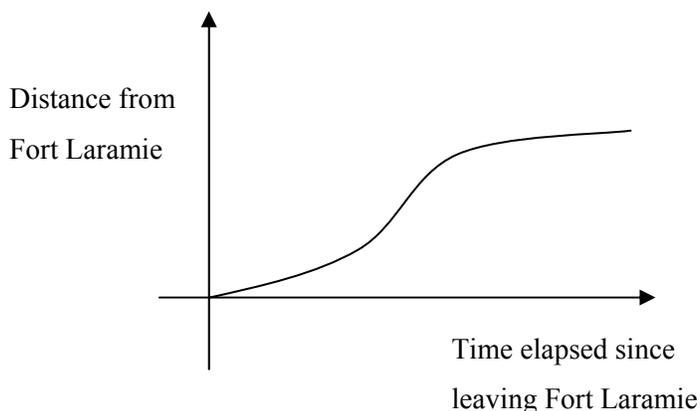
Table 3

Three components of Ms. Nelson's role and corresponding instructional strategies

Supporting Students in Making Contributions	Establishing and Monitoring a Common Ground	Guiding the Mathematics
<i>Eliciting student ideas</i>	<i>Creating a shared context</i>	<i>Guiding high-level task implementation</i>
Request & press	Establishing prerequisite concepts	Modifying tasks
Providing time	Verbally marking	Providing "food for thought"
Giving participation points	Affording multiple opportunities to access ideas	Ongoing assessing and diagnosing
<i>Scaffolding the production of student ideas</i>	<i>Maintaining continuity over time</i>	<i>Guiding with a developmental map of algebra learning</i>
Representing	Using the board over time	Attending to "pressure points"
Providing structure	Keeping the purpose salient	
Extending	Pursuing discrepancies	
	Grounding, then building	
<i>Creating contributions</i>	<i>Coordinating the collective</i>	<i>Guiding by following: "going with the kids"</i>
Expanding what counts	Positioning students for collective work	Flexibly following a student's thinking
Demonstrating the logic	Controlling the flow	Keeping students positioned as the thinkers and decision-makers
Linking		

One noticeable feature of Ms. Nelson's practice was her *press* and persistence in eliciting students' thinking, even when they were reluctant or showed signs of hesitancy. Her press was a well-balanced mixture of commitment and pressure, indicating to the student that she valued what he had to say while also indicating that he was expected to articulate or further explicate his thought. Encouragement and praise were often part of the way she supported the press situation. With her press and requests, her goal was to make students' thinking public, and she pushed until she was satisfied that both she and others in the class had access to the student's ideas. Access required enough detail and clarity so the idea subsequently could be commented on or built upon.

In this next excerpt, from a lesson in April, the class is trying to determine the point at which a wagon train is moving the fastest by considering a graph of its distance (from Ft. Laramie) verses time (days). (See Figure 2.) This was one of their first days working with graphs of situations and interpreting these graphs, Dontay claimed, from his seat, that somewhere in the middle of the graph is where it is moving fastest. Indeed, this is correct. Ms. Nelson follows up, trying to learn more about Dontay's thinking:



*Figure 2:* Graph of distance from Fort Laramie verses Time

- (1) EN: Somewhere in the middle you think it was moving fast? Why do you say that?
- (2) D: It's a fast looking line.
- (3) EN: What does a fast looking line look like, Dontay?
- (4) D: Like that.

(5) EN: Well, show, can you come up here and show me what you think was a fast looking line and slow looking line.

*He motions no.*

(6) EN: Come on, I'm really interested, come on, you can do it. I wanna see this fast looking line.

In his first two turns (lines 2 and 4), Dontay does not provide any additional information about his thinking, despite Ms. Nelson's requests. He has an idea, but there is no elaboration. His passive resistance notwithstanding, Ms. Nelson presses, expressing her interest in Dontay's thinking and her confidence in his ability to articulate it. After line 6, Dontay goes to the board to explain where he thinks the graph shows the wagon train was going the fastest.

(7) D: Right here, because, because it looks like they slow down here (*He points to the top part where the graph begins to level out*) cause it, yeah.

(8) EN: And how about the bottom.

(9) D: (I'm gonna sit down. (*He starts to sit.*))

(10) EN: Bupbupbup (*Ms. Nelson puts her hands on his shoulders and turns him back to face the graph.*)

(11) EN: Tell me about the bottom.

(12) D: Uh I don't know (inaudible)

(13) EN: Why, why, why, you know right here? (*She points to the bottom part of the graph, before it gets steep.*) ... Why wouldn't here be the fast looking line. Why are you picking here to be the fast looking line.

(14) D: Because it looks the fastest?

(15) EN: Well come on, tell me why.

(16) D: (*low laugh*)

(17) EN: What is a fast looking, you were saying it before. You said they look like they were slowing down here. What made you think they were slowing down.

This is fairly intense press on Dontay. He has an idea, and Ms. Nelson is questioning him and really pressing to uncover more of his reasoning behind his thought. Even after he offers some more of his thinking (line 7), Ms. Nelson still presses, even physically guiding him to turn back to the board (line 10). Clearly these interactions are supported by her relationship with Dontay and the class which allowed for this extended exchange. The exchange with Dontay concludes with his next turn where Dontay offers that they look like they are slowing down at the top because it looks like they are going “uphill.” (This is a common misconception.) At this point, Ms. Nelson seems to be satisfied that she (and the class) has gained some insights into his reasoning, and another student offers her idea.

There are two interesting points about Ms. Nelson’s requests and press worth noting which were characteristic of her interactions with students and seemed to help to make these exchanges with students successful (i.e., they were able to articulate their ideas further). First, Ms. Nelson consistently made requests, as she does here, that targeted the *student’s thinking*. In the above excerpt, she did not ask Dontay to show her where the wagon train *is* fastest and why. Rather he is being held accountable for sharing *his ideas* and his thinking and reasoning, and not necessarily for producing correct mathematical ideas and right answers. This perhaps lowered the risk of participation. It also centralized students’ ways of thinking and understanding as the focus of class discussions. Second, Ms. Nelson used a variety of strategies to leverage her way into a student’s thinking. Her requests and press did not involve simply repeating generic questions such as, “Why?” or, “How do you know?” although she used these prompts as well. For example, in line 7, Dontay reveals that he is distinguishing between faster and slower parts of the graph by comparing the middle of the graph (the fastest part) with the top of the graph (where it slows down). Ms. Nelson picks up on this, and she follows his thought trajectory and asks him about the bottom of the graph (line 13). This is a subtle but important aspect of her practice, whereby Ms. Nelson attunes her requests for elaboration with the student’s approach to the problem. This is related to her work scaffolding the production of ideas which is discussed in the next section.

It is important to note that Ms. Nelson continued her requests and press throughout the year. Although students were more comfortable sharing their ideas and participating publicly, it was often the case that Ms. Nelson worked with the student to learn more about his reasoning and to help him share and articulate important aspects of the student’s thinking with the class.

Other strategies Ms. Nelson used to elicit student ideas were *providing time* and *offering participation points*. Ms. Nelson provided students with ample time to formulate and articulate their thinking.

Like press, providing time demonstrated both a commitment and an expectation. It signaled that Ms. Nelson wanted students to seriously consider the question or idea, and that she recognized that thinking and articulating ideas took time. Particularly in a class of lower-attaining students, who were hesitant to participate or needed more time to consider the mathematics, offering time seemed to be a critical component in garnering their participation and ultimately hearing their ideas. Later excerpts highlight this approach.

Participation points were given for student contributions, including both going to the board as well as asking questions of others. Although some students showed little interest in their grades, this strategy seemed influential in garnering contributions. At times it seemed to give students an impetus or justification for their participation (e.g., “OK, I’m gonna get a participation point.”). This strategy encouraged broad-based participation and reinforced the idea that it was participating, and not correct responses, that were valued.

*Scaffolding the production of ideas.* An important part of Ms. Nelson’s success in eliciting students’ ideas was that she used a variety of instructional strategies to uncover their thinking and support them in expressing their ideas. I refer to these as scaffolds, using the term in a manner complementary to Vygotsky (1978) by considering the teacher a more experienced or capable other who assists students in a performance they would not have achieved on their own<sup>3</sup>.

Making use of representations (*representing*) was an important instructional strategy that supported students in sharing their ideas and making contributions. In the above excerpt, one scaffold Ms. Nelson offered Dontay was the use of the visual representation on the board (line 5), so that he might point or use the aid otherwise to communicate his ideas. Throughout the episode the availability of the graph on the board supports their interactions and Dontay’s public sharing of his ideas.

In scaffolding students’ sharing of ideas, Ms. Nelson often provided guidance for students’ work by asking them to consider a particular question or aspect of a problem. An important characteristic of these kinds of questions and prompts was that they provided structure to support the student in developing and/or articulating her idea, but did not overly constrain or override the student’s thinking. The questions and prompts were posed with careful attention to, and in response to, the student’s ideas and approach. I call this *providing structure* without constraining, as Ms. Nelson often guided in a manner that helped the student consider the broader goals they were trying to accomplish (the structure), but left the details of how to accomplish this goal to the student to work out (unconstrained). Thus she provided a kind of cognitive, structural support while maintaining space for the student to contribute and develop his or her ideas.

For example, in a lesson in September, Ken was asked to show how he knew that the perimeter of a trapezoid was 5, even though it had 4 sides. (The longer base was twice the length of the other three sides.) He had some trouble formulating the argument. His first explanation, which referenced the longer base, was “this side’s two for some reason.” Ms. Nelson asked if he could figure out the “some reason.” His second explanation was, “because that was what was written on the worksheet.” Ms. Nelson asked again if there were some way for him to demonstrate it, this time adding that perhaps he could do some measuring with the blocks. With this suggestion, he picked up two trapezoid blocks and considered what he might do. He then showed how two of the shorter sides were equivalent to the longer base.

*Creating contributions.* In some mathematics classes, the set of student comments that count as contributions comprises right answers and clear articulations of mathematical ideas. In Ms. Nelson’s class, however, she constantly took up a broader array of statements. A half-articulated thought, an idea, a question, a way of recording, and anything else that might further the class’s mathematical journey were regularly positioned as contributions (see also Staples & Hand, under review). Half-articulated and seemingly illogical ideas can be difficult to manage in a public forum, as can be mathematically incorrect answers (Ball, 1993; Chazan, 2000), in part because of the salience of right and wrong in mathematics and the stigma generally associated with being incorrect. Ms. Nelson encouraged students to share ideas, and emphasized that making errors was part of the learning process, with comments such as “Remember the idea is to go up and give us some good discussion ... that helps the class move along regardless of whether it’s right or wrong, it enables us to have good discussion.” Her practices went well beyond this, however, to fundamentally reshape what counted as a contribution. I discuss the important aspect more fully in the second section on the development of practices over time.

One strategy Ms. Nelson used to create contributions was to find and *demonstrate the logic* of a student’s thinking. Her assumption was that behind a partial or seemingly erroneous comment, there was a logical thought process that could be uncovered and used to advance the class’s inquiry. By using questioning and other techniques, she would dig underneath the surface to find sound mathematical thinking. For example, students were asked the meaning of  $C$  in the equation  $C + (C+20) + (C+40) = 90$  in relation to a particular word problem. At one point Lauren offered, “It could be change maybe.” This idea is incorrect, and in fact did not seem very reasonable ( $C$  represented “child’s age” for this problem). As with all teaching moments, Ms. Nelson had a choice whether to solicit additional ideas or follow up with Lauren.

EN: It could be change maybe... So you think maybe the  $C$  has something to do- Why did you pick the word change?

L: I don't know. Because it starts with a C.

EN: Because it starts with a C. And mathematicians like to pick variables that are easy for us to remember.... and if we pick C for change, or C for, what was her idea? Wendy's idea?

W & O: Child's age.

EN: Child's age, because that begins with C, then our brain connects a letter with a word and it helps us understand things better.

Although Lauren's idea that C stands for change is incorrect, there is a logic behind her suggestion. Ms. Nelson's follow-up question, "Why did you pick the word change?" was critical in uncovering Lauren's reasoning. Lauren's erroneous comment is positioned as a contribution as an appropriate way to reason about the potential meaning of a variable. Ms. Nelson then used the logic of Lauren's response to reflect on Wendy's (correct) response and reinforce the idea that often the choice of letter for the variable is made to indicate the quantity it represents.

Ms. Nelson did not shield students from articulating something that might not be complete or correct. Nor did she try to pursue an agenda of "protection" or "face saving" by manipulating public conversations to minimize attention to incorrect information or students' incomplete understandings. Although she recognized students might become uncomfortable, she prioritized uncovering their mathematical thinking, pressing them to articulate their ideas. This mode of teaching must be supported by norms of trust and mutual respect (Lampert, 2001). This trust evolved over time as Ms. Nelson repeatedly treated student ideas respectfully and as they experienced success contributing to class. The students' successes were a product of many factors: the openings created, the supports they received to take up those openings, and the ways Ms. Nelson interacted with students around their ideas to uncover their valuable thinking and create contributions.

#### *Establishing and Monitoring Common Ground*

A second component of Ms. Nelson's role in supporting collaborative activities was *establishing and monitoring a common ground*. Common ground (Clark, 1996) comprises the suppositions, ideas, and objects that participants take as mutually held or recognized. It is that which an individual *assumes others* hold in common with him. Common ground provides the basis of all joint activity (Clark, 1996), including collaboration, as it supports participants in coordinating their actions to advance the activity. In a mathematics class, common ground might comprise the algebra problem on the board and the question the teacher just posed, as well as mathematical terminology and class routines<sup>4</sup>. Common ground is a dynamic, fluid construct – it *accumulates* and changes as a joint activity advances over time.

From the analysis, two important characteristics of common ground emerged as critical in supporting collaborative inquiry in Ms. Nelson's classroom. First, Ms. Nelson attended to establishing and supporting a common ground shared *among students*. Second, Ms. Nelson organized common ground that significantly comprised *students' ideas*. These two characteristics are not required of more traditionally organized instruction, where the focus is establishing common ground between the teacher and individual students (Edwards & Mercer, 1987), and primarily comprises the teacher's (or textbook's) predetermined mathematical ideas and ways of understanding. Even in more student-centered classrooms, many teachers elicit students' ideas but may not attend to ensuring that the ideas become part of a common ground. These two characteristics, however, are necessary for collaborative inquiry where students must respond to and build on each other's contributions.

*Creating a shared context.* In order to establish a student's idea as part of common ground, the class needs reasonable opportunity to access and comprehend the ideas of others, despite differences in how they process information, their levels of attention, and their background knowledge. Towards this end Ms. Nelson organized classroom exchanges and activities so that students were regularly *afforded multiple opportunities to access ideas* and make sense of other students' ideas. There were two primary means by which this was accomplished: redundancy, or repetition (multiple statements) and the use of multiple modes of representing the ideas (e.g., auditory, visual symbolic, visual graphical). Visual representations were particularly important as they were an enduring record of students' ideas and the relationships among the ideas. Ms. Nelson also carefully organized board use, directing students to use particular portions of the board, or different colored markers, to distinguish their individual contributions from what was already there. This semi-permanent record tracked the development of the ideas as the lesson unfolded and provided strong evidence of what was part of the common ground (Clark, 1996). All of these strategies helped to align students' attention and recognize what comprised common ground.

*Maintaining continuity over time.* Students' continued collaboration depends upon the maintenance of a common ground over time. It is easy for students to become misaligned, which inhibits communication and may cause students to talk past one another. During collaborative work, ideas evolve over time and the focus of the conversation shifts, sometimes quite quickly. If the progression of the conversation becomes unclear, collaboration is compromised. Only a small number of students remains positioned to contribute and collaborate. Even if the result is subsequently explained, students will have lost the opportunity to consider some more subtle points that went into the *development* of the idea. The intellectual process of tracking the development of the ideas is an important component of mathematical thinking afforded by

collaborative inquiry. Clark (1996) refers to strategies or other supports that maintain common ground as *coordinating mechanisms*. The strategies discussed above that create a shared context, particularly the board use, also serve as coordinating mechanisms.

To maintain continuity over time, Ms. Nelson also proactively monitored the common ground, eliciting evidence of alignment and *pursuing discrepancies* and any indicators of misalignment. Often teachers gloss over indications that students do not fully understand or are not following a line of reasoning. Edwards and Mercer (1987) attribute this to practicalities of joint action in a classroom that get prioritized in an effort to sustain the teacher's agenda for advancing the activity (p. 119). Rather, Ms. Nelson would ask a follow up question or organize some further work to ensure that students knew what ideas or questions they were working on and understood any ideas that had been made public. Naturally, teachers are always working with incomplete information and it is not possible to have complete alignment at any point in time.

Emphasizing the *purpose* of the class's current work or particular student's contribution was another means Ms. Nelson used to help maintain continuity of common ground. For example, Ms. Nelson often noted the purpose of the student's upcoming participation: "Come up please. Ron says that there are more diagonal lines. That Oscar didn't put enough in." With this simple phrase, she indicates what they are working on (finding all the diagonals) and what Ron's goal is (to add more, as he thinks Oscar's work is incomplete). With a clear sense of the current activities, students are better able to reflect on and evaluate how Ron's participation advances their work towards their goal and to contribute to the next steps if the goal is not met. Similarly, Ms. Nelson regularly highlighted for the class what they had figured out and what the current state of their inquiry was. These pedagogical moves, which kept the purpose salient, helped students make sense of a new contribution and created a link between what was about to take place and the work they had already done. Furthermore, these moves aligned students' attention on the same issues and concentrated their efforts towards a common goal.

*Grounding, then building* was another important instructional strategy Ms. Nelson used regularly throughout lessons to maintain continuity and support whole-class discussions. With this strategy, she would *ground* a contribution before having students do further intellectual work on the idea. (See also Clark, 1996, Chapter 8.) Ms. Nelson did not assume that students readily understood a new proposal, idea or question. Rather, she recognized that students needed an opportunity to become familiar with the idea or issue at hand before they could contribute to its development or resolution. Consequently, she created opportunities for students to make sense of, and gain clarity on, a new proposal or another student's work

(*grounding*) before asking them to take it up for active query and consideration (*building*). The opportunities were more extensive than just hearing the idea once or twice, or seeing a diagram on the board.

As an example, in this excerpt, Emily had gone to the board to complete a problem with an In-Out chart, filling in values and finding a rule for the function. The bolded entries are Emily's responses; the other entries and text were given.

IN	OUT
3	8
6	4
2	12
<b>1</b>	24
.5	<b>48</b>
n	<b><math>n \div 24</math></b>

**Rule:  $\mathbf{In \times Out = 24}$**

*Figure 3:* In-Out chart with Emily's work. Note that the numerical values and rule are correct. For  $IN = n$ , however, the out should be  $24 \div n$  (or  $24/n$ ) and not  $n \div 24$ .

Ms. Nelson begins by asking Emily about how she approached the problem. There is no comment from Ms. Nelson or any student about the correctness of the work. Emily talks about how her dad helped her, which Ms. Nelson praises as a good strategy. She then asks Emily to explain more about her thinking and how she and her dad figured out the problem. Emily seems to find this difficult to articulate.

(1) E: I described [to my dad] what we had been doing in class, and, just like, kind of like (doing) something to number 2, 4, number 8

...

(3) E: and then somehow I was looking at it and then it just, just like clicked. It was like, 3 times 8 is 24, and 6 times 4 is 24.

(4) EN: So how did you do that .5 one. Cause I stuck that in there to trick you guys a little. What did you do to decide what number went with .5? Do you remember, how you figured out it was 48?

Ms. Nelson directly probes Emily's thinking about how she produced the 48. She is trying to elicit more about her thinking and also potentially uncover what led her to write  $n/24$  instead of  $24/n$ . This probe

simultaneously draws the collective's attention to this input-output pair of Emily's work. This is the beginning work of grounding, which focuses the students on a piece of mathematical work that may be important to consider as they wrestle with formulating the generalization of this rule.

Ms. Nelson's question to Emily does not elicit much more information about her thinking however. Ms. Nelson then continues grounding by asking other students about their approaches.

(5) E: No, we were just thinking about it and it's like, oh, that works. My dad told me but I can't exactly remember what

*Ms. Nelson turns to the class.*

(6) EN: So, Mary Jane, when you did .5, how did you know it was 48?

(7) M: I divided it into 24, and then

(8) EN: Did you divide it, [M: yeah] like dividing, like long division?

(9) M: Yeah

(10) EN: So you went ahead and did long division, and that's a very good method to do.

At this point, Ms. Nelson has attended to Emily's ideas and created the opportunity for the class to understand her work and approach to the problem. (She has found the limits to Emily's explanation of her thinking (line 5) as well.) She also has gathered information about Mary Jane's thinking (line 7) and a third student's thinking (not in the dialogue). Finally, she has drawn students' attention to, the use of division in solving this problem (line 10), which is operation they will need to write an explicit rule for this function. This set of exchanges has focused the class's attention on a particular piece of the mathematics – here the decimal values and the idea of division—thus grounding these ideas. Previously only the operation of multiplication had been made explicit.

As the students seem to remain unaware of the error, Ms. Nelson raises the issue that something in Emily's solution needs to be "tweaked slightly."

(11) Does anybody see that there's something that's just, tweaked slightly there, that should be changed? Mary Jane?

(12) M: Oh, tweaked slightly? That should be changed?

(13) EN: That, that should be changed up there.

(14) M: No

(15) EN: Does everybody agree that everything she has there is correct, or does anybody see that there's a mistake somewhere.

No one seems to recognize the error, but the grounding has set the stage for their building work to figure out what is incorrect, namely  $n/24$  as the OUT corresponding to the input  $n$ . Notice also that by grounding first, Ms. Nelson has positioned Emily's solution and thinking as a contribution to the class, despite the fact that it was not entirely mathematically correct. It is now the focus of their collective intellectual work.

As the lesson continues, Jay suggests that it needs to be  $24/n$ , and Ms. Nelson presses him to articulate a reason. Subsequently, Jay, Mary Jane and Emily co-construct (with Ms. Nelson's support) an argument as to why  $n/24$  does not make sense. Part of their argument is that  $n/24$  would produce decimal values for the OUT values given the IN values they have (e.g.,  $3/24$  is a decimal), so  $n/24$  cannot be correct. This argument seems to pick up on Ms. Nelson's questioning about the .5 and 48. Thus students are building upon the common ground that had been established.

(19) J: oh, um, (*pause*) [it's gotta be 24 divided by n

(20) M: [In times (*inaudible*)

(21) EN: it has to be 24 *divided* by n, why is that? That's very nice Jay. Why?

(22) J: uh because, if you (*pause*) Huh. Uh, god. My brain isn't moving OK.

(23) EN: Now your brain isn't movin' because you had that soda there.

(24) J: No it's not! *Laughter* It's not the soda.

*(There are some brief exchanges around eating breakfast and sugar)*

(33) J: 'cause it just doesn't work

(34) EN: Why doesn't it work Jay? *EN moves to the board*

(35) J: 'cause if you divide IN into 24,-

(36) M: you get (*inaudible*) -

(37) J: yeah, you get point, you get like point something

(38) E: oh yeah, because 24 divided by, the numbers in here (*points to the set of "Ins"*) could be smaller than 24, and then you'd probably get negative numbers or something

(39) EN: You wouldn't get a negative number, you'd get a - ?

(40) J: (*inaudible*)

(41) M: a decimal

(42) EN: a decimal, very nice, you'd get a decimal number, and that's not what these numbers were over here. These were regular numbers. That is EXCELLENT. I like how you guys explained that, and Emily did a lovely job. Very nice.

One typical pattern evidenced in grounding excerpts is that Ms. Nelson first gains clarity on the idea(s) presented and student's thinking in relation to these ideas. If incorrect, Ms. Nelson then focuses them on thinking why it is *not* correct and then pursues how to rework it so that it *is* correct. Ms. Nelson at times would explicitly direct the class's attention to the difficulty, insisting that the next contributor tell her what was incorrect and not what was correct or what he had as a result. This treatment of a problem moves the class in a reasonable progression from the particular student's (or students') work, to identifying and analyzing the mathematics in the problem, to addressing and building on it. Even though the idea in the initial contribution is not correct or complete, Ms. Nelson maintains continuity of their inquiry by working off of what the student presented.

*Coordinating the collective.* The discussion thus far has focused on coordinating mechanisms that helped keep students aligned by maintaining a shared intellectual context with respect to the mathematics they are working on. Other coordinating mechanisms attended to the more social components of joint action. These strategies helped align student participation structures with the activity of collaborative inquiry. Next, I discuss two strategies related to coordinating the collective: *positioning students to work collectively* and *controlling the flow*.

Ms. Nelson played a prominent role in orchestrating discussions. She positioned students (Harré and van Langenhove, 1999) for collective work by having them attend to and respond to one another's ideas, thereby establishing certain ideas as part of the common ground and organizing their exchanges directly in support of collaborative inquiry. One way Ms. Nelson accomplished this was with verbal statements that set up the possibility(s) for the next interaction by explicitly encouraging continued attention to a particular idea. For example, in one lesson, Jay was having some difficulty fully articulating a justification. Ken raised his hand and Ms. Nelson acknowledged his bid for the floor: "OK, do you wanna explain some more Ken? Or do you have a question for Jay?" In her invitation for Ken to contribute, Ms. Nelson organized two options: Ken could explain more, or ask Jay a question. Both presume that Ken has heard and understood (to some extent) what Jay has said and encourage Ken to extend or build upon Jay's thinking. Other phrases Ms. Nelson used included, "Who can add to what she's saying?", "Who can try to say what she's thinking?" and "Don't tell me what's right. Tell me what's wrong". Each of these positions the current idea for continued work by the collective by inviting a particular kind of participation from a student, and thus is also contributes to maintaining continuity over time.

Ms. Nelson also actively controlled the flow of information, sometimes intervening to allow some time to linger on an idea, record it, or to allow students to process what was said. This often required

socially managing turn-taking during class. Both of these strategies also served as a coordinating mechanism with respect to the ideas to help maintain the continuity in the discussion.

In this next lesson segment, Ms. Nelson contributions are bolded to demonstrate her work establish and maintain common ground. Ron was using a method Frank developed to generate a set of ages for three family members. In this episode I highlight Ms. Nelson's work positioning students to work collectively and controlling the flow of ideas. Ron's new set of ages (13, 27, 50) however did not satisfy all the problem constraints. Although the ages added to 90 (constraint 1), the family members were not of different generations, which meant at least 15 years apart in age (constraint 2). The excerpt begins with Mary Jane noting that the ages do not satisfy the constraints. Oscar and Halo contribute as well.

(1) M: It doesn't work. There's only 14 years (between 13 and 27)

...

(2) R: What are you talking about

(3) H: The kids have to be at least 15 to have a kid

(4) R: Oh, they have to be 15? I thought they had to be less than 15

(5) O: Yeah, kids that are 14 just don't have babies Ron

**(6) EN: To Ron I think you're misunderstanding what she's saying. To class Is there anything wrong with a 13 year-old? Can we not be 13?**

(7) M: No, you can be 13.

**(8) EN: You can be 13, but what she means is. Where's the problem Halo?**

(9) H: Because it's not a generation, well

**(10) EN: Between which two isn't a generation**

(11) H and O: Between the 13 and 27

(12) M: *to Ron* Just (make that) 49 and make that (28)

*Some low chatter*

**(13) EN: And why is that happening Ron?**

(14) R: I don't know. ... Wait, she is too young to have that one? *He's pointing the 27 and 13, referencing the mother and the child.*

**(15) EN: That's what Halo's saying**

(16) R: It'd be 26. She was (*inaudible*) ...

(17) H: It's gotta be like 15, in between *She motions with her hands*

**(18) EN: So use Frank's strategy again.**

Ms. Nelson first contributes, in line 6, to control the flow of information, which creates an opportunity for Ron and the class to understand what Halo and Mary Jane have said, as Ms. Nelson does not think that Ron has understood her. Furthermore, she asks the class if there is anything wrong with being a 13 year-old, thus focusing their collective attention on the mathematical issue germane to their discussion. After she obtains this clarification from the class, she turns the conversation back over to Halo to directly address Ron (line 8), repositioning them to work collectively. Line 10 again focuses the class on a particular piece of mathematics, a move to gain clarity and in line 13, Ms. Nelson simultaneously assesses Ron's understanding and gives him space to process the information, again slowing the progression of ideas and creating space for Ron to think (controlling the flow). This moment for reflection seems important in helping him process the implications of Halo's comment for his solution (line 14). Ms. Nelson affirms Ron's understanding of Halo's comment (line 15). Subsequently, Ms. Nelson positions Ron's continued work in relation to Frank's idea (line 18), a building move, to further support collaborative inquiry.

Importantly, throughout this segment, Ms. Nelson does not generate any of the mathematical ideas. She even seems to catch herself in line 8, perhaps as she is about to explain Halo's comment, and instead positions Halo to contribute by restating her idea. The students are constantly positioned as the knowers and doers of mathematics. Ms. Nelson positions herself in a supportive role by intervening to facilitate the exchange of ideas and students' thinking. We can also see in this segment Ms. Nelson's work to guide the mathematics. Her insertions keep certain pieces of mathematics central to the conversation, as will be explore in more detail in the next section.

### *Guiding the Mathematics*

A third component of Ms. Nelson's role during collaborative inquiry activities is guiding the mathematics. Attending to the development of mathematical ideas during lessons is a critical component of the teacher's role. The teacher is no longer in a position to transmit mathematical understanding and yet maintains responsibility for supporting students in developing proficiency within various domains of the discipline.

To understand how Ms. Nelson guided the development of mathematical ideas during whole-class discussions, a conceptual scheme was developed that frames her work as simultaneously guiding by maintaining high-level task implementation; guiding with a "map" of students' algebra learning; and guiding by following—"going with the kids." Ms. Nelson consistently acted deliberately towards each of these three ends. This was evidenced in her questioning, the ways she organized students' interactions, how she responded to students' comments, and the examples she chose. Each of these three aspects of her role

operates on a different time scale—students' ideas are worked with in the moment; a task is implemented over a lesson; and the development of students' proficiency with mathematics occurs over a larger time frame, such as a series of connected lessons, a unit, or the course. Yet, each of these had a presence in any given moment: as Ms. Nelson worked with students' ideas she attended to advancing their progress on the task and guided them towards productive engagement with the practices and concepts of mathematics.

*Implementing tasks at a high-level of cognitive demand.* A significant dimension of Ms. Nelson's role in guiding the mathematics was selecting and implementing tasks at a high level of *cognitive demand* (Stein, Grover, & Henningsen, 1996). Ms. Nelson successfully guided the task by keeping students positioned as the mathematical thinkers and by skillfully supporting and scaffolding their work, not taking away the opportunity to construct and contribute ideas. Stein and her colleagues (Stein et al., 1996; Stein, Smith, Henningsen & Silver, 2000) have demonstrated that tasks initially set at a high level of cognitive demand, which lend themselves to collaborative inquiry, likely decline in demand during implementation as the challenging aspects of the task are removed or as a lack of direction or focus leads to unsystematic exploration. Teachers seem to take over the challenging aspects when there is evidence that students have stopped making progress or when they seem to be struggling with the ideas, potentially showing signs of frustration.

Ms. Nelson supported and sustained students' work on tasks, even when they experienced some difficulty, with a variety of instructional strategies. One such strategy was to *provide students with "food for thought."* Ms. Nelson emphasized some aspect of the work they had done, raised a new perspective, or highlighted the intellectual issue with which they were wrestling. These means of "food for thought" served as scaffolds that generally did not reshape the task in a significant way. For example, Ms. Nelson would recount their intellectual journey, describing how they came to their current point in the problem solving process. She often used this approach when the class seemed to no longer be making progress. It generally prompted students to generate some new thoughts, even if very partial, which Ms. Nelson could then extend.

Alternately, Ms. Nelson offered or requested new representations as a means of providing food for thought to help students think about the mathematics from another perspective. For example, in a lesson in January, students were finding it challenging to make sense of why the number of arrangements of  $n$  different flavors of ice cream on a cone was  $n!$ , a conjecture they had empirically derived. As she guided their work, Ms. Nelson focused their attention first on a table of values, then on the diagrams they had drawn for the particular solutions for 2- and 3-scoop cones to the pictures, and then finally turned their

attention to the situation, elaborating on the word problem and creating a scenario where she visited an ice cream shop and ordered a triple-decker cone. This work – perhaps most significantly their thinking about the situation and how a cone would actually be produced (a less static representation) - seemed to enable them to finally connect the factors producing the number of arrangements of scoops ( $1 \times 2 \times 3 \dots \times n$ ) with the number of scoops ( $n$ ). Despite students' lack of early progress, Ms. Nelson had persisted, moving through several representations. In this manner she sustained the students' thinking and did not take over the demanding aspects of the task.

An overarching theme in Ms. Nelson's work was that she fostered students' thinking *about* the problem and not their progression towards task completion, by offering hints, next steps or other information. Her interactions in this role are similar to those described by Lobato et al. (2005) in their reformulation of telling that explicates how teachers' "telling moves" can support students' construction of mathematical ideas, as opposed to close off or overly direct the students' thinking. Behind this approach was Ms. Nelson's stance that students *can* do the math. When students experienced difficulty, Ms. Nelson interpreted this to mean that they had not yet had enough experience with the problem, or perhaps were not clear on the parameters of the problem. She did not take that to mean that they needed the mathematics explained to them.

*Ongoing assessing and diagnosing* of students' understanding was another key practice Ms. Nelson used to implement tasks at a high level of cognitive demand. She required high standards of evidence before concluding that a student understood an idea, which produced a level of rigor in discussions, as students were required to fully articulate their reasoning. Her ongoing assessing also involved diagnosing any underlying difficulties that students were having, which she used to guide the subsequent discussion (or a future discussion).

The excerpt with Emily and the In-Out exercise demonstrates Ms. Nelson's ongoing assessing and diagnosing. In that episode, after hearing Emily's initial response, Ms. Nelson organized a discussion to explore Emily's and others' thinking about the problem. She did not immediately focus on Emily's error. Rather, Ms. Nelson asked Emily to explain her work, and then asked Emily and others for the strategies they used to generate values, often inquiring specifically about the ordered pair (.5, 48). Emily's responses revealed that she had considered the multiplicative relationship ( $\text{In} \times \text{Out} = 24$ ) and determined the missing values using this relationship, but her thinking did not flexibly include the inverse operation (e.g., conceptualizing 8 as  $24/3$  or  $\text{OUT} = 24/\text{IN}$ ). Other students had used the division operation. However, no one (at least initially) noticed that  $n \div 24$  (or  $n/24$ ) was incorrect, perhaps because of the newness of

variables and the use of variables in expressing general relationship. It seemed that some students could not yet proficiently interpret the meaning of this expression. With this assessed information, Ms. Nelson then pressed them to consider what might “be tweaked” which then led them to be able to formulate the correct representation and an argument for why the original generalization was incorrect. Ms. Nelson guided the discussion to ensure that they focused on making sense of the meaning of  $24/n$  versus  $n/24$ .

It is revealing to contrast Ms. Nelson’s explorations of students’ thinking with what Ms. Nelson did *not* do during this episode. First, she did not treat the problem as routine. By examining Emily’s thinking, she raised the demand of a potentially lower-level task that might only have focused on the correctness of the solution. Second, she did not treat the incorrect expression  $n \div 24$  cursorily as a typo, which would have been reasonable, given that the rest of Emily’s work was correct. Both of these moves would have closed space for collaboration and engaged students in different, less demanding, mathematical thinking.

*Guiding with a map of students’ algebra learning.* In analyzing the ways Ms. Nelson guided tasks and extended students’ thinking beyond their initial responses to the task, it became clear that Ms. Nelson was acting on a particular understanding of how students develop competence with algebra. These understandings of how students learn algebra were evident in her interactions with students both during discussions and at other times. She saw today’s task and ideas in relation to tomorrow’s and next week’s.. To capture and describe this aspect of Ms. Nelson’s teaching, I developed what I refer to as Ms. Nelson’s *map of students’ algebra learning*.

To illustrate this aspect of Ms. Nelson’s teaching consider the topic of patterns and functions, which was the primary focus during the first 12 weeks of school. One learning goal was for students to recognize a pattern in a table of values and represent that relationship using symbolic notation, ultimately as  $y = f(x)$ . Over these weeks of instruction, Ms. Nelson guided the students towards increased proficiency. During lessons, Ms. Nelson requested, pushed for, or pursued specific ways of representing and thinking that seemed tied to how she understood students to develop proficiency with algebra over time. These *pressure points* corresponded to transitions between ways of reasoning that Ms. Nelson seemed to identify as critical steps for students as they developed proficiency with various topics. Figure 4 represents this trajectory with six pressure points that were revealed during lessons.

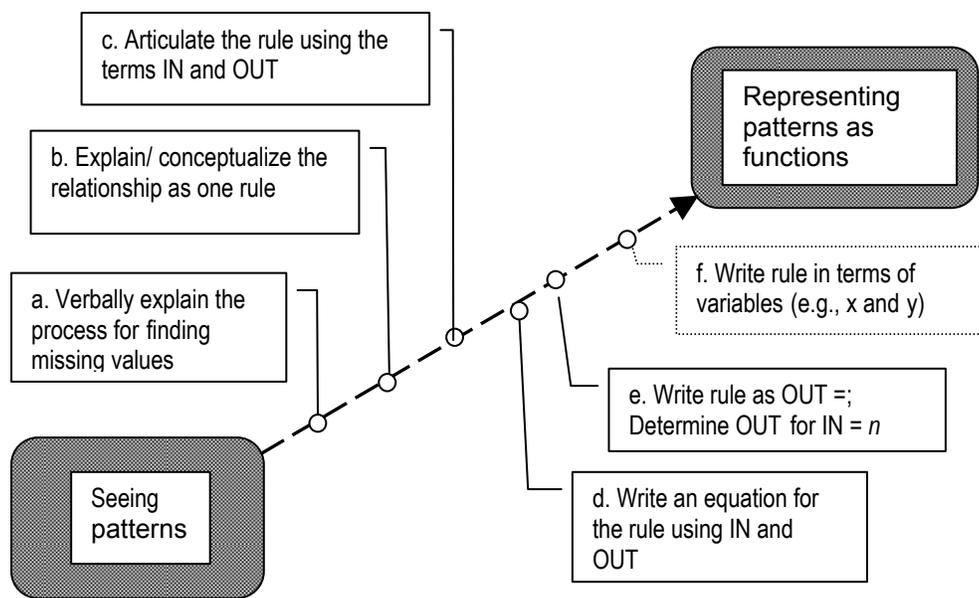


Figure 4: Map of students' algebra learning

An example will help clarify. In explaining how he found the numbers he used to correctly complete an IN-OUT table (see Figure 5), Kurt stated: "Multiply the bottom by 7 if the top is missing, and if the bottom's missing, divide the top by 7." This is correct.

Human yrs	2	4	5		6
Dog yrs	14		35	70	

Figure 5: Horizontal In-Out chart

Kurt saw a valid relationship and articulated it as two (equivalent) rules with a general reference to the "top" and "bottom." Although correct, Ms. Nelson pressed on his thinking, asking him to articulate "one rule" that worked for all the missing values (pressure point b), and to then use the words IN and OUT (pressure point c) to explain. In this manner, she was moving him towards new ways of thinking, namely, to conceptualize a single relationship (which encompass the two he identified) and to do so using the abstracted entities IN and OUT.

Similarly, later in the unit, Ms. Nelson explained to students that a rule such as  $IN = OUT \times 7$  or  $IN + OUT = 18$  is correct, but that the "best way" is to write " $OUT =$ ". She continued to honor students' present approaches that used the implicit rule, which reflected the patterns students saw in the data tables, but also

pressed them to recast the relationship they saw in a manner more closely aligned with how explicit functions are conceptualized and represented algebraically. The difference between conceptualizing a pattern as  $IN = OUT \times 7$  and  $OUT = 7 \div IN$ , while trivial for those well versed in mathematics, is significant for students. (The same issue also arose in the previous excerpt with Emily.) Ms. Nelson recognized the different demands and the different thinking processes behind these conceptualizations, and carefully attended to these as she guided students' towards an increased proficiency with the notational and reasoning practices of algebra.

The depth and breadth of Ms. Nelson's knowledge of students' thinking and mathematics was quite remarkable. Ms. Nelson's developmental map of students' algebra learning is a particularly interesting representation of her knowledge. It lies at the intersection of understanding what is important mathematically (algebraically), and how students come to understand and use this important body of mathematics. Significantly, it is developmental, not static, capturing a progression of refinement of students' thinking that might plausibly be traversed as they increase their proficiency. The understandings and ways of thinking represented in this developmental map are similar in some respects to what Ma (1999) has called *profound understanding of fundamental mathematics* (PUFM). The teachers in her study with PUFM had an intricate conceptualization of the relationships between and among mathematical ideas that were informed by students' thinking as well as a developmental sequence of how students come to understand and learn various topics in arithmetic. Ms. Nelson's map also evinces a deep understanding of algebra and is organized based on how students learn and come to know<sup>5</sup>. Some of what is included in this map is not even standard mathematics. For example, the idea of representing a pattern as an equation using the terms IN and OUT (e.g.,  $IN + OUT = 18$ ) is not particularly un-mathematical, but it is not a topic covered in a standard algebra textbook. Rather, it is a step in students' thinking as they refine their mathematical tool use and ability to conceptualize functions. In her practice, Ms. Nelson guided the mathematical discussions by attending to these pressure points which seemed to push students into new territory, where they had to work out new meanings and make sense of their, or their peers', current thinking in a new way.

*Guiding by following* – “going with the kids.” Limiting the discussion to the above two categories overlooks Ms. Nelson's work guiding by following, and “going with the kids.” This was a phrase she used to describe her teaching. She had learned to “let go” of her agenda and follow the students' thinking, trusting that a valuable point would arise or that she would be led to the ideas the students needed to work on. Indeed, aspects of her teaching described in earlier sections were in service of this goal. This also required

her to constantly position students as the thinkers and mathematical decision makers. Ms. Nelson attended to this in some important and subtle ways.

For example, during a lesson where students were determining the number of diagonals of an  $n$ -sided polygon, Ms. Nelson needed to help a student, Ike, draw the diagonals of a large octagon on the board using a meter stick. Initially, she suggested that he draw and she hold the ruler, but then changed her mind and had him hold the ruler remarking “Then I know which ones you’re connecting.” Whereas holding the ruler seems like the less active role for Ike, it was placing the ruler for the diagonal that required the mathematical thinking. Thus having Ike position the ruler gave her insight into his thinking about constructing diagonals (e.g., whether he had a systematic way for determining which diagonal to draw next, or how to determine if the diagram was complete). Notice her work in assessing and diagnosing is present here as well. This seemingly small decision about whether to draw or hold the ruler allowed her to keep “going with the kids” and provided her with ongoing access to their thinking.

Ms. Nelson’s ways of guiding the mathematics for collaborative whole-class discussion produced lessons that had an emergent quality. The overarching goals of task implementation and learning algebra guided much of their work on a broad scale, but the particular path they took – the arguments constructed, the ways of reasoning, the emergence of systematic ways to explore patterns—was shaped by students’ ideas and thinking. Furthermore the aimed-for destination was open to revision, based upon what Ms. Nelson learned about the students’ thinking as the lesson unfolded.

#### *The Development of a Community of Collaborative Learners*

I now turn attention to the second research question: What is the process by which these collaborative learning practices develop over time? The ways the Math A students worked together on mathematics looked markedly different in April than September. In this section, I explore how it was that the students came to participate differently in doing mathematics together.

#### *The development of practices over time*

In trying to establish a new culture and classroom community in this Math A class, Ms. Nelson faced significant challenges. The students had experienced traditional models of teaching and learning in middle school, and many reported that they did not enjoy mathematics nor math class. Many students also did not see themselves as competent mathematically, which also creates challenges for participation, particularly in situations where notions of “being right” and “being wrong” are salient.

Sociocultural theories, including the literature on communities of practice, offer little guidance in understanding how these practices of inquiry and collaboration around mathematics develop over time

(Cobb, 2000). These theories tend to assume stability in the practices of a community. The processes of change described by Wenger (1998, Lave & Wenger, 1991) generally focus on a shift in an individual's participation—essentially the process of enculturating or apprenticing an individual into a well established community of practice. This is not an option for the Math A community's members. The target practices are not part of the community's shared repertoire (at least initially) so the students do not experience them directly nor peripherally. In fact, it is likely that only the teacher has been part of such a collaborative learning community before. While these theories acknowledge that individuals shape culture and influence the community in addition to being shaped by it, they do not attend significantly to how an individual can shape culture, particularly in a deliberate manner, nor how a community ultimately generates new practices that appear quite distinct from former ones.

Wenger (1998) discusses the stability of practices with implications for how they might change over time. He asserts that practices are always being recreated and renegotiated, whether or not they are perceived as stable or changing. Perceived stability results when people constantly re-negotiate the meaning of that practice in a manner that is similar to how it had been previously. Likewise, perceived change results when people re-negotiate the meaning of a practice and enact it in a way that differs from the past. Wenger notes that "stability and destabilization can occur, but they cannot be assumed. They must be explained" (1998, p. 97). This draws attention to the fact that there may be more similarities between how practices change and how practices perpetuate themselves: both stability and change may be products of the same mechanisms.

Turning to the educational research literature, some instructional strategies have been identified that support the establishment or enactment of inquiry practices, such as explicitly stating expectations and modeling practices (Lampert, 2001; Wood, 1999) as well as assigning roles (Herrenkohl & Wertsch, 1999). These strategies invite new forms of participation and implicitly ascribe meaning to them. Several research studies document that these new forms of participation are taken up by students and influence on how the community learns together. For example, Wood (1999) describes how the teacher of a second-grade class explicitly modeled how to productively disagree with others' answers and reinforced this practice during classroom activities. Teaching students how to disagree seems to be an important component of supporting collaborative practices, as students are inevitably in situations where they need to manage discrepant answers in an intellectually and socially productive way.

The development of complex learning practices and new roles by students however is not always straightforward. Students do not always respond to meet stated expectations and adopt new roles and

practices unproblematically (Chazan, 2000; Corbett & Wilson, 1995; Heaton, 2000). Challenges arise as students are not fully aware of the shift required for successful engagement in these practices, or may not be comfortable or willing to participate in such a manner. Furthermore, the students may not have had the opportunity to develop proficiency with the various requisite practices, and thus, even if inclined, may lack facility with the desired forms of participation. In her description of her attempts to organize collaboration among her fourth-grade class, Heaton (2000) discusses how she could open up the class to allow for mathematical discussion, but did not have tools for engaging all students with the activity, in part because of students' resistance. Thus while explicitly stating expectations, modeling practices and assigning roles can facilitate collaborative learning practices, they do not comprise sufficient conditions for collaboration across all classrooms.

In her book *Teaching with Problems and the Problems of Teaching*, Lampert (2001) addresses issues related to student participation in novel practices from a different perspective. Like others, Lampert noted the importance of stating expectations explicitly and modeling, but she also emphasized the importance of generating *meaning* of those expectations with the students. She recognized that certain practices, such as revising, might be unfamiliar to her students. She did not expect the practices to be readily adopted, as students might not understand how the practice related to their learning of mathematics.

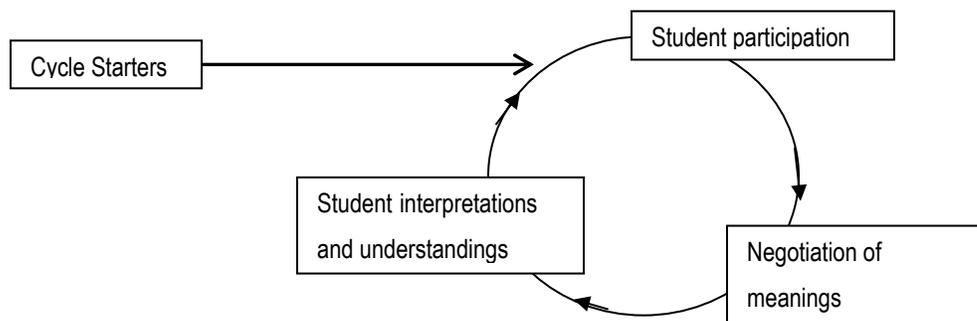
Introducing revising into the study of mathematics would require a change in how I imagined students would typically think about what one does to study mathematics. It would probably also require some changes in what they thought about the roles of "smart" and less smart classmates and about how to interact with them. I did not expect that my students would come to fifth grade knowing how to evaluate their own assertions or those of their peers in order to decide whether or not such assertions needed revising. Nor did I expect that they would see such evaluation and revision as activities that would contribute to their learning. (Lampert, 2001, p. 65)

In considering this specific case of introducing the new practice of revising<sup>6</sup>, Lampert notes how this practice might run counter to fifth-graders' understanding of how to do mathematics and some of their beliefs about others in the class. She points to the importance of students' understandings of the meaning of the practice of revising for their participation in this practice in a manner that supports their mathematics learning. Establishing the practice of revising in her classroom would not be automatic, and would require her to discuss and work on this practice with her students on multiple occasions.

The high school setting may add to the challenges of developing a complex set of learning practices, such as collaborative inquiry, among a group of students. High school classes meet for a limited amount of time each day, and students have much longer histories in math classes and perhaps more firmly established perceptions of themselves, their relationship to mathematics, and the nature of the activities in mathematics classrooms. Students' senses of selves and peer group affiliations can also affect the classroom social dynamics and potentially impede the exchange of ideas and students' willingness to engage in such collaboration (Chazan, 2000).

### *The model*

The process of the development of collaborative inquiry practices in Ms. Nelson's class is described by the conceptual model presented in Figure 6.



*Figure 6:* The process by which community's capacity to participation in collaborative practices changes over time

The model represents a working hypothesis regarding the process by which a community's capacity to engage in collaborative inquiry practices develops over time. The process is iterative and the model proposes a relationship among students' participation, the negotiation of meanings of various practices, and students' understandings and interpretations. From analyses of Ms. Nelson's class, it is hypothesized that students' participation in various practices led to opportunities to negotiate the meanings of those practices, often at points when there was a rupture or evidence of conflicting interpretations. A set of strategies, which I call "cycle starters," were often used as a means to encourage and initiate new practices and ways of participating. Students' participation and the opportunities to negotiate meanings seemed to influence their interpretations and understanding of practices in ways that facilitated their participation.

In discussing the model, I centralize the negotiation of meaning and address the other components as they relate. First I demonstrate the importance of students' meaning for their participation, and then turn my attention to the negotiations of meanings. Second, I discuss the development of new practices, which also required the negotiation of meaning as well as additional efforts on the part of Ms. Nelson to introduce these new forms of participation to the class.

### *Negotiation of Meanings.*

The meanings and interpretations that students hold of practices and classroom events can have a profound impact on their participation. For example, the following exchange from September was significantly shaped by the student's interpretations. Dontay had volunteered to share his answer to a problem:

D: I got 41.

EN: OK, you got 41. So why do you think it's 41.

D: Um, no, it's 42.

Dontay provided an answer, but immediately backed away from his answer, seeming to interpret Ms. Nelson's question as meaning he is incorrect. His interpretation of this exchange, at least for the moment, keeps him from sharing his thinking about the problem. This interpretation – a question means I'm wrong – may be a reasonable product of past experiences. For Dontay to respond in a manner that would be conducive to collaborative inquiry, he needed to hold a different interpretation of Ms. Nelson's question.

Like Lampert, Ms. Nelson was keenly attuned to students' understandings of classroom practices, particularly interpretations that might impede participation, and tried to account for these as she interacted with students. In the above excerpt, Ms. Nelson responded, "I didn't say you were wrong, Dontay. I just asked how you got 41." He then responded, "Oh" and then explained his thinking in producing the value of 41. Ms. Nelson anticipated and countered Dontay's interpretation, which led to further information about his thinking. Ms. Nelson regularly responded to and took up points where she saw students' held certain perceptions or understandings of practices or their activities which were not conducive to collaborative inquiry.

In the Math A class, there were many practices for which the meanings needed to be negotiated in order to support student participation in inquiry practices. Some were familiar practices that needed new interpretations, such as the meaning of a teacher's question and the meaning of going to the board to share one's solution (e.g., to show others a correct way verses to create learning opportunities). Others were practices that needed to be established as part of doing math class, such as asking a "good question." Not

only were many of the practices of collaborative inquiry unfamiliar to students, some of them did not necessarily even *make sense* to the students as ways to go about learning mathematics.

*Helping students make sense of practices.* A key theme in the negotiation of meanings was helping students *make sense* of classroom interactions and their participation. To accomplish this, Ms. Nelson made efforts to help students understand the nature of the activity and why it was a reasonable approach for learning mathematics. For example, in a lesson from September, Ken volunteered to do a problem at the overhead. He apparently thought he had volunteered to do a different problem. When this came to light, he was still willing to attempt the problem. As he worked, filling in the missing values in a table, his pace was fairly slow and deliberate. At one point, he found an error and had to then figure out what the correct value was. Ms. Nelson provided commentary intermittently during the silent times – when Ken was thinking or working out something –describing the value and nature of the work that was taking place. Her comments included the following:

EN: He is noticing a pattern over here.

And a few turns later:

EN: *to Ken* So now you've gotta figure out that number that goes with 101.

...

EN: *to class* So now we're going to watch him brainstorm, let's watch him problem solve.

*Someone starts to tell him what to do.*

EN: No I want him to do this himself. ... He's gonna try some things, and we're gonna watch how his brain is working.

Ms. Nelson legitimizes these times that could be construed as “just waiting” and unproductive as times when thinking and purposeful observing were taking place. Not only do her comments fill some of this silent space, perhaps alleviating the social awkwardness simply by removing the silence, they also identify the nature of the activity, highlighting the productive aspects of this use of time. Ms. Nelson indicates that Ken is making efforts to “figure out,” “brainstorm,” and “problem solve.” Her commentary asserts that these are valued activities in this class; they take time and will be given the necessary time. Furthermore, some of

her comments indicate that the class should be watching what he is doing and noticing his process. This is a learning opportunity for the rest of the class as well.

These strategies were particularly important at the beginning of the year, as students did not expect that doing math would, or perhaps should, take so long. As Ken worked on the problem, there were several student comments that indicated impatience and revealed that at least some of the students expected things to be moving along more quickly. These included:

R: Alright. *Said impatiently.*

O: Any time now.

J: Are you ever gonna write anything?

Students also tried to “help” by providing Ken with the answer or a next step. From the students’ perspective, this was helpful: the problem would be completed more quickly, and the presenting student would turn in a successful performance. Ms. Nelson however did not view that kind of interaction as helpful and actively addressed this. She emphasized the importance of being able to think independently as well as the importance of giving someone else a chance to think for himself and share his ideas.

Over time, student comments that demonstrated impatience or offered help by answering subsided, and students seemed to develop patience and provide others with more time to think. For example, in the earlier excerpt described from a lesson in April, Ron had used Frank’s strategy to rework the set of three ages. The students apparently watched and waited patiently until Ron finished before commenting on his work, and specifically his error. Six students had their hands raised when Ron finished, presumably because they saw there was still an error in the revisions. The exception to this stance was one student who quietly made a buzzer sound as Ron wrote his new ages<sup>5</sup>.

*Providing evidence of the value for learning.* Another way Ms. Nelson negotiated the meanings of practices with students was by providing evidence of the role various practices directly played in their learning. She regularly took the opportunity to make salient to students how their engagement in particular interactions or practices advanced their mathematical understanding or problem solving process. She also pointed out how specific student contributions, particularly errors, served a critical role in advancing the class’s understanding.

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<sup>5</sup> In the focal set of April lessons, there are no public instances of students helping another by giving an answer unless Ms. Nelson had made it clear that the answer was an acceptable contribution.

For example, the class had been working on interpreting graphs. They were considering a set of four graphs, one at a time, matching the graph with an appropriate scenario. One of the graphs displayed a sinusoidal curve. Oscar asserted (incorrectly) that the graph represented the temperature over the course of the day. As the class examined this idea, they found it lacking, as they could not come up with a reason why the temperature would fluctuate so many times in 24 hours. A few minutes later, Mary Jane gave a sound explanation of why another graph represented the temperature over the course of the day. Ms. Nelson responded: “And you know what might have helped was all that discussion we had before about the other one that wiggled, because it tunes you in to how to look at a graph and what was wrong with that other one so it couldn’t possibly be [the temperature] unless we were talking about multiple days.” Ms. Nelson saw a connection between this earlier discussion and Mary Jane’s clear articulation of a meaning of this graph. She took the opportunity to highlight this for students, thus demonstrating the value of their collaboration in the development of their understanding and skill interpreting graphs. Note also that Ms. Nelson’s commentary positioned the discussion occasioned by Oscar’s idea as productive for their learning.

To explore these themes further, I examine excerpts from a lesson in November where students were working on determining the number of diagonals in an  $n$ -sided polygon as a whole class. This is the first time Ms. Nelson engaged them whole-class problem solving. She posed a question at the beginning of the lesson – *how many diagonals does a polygon have?* –and explained to them that this was the question they were going to work together during the class period. She further offered an animated description of how they would do this, explaining how a student would come up and offer an idea, and then others would look at it and ponder it. Then someone else might come up with another idea, and so on.

Part way into the lesson, it emerged that there were two different ideas about the number of diagonals in a square. From their earlier discussion, it seemed that they had resolved a square had 2 diagonals. As Ms. Nelson was recapping and writing on the board what they knew up to that point, a few students, including Vanessa and Dontay, commented that a square had 4 diagonals. Dontay was willing to go up and show why he thought it was 4.

- (1) EN: You’re gonna come up. Okay.
- (2) (*Dontay goes to the board.*)
- (3) WI: uhhhh ...
- (4) *Some laughter*
- (5) O: Dontay, (give you a hint)

- (6) D: There's a diagonal, there's a diagonal. (*He is tracing the 4 line segments created by the intersection of the two diagonals of the square.*)
- (7) EN: Keep going Dontay.
- (8) O: Dontay, stop before you look stupid. Please.
- (9) D: There's a diagonal.
- (10) O: Oh, you're already there now. You're there now.
- (11) EN: Ok, so now, now Dontay's agreeing with you Vanessa.

In this exchange, we see Ms. Nelson supporting Dontay's participation (line 7) and her validation of his thinking by verbally connecting his thinking with Vanessa's (line 11). While Ms. Nelson clearly supports this participation and focuses attention on Dontay's ideas, Oscar's comments indicate a different understanding of the events. He warns Dontay he should terminate his participation as he is about to look stupid (line 3), and then, as Dontay proceeds, Oscar indicates that he indeed does look stupid (line 5). Oscar's understanding of these practices runs counter to those of Ms. Nelson where one's thinking is offered to support the class's learning as opposed to identifying the correct answer and demonstrating competence.

As the lesson continued, Ms. Nelson worked to *ground* Dontay's idea for the group to consider. As she did, Dontay demonstrated some uncertainty around his ideas claiming "I don't even know what I'm talking about." Ms. Nelson takes this as an opportunity to try to pursue a point of confusion with respect to diagonals that the class may be having that they need to sort out.

- (31) EN: Now Dontay, can you ask a question about what, what is confusing to you. Do you have a question that, do you think the class has a question in general that they, that they don't understand, that's causing the confusion.

After a few more exchanges, through which Ms. Nelson revisits Dontay's thinking, Jay raises his hand, responding to Ms. Nelson's efforts to try to identify the point of confusion.

- (46) EN: Yes Jay.
- (47) J: They have to go all the way through.
- (48) EN: Ah.
- (49) J: They have to be a continuing line. They can't be cut.

- (50) D: (*quietly*) That's why I feel stupid.
- (51) EN: Aah! That's the question! But you know what Dontay, we needed you to do that for Jay to make his point.
- (52) D: She did it first! (*Points to Vanessa*)
- (53) EN: Yes but-
- (54) J: Thank you Dontay!
- (55) D: (I messed up.) Why didn't you guys stop me!?
- (56) O: I tried.

In this exchange the meaning of Dontay's participation is being negotiated. Oscar, and likely others in the class, is operating from the stance that saying something that is mathematically incorrect in class should be avoided. Dontay seems to agree with Oscar by indicating that the class should have kept him from participating in this manner (line 55). There is an indication of an implicit agreement where members of the class should protect each other from making a public mistake. Oscar asserts that he tried (line 56), but his efforts were not successful. Dontay also references that Vanessa had the same idea (line 52). One interpretation of this move is that he was trying to give credit where credit was due. Given his remarks about feeling stupid, however, it is more likely that he was trying to reject Ms. Nelson's comment in line 51 that gave him ownership of the contribution by attributing it instead to Vanessa. In support of Ms. Nelson's position, Jay seems to accept her claim (line 51) about the unfolding of events as he thanks Dontay (line 54). Thus we see the active negotiation of meanings of Dontay's participation.

One point of negotiation is the value of offering an idea that turns out to be mathematically incorrect. Ms. Nelson worked to reshape what errors meant for this class. Indeed, from her perspective, they were not then errors at all, but a necessary and productive part of the learning process. She positions Dontay's work as a contribution, which is contested by Oscar's position that it was an unproductive and embarrassing move. Although Ms. Nelson's interpretation is not accepted at this point by Dontay, later in the lesson this instance is revisited at which point Dontay seems to be more open to her perspective, although not fully accepting of it.

In Ms. Nelson's class, it was not just that errors were accepted and OK (everybody makes mistakes), which is a well-documented norm in reform-oriented classrooms (Hiebert et al., 1997). Rather,

errors or ideas that were not fully mathematically correct fundamentally organized new learning opportunities for the class. Classrooms can support the norm that mistakes are acceptable, but they may still not be desired. Ms. Nelson indicated that these were desirable contributions.

It should be emphasized that Ms. Nelson's did not offer her interpretations detached from students' participation, referencing abstract activities or situations (e.g., "You learn from discussions" or "It's helpful to see other ways of reasoning."). Instead, Ms Nelson *demonstrated* the value of practices. She identified examples of success and made these salient for students. In the above excerpt, Ms. Nelson took up Dontay's comment, which demonstrated a particular interpretation of his participation, and specifically identified how his participation provided a learning opportunity for the class by referencing Jay's contribution. Thus she not only countered Dontay's claim, but also offered justification for her counterclaim based on their collective experiences, thus concretely refuting his assertion. (See also Staples, in preparation, for an extended analysis of this full episode.)

*Negotiation of the joint enterprise.* In Ms. Nelson's Math A class, it was not just particular practices and forms of participation that needed to be negotiated. In addition, the productiveness of the overarching approach to learning collaboratively was negotiated. Students were used to doing seatwork and going over answers as a whole-class. Engaging in joint problem solving activities and discussing answers was not something familiar to them as a way to do and learn mathematics.

In the lesson described above, at one point a student commented, "I'm not like learning anything here." Ms. Nelson responded:

Well, hang on, this is a different, this is a different way of learning ... I'm not standing up here telling you everything, we're kind of experimenting here, which ...is my different way of learning that [is how] I learn with my seniors right before they go to college.

Here Ms. Nelson indicates that this is a different approach to learning that is more like experimenting. She aims to legitimize their activity by noting it is something she does with her seniors who will soon go to college. Ms. Nelson's remarks and commentaries offered alternate interpretations of their experiences framing the activity as a reasonable way to engage in doing math class.

There were other instances where the students rejected the vision of a learning community Ms. Nelson was putting forth as well. At one point when Ms. Nelson referenced the ways "her seniors" did mathematics together, one student countered "we're not seniors." At another point when she was legitimizing the mathematics they were working on, they explained to her that she did not have to "sugar coat" it for them – they knew they were the dumb class. Ms. Nelson actively countered these assertions,

demonstrating the importance and legitimate challenge in the mathematics as well as their approaches to learning.

Episodes such as the one with Dontay and Oscar revealed the conflict in interpretations, which in turn impeded the students' participation in collaborative inquiry mathematics. These episodes, which were recurrent events in the fall, were not present in the spring class. There were still some instances where the students' assertions of their broader identities as "not smart" or "the dumb class" were noticeable, but these significant ruptures with respect to collaborative inquiry practices and students' participation did not occur.

Through the fall, Ms. Nelson supported the active negotiating regarding the organization of their collaborative inquiry work and the meaning of various practices. She opened up discussions about how the class was doing mathematics together and directly addressed students' views and perceptions based on their prior experiences. The initiations of such episodes was reminiscent of the activities described in Gutiérrez, Baquedano-López & Tejada (1999) of a teacher who created a *third space*, or a zone of proximal development, by taking up student discourse and participation that most often would have been construed as off-task and part of the "unofficial space" of the classroom. These "ruptures" served as "points of negotiation rather than disruptions" (p. 294). The general value of recognizing students comments that might otherwise be seen as irrelevant or off-task has been recognized by other researchers as well and connected with productive learning environments that garner a broader base of participation and capitalize on a wider range of student contributions (Warren, Ballenger, Ogonowski, Rosebery, & Hudicourt-Barnes, 2001). By actively taking up students' comments in the "unofficial space," Ms. Nelson was able to create opportunities to negotiate meanings and transform the ways the class worked together.

### *Cycle Starters*

The opportunities to negotiate meanings arose out of students' participation. Consequently, Ms. Nelson needed to find ways to support students in participation in some semblance of the desired learning practices –no matter how rough or approximate the form – in order to create these opportunities. This required extensive effort and scaffolding on the part of Ms. Nelson, particularly in the fall. The previously discussed instructional strategies (see Table 3) were utilized as she supported students' participation in collaborative work. In addition, she supported their participation in new practices using "cycle starters." These strategies introduced students to practices or participation structures, and induced their initial attempts at these. These included: creating a vision of the desired practices (e.g., through describing ways of participating, showing a video of older students collaborating to solve a problem and discussing their ways of working together), explicitly introducing new practices (e.g., by naming, modeling, and identifying

them when they occurred), and providing incentives for participation in particular desired practices (e.g., by offering participation points) which seemed particularly important when there were social barriers for participation. Once Ms. Nelson had some initial efforts, she could then further foster, revise and refine these practices with students.

### *Students' Interpretations and Understandings of Practices*

As noted earlier, at the beginning of the year, students' interpretations were not necessarily conducive to collaborative inquiry. Over the course of the year, students' *interpretations and understandings* of various learning practices changed in a manner that was congruous with their changed participation. Whereas in the fall it was a regular occurrence for students to make comments during classtime that demonstrated views that were more likely to impede than enhance collaborative work, such instances were uncommon in the spring. Although this observed difference could result from students developing a view that making such comments were not worth their energies, my assertion is that this shift was connected with changes in their interpretations and perceptions of learning practices.

One change in many students' perceptions that is germane to collaborative inquiry practices was the students' orientation towards others and the role they saw others playing in their learning. At the beginning of the year, the role for others in their learning was rather limited, and primarily social in nature. For example, students reported that working with others helped them learn because this arrangement made work less boring. Consequently, they would do their work, or at least were more likely to do their work, and thus they would learn more. Peers were also implicated as potential substitute instructors when the teacher was busy working with another student or otherwise occupied. Students reported that they liked working with others because they may not have to wait as long if they had a question.

At the end of the year, students generally reported a much broader role for others, encompassing social and intellectual dimensions (Staples, in preparation). Most students noted that they learned by listening to or working with others. They could figure out problems with others, others might give them an idea, or a peer might also explain something a different way, even better than the teacher, which would support their learning. Thus they saw working with others productive and potentially generative.

This change in students' orientation towards their peers supports collaborative inquiry practices. To collaborate, students must listen to, comprehend, and respond to other students. When students identify these practices as beneficial, they engage differently when another student is sharing his thinking and see their responsibilities towards others in a different way as well. Students also reported changes in how they

felt about going to the board and their preferences for group members, which shifted from friends to those who participated and were willing to talk about the math.

### *Transforming a Community's Repertoire*

I have proposed a model to describe the process by which a community can expand its shared repertoire and documented some of the strategies Ms. Nelson employed to support the transformation of students' modes of participation. Central to her work was creating and/or taking up opportunities to explicitly negotiate the meanings of various practices and how they contributed to the joint enterprise of learning mathematics. Along with this, the overall nature of their work together was negotiated. It is important to emphasize that there are likely many reasons for these observed changes, and I do not suggest that these shifts can be attributed exclusively to students' experiences in this Math A class. Their experiences and participation in Ms. Nelson's class comprised only a small portion of their day, and thus there were many other influences on their perceptions of various learning environments and their participation. For example, several of the students were supported in the school's resource program which provided students with opportunities to reflect on their own learning and productive ways of participating in class. Regardless of the full complement of sources, students' perceptions and interpretations of classroom practices in the spring supported student participation in collaborative inquiry mathematics.

Though perhaps a naïve stance, I had expected Ms. Nelson to play a significantly less prominent role organizing collaborative work in the spring than in the fall. Her active role persisted throughout the year, however, even in the spring when norms of collaboration had been established. It was not the case that students came to collaborate independent of her guidance. She remained a strong presence, actively working throughout the lesson to elicit students' ideas, maintain a common ground, and guide the class's mathematical work.

One set of explanations for this difference is that it can be attributed to the maturity of students; the more challenging mathematical materials of the spring; or other similar factors. Another explanation is that this work simply takes an extensive amount of time, and one year is not enough to develop that level of independence in whole-class discussion particularly with a group of students who begins the year with negative attitudes towards school and math. Schoenfeld (1992) notes the lengthy timeframe for change in students' approaches to doing mathematics. He reports that it took nearly the full semester for his class of highly academic first-year Berkeley students to shift in their approaches to problem solving. Although the Math A class did not come to collaborative independent of Ms. Nelson's support, there was a clear change

in the norms, the repertoire of the community of practice, and how they were able to go about “doing math class” together.

### Discussion

In this article, I have undertaken a careful examination of the pedagogical strategies used by one teacher, Ms. Nelson, to organize and support collaborative inquiry mathematics practices in a ninth-grade classroom of lower-attaining students. I have presented two conceptual models: one representing the teacher’s role during whole-class collaborative discussions and the other representing the process by which the class’s capacity to enact collaborative practices developed over time. For the purposes of highlighting particular aspects of teaching, these models partition the role of the teacher into two temporal frames: the work of teaching “in-the-moment,” as Ms. Nelson and the students engaged directly in collaborative inquiry, and the work of teaching over a longer period of time to augment the class’s capacity for collaborative inquiry. Naturally these are mutually influential and nested, with the former embedded in the latter. These models offer one conceptualization of teaching in an effort to provide insights into the nature of collaborative inquiry practices, how they might be organized in a high school mathematics classroom, and how a community can shift and expand its shared repertoire over time under the guidance of a teacher.

#### *The relationship between the demands of collaborative inquiry and the teacher’s role*

A major focus on this article has been to understand the nature of the teacher’s work in relation to the demands of a specific participation structure, namely, whole-class collaborative inquiry. At the outset, I contrasted the features of collaborative inquiry mathematics discussions with discussions that merely evinced cooperation and the sharing of ideas. These different qualities in discussions have also been noted by other researchers. Wood, Williams & McNeal (2006) distinguish between strategy reporting (sharing) and inquiry/argument classroom cultures and document the different teacher-student interaction patterns comprising each culture. Enacting these classroom cultures necessarily places different demands on the teacher’s pedagogy.

One question that arises is: What distinguishes the teacher’s role in a collaborative inquiry mathematics community relative to a sharing/strategy reporting mathematics community? Exploring this question illuminates some of the differences between the teaching in the two kinds of communities, both of which align with our current visions of reform, but which support different student learning practices and mathematical reasoning (Wood et al., 2006). Some aspects of the teacher’s role seem to span across these different kinds of discussions, such as eliciting students’ thinking. Indeed, it seems reasonable to claim that these three components, Supporting students in making contributions; Establishing and

monitoring a common ground; and Guiding the math, are also components of the teacher's role in sharing/strategy reporting classrooms. What aspects of the teacher's pedagogy, then, address the specific, and perhaps unique, demands of organizing and sustaining collaborative inquiry mathematics practices? While this question cannot be definitively answered by this study, it is possible to articulate some reasonable conjectures in relation to empirically derived findings.

*Collaborative inquiry practices and common ground.* As discussed, all mathematics classrooms have a common ground which provides the basis of their joint activity (Clark, 1996). However, what comprises common ground varies from setting to setting. In class discussions that support sharing or strategy reporting, students do not need to analyze, evaluate and build upon their classmates' thinking, as they must for a collaborative inquiry discussion. Student-student exchanges play a minimal or non-existent role in instruction. Common ground *shared among students comprising students' ideas and thinking* then is not a requirement of sharing discussions. In collaborative inquiry classrooms, a common ground shared among students, comprising their ideas, is a necessity. This has implications for differences in the demands on the teacher's pedagogy in collaborative and non-collaborative settings.

It is important to note that the organization of a common ground is inherently problematic. Common ground is problematic even in everyday conversations among two or three people (Clark, 1996). In a classroom, the challenges are amplified as the number of participants increases along with the range of mathematical backgrounds and learning styles. In a whole-group discussion format, misalignments in common ground are likely and, given the participation structure, they are not easy to repair. The teacher must proactively make efforts to establish a shared intellectual context and maintain continuity over time. In addition, she must have strategies to ascertain when there is a misalignment (that is impeding the class's inquiry) and to repair these. Ms. Nelson's strategies aimed to maintain and establish ground shared among students with respect to students' ideas. These included ongoing assessing and proactive work to record ideas publicly, ground before building, and control the flow and pacing of information. There are likely other effective strategies not documented in this case study.

*Collaborative inquiry practices and guiding the mathematics.* Teachers of course play a role in guiding the mathematics regardless of the kind of classroom culture their teaching supports. One critical aspect for supporting collaborative inquiry practices seems to be the teacher's role in upholding a level of mathematical rigor during discussions. Maintaining high standards of evidence for understanding and adhering to disciplinary norms for justification opens up more intellectual space for collaborative inquiry work, as the students and teacher press for a deep level of understanding. This difference is evident in

Kazemi & Stipek (2001) comparison of the nature of conversations in classrooms where the teacher *pressed* for explanation as compared to one where explanations were shared, but rarely queried or evaluated. (See also Inagaki, Hatano & Morita, 1998 for a comparison of discussions with and without teacher intervention.)

The influence of the teacher's role in guiding the mathematics on student collaborative practices is also illustrated in Nathan & Knuth's (2003) two-year study of an elementary teacher who actively tried to instantiate a more student-centered classroom. In the second year, Nathan & Knuth documented a significant increase in the number of student-to-student exchanges in her classroom, suggesting a degree of collaboration and the presence of common ground with respect to students' ideas and reasoning. However, this teacher's new role, which greatly enhanced the presence of students' voices and ideas, simultaneously left a void with respect to the mathematics, as the teacher chose not to use her position in the classroom to shape the mathematical trajectory of the lesson. In one instance, Nathan & Knuth reported that the class, unable to resolve their discussion independently, put the mathematical issue at hand to a vote. Without the teacher's guidance with respect to the mathematical ideas being discussed, the class continued cooperating, maintained a common ground with respect to students' ideas, and sustained their discussion for a time, but was not able to independently support their inquiry with and into mathematics.

Other researchers have similarly demonstrated the importance of the teacher strategies that adhere to disciplinary standards of justification and argumentation in promoting student collaborative inquiry (e.g., Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Ball & Bass, 2000). The teacher's awareness of and upholding of these criteria as she guides students' participation in inquiry practices supports students in collaboratively exploring mathematics by drawing upon disciplinary resources rather than becoming entrenched in two or more views, or accepting a conclusion or generalization that is mathematically unsound.

Ms. Nelson's instructional approach of "going with the kids" also seems to be a critical feature in collaborative inquiry classrooms. Eliciting and pursuing their ideas is a first step in organizing collaborative inquiry, as the focus of discussion is the students' reasoning. In eliciting students' ideas, Ms. Nelson made extended efforts to make the students' *thinking processes* and *ways of reasoning* available to others in the class. For example, in follow up questions, Ms. Nelson often asked students why they had chosen a particular approach, or would ask them to make a record of their process in addition to explaining their ideas. Some have argued that making individuals' thought processes available to others needs to be at the heart of collaboration (Engeström, 1999; Schwartz, 1999). Ms. Nelson's particular forms of requesting and

scaffolding often uncovered the presenter's thinking processes and provided students with insight into the particular approach, thus creating a broader basis for the collaborative work and more opportunities to consider others' reasoning. Thus there are particular qualities of the way Ms. Nelson would "go with the kids" that may have facilitated collaborative inquiry practices.

It is important to reiterate that nearly all of the work that the teacher does with respect to *Supporting students in making contributions* and *Establishing and monitoring a common ground* also shapes the trajectory of the mathematical discussions while centralizing students' thinking and can be considered part of the teacher's role in guiding the mathematics. For example, a teacher's follow-up question affects what mathematics is made public for collaborative work, as does the teacher's verbal marking of certain ideas. Similarly, how the teacher has students record their ideas is also critical. Representations can strategically make salient certain features of one's mathematical thinking and background others.

The ways these critical but subtle moves shape the trajectory of mathematical discussion is an area where additional research could be quite productive. For example, Rasmussen & Marongelle (2006) offer an illuminating analysis of teaching that demonstrates the role of particular *pedagogical content tools*, which are devices teachers use "to connect to students thinking while moving the mathematical agenda forward" (p. 288). The use of such tools, which teachers build in response to student contributions, exemplifies the overlapping roles of supporting contributions and guiding the mathematics. It further exemplifies how the particular ways teachers enact these roles can set up collaborative discussions and opportunities for sense making as well as influence the mathematical trajectory of the lesson.

*Collaborative inquiry and a developmental map of learning.* Although pursuing students' thinking and maintaining a level of rigor of discussions seem essential for collaborative work, it is not immediately clear why or how Ms. Nelson's instruction that guided class discussions using a developmental map of students' thinking is *required* for collaborative work. This category of work was correlated with particular interjections of the teacher – a question, a clarification, a request for a particular form for an expression—that revealed her pursuit of engaging students in increasingly sophisticated uses of algebraic notation and mathematical ways of reasoning. This may facilitate collaboration: by having in mind a long-term trajectory of student learning, such as that which is represented by this developmental map, the teacher can organize discussions that are likely to raise issues related to the particular objectives or "pressure points." Likewise, the teacher can recognize opportunities to address these ideas that are authentically occasioned as students share their thinking and work collaboratively. For example, in the excerpt with Emily, it was Ms. Nelson's recognition of a problematic piece of mathematics for students – using symbolical representations

and writing rules explicitly – that guided her work and subsequently prompted the students to reformulate on aspect of Emily's solution and explain the inadequacies of her original notation. The importance of problematizing subject matter has been highlighted by other researchers studying collaborative inquiry environments as well (e.g., Chazan, 2000; Engle & Conant, 2002; Lampert, 2001). From this research, however, it is not clear how this particular aspect of her role in guiding the mathematics was needed for collaborative inquiry. This is another area where additional research is needed to help us further understand the teacher's role and how it functions in collaborative inquiry settings.

To return to the question of distinguishing features of the teacher's role in collaborative inquiry classrooms relative to sharing or strategy reporting cultures, it seems that what differs is how a teacher inhabits these roles and the purpose of each component. The teacher's work, for example, establishing common ground, must be done with an eye to the purpose of the role in relation to collaborative inquiry mathematics practices. If the teacher is aware of how the role contributes to collaborative inquiry, and the specific demands of collaborative inquiry practices, she is more likely to be able to successfully attune specific instructional strategies towards these ends and support students' participation in these practices.

#### *Applicability to other settings*

Given the case study design, it is important to consider how these models may be useful in understanding collaborative inquiry learning practices in other settings – at other grade level, in other school environments, with students with different prior experiences. As argued above, many of the categories of work respond to particular demands of organizing and supporting whole-class collaborative inquiry practices. Thus I would hypothesize that, regardless of the setting, the functions implicated by the roles (e.g., making students' ideas public; creating a common ground shared among students; ensuring the discussion adheres to disciplinary standards or argumentation and justification) would need to be fulfilled.

Although the components of the teacher's role may have broad applicability, the specific instructional strategies required to meet the pedagogic demands of each component in support of whole-class collaborative inquiry may vary significantly across different settings. For example, the directed efforts teachers must make towards having students clearly articulate their thinking will vary. This can vary with a range of factors: students' prior experiences and comfort sharing their thinking; the nature of the mathematics or particular task; students' proficiency with representations; how long the class has been working together; to name but a few. It is important to note that the interrelationships among mathematics, the students, their history with mathematics, their relationship with each other and the teacher, the teacher's goals, the students' goals, and so on, create a complex landscape that does not lend itself to

determinate analysis. Consequently, although the model of the teacher's role presented here may be able to "travel," the specific instructional strategies required to enact various role components may need to be worked out in response to the particular demands of the contexts. The conceptual models presented here then should be looked at as generative, not prescriptive, offering a vision of practice and possible instructional strategies that might productively inform teaching in other settings. Continued efforts are needed to refine and revise this model, and to understand the variability across contexts that influences the strategies teachers use to successfully enact their role.

The particulars of the process of developing the class's capacity to work collaboratively may be even more context dependent and thus greatly vary across settings. Students bring a range of prior experiences and orientations towards mathematics and school with them to mathematics class. These have a significant influence on the work the teacher will need to do to engage them in collaborative inquiry mathematics practices. The set of practices that needs to be negotiated and established as part of this process is likely to be dependent upon the activity or participation structures the teacher selects (e.g., whole-class problem solving vs. whole-class discussion that follow groupwork) as well as students' past experiences and interpretations of various teaching and learning practices. Indeed, this kind of teaching fundamentally requires responsiveness to students, and the students' initial approaches to the classroom will significantly shape the work that the teacher must do with the class to develop its capacity for collaborative inquiry. That said, collaborative learning practices require students to share their ideas, take account of each others' ideas, and build upon others' thinking, which indicates some promising candidates for practices and meanings that need to be negotiated in a mathematics class (e.g., what counts as a contribution, and what a question means). Given the literature documenting the prevalence of traditional models of teaching, it is reasonable to expect that many of the practices of a collaborative community are novel to the students, and the magnitude and scope of the requisite changes are not small.

As with the first model, the main components of the second model, which centralize the process of meaning-making and students' perceptions of practices, may hold across settings. Lampert's research attends to students' processes of meaning-making, an issue also raised by Chazan (2000) in his work with a lower-attaining group of high school students. In general, we do not yet have many detailed accounts of significant changes in the practices of learning communities, particularly those for which the transition involved some tensions. Future studies are needed to further illuminate this process of developing a community's capacity to engage in collaborative inquiry mathematics.

## CONCLUSIONS AND IMPLICATIONS

Ms. Nelson's class is a case of successful enactment of reform practices with a group of students who had not found prior success with school mathematics. The study reinforces the importance of the teacher's work in reform-oriented mathematics classrooms and offers a vision of the possible. Prior studies of such classrooms have generally examined teachers with unusual backgrounds or unusual teaching situations, and rarely have looked at high school classrooms. Questions have been raised about the feasibility of enacting such practices in more typical school settings (e.g., diSessa & Minstrell, 1998). Ms. Nelson was an exceptional teacher, but not in an unusual teaching situation. While this case paints a hopeful picture, it also leads us to consider the depth of the challenges that teachers face in working to realize such learning environments.

Organizing collaborative inquiry mathematics classrooms requires teachers to be responsive to students. Only by opening up space for students' contributions and pursuing their thinking can their ideas provide a basis for inquiry activities. To be responsive to students and the setting, a teacher must constantly interpret, take in and process information, and then make fairly immediate decisions as to how to proceed. She must balance providing a clear structure and trajectory for their work with being responsive to students' needs. This kind of teaching also requires a deep understanding of mathematics, students' thinking, curricular materials, as well as how students reason and potentially develop proficiency with a mathematical domain and its practices. The structures of schools presently seem to mediate against this kind of teaching. In some schools, new curricula are introduced with some frequency as various battles regarding what is best for students or test scores are waged. Such a situation precludes the development of extensive curriculum-specific knowledge which facilitates this kind of teaching. Teachers are under pressure to cover a defined body of course content and have non-trivial portions of their instructional time reallocated for other purposes such as state testing. The time and energy required to foster new norms of participation and help students rethink themselves as learners is also remarkable. Furthermore, opportunities to negotiate the meaning of practices cannot be scheduled and put into a lesson plan, thus increasing the uncertainty in pursuing the reorganization of the classroom learning community.

Nevertheless, there are steps that can be taken to support teachers in their efforts to instantiate collaborative inquiry practices in their classrooms. This study affords a comprehensive description of one case of successful enactment which may be useful for teachers in generating a new vision of the possibilities for their own classes and instruction. In considering out current imagines of reform teaching, the teacher's role to elicit students' thinking and establish and monitor common ground seems closely linked to the role of a facilitator. Contrary to common images of facilitation, which represents a more

passive stance by the teacher with respect to discussions, this case represents a very active and present role of the teacher throughout, providing a well defined structure within which students conduct their mathematical work. Indeed, the teacher was very directive in this respect. At the same time, the teacher's guidance is fundamentally shaped by students' mathematical contributions, as well as their socio-emotional states, levels of participation, and even side comments. Thus the strong frame she provides simultaneously accounts for the centrality of student thinking and participation and for the particulars of the context. These qualities of the role of the facilitator present a different vision for teachers in reform-oriented classrooms which may be useful in guiding teaching practice.

This study also offers conceptual tools to illuminate the teacher's role, as well as detailed analyses of deliberate pedagogical actions that support collaborative work and develop a community's capacity to engage in such practices over time. Organizing professional development activities around any one of a number of these concepts or practices, such as monitoring a common ground or creating contributions, might productively support teachers' reflection on and development of their own practice teaching towards collaborative inquiry. It is expected that these findings might offer productive starting points for discussions of practice, which would then be contextualized and developed in relation to a particular group of teachers' setting. The teachers, like the Math A students, will also need opportunities to participate in these practices, negotiation and make sense of their meanings in relation to their teaching, and perhaps change their interpretations and understandings of various aspects of their instruction. The usefulness of the particular conceptualization and rendering of practice presented here, as well as other conceptualizations and analyses offered by researchers, for fostering productive dialogue and efforts towards the instantiation of collaborative classrooms is an important focus of future research.

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<sup>1</sup> Class sizes of 20 were not atypical as state funding supported class-size reduction for Grade 9 math and English courses.

<sup>2</sup> Video content logs comprised a record of the events of the lesson, at thirty-second intervals, including segments of dialogue.

<sup>3</sup> It is of course difficult or impossible to draw a line between when the teacher is merely supporting the student in articulating his or her thinking more fully, and when the student is actually articulating new thoughts, prompted in the interaction. From a Vygotskian perspective, there is no distinction to be made. Language and thought are intertwined and co-constitutive, and the act of articulating a thought cannot be separated from having the thought itself (Vygotsky, 1978).

<sup>4</sup> Common ground also includes the class's shared history, normative practices, and the mathematics content they have learned prior to the given lesson. These too play a critical role in supporting the collaborative inquiry. I focus less on these aspects, however, in an effort to foreground processes *during* collaborative inquiry that support the accumulation of common ground in moment-to-moment interactions over the course of a lesson.

<sup>5</sup> The particulars of this "map" may also be tightly linked to the specific approach of the curriculum.

<sup>6</sup> While revising itself is not a collaborative practice *per se*, the opportunity to revise opens space for students' continued work with their own and others' ideas, which can support collaborative inquiry.

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