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Sequential Pre-Marital Investment Games: Implications for Unemployment

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Abstract

Agents on the same side of a two-sided matching market (such as the marriage or labor market) compete with each other by making self-enhancing investments to improve their worth in the eyes of potential partners. Because these expenditures generally occur prior to matching, this activity has come to be known in recent literature (Peters, 2007) as pre-marital investment. This paper builds on that literature by considering the case of *sequential* pre-marital investment, analyzing a matching game in which one side of the market invests first, followed by the other. Interpreting the first group of agents as workers and the other group as firms, the paper provides a new perspective on the incentive structure that is inherent in labor markets. It also demonstrates that a positive rate of unemployment can exist even in the absence of matching frictions. Policy implications follow, as the prevailing set of equilibria can be altered by restricting entry into the workforce, providing unemployment insurance, or subsidizing pre-marital investment.

1 Introduction

In two-sided matching scenarios, agents often put a great deal of effort into making themselves more attractive, hoping to improve their ranks in the preference orders of potential partners. The labor market provides an excellent example of this behavior, as most folks devote the better part of their younger years to the pursuit of various educational activities. They do so primarily because they want to be gainfully employed in adulthood. Firms then respond in kind by setting aside funds for superior compensation packages and making physical investments in new equipment or amenities, as they strive to recruit the best and brightest employees from the field of applicants.

The upshot to all this competitive preening is that it tends to improve the total productivity of relationships once matching has occurred. Unfortunately for the agents involved, however, the returns to their investments are seldom guaranteed. The sunk cost aspect of relationship-specific capital combined with conflicting incentives on both sides of the market makes agents vulnerable to economic holdup. In the labor market setting, workers may end up employed but underemployed given the skills they have acquired. Firms may end up with workers that are unable to take full advantage of advanced equipment. Nevertheless, agents often willingly incur these costs prior to any sort of contractual obligations because of the potential net benefits yielded by superior match outcomes.

These types of self-enhancing efforts made by match market participants have recently been identified as *pre-marital investments* in work by Peters (2007) and Peters and Siow (2002). That literature treats pre-marital investment as a simultaneous game played by agents on both sides of the market. In particular, Peters (2007) finds that investments on both sides of the market are inefficiently high, a rather dismal result for the world of matching.[1]

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[1]Other works on pre-contractual investment, though with slightly differing model specifications and contexts, do suggest some potential for efficiency. These include Cole, Mailath and Postlewaite (2001) and Han (2001).

Alternatively, however, the game can be formulated sequentially, with one side of the market investing first and the other side responding. The sequential version of the pre-marital investment game is appealing firstly because it provides a free-market solution to the holdup problem: when one side of the market is smaller than the other (for example, in the case of firms matching with workers or schools matching with students), those agents can wait for the other side to invest first because a scarcity of partners promotes a competitive response from the larger side[2].

A sequential structure also seems more appropriate for the particular case of the worker-firm matching problem. The human capital investment process can be both intense and time-consuming. That means that decisions by prospective workers inevitably occur prior to full knowledge of the market conditions that will prevail once their investments are completed. Indeed, the bulk of human capital investment tends to occur quite early on in an individual's life. Therefore, the best that a person can do is simply try to anticipate how firms will react to, and hopefully reward, their efforts.

Firms, meanwhile, have the luxury of being more selective. Unlike human beings, who rely on employment for sustenance, firms can often wait to be sure deserving candidates are available before funding projects to attract them. To take an example from the economics profession, experimental labs need not be built unless qualified experimental economists exist to make productive use of them. More generally, exceptional amounts of capital need not be set aside for salaries if exceptional candidates are not available.

This paper describes a two-sided economy consisting of firms and workers that engage in sequential pre-marital investment. Workers move first, knowing that firms desire human capital, and then firms make their own investments to attract workers. Matching then occurs assortatively based on those investment levels. Formulated in this manner, the pre-marital invest-

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[2]For a more in depth treatment of this specific phenomenon, see Felli and Roberts (2002).

ment game involves leader-follower competition among agents across different sides of the matching market, as well as simultaneous competition between agents on the same side of the market. The goal of this paper is therefore to reveal the implications of this underlying incentive structure for matching markets in general and for the labor market in particular.

Conceptualizing the labor market as a sequential pre-marital investment game provides a new microfoundation for a positive natural rate of unemployment. Though there is an existing literature that attempts to explain the empirically observed phenomenon in terms of job matching, these explanations are almost exclusively based on friction. Diamond (1982), Howitt and McAfee (1987), Mortensen (1989), Pissarides (1989), Rocheteau (1999a, 1999b), Salop (1979), and Weitzman (1982) all make use of search-theoretic labor market models that incorporate various costs and problems of coordination into the matching procedure[3]. While these types of frictions are undoubtedly important traits of labor markets, and are well deserving of the consideration they have received, the results presented in this paper suggest that they alone may not tell the entire story of long-run unemployment.

Unlike the search literature, the model here takes efficient (assortative) matching as given. After investments are made the highest quality workers are matched with the highest paying firms in a frictionless manner. A positive level of unemployment is inevitable in this setting not because of any market imperfections, but simply because there is always an incentive for more workers than firms to actively participate in the market. Additional workers are always willing to play the game if given the chance, simply because the available prizes (jobs) are worth their effort.

Unemployment in this context is therefore also different from the type described by Shapiro and Stiglitz (1984), which is caused by firms paying efficiency wages in order to prevent workers from shirking. Like market

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[3]See Rogerson, Shimer and Wright (2005) for a general survey of the search-theoretic labor market literature.

friction, this too is surely an important feature of labor markets, but is not a feature of this model. Rather, here it is workers who over-invest themselves to acquire jobs in the first place.

As with classic frictionless matching models (e.g. those described by Roth and Sotomayor (1990)), multiple equilibria in the sequential pre-marital investment game can be collectively ranked by each side of the market and involve an oppositional duality: the equilibrium most preferred by workers is worst for firms and vice versa. This also marks an important separation from the search-theoretic labor market literature, as multiple equilibria in those models tend to be ranked in terms of aggregate efficiency (Rocheteau, 1999a and 1999b).

In terms of policy advice, the contrast between the interests of firms and workers is critical because of the implied tradeoffs. Individual policies targeted toward helping one group may come only at the expense of the other. A free market approach to the labor market, for example, causes fierce competition among workers for jobs, leading to relatively high payoffs for firms but low payoffs for the workers themselves. Policies such as unemployment insurance or labor market restrictions can lessen the impact or incidence of unemployment, reducing the intensity of worker competition and thereby raising their payoffs, but in the process hinders firm productivity since workers do not invest as much.

On the bright side, and in spite of the tradeoffs imposed by individual policies, certain policy combinations can lead to Pareto-improvements. The key is to combine them in a way that takes advantage of complementarities between worker and firm investment. Social safety nets, restrictions on labor market entry, and investment subsidization packaged together, for example, can work to both insulate workers from the harsh consequences of unemployment and encourage firm productivity. In that sense, social welfare and productivity need not be sacrificed for one another.

After laying out the sequential pre-marital investment game's basic as-

sumptions, the next section of the paper establishes sufficient market conditions for the existence of subgame perfect Nash equilibria in which all agents play pure strategies on the equilibrium path. The third section then describes how full employment is impossible in any subgame perfect Nash equilibrium of the game with free entry, and the fourth section compares the effects of various policies in the context of possible equilibria. The fifth section concludes with a brief discussion.

2 A Sequential Game

2.1 Setup and Assumptions

Consider two disjoint sets of agents known as firms and workers, having sizes of m and n respectively with $m < n$. Individual firms wish to match with individual workers in order to engage in a mutually beneficial relationship known as employment. Prior to matching, firms and workers make pre-marital investments to increase their post-match productive potential, making them more attractive to one another. Aside from their investment levels, agents belonging to the same set are homogeneous.

Workers move first, investing in a type of capital $h \in H$, which may be thought of as some skill enhancement that appeals to employers. After the workers invest, firms move next with full knowledge of the workers' actions in the previous round. Firms invest in a different type of capital, $k \in K$, that appeals to potential employees. This action may be thought of as firms committing to wage offers or announcing costly vacancies to applicants. Both action spaces H and K are discrete, bounded below by zero, and unbounded from above. The size of the increments in each set, specified by ϵ_h and ϵ_k, will be noted when restrictions are necessary.

After all agents have made their investments, workers and firms match in a frictionless manner so that the highest paying firm always employs the most qualified worker and so forth. Tie-breaking is random, so if two workers

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have the same level of qualification, each worker is equally likely to end up with the superior post.

In making their choices, individual agents are motivated by payoff functions; $v(k, h)$ for firms and $u(k, h)$ for workers, where one argument indicates the agent's own investment and the other indicates the amount of investment by the agent's partner. Both functions are assumed concave in both arguments, increasing in partner's investment, and for any given amount of partner investment reach a maximum in own investment. This optimum is designated $k^*(h)$ for firms and $h^*(k)$ for workers. Investment any higher than the optimum decreases payoffs, so there is a definite cost to excessive own investment. Payoffs are supermodular, however, so $k^*(\cdot)$ and $h^*(\cdot)$ are both increasing, $v(k^*(h_1), h_1) > v(k^*(h_2), h_2)$ for all $h_1 > h_2$, and $u(k_1, h^*(k_1)) > u(k_2, h^*(k_2))$ for all $k_1 > k_2$. The optimal investments functions are also assumed concave. An agent who is left unmatched is designated by a partner investment of 0, and to simplify notation assume that $h^*(0) = k^*(0) = 0$. The best payoff an unmatched agent can receive is therefore $v(0, 0)$ or $u(0, 0)$.

To summarize, worker and firm investments in this scenario are costly but complementary. More qualified workers are worth more to firms, but also benefit more from firm investments. This type of interaction between agents' payoffs makes sense in the context of a matching game, since it increases the potential for net gains on both sides of the relationship. A well-equipped laboratory, for example, can be more helpful to the career of an above average scientist than an average one, but that benefits both the scientist and the institution funding the lab. Over-investment is thus always a threat, but the threshold of how much is too much increases with the quality of an agent's partner.

Similarly, discretization of the action spaces also makes a bit of intuitive sense, as costs of investment are often rounded to the smallest monetary denomination available (e.g. to the nearest penny). It should be made

clear before moving on that this feature is included to make equilibria more tractable, but is *not* a "friction" necessary for unemployment to arise in the model. In fact, it turns out to be quite the opposite, as unemployment fails to arise only in the event that the action spaces are exceptionally sparse.

2.2   Equilibrium

Before tackling the issue of entry, it is first helpful to consider equilibrium behavior for a fixed number of agents in the sequential pre-marital investment game. The equilibrium concept is subgame perfect Nash equilibrium (SPNE), as befits the sequential environment, with focus on symmetric equilibria in which all agents play pure strategies. Understanding pure-strategy equilibria in this version of the game is instructive because they prominently display two fundamental characteristics: workers are prone to over-investment, and firms take advantage of that over-investment. Equilibria can therefore be ranked by the degree to which workers over-invest, a fact that facilitates policy analysis later.

Proposition 1. *Symmetric, subgame perfect Nash equilibria in which all workers invest the same h and all firms invest $k^*(h)$ along the equilibrium path can exist for the sequential pre-marital investment game, for some m/n.*

An SPNE for this scenario is established in the conventional manner, working backward and restricting attention to Nash equilibria at each stage of the game. Taking the n worker investments from the first stage as given, then, define $\{h_{1:n}, \ldots, h_{n:n}\}$ as a reordering of those investments so that $h_{1:n} \geq \ldots \geq h_{n:n}$, and consider the firms' response.

Lemma 1. *If $h_{1:n} = \ldots = h_{\ell:n}$ with $\ell \geq m$, then the firm subgame has a pure-strategy Nash equilibrium in which all firms invest $k^*(h_{1:n})$.*

Proof. Since all firms are assured a match of the same quality they have no

need to compete, and can simply optimize their payoffs accordingly. $\square$

Of course, if the m highest worker investments are not all the same, then the firm subgame may fail to have a pure-strategy Nash equilibrium. Consider, for example, the case in which $h_{1:n} = h_{2:n} = \ldots = h_{m-1:n} > h_{m:n}$. Individual firms would rather receive the higher payoff of $v(k^*(h_{1:n}), h_{1:n})$, but with a scarce supply of talent they must weigh that potential gain with the possibility of losing out and ending up with the lower quality worker.[4]

More generally, for the case of $h_{1:n} \geq h_{2:n} \geq \ldots \geq h_{m:n}$ with $h_{1:n} > h_{m:n}$, a symmetric mixed-strategy Nash equilibrium will always exist for the firm subgame. The strategy played by all firms in that equilibrium will be a probability distribution, $F(k)$, supported on

$$\text{supp}(F) = [k^*(h_{m:n}), k^*(h_{m:n}) + \epsilon_k, \ldots, k'(h_{1:n}) - \epsilon_k, k'(h_{1:n})],$$

where $k'(h_{1:n})$ is the highest value of k that is greater than $k^*(h_{1:n})$ and that still satisfies $v(k, h_{1:n}) > v(k^*(h_{m:n}), h_{m:n})$. Formally, $F(k)$ (and its associated probability mass function, $f(k)$) solves

$$\sum_{t=0}^{m-1} \sum_{s=0}^{m-1-t} \frac{(m-1)!}{t!s!(m-1-t-s)!}(1-F(k))^t(f(k))^s(F(k)-f(k))^{m-1-t-s}\left[\sum_{r=0}^{s}\frac{1}{s+1}v(k, h_{t+1+r:n})\right]$$
$$= \sum_{t=0}^{m-1} \frac{(m-1)!}{t!(m-1-t)!}(1-F(k^*(h_{m:n})))^t(F(k^*(h_{m:n})))^{m-1-t}\left[\sum_{r=0}^{m-1-t}\frac{1}{m-t}v(k^*(h_{m:n}), h_{t+1+r:n})\right] \quad (1)$$

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[4]Note that a pure-strategy Nash equilibrium may still be possible for the firm subgame in such a case, depending on further specifications of circumstance. If, for the example of $h_{1:n} = h_{2:n} = \ldots = h_{m-1:n} > h_{m:n}$, payoffs were such that

$$\frac{m-1}{m}v(k^*(h_{1:n}), h_{1:n}) + \frac{1}{m}v(k^*(h_{1:n}), h_{m:n}) > v(k^*(h_{m:n}), h_{m:n})$$

and
$$\frac{m-1}{m}v(k^*(h_{1:n}), h_{1:n}) + \frac{1}{m}v(k^*(h_{1:n}), h_{m:n}) > v(k^*(h_{1:n}) + \epsilon_k, h_{1:n})$$

then all firms investing $k^*(h_{1:n})$ would be a pure-strategy Nash equilibrium.

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for all points $k > k^*(h_{m:n})$ within its support, subject to the constraint that $\sum_{k \in \text{supp}(F)} f(k) = 1$. Though cumbersome in terms of notation, in words the logic of the strategy is rather elementary: firms mix so that they are indifferent to all points within the support of $F(k)$, while any deviation outside of its support can only be detrimental. Indicative of the firms' second-mover advantage, the expected payoff to firms playing $F(k)$ must be greater than $v(k^*(h_{m:n}), h_{m:n})$, since a firm playing $k^*(h_{m:n})$ always has some chance of matching with a worker investing more than $h_{m:n}$.

With firm behavior sufficiently established for now, consider the incentives facing workers in the game's first stage. Begin by imagining that all $n > m$ workers invest at the same h. Given the firms' response, by lemma 1 this would entail an expected payoff of

$$\frac{m}{n} u(k^*(h), h) + \frac{n-m}{n} u(0, h).$$

If one worker deviates by playing $h + \epsilon_h$, however, firms mix according to $F(k)$. The deviating worker is then assured a match with the firm that has the highest of the m realized investments. Hence, let $\mathbb{E}u(k_{1:m}, h + \epsilon_h)$ be the expected payoff to that deviating worker when all m firms employ $F(k)$ as a best response to the n worker investments made in the game's first stage. Also, define the firm strategy s_k as specifying that all firms play $k^*(h_{1:n})$ if $h_{1:n} = \ldots = h_{m:n} \geq h_{m+1:n} \ldots$, and $F(k)$ otherwise, where $F(k)$ depends on $\{h_{1:n}, \ldots, h_{n:n}\}$.

Lemma 2. *In order for (h, s_k) to constitute a symmetric Nash equilibrium in which all agents play pure strategies on the equilibrium path, it is necessary and sufficient for the following conditions to hold concurrently:*

i. $\frac{m}{n} u(k^*(h), h) + \frac{n-m}{n} u(0, h) \geq u(0, 0)$

ii. $\frac{m}{n} u(k^*(h), h) + \frac{n-m}{n} u(0, h) \geq \mathbb{E}u(k_{1:m}, h + \epsilon_h).$

Proof. For necessity, simply note that if either condition were not satisfied,
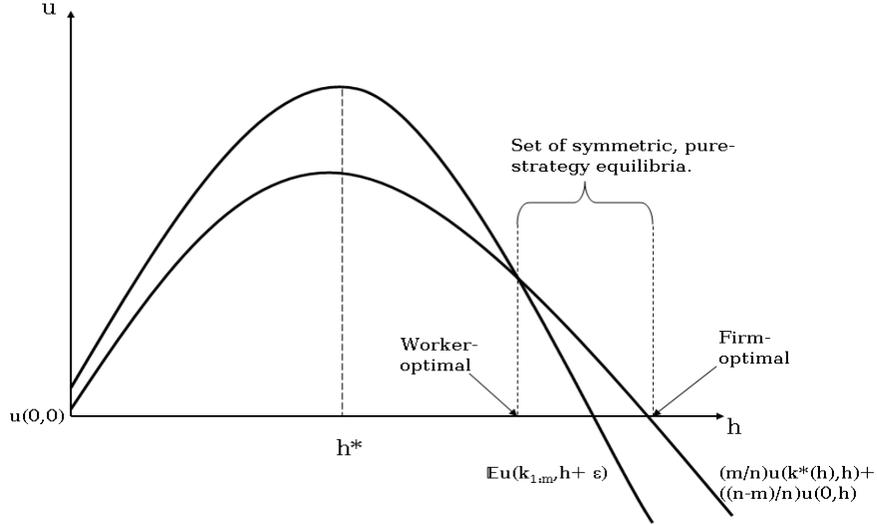
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Figure 1: Satisfaction of Conditions i and ii

a unilateral deviation would be profitable for a worker. For sufficiency, no worker can expect a higher payoff via unilateral deviation if both conditions are satisfied. With all $n > m$ workers investing the same h, firms then respond with $k^*(h)$ by lemma 1. $\square$

Figure 1 illustrates the expected payoff for a worker who invests the same h as all other workers, and the expected payoff for a worker deviating upward by ϵ_h. According to lemma 2, for a symmetric SPNE in which all agents play pure-strategies along the equilibrium path to exist, it is necessary and sufficient for the two curves in Figure 1 to cross above the $u(0,0)$ line. The question remains, however, whether or not that can occur.

To that end (and with slight abuse of notation), define h^* as the highest investment level that provides a worker with the highest payoff possible, given

that all other workers invest $h - \epsilon_h$ and all firms accordingly play $F(k)$;

$$h^* = \sup\{\arg\max_{h \in H} \mathbb{E}u(k_{1:m}, h)\}.$$

Concavity assumptions on $u(\cdot, \cdot)$ and $k^*(\cdot)$ ensure that h^* will exist. As h increases beyond h^*, the cost of a worker investing an additional unit becomes more and more severe, while the additional response from firms becomes smaller and smaller. Accordingly, for some h far enough beyond h^*, $u(k^*(h), h)$ will be sufficiently greater than $\mathbb{E}u(k_{1:m}, h + \epsilon_h)$ to offset the risk of being left with $u(0, h)$. This is, again, due to the concavity of agents' payoff functions, but also because of the discrete action spaces. The concavity of firms' responses means that the reward a worker gains from employment is increasingly similar irrespective of deviation. The discrete action space, meanwhile, along with concavity of the workers' payoffs, ensures that the cost to any deviation is increasingly large.

So at some point condition ii will be satisfied. Lemma 2 specifies that condition i must be satisfied as well, however, and this may not be the case for all payoff specifications. Though the curves depicted in Figure 1 must eventually cross, they may do so far below the $u(0, 0)$ line.

Remark 1. *Potential non-existence of symmetric equilibria in which all agents play pure strategies along the equilibrium path.* If condition ii is satisfied only after $u(k^*(h), h) < u(0, 0)$ for all $0 < m/n < 1$, there is no symmetric SPNE in which workers play pure-strategies along the equilibrium path. In that scenario, one worker can always profit by deviating upward unless all workers are investing h such that $u(k^*(h), h) < u(0, 0)$. But if all workers were investing such an h, at least one could always profitably deviate by not investing at all. Symmetric equilibria therefore only exist in mixed-strategies in this situation, a possibility to be discussed more in depth in section 4.

For the final portion of proposition 1, note that if condition ii can be satisfied for some h prior to workers' payoffs declining below $u(0, 0)$, a critical

ratio of m/n is required. In fact, rearranging conditions i and ii algebraically yields not one but two restrictions on the ratio of firms to workers. Since only the larger of the two will bind,

$$1 > \frac{m}{n} \geq \max\left\{\frac{u(0,0) - u(0,h)}{u(k^*(h),h) - u(0,h)}, \frac{\mathbb{E}u(k_{1:m}, h + \epsilon_h) - u(0,h)}{u(k^*(h),h) - u(0,h)}\right\} \quad (2)$$

is a necessary condition for the existence of a symmetric SPNE in which all agents play pure-strategies along the equilibrium path.

Remark 2. *Potential multiplicity of symmetric equilibria in which all agents play pure strategies along the equilibrium path.* Though the conditions in lemma 2 and the relationship in (2) guarantee the existence of at least one of the desired equilibria, they do not preclude the existence of others. Furthermore, in the event that multiple elements in H do constitute symmetric Nash equilibria, coordination problems abound for the workers. For example, suppose m/n is large enough that $\tilde{h}$ and $\tilde{h} + \epsilon_h$ both satisfy i and ii. Both values must be greater than h^*, so $\tilde{h}$ entails the higher expected payoff for workers in equilibrium, but this is not possible to attain via unilateral deviation if all other workers are investing $\tilde{h} + \epsilon_h$. Figure 1 illustrates this phenomenon.

Also apparent in Figure 1, and inherent in the discussion of potential coordination problems among workers, is the fact that the game's equilibria can be collectively ranked by each side of the market. Since an h must be at least h^* to be a symmetric equilibrium for workers, the best such SPNE for the workers is the one in which they all invest the lowest possible h that still satisfies condition ii. Equilibria are then less and less preferred by workers as the entailed h increases. Similarly, firms can also rank multiple equilibria, should they exist, but their ordering is exactly opposite that of the workers. Firms prefer all workers to invest as much as possible since they are able to capitalize in the second stage. The best symmetric SPNE for firms that features only pure-strategies along the equilibrium path is therefore the one

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that involves the largest possible h that still satisfies condition i.

The opposing nature of the market's equilibria is a distinguishing feature of this model. Along with multiplicity, it implies that the same level of unemployment can accompany drastically different outcomes for firms and workers. Yet so far the presence of unemployment itself has been exogenously imposed by assuming $n > m$. The next section demonstrates why this assumption is well justified.

3   The Game with Entry

To incorporate workers' entry decision into the game, continue to assume that the potential number of workers is larger than the existing number of firms, m, and redefine n as the actual number of workers that invest positively (with positive probability) in equilibrium. In other words, *entry* for workers is defined by the act of investment. Those workers that invest positively are those that actively seek employment, while those who do not can be thought of as not a part of the official labor force, or perhaps even as discouraged workers. Accordingly, the unemployment rate in the market is based solely upon the number of workers actively investing.

Proposition 2. *For ϵ_h small enough, the number of workers that invest positively must exceed the total number of firms along any non-trivial equilibrium path of the sequential pre-marital investment game with entry.*

The remainder of this section first proves proposition 2, and then builds upon the results of the previous section to fully categorize possible sets of equilibria in the game with entry. The proof is by way of two constructive lemmas that demonstrate the necessity of excess workers with mild restrictions on the increments of H.

For the characterization of market entry to make sense, of course, those workers who do not enter the market should be unable to gain employment in equilibrium. If a worker who does not invest has a chance to be em-

ployed, it would not seem reasonable to consider them out of the labor force. Any worker not investing positively must therefore remain unmatched in any SPNE. Fortunately, this proves to be true for small enough ϵ_h as long as trivial equilibria are ruled out.

A trivial equilibrium in this setting is one in which no workers (and therefore no firms) invest positively along the equilibrium path, thus rendering matching meaningless. Since that case is not very interesting, it is assumed that there exists at least one $h \in H$ and one $k^*(h) \in K$ such that $u(k^*(h), h) > u(0, 0)$ and $v(k^*(h), h) > v(0, 0)$, so the employment relationship can in fact be mutually beneficial.

Lemma 3. *For small enough ϵ_h, there is no SPNE of the game with entry in which only $1 \leq n < m$ workers invest along the equilibrium path.*

Proof. If only $n < m$ workers invest positively along the equilibrium path, each worker's expected payoff must at least be greater than or equal to $u(0, 0)$. Non-investing workers may also have an expected payoff greater than $u(0, 0)$, however. If firms employ a mixed-strategy response, there is positive probability that more than n firms will end up investing positively, and in that event each non-investing worker has a chance to be matched with one of the $m - n$ lowest investing firms. There are then two possible cases.

First suppose that the expected payoff to the highest investing worker is greater than that of non-investing workers, so non-investing workers receive a payoff of less than $\mathbb{E}u(k_{1:m}, h_{1:n})$. In that case, with small enough ϵ_h, one of the non-investing workers could profitably overbid the investing workers and earn a payoff of $\mathbb{E}u(k_{1:m}, h_{1:n} + \epsilon_h)$. In addition to potentially increasing the firms' $k_{1:m}$, with a small enough ϵ_h the disutility of investing isn't severe enough to prevent the deviation from being profitable.

Next suppose that the expected payoff to the highest investing worker is actually less than the expected payoff to non-investing workers. In that case, even though overbidding may not yield a profitable deviation, one is still possible for fine enough H. Instead, a worker could invest just ϵ_h for an

expected payoff of $\mathbb{E}u(k_{n+1:m}, \epsilon_h)$. Since the $(n+1)$th firm investment is the highest reward a non-investing worker could possibly expect, small enough increments would allow a profitable deviation just by securing that reward.

Thus, if it is worthwhile for $1 \leq n < m$ workers to invest $h_{1:n} \geq h_{2:n} \ldots \geq h_{n:m} > 0$, for small enough ϵ_h it is always profitable for one more to invest. $\square$

By lemma 3, at least m workers must enter the market if any enter at all, as long as there is room for them to maneuver. The next result establishes the impossibility of full employment when workers are allowed to enter the market, meaning that $n = m$ workers investing positively can never occur along the equilibrium path of the game with entry, as long as H is not excessively sparse. Combined with lemma 3, this implies that even in the frictionless matching environment laid out here, more workers will seek jobs than there are jobs available.

Lemma 4. *For small enough ϵ_h, the number of workers investing will never be equal to the number of firms on the equilibrium path of the game with entry.*

Proof. Suppose to the contrary that there is an SPNE in which exactly m workers enter the market. On the equilibrium path, it must be the case that they all earn a payoff greater than $u(0,0)$ since all m are guaranteed a match. In fact, all workers must receive at minimum

$$\max_{h \in H} u(k^*(h), h). \tag{3}$$

Call the solution to (3) h^m. By assumption, $u(k^*(h^m), h^m) > u(0,0)$. If H is fine enough that $u(k^*(h^m), h^m + \epsilon_h) > u(0,0)$, however, there must be a contradiction of SPNE. Again, there are two cases.

If $u(k^*(h^m), h^m) > \mathbb{E}u(k_{1:m}, h^m + \epsilon_h)$ and only m workers invest along the equilibrium path then it must be that all of them invest h^m. But as long as $u(k^*(h^m), h^m + \epsilon_h) > u(0,0)$, it would be profitable for an additional worker

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to enter with $h^m + \epsilon_h$, contradicting the supposition of SPNE.

If $u(k^*(h^m), h^m) < \mathbb{E}u(k_{1:m}, h^m + \epsilon_h)$ then all m workers can not invest the same amount on the equilibrium path. It can not be that all m workers invest h^m because there is incentive to deviate upward, and it can not be that all m invest the same $h > h^m$ because then firms would simply respond with $k^*(h)$ which would entail a payoff lower than $u(k^*(h^m), h^m)$. There must therefore always be a profitable opportunity for an additional worker to enter the market and invest more than the lowest of the m already entered workers, guaranteeing employment and a payoff greater than $u(0,0)$. $\square$

So unless the game's specifications are extremely rigid, more workers than firms enter the market on the equilibrium path of an SPNE. Rather than friction in the matching process, it is simply competition that leads to a positive level of unemployment.

Appealing to the previous section's results now reveals more about the types of equilibria that may exist for the game with entry. As before, much hinges on the range of possibilities for the ratio of firms to workers. More workers than firms will enter, but how many more?

Proposition 3. *If it is non-empty to begin with, the set of symmetric, subgame perfect Nash equilibria in which all agents play pure strategies along the equilibrium path shrinks from both ends of the spectrum as n increases.*

Proof. This can be pictured as a contraction of the flatter curve in Figure 1. As n increases with a fixed number of firms, the likelihood of ending up unemployed increases when all workers invest the same h. Higher levels of h will therefore no longer satisfy condition i since there is a greater likelihood of workers receiving $u(0, h)$, and a lesser likelihood of receiving $u(k^*(h), h)$. At the same time, however, lower values of h will no longer satisfy condition ii. Because payoffs are concave in h and ϵ_h is of fixed size, lower values of h are more restrictive in terms of upward deviations being profitable. When employment is less likely, upward deviations that guarantee employment be-

come more appealing. □

A corollary to proposition 3 is that if the right number of workers enter a market in which symmetric equilibria do exist in which all agents play pure strategies along the equilibrium path, a unique such equilibrium emerges. Workers are always willing to enter the market if they can either profitably outbid those workers currently in the market, or if they can invest the same as those currently in the market and still receive a payoff higher than $u(0,0)$. That is, workers cease entering only if $n > m$ workers are already investing h such that $u(k'(h + \epsilon_h), h + \epsilon_h) \leq u(0,0)$, and the addition of one more worker investing h brings the expected payoff down below $u(0,0)$. Even if such an equilibrium does exist, however, it does not negate the possibility of mixed-strategy equilibria that allows for even more workers.

Proposition 4. *A symmetric SPNE in which all workers employ a mixed-strategy along the equilibrium path will always exist for the sequential pre-marital investment game, for any $n > m$.*

Proof. The proof is by construction. All n workers play according to a probability distribution, $G(h)$, that is supported on

$$\text{supp}(G) = [0, \epsilon_h, 2\epsilon_h, \ldots, h'(k_{1:m}) - \epsilon_h, h'(k_{1:m})],$$

where $h'(\mathbb{E}k_{1:m})$ is the highest worker investment level such that

$$\mathbb{E}u(k_{1:m}, h'(k_{1:m})) > u(0,0).$$

For all points within its support, $G(h)$ must solve

$$\sum_{t=0}^{m-1} \sum_{s=0}^{n-1-t} \frac{(n-1)!}{t!s!(n-1-t-s)!} (1-G(h))^t (g(h))^s (G(h)-g(h))^{n-1-t-s} \left[\sum_{r=0}^{s} \frac{1}{s+1} \mathbb{E}u(k_{t+1+r:m}, h) \right]$$

$$+ \sum_{t=m}^{n-1} [1-G(h)]^t u(0,h) = u(0,0), \ (4)$$

subject to the constraint that $\sum_{h \in \text{supp}(G)} g(h) = 1$. Here $\mathbb{E}u(k_{q:m}, h)$ is the expected payoff from a worker investing h and matching with qth highest expected firm investment, given that the m firms invest according to their symmetric best response, and $g(h)$ is the probability mass function associated with G. When all workers employ this strategy they receive an expected payoff of $u(0,0)$, the highest payoff any individual worker can unilaterally guarantee for themselves. By definition they are indifferent among all points within $\text{supp}(G)$, and can not improve with any deviation outside of that interval.

The firms' best response depends on the realized investments of workers. If the highest m or more worker investments are all equal, firms all invest $k^*(h_{1:n})$. Otherwise, firms all play the mixed strategy $F(k)$, defined on the interval $\text{supp}(F)$, that is the solution to equation (1). $\square$

Henceforth for brevity, the type of equilibrium defined in proposition 4, in which workers play mixed-strategies along the equilibrium path, will simply be referred to as a mixed-strategy equilibrium. Similarly, the type of equilibrium defined in proposition 1, in which all agents play pure-strategies along the equilibrium path, will be referred to as a pure-strategy equilibrium.

In spite of the fact that a mixed-strategy equilibrium can always prevail in a market with unrestricted entry, the monstrous expressions in (1) and (4) provide an obvious reason why equilibria featuring pure-strategies have been the primary focus of the paper thus far. Even die-hard game theorists may question the validity of behavioral implications from strategies so

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complex. Perhaps more importantly, however, and as the next section emphasizes, pure-strategy outcomes are also more appealing because of their welfare implications.

4   The Role of Policy

A natural starting place when making policy comparisons is the establishment of baseline welfare standards. In the sequential pre-marital investment game, the mixed-strategy equilibrium defined in proposition 4 provides one such reference point for two reasons. Firstly it will always exist, no matter the degree to which workers outnumber firms. Second, and perhaps more importantly for welfare considerations, the equilibrium's payoff to workers, $u(0,0)$, represents a worst-case scenario for those agents.

Another good reference point is the polar-opposite outcome: a best-case scenario for workers, worst-case scenario for firms, and an equilibrium that will not exist in most circumstances. Full employment, meaning that the number of workers active in the market is equal to the number of firms, can not occur on the equilibrium path of the game with entry as long as more workers have the option to enter and H is not too sparse. But if it were somehow possible to restrict the number of workers entering the market to precisely m, perhaps by restricting both population growth and immigration, any SPNE would entail workers investing no more than h^* on the equilibrium path. Since every other SPNE outcome must have workers investing h^* or greater, this is indeed the worst equilibrium outcome for firms and the best for workers.

With these two extremes in mind, and the corresponding orderings of all equilibria in between, what types of policy should planners pursue in the context of a sequential pre-marital investment game? The answer, of course, depends on their underlying objectives. The role of policy in this setting is one of equilibrium selection; by influencing the worker-firm ratio or

agents' payoffs, a central authority can determine which equilibria are or are not attainable for the market. The trick is that, given the polarized nature of the game's equilibria, pushing the market toward any one in particular involves a tradeoff between the interests of the two sides.

As previously mentioned, full employment may not be a desirable policy goal even if a central authority does have complete control over both job creation (m) and worker entry (n). Certainly it represents the least productive outcome from the standpoint of the firms. With some control over either factor, however, improvement could be possible for both sides relative to the mixed-strategy equilibrium that obtains in the absence of intervention. In particular, the pure-strategy equilibrium that involves the highest possible h while still satisfying condition i with strict inequality is certainly an improvement for workers. Whether or not it benefits firms as well, however, depends on the expected value of the mth highest investment when workers play according to $G(h)$. If $\mathbb{E}h_{m:n} < h$, then enforcing the m/n that allows that pure-strategy equilibrium to exist is a Pareto improvement. If not, then any pure-strategy equilibrium satisfying conditions i and ii with strict equality would still represent a compromise between the the interests of both sides, relative to the two extreme outcomes of full employment and unrestricted entry.

Alternative to, or in conjunction with, restrictions on market entry is the possibility of directly altering agents' payoffs. Increasing $u(0, \cdot)$, for example, could easily be justified for the sake of the workers. Such a policy is analogous to unemployment insurance or some other kind of social safety net, and in this context actually improves the welfare of all workers in SPNE by mollifying the effects of being left unmatched. Again there is a tradeoff to face, however, as the very effects that are beneficial for workers are detrimental to firms.

To illustrate, suppose that all employed workers have a tax of τ extracted from their payoff and unmatched agents receive a benefit of $\tau' = \frac{m}{n-m}\tau$. Although the expected payoff to workers in symmetric, pure-strategy equilibria
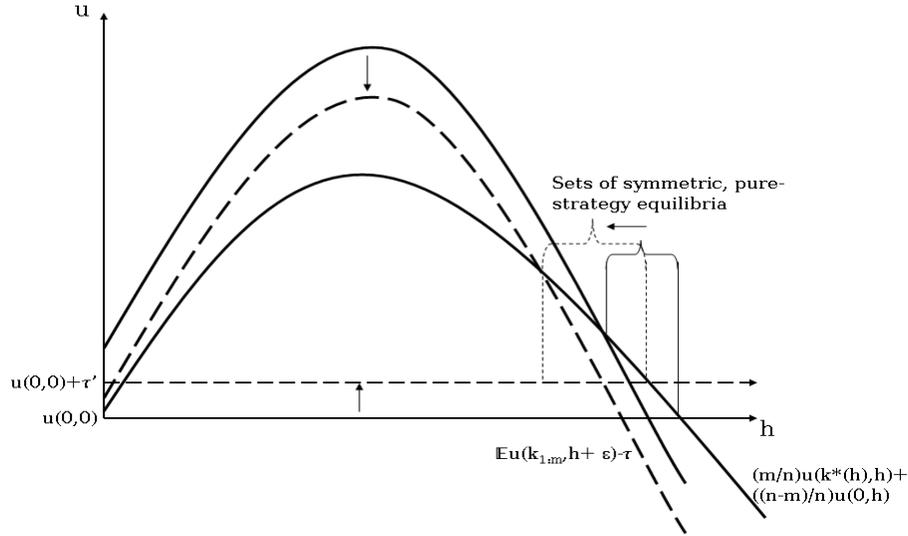
Figure 2: The Effects of Unemployment Insurance

remains unchanged, the payoffs for deviation do change. Upward deviations become less profitable since guaranteed employment is taxed, while deviation to zero becomes more appealing. Figure 2 depicts how the policy alters the set of equilibria. Pure-strategy equilibria entailing higher h levels are eliminated while some with lower h levels are admitted. Also, the upper bound of $\text{supp}(G)$ declines, since workers are not as willing to over invest.

But safety net programs are not the only way to modify agents' payoffs. Another option is to subsidize the investments made by workers, firms, or both. Interestingly, and unlike the previous two policy choices, subsidization of investment on either side of the market is almost exclusively to the benefit of firms.

In a market with unrestricted entry, making investment for workers less costly does not change the fact that they are forced to compete themselves to indifference. The only change is that higher levels of h have to be included in

supp(G), raising the value of $\mathbb{E}h_{m:n}$ and thereby increasing firms' expected payoffs. Similarly, if pure-strategy equilibria can be maintained, making investment less costly for workers only shifts the set of pure-strategy equilibrium h values upward. Since upward deviations can not be profitable in an SPNE, payoffs for workers remain held down despite cheaper investment. Also, because of the game's sequential structure, subsidizing firm investment yields the same effects. Knowing that firms' responses will be higher, workers are forced to over-invest in any SPNE, allowing firms to capitalize.

Again, the optimal policy combination will depend on the desired balance across the market. Complementary policies, however, can work together to benefit both firms and workers. Bundling safety net benefits along with subsidized human capital investment is one example with definite Pareto-improving potential. This possibility can be envisioned by an expansion of all curves in Figure 2. Unemployment insurance serves to benefit all workers, while subsidizing investment prevents the deterioration of workers' investment incentives, perhaps even increasing it beyond its initial level to improve the welfare of firms.

5   Discussion

By enhancing the productivity of relationships, pre-marital investment does much to benefit both sides of a matching market. It also leads to conflict, however. Agents on the same side of the market tend to compete each other to over-investment, imposing a type of negative externality on each other. The opposing side of the market, meanwhile, welcomes and feeds off that excess as a positive externality. When multiple equilibria are possible, the two sides therefore have opposing orders of preference over them. Each side would prefer the other to invest more.

The labor market offers a particularly interesting case of pre-marital investment, since the two sides of the market differ in the timing of their

decisions. The prospect of employment causes many individuals to make costly investments in spite of the likelihood that some may go unrewarded, or at least under-rewarded. Simply because of competition, some individuals inevitably end up ready and willing to work but without a position. Again, this paper's assertion is not that sequential pre-marital investment is the sole explanation for long-run involuntary unemployment, but only that it is a plausible contributing factor. And one with important consequences for policymakers.

Because unemployment is such a perpetual problem, and more broadly because of the vital role it plays in any economy, the labor market is often a target of policy. Vast amounts of government resources are put toward improving both the welfare of workers and the productivity of firms. The results of this paper suggest that policymakers take into account possible tradeoffs between the two, and be aware that benefitting one group may come at the expense of the other. Despite the somewhat Marxian conflict that pre-marital investment fosters across the two sides of the market, however, the right combination of policies can lead to Pareto improvements.

The benefits of appropriately combined policies may be exemplified by the success of Northern European countries such as Norway, Sweden, and Denmark. All of these countries have relatively restrictive labor markets and high unemployment benefits, but also tremendous public spending on education. Other European countries that also have restrictive labor markets (Spain and Italy, for example), but with lesser unemployment benefits and much less public spending on education, have not shared the same levels of economic success. More in depth cross-country empirical investigation on this matter offers a promising area for future research, as it may reveal more detailed support for the effectiveness of labor market policy combinations, and the importance of pre-marital investment considerations in general.

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