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## **Abstract**

The capital structure and regulation of financial intermediaries is an important topic for practitioners, regulators and academic researchers. In general, theory predicts that firms choose their capital structures by balancing the benefits of debt (e.g., tax and agency benefits) against its costs (e.g., bankruptcy costs). This paper studies the impact and interaction of deposit insurance, capital requirements and tax benefits on a bank's choice of optimal capital structure. Using a contingent claims model to value the firm and its associated claims, we find that there exists an interior optimal capital ratio in the presence of deposit insurance, taxes and a minimum fixed capital standard as long as there is a significant financial burden associated with violating capital requirements. Banks voluntarily choose to maintain capital in excess of the minimum required in order to balance the risks of insolvency (especially to future tax benefits) against the benefits of additional debt. Because our model includes all three contingent claims, our results differ from those of previous studies of the capital structure of banks that have generally found corner solutions (all equity or all debt) to the capital structure problem.

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## 1. Introduction

Since the seminal work of Modigliani and Miller (1958), the corporate finance literature has developed a number of different theories to explain the observed cross-sectional variation in capital structure. Prominent among these explanations is that firms choose their leverage by balancing the benefits and costs of an additional dollar of debt.<sup>1</sup> The benefits of increasing debt include tax benefits and the reduction of agency costs associated with free cash flow (Jensen and Meckling, 1976). The costs of additional debt include bankruptcy costs and agency costs associated with conflicts of interest between shareholders and debtholders (Myers and Majiluf, 1994). Banks, as financial intermediaries, are different than other firms. Significantly, banks have the unique benefit of being able to issue federally insured debt; but they also bear the cost of strict capital regulations, including the threat of being placed in receivership and wiping out the investment of the shareholders. This paper examines how these special characteristics influence the optimal capital structure of banks.

To date, academic studies of bank capital have generated conflicting predictions. First, traditional moral hazard theory has been applied to predict that banks with deposit insurance will choose extremely high levels of leverage. Moral hazard arises when the insurance premium does not reflect the underlying risk of the insured's activities. Insured deposits represent the lowest cost capital source for banks and, in recent years, most solvent banks have paid almost no premium for their insurance. Thus, based strictly on the explicit pricing of deposit insurance, theory predicts that banks with deposit insurance should choose high leverage. These models

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<sup>1</sup> See Fama and French (2002) for a general description of the major competing theories and a test of the predictive power of those theories. They identify two major theories—the trade off theory described in the text and the pecking order theory proposed by Myers (1984). Because, under our assumptions, banks can issue riskless debt without market constraints, the Myers' pecking order theory is less applicable to banks.

suggest that banks should set capital ratios to the lowest level permitted by capital regulations. (See Keeley, 1980, Marshall and Prescott, 2000, Gueyie and Lai, 2003, for further discussion.)<sup>2</sup>

However, the insurance premium paid by banks for deposit insurance is only one component of the total regulatory cost associated with deposit insurance and other studies that consider these regulatory costs generally predict that banks will not choose high leverage. Banks are highly regulated entities and the regulators establish minimum capital requirements, investment regulations and monitoring mechanisms to restrict a bank's ability to take on excessive risk and/or leverage. Buser, Chen and Kane (1981) point out that banks face significant costs that are not explicitly priced attributable to regulations, investment restrictions and monitoring. Furthermore, the regulatory and monitoring functions of the regulator are influenced by a bank's choice of leverage. In general, the implicit cost of the regulatory burden involved increases with leverage and the increased cost serves to offset the bank's incentive for higher leverage. Merton (1978) develops a contingent claims model of bank leverage that includes explicit regulatory costs for insolvent banks. He shows that this regulatory burden can be significant enough to create a preference for equity among solvent banks. Based on Merton's model, Marcus (1984) explicitly examines bank capital structure under capital regulation and argues that "for solvent banks, increases in capital are wealth-increasing, while for sufficiently insolvent banks, capital withdrawals increase owner's wealth."<sup>3</sup> Thus, both models seemingly contradict the moral hazard prediction that banks will choose the highest possible leverage.

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<sup>2</sup> Other authors have examined the optimal capital structure of banks from different perspectives and concluded that banks will choose high leverage. For instance, Diamond and Rajan (2000) and Diamond (2001) consider the optimal capital structure of banks as the result of the tradeoff between liquidity creation, costs of bank distress, and the ability to force borrower repayment. Their model suggests an optimal capital structure with high leverage.

<sup>3</sup> Marcus extends his model to consider the value of a bank's charter—a value that is lost if the bank fails and is arguably in part attributable to the availability of deposit insurance. He finds that this extension reinforces his prediction that solvent banks should choose low leverage. Hughes, Lang, Moon and Pagano (2003) find empirical evidence in support of Marcus's proposition about the importance of charter value.

However, neither Merton's nor Marcus's model explicitly includes the benefits of deposit insurance.

The conflicting predictions about bank capital structure arise because each theory focuses on one side of the story: either the benefits or costs associated with the deposit insurance system. This paper builds upon the model developed by Leland (1994) to provide a comprehensive framework of bank capital structure decisions under a deposit insurance system. Leland derives a closed-form expression for the optimal capital structure of a firm that issues risky debt in the presence of bankruptcy costs and tax-advantaged debt. The first case that we consider is where banks operate without any capital regulations and choose their bankruptcy threshold endogenously. In our second case, the bank's insolvency threshold is established by a regulator, and the capital regulations require the liquidation of the bank as soon as the capital ratio falls below the threshold. This case is similar in spirit to Leland's model of protected debt.

We find that there exists an interior optimal capital ratio in banks with deposit insurance, a minimum capital ratio and tax-advantaged debt. That is, banks voluntarily choose to hold capital in excess of the required minimum. This does not mean that minimum capital requirements are ineffective. Rigid capital requirements threaten all banks with the prospect of losing the value of their equity if the bank violates the requirement as the result of random fluctuations in asset values. Accordingly, banks choose capital ratios well above the minimum requirement to maximize the expected value of their equity. If there are no capital requirements, we find that banks choose a corner solution with very high leverage. Hence the real function of capital requirements is to create a cost of insolvency that replaces bankruptcy costs.

The basic model is examined with and without tax-advantaged debt, and we find that tax-advantaged debt is central to the existence of an interior optimal capital ratio. Bankruptcy costs

and insurance benefits are fairly small and increase monotonically with deposits. This behavior leads to a corner solution with the firm choosing all debt or equity depending upon which factor dominates. For reasonable choices of model parameters, the value of the contingent claim associated with tax-advantaged debt is much larger than either the insurance benefits or the bankruptcy costs and capital regulation places this value at risk due to the possibility of forced liquidation. Therefore, in the presence of tax-advantaged debt, banks voluntarily choose to maintain capital to protect the value of their tax benefits – even under regulatory regimes that would lead to banks choosing to finance its assets with all debt in the absence of tax benefits. The risk of the loss of tax benefits is a significant part of the cost of insolvency.

In the final section of the paper, we extend the model to analyze the case with two regulator-set bankruptcy thresholds: a high “warning” level and a lower insolvency level where the bank is liquidated. The higher warning threshold results in increased monitoring and scrutiny that imposes additional costs on the bank. We find that the imposition of this second threshold has only a small impact on the optimal level of leverage of banks even in the face of substantial regulatory costs. This finding emphasizes the importance of the threat of forced liquidation when considering the effectiveness of capital regulation applied to insured financial institutions.

The remainder of this paper is organized as follows. The second section develops a model of the capital structure of banks with and without capital regulations. The third section analyzes the bank’s optimal capital structure in an environment without tax-advantaged debt.<sup>4</sup> The fourth section extends the analysis to include the effects of tax benefits. The fifth section discusses a numerical example and the sixth section extends the model to include multiple

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<sup>4</sup> We consider the case where debt is not tax-advantaged because, historically, many depository institutions in the U.S. have been granted tax provisions that lower effective tax rates or been exempt from the income tax. For example, federally chartered savings and loan associations were originally exempt from taxation and subsequently were granted special tax provisions (e.g., bad debt provisions) that lowered their effective tax rate. Currently, credit unions are the only major group of depositories that are exempt from the corporate income tax.

insolvency thresholds. Section seven discusses the policy implications of the models and concludes the paper.

## **2. Bank Capital Structure with Deposit Insurance and Capital Regulation**

In developing our model of the capital structure of banks, we follow the derivation of Leland (1994). In Leland's framework, a firm's assets are financed with a combination of debt and equity. Uncertainty enters the model because the firm's assets are assumed to evolve stochastically. To assure that the stochastic process for the assets is unaffected by the capital structure choices of the firm, debt service payments are made by selling additional equity. This implies that the face value of deposits is static over time. In applying this framework to banks, we assume that banks have only one form of debt—fully insured deposits and that these deposits are deemed by investors to be riskless. Consistent with recent experience in the U.S., we further assume that banks do not pay an insurance premium for deposit insurance. Under these assumptions, banks pay the riskless rate on all deposits. As in Leland, we assume that values evolve continuously and that the firm's capital structure decision is summarized by its choice of a promised continuous payment  $C$ .<sup>5</sup>

We assume that the firm's portfolio of assets,  $V$ , comprises continuously traded financial securities<sup>6</sup>, the market value of which follows a standard geometric Brownian motion process:

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<sup>5</sup> Consider an investor who is forming a bank. We assume that the firm's initial book of assets,  $V$ , is fixed and the bank owner must choose how to best finance those assets—with either debt or equity. In our framework, the bank owner must first choose the optimal amount of deposits to issue to the public. The bank's owner must then contribute the remaining funds needed to purchase the initial assets. Thus, given that the total cost of the bank's assets is fixed, the choice of the deposit amount simultaneously determines the equity contribution and capital structure. We assume that, as a practical matter, local market conditions and federal regulations impose upper bounds on firm size and the amount of deposits that can be raised.

<sup>6</sup> A bank's assets comprise two major categories: loans and securities. While the assumption of active trading is valid for the securities component, we assume that the loan component is perfectly correlated with some actively traded benchmark security. We believe this assumption is reasonable given the close linkage between loan rates and capital market rates.



$$dV = \mu V dt + \sigma V dW \quad (1)$$

Following Cox, Ingersoll and Ross (1985), assuming a fixed riskless rate of  $r$ , any claim,  $F(V,t)$ , contingent on the firm's asset value with a continuous payment,  $C$ , must satisfy the standard partial differential equation (with boundary conditions determined by payments at maturity and/or time of insolvency):

$$\frac{1}{2} \sigma^2 V^2 F_{VV} + r V F_V + F_t - r F + C = 0 \quad (2)$$

While, in general, this partial differential equation does not have a closed form solution for arbitrary boundary conditions, if  $F_t=0$ , then equation (2) becomes an ordinary differential equation with the general solution:

$$F(V) = A_0 + A_1 V + A_2 V^{-X} \quad (3)$$

where  $X=2r/\sigma^2$  and  $A_0$ ,  $A_1$  and  $A_2$  are determined by the boundary conditions. In the context of corporate debt (Leland, 1994), the assumption  $F_t=0$  can be justified by considering only long maturity debt or debt that is continuously rolled over at a fixed rate or a fixed spread to a benchmark rate.<sup>7</sup> The latter justification is also applicable to banks. Ever though most bank deposits technically have short maturities, as long as the bank is solvent and maintains competitive pricing, it can rollover deposits at the riskless rate because depositors do not have an incentive to monitor a bank's financial condition. For example, although demand deposits at banks can be withdrawn at any time by the customer, in bank acquisitions, these deposits are generally viewed as a long-term, stable source of funds and hence part of the charter value of the bank.

Using the general solution in equation (3), we can obtain an expression for the major claims that influence the market value of a firm and the market value of the equity claim held by

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<sup>7</sup> See Leland (1994) for a detailed explanation, and Leland and Toft (1996) for a model with finite maturity debt.

the owners of the firm. The market value of a firm that is financed entirely by equity is equal to the market value of its assets. However, when a firm is financed by debt and equity, additional factors must be considered. First, the issuance of debt raises the possibility of forced liquidation in the event of insolvency. If there are deadweight costs associated with liquidation in the event of insolvency (defined by the market value of assets hitting some threshold  $V_B$ ), firm value is reduced by the current market value of those bankruptcy costs. If the firm is subject to taxation and interest payments on debt are tax deductible expenses, then firm value is increased by the tax benefits associated with debt financing. In addition, banks have a third factor to consider—the value of the insurance provided by the federal government. We view the insurance as a contingent claim that makes a payment to the firm equal to the difference between the realizable value of the firm's assets and the face value of the debt. These costs can be viewed as a contingent claim on  $V$  and valued using equation (3) and boundary conditions, and we can define the market value of the bank,  $v$ , as.

$$v = V - BC(V) + TB(V) + IB(V) \quad (4)$$

### *Bankruptcy Costs*

To apply equation (3) to value bankruptcy costs, we need to identify the appropriate boundary conditions that reflect the actual payments associated with the claim. When a bank becomes insolvent, its assets are liquidated. The liquidation is triggered when the value of the firm's assets,  $V(t)$ , falls to a specified level,  $V_B$ . For our current purpose, it does not matter how  $V_B$  is set—only that it is an observable constant. We assume that when liquidation occurs, the firm will receive a fraction of the current market value of the assets,  $(1-\alpha)V_B$ , where  $0 < \alpha < 1$ . The inability to realize full market value can be attributed to the need for large bulk sales in a short time and problems of asymmetric information associated with the sale of loans. The second

boundary condition is established by the fact that as  $V(t)$  gets very large, the possibility of liquidation becomes increasingly remote and the market value of the bankruptcy costs approaches zero. We thus have the following two boundary conditions:

$$\begin{aligned} BC(V) &= \alpha V_B, \text{ when } V = V_B \\ BC(V) &= 0, \text{ as } V \rightarrow \infty \end{aligned}$$

Using these conditions with equation (3) results in the following expression for  $BC(V)$ :

$$BC(V) = \alpha V_B \left( \frac{V}{V_B} \right)^{-X}, \text{ where } X = 2r/\sigma^2 \quad (5)$$

The term  $\left( \frac{V}{V_B} \right)^{-X}$  can be interpreted as the present value of \$1 payable when the random variable  $V(t)$  first reaches  $V_B$ . With this interpretation, the market value of bankruptcy costs can be viewed as the expected present value of the deadweight costs associated with liquidation,  $\alpha V_B$ . Because  $X > 0$ , as the current value of  $V$  increases, the expected present value factor declines and the market value of the claim falls. As  $V$  approaches  $V_B$  from above, the present value approaches its maximum of 1 and the market value of the claim approaches  $\alpha V_B$ .<sup>8</sup>

### *Tax Benefits*

According to the U.S. tax code, interest payments are deductible from corporate earnings when computing a firm's corporate income tax. Thus each dollar of interest paid results in a savings in taxes for a taxpaying firm equal to its marginal tax rate times the interest paid. Following Leland (1994), we assume that tax benefits for a solvent firm are proportional to the interest payment on its debt and are terminated at the insolvency threshold,  $V_B$ .<sup>9</sup> Thus, the

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<sup>8</sup> Because the bank is liquidated as soon as  $V$  first reaches  $V_B$ ,  $V$  can never be less than  $V_B$ .

<sup>9</sup> Because the tax deductibility of interest results in a tax savings only when the firm has taxable income, the tax benefits generally stop before firm liquidation. However, the tax code also provides firms that are currently not earning enough to utilize their tax benefits limited flexibility to carry taxable losses forward until they return to profitability.

market value of future tax benefits is zero when  $V$  hits  $V_B$ . For the second boundary condition, we again consider the case as  $V$  gets very large. As  $V$  increases relative to  $V_B$ , the likelihood of insolvency declines and the possibility of losing the tax benefits becomes remote. In this case, the tax benefits have a market value equal to the present value of a continuously paid perpetuity of  $\tau C$ , where  $\tau$  represents the marginal tax rate and  $C$  denotes the continuously interest on the debt. Therefore the boundary conditions for valuing the tax benefits are:

$$\begin{aligned} TB(V) &= 0, & \text{when } V &= V_B \\ TB(V) &= \frac{\tau C}{r}, & \text{as } V &\rightarrow \infty \end{aligned}$$

Using these conditions with equation (3) results in the following expression for the market value of tax benefits:

$$TB(V) = \frac{\tau C}{r} \left( 1 - \left( \frac{V}{V_B} \right)^{-X} \right) \quad (6)$$

which is unambiguously positive for  $V > V_B$ . The form of the expression for tax benefits is identical to that for the bankruptcy costs. The only difference lies in the expected present value factor which simply reflects the reversal of the boundary conditions.

### *Insurance Benefits*

Deposit insurance covers the gap between the realizable value of assets and the face value of deposits if a bank must be liquidated. Thus, when  $V = V_B$ , the insurer must pay the  $\text{Max}[(D - (1 - \alpha)V_B), 0]$ , where  $D$  is the face value of deposits. Given the earlier assumptions and denoting the promised, continuous, interest payment as  $C$ , then the market value of the debt is simply  $D = C/r$ . However, when  $V$  becomes very large and the likelihood of insolvency declines, the market value of the insurance payment claim falls. Therefore, we have the following boundary conditions for the insurance benefit:

$$\begin{aligned}
IB(V) &= \text{Max}\left[\left(\frac{C}{r} - (1-\alpha)V_B\right), 0\right] && \text{when } V = V_B \\
IB(V) &= 0, && \text{as } V \rightarrow \infty
\end{aligned}$$

The market value of the insurance benefit claim is:

$$IB(V) = \text{Max}\left[\left(\frac{C}{r} - (1-\alpha)V_B\right)\left(\frac{V}{V_B}\right)^{-x}, 0\right] \quad (7)$$

As long as the insolvency threshold,  $V_B < \frac{C/r}{(1-\alpha)}$ , the insurance payment will be greater than zero and the max operator can be ignored in equation (7). If  $V_B > \frac{C/r}{(1-\alpha)}$ , the market value of insurance benefits is uniformly zero. We will show shortly that when the bank is free to select  $V_B$ , it will choose a level strictly less than  $C/r$  and so the value of insurance benefits is strictly positive in that case. When the insolvency threshold is determined by the regulator, it is reasonable to assume that the regulator will also set  $V_B \leq \frac{C/r}{(1-\alpha)}$  since a higher threshold totally eliminates all insurance benefits.

Also, conditional on  $V_B \leq \frac{C/r}{(1-\alpha)}$ ,

$$IB(V) - BC(V) = \left(\frac{C}{r} - V_B\right)\left(\frac{V}{V_B}\right)^{-x} \quad (8)$$

The canceling out of the term  $\alpha V_B$  in equation (8) confirms that deposit insurance has the effect of transferring the burden of bankruptcy costs from the firm to the insurer. In this case, the deadweight cost factor,  $\alpha$ , does not affect the overall firm value in the presence of deposit insurance.

### *Firm Value and Equity Value*

Substituting equations (5), (6) and (7) into the expression for the market value of the bank gives the following:

$$v(V) = V + TB(V) - BC(V) + IB(V) = V + \left[ \left(1 - \tau\right) \frac{C}{r} - V_B \right] \left( \frac{V}{V_B} \right)^{-X} + \frac{\tau C}{r} \quad (9)$$

Since  $v(V)$  must equal the sum of the market values of debt and equity and under our assumptions  $D(V)=C/r$ , the market value of equity is simply the market value of the firm less  $C/r$ . Thus,

$$E(V) = V + TB(V) - BC(V) + IB(V) - \frac{C}{r} = V + \left[ \left(1 - \tau\right) \frac{C}{r} - V_B \right] \left( \frac{V}{V_B} \right)^{-X} - (1 - \tau) \frac{C}{r} \quad (10)$$

Table 1 summarizes the assumptions made in the above derivation.

## **3. Optimal Capital Structure of Banks without Tax-Advantaged Debt**

### *3.1 Without Capital Regulation*

To study the capital structure choice in the absence of tax benefits, we set  $\tau=0$  in equations (9)

and (10). Conditional on  $V_B < \frac{C/r}{(1-\alpha)}$ , the value of the firm simplifies to the following<sup>10</sup>:

$$v(V) = V + TB(V) - BC(V) + IB(V) = V + \left[ \left( \frac{C}{r} \right) - V_B \right] \left( \frac{V}{V_B} \right)^{-X} \quad (11)$$

$$E(V) = V - \frac{C}{r} = \left[ \left( \frac{C}{r} \right) - V_B \right] \left( \frac{V}{V_B} \right)^{-X} - \frac{C}{r} \quad (12)$$

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<sup>10</sup> The case where  $V_B > \frac{C/r}{(1-\alpha)}$  is not explicitly studied here because as will be shown, bank managers would not set  $V_B$  in that range, and if they did, the results would be uninteresting in that bank managers would then choose  $C=0$  (i.e., choose not to issue deposits) to maximize firm value. Similarly, bank regulators would not set  $V_B > \frac{C/r}{(1-\alpha)}$  because no firm would voluntarily choose to become an insured bank. That is, all firms would choose  $C=0$ .

As in Leland (1994), consider a value maximizing bank that is free to choose the optimal  $V_B$ .<sup>11</sup> Since  $V$  is exogenously determined, the choice of  $V_B$  is fully determined by the second term in equation (11), which as noted above is equal to the difference between insurance benefits and bankruptcy costs. We denote that term as the net insurance benefits. The higher one sets the insolvency threshold, the lower the net insurance payout in the event of insolvency,  $[C/r - V_B]$ , but the higher is the likelihood of insolvency and the expected present value factor  $\left(\frac{V}{V_B}\right)^{-X}$  (which we denote as the “discount” factor). The behavior of the product of the net insurance payout and the discount factor is demonstrated by considering two cases:  $V_B=0$  and  $V_B=C/r$ . As  $V_B$  approaches zero from above, the discount factor approaches zero making the product approach zero.<sup>12</sup> Similarly, when  $V_B=C/r$ , the net insurance payout is zero and hence the product is zero. Between those points, the product is positive for all intermediate values of  $V_B$ . Thus, there is clearly an interior maximum value of  $V_B$ .<sup>13</sup> Equation (12) shows the first order condition for a maximum of  $v$  with respect to  $V_B$ .

$$\frac{\partial v}{\partial V_B} = (1 - X) \left(\frac{V}{V_B}\right)^{-X} + X \left(\frac{C}{r}\right) \left(\frac{V}{V_B}\right)^{-X} V_B^{-1} = 0 \quad (13)$$

Solving equation (13) for  $V_B$  provides the following expression for the optimal insolvency threshold.

$$V_B^* = \left(\frac{X}{1 + X}\right) \left(\frac{C}{r}\right) \quad (14)$$

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<sup>11</sup> In our model, firm value and equity value differ by  $C/r$ . Thus for a fixed  $C$  and  $r$ , there is no difference between choosing an insolvency threshold that maximizes firm value and one that maximizes equity value.

<sup>12</sup> The discount factor approaches zero as  $V_B$  approaches zero because the probability of liquidation goes to zero. The fact that all future values of  $V$  must be non-negative is a standard property of models where the market value of the firm’s portfolio of assets is assumed to follow a standard geometric Brownian motion process.

<sup>13</sup> The concavity of equation (11) with respect to  $V_B$  can be verified by taking the second derivative with respect to  $V_B$ .

As pointed out by Leland (1994), we must consider an additional condition. Because of the limited liability of shareholders, we need to consider the credibility of a corporate promise to liquidate its assets precisely when  $V$  first falls to the level  $V_B$ . Specifically, we must assure that the market value of equity is zero when  $V=V_B$  and nonnegative for all values of  $V>V_B$ . If the market value of equity were positive when  $V=V_B$ , the firm could not credibly commit to liquidate at  $V_B$  even though establishing that liquidation threshold in advance maximizes the present value of the firm. In addition, if the market value of equity were to become negative as  $V$  falls toward  $V_B$ , the firm would no longer be able to raise equity to pay the promised debt service payment,  $C$ , and would default triggering liquidation above  $V_B$ . We begin by repeating equation (12) which provides the value of equity when  $\tau=0$ :

$$E(V) = V + \left[ \left( \frac{C}{r} \right) - V_B \right] \left( \frac{V}{V_B} \right)^{-X} - \frac{C}{r} \quad (12)$$

The equation confirms that equity must have zero value when  $V=V_B$ . We next examine the first and second derivatives of  $E(V)$  with respect to  $V$ :

$$\begin{aligned} \frac{\partial E}{\partial V} &= 1 - X \left( \frac{C}{r} - V_B \right) V_B^X V^{-X-1} \\ \frac{\partial^2 E}{\partial V^2} &= X(X+1) \left( \frac{C}{r} - V_B \right) V_B^X V^{-X-2} \end{aligned} \quad (15)$$

Evaluating these derivatives at  $V=V_B^*$  confirms that  $E(V)$  is strictly convex for values of  $V_B < C/r$ . Thus the market value of equity is zero when  $V=V_B^*$  and is increasing in  $V$  at that point. Therefore, the optimal choice of insolvency threshold,  $V_B^*$ , is credible in that rational firm owners will never liquidate when  $V>V_B^*$  and have no reason not to liquidate when  $V=V_B^*$ .

Substituting the optimal insolvency threshold  $V_B^*$  into equations (11) and (12), we obtain the following expressions for the market value of the firm and the market value of equity.



$$v(V) = V + \left(\frac{C}{r}\right)\left(\frac{C}{V}\right)^X h, \text{ where} \quad (16)$$

$$h = \frac{1}{(1+X)} \left[ \frac{X}{r(1+X)} \right]^X > 0$$

and

$$E(V) = V + \left(\frac{C}{r}\right) \left[ \left(\frac{C}{V}\right)^X h - 1 \right] \quad (17)$$

Since the second term in equation (16) is always positive, the market value of the firm is always greater than the market value of its financial assets, V, as long as the bank raises deposits. More significantly, the partial derivative of v with respect to C is positive:

$$\frac{\partial v}{\partial C} = \frac{h}{r} V^{-X} (1+X) C^X > 0 \quad (18)$$

so the bank has an incentive to increase C to its maximum possible value. Given a fixed value of V, C would be set equal to the product rV so that the firm is entirely debt financed.

### 3.2 Optimal Capital Structure of Banks with Capital Regulation

Bank capital requirements have grown increasingly complex in recent years and generally include a requirement to hold “capital” that exceeds a specified percentage of the book value of assets and a different percentage of “risk-weighted” assets.<sup>14</sup> Two categories of capital are defined in the regulations, Tier I and Tier II. Tier I capital includes paid-in-capital, common stock, retained earnings, noncumulative preferred stock and certain other elements and is closest to the notion used in this paper of capital as equity. Tier II capital includes Tier I capital plus

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<sup>14</sup> To calculate a bank’s risk weighted assets, one multiplies each category of assets by a factor that reflects the credit risk of that type of assets. The factors range from 0 to 1. In addition, certain off-balance sheet activities (e.g., letters of credit and derivatives) also contribute to the sum of risk weighted assets. For most banks, the risk weighted assets total less than the book value of their assets. In recent years, bank regulators from major countries have been working on a new accord (Basel II) that would refine the risk weights.

subordinated debt, loss reserves, cumulative preferred stock and certain other debt instruments that are subordinate in priority to deposits. To be deemed “Well Capitalized” (the highest category), a bank must maintain Tier I capital that exceeds five percent of its book assets and six percent of its risk weighted assets and Tier II capital that exceeds ten percent of its risk weighted assets.

We consider a simplified version of these regulations where a bank is required to maintain the market value of its assets,  $V$ , above some threshold that is related to the face value of its deposits.<sup>15</sup> That is, the bank is required to maintain  $V > \beta D$ , where the parameter  $\beta$  measures the stringency of the capital requirement. At this time, we assume a single capital threshold and further assume that the bank is liquidated if it does not meet the specified requirement—i.e., when  $V$  first falls to  $\beta D$  or  $\beta(C/r)$ . Therefore, once the bank chooses  $D$ ,  $\beta D$  can be viewed as the insolvency threshold.<sup>16</sup> For the regulatory constraint on capital to have any effect,  $\beta > \frac{x}{1+x}$ . If  $\beta$  were set below this level, it would have no effect on the bank because it would be below the insolvency threshold the bank would select in the absence of capital regulation.

This regulatory environment can be expressed in the more traditional language of minimum capital requirements and maximum leverage using the basic accounting identity that  $V = D + Eq$ , where  $Eq$  denotes the book value of equity not the market value of equity,  $E(V)$ . A requirement to maintain a minimum level of capital can be thought of as requiring  $(Eq/V)$  to remain above the specified threshold  $\underline{c}$ . Using the accounting identity, this establishes a maximum leverage ratio,  $D/V < 1 - \underline{c} = \ell^*$  and, in turn, that  $V > D/1 - \underline{c}$ . Thus,  $\beta = 1/1 - \underline{c}$ .

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<sup>15</sup> This simplified regulation structure is equivalent to considering a bank that only has Tier I capital and a low risk portfolio for which the book assets capital ratio is the binding constraint.

<sup>16</sup> Note that  $\beta = 1$  is equivalent to Leland’s (1994) case of protected debt.

To distinguish between the endogenously chosen  $V_B$  and the regulator-imposed insolvency threshold, we use the term  $V_R$  to denote the regulator-imposed insolvency limit,  $V_R = \frac{\beta C}{r}$ . Substituting  $V_R$  (and its definition) into equation (11) for the value of the firm, we have:

$$v(V) = V + \left[ \frac{C}{r} - V_R \right] \left( \frac{V}{V_R} \right)^{-X} = V - \left( \frac{C}{r} \right) \left( \frac{C}{V} \right)^X p, \text{ where } p = (\beta - 1) \left( \frac{\beta}{r} \right)^X \quad (19)$$

Similarly, substituting  $V_R$  into equation (12) yields:

$$E(V) = V - \left( \frac{C}{r} \right) \left[ \left( \frac{C}{V} \right)^X p + 1 \right] \quad (20)$$

It is interesting to note that the bank's optimal choice of  $C$  and hence leverage depends in a "knife-edge" way on  $\beta$ . When  $\beta > 1$ , then  $p > 0$  and  $\underline{c} > 0$  and the market value of the bank,  $v(V)$  is a monotonic decreasing function of the coupon payment,  $C$ .<sup>17</sup> For this level of  $\beta$ , the bank is liquidated before  $V$  falls below the market value of deposits and thus while the shareholders still have positive equity. This equity is wiped out by the liquidation and used to pay a portion of the deadweight bankruptcy costs. The "cost" to the equity holders from this expected loss exceeds the market value of the small insurance payout when  $\beta > 1$ .<sup>18</sup> The model only contains two contingent claims: bankruptcy costs that favor equity financing and insurance benefits that favor debt financing. Both claims payoff when the firm's assets,  $V$ , reach  $V_R$ . When  $\beta > 1$ , the losses covered by insurance are less than the total bankruptcy cost and the equity-favoring bankruptcy cost contingent claim dominates the debt-favoring insurance benefits. As a result, the bank would set  $C^* = 0$ .

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<sup>17</sup> Note that  $\frac{\partial v}{\partial C} = -\frac{p}{r} V^{-X} (1+X) C^X < 0$ , for  $p > 0$ .

<sup>18</sup> This effect can also be seen in equation (11) with  $V_B$  set equal to  $V_R > C/r$ . The second term on the right hand side of the equation is negative, implying an expected loss from liquidation, net of the insurance benefits

When  $\beta < 1$ , then  $p < 0$  and  $c < 0$ <sup>19</sup>, the market value of the bank,  $v$  becomes a monotonic increasing function of  $C$  since  $\frac{\partial v}{\partial C} > 0$ . In this case, the value of the insurance benefits contingent claim always exceeds the market value of the bankruptcy cost contingent claim. This leads to the same bank behavior as when it could set the insolvency threshold itself: the bank prefers the highest leverage possible. When  $\beta = 1$ , then  $p = 0$  and the market value of the bank always equals  $V$  and the bank is indifferent between funding its operations with debt or equity.

#### 4. Optimal Capital structure of Banks with Tax-Advantaged Debt

In this section we consider the full model with insurance benefits, bankruptcy costs and tax-advantaged debt. As in Section 3, we will first consider the case without capital regulations and then the case with a regulator-imposed insolvency threshold.

##### 4.1 Without Capital Regulation

The equations from Section 2 for the market value of the bank and the market value of the bank's equity are repeated here for convenience.

$$v(V) = V + TB(V) - BC(V) + IB(V) = V + \left[ \left( (1 - \tau) \frac{C}{r} \right) - V_B \right] \left( \frac{V}{V_B} \right)^{-X} + \frac{\tau C}{r} \quad (9)$$

$$E(V) = V + TB(V) - BC(V) + IB(V) - \frac{C}{r} = V + \left[ \left( (1 - \tau) \frac{C}{r} \right) - V_B \right] \left( \frac{V}{V_B} \right)^{-X} - (1 - \tau) \frac{C}{r} \quad (10)$$

As in Section 3, the endogenous insolvency threshold is selected by the bank to maximize the market value of the firm given by equation (9). The derivative  $v$  with respect to  $V_B$  has nearly an

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<sup>19</sup> A value of  $c < 0$  can arise if the definition of capital is based on an inappropriate accounting method. For example, if the available capital is based on the historical cost (e.g., book value) of assets and not the market value, then even though the technical capital regulation calls for positive capital, the effective capital requirement can be negative. This type of situation arose in the early 1980s when savings and loan associations were allowed to operate with negative market value of equity. Their rapid growth funded with new deposits in that era is consistent with the prediction of our model.

identical form except for the factor  $(1-\tau)$  and the optimal bankruptcy threshold takes a form nearly identical to equation (13):

$$V_B^* = \left( \frac{X}{1+X} \right) \left( (1-\tau) \frac{C}{r} \right), \text{ where } X = \frac{2r}{\sigma^2} \quad (21)$$

Inspection of equation (10) confirms that  $E(V_B^*)=0$ . As in Section 3.1, without taxes, the first derivative of equation (10) with respect to  $V$  is zero at  $V_B^*$ , and the second derivative is positive for all  $V > V_B^*$ , thereby assuring that the smooth pasting/credible bankruptcy threshold condition is satisfied at  $V_B^*$ .

Conditional on the firm's choice of this insolvency threshold, the choice of optimal capital structure is the same as it was without tax-advantaged debt - the firm will choose to increase leverage to the upper limit by setting  $C=rV$  and total debt,  $D=V$ .<sup>20</sup> Relative to the results in Section 3.1, the addition of tax benefits simply reinforces the excess of insurance benefits over bankruptcy costs associated with increased leverage and thus enhances the bank's preference for additional deposits.

#### *4.2 Optimal Capital Structure of Banks with Capital Regulation*

As in Section 3.2, we assume that the regulator sets  $V_R=\beta(C/r)$ . Substituting this into equation (9) yields a general expression for the market value of a bank facing this form of insolvency threshold. The market value of the firm over time is:

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<sup>20</sup> The basic form of the partial derivative of  $v(V)$  with respect to  $C$  (equation (18)) remains essentially the same as it was without tax-advantaged debt, with an additional term representing tax benefits, which is also unambiguously positive.

$$v(V) = V + IB(V) - BC(V) + TB(V) = V + \left[ \tau - k \left( \frac{C}{V} \right)^X \right] \left( \frac{C}{r} \right), \text{ where} \quad (22)$$

$$k = (\tau + \beta - 1) \left( \frac{\beta}{r} \right)^X$$

Depending on the magnitudes of  $\tau$  and  $\beta$ ,  $k$  can be positive, negative or zero. In practice, however, we expect  $(\tau + \beta) > 1$ <sup>21</sup> and thus we expect that  $k > 0$ . In fact, as will be seen later in equation (23), an interior optimal value of  $C$  does not exist if  $k \leq 0$ . With  $k > 0$ , the sign of the second term in equation (22) is indeterminate and the market value of the firm can be greater than or less than the value of its assets,  $V$ , depending on the magnitudes of  $C$  and  $\beta$ .

To find the optimal value of  $C$  (and hence the optimal leverage), we calculate the first two derivatives  $v$  with respect to  $C$ .

$$\frac{\partial v}{\partial C} = \frac{\tau}{r} - (1 + X) \left( \frac{k}{r} \right) \left( \frac{C}{V} \right)^X \quad (23)$$

$$\frac{\partial^2 v}{\partial C^2} = -X(1 + X) \left( \frac{k}{r} \right) V^{-X} C^{X-1}$$

The second derivative is negative for  $k > 0$  and so the market value of the bank is a strictly concave function of  $C$ . Setting the first derivative equal to zero and solving for the optimal  $C^*$ , yields:<sup>22</sup>

$$C^* = gV(0), \text{ where } g = \left[ \frac{\tau}{k(1 + X)} \right]^{\frac{1}{X}} \quad (24)$$

<sup>21</sup> For example,  $\tau$  should be on the order of .2 to .4 and  $\beta$  should be close to one if not greater than one since setting it below one obligates the regulator to make an insurance payout in excess of bankruptcy costs in the event of insolvency.

<sup>22</sup> The optimal coupon  $C^*$  is set at time zero based on the initial value of the firm. The value of the firm evolves over time, but these time subscripts have been suppressed throughout the paper since the model investigates the one time capital structure decision of a forward looking firm.

The interior optimum exists because at low levels of leverage, adding additional debt increases the value of the firm by increasing the tax benefits while adding little to the net of expected bankruptcy costs and insurance benefits since insolvency is remote. However, as leverage increases, the incremental immediate tax benefits are outweighed by the risk to future tax benefits with the possibility of insolvency.<sup>23</sup> This is quite different from the result in the case without-tax-advantaged debt (section 3.2), where equation (18) showed that the market value of the firm monotonically decreased with  $C$  when  $\beta$  was greater than one and monotonically increased with  $C$  when  $\beta$  was less than one.

Figure 1 demonstrates the balancing of insurance benefits, tax benefits and bankruptcy costs as a function of the bank's choice of  $C$  at time zero under the assumptions that bankruptcy costs or  $\alpha=0.10$ , regulator insolvency threshold or  $\beta=1.05$ , variance of asset return or  $\sigma=0.15$ , risk free interest rate or  $r=0.06$ , effective tax rate or  $\tau=0.25$ , and initial firm value or  $V=100$ .<sup>24</sup>

The high concave (solid) line represents the market value of the tax benefits, the convex (dash-dotted) line represents the bankruptcy costs, and the dashed line represents the insurance benefits, which are all plotted against the coupon payment  $C$ . The figure shows that tax benefits increase with  $C$  when leverage is very low. However, at higher levels of leverage, the tax benefits begin to decline as the result of the increased likelihood of insolvency as  $V_B$  (which equals  $\beta*(C/r)$ ) approaches  $V$  from below. Tax benefits are zero when  $V_B=V$  (or  $C=rV/\beta$ ). The figure also points out the critical role that the tax benefits play in determining the optimal

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<sup>23</sup> An interior optimal capital structure exists even with a fairly weak capital standard, as long as  $1/(1-\alpha) > \beta > 1-\tau$ . When the regulator establishes a standard where  $\beta < 1$ , it is allowing the bank to stay in operation even though the market value of the bank's financial assets is less than the market value of its deposits—a condition most likely associated with negative book net worth.

<sup>24</sup> These assumptions are similar to those used in Leland's (1994) case of protected debt except that we assume a higher insolvency threshold consistent with conservative regulation that requires strictly positive capitalization and lower bankruptcy costs consistent with financial institutions whose assets are largely marketable loans and securities.

leverage because they are relatively much larger than either of the other claims over much of the relevant range for  $C$ . With  $\beta$  set conservatively at 1.05, the bankruptcy costs increase with  $C$  somewhat more rapidly than do the insurance benefits, but the net insurance benefit is small over the entire range of  $C$ .

Finally, Substituting  $C^*$  into equations (9) and (10) yields the market value of the firm and its component debt and equity claims.

$$v^*(V) = V + \left( \frac{X}{1+X} \right) \left( \frac{\tau g V}{r} \right) \quad (25)$$

$$E^*(V) = V + \left[ \tau - 1 - k \left( \frac{C}{V} \right)^X \right] \left( \frac{C}{r} \right) = V - \left[ k g^X + 1 - \tau \right] g \left( \frac{V}{r} \right) \quad (26)$$

In conclusion, we find that a weak minimum capital requirement ( $\beta < 1$ ) creates a preference for debt in the absence of tax-advantaged debt. Somewhat surprisingly, tax benefits can partially offset this preference by introducing the risk of losing those benefits in the event of insolvency. As a result, there is an internal optimum debt service payment and market value leverage. Furthermore, banks voluntarily choose a level of deposits that is significantly less than the maximum permitted under the capital regulation. Although the apparent non-binding nature of the minimum capital requirement might suggest that the capital requirement is irrelevant or unnecessary, this is definitely not the case. Without capital regulation, banks with insurance benefits will seek to take on as much leverage as possible with or without tax benefits. It is the combination of a capital requirement and the associated threat of liquidation with a significant potential loss in the event of liquidation that creates an incentive for banks to limit their use of deposits.



#### 4.2.1 Factors that Influence Optimal Capital Structure

The optimal coupon  $C^*$  is monotonically increasing in  $g$ , and  $g$  can be thought of as an optimal “asset payout rate” to debt holders from the bank’s financial assets  $V$ . This optimal asset payout rate and therefore leverage based on book value is an increasing function of the riskless rate  $r$  and a decreasing function of the volatility of the assets  $\sigma$  and the insolvency threshold factor  $\beta$ , as long as  $\beta > 1 - \tau$ .<sup>25</sup> Increases in  $\sigma$  and  $\beta$ , as well as a decrease in  $r$ , lead to a higher likelihood of insolvency, *ceteris paribus*, and so lead the bank to select a higher coupon rate and lower leverage.

It is also interesting to look at the optimal payout rate divided by the riskless rate ( $g/r$ ) to gain an understanding of the driver’s of a bank’s choice of  $C$ . Figure 2 plots  $g/r$  as a function of the riskless rate  $r$ , and is based on the same parameter assumptions as figure 1. The figure shows that the bank’s optimal debt payout rate (as a percent of assets  $V$ ) ranges from slightly below half the riskless rate to three-quarters of the riskless rate. One conclusion we can draw from the figure is that, in general, the bank will establish a payout rate,  $C^*$ , that is less than the riskless rate.

Next, we investigate the relationship between the bank’s capital structure decision and the corporate tax rate. Taking the partial derivative of the optimal coupon (equation 24) with respect to the tax rate, we obtain:

$$\frac{\partial C^*}{\partial \tau} = \frac{gV}{X\tau} \left( \frac{\beta - 1}{\tau + \beta - 1} \right) \quad (27)$$

We first observe that for values of  $\beta$  close to one, the sensitivity of  $C^*$  to changes in the tax rate is small. Nevertheless, it is instructive to consider how the tax rate interacts with the nature of

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<sup>25</sup> The results for the riskless rate and volatility of assets also require the assumption that  $\log(1+X) > 1/X$  where  $X = 2r/\sigma^2$ . This assumption is satisfied whenever  $2r/\sigma^2 > 1$ , which is a fairly standard assumption in financial models.

the regulatory environment. The first term in equation (25) is positive as long as  $\tau + \beta > 1$ . The second term is positive only when the regulator imposes a strong capital standard with  $\beta > 1$ . Thus, in a stringent regulatory capital environment, the optimal debt service payment,  $C^*$ , is increasing in the tax rate. However, if  $\beta < 1$ , the opposite is true—a higher tax rate would lead bank owners to choose a lower debt service payment. To understand this behavior on the part of the bank's owners, one must recall that without tax benefits, when  $\beta > 1$ , the bank would prefer to not use any debt in order to avoid the risk of insolvency costs. The use of debt is motivated entirely by the desire to capture the tax benefits of debt. In that environment, larger tax benefits associated with a higher tax rate provide more incentive to use debt. When  $\beta < 1$ , with no tax benefits, the bank would choose the highest possible leverage because with insurance the bank lost nothing in the event of insolvency. However, the introduction of tax benefits means that the firm now has something to lose in the event of insolvency. The higher the tax rate, the larger the possible loss of future tax benefits; this leads to a more conservative choice of debt service payment by the bank as the tax rate increases.<sup>26</sup>

We define the bank's optimal leverage in terms of market values:  $L^* = D^*/v(V)^*$  or  $(C^*/r)/v(V)^*$ . Note that the bank does not choose  $L^*$  directly. Rather, the bank chooses  $C^*$ . The market value of the bank, including its contingent claims for insurance benefits and tax benefits and bankruptcy costs, is determined by that choice. Using the definitions above, we find:

$$L^* = \frac{D^*}{v^*(V)} = \frac{1}{\frac{r}{g} + \frac{X}{1+X}\tau} \quad (28)$$

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<sup>26</sup> It should be noted, however, that, *ceteris paribus*, a bank with  $\beta < 1$  will choose a higher  $C^*$  than the equivalent bank with  $\beta > 1$ . The difference we are talking about here is in the response of the bank to a change in tax rate. The first bank would lower its very high leverage while the second would increase its lower leverage.

Notice that the optimal leverage is independent of the value of the bank's financial assets,  $V$ . Under the standard condition that  $\tau + \beta > 1$ ,  $g$  exists and  $L^* > 0$ . However, for the optimal capital structure to be consistent with leverage less than one, the denominator must be greater than one. The first term,  $r/g$ , is the inverse of the payout factor ratio portrayed in figure 2. There we saw that banks optimally choose a debt service factor,  $g$  that represents a smaller percentage of its assets than the riskless rate.<sup>27</sup> As a result, under stringent regulatory conditions, (i.e.,  $\beta > 1$ ), the first term will be greater than one. The second term adds a fraction of the tax rate and thus the optimal leverage will be between zero and one.<sup>28</sup>

Recall that the sensitivity of the bank's choice of  $C^*$  to changes in the tax rate depended critically on whether  $\beta$  was greater or less than one. In stringent regulatory environments ( $\beta > 1$ ), an increase in the corporate tax rate led to a choice of a higher  $C^*$ , while the reverse was true in lax regulatory environments. However, when we analyze the sensitivity of leverage, measured in terms of the ratio of market value of debt to market value of the firm, we find that  $L^*$  is inversely related to the tax rate in both regulatory environments. The first order partial derivative of  $L^*$  with respect to the tax rate is:

$$\frac{\partial L^*}{\partial \tau} = -(L^*)^2 \left[ -\frac{r}{Xg\tau} \left( \frac{\beta - 1}{\tau + \beta - 1} \right) + \frac{X}{1 + X} \right] \quad (29)$$

When  $\beta < 1$ , both terms in the square brackets are positive and so the sum is clearly positive. Thus, the partial of  $L^*$  with respect to the tax rate is negative. When  $\beta > 1$ , the partial is still negative for reasonable parameter values because the second term inside the bracket is larger

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<sup>27</sup> Under the risk neutral probability measure, all assets have a drift equal to the riskless rate. If banks chose to commit to a debt service payment rate (as a percentage of its assets) that is in excess of the asset drift, this would imply market value of debt in excess of the market value of assets under the risk neutral measure and the bank would be unable to raise new equity to service the debt commitment.

<sup>28</sup> The requirement,  $\beta > 1$ , is sufficient but not necessary. The optimal coupon payment leads to leverage between zero and one for most values of  $\beta$  between  $(1 - \tau)$  and one. Leverages above one can arise for values of  $\beta$  near  $(1 - \tau)$ . i.e., values of  $\beta$  near the point where an optimal coupon does not exist and  $g$  is very close to zero. Recall that we must still impose the restriction that  $\beta < 1/(1 - \alpha)$  to assure that insurance benefits are positive.

than the first.<sup>29</sup> Thus, when the tax rate increases, the bank's market value leverage unambiguously decreases. While this finding is intuitive for the case ( $\beta < 1$ ) where  $C^*$  is decreasing in the tax rate, it is less so for the case where  $\beta > 1$  and the bank optimally increases  $C^*$  as a result of an increase in the tax rate. The result arises because although an increase in  $C^*$  in this environment, directly increases  $D^*$ , it also increases  $v^*(V)$  because of the inclusion of the contingent claims in  $v(V)$ . In fact,  $v^*(V)$  increases more rapidly than  $D^*$  as a function of the tax rate, with the result that  $L^*$  actually declines. These results suggest that a change to lower corporate tax rates may have the unintended consequence of increasing the optimal leverage ratio for banks.

## 5. A Numerical Example

In this section, we demonstrate certain policy implications of our model through numerical examples. In all cases, we assume the initial value of the bank's financial assets,  $V(0)$ , is one hundred dollars. For our base case, we set  $\beta=1$  because at that value bankruptcy costs and insurance benefits perfectly offset each other. We further assume that the risk free rate  $r=0.06$ , volatility of asset returns  $\sigma=0.15$ , and the effective corporate tax rate  $\tau=0.25$ . Although, the bankruptcy cost parameter,  $\alpha$ , does not enter the formulas for firm value directly, it does influence the maximum level of  $\beta$  that the regulator can choose since we have assumed throughout that  $\beta < 1/(1-\alpha)$ . We consider a base case of  $\alpha$  equaling ten percent, which in turn leads to a maximum for  $\beta$  of 1.11.

Figure 3 plots the market value of the firm as a function of the choice of debt service payment,  $C$  (and implied leverage) for the selected base case. The solid line represents the value of the bank with no capital requirement and no tax-advantaged debt. As discussed previously, in

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<sup>29</sup> As in the previous footnote, in practice, this relationship only breaks down for values of  $\beta$  near  $(1-\tau)$ .

this case, bank value increases monotonically with  $C$ . The line with alternating dashes and dots represents the value of the bank with an insolvency threshold set equal to the value of the deposits ( $\beta=1$ ). This can be thought of as triggering insolvency at the point where the market value of the bank's financial assets drops to the level of the deposits – roughly a zero GAAP net worth since generally accepted accounting principles do not reflect the other contingent claims that, together with  $V$ , comprise the market value of the bank. This line is perfectly horizontal because with  $\beta=1$ , the bank is indifferent between using equity or debt to finance  $V(0)$ . Finally, the concave dashed line represents the market value of a bank with both tax-advantaged debt and a minimum capital requirement ( $\beta=1$ ). As was shown earlier in Figure 1, the concavity of firm value is clearly driven by tax benefits.

Table 2 illustrates the nature of bank capital structure decisions in more detail by varying  $\beta$  between .9 and 1.1 across the columns 1 through 5 and varying the tax rate,  $\tau$ , across the three panels of the Table. The assumed tax rates range from fifteen percent in panel A to thirty-five percent in Panel C.<sup>30</sup> Each panel of the Table presents the firm's optimal choice of  $C^*$  along with the corresponding time zero value of the firm and the associated values of the three contingent claims: insurance benefits, bankruptcy costs and tax benefits. We also report both the market leverage and book leverage for this optimal choice of  $C$ . For all three panels in Table 1, we hold constant the risk free rate  $r=0.06$  and asset volatility  $\sigma=0.15$ .

Looking across the row labeled  $C^*$  in each panel, we see that  $C^*$  falls as the capital requirement becomes more stringent. The magnitude of this effect can be substantial with  $C^*$  falling by 39.5 percent as  $\beta$  increases from 0.90 to 1.1 when the tax rate is 15 percent. The effect

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<sup>30</sup> Although the top marginal corporate tax rate in the U.S. is forty percent, based on FDIC Call reports, the average effective tax rate for all commercial banks in the U.S. is 31.9%. However, the effective tax rate for small commercial banks (assets less than \$100 million) is 19.8% while the rate for large banks (assets > \$1 billion) is 32.7%.

of  $\beta$  falls, however, as the tax rate increases, and when the tax rate is 35 percent  $C^*$  only falls by 26.7 percent as  $\beta$  increases from 0.90 to 1.1. The differential effect of capital regulation for different tax rates is consistent with equation (27), which showed that the derivative of  $C^*$  with respect to  $\tau$  changes sign at  $\beta=1$ . For example, comparing Panel A with Panel C,  $C^*$  falls from 5.80 to 5.02 as the tax rate increases when  $\beta=0.9$  but increases from 3.51 to 3.68 with the tax rate when  $\beta=1.1$ . The bank's market value of leverage is highest when the insolvency threshold is set low and lowest for high  $\beta$  for any tax rate. As noted before, because GAAP does not include any of the three contingent claims in its measure of firm size, the market value of leverage ( $L_1$ ) understates the "book" value calculation of leverage ( $L_2$ ).

The last two columns in Table 2 present the value and capital structure of a comparable, unregulated, firm as studied by Leland (1994) using bankruptcy costs based on  $\alpha=0.10$ . The first column of the pair presents the results for an unregulated firm that is free to choose its own bankruptcy threshold endogenously. The last column presents the results for a firm with protected debt that results in a positive net worth requirement ( $\beta=1$ ). The first firm type (with unprotected debt) chooses a higher coupon than all the banks with the exception of the least constrained bank with low tax rates. It chooses a very low bankruptcy threshold which contributes to the very high tax benefit reported for these firms. This enhanced tax benefit outweighs the fact that the unregulated firm has no insurance benefit and the overall firm value is higher than all but the least regulated bank.<sup>31</sup> However, if the firm must provide debtholders with a positive net worth covenant, the firm voluntarily chooses a lower level of debt service. In

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<sup>31</sup> The possibility that a regulated bank might choose a leverage level that is higher than an unregulated firm without protected debt deserves some discussion. For the regulated bank that operates under the weakest capital standards, the insurance benefits net of bankruptcy costs provide an incentive for taking on more debt. It is only the risk of losing tax benefits due to forced liquidation that yields an interior optimal capital ratio. As tax rates decrease, this risk becomes less important and bank leverage increases, while for the unregulated firm tax benefits create an incentive for taking on more leverage even in the face of bankruptcy costs and so firm leverage decreases as tax rates fall.

turn this lowers the tax benefit and the value of the firm. The firm that issues protected debt has a lower optimal firm value than all but the bank with the highest capital requirement. These results suggest a positive charter value for operating an insured bank—even in the face of significant capital regulation.

Table 3 presents a similar set of panels varying the level of asset price volatility,  $\sigma$ , from 10% to 25% while holding the tax rate fixed at 25%. In general, the optimal debt service payment,  $C^*$ , declines as volatility increases. For  $\beta=1$ ,  $C^*$  falls by 29.3 percent as  $\sigma$  rises from 10 to 25%. Holding other factors equal, as volatility increases, the expected first passage time for  $V(t)$  to strike  $V_B$  decreases thereby increasing the value of contingent claims payable at insolvency. The primary factor behind the decline in  $C^*$  is the increased risk of forced liquidation by regulators and the associated loss of tax benefits, which as discussed previously is the largest contingent claim. For all values of  $\beta$  considered, tax benefits fall by approximately 50 percent as volatility rises from 10 to 25%. The effect of insurance benefits net of bankruptcy costs on  $C^*$  depends upon  $\beta$ . For lax regulatory environments, where  $\beta < 1$ , insurance benefits exceed bankruptcy costs, and this positive net benefit increases in magnitude with volatility leading to higher levels of leverage being chosen. With  $\beta > 1$ , the insurance benefits are smaller than bankruptcy costs, and the magnitude of this negative net benefit increases with volatility leading to lower leverage. In terms of magnitude changes as volatility rises from 10 to 25%, insurance benefits net of bankruptcy costs increases by 3.54 when  $\beta=0.9$  and decreases by 0.67 when evaluated at  $C^*$ .

Further, these factors have clear implications for the market value of the firm. As volatility increases, the risk of loss of future tax benefits because of insolvency dominates any increases in the value of the firm due to insurance benefits even when capital regulation is lax,

$\beta < 1$ . The combination of less debt as  $C^*$  falls and smaller tax benefits per dollar of debt results in a significant decline of the tax contingent claim, TB, as volatility increases. This decline in the tax benefit contingent claim dominates the small net increase from the other two contingent claims in a lax regulatory environment and reinforces the net decrease in the stringent regulatory environment. The net result is an unambiguous decline in firm value with volatility. This example demonstrates the importance of having a comprehensive model that explicitly incorporates insurance benefits, bankruptcy costs and tax benefits. Ignoring tax benefits, we might conclude that banks subjected to lax capital regulation ( $\beta < 1$ ) would have a tendency to invest in riskier assets. It also reinforces the earlier observation that corporate tax policies interact with capital regulation and that non-zero taxation contributes to more conservative choices of both leverage and asset risk.

The book leverage levels reported in Tables 2 & 3 for banks tend to be lower than the industry average of approximately ninety percent.<sup>32</sup> Focusing on  $\beta = 1.05$  (which roughly corresponds to the highest “Well Capitalized” bank standard) and ten percent asset volatility, the model predicts a debt to book value of assets of approximately seventy-five percent. There are several possible explanations for this difference. First, the overall industry average is somewhat distorted by the inclusion of the large international banks. The industry average leverage for small banks (assets less than \$100 million) is somewhat lower at 87%. However, more significantly, our model does not consider multiple types of debt which may lead to an underestimate of the total leverage. On average, banks make extensive use of non-depository debt. Deposits represent only 67% of their assets – a figure lower than our predicted ratio. The

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<sup>32</sup> The industry average is based on FDIC reports for all commercial banks. The ninety percent figure is based on the ratio of the book value of total bank liabilities (including non-deposit debt) to the book value of total assets. Consequently, it is not directly comparable to either leverage ratio generated by the model. However, it should be reasonably close to  $L_2$  in the tables.



use of other forms of debt is motivated by a number of different factors not captured in our model. For example, after considering operating costs, short – term borrowing via Fed Funds or repurchase agreements may be less costly than deposits which entail providing customer service as well as interest. On average, these items represent approximately 15% of the typical banks assets. Subordinated debt provides another example because banks can include some subordinated debt in their Tier II capital. Finally, our model does not incorporate any of the agency cost related motives for issuing debt. Jensen (1986) and Jensen and Meckling (1976) argue that issuing debt can mitigate the agency problems associated with free cash flow. Our model does not include this debt-favoring factor and should be expected to understate the optimal leverage for that reason.

## **6. Multiple Bankruptcy Thresholds**

In practice, banking regulators use multiple thresholds that vary in their regulatory significance. For example, as noted above, as long as a bank’s Tier I capital exceeds five percent of its assets (and the risk weighted thresholds are also met) it is deemed to be “Well Capitalized” and can operate without any unusual restrictions. If the bank’s capital ratio falls below four percent, it is deemed to be Undercapitalized and is subject to significant regulatory constraints. Within forty-five days of falling below the threshold, the bank must submit a capital improvement plan and will most likely be subjected to limits on growth. As its capital falls further, the regulatory burdens increase. For example, a significantly undercapitalized bank can be subjected to caps on the interest rate it can pay on deposits, and may be forced to dispose of assets or change management. However, there is a lower limit on regulatory forbearance and once a bank’s capital falls to this lower limit, the regulators can seize and liquidate the bank.

In this section, we extend the model to include two fixed thresholds: a “warning” threshold,  $V_C$ , and an “insolvency” threshold,  $V_R$ , where  $V_C > V_R$ . We assume that the warning threshold can be expressed in the same manner as the original insolvency threshold, i.e.,  $V_C = \gamma(C/r)$ . If the bank’s assets,  $V$ , fall to this level, the bank incurs additional regulatory costs but is able to continue to operate. Our regulatory costs are similar in spirit to the monitoring costs in Merton’s (1978) model. For tractability reasons, we model these payments as a one time cost paid at the first instance when  $V$  falls below  $V_C$  and assume the payment is proportional to the warning threshold (i.e.  $=\delta V_C$ ).<sup>33</sup> The insolvency threshold,  $V_R$ , is modeled as in previous sections:  $V_R = \beta(C/r)$  ( $\beta < \gamma$ ). When the bank’s assets first reach  $V_R$ , the bank is liquidated immediately and the value of equity falls to zero. As long as the bank’s assets,  $V$ , are above  $V_R$ , the contingent claims for tax benefits, insurance benefits and bankruptcy costs are unaffected by whether the bank is above or below  $V_C$ . Further, under the assumptions that  $C^*$  satisfies the requirement that  $\gamma C^*/r < V(0)$ ,<sup>34</sup> we can model the extra regulatory costs (ERC) in a manner similar to the other contingent claims. Namely, this claim must satisfy the ordinary differential equation in equation (2) with boundary conditions:

$$\begin{aligned} ERC(V_C) &= \delta V_C, & \text{when } V \text{ first hits } V_C \text{ from above} \\ ERC(V) &= 0, & \text{as } V \rightarrow \infty \text{ and as long as } V \geq V_C \end{aligned} \quad (30)$$

where the fixed regulatory cost is applied if  $V$  ever falls below the warning threshold and the boundary condition of zero holds as  $V$  tends towards infinity presuming that  $V$  has not previously dipped below the warning threshold.<sup>35</sup>

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<sup>33</sup> These assumptions are required to obtain a closed form solution to the problem and are clearly capable of capturing a significant penalty for a forward looking firm in terms of possible loss of a significant share of the firm’s equity as well as all future returns on that lost share.

<sup>34</sup> For reasonable  $\gamma$ , this condition is usually satisfied in the one threshold model shown by  $L_2$  in Tables 1 and 2. The second threshold will further decrease leverage, and so this condition should be satisfied for this model, as well.

<sup>35</sup> The boundary conditions in Equation (30) may appear slightly different than the contingent claims presented in section 2 in that the regulatory cost cannot go to zero if  $V$  drops below  $V_C$  and then later increases to infinity.

Applying the same approach as for the other contingent claims yields the present value of the extra regulatory costs presuming that the initial  $V$  exceeds the warning threshold:

$$ERC = \delta\gamma \frac{C}{r} \left( \frac{V}{V_C} \right)^{-X} \quad (31)$$

As in the prior development, we must have  $V_R > V_B^*$  and  $\gamma > \beta > X/(1+X)$ . We assume that the regulators choose  $\gamma > 1$  in order to assure that corrective action can be taken while the market value of the bank's assets still exceed the market value of the insured deposits. After taking into account the extra regulatory costs associated with the warning threshold, the total value of the bank becomes  $v = V + IB - BC + TB - ERC$ , which is continuous at  $V_C$ .

$$v = V + \left[ \tau - n \left( \frac{C}{V} \right)^X \right] \frac{C}{r} \quad (32)$$

where,  $n = k + \delta\gamma \frac{1+X}{r} r^{-X} > k$

Taking the first order derivative of the total value of the bank,  $v$ , with respect to  $C$  yields the optimal debt service  $C^*$

$$C_m^* = qV, \text{ where } q = \left[ \frac{\tau}{n(1+X)} \right]^{\frac{1}{X}} \quad (33)$$

The new payout rate,  $q$ , is less than the original,  $g$ , because  $n > k$ . As a result,  $C_m^* < C^*$ . In other words, a bank that faces a “warning” threshold set above the insolvency threshold chooses a lower level of debt service payments than it would without the warning threshold. Similarly, the total value of the bank is reduced as is its optimal leverage.

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However, this irreversibility is not unlike the fact in the other boundary conditions that  $V$  cannot limit to infinity if it first crosses  $V_B$  because the firm is liquidated at the point that  $V$  crosses the  $V_B$  threshold.

Table 3 presents numerical results using the same parameter values as were used in the second panel and the middle column of Table 1 ( $\beta=1$  and  $\tau=0.25$ ). The first column replicates the results from Table 1 with no warning threshold. The next three columns present the results applying increasing stringent warning thresholds moving from capital requirements of 5 to 15 percent of  $V$ . The three panels represent varying penalties ranging from 1 to 5 percent of  $V$ . The striking finding from this analysis is that this warning threshold has little impact on the firm's optimal leverage. Even the most onerous regulatory regime of a 15 percent standard and a penalty of 5 percent of  $V$  only reduces book leverage ( $L_2$ ) from 70.7 to 65.7 when compared to the case with no warning threshold. The small effect of the warning threshold on firm leverage again arises from the importance of tax benefits, which are only minimally affected by this warning threshold, in establishing an interior optimal capital ratio. This finding further emphasizes the importance of the threat of liquidation if capital regulation is expected to create an incentive for banks to limit their use of deposits.

## **7. Conclusions**

This paper examines the optimal capital structure of insured banks under four regulatory environments categorized by whether the regulators establish a binding liquidation threshold and whether there are tax benefits associated with the issuance of insured deposits. We first consider the two cases without tax-advantaged debt. In an unregulated environment without tax benefits, banks with deposit insurance unambiguously prefer maximum leverage because the insurance has the effect of insulating the bank and its owners from bankruptcy costs. In this case, banks are motivated to maximize the benefits from insurance by choosing high leverage. In a regulated environment (without tax benefits) where regulators liquidate banks that fall below a

predetermined capital standard, we find that a bank's capital structure choice depends critically on the nature of the capital standard. If the standard is set such that the market value of the bank's financial assets can fall below the market value of deposits before the bank is liquidated, the bank prefers high leverage. However, if the regulators establish a capital standard which results in liquidating the bank at a point where the market value of the assets exceeds the market value of deposits, the bank's owners prefer to fund all assets with equity. The cost associated with early liquidation more than offsets the minimal benefits of insurance with such a high liquidation threshold.

The inclusion of tax benefits in the model significantly alters these results. Because the tax benefits favor debt issuance, the inclusion of those benefits reinforces the bank's preference for debt in an unregulated environment. However, when the regulator establishes a capital requirement, we find that there is an optimal level of leverage for all banks, even those that face relatively weak capital regulation. With capital regulation comes the threat of liquidation and, consequently, the market value of a claim to future tax benefits does not monotonically increase with leverage. When banks choose higher leverage, the amount of debt (and its associated tax benefit flow) increases but the risk of losing all future tax benefits increases as well. At some point, the second factor dominates and the market value of a claim to all future tax benefits declines, pulling down the market value of the firm. This non-monotonic price behavior of the tax benefit contingent claim introduces a new risk for banks that are subject to only a weak capital regulation and that would otherwise seek very high leverage. The new risk is the loss of future tax benefits. The importance of a liquidation threat is further emphasized by our model extension that adds a second higher, warning threshold where firms face penalties, but are not

liquidated. The addition of a warning threshold with stringent criteria for allowable capital ratios and significant penalties had only moderate impact on bank leverage.

Our results are significant because there are still a large number of insured depositories (credit unions) that are exempt from the corporate income tax. For these institutions, the nature of the capital requirement is crucial. There is a knife-edge type of response with banks choosing extremely high leverage when the capital standard allows for negative net worth on a market value basis and preferring no leverage at all when the standard requires positive market value net worth. For the institutions that are subject to the corporate tax, the tax benefits associated with debt financing result in banks choosing an interior optimal capital structure. That optimal capital structure entails maintaining capital in excess of the minimum specified by the regulators as a cushion for uncertain future fluctuations in asset value. When the regulatory environment includes an earlier “warning” level for capital that is associated with an increased regulatory burden of some sort, banks will tend to choose even lower leverage ratios, but these effects are small relative to the impact of the liquidation threat associated with capital regulation.

Our results are also relevant to the current debate over GSE regulation. Currently, the regulator of the two large housing GSEs does not have the clear power to liquidate them for violating capital standards. Our model suggests that significant incentive problems arise when the regulator does not have the power to strip equity holders of the remaining market value of their ownership. The same argument can be made for any financial institution deemed “too big to fail” and therefore exempt from liquidation when asset values fall. On the other hand, in the presence of tax-advantaged debt, even fairly weak capital standards, if truly enforced by liquidation when violated, can lead to regulated firms choosing reasonable leverage levels and taking protective actions when market volatility increases.

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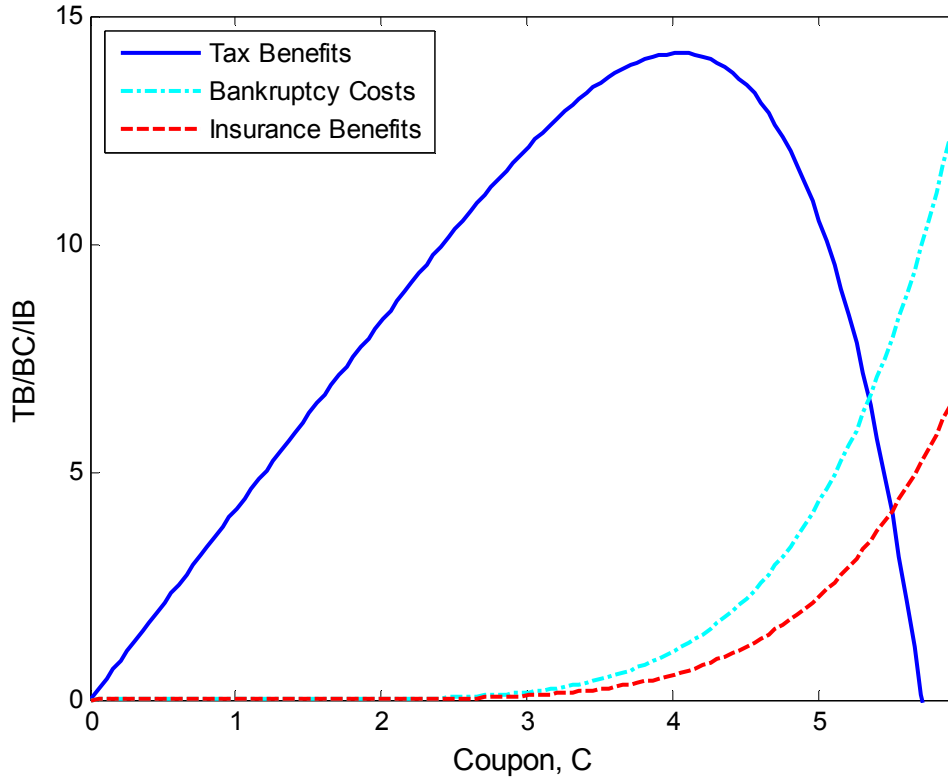
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## Value of Tax Benefits, Bankruptcy Costs and Insurance Benefits

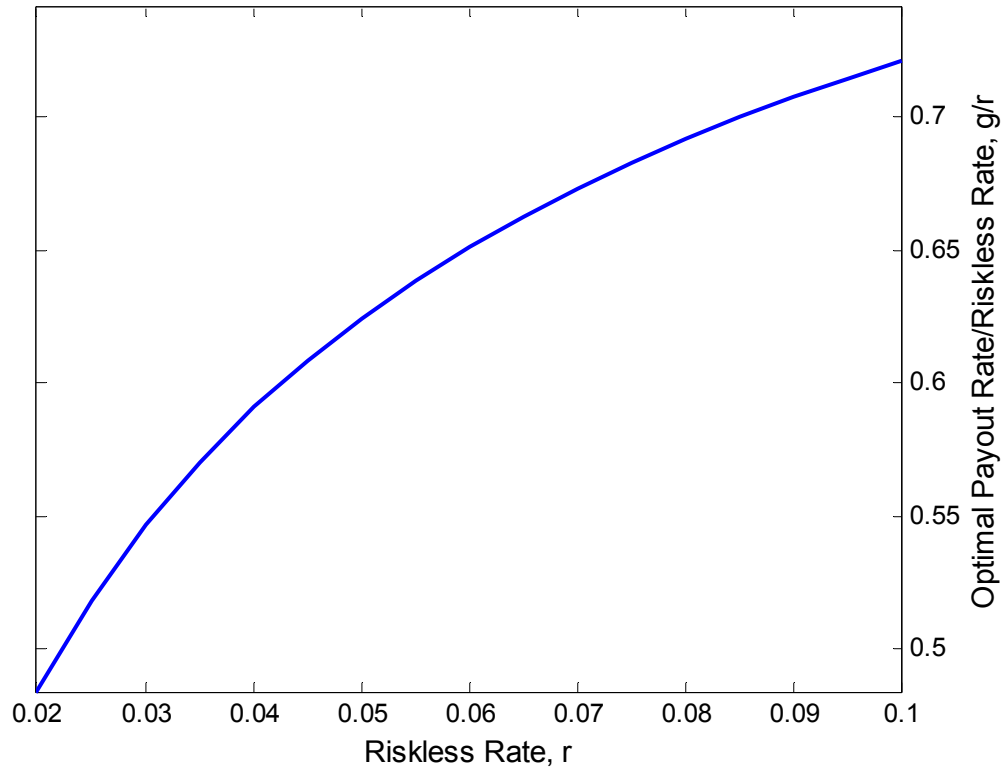
( $V=100$ ,  $\sigma=.15$ ,  $r=.06$ ,  $\beta=1.05$ ,  $\alpha=.10$  and  $\tau=.25$ )



**Figure 1. Market Value of Tax Benefits, Bankruptcy Costs and Insurance Benefits as a Function of the Firm's Choice of Debt Payment, C.** Figure 1 shows the response of the value of the three contingent claims to changes in the firm's choice of continuous debt service payment, C. All three lines assume V is fixed at 100 and the firm chooses how to finance those assets by selecting C, which determines the deposit level, D. The solid line represents the market value of future tax benefits; the dash-dotted line represents the value of the bankruptcy costs and the dashed line represents the value of future insurance benefits. All three are calculated using the listed parameter values and equations 6, 5 and 7, respectively.

Ratio of the Optimal Payout Rate Divided by the Riskless Rate

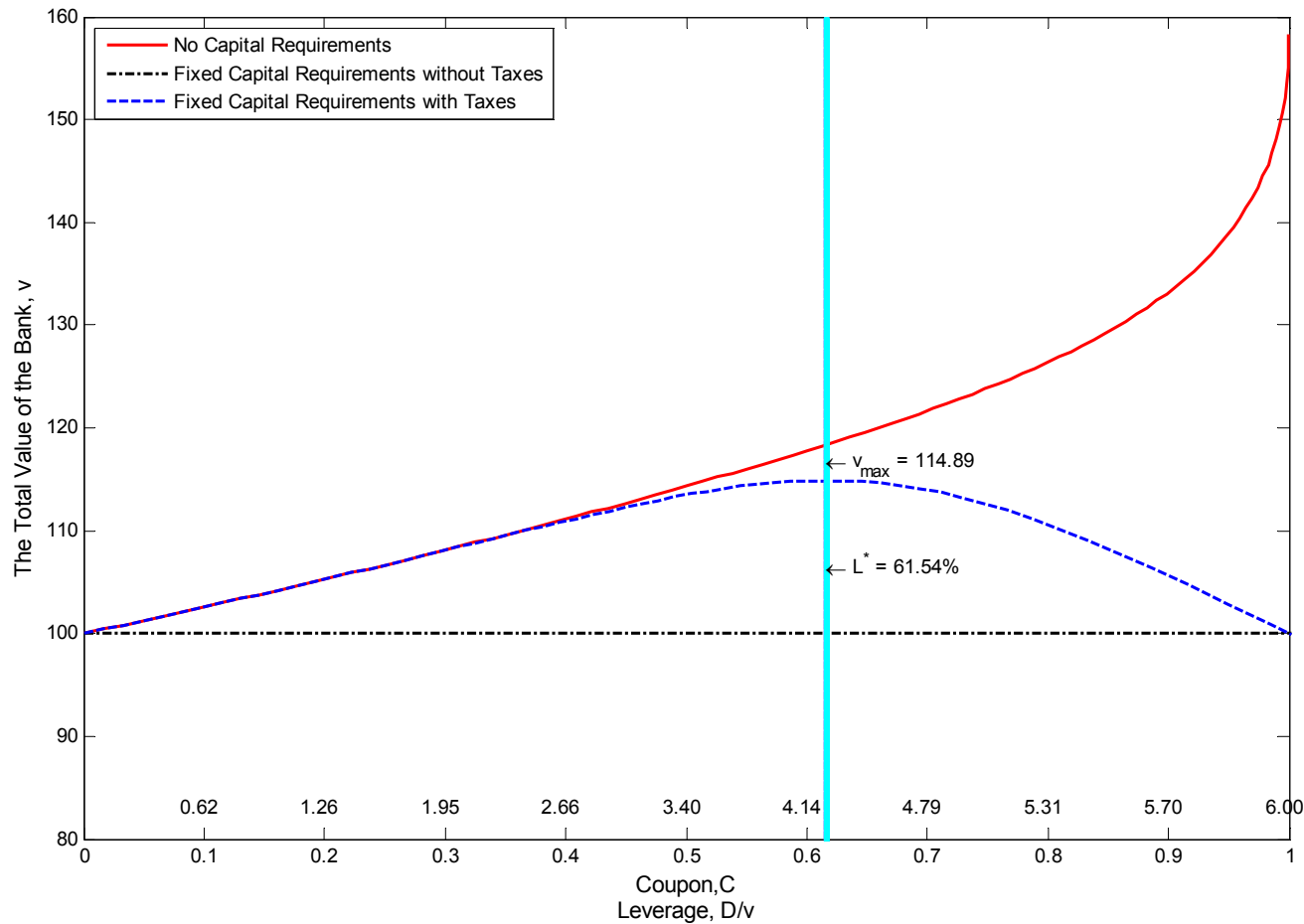
( $\sigma=.15$ ,  $r=.06$ ,  $\beta=1.05$ , and  $\tau=.25$ )



**Figure 2. Sensitivity of Bank Payout Rate to Changes in the Riskless Rate.** The bank's optimal coupon rate,  $C^*$ , is determined at time zero based on the relationship,  $C^*=gV(0)$ , where  $g$  is defined in equation (24). The variable,  $g$ , can be thought of as the optimal payout rate expressed as a percentage of the firm's initial assets. By dividing  $g$  by the riskless rate, the figure shows that the bank's optimal payout rate ranges between roughly one-half to three quarters of the riskless rate.

### Market Value of Bank Under Different Capital and Tax Regimes

( $V=100$ ,  $\sigma=.15$ ,  $r=.06$ ,  $\beta=1.0$ ,  $\alpha=.10$  and  $\tau=.25$ )



**Figure 3. Market Value of the Bank as a Function of the Choice of Debt Service Payment, C.** The solid line represents the value of a bank with no external capital requirement and tax-advantaged debt. The dashed line represents the value of a bank with  $\beta=1$  and tax-advantaged debt. The dash-dotted line represent the value of a bank with  $\beta=1$ , but without tax-advantaged debt. The vertical line shows the optimal debt service payment for the firm with  $\beta=1$  and tax-advantaged debt.

**Table 1 Summary of Assumptions**

<b>Assumption 1</b>	All securities can be traded continuously in time.
<b>Assumption 2</b>	There exists a riskless asset that pays a constant rate of interest, $r$ .
<b>Assumption 3</b>	The market value of a bank's unlevered assets, $V$ , follows a diffusion process with constant drift, $\mu$ , and constant volatility of rate of return, $\sigma$ : $\frac{\partial V}{V} = \mu dt + \sigma dW$ , where $W$ is standard Brownian motion.
<b>Assumption 4</b>	Bankruptcy costs are proportional to the unlevered value of the firm or equal to $\alpha V$ where $0 < \alpha < 1$ . There are no transaction costs, other than bankruptcy costs
<b>Assumption 5</b>	Benefits associated with tax advantaged debt are proportional to the total debt service payment, $C$ , and equal zero when the $V = V_B$
<b>Assumption 6</b>	Deposits are the only type of debt issued by the bank and are fully insured. Deposits are insured by the federal government and provide depositors riskless assurance of full repayment should the bank become insolvent. The bank pays interest on deposits at the riskless rate, $r$ .
<b>Assumption 7</b>	The bank does not pay any premium for deposit insurance.
<b>Assumption 8</b>	Capital regulation requires that firms be liquidated if $V \leq \beta C/r$ where $\beta$ is set by the regulator to a positive number less than $1/(1-\alpha)$ .
<b>Assumption 9</b>	All capital structure decisions are made at the creation of the bank. The bank's financial assets, $V$ are specified exogenously and the capital structure decision entails how the financing of those assets is allocated between debt and equity. Once made, the capital structure decisions cannot be changed and the face value of deposits is static over time. The bank services its debt by promising a total debt service payment, $C$ , which is paid continuously by issuing new equity.

Note: Assumptions 1, 2, 3, 4, 5 and 9 are identical to those made by Leland (1994). Assumptions 6, 7 and 8 are unique to the analysis of banks.

**Table 2**  
**The Impact of Insolvency Threshold and Tax Rate on Bank Capital Structure Decisions**

<b>Panel A: <math>\tau = 15\%</math></b>								
<b>Banks with Deposit Insurance and Capital Requirements</b>						<b>Non-regulated firms</b>		
$\beta$	Insolvency Threshold	0.90	0.95	1.00	1.05	1.10	Endog	1.00
$\underline{c}$	c, Minimum Capital Requirement	-11%	-5%	0%	5%	9%	--	--
C*	Optimal Debt Service Payment	5.8	4.82	4.24	3.83	3.51	5.51	3.86
v*	Firm Value at C*	112.2	110.15	108.94	108.06	107.38	111.6	108.12
IB	Insurance Benefit	8.69	2.76	1.12	0.42	0.06	--	--
BC	Bankruptcy Costs	4.12	1.81	1.12	0.79	0.61	0.7	0.61
TB	Tax Benefits	7.63	9.2	8.94	8.44	7.94	12.31	8.73
L1	Market Value Leverage = $C^*/rV^*$	86.08%	72.95%	64.94%	59.07%	54.42%	82.31%	58.46%
L2	Book Value Leverage = $C^*/rV$	96.59%	80.35%	70.74%	63.84%	58.44%	91.86%	64.28%

<b>Panel B: <math>\tau = 25\%</math></b>								
<b>Banks with Deposit Insurance and Capital Requirements</b>						<b>Non-regulated firms</b>		
$\beta$	Insolvency Threshold	0.90	0.95	1.00	1.05	1.10	Endog	1.00
$\underline{c}$	c, Minimum Capital Requirement	-11%	-5%	0%	5%	9%	--	--
C*	Optimal Debt Service Payment	5.19	4.66	4.24	3.91	3.62	6.44	3.99
v*	Firm Value at C*	118.21	116.35	114.89	113.71	112.71	122.61	113.98
IB	Insurance Benefit	4.33	2.22	1.12	0.47	0.07	--	--
BC	Bankruptcy Costs	2.05	1.46	1.12	0.9	0.75	0.85	0.75
TB	Tax Benefits	15.94	15.58	14.89	14.14	13.39	23.46	14.73
L1	Market Value Leverage = $C^*/rV^*$	73.18%	66.74%	61.57%	57.26%	53.57%	87.58%	58.27%
L2	Book Value Leverage = $C^*/rV$	86.51%	77.65%	70.74%	65.11%	60.38%	107.38%	66.42%

<b>Panel C: <math>\tau = 35\%</math></b>								
<b>Banks with Deposit Insurance and Capital Requirements</b>						<b>Non-regulated firms</b>		
$\beta$	Insolvency Threshold	0.90	0.95	1.00	1.05	1.10	Endog	1.00
$\underline{c}$	c, Minimum Capital Requirement	-11%	-5%	0%	5%	9%	--	--
C*	Optimal Debt Service Payment	5.02	4.6	4.24	3.94	3.68	7.55	4.05
v*	Firm Value at C*	124.68	122.59	120.85	119.37	118.08	137.07	119.89
IB	Insurance Benefit	3.52	2.05	1.12	0.5	0.08	--	--
BC	Bankruptcy Costs	1.67	1.34	1.12	0.95	0.83	0.94	0.83
TB	Tax Benefits	22.83	21.89	20.85	19.82	18.84	38.01	20.72
L1	Market Value Leverage = $C^*/rV^*$	67.15%	62.53%	58.54%	55.05%	51.96%	91.76%	56.29%
L2	Book Value Leverage = $C^*/rV$	83.72%	76.65%	70.74%	65.71%	61.35%	125.77%	67.49%

Notes:

1. All results for bank capital structure were calculated based on the formulas derived in Section 4.2. For all three panels, the riskless rate,  $r = 6\%$ , the volatility of assets returns  $= 15\%$  and the deadweight loss in insolvency,  $\alpha = 10\%$ .
2. The values in the columns labelled Non-regulated firms are calculated using the formulas derived by Leland (1994). The first column corresponds to the case with unprotected debt and an endogenous bankruptcy threshold. The second column represents the case with protected debt where the firm provides a positive net worth covenant. The values in these columns are based on the same assumptions for the riskless rate, asset volatility and bankruptcy costs.

**Table 3**  
**The Impact of Insolvency Threshold and Asset Return Volatility on Bank Capital Structure Decisions**

Panel A: $\sigma = 10\%$								
Banks with Deposit Insurance and Capital Requirements						Non-regulated firms		
$\beta$	Insolvency Threshold	0.90	0.95	1.00	1.05	1.10	Endog	1.00
$\underline{c}$	c, Minimum Capital Requirement	-11%	-5%	0%	5%	9%	--	--
C*	Optimal Debt Service Payment	5.62	5.2	4.85	4.55	4.28	6.86	4.71
v*	Firm Value at C*	121.61	119.98	118.64	117.48	116.47	126.38	118.12
IB	Insurnace Benefit	2.28	1.21	0.62	0.27	0.04	--	--
BC	Bankruptcy Costs	1.08	0.79	0.62	0.51	0.43	0.48	0.43
TB	Tax Benefits	20.41	19.57	18.64	17.72	16.87	26.85	18.55
L1	Market Value Leverage = C*/rv*	76.99%	72.18%	68.07%	64.48%	61.29%	90.44%	66.48%
L2	Book Value Leverage = C*/rV	93.63%	86.60%	80.76%	75.75%	71.38%	114.29%	78.52%

Panel A: $\sigma = 15\%$								
Banks with Deposit Insurance and Capital Requirements						Non-regulated firms		
$\beta$	Insolvency Threshold	0.90	0.95	1.00	1.05	1.10	Endog	1.00
$\underline{c}$	c, Minimum Capital Requirement	-11%	-5%	0%	5%	9%	--	--
C*	Optimal Debt Service Payment	5.19	4.66	4.24	3.91	3.62	6.44	3.99
v*	Firm Value at C*	118.21	116.35	114.89	113.71	112.71	122.61	113.98
IB	Insurnace Benefit	4.33	2.22	1.12	0.47	0.07	--	--
BC	Bankruptcy Costs	2.05	1.46	1.12	0.9	0.75	0.85	0.75
TB	Tax Benefits	15.94	15.58	14.89	14.14	13.39	23.46	14.73
L1	Market Value Leverage = C*/rv*	73.18%	66.74%	61.57%	57.26%	53.57%	87.58%	58.27%
L2	Book Value Leverage = C*/rV	86.51%	77.65%	70.74%	65.11%	60.38%	107.38%	66.42%

Panel A: $\sigma = 20\%$								
Banks with Deposit Insurance and Capital Requirements						Non-regulated firms		
$\beta$	Insolvency Threshold	0.90	0.95	1.00	1.05	1.10	Endog	1.00
$\underline{c}$	c, Minimum Capital Requirement	-11%	-5%	0%	5%	9%	--	--
C*	Optimal Debt Service Payment	4.98	4.29	3.78	3.39	3.07	6.28	3.38
v*	Firm Value at C*	115.56	113.39	111.81	110.59	109.6	119.63	110.56
IB	Insurnace Benefit	6.57	3.24	1.57	0.65	0.09	--	--
BC	Bankruptcy Costs	3.11	2.12	1.57	1.24	1.01	1.2	1.01
TB	Tax Benefits	12.1	12.28	11.81	11.17	10.51	20.83	11.56
L1	Market Value Leverage = C*/rv*	71.81%	62.99%	56.34%	51.05%	46.71%	87.50%	50.93%
L2	Book Value Leverage = C*/rV	82.99%	71.43%	63.00%	56.46%	51.19%	104.67%	56.31%

Panel A: $\sigma = 25\%$								
Banks with Deposit Insurance and Capital Requirements						Non-regulated firms		
$\beta$	Insolvency Threshold	0.90	0.95	1.00	1.05	1.10	Endog	1.00
$\underline{c}$	c, Minimum Capital Requirement	-11%	-5%	0%	5%	9%	--	--
C*	Optimal Debt Service Payment	4.98	4.06	3.43	2.97	2.62	6.34	2.88
v*	Firm Value at C*	113.64	111.12	109.41	108.15	107.18	117.37	107.9
IB	Insurnace Benefit	9	4.2	1.96	0.78	0.11	--	--
BC	Bankruptcy Costs	4.26	2.75	1.96	1.49	1.17	1.49	1.17
TB	Tax Benefits	8.9	9.67	9.41	8.86	8.25	18.86	9.07
L1	Market Value Leverage = C*/rv*	73.01%	60.89%	52.31%	45.83%	40.74%	90.02%	44.51%
L2	Book Value Leverage = C*/rV	82.97%	67.66%	57.23%	49.57%	43.66%	105.66%	48.03%

Notes:

1. All results for bank capital structure were calculated based on the formulas derived in Section 4.2. For all four panels, the riskless rate,  $r=6\%$ , the tax rate= $25\%$  and the deadweight loss in insolvency,  $\alpha=10\%$ .
2. The values in the columns labelled Non-regulated firms are calculated using the formulas derived by Leland (1994)  
The first column corresponds to the case with unprotected debt and an endogenous bankruptcy threshold  
The second column represents the case with protected debt where the firm provides a positive net worth covenant  
The values in these columns are based on the same assumptions for the riskless rate, asset volatility and bankruptcy costs.

**Table 4**  
**The Impact of Dual Insolvency Thresholds on Bank Capital Structure Decisions**

<b>Panel A: <math>\delta = 1\%</math></b>					
<b>Banks with Deposit Insurance and Capital Requirements</b>					
$\gamma$	Warning Threshold	--	1.05	1.1	1.15
$\beta$	Insolvency Threshold	1.00	1.00	1.00	1.00
$\underline{c}$	c, Minimum Capital Requirement	0%	0%	0%	0%
C*	Optimal Debt Service Payment	4.24	4.20	4.19	4.17
v*	Firm Value at C*	114.89	114.75	114.70	114.64
IB	Insurnace Benefit	1.12	1.05	1.03	1.00
BC	Bankruptcy Costs	1.12	1.05	1.03	1.00
TB	Tax Benefits	14.89	14.89	14.89	14.88
ERC	Extra Regulatory Cost		0.14	0.19	0.24
L1	Market Value Leverage = C*/rv*	61.57%	61.04%	60.87%	60.65%
L2	Book Value Leverage = C*/rV	70.74%	70.04%	69.81%	69.53%
<b>Panel B: <math>\delta = 3\%</math></b>					
<b>Banks with Deposit Insurance and Capital Requirements</b>					
$\gamma$	Warning Threshold	--	1.05	1.1	1.15
$\beta$	Insolvency Threshold	1.00	1.00	1.00	1.00
$\underline{c}$	c, Minimum Capital Requirement	0%	0%	0%	0%
C*	Optimal Debt Service Payment	4.24	4.13	4.09	4.05
v*	Firm Value at C*	114.89	114.48	114.35	114.20
IB	Insurnace Benefit	1.12	0.93	0.88	0.82
BC	Bankruptcy Costs	1.12	0.93	0.88	0.82
TB	Tax Benefits	14.89	14.86	14.83	14.80
ERC	Extra Regulatory Cost		0.38	0.48	0.60
L1	Market Value Leverage = C*/rv*	61.57%	60.07%	59.61%	59.05%
L2	Book Value Leverage = C*/rV	70.74%	68.76%	68.16%	67.44%
<b>Panel C: <math>\delta = 5\%</math></b>					
<b>Banks with Deposit Insurance and Capital Requirements</b>					
$\gamma$	Warning Threshold	--	1.05	1.1	1.15
$\beta$	Insolvency Threshold	1.00	1.00	1.00	1.00
$\underline{c}$	c, Minimum Capital Requirement	0%	0%	0%	0%
C*	Optimal Debt Service Payment	4.24	4.06	4.00	3.94
v*	Firm Value at C*	114.89	114.24	114.05	113.83
IB	Insurnace Benefit	1.12	2.05	1.12	0.50
BC	Bankruptcy Costs	1.12	1.34	1.12	0.95
TB	Tax Benefits	14.89	14.81	14.75	14.68
ERC	Extra Regulatory Cost		0.57	0.71	0.85
L1	Market Value Leverage = C*/rv*	61.57%	59.19%	58.51%	57.71%
L2	Book Value Leverage = C*/rV	70.74%	67.62%	66.73%	65.69%

Notes:

1. All results for bank capital structure were calculated based on the formulas derived in Sections 4.2 & 6. For all panels, the riskless rate,  $r = 6\%$ , the volatility of assets returns  $= 15\%$ , the deadweight loss in insolvency,  $\alpha = 10\%$  and the tax rate,  $\tau = 25\%$