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Eliciting Student Judgments of Intrinsic Difficulty for Studying Student Misconceptions in Solving Basic Mathematics Items

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Developmental mathematics students are one of the most challenging groups of students to work with. This particular group of students traditionally score lower on placement exams at the community college level. They have conceptions and ideas about how to solve an arithmetic problem presented to them. Some of the conceptions lead to solution strategies that result in correct answers. However, many of the students have misconceptions of solution strategies that lead to incorrect responses to the questions. This creates frustration for both the students and instructors. This is becoming a national problem as evidence from the attention it is getting the federal government and the media. The project being worked on started with an effort to determine if the notion of *intrinsic difficulty*, as developed by Sweller (2010) in cognitive load theory, could be quantified and compared to proportion correct scores (p-values). A greater understanding of this concept was deemed important for improving instruction in college-level developmental mathematics. If instructors better understand the conceptions that students have that lead to incorrect responses when solving arithmetic problems, instruction of the material presented to the students can potentially improve. The present study sought to examine developmental math students' thought processes while attempting to solve arithmetic problems. Understanding how a student thinks when solving a problem is imperative in correcting the misconceptions a student has about a given topic.

Previous Research on the Topic on Understanding Student Misconceptions of Mathematics Subject Matter

Tatsuoka (2009) describes a methodology called Rule Spaced Method (RSM) and Q-Matrix Theory to understand elementary algebra operations diagnostically. This is one of example of efforts being undertaken by cognitive researchers to understand student misconceptions. Sweller (2010) presents a theory of cognitive load that differentiates intrinsic and extrinsic cognitive load. In this paper, we introduce the notion of *intrinsic difficulty* to represent individual cognition that results from non-germane aspects of tasks. This is particularly important for developmental mathematics testing because standard measurement techniques have been known to focus on proportion correct scores as item difficulty. Here, we are trying to understand the many processes examinees use to solve math problems.

Method

The use of think-aloud protocols as described by Ericsson and Simon (1993) for eliciting how 20 students thought about how to solve 20 fraction and decimal questions was important to the project. Four basic math skills classes at Rockland Community College were used in the study. From those four courses, five students from each class volunteered for a total of 20 students. Each group of five students was given the same five math questions totaling to the 20 questions in the study. The interview of the students took place in a classroom at Rockland Community College during the meeting time of each of the classes. The only instructions the students were given was to solve the problems out loud and to please continue talking. They were not given any response from the researcher administering the questions as to how the student answered each problem. Each student was interviewed separately. The responses from the students were tape-recorded and notes were taken concurrently. The tape recordings were later transcribed. Misconceptions on how to solve these problems were then identified and grouped for each item, resulting in approximately four conceptions (including student misconceptions) for each item.

Analyses

The transcriptions were sorted by question and the student response to each question. The think-aloud protocols were categorized into four solution strategies for each of the 20 open-ended arithmetic items for a total of 80 solution strategies. Students had a vast array of misconceptions for these questions. The authors had pre-conceived notions of a number of misconceptions students might have had, but did not anticipate all the misconceptions encountered. An example of this would be adding numerators and denominators across in an addition of fractions problem. The following are examples of six questions of what the students were asked to solve in the interview they had, and their thought processes. These questions were selected from the 20 questions by examining the proportion of variation explained in the actual item scores by each of the solution strategy options (see Secolsky and Magaram, 2011, this conference). This is followed is by two additional questions that had interesting responses in the think aloud protocols.

Results

Item #1

Simplify. $\frac{9}{15}$

a) I divide 3 into 9 and divide 3 into 15 to get 3/5.

Student #6 said, "I haven't done this stuff in a long time. Simplify by like 3, I hope this is right, I'm going to feel like an idiot if it's not. 3, so it would be, 3 goes into 9 3 times, 3 goes into 15 5 times, so it would be, $\frac{3}{5}$."

b) 9 into 15 goes 1 time with 6 as the remainder. So I go 1 6/9 or 1 2/3

Item #3

Multiply and simplify $\frac{3}{10} \bullet \frac{43}{100}$

a) I'm going to cross multiply and get 430 over 300.

b) After I got 430 and 300, I should have added to get 730.

c) I should have gotten (4300/300) which becomes 43/3 or 14 remainder 1.

Student #4 said, "Cross multiply and get 4300 over 300. I'm going to 4300÷300, that's the easy way out for me."

d) I should have multiplied across: 3 times 43 and 10 times 100.

Student #12 said, "3 x 43 = 129, over 100 x 10 = 1000."

Item #5

Divide and simplify $\frac{7}{4} \div 7$

a) You would have to multiply by ¼. So you get 7/4. 7/4 divided by 7/4 equals 1.

Student #1 said, "And then, $\frac{7}{4} \div 7$, so you would have to get this, you have to multiply by $\frac{1}{4}$ and cause that's by 1, so you get uh, $\frac{7}{4}$ divided $\frac{7}{4}$ equals 1."

b) You start out by changing 7/4 to 1 ¾ and then dividing by 7.

Student #3 said, "Divide and simplify, make them both a whole number. So it would be 1 $\frac{3}{4}$ and to divide you have to make 7 into a fraction too so, you have to take 7 and make a 6 $\frac{4}{4}$ and you subtract the two. So, you get rid of the 1 and make it all $\frac{4}{4}$ and then you're left with 6."

c) You should start out by changing to a multiplication problem: 7/4 times 1/7.

d) After you have 7/4 times 1/7 you cross multiply to get 49/4 or 12 ¼.

Item #7

Add and simplify. $\frac{7}{9} + \frac{5}{6}$

a) I add the numerators and add the denominators to get 12/15. Then I simplify to get 4/5.

b) First, I find the lowest common denominator by multiplying 9 by 6 =54.

c) The lowest common denominator is 18. I then multiply 2 by 7 and 3 by 5 = 14+ 15= 29/18 = 1 11/18.

Student #7 said, "Subject: I'm gonna get a common denominator for them. So, I'm going to use 18. I'm multiplying the 9 by 2 and the 7 by 2, so I get $\frac{14}{18}$. Then on the other side I'm multiplying them by 3, so I'm getting $\frac{15}{18}$ and I should be getting $\frac{29}{36}$."

d) The lowest common denominator is 36. 28/36 + 30/36 = 58/36 = 1 22/36 = 1 11/18.

Item #8

Subtract and simplify. $\frac{7}{10} - \frac{13}{25}$

a) I need to get the denominator to be 100.Then I get 70/100and 52/100, which is 18/100 and simplified to 9/50.

   Student #10 said, "Subject: I am just going to get the denominator to 100, I times the bottom one by 10 and the top by 10 and give me $\frac{70}{100}$. I times the bottom by 4 and the top by 4 and give me $\frac{52}{100}$. 70-52=18. Gives me $\frac{18}{100}$ which can be simplified to $\frac{9}{50}$."

b) I have to find the lowest common denominator which is 250.

c) First, I cross multiply and get (10)(13) and (7)(25). It gives me 130/175. Then I subtract to get 1/45.

d) Cannot be done. 13/25 is greater than 7/10. The answer could be negative.

Item #14

Divide. Write a mixed numeral for the answer. $12 \div 1\frac{1}{13}$

a) I change 12 to 12/1 and 1 1/13 to 13/14 and then cross multiply.

b) I'm not sure what a mixed numeral is.

   Student #5 said, "I haven't done this stuff. So, 12 divided into 1 is 12. So that would be 12 over 13."

c) I change 1 1/13 to 14/13 then I multiply 12 by 13/14.

Student #8 said, "Subject: $\frac{12}{1} \div 1\frac{1}{13}$. 13x1=13+1=14$\frac{14}{13}$. Keep, change and that becomes$\frac{13}{14}$. 12x14=168, 168x13. I'm sick of math. I know how to do up to a certain point."

d) The answer is 11 1/7.

Two additional items.

Item # 19: Find percent notation for 0.372

Subject: Find the percent notation for 0.372. Move the decimal place over 1, 2, 1, 2. I guess I move it over 2 places. That's 37, I think it is just 37%; you get rid of the 2.

Item # 2: Multiply and simplify $\frac{2}{5}$ x 35

Subject: Multiply and simplify $\frac{2}{5}$ x 35. Uh, you make the 35 a fraction, I think and then you cross multiply. So, no you don't. Oh no, you got to make uh…

Subject: Yes, you cross multiply. I have to cross multiplying, 35 x 5, 5x5=25 and 2 up top 5x3=15 and then the 2 up top is 17, so that is 175. I think its 175 and 2 x 175 is. 175 x 2 2x5 is 10, bring the 1 up, 2 x 7 is 14, 15 bring the 5 down, 1 up, 2 x, + 1 is 3 the answer is 350.

The thought processes for the students when solving these questions are very interesting. They have ideas about how they should go about solving the problems, but their conceptions about the solution strategy are not always correct. Solution strategies were added to each of the

20 arithmetic problems based on the students' thinking for each question. The following are the solution strategies for the two additional examples as can be seen above.

19) Find percent notation for 0.372.

a) I didn't know what percent notation meant.

b) I moved the decimal place over two places to the right. That's 37 or 37%.

c) I moved the decimal over one place to the right which gave me 3.72%.

d) I dropped the 2 to make it 0.37%.

2) Multiply and simplify $\frac{2}{5} \bullet 35$

a) I'm supposed to put a 1 under 35 and then cross multiply.

b) I'm supposed to multiply 5 by 35 = 175 times 2 =350.

c) I go 2 times 35=70 and my answer is $\frac{70}{5}$.

d) I didn't know that the dot meant multiply.

The four solution strategies and the proportion correct scores for the actual items are presented in Table 1.

[INSERT TABLE 1 ABOUT HERE]

Discussion

Some students had more than one misconception, and other students had individualistic and unique misconceptions. For example, a student not only added numerators and denominators in an addition of fractions problem, but also did not know how to simplify his/her answer. As you can see from the data from question 2, the p-value was 0.54 and the proportion selecting the incorrect solution strategy JA was 0.69. From findings like that of question 2, solution strategies enable the determination of the significance of misconceptions that arise in attempting to find the correct answer to an arithmetic item. Instructors often look at a student's solution to a given problem and may become confused by the solution strategy used by the student. If an instructor could better understand the thought processes of students that lead to misconceptions about how to solve a problem, then instruction for these students could become more concentrated with greater focus to the students previous conceptions about a given topic.

Limitations

There was no basis for establishing agreement for the solution strategies. One person transcribed the interviews and formed the solution strategy for each question. More input on the solution strategies will soon take place.

Implications

If teachers could begin to become informed as to which misconceptions were relatively prevalent in solving these types of questions, then instruction for these students could become more focused with greater sensitivity to existing knowledge structures of students. This could be used in faculty development so that instructors can understand how to incorporate student misconceptions into instruction rather than showing the best way to solve a problem. Cognitive psychologist's often use best example theory to explain how instruction should take place. We do not believe that is the case. We have to uncover the strands of cognitive thinking as we proceed in instruction.

Conclusion

An attempt was made to collect students' solution strategies that were associated with their solving of math questions. For all 20 items, there were several solution strategies that pointed to common student misconceptions. Future research should consist of studies on other content domains.

References

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Sweller, J. (2010). Element interactivity and intrinsic, extraneous and germane cognitive load. *Educational and Psychological Review*, *22*(*2*), 123-38.

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Table 1: Proportion Correct Scores for Question1 – Question 20 and Frequency of Solution Strategies JA1 –JD20 (Please note that the judgment proportions may not sum to 1.00).

	p-value	JA	JB	JC	JD
Q1	0.84	0.79	0.04	0.05	0.01
Q2	0.54	0.69	0.06	0.15	0.02
Q3	0.47	0.27	0.02	0.15	.42
Q4	0.43	0.09	0.49	0.17	0.14
Q5	0.54	0.07	0.07	0.52	0.23
Q6	0.34	0.10	0.03	0.03	0.33
Q7	0.37	0.25	0.13	0.34	0.12
Q8	0.41	0.30	0.07	0.12	0.10
Q9	0.18	0.22	0.19	0.16	0.21
Q10	0.25	0.17	0.14	0.28	.04
Q11	0.23	0.06	0.05	0.23	0.35
Q12	0.28	0.07	0.25	0.03)	0.31
Q13	0.10	0.16	0.09	0.08	0.23
Q14	0.11	0.20	0.09	0.17	0.16
Q15	0.09	0.08	0.06	0..32	0.13
Q16	0.24	0.16	0..17	0.30	0.04
Q17	0.11	0.08	0.19	0.13	0.13
Q18	0.11	0.13	0.06	0.08	0.25
Q19	0.28	0.10	0.28	0.18	0.05
Q20	0.10	0.13	0.11	0.10	0.21